Nature of $X(2370)$

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We address the nature of the $X(2370)$ resonance observed in the J/ψ radiative decays, $J/\psi \to \gamma K^+K^-\eta'$, $J/\psi \to \gamma K_S K_S \eta'$, and $J/\psi \to \gamma\pi^+\pi^-\eta'$. By studying the invariant mass spectra, we confirm that the decays of the $X(2370)$ into three pseudo-scalars are well described by an effective chiral Lagrangian. We extract the branching ratio of $J/\psi \to X(2370)\gamma$ and show that it is an order of magnitude larger compared to the glueball production rate predicted by lattice QCD. This indicates that $X(2370)$ is not likely to be a glueball candidate.

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I. INTRODUCTION

Over the past decade, important discoveries have been made about strong interactions, especially in regards to the spectrum of hadrons. The observation of charged mesons and baryons with hidden charm, *e.g.,* the Z_c [1,2] and P_c states [3,4], indicates the possibility of existence of compact multiquark bound states that cannot be explained by the quark model. Furthermore, there is a growing evidence that gluons, besides confining quarks, can also act as hadron constituents resulting in quark-gluon, also known as hybrid states or pure glue made glueballs. For a recent review of the hybrid meson signatures and the phenomenological studies addressing the role of gluons as constituents of hadrons, see, for example, [5–12] and references therein.

Recent analysis of the BESIII data on J/ψ radiative decays to two pseudoscalars have identified a multitude of isoscalar states, and arguments have been put forward that there is a colorless, C-even pure glueball among them [13–15]. Furthermore, the recent observation of the odderon [16], in the high-energy pp and $p\bar{p}$ collisions, may be related to existence of a C-odd glueball resonance in the direct channel. It is the gluon compound in J/ψ radiative decaying to three pseudoscalars that we address in this paper. Lattice QCD (LQCD) predictions of [17–21] place a pseudoscalar glueball mass above 2 GeV. For example, the most recent computation from [21] gives $M_G \simeq 2395 \pm 14$ MeV. However, since these are quenched calculations, it is difficult to access the systematic uncertainties.

Glueballs are expected to be produced in radiative decays of the J/ψ [22] because annihilation of the $c\bar{c}$ pair leaves behind a gluon rich component of the J/ψ wave function. Recently, the BESIII Collaboration reported several, high statistics measurements of exclusive J/ψ radiative decays. A structure with a mass around 2.37 GeV, referred to as the $X(2370)$, has been seen in $\pi^+\pi^-\eta'$ [23] and $\bar{K}K\eta'$ [24] invariant mass distributions. In the former, the mass and width of the $X(2370)$ are measured to be $M = 2376.3 \pm 8.7(stat)^{+3.2}_{-4.3}(syst)$ MeV and $\Gamma = 83 \pm 17(stat)^{+44}_{-6}(syst)$ MeV, with the statistic significance of 6.4σ, while in the latter decay, it was found that $M = 2341.6 \pm 6.5(stat) \pm 5.7(syst)$ MeV and $\Gamma = 117 \pm 10(stat) \pm 8(syst)$ MeV with the 8.3σ statistical significance. Notice that the $\eta(2225)$ has a comparable mass and width, but it has been seen in the $J/\psi \to \gamma\phi\phi$ [25], decaying into four kaons. It is not our intention to examine all glueball candidates. Instead, we focus here on the study of the three-pseudoscalar meson spectrum given the recent measurements of several such channels by BESIII and the presence of a clear resonance signal in this spectrum.

There is no first principle method that would enable us to distinguish a glueball from other inner components of a physical resonance. The two, however, are expected to have

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different phenomenological consequences, and here, we propose to compare the measured J/ψ radiative decay branching ratios to those predicted by LQCD to investigate if the recently observed $X(2370)$ resonance is a good candidate for the pseudoscalar glueball. The $X(2370)$ has been considered in previous studies [26,27], where it was concluded that it may indeed correspond to the pseudoscalar glueball that was found in the LQCD simulations. To verify this interpretation, it is necessary, however, to consider the production characteristics. Following [26], we postulate the effective interactions between the $X(2370)$ and the light scalar and pseudoscalar meson resonances that appear to dominate its decay spectrum. The subsequent decays of these resonances are well studied, *e.g.,* using the chiral theory 28]]. Combining these processes, we construct a reaction model to describe the J/ψ radiative decays for all the measured channels that contain the X, $J/\psi \to \gamma K^+K^-\eta'$, $J/\psi \to \gamma K_S K_S \eta'$, $J/\psi \to \gamma\pi^+\pi^-\eta'$, and $J/\psi \to \gamma\eta\eta\eta'$. With the branching ratios $\mathrm{Br}[X \to PPP]$ of X decaying into three pseudoscalars fixed by the reaction model and the branching ratio for the J/ψ radiative decay in the mass region of the X, $\mathrm{Br}[J/\psi \to \gamma X \to \gamma \bar{K}K\eta']$ determined by the experiment [24], we extract the branching ratio $\mathrm{Br}[J/\psi \to \gamma X]$,

$$\mathrm{Br}[J/\psi \to \gamma X] = \frac{\mathrm{Br}[J/\psi \to \gamma X \to \gamma PPP]}{\mathrm{Br}[X \to PPP]}. \tag{1}$$

By comparing our results with those of the QCD predictions [21] and the models [26,27], we can therefore distinguish whether the $X(2370)$ is more likely to be a glueball or a $q\bar{q}$ resonance.

II. FORMALISM

The effective Lagrangian for $J/\psi \to \gamma X$, constrained by chiral and discreet symmetries, *e.g.,* charge conjugation and parity, can be written as

$$\mathcal{L} = g_\gamma \epsilon^{\mu\nu\alpha\beta} D_\mu \psi_\nu F_{\alpha\beta} X, \tag{2}$$

where $F_{\alpha\beta}$ is the usual electromagnetic field strength tensor. Following [26,29,30], we apply the chiral effective theory to describe the interactions between the X and nonets of scalar, S, and pseudoscalar, P, fields which is given by

$$\mathcal{L} = i g_X X(det\Phi - det\Phi^\dagger), \tag{3}$$

with $\Phi = \Phi_0 + Z_S S(x) + iZ_P P(x)$, where Φ_0 is a constant matrix that contains various condensates and Z_S, Z_P are the wave function renormalization constants, For the pseudoscalar nonet, P, we take the lightest π, K mesons, and include the η and η' in the standard way as a result of mixing between the η_0 and η_8.[1] Decays of $X \to KK\eta$ to heavier η mesons have smaller phase space, and their mixing with the lighter η's is small; thus, they will be ignored in our analysis. For the scalar nonet, S, there are two sets of resonances with a mass below 2 GeV [40] that are relevant. The lighter set is associated with $\{\sigma, \kappa, a_0(980), f_0(980)\}$, and the heavier one with the $\{f_0(1370), K_0^*(1430), a_0(1450), f_0(1500)\}$ resonances. There is phenomenological evidence that the states in the heavier set are dominated by $\bar{q}q$ configurations [28], though a mixing with a glueball can not be neglected. The structure of the lighter scalars is still a mystery, but they have nonignorable valance quark components [41–44]. Out of these, we construct two scalar nonets $S = S_L, S_H$. Specifically, we use the heavier isoscalar, scalar mesons $f_0(1370)$, $f_0(1500)$, and the $f_0(1710)$, mixed according to the model of [28], to extract the two isoscalar elements of the heaver multiplet $S = S_H$ as well as the scalar glueball G,[2]

$$\begin{pmatrix} \sigma_n \\ \sigma_s \\ G \end{pmatrix} = \begin{pmatrix} \sqrt{1/3} & \sqrt{2/3} & 0 \\ -\sqrt{2/3} & \sqrt{1/3} & 0 \\ 0 & 0 & 1 \end{pmatrix} A \begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix}.$$

where the 3×3 mixing matrix A is given by in [28]. The other components of S_L and S_H originate from mixing between the physical states from the lighter set of resonances, $a_0(980)$ and κ, and the $a_0(1450)$ and the $K_0^*(1430)$ for the heavier ones. They are given as [28,50]

$$a_{0,H} = \cos\varphi_a a_0(1450) - \sin\varphi_a a_0(980),$$
$$K_{0,H}^* = \cos\varphi_k K_0^*(1430) - \sin\varphi_k K_0^*(700).$$

The decays of the two nonets of the scalar resonances is described by the chiral effective Lagrangian from [28],

$$\begin{aligned} \mathcal{L} = {} & c_d^H \langle S_H u_\mu u^\mu \rangle + c_m^H \langle S_H \chi_+ \rangle + \alpha_H \langle S_H u_\mu \rangle \langle u^\mu \rangle \\ & + \beta_H \langle S_H \rangle \langle u_\mu u^\mu \rangle + \gamma_H \langle S_H \rangle \langle u_\mu \rangle \langle u^\mu \rangle \\ & + c_d' G \langle u_\mu u^\mu \rangle + c_m' G \langle \chi_+ \rangle + \gamma' G \langle u_\mu \rangle \langle u^\mu \rangle, \\ & + c_d^L \langle S_L u_\mu u^\mu \rangle + \alpha_L \langle S_L u_\mu \rangle \langle u^\mu \rangle + c_m^L \langle S_L \chi_+ \rangle. \end{aligned} \tag{4}$$

The interactions among the pseudoscalars are described by the chiral Lagrangian taken from [51,52],

[1]For $\eta - \eta'$ mixing, as discussed in [31], it is not clear that the double angles mixing scheme [32,33] will improve the analysis of $\eta' \to \pi^+\pi^-\gamma$. Thus, we simply use one angle mixing scheme as what is done in Refs. [34,35]. For mixing between the η, η' and gluon component, it is ignored here but can be found in Refs. [36–39].

[2]Notice that the scalar glueballs, mixing with f_0 states, are also discussed in Refs. [45–49].

$\mathcal{L} = F^2\langle u_\mu u^\mu + \chi_+\rangle/4$. Notice that the vector meson resonances, such as $\rho(770)$, $\omega(782)$, and $\phi(1020)$, do not appear as in the decay of the $X(2370)$. The reason is that the XVP vertex violates the C parity conservation. Similarly, the axial vectors such as $a_1(1260)$, $b_1(1235)$ do not appear as the intermediate states because APP is not allowed by the parity conservation.

As mentioned in the Introduction, we need to extract the radiative decay width of $J/\psi \to \gamma X$ to judge whether the $X(2370)$ is dominated by the glueball. The experiment [24] measured the product of the branching ratios $\mathrm{Br}[J/\psi \to \gamma X]\mathrm{Br}[X \to PPP]$. The total width of the $X(2370)$, irrespective of its nature, can be estimated by summing over all the decay channels with three light pseudoscalar final states,

$$\Gamma_X(Q^2) = \sum_i \Gamma_{X\to(PPP)_i}, \tag{5}$$

with the sum ruining over $KK\pi$, $\pi\pi\eta$, $KK\eta$, $KK\eta'$, $\pi\pi\eta'$ $\eta\eta\eta$, $\eta\eta\eta'$, $\eta\eta'\eta'$ [26]. Since $\mathrm{Br}[X \to PPP]$ is given by a ratio of partial to total widths, the dependence on g_X cancels, and it can be predicted directly from the amplitudes constructed from the effective Lagrangians in Eqs. (3), (4). Thus, the branching ratio $\mathrm{Br}[J/\psi \to \gamma X]$ for the production of the $X(2370)$ can be extracted directly using Eq. (1) with the denominator fixed by the dynamics governing the $X(\to SP) \to PPP$ decays and the numerator given by the experiment.

TABLE I. Predictions of branching ratios of the $X(2370)$ decays.

	Solution I	Solution II
$\mathrm{Br}[X \to KK\eta']$	$0.981 \pm 0.05 \times 10^{-2}$	$0.983 \pm 0.04 \times 10^{-2}$
$\mathrm{Br}[X \to KK\pi]$	0.522 ± 0.01	0.525 ± 0.01
$\mathrm{Br}[X \to \pi\pi\eta']$	$2.64 \pm 0.13 \times 10^{-2}$	$2.71 \pm 0.16 \times 10^{-2}$
$\mathrm{Br}[X \to KK\eta]$	$6.73 \pm 0.13 \times 10^{-2}$	$6.55 \pm 0.13 \times 10^{-2}$
$\mathrm{Br}[X \to \eta\eta\eta']$	$3.27 \pm 0.26 \times 10^{-3}$	$4.62 \pm 0.23 \times 10^{-4}$
$\mathrm{Br}[X \to \pi\pi\eta]$	0.356 ± 0.01	0.359 ± 0.01
$\mathrm{Br}[X \to \eta\eta'\eta']$	0.0	0.0
$\mathrm{Br}[X \to \eta\eta\eta]$	$1.56 \pm 0.05 \times 10^{-2}$	$1.28 \pm 0.04 \times 10^{-2}$

III. RESULTS AND DISCUSSIONS

The experiment gave the branching ratios of the $J/\psi \to \gamma X \to \gamma K^+K^-\eta'$, $J/\psi \to \gamma X \to \gamma K_S K_S \eta'$, and upper limit of the branching ratio of $J/\psi \to \gamma X \to \gamma\eta\eta\eta'$. To estimate $\mathrm{Br}[X \to PPP]$, one needs to fix the couplings in $S \to PP$. The mixing angles of scalars and that of $\eta - \eta'$, as well as the coupling constants in Eqs. (4) were determined in [28] through analysis of PP production. In that analysis, the scalar resonances do not appear as isolated Breit-Wigner amplitudes but decay into PP, where the final state interactions (FSI) are taken into account. We perform two analyses in the present paper according to different ways to deal with $S \to PP$. In what we refer to as solution I, the parameters of $S \to PP$ are taken from [28], and then they are input into the analysis of J/ψ radiative decays. In contrast, in solution II, the $S \to PP$ decays are refitted without the FSI. Note that the FSI of $\pi\pi - K\bar{K}$ has been partly restored by the Breit-Wigner forms of the scalars appear in the intermediate states. The results of $\mathrm{Br}[X \to PPP]$ are shown in Table I. The results of solution I are only a slightly different from that of solution II, except for the $\mathrm{Br}[X \to \eta\eta\eta']$. The latter is caused by the difference in $f_0(1370) \to \eta\eta$ decay widths found between solution I and solution II with the former almost 4 times larger. From these branching ratios, it is easy to compute the branching ratios of the $J/\psi \to \gamma X$ directly from the experimental results for $\mathrm{Br}[J/\psi \to \gamma X \to \gamma PPP]$ through Eq. (1). In practice, to do so, we perform a combined fit of $\mathrm{Br}[J/\psi \to \gamma X \to \gamma K^+K^-\eta']$, $\mathrm{Br}[J/\psi \to \gamma X \to \gamma K_S K_S \eta']$, and impose the upper limit of $\mathrm{Br}[J/\psi \to \gamma X \to \gamma\pi^+\pi^-\eta']$ to extract the single parameter, $\mathrm{Br}[J/\psi \to \gamma X]$, The results are also named as sols. I and II according to different $S \to PP$ inputs and are shown in Table II. As can be found, when the fitted value for $\mathrm{Br}[J/\psi \to \gamma X]$ is used to compute $\mathrm{Br}[J/\psi \to \gamma X \to \gamma PPP]$, the result agrees with the experiment within the experimental uncertainties. In solution I, and II, we find $\mathrm{Br}[J/\psi \to \gamma X] = 2.87 \pm 0.68 \times 10^{-3}$ and $\mathrm{Br}[J/\psi \to \gamma X] = 3.95 \pm 0.71 \times 10^{-3}$, respectively, while in quenched LQCD, the pure glueball production rate is found to be $\mathrm{Br}[J/\psi \to \gamma X] = 0.231 \pm 0.090 \times 10^{-3}$ [21], for $M_G = 2.395$ GeV. Our result is almost 1 order larger than that of LQCD, and it implies that the glueball can not be the dominant component of the $X(2370)$.

TABLE II. Predictions of branching ratios of J/ψ radiative decays from our fit. Here, the superscripts "1,2,3" represent for the processes of $J/\psi \to \gamma X \to \gamma K^+K^-\eta'$, $J/\psi \to \gamma X \to \gamma K_S K_S \eta'$, and $J/\psi \to \gamma X \to \gamma\eta\eta\eta'$, respectively. The label "tot" is for the process of $J/\psi \to \gamma X(2370)$. The $\chi^2_{\mathrm{d.o.f}}$ is 0.15 and 0.60 for solution I and solution II, respectively.

	Solution I	Solution II	Results from experiment or LQCD
$\mathrm{Br}^{(1)}(10^{-5})$	1.45 ± 0.23	1.99 ± 0.38	$1.79 \pm 0.23 \pm 0.65$ [24]
$\mathrm{Br}^{(2)}(10^{-5})$	0.68 ± 0.11	0.94 ± 0.18	$1.18 \pm 0.32 \pm 0.39$ [24]
$\mathrm{Br}^{(3)}(10^{-6})$	9.20 ± 1.26	1.79 ± 0.25	<9.2 [53]
$\mathrm{Br}^{\mathrm{tot}}(10^{-3})$	2.87 ± 0.68	3.95 ± 0.71	0.231 ± 0.090 [21]

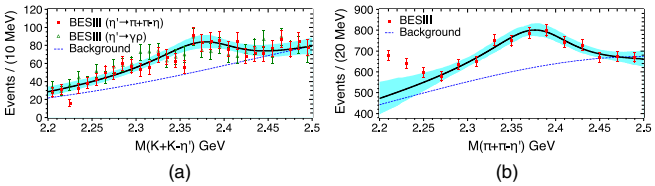

FIG. 1. Combined fit results of $K^+K^-\eta'$ and $\pi^+\pi^-\eta'$ invariant mass distributions. The experimental data displayed are from BESIII Collaboration [24] for the $J/\psi \to \gamma K^+K^-\eta'$, and [23] for the $J/\psi \to \gamma\pi^+\pi^-\eta'$, respectively. Two η' decay modes, *i.e.,* $\eta' \to \pi^+\pi^-\eta$ and $\eta' \to \gamma\rho^0$ are displayed for $J/\psi \to \gamma K^+K^-\eta'$ process with red filled square and green open triangle. Our results are shown by the black solid lines with cyan bands. The backgrounds are shown with blue dotted lines.

To further study the nature of the $X(2370)$ and as a check of the amplitude model, we also perform a combined analysis of the branching ratios and the invariant mass spectra in $J/\psi \to \gamma K^+K^-\eta'$, $J/\psi \to \gamma K_S K_S \eta'$ and $J/\psi \to \gamma\pi^+\pi^-\eta'$. The differential decay rate is given as

$$\frac{d\Gamma_{J/\psi\to\gamma(PPP)_i}}{dQ} = \frac{(M_{J/\psi}^2 - Q^2)}{128 Q M_{J/\psi}^3 (2\pi)^5} \times \int ds \int dt |\mathcal{M}_X^{(i)} + \mathcal{M}_{b.g.}^{(i)}|^2. \quad (6)$$

Here, $\mathcal{M}_X^{(i)}$ comes from chiral effective field theory, describing the amplitudes of $J/\Psi \to \gamma X \to \gamma(PPP)_i$, where "$i$" denotes the ith channel. $\mathcal{M}_{b.g.}^{(i)}$ is a background [not from the intermediate state X(2370)] parametrized by first-order polynomials of s. This distribution is fitted to the (unnormalized) invariant mass spectrum, and one can obtain the branching ratio of $J/\Psi \to \gamma X \to \gamma(PPP)_i$ by integrating Eq. (6) under the peak (*i.e.,* with the background removed).

In particular, in Fig. 1, we show a sample fit result obtained for $J/\psi \to \gamma K^+K^-\eta'$ and $J/\psi \to \gamma\pi^+\pi^-\eta'$ mass spectra. In our analysis, we also include backgrounds, which are caused by processes where thresholds for production of intermediate states are far away from the $X(2370)$ region, for instance, $J/\psi \to \gamma\eta(\eta') \to \gamma\bar{K}K\eta'$. These contribute with a smooth function of the invariant mass, and when subtracted from the overall intensity, the contribution of the $X(2370)$ is found to be very well described by a Breit-Wigner resonance with a mass and width 2387.8 ± 1.3 MeV, and 119.6 ± 6.7 MeV, respectively. These values are close to those of the experimental analysis from [23,24]. Finally, the combined analysis gives $\mathrm{Br}[J/\psi \to \gamma X] = 3.26 \pm 0.81 \times 10^{-3}$, very close to that of solution II.

IV. A PHENOMENOLOGICAL ANALYSIS OF THE GLUEBALL PRODUCTION RATE

As is known, the glueball is a pure state (G), but it mixes with quark components to form a physical state, X. Based on the $U(1)_A$ anomaly, the dominant underlying mechanism of pseudoscalar production in J/ψ radiative decay is via $c\bar{c}$ annihilation into two gluons and a photon [54–56], and the production rate fraction of the physical state X and pseudoscalar meson η can be expressed as [27]

$$\frac{\mathrm{Br}[J/\psi \to \gamma X]}{\mathrm{Br}[J/\psi \to \gamma\eta]} = \left(\frac{\alpha_X}{\alpha_\eta}\right)^2 \left(\frac{M_{J/\psi}^2 - M_X^2}{M_{J/\psi}^2 - M_\eta^2}\right)^3. \quad (7)$$

Here, α_i, $(i = X, \eta)$ stand for the matrix elements $\langle 0|\alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}|i\rangle$; $G_{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ denote the gluon field strength tensor and its dual, respectively. We first assume that X is dominated the glueball, *i.e.,* $|X\rangle = |G\rangle$. Then we can use the pure gauge lattice results from [21] for $\mathrm{Br}[J/\psi \to \gamma G] = 0.231 \pm 0.090 \times 10^{-3}$ and for $\alpha_X = \alpha_g = -0.054\ \mathrm{GeV}^3$ [57]. Then using the experimental data for $\mathrm{Br}[J/\psi \to \gamma\eta] = 1.11 \pm 0.03 \times 10^{-4}$ [57], from Eq. (7), we obtain $\alpha_\eta = 0.031\ \mathrm{GeV}^3$. Now that we have determined α_η, we input this value into the model of [27], which enables us to predict the physical matrix elements α_X for the physical state, that is a mixture of the glueball and the $q\bar{q}$ component. If this state was to be identified with the X, we can determine using Eq. (7), the production rate, obtaining $\mathrm{Br}[J/\psi \to \gamma X] = 0.487 \pm 0.143 \times 10^{-3}$. When compared with the result from our analysis of the experimental data, this further confirms our conclusion that the $X(2370)$ is not likely to be a glueball.

V. SUMMARY

In this work, we constructed $J/\psi \to \gamma K^+K^-\eta'$, $J/\psi \to \gamma K_S K_S \eta'$, $\gamma\pi^+\pi^-\eta'$, and $\gamma\eta\eta\eta'$ decay amplitudes using chiral effective Lagrangians. It is found that the $\mathrm{Br}[J/\psi \to \gamma X]$ can be directly extracted from the experiment measurement of $\mathrm{Br}[J/\psi \to \gamma X \to \gamma PPP]$, where the $\mathrm{Br}[X \to PPP]$ is fixed by these amplitudes. The branching ratio of $\mathrm{Br}[J/\psi \to X\gamma]$ is found to be $2.87 \pm 0.68 \times 10^{-3}$ or $3.95 \pm 0.71 \times 10^{-3}$ depending on the choice of parameters and are 1 order of magnitude larger than that of LQCD, $0.231 \pm 0.090 \times 10^{-3}$, or $0.487 \pm 0.143 \times 10^{-3}$ from a LQCD motivated phenomenology analysis. This is confirmed by a refined analysis of the amplitude model that also considers the invariant mass spectra. Our result is a strong evidence that the $X(2370)$ is not dominated by a glueball component. Future experiments by BESIII at BEPCII and BelleII at superKEKB, which focus on the branching ratios of X decays, would be rather helpful to study the nature of the $X(2370)$.

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