

ABERRATION COMPENSATION IN A SKEW PARAMETRIC- RESONANCE IONIZATION COOLING CHANNEL

A. Sy[#], Y. Derbenev, V.S. Morozov, Jefferson Lab, Newport News, VA 23606, U.S.A.
 A. Afanasev, George Washington University, Washington, DC 20052, U.S.A.
 Y. Bao, University of California, Riverside, Riverside, CA 92521, U.S.A.
 R. P. Johnson, Muons, Inc., Batavia, IL 60510, U.S.A.

Abstract

Skew Parametric-resonance Ionization Cooling (Skew PIC) represents a novel method for focusing of highly divergent particle beams, as in the final 6D cooling stage of a high-luminosity muon collider. In the muon collider concept, the resultant equilibrium transverse emittances from cooling with Skew PIC are an order of magnitude smaller than in conventional ionization cooling. The concept makes use of coupling of the transverse dynamic behavior, and the linear dynamics are well-behaved with good agreement between analytic solutions and simulation results. Compared to the uncoupled system, coupling of the transverse dynamic behavior purports to reduce the number of multipoles required for aberration compensation while also avoiding unwanted resonances. Aberration compensation is more complicated in the coupled case, especially in the high-luminosity muon collider application where equilibrium angular spreads of the muon beam in the cooling channel are on the order of 200 mrad. We present recent progress on aberration compensation for control of highly divergent muon beams in the coupled correlated optics channel, and a simple cooling model to test the transverse acceptance of the channel.

INTRODUCTION

Skew Parametric-resonance Ionization Cooling (Skew PIC) [1-3] has been explored for focusing of highly divergent particle beams, especially for muon beams with large equilibrium emittances. The concept could make possible a high-luminosity muon collider by providing an additional order of magnitude in the luminosity with a factor of ten reduction in the transverse emittance. The PIC [4] concept uses correlated optics and a periodically-driven parametric resonance to provide natural periodic points at which the beam is focused in both transverse planes; ionization cooling can then occur at these periodic focal points with the use of thin absorber plates or wedges to cool the transverse and longitudinal momentum components, and RF cavities to restore the longitudinal momentum only. The correlated optics condition requires the free betatron oscillation periods of the two transverse planes to be integer multiples of a characteristic channel period λ , i.e. $\lambda = m\lambda_x = n\lambda_y$, where both m and n are integers. The parametric resonance forces the x - x' and y - y' transverse phase space trajectories from stable elliptical motion to unstable hyperbolic motion at the periodic focal points; particle positions are periodically damped, while

particle angles periodically grow. The overall beam size is kept stable and is cooled through ionization cooling. Skew PIC expands on the concept by coupling the dynamics in the two transverse planes with a skew quadrupole field; the coupling moves the natural betatron tunes away from the resonant values enforced by the correlated optics, and the transverse dynamics exhibit rotational behavior at the periodic focal points. The parametric resonance is driven as in the uncoupled case, but the coupled behavior only requires the parametric resonance to be driven in one of the two transverse planes.

Recent work has focused on compensating the nonlinear beam aberrations with higher order multipoles [3]. This work discusses multipole optimization progress and implementation of a simple cooling model into a Skew PIC channel with compensating multipoles.

MULTIPOLE OPTIMIZATION

Compensation of nonlinear effects using higher order multipoles is vital to the success of the Skew PIC technique for cooling of highly divergent muon beams; the initial RMS angular spread of the muon beam is on the order of 100 mrad, and poor control of large amplitude particles will result in substantial beam loss. Aberration compensation in the Skew PIC channel is complicated due to the coupled dynamic behavior and the oscillating dispersion functions along the entire length of the channel. MADX implementation of higher order multipoles for compensation of nonlinear effects utilizes the various multipole coefficients of the sector bends that define the linear orbit. Previous work constrained the multipole forms to explicitly follow the periodicity of the channel's dipole and skew quadrupole fields, requiring one sextupole, one octupole, and one decapole harmonic to control a particle distribution with $|\theta_{\max}|=82$ mrad and $|\Delta p/p|$ up to 2%.

Increasing the number of harmonics per multipole order by setting four independently-varying coefficients per Skew PIC channel period allows for greater control of the same particle distribution with lower order fields. Figures 1 and 2 compare the x - y and x' - y' phase space distributions at an absorber location after 1, 51, and 101 channel periods for a 400 particle distribution with $|\theta_{\max}|=82$ mrad and $|\Delta p/p|=2\%$ for the single sextupole-decapole harmonic case, and for a channel with four independently-varying components for sextupoles and octupoles only. The flexibility in the sextupole and octupole fields allows for transport of all particles through

[#]amysy@jlab.org

the channel with no particle losses and a reduction in the order of multipoles required. The average radial offset of particles in the absorber plane after 101 periods through the channel is 1.25 mm, a 20% decrease as compared to the single sextupole-decapole implementation.

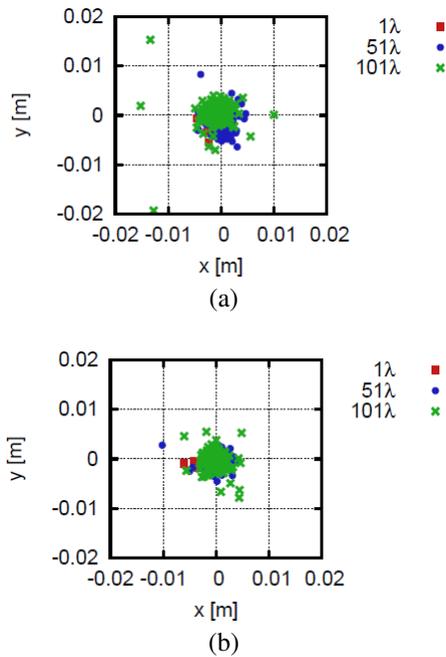


Figure 1: (a) x-y phase space in the absorber plane after 1, 51, and 101 periods of the single sextupole-decapole channel. (b) x-y phase space in the independently-varying sextupole and octupole channel.

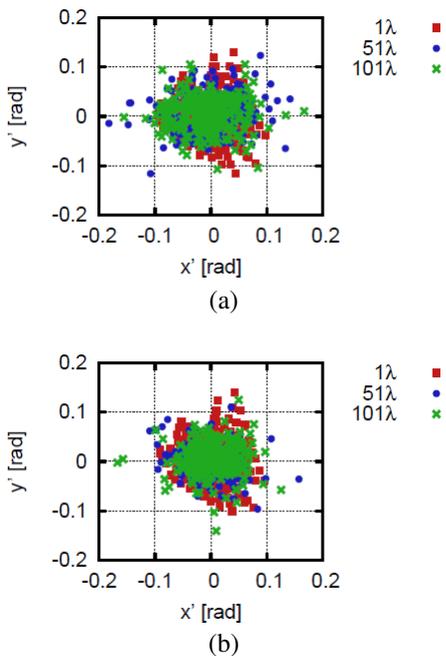


Figure 2: (a) x'-y' phase space in the absorber plane after 1, 51, and 101 periods of the single sextupole-decapole channel. (b) x'-y' phase space in the independently-varying sextupole and octupole channel.

Previous simulations used an incremental approach to expanding the particle distribution and optimizing multipoles to control all particles. The general procedure involved fixing the angular distribution and optimizing coefficients of one multipole order, due to the inflexibility of using one or two harmonic forms per order, and then incrementing both the angular distribution and the multipole order for the next round of optimization. The simultaneous optimization of several coefficients across several multipole orders for large angular distributions approaching the values required for the final design is found to be a more suitable approach for the multipole optimization. This simultaneous optimization requires large variable sets that are not efficiently handled by optimization routines such as MINUIT, and a genetic algorithm approach is currently being explored for efficient optimization over the wide parameter space.

SIMPLE COOLING MODEL IMPLEMENTATION

A simple cooling model was implemented in a Skew PIC channel to test the dynamic behavior with energy damping. The ionization cooling effect was simulated as a momentum kick at each assumed absorber location at the end of the channel period. The damping kick to the momentum is characterized by the momentum loss at each absorber $\delta p/p$, and the particle distribution at each absorber location is modified by the kick:

$$\left(\frac{\Delta p}{p}\right)_f = \left(\frac{\Delta p}{p}\right)_i - \frac{\delta p}{p};$$

$$x'_f = x'_i \left(1 - \frac{\delta p}{p}\right); \quad y'_f = y'_i \left(1 - \frac{\delta p}{p}\right)$$

as each absorber location is traversed over the course of the simulation. A thin RF cavity after the damping kick restores the longitudinal momentum component only. For $\delta p/p=0.005$, the energy loss per absorber is ~ 9 MeV, corresponding to a beryllium absorber thickness of 3 cm. Figure 3 shows the x, x' and $\Delta l=-ct$ projections in the absorber plane over 300 periods in a Skew PIC channel including multipoles up to decapole for a single particle with $\theta=90$ mrad, $x=x'=y=y'=\Delta p/p=0$, $\phi_s=0$ for the undamped case. The particle exhibits stable motion through the channel. Figure 4 shows similar plots with damping effects included for a single particle with $\theta=120$ mrad, $x=x'=y=y'=0$, $\Delta p/p=1\%$, $\phi_s=-30^\circ$.

The damping effect is readily seen as the particle traverses the length of the channel, and even allows for increased transverse acceptance of the channel. Apparent longitudinal damping can also be seen in Figure 4(c); this is likely due to transverse-longitudinal coupling due to the dispersion throughout the channel. We note that these are idealized effects due to the lack of stochastic effects and that a parametric resonance has not been explicitly induced in the damping simulations. Future work will focus on more realistic cooling simulations with particle distributions.

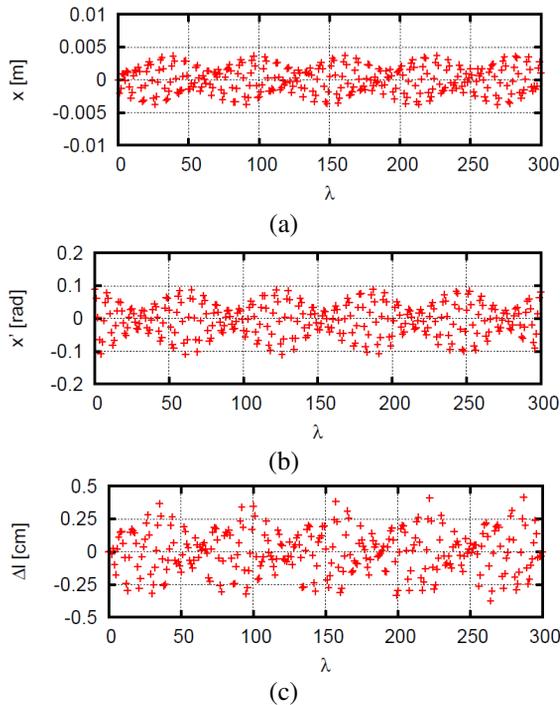


Figure 3: (a) Transverse position x in the absorber plane over 300 periods in the Skew PIC channel with no damping effects included. (b) Transverse angle x' . (c) Path length difference Δl .

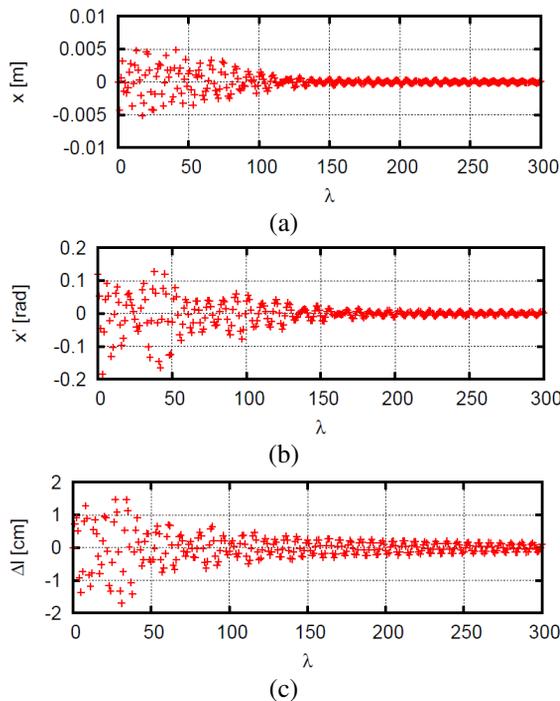


Figure 4: (a) Transverse position x in the absorber plane over 300 periods in the Skew PIC channel with damping effects included. (b) Transverse angle x' . (c) Path length difference Δl .

SUMMARY

Skew parametric-resonance ionization cooling (Skew PIC) continues to be developed and understood in the context of transporting particle beams with large angular divergences and minimal beam losses. For the high-luminosity muon collider application, the equilibrium angular spread of the muon beam in the cooling channel is on the order of 200 mrad, and current multipole optimization results have controlled particles with angles up to 82 mrad. More sophisticated optimization routines will be explored to search the wide parameter spaces available to the large variable sets of the Skew PIC channel. A simple cooling model has been implemented to test the transverse acceptance of a channel with recent multipole settings. The ionization cooling is implemented as a damping kick to the transverse and longitudinal momentum components, and a thin RF cavity restores the longitudinal momentum component only. Both transverse and longitudinal damping are apparent from the simple cooling model simulations. Future work will test the channel with more realistic cooling simulations.

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