EXTRACTION OF PROTON AND DEUTERON $F_2$ STRUCTURE
FUNCTION FROM INCLUSIVE ELECTRON-NUCLEON
SCATTERING AT LARGE $BJORKEN - X$

A Dissertation
by
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This dissertation submitted by Debaditya Biswas in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Hampton University, Hampton, Virginia is hereby approved by the committee under whom the work has been completed.

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ABSTRACT

Extraction of Proton and Deuteron $F_2$ Structure Function from Inclusive Electron-Nucleon Scattering at Large $Bjorken – X$

(May 2022)

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The E12-10-002 experiment measured the inclusive electron-nucleon cross section at Jefferson Lab at large $Q^2$ (up to 16 GeV$^2$) and a large range of $x$ (up to 0.99). The data is analyzed and the cross sections are extracted for hydrogen and deuterium. Then the $F_2$ structure function is extracted for hydrogen and deuterium. Finally the quark-hadron duality is tested for the proton $F_2$ structure function for the previous world data and this experiment.
Dedicated to all my teachers from whom I ever learned anything.
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also like to take this opportunity to remember the good time I spent in the lab with Chuck Long.

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Now at the end it’s time to pay tribute to some of the victims of my Ph.D., namely my family in India. My parents, Giridhar Biswas and Gita Biswas, would be happy to see their son more often. Dad, you always asked to dream high, and mom you made sure I didn’t lose my grip on life’s reality. My dearest friend (and wife) Antara Bhattacharjee, I hope you still remember those times in high school we spent together discussing physics problems. Thank you for always being supportive of my research career. Dibyendu Biswas, my little brother, thank you for being one of my biggest morale supporters in crisis and a friend with unbiased opinion.

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CHAPTER 1
INTRODUCTION

Quarks and gluons are the fundamental building blocks of matter within the realm of the most successful subatomic model of matter to date, the standard model. Even though quarks and gluons are not the direct observables in experiments, knowledge of Parton Distribution Functions (PDF) can tell us a lot about them. Experiments on hard scattering processes, along with the QCD theory and phenomenology, are the best way to learn about PDFs. Lepton-nucleon deep inelastic scattering experiments have been used for a long time in particle physics to extract information about the quark PDFs.

But our knowledge about the PDFs is limited at large Bjorken X (defined in Section 1.1.1, and from now on abbreviated as $x$). It was recently shown by theoretical studies [5] that loosening the $Q^2$ and $W^2$ (defined in Section 1.1.1) cuts on the data and considering the target mass correction, higher twist contributions, and nuclear effects for deuterium can result in very good PDF fits. For a fixed $Q^2$, as $x$ approaches to large values, the invariant mass squared $W^2$ reaches towards the resonance region and the non-perturbative effects become significant. Going to large $x$, while keeping the $W^2$ in DIS region, means pushing the data to high $Q^2$, which was not possible before the 12 GeV era at Jefferson Lab. Jefferson Lab is now able to provide a high current beam with energy up to 11 to 12 GeV, making it possible to measure high $x$, high $Q^2$ data.

Pushing the existing data to large ($Q^2, x$) also helps in modeling the electron-nucleon cross section in the DIS and resonance regions. Figure 1.1 shows the ratio of data to model cross section for the proton. As we can see, the $Q^2$ goes up to 8 GeV$^2$. 
Pushing the data to higher $Q^2$ will certainly help in constraining the cross section fit. More detail about the DIS and resonance modeling can be found here [14].

![Figure 1.1. The electron-proton cross section ratio of data to fit [14].](image)

The neutron cross section cannot be measured directly, as the neutron can not be found in the free state as a target. For that the deuteron cross section needs to be measured along with the proton cross section [7]. This experiment, E12-20-002, measured both the proton and deuteron cross sections, and hence the neutron cross section can be extracted from this data.

The large $x$ data is important for two more reasons apart from constraining PDFs at large $x$, as mentioned above. One is the non-singlet moment. The study of the non-singlet moment is a fundamental test of QCD which is also possible with the data generated in this experiment [20].
Also, several decades of SLAC and JLab data already showed that the average resonance region behaviour always wiggles around the scaling curve. This phenomenon is known as quark-hadron duality and will be discussed in detail in the later sections. The quark-hadron duality can be used as a tool in the resonance region to find the average scaling behaviour in that region, which then can be used to constrain the PDFs. By the end of the 6 GeV era, the proton and deuteron structure functions were measured for large $x$ up to 0.9 and $Q^2 \lesssim 7 \text{ GeV}^2$. For the high $Q^2$ and large $x$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The statistical uncertainty in percentage plotted against $x$ in the top panel. The bottom plot shows the kinematic coverage of the data. The red circles are from the current experiment E12-10-002, the blue dots corresponds to 6 GeV era experiment E00-116, and the green triangles corresponds to the Whitlow reanalysis of SLAC data. The black line shows the demarcation between the DIS and resonance regions. The figure was generated by William Henry.}
\end{figure}
there were some data provided by SLAC experiments but with rather large statistical uncertainty as shown in Figure 1.2. For a \((Q^2, x)\) where both SLAC and E12-10-002 have measurements, it is evident that the SLAC data had a statistical uncertainty of 20-40\%, whereas the current experiment reduces the statistical uncertainty to under 3-4\%. The kinematic region \(Q^2 > 10 \text{ GeV}^2\) and \(x > 0.8\) was also not very well explored before this experiment. The kinematic coverage of E12-10-002 for \(Q^2\) is from 4 to 17 \(\text{GeV}^2\), and for \(x\) the range is from \(\sim 0.2\) to 1.0.

With all these physics motivations, the E12-10-002 experiment was conducted at Jefferson Lab in the spring of 2018. As described in this section the data of this experiment is valuable for many purposes, but for this thesis I will focus on the quark-hadron duality using a new method. For that I will do a global data analysis where this experiment will help to push the \(Q^2\) up to 17 \(\text{GeV}^2\) for a large range of \(x\). More detail about the motivations for proposing this experiment can be found here [33].

1.1 Theory

In this section the underlying theory important for this experiment will be discussed. This section will start with defining the kinematics of the experiment, then the relation between the structure functions and cross sections will be discussed. Then the following subsections will be about Bjorken scaling, scaling violation, discussion of quark-hadron duality and finally the current experimental status of quark-hadron duality.

1.1.1 Kinematics

The experiment I will discuss in this thesis explores the deep inelastic electron-nucleon scattering process. The one photon exchange Feynman diagram of the above mentioned scattering process is shown in Figure 1.3. In this figure \(e(k)\) represents the
Figure 1.3. The single photon exchange (first order in fine structure constant $\alpha$) Feynman diagram for inclusive deep inelastic electron-nucleon scattering.

incoming electron with four momentum $k$. The incoming electron interacts with the nucleon via the exchange of a virtual photon, where $e'(k')$ represents the outgoing (scattered) electron with four momentum $k'$. If $E$ is the energy of the incoming electron and it scatters from the target with an energy of $E'$ at an angle $\theta$ (with respect to the incoming electron direction), then the virtual photon energy is given by

$$\nu = E - E'. \quad (1.1.1)$$

The negative squared mass of the virtual photon ($-q^2$) can be expressed in terms of electron energy and scattering angle as

$$Q^2 \equiv -q^2 = 4EE' \sin^2 \left(\frac{\theta}{2}\right). \quad (1.1.2)$$
neglecting the electron mass. The invariant mass squared of the final hadronic state is given by
\[ W^2 \equiv (p + q)^2 = M^2 + 2M\nu - Q^2, \]  
where \( p \) is the four momentum of the target and \( M \) is the target mass. The dimensionless quantity \( x \) can be defined as
\[ x = \frac{Q^2}{2p.q} = \frac{Q^2}{2M\nu}. \]  
The physical interpretation of \( x \) is the fraction of the nuclear momentum carried by the struck parton (quark). Another important dimensionless quantity is
\[ y = \frac{p.q}{p.k} = \frac{\nu}{E}. \]  
The physical interpretation of \( y \) is the fraction of the beam energy transferred by virtual photon. For elastic scattering, \( W^2 = M^2 \) and \( x = 1 \).

1.1.2 Unpolarized Scattering Cross Sections and Structure Functions

In the previous section, I introduced the kinematic variables for unpolarized electron-nucleon scattering of which \( \theta, E, E' \), and the total number of incoming and scattered electrons are the quantities measured in this experiment. With the help of these measurements, the most important physical quantity we want to calculate is the scattering cross section. Then our goal is to learn about the internal structure of nucleon by exploring the relationship between the scattering cross sections and the structure functions (defined later in this section). In this section, I will explain the scattering cross sections, the structure functions, and the relationship between them.
The scattering cross section ($\sigma$) is proportional to the number of scattering events per unit time per unit incident particle flux. The unpolarized differential scattering cross sections (i.e. both the incoming lepton and the target hadrons are unpolarized) can be expressed as
\[
\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 E'}{Q^4 E} L^{\mu\nu} W_{\mu\nu};
\]
where, $\sigma$ is the total cross section, $\Omega$ is the lab solid angle, $\alpha$ is the fine structure constant, $L^{\mu\nu}$ is the leptonic tensor which represents the lepton-virtual photon interaction vertex, and $W_{\mu\nu}$ is the hadronic tensor which represents the virtual photon-nucleon interaction vertex.

The leptonic tensor can be expressed as
\[
L^{\mu\nu} = \pi (k') \gamma^\mu u(k) \overline{\pi} (k) \gamma^\nu u(k').
\]
For the unpolarized leptons, averaging over initial spins and summing over final spins
\[
L^{\mu\nu} = 2 \left( k'^\mu k'^\nu + k'^\mu k'^\nu - g^{\mu\nu} k' k \right).
\]
The hadronic tensor can be expressed as
\[
W_{\mu\nu} = \frac{1}{4} \sum_X P j_\mu X X j_\nu P (2\pi)^4 \delta^4 (P + q - P_X)
\]
where $X$ is any hadronic final state. As in the inclusive process, all the hadronic final states are summed over, and $W_{\mu\nu}$ depends only on the incoming hadron momentum $P$ and virtual photon momentum $q$. The electromagnetic current $j_\mu$ is conserved: $\partial_\mu j_\mu = 0$. Hence,
\[
q_\mu W_{\mu\nu} = q_\nu W_{\mu\nu} = 0.
\]
Along with this condition, together with parity conservation and time reversal invariance, one can show $W_{\mu\nu}$ can be expressed in terms of two structure functions $W_1$ and $W_2$ as

$$W_{\mu\nu} = W_1 (\nu, q^2) \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2 (\nu, q^2)}{M^2} \left( P_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{p \cdot q}{q^2} q_\nu \right).$$

(1.1.11)

Substituting into equation 1.1.6 we get

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ W_2 (\nu, Q^2) + 2W_1 (\nu, Q^2) \tan^2 \left( \frac{\theta}{2} \right) \right],$$

(1.1.12)

where the term $\sigma_{\text{Mott}}$ represents the elastic scattering cross section of the electron from a point-like spinless charged particle and is given by

$$\sigma_{\text{Mott}} = \frac{\alpha^2 \cos^2 \left( \frac{\theta}{2} \right)}{4E^2 \sin^4 \left( \frac{\theta}{2} \right)}.$$

(1.1.13)

$W_1$ and $W_2$ cannot yet be calculated directly in the QCD framework. These two structure functions must be determined experimentally. $W_1$ and $W_2$ contain the information about the internal structure of the nucleon and can be expressed in terms of two other dimensionless structure functions $F_1$ and $F_2$ as

$$F_1 (x, Q^2) = MW_1 (\nu, Q^2),$$

(1.1.14)

$$F_2 (x, Q^2) = \nu W_2 (\nu, Q^2).$$

(1.1.15)

The $F_2$ structure function has been measured in many experiments worldwide. An $F_2$ vs $Q^2$ for the proton plot for the world data is shown in Figure 1.5.
The differential scattering cross section can also be expressed in terms of the longitudinal \( (\sigma_L) \) and transverse \( (\sigma_T) \) scattering cross sections as

\[
\frac{d^2\sigma}{d\Omega dE'} = \Gamma \left[ \sigma_T \left( x, Q^2 \right) + \epsilon \sigma_L \left( x, Q^2 \right) \right]; \quad (1.1.16)
\]

where \( \sigma_T \) \( (\sigma_L) \) represents the cross section when the purely transverse (purely longitudinal) virtual photon scatters of the target, \( \Gamma \) is the virtual photon flux expressed as

\[
\Gamma = \frac{\alpha \kappa}{2\pi^2 Q^2} \frac{E'}{E} \frac{1}{1 - \epsilon}; \quad (1.1.17)
\]

with

\[
\kappa = \frac{W^2 - M^2}{2M}. \quad (1.1.18)
\]

and the virtual photon polarization parameter \( \epsilon \) is given by

\[
\epsilon = \left[ 1 + 2 \left( 1 + \frac{Q^2}{4M^2 x^2} \right) tan^2 \left( \frac{\theta}{2} \right) \right]^{-1}. \quad (1.1.19)
\]

For the purely transverse polarization, \( \epsilon = 0 \). The ratio of longitudinal and transverse virtual photon cross sections is expressed as \( R \)

\[
R = \frac{\sigma_L}{\sigma_T}. \quad (1.1.20)
\]

Alternate structure functions \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) can be expressed in terms of \( \sigma_T \) and \( \sigma_L \) as

\[
\sigma_T = \frac{4\pi^2 \alpha}{\kappa M} F_1, \quad (1.1.21)
\]

\[
\sigma_L = \frac{4\pi^2 \alpha}{\kappa M \nu} \left[ 1 + \frac{\nu^2}{Q^2} \right] M F_2 - \nu F_1 \right] = \frac{4\pi^2 \alpha}{\kappa M \nu} \frac{1}{2} \frac{1}{2} \frac{F_L}{F_L}. \quad (1.1.22)
\]
Figure 1.4. The proton magnetic form factor $G_M(Q^2)$ divided by $\mu_p$ plotted against $Q^2$. The important feature is to notice that $G_M(Q^2)/\mu_p$ drops rapidly with increasing $Q^2$ [34]. This means that proton has a substructure and in this energy scale cannot be considered as a dimensionless point like particle.

1.1.3 Bjorken Scaling

In the previous section the structure functions and their relation with cross sections were established. In this section I will discuss the $F_2$ behavior with varying momentum transfer squared $Q^2$.

Figure 1.4 shows the magnetic form factor (equivalent to inelastic structure function) vs $Q^2$ for the elastic scattering. The form factor drops rapidly with $Q^2$. This means that the proton has a substructure and in this energy scale cannot be considered as a dimensionless point like particle.

From DIS scattering data, the proton $F_2$ at fixed $x$ is plotted against $Q^2$ from many different experiments in Figure 1.5. This shows that for a particular $x$, proton $F_2$ is “almost” independent of $Q^2$. Compared to the elastic form factor, as shown in Figure 1.4, which drops by almost two orders of magnitude between $5 < Q^2(GeV^2) < 30$, 
Figure 1.5. The $F_2$ structure function of proton measured in different experiment worldwide from Particle Data Group(year 2018) [22]. This is a representative plot and might not include all the reliable data worldwide.

The inelastic structure function shows an approximate scale invariance. That means

$$F_2(x, Q^2) \approx F_2(x) \text{ when } Q^2 \gg M^2. \quad (1.1.23)$$

This approximate scale invariance can be interpreted as the incoming virtual photon striking a charged, point-like constituent inside the proton. This point-like constituent is referred to as a Parton.
1.1.4 Parton Model

This parton model can be understood in the reference frame where the nucleon (e.g. proton) has a very large momentum so the constituent partons’ momenta are almost parallel to the proton momentum. For \( Q^2 \gg M^2 \) it can be shown that the fraction of proton momentum carried by the struck parton is given by \( x = Q^2/2M\nu \).

The structure function \( F_1 \) depends on the spin of the partons and it can be shown that if partons are spin 0,

\[
F_1(x, Q^2) = 0; \tag{1.1.24}
\]

Otherwise, if the partons are spin \( \frac{1}{2} \) particles,

\[
2xF_1(x, Q^2) = F_2(x, Q^2) \quad \text{(Callan-Gross relation).} \tag{1.1.25}
\]

The experimental data favoured heavily the equation 1.1.25 over the equation 1.1.24 hence confirming that the partons are spin \( \frac{1}{2} \) particles.

\( F_2 \) can be expressed as

\[
F_2(x, Q^2) = \sum_a e_a^2 x f_a(x), \tag{1.1.26}
\]

where \( f_a dx \) is the probability of finding a parton \( a \) of charge \( e_a \) with fractional momentum between \( x \) and \( x + dx \). Together with 1.1.21-1.1.22, the measured cross section at a particular \( x \) is proportional to the probability of finding a parton which carries the fraction \( x \) of the proton momentum. The proton structure functions measurements along with neutron structure functions measurements and data from neutrino scattering also confirmed that the \( e_a^2 \) is either \( \frac{1}{9} \) or \( \frac{4}{9} \).

In summary, (1) the scaling invariance of \( F_2 \), (2) the Callan-Gross relation, and (3) the fractional charge (\( \frac{1}{3} \) or \( \frac{2}{3} \)) of partons proved that partons were actually the already hypothesized quarks beyond any reasonable doubt.
1.1.5 Scaling Violation

The scaling behaviour of the structure function was established when there was not much data available for a wide range of \((Q^2, x)\). As more and more experimental data accumulated, it was observed that \(F_2\) is not completely independent of \(Q^2\). This dependence of \(F_2\) on the \(Q^2\) is known as a scaling violation. This scaling violation cannot be fully explained by the quark-parton model. One of the biggest triumphs of QCD was established when the scaling violation was explained.

The generation of a new quark from a parent quark can mostly be explained by gluon radiation. The gluons in the interaction create a sea of quark-antiquark pairs. So the higher order interaction need to be considered. The Altarelli-Parisi equation [23] describes the \(Q^2\) evolution of the \(F_2\) structure function rigorously,

\[
\frac{d q(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y, Q^2) P_{qq}(\frac{x}{y}).
\] (1.1.27)

This equation tells us that a quark with momentum fraction \(x\) \([q(x, Q^2)]\) can be generated from the parent quark \(q(y, Q^2)\) with momentum fractions \(y\), with a probability proportional to \(\alpha_s P_{qq}(x/y)\). The integral is a sum over all the possible momentum fraction \(y(> x)\) of the parent quark. Figure 1.6 shows the interactions corresponding to leading order (LO) and next-to-leading order (NLO).

In short, if we know the structure function at particular \((Q^2, x)\), the Altarelli-Parisi evolution equation can predict the structure function at any other \(Q^2\). Going back to Figure 1.5, at around \(x = 0.25\) the scaling holds very well. For \(x \lesssim 0.25\) the structure function increases with increasing \(Q^2\), but for \(x \gtrsim 0.25\) it decreases. If the partons were non interacting with increasing \(Q^2\), no further structure would have been seen and scaling would hold perfectly. In that case, the parton model would
Figure 1.6. (a), (b), and (c) are the examples of leading order (LO) QCD corrections while (d) is an example of a next-to-leading order (NLO) correction [32].

be satisfactory. But for very high $Q^2$, due to the gluon radiation it becomes more probable to find a “soft” quark.

1.1.6 Asymptotic Freedom

We already established that nucleons (e.g. proton and neutron) are not fundamental particles and are made up of point-like quarks. The quarks are spin $\frac{1}{2}$ fermions interacting in the strong force field through the exchange of spin 1 gauge bosons called gluons. In this section a remarkable feature of the strong force called Asymptotic Freedom will be discussed.

At this point we should recognize that the manifestation of the internal structure of the nucleon in experimental data to some extent depends on the energy scale with the sea of quark-antiquark pairs generated by the gluon emissions form a cloud around the “parent” quark.

$^1$soft quark : when the sea of quark-antiquark pairs generated by the gluon emissions form a cloud around the “parent” quark.
which we are probing the nucleon. A high energy scale generally means probing with large momentum transfer and short distance scale. At very short distances, the interaction of the strong force is weaker, and around 0.1 fm the lowest-order diagram via one gluon exchange dominates.

This phenomenon can also be understood with the help of the QCD running coupling constant $\alpha_s(\ll 1)$. Any observable $f$, can be expanded in terms of the powers of $\alpha_s$ as

$$f = f_0 + \alpha_s f_1 + \alpha_s^2 f_2 + \ldots \quad (1.1.28)$$

where $f_0$ is the leading order term, $\alpha_s f_1$ is the next-to-leading order (NLO) term etc. These individual terms can be calculated by Feynman diagrams. The coupling con-

![Figure 1.7. The running coupling constant $\alpha_s(\mu)$ is plotted against energy scale $\mu$. $\alpha$ decreases with increasing $\mu$. The solid circles are different experimental values, the solid lines are QCD prediction. This figure is reprinted from W.-M. Yao et al., Journal of Physics, G33, 1, 2006. [16]](image)

stant $\alpha_s$ decreases with increasing momentum transfer as seen in Figure 1.7. Hence for a very high momentum transfer, the leading order term of equation 1.1.28 dominates.
So, probing with high momentum transfer at a shorter distance scale, the quarks can be seen as asymptotically free particles, and hadronic scattering cross sections can be realized in terms of quark-gluon basis states.

1.1.7 Color Confinement

In the previous section, the shorter distance (high momentum transfer) interaction regime (0.1 fm) is discussed. But the quark picture inside the hadrons is not that “simple” every time. In the larger distance scale (hence comparatively low momentum transfer) another striking feature of the strong force dominates: color confinement. In this section I will discuss color confinement.

Different kinds of quarks come in 6 flavors: up (u), down (d), charm (c), strange (s), top (t) and bottom (b). They are grouped by increasing mass into 3 generations as shown below.

\[
\begin{pmatrix}
u \\ d \\ c \\ s \\ t \\ b
\end{pmatrix}
\]

The u, c, t quarks have charge of \(+ \frac{2}{3}e\) and d, s, b quarks have charge of \(- \frac{1}{3}e\). where e is the electronic charge unit. Apart from their spin and charge, the quarks have another degree of freedom known as color. Any quark can exist in three different color states r (red), g (green), and b (blue). The net color charge is always zero in any quark bound state (hadrons). This is known as color confinement. An unusual feature of the strong interaction is that the gluon mediator also carries a color charge. Hence in the strong interaction, the gluon-gluon interaction is also possible.

As the distance between two quarks increases, the strong interaction also increases and the higher order diagrams become more and more important. In this regime the strong interaction is more complicated we cannot visualize quarks as asymptotically free particles, and cross sections must be interpreted with hadrons degrees of freedom.
1.1.8 Quark-Hadron Duality

The two most striking features of QCD, asymptotic freedom at short distances and color confinement at large distances, are established by several DIS lepton-nucleon scattering experiments. At low energies, the inclusive lepton-nucleon cross sections are dominated by the nuclear resonances. Bloom and Gilman established a bridge between the low and high energy regime by analyzing the early SLAC data of electron-proton scattering [11]. This phenomenon, which bridges the low and the high energy regimes, is known as quark-hadron duality. In this section I will discuss quark-hadron duality.

In the 1970’s, Bloom and Gilman observed that with increasing momentum transfer \( Q^2 \) the three prominent resonance peaks do not disappear for the proton structure function \( W_2(\nu, Q^2) \) relative to background but fall roughly at a same rate. The resonance region can be defined by \( W^2 \), and with then available SLAC data the resonance region is defined by \( W^2 < 4 \). Another observation was that the structure function in the resonance region approximately averages to the DIS region structure function \( (F_2) \). As Bloom and Gilman emphasized, this behaviour becomes particularly obvious when structure functions are plotted against

\[
\omega' = \frac{2M\nu + M^2}{Q^2}.
\]  

(1.1.29)

This new scaling variable \( \omega' \), named by Bloom-Gilman as the “improved scaling variable” allowed the easy comparison between high \( W^2 \) data and low \( W^2 \) data over a range of \( Q^2 \). Figure 1.8 shows the early SLAC data ranging from \( Q^2 \) of 0.75 to 3 GeV\(^2\) analyzed by Bloom and Gilman. The figure shows the resonance region for \( W^2 < 4 \) GeV\(^2\) where almost twenty different nucleon resonances give rise to three prominent resonance peaks. Here \( \nu W_2(\nu, Q^2) \) is extracted for a fixed \( R = \sigma_L/\sigma_T = 0.18 \). The scaling curve shown here is a parameterization of the high-\( W \) (high \( Q^2 \)) data available
The function $\nu W_2$ plotted against $\omega'$. The dashed line is the scaling curve. The solid line is the resonance curve. As the energy increases, so the momentum transfer, hence the resonance peaks slide down the scaling curve. This picture is reprinted from the famous paper “Scaling, duality, and the behavior of resonances in inelastic electron-proton scattering” by Bloom and Gilman. [11]

at that time. Hence the observations of Bloom and Gilman were:

1. The resonance peaks oscillate around the scaling curve.
2. The resonance data, on average, follow the scaling curve.
3. The resonance peaks slide along the DIS curve with increasing $Q^2$. [36]

Bloom and Gilman tried to quantify this phenomenon with finite energy sum rules by equating the integral of $\nu W_2$ over $\nu$ in the resonance region to the integral of the scaling function over $\omega'$. 

\[ \int \nu W_2 \, d\nu = \int \text{scaling function} \, d\omega' \]
\[
\frac{2M}{Q^2} \int_0^{\nu_m} d\nu W_2 (\nu, Q^2) = \int_{1+\frac{W_m^2}{Q^2}}^{1+\frac{W_m^2}{Q^2}} d\nu' \nu W_2 (\nu'), \tag{1.1.30}
\]

with \( W_m \approx 2 \text{ GeV} \) denoting the resonance region boundary. The maximum value of \( \nu \) is \( \nu_m = (W_m^2 - M^2 + Q^2)/2M \) which corresponds to the maximum value of \( \nu' = 1 + \frac{W_m^2}{Q^2} \). The relative difference of the two sides of the equation 1.1.30 was shown to range from \( \approx 10\% \) at \( Q^2 = 1 \text{ GeV}^2 \), to \( \leq 2\% \) at \( Q^2 = 2 \text{ GeV}^2 \) [36], quantifying the degree to which duality holds.

### 1.1.9 QCD and Quark-Hadron Duality

The phenomenon of quark-hadron duality was known even before the full development of QCD. Bloom and Gilman tried to quantify the phenomenon but the theoretical foundation of duality based on QCD was first established by De Rújula, Georgi and Politzer [18] [19]. They interpreted the Bloom-Gilman duality in terms of the structure function moments. Starting with the definition of structure function moments in this section, the QCD origin of duality will be briefly explained.

The \( x \) weighted integrals of structure functions are known as structure function moments where the \( n^{\text{th}} \) moment of the structure functions can be defined as

\[
M_1^{(n)} = \int_0^1 dx x^{n-1} F_1 (x, Q^2) , \tag{1.1.31}
\]

\[
M_{2,L}^{(n)} = \int_0^1 dx x^{n-2} F_{2,L} (x, Q^2) . \tag{1.1.32}
\]

The moment of \( F_1 \) at \( n = 1 \) is a count of the quark charges in the parton model. The moment of \( F_2 \) at \( n = 2 \) corresponds to a momentum sum rule. For \( n = 2 \) and replacing \( F_2 \) by the asymptotic structure function \( F_2^{\text{asy}} \), an analogous equation to
1.1.30 can be formulated in terms of moments

\[ M_2^{(2)} = \int_0^1 dx F_2(x, Q^2). \]  

(1.1.33)

Hence the moment can be thought of as the integral over the whole \( x \) range for a particular \( Q^2 \). At the large \( Q^2 \), the DIS contribution is bigger than the resonance contribution, and vice versa for smaller \( Q^2 \). The moment at high \( Q^2 \) (\( \approx 10 \text{ GeV}^2 \)) and the moment at low \( Q^2 \) (\( \approx 2 \text{ GeV}^2 \)) are almost equal and the small difference can be accounted for by the scaling violation. This equality of the \( F_2 \) structure function moments at high and low \( Q^2 \) is the essence of quark-hadron duality.

As experimental discovery of quark-hadron duality preceded QCD, many theoretical efforts were made to explain the phenomenon. But the standard theoretical framework of QCD which explains quark-hadron duality is known as the operator product expansion (OPE) (or twist expansion) developed by Wilson [53] and others [12] [39].

At large \( Q^2 \), the moments can be expanded in powers of \( 1/Q^2 \). The coefficients (matrix elements) of the expanded terms correspond to the twist \( \tau \), where \( \tau \) is defined as the mass dimension minus the spin \( (n) \) of the operator. The \( n^{th} \) moment of \( F_2 \) is given by

\[ M_2^n(Q^2) = \sum_{\tau=2,4,...}^{\infty} \frac{A_{\tau}^{(n)}(\alpha_s(Q^2))}{Q^{\tau-2}}. \]  

(1.1.34)

\( A_{\tau}^{(n)} \) is the matrix element with twist \( \leq \tau \).

![Figure 1.9](image)

*Figure 1.9. Figure (a) shows the leading twist contribution to the structure function, (b) corresponds to higher twist (four quark) and (c) corresponds to a higher twist quark-gluon interaction.*
The leading twist term as shown in Figure 1.9 (a) corresponds to the incoherent scattering of the virtual photon from a single parton. The higher twist terms as shown in Figures 1.9 (b), (c) are proportional to powers of $1/Q^2$.

1.1.10 Quark-Hadron Duality: Experimental Status

During the discovery of the Bloom-Gillman duality, the kinematic coverage of the data sets were very limited. Since then the availability of high luminosity experiments made it possible to collect high quality of data over a large range of $Q^2$ and $x$. In this section I will discuss the results on spin-averaged $F_2$ from different experiments in the context of quark-hadron duality.

Figure 1.10. Proton $F_2 = \nu W_2$ plotted against $\xi$ at $Q^2$ of 0.45, 0.85, 1.70, and 3.30 GeV$^2$. The arrows indicate the elastic point where $\xi = \xi(x = 1)$. The dashed line is the NMC curve [9] at $Q^2 = 5$ GeV$^2$ at the same $\xi$ but higher ($W^2, Q^2$) and the solid line is the same for $Q^2 = 10$ GeV$^2$. 
Quark-hadron duality was again restudied by one of the 6 GeV era JLab experiments [42]. As shown in Figure 1.10, the proton $F_2 = \nu W^2$ data is plotted against the Nachtman variable $\xi$ in the resonance region. This resonance region data is compared with the fit to a large data set of higher $W^2$ and $Q^2$ from the New Muon Collaboration [9]. The important feature of this plot is that the resonance structure wiggles around the DIS curve. Quantitatively, the average of the resonance curve follows the DIS curve within 10% hence confirming the Bloom-Gilman duality. As the $Q^2$ increases the resonance structures slide on the DIS curve towards higher $\xi$.

To plot the resonance curve and DIS curve at the same $\xi$ scale, low $Q^2$ resonance data were needed to compare with high $Q^2$ DIS data. As shown in Figure 1.10, the resonance data for $Q^2$ of 0.45, 0.85, 1.70, and 3.30 GeV$^2$ were compared with NMC curves for $Q^2$ of 5 or 10 GeV$^2$. For a better understanding of the duality it is desirable to compare the resonance data with the DIS predictions at the same $Q^2$. This is possible by using the results of global QCD fits which include several hard scattering processes like lepton-nucleon deep inelastic scattering, Drell-Yan measurements, jet production, etc. Different groups like Coordinated Theoretical-Experimental Project (CTEQ) [1]; Martin, Roberts, Stirling, and Thorne (MRST) [3]; Gluck Reya, and Vogt(GRV) [4], and others provide the global QCD fits.

For the later JLab experiment E94-10-110 [27], the resonance region data were compared with the PDF fits for the same values of $(Q^2, x)$ as shown in 1.11. This experiment measured the longitudinal and transverse cross sections, making the $F_2$ more precise than that of the previous data shown in Figure 1.10. The smooth curves in Figure 1.11 are QCD fits from MRST (dark blue) [6] and CTEQ [26]. The red smooth curve is the fit to the SLAC DIS data [52]. The pQCD curves (target mass corrected) describe the average resonance strength for each $Q^2$. This shows very clearly for different $Q^2$ the relation between the very non-perturbative resonance region and the perturbative DIS region.
Another way of quantifying duality is to calculate the integral of the structure function over $x$ for a fixed $Q^2$ and compare it the integral of the DIS scaling curve over the same $x$ region and for the same $Q^2$. The integral of the structure function is given by

$$\int dxF_2(x, Q^2).$$

(1.1.35)

Figure 1.12 shows the ratios of the integrals for the proton structure functions $F_2$, $F_1$, $F_L$, and $R$. Except for $F_2$, the discussions about the other structure functions are beyond the scope of this thesis. The scaling or perturbative strength was calculated by MRST (with target mass correction) and fitted to the SLAC DIS data. From the Figure 1.12 it is evident that the DIS strength agrees with the resonance region strength to better than 5% above $Q^2 = 1$ GeV$^2$. This proves unambiguously that
duality holds on average for the unpolarized structure functions, thus the average strength of the resonance region over a range of $x$ is equivalent to the predictions from the PDF fits.

![Figure 1.12. Ratio of the $F_2$ integral over $x$ at fixed $Q^2$ plotted against $Q^2$. The scaling strength corresponding to the black box points are from the fit to the SLAC DIS data, whereas for the blue inverted triangles the scaling strength is calculated by MRST (with target mass correction).]

In general the global PDF fits completely ignore the resonance region and extrapolate the PDF fit from the DIS region to the resonance region. If duality holds, the large $x$ resonance region data can also be utilized for the PDF fits. To do that it becomes more important to precisely measure the data for the $(x, Q^2)$ region where the perturbative calculation is no longer the most significant. For the pQCD analysis, the higher twist terms can be extracted from data with a cut $W^2 > 10$ GeV$^2$. It can be shown that $xW^2$, rather than $Q^2$ [13] is the more natural scale for the twist expansion.
The low $W^2$ region (dominated by the resonances) was analyzed by Liuti et al [28]. This analysis is done within a fixed $W^2$ framework. The higher twist contributions were found to be similar to those of $W^2 > 10 \, GeV^2$. More detail about the work can be found in Ref. [28].

The last experiment in the 6 GeV era at JLab to study quark-hadron duality for the spin-averaged structure function was E00-116 [31]. In this work, quark-hadron duality was quantified as the ratio

$$I(x_{\text{ave}}, Q^2_{\text{ave}}) = \frac{\int_{x_{\text{min}}}^{x_{\text{max}}} F_2^{\text{data}}(x, Q^2) dx}{\int_{x_{\text{min}}}^{x_{\text{max}}} F_2^{\text{param}}(x, Q^2) dx}. \quad (1.1.36)$$

To do both local and global duality studies it is important to define the resonance regions defined by $W^2$ as,

1) first resonance region ($\Delta$) $\rightarrow W^2 \in [1.3, 1.9]$

2) second resonance region (S) $\rightarrow W^2 \in [1.9, 2.5]$

3) third resonance region (F) $\rightarrow W^2 \in [2.5, 3.1]$

4) fourth resonance region $\rightarrow W^2 \in [3.1, 3.9]$

5) deep inelastic scattering region $\rightarrow W^2 \in [3.9, 4.5]$

Figure 1.13 shows the results of Equation 1.1.36 for the local duality, global duality, and DIS regions. The figure includes data from JLab experiments E00-116, E00-110, and from SLAC. The data is also compared with CTEQ6M+TM. The comparison with other models can be found here in Ref. [31]. It is important to notice that the ratio $I$ is almost equal to 1 near $Q^2 = 1.25 \, GeV^2$, then rises above unity with increasing $Q^2$. The $Q^2$ dependence saturates from 4 GeV$^2$ and above. This behaviour is similar for DIS, global, and three of four resonance regions except the first resonance region. The $Q^2$ independence (for the larger $Q^2$) of the ratio $I$ indicates that, the discrepancy between the data and model might not be coming from the $Q^2$. The
Figure 1.13. Local and global duality plots for hydrogen where CTEQ6M+TM is used for the comparison.

large $Q^2$ also pushes the data towards large $x$. It is most likely that the PDF fits are poorly constrained by data at large $x$. 
CHAPTER 2

EXPERIMENTAL SETUP

Figure 2.1. Aerial view of the Continuous Electron Beam Facility (CEBAF), Thomas Jefferson National Accelerator Facility, Newport News, Virginia, USA.

2.1 Overview

The commissioning experiment E12-10-002 was conducted at the Thomas Jefferson National Accelerator Facility (Jefferson Lab) in the spring of 2018, using its Continuous Electron Beam Accelerator Facility (CEBAF). The main goal of this experiment was to measure the differential scattering cross-sections of electron-nucleon inclusive scattering for a large range of $Q^2$ ($\approx$ from 4 to 16 GeV$^2$) and $x$ (from 0.2 to 1.0). To achieve this goal, an electron beam of the desired energy was produced by the CEBAF accelerator and data were collected using the facility of experimental
Hall C. In this chapter I will discuss the CEBAF’s operation in brief along with the details of beamline hardware, target, and spectrometers of Hall C.

2.2 CEBAF

![Schematic of the Continuous Electron Beam Facility (CEBAF)](image)

As indicated in the previous section, CEBAF stands for the Continuous Electron Beam Accelerator Facility. CEBAF produces a continuous wave (cw), high intensity, electron beam for high energy nuclear physics scattering experiments. During this experiment, CEBAF produced beam currents up to 80 \( \mu \text{A} \) and electron energies up to 11 GeV. The general setup of CEBAF is shown in Figure 2.2.

The electron beam’s journey starts from the injector (shown as the green box in Figure 2.2) and ends in one of the experimental Hall beam dumps. In the injector, a laser pulse of frequency \( f_0/n \) (where \( f_0 \) is the fundamental machine frequency and \( n \) is an integer) illuminates a Gallium arsenide (GaAs) photocathode, which creates the electron bunches. Then the electron bunches are sent to the injector beam line
and get accelerated to 123 MeV by a C100 cryomodule. To accelerate the electrons inside the linac, SRF technology is used which will be discussed briefly next.

![Figure 2.3. Electron accelerating through a 5 cell SRF cavity.](image)

To accelerate the electrons inside the linacs, CEBAF uses Niobium SRF (Superconducting Radio Frequency) resonant cavities. Eight such 7-cell SRF cavities make up a C100 cryomodule. The fundamental frequency of the SRF cavities is 1497 MHz, and they operate at a very low temperature of 2K in liquid $^4\text{He}$. The oscillating electric field inside the cavities accelerates electrons with the right phase. When the electron bunches cross the cell boundary, the electric field changes polarity hence pushing the electrons further downstream as shown in a 5-cell SRF cavity in Figure 2.3.

From the injector beam line, the electron bunches are sent to the north linac and accelerated further with a gain of 1.1 GeV by the end of the north linac. There are bending arcs (80 meters in radius) at the end of the north linac to steer the beam by 180° and send it to the south linac. The south linac accelerates the electron beam an additional 1.1 GeV. Hence, in 1 pass (when the electron passes through both the linacs) the electron beam gains 2.2 GeV energy. Each of the linacs has 25 cryo modules. Inside the arcs there are many quadrupoles and dipoles to focus and steer the beam, respectively.

The number of passes is determined by the desired beam energy for the experiment. After the desired beam energy is reached, at the end of the south linac, separators send the beam to the different halls (Hall A, B or C).
2.3 Hall C

The desired beam energy (10.602 GeV) for the experiment E12-10-002 was achieved after 5 passes in accelerator. Then the beam was steered from the Beam Switchyard towards Hall C through the beam line. After leaving the south linac, the beam enters the Hall C beam line then the Hall C arc. The curvature of the arc is given by the angle $\theta_{\text{arc}}$ as shown in Figure 2.4. There are eight dipole magnets (along with 5 other small beam correctors) inside the arc which bend the beam into Hall C. From the beam switch yard to the hall C entrance (starting from green shield wall towards the hall as shown in Figure 2.4) the beam bends by $37.5^\circ$.

![Figure 2.4. Hall C arc and beamline along with different beamline components. This figure is reprinted from the thesis by Carlos Yero [46].](image)

The Hall C beam line has several monitors to measure the beam energy, beam current, beam position, and beam profile. After passing through the beam line, the beam reaches the target. The targets used for this experiment were liquid hydrogen
(LH2), liquid deuterium (LD2), carbon (C), and aluminum (Al). The beam interacted with the target and the scattered electrons were detected by the Hall C detector systems. Hall C has two magnetic spectrometers: (a) the High Momentum Spectrometer (HMS) and (b) the Super High Momentum Spectrometer (SHMS) which was newly built in the 12 GeV era of Jefferson Lab. Superconducting magnets in the spectrometers focus the scattered particle of desired angle and momentum bite towards the detector hut. The unscattered beam directly goes to the beam dump. The details of the different spectrometers’ components will be discussed in the later sections. In the next section I will discuss several beam line components in detail.

2.4 Hall C Beam Line Instrumentation

There are different components along the beam path for beam diagnostics, among which the most relevant are harps, beam position monitors, and beam current monitors [48]. As shown in Figure 2.5, upstream of the target chamber, the beamline components are: fast raster (FR), beam position monitors (BPM), beam current monitors (BCMs, UNSER), and harps. As shown in Figure 2.6, downstream of the target chamber, the beam pipe is 27.4 meters long which ends at the beam dump tunnel entrance. Near the beam dump, two big BPMs can be placed to measure the beam position downstream of the target. The big BPMs are necessary because the fringe field from the SHMS can change the direction of the beam downstream of the target chamber.

In the next few sections, I will discuss details of the few beam line components, such as: harps, beam position monitors, and beam current monitors. Along with these, the beam energy measurement procedure, beam rastering system and target scattering chamber will also be discussed in more detail.
Figure 2.5. Schematic diagram of the Hall C beamline upstream of the target chamber. The target chamber can be seen at the very right of the figure. The pivot center is situated at the center of the target chamber. All distances shown in the figure are measured from the pivot. The main beam line components upstream of the target are: fast raster, BPMs, BCMs, and harps. [46]

Figure 2.6. Schematic diagram of the Hall C beamline downstream of the target chamber. Here the target chamber is shown at the very left of the figure. The main beam line components in this part are: beam line to beam dump, beam dump entrance and big BPMs. [46]

2.4.1 Harps

Harps provide the (a) absolute position of the beam and (b) beam size information. The general structure of a harp is indicated in Figure 2.7.
Figure 2.7. Schematic diagram of a harp used in Hall C [8].

The main parts of a harp are three wires (two vertical wires and one horizontal wire, indicated by blue lines in picture), which are mounted in a frame. The frame is moved diagonally by a stepper motor. A position encoder, attached to the stepper motor, precisely records the position of the wires. When the wires pass through the beam, secondary emission produces current in the wires. After amplification, the current signals are sent to an ADC (analog to digital converter). The vertical wires measure the horizontal beam position and profile, and the horizontal wire measures the vertical beam position and profile. There are several harps in the Hall C beamline for different purposes. All the harps are listed in Table 2.1.

Table 2.1

Different harps in Hall C beam Line:

<table>
<thead>
<tr>
<th>Harp name</th>
<th>Harp application</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C07</td>
<td>Beam Energy measurement procedure (start of arc)</td>
</tr>
<tr>
<td>3C17</td>
<td>Beam Energy measurement procedure (end of arc)</td>
</tr>
<tr>
<td>3C20</td>
<td>Relevant for Moller and Compton polarimeter Measurements</td>
</tr>
<tr>
<td>3CH07A</td>
<td>a) position information for BPM calibration</td>
</tr>
<tr>
<td></td>
<td>b) beam size information</td>
</tr>
<tr>
<td>3CH07B</td>
<td>a) position information for BPM calibration</td>
</tr>
<tr>
<td></td>
<td>b) beam size information</td>
</tr>
</tbody>
</table>
Even though the harps can measure the beam position very precisely, it is an invasive procedure hence cannot be used during the data collection. A non-invasive measurement is performed by the Hall C Beam Position Monitors (BPMs).

2.4.2 Beam Position Monitors

The BPMs of the Hall C beamline are used for the continuous, non-invasive beam position monitoring. In Hall C there are three BPMs immediately upstream of the target. Each BPM has four antennae placed at 45 degrees relative to the EPICS coordinate system (see Figure 2.8). When the beam passes through the BPM cavities, antennae pick up signals with amplitude inversely proportional to the distance from the beam to each antenna. Of course, the beam position at the target is of primary interest. Any two of the three BPMs can be projected to the target to get the beam position at the target, as shown in the Figure 2.9. As per the fall 2017 survey, the BPM A, B and C are at a distance of $Z_{BPM\ A} = 320.42$ cm, $Z_{BPM\ B} = 224.86$ cm.
and $Z_{BPM\,C} = 129.44$ cm respectively from the target. If $X_{tar}$ is the x position of the target, then

$$X_{tar} = Z_{BPM\,A} \left( \frac{X_{BPM\,C} - X_{BPM\,A}}{Z_{BPM\,A} - Z_{BPM\,C}} \right) + X_{BPM\,A},$$

(2.4.1)

and similarly, $Y_{tar}$ can be calculated as

$$Y_{tar} = Z_{BPM\,A} \left( \frac{Y_{BPM\,C} - Y_{BPM\,A}}{Z_{BPM\,A} - Z_{BPM\,C}} \right) + Y_{BPM\,A}$$

(2.4.2)

Figure 2.9. A schematic diagram of the birds eye view including the target and two BPMs. The beam positions measured by the two BPMs can be projected to the target as shown in the figure by red line.

For the better determination of the beam position at the target, all three BPMs were used. In general, the average beam position varied by less than 0.2 mm at the target during the run.
2.4.3 Determination of Beam Energy

The Beam Energy Measurement procedure is performed by the MCC (Machine Control Center). It is done using the Hall C arc and cannot be done simultaneously with data taking. At the time of this procedure, the arc quadrupoles and beam corrector magnets are turned off, and only the 8 dipole magnets are left on.

Figure 2.10. Beam energy is measured in the Hall C arc. The arc nominal net bend angle is $\theta_{arc} = 34.3^\circ$. Eight dipole magnets steer the beam along the arc. $\vec{v_e}$ is the electron velocity always tangential the arc curvature, $\vec{B}$ is the magnetic field directed into the page, $\vec{F}$ is the force on the electron in the magnetic field $\vec{B}$.

The electrons bend in the arc under the influence of the magnetic field from the dipole magnets. Hence the beam electrons are subjected to the Lorenz force and the radius of the electron path is inversely proportional to the strength of the magnetic field. If we know the magnetic field strength and the electron trajectory, we can
calculate the electron momentum as:

\[ P = \frac{e}{\theta} \int B dl \]  \hspace{1cm} (2.4.3)

where \( P \) is the electron momentum, \( e \) is the electron charge, \( B \) is the magnetic field and \( dl \) is the infinitesimal path traversed by electron. Four super harps (two at the arc entrance and other two at the arc exit) accurately determine the beam angle change \( \theta \). The dipole fields are accurately mapped to get the correct field integral.

### 2.4.4 Beam Current Monitors

There are six current monitors along the Hall C beam line. BCM1, BCM2 and the Unser are situated at \( \sim 7.5 \) m upstream the target and BCM4A, BCM4B, BCM4C are at \( \sim 10.5 \) m upstream the target. The latter employ digital receivers. More about the digital receivers can be found here [38].

BCMs are cylindrical microwave cavities. These cavities are placed along the beam line and the beam goes along the axis. Beam excites the resonant modes in the cylindrical cavity. There is a wire loop antenna inside, which picks up the signal. The power is proportional to the beam current squared, but for certain modes (e.g. TM\(_{010}\)) is relatively insensitive to the beam position. To optimize these measurements, the cavity size and the material can be chosen in such a way that TM\(_{010}\) mode is excited at the accelerator frequency. The RF signal then converted to lower frequency by a downconverter, and a receiver yields a DC voltage proportional to the beam current. The BCMs have stable gain and high signal to noise ratio. On the other hand, their absolute gain must be calibrated since the output power depends on the cavity impedance, quality factor, signal cable attenuation, etc.

We use an Unser monitor to cross-calibrate the BCMs. The Unser is a toroidal transformer placed in a feedback loop of an operational amplifier. The Unser is
sensitive to the DC current but also to environmental effects like magnetic fields, mechanical vibrations, etc. The Unser gain is very stable and can be confirmed with a wire current. But its zero offset can drift $\sim 0.5 \, \mu A$ in a half hour, hence cannot be used solely to measure the beam current during production data-taking.

2.4.5 Beam Rastering System

To measure cross-sections precisely we need to know the target density precisely. Two challenges in knowing the target density correctly are the high beam intensity and $\sim 100 \mu m$ beam spot size.

During the experiment, when this high intensity beam passes through the cryogenic target, it generates heat. This changes the local density of the target by $0(1)\%/K$ and hence reduces the yield. Bubble formation at the windows, due to boiling, may also be possible. The beam rastering system minimizes these effects.

The fast raster (FR) is located $\sim 14 \, m$ upstream of target. The main components of the FR are two magnets (X magnet and Y magnet) which have an air core. An alternating current ($\sim 25 \, kHz$) passes through the magnet coils to produce an oscillating magnetic field as shown in Figure 2.11. The beam is rastered in a square pattern of $2 \, mm \times 2 \, mm$ with high uniformity. The optical effect of the FR motion is corrected in the offline analysis. This way it was possible to distribute the heat produced by beam interaction into a larger volume, keeping the local temperature more constant.

2.5 Target

In this experiment there were two main targets: liquid hydrogen and liquid deuterium. Both the liquid hydrogen and liquid deuterium were kept in $\approx 10 \, cm$ long cells with aluminum windows. Apart from those an aluminum empty cell (dummy) was used to estimate the background from the windows, and a carbon foil target was
Figure 2.11. Conceptual drawing of beam rastering operation with Hall C Fast Raster. The electron beam is shown in blue line along the +Z direction. The pink arrow shows the magnetic field direction of the FR-X magnet. The green arrow shows the magnetic field direction due to FR-Y magnet. These two magnetic fields are perpendicular to each other. This figure depicts a particular moment in time when FR-X pushes the beam towards right (+X direction) by $\Delta X$ and FR-Y pushes it down (-Y direction) by $\Delta Y$. The light blue 2 mm by 2 mm region depicts the rastered beam spot on the target.

used to verify the beam vs target alignment. All these solid targets and the cryogenic target cells were mounted on a target ladder which could be moved vertically.

This target ladder is kept in a scattering chamber, which is an evacuated cylindrical aluminum tank. During the experiment when the beam electrons hit the targets a significant amount of heat is produced. So we also need a system to dissipate this heat either by conduction and/or convection. In the next few paragraphs the target ladder, cryogenic loop anatomy, and target chamber will be discussed in further detail.

**Target Ladder** The target ladder is a vertical frame on which the target cells for the different cryo targets and several solid targets are attached as shown in the picture. The target ladder can be moved remotely up and down to position the correct
targets in the beam path. There are three cryogenic target cells: liquid helium (loop 1), liquid hydrogen (loop 2), and liquid deuterium (loop 3). The loop 2, loop 3 and dummy dimensions are given in Figures 2.13, 2.14, 2.15, respectively, and are particularly important for the dummy subtraction. The liquid hydrogen in loop 2 is kept at a temperature of 19±0.1 K \(^1\). The liquid deuterium is kept at a temperature of 22±0.1 K \(^2\).

\(^1\)The freezing and the boiling temperatures of hydrogen are 13.8 K and 22.1 K respectively.
\(^2\)The freezing and the boiling temperatures of liquid deuterium are 18.7 K and 25.3 K respectively.
Target Scattering Chamber  As shown in Figure 2.12, the target ladder resides inside an evacuated aluminum cylindrical container. The pressure inside the chamber is kept at about $10^{-6}$ torr with the help of a vacuum pump. There is an exit window for the particles, scattered from the target, to come out. There also an opening connected to the beam exit pipe. The target chamber exit window is shared by the SHMS and HMS beam entrances. Although the spectrometer entrance windows are very close, there is an air gap between the target chamber exit and the spectrometer entrance windows. The unscattered beam passes through the opening and enters the beam exit pipe for its final destination at the beam dump.

Cryogenic Target Loop Anatomy  During the experiment, beam electrons continuously hit the target which causes the generation of heat. For the solid targets there is a chance of melting, and the liquid targets could potentially boil. To avoid
these circumstances, in addition to beam rastering, the target system is engineered in a special way.

The solid targets dissipate the heat by conduction. On the other hand to keep the cryo target at low temperature the fluid needs to be recirculated. Figure 2.17 shows a schematic diagram of the loop anatomy. LH2 and LD2 are supplied from a
gas panel outside the hall to the heat exchanger. From the ESR, LHe coolant (at 15 K temperature) comes into heat exchanger. LH2 cools down to 19 K and LD2 cools down to 22 K. Joule-Thompson (JT) valves control the amount of coolant flow. Then the LH2 (or LD2) is sent to the target cell 2 (cell 3) through an inlet. There is a diverter inside the target cell. The diverter is used to separate the inlet target fluid inlet from the outlet and also minimize the local boiling of the target. After exiting the target cell, the fluid is sent back to the heat exchanger. Inside the heat exchanger it gets cooled. This cycle repeats continuously during the beam operation.

2.6 Hall C Spectrometers

![Figure 2.18](image)

*Figure 2.18. The two spectrometers of Hall C: SHMS and HMS. The white arrow indicates the beam direction.*

Hall C has two magnetic spectrometers: the Super High Momentum Spectrometer (SHMS) and the High Momentum Spectrometer (HMS) as shown in Figure 2.18. Each spectrometer can be divided into two sections: (a) optical elements and (b) the particle detector system.

The purpose of the optical elements is to transport the scattered particles from the target chamber to the detector hut and select the particles in a desired momentum...
bite. The momentum bite is typically $p_0 \pm O(10)\%$. The central momentum can be selected by setting the magnets of the optical system, where $\delta(\delta \equiv \delta p/p_0)$ is defined as the fractional deviation of particle momentum from the central momentum. The range of $\delta$ somewhat depends on the spectrometer design. The optical system is comprised of quadrupoles and dipoles and will be discussed in detail in a later section.

After the optical system selects the particles with desired momenta, the job of the particle detectors is to identify the particles from background, then reconstruct the scattering particle tracks at the focal plane so that we can reconstruct the tracks back to the scattering vertex. To do this job, the detector system has different kinds of particle detectors and the whole detector system is shielded from radiation inside a hut with thick walls.

The spectrometer system sits on a frame which aligns the two spectrometers with each other and to the target chamber. Both the spectrometers can be rotated around the pivot on a steel rail to a desired angle relative to incoming beam. There is a scale attached to the rail to measure the rotation angle. This rotation can be controlled remotely from the Hall C counting house. The camera, attached to the rear end of the spectrometer, sends the picture of the scale to the counting house where the angle can be read on a TV screen. Some important parameters of the SHMS and HMS are shown in Table 2.2.

<table>
<thead>
<tr>
<th></th>
<th>HMS</th>
<th>SHMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Momentum Range</td>
<td>0.4 to 7.4 GeV/C</td>
<td>2 to 11 GeV/C</td>
</tr>
<tr>
<td>Momentum Acceptance</td>
<td>$\pm 10.0%$</td>
<td>-10.0$%$ to +22.0$%$</td>
</tr>
<tr>
<td>Momentum Resolution</td>
<td>0.1 to 0.15$%$</td>
<td>0.03 to 0.08$%$</td>
</tr>
<tr>
<td>Angular Range</td>
<td>10.5$^\circ$ to 90$^\circ$</td>
<td>5.5$^\circ$ to 40.0$^\circ$</td>
</tr>
<tr>
<td>Horizontal Angle Resolution</td>
<td>0.8 mrad</td>
<td>0.5 to 1.2 mrad</td>
</tr>
<tr>
<td>Vertical Angle Resolution</td>
<td>1.0 mrad</td>
<td>0.3 to 1.1 mrad</td>
</tr>
<tr>
<td>Maximum Flux within Acceptance</td>
<td>$\sim$5 MHz</td>
<td>$\sim$5 MHz</td>
</tr>
</tbody>
</table>
The general idea of the optical design and the detector design of the two spectrometers are the same. Hence in the next few sections I will discuss the optical and detector components of the two spectrometers together. The differences between the components of two spectrometers will also be discussed.

2.6.1 Optical Systems of the Spectrometers

![CAD drawing of the Super High Momentum Spectrometer in Hall C. The spectrometer magnets HB, Q1, Q2, Q3 and dipole are shown in the picture. The right side of the yellow room is the heavily shielded detector hut and the left side of it is the electronics room. The detectors reside inside the hut. On the right bottom corner the wheels (in red) can be seen with which the spectrometer rotates on a rail about the pivot. [48]](image)

In this section the SHMS optical system will be discussed in detail. The main elements of the SHMS optical system are several super conducting magnets. The electro-magnetic fields generated in the magnets steer the scattered electrons from the target to the SHMS detector system. To keep the magnet coils at the superconducting
temperature of 4.5 K, liquid $^4$He needs to be supplied to Hall C from the ESR (End Station Refrigerator).

As shown in Figure 2.19, there are three quadrupoles ($Q_1$, $Q_2$, $Q_3$) and two dipoles ($D$ and horizontal bender (HB)). Starting from the target towards the detector system, the magnets are arranged in the following sequence—HB, $Q_1$, $Q_2$, $Q_3$, $D$. As evident from its name, the HB bends the scattered particles horizontally to the left by $3^\circ$ so that the rest of the SHMS elements stay further away from the primary beam line. After being steered by the HB, the scattered particle is focused by quadrupole magnets $Q_1$, $Q_2$, $Q_3$ and then vertically momentum dispersed by the dipole magnet $D$. It is possible to select either positively or negatively charged particles by changing the polarity of the magnets. The central momentum of each magnet can be set remotely from the counting house using a magnet setting program.

![Figure 2.20. High Momentum Spectrometer side view. [46]](image)

As shown in Figure 2.20, the HMS has three quadrupoles ($Q_1$, $Q_2$, $Q_3$) and a dipole ($D$). Unlike the SHMS, the HMS does not have a horizontal bender. Other than that the HMS optical system is similar to that of the SHMS.
2.6.2 Spectrometer Detectors

Both the spectrometers in Hall C have similar sets of detectors: a pair of drift chambers for particle track reconstruction, hodoscope planes for efficient particle triggering, a calorimeter to separate pions from electrons, and Cherenkovs for additional particle identification.

![Figure 2.21. The SHMS detector stack.](image)

The detector stack for the SHMS is shown in Figure 2.21. The scattered particles first enter through the Noble Gas Cerenkov (NGC). After leaving the NGC, the particles pass through other detectors in the following order: a pair of drift chambers, a pair of XY hodoscope planes, the Heavy Gas Cerenkov (HGC), aerogel Cerenkov, another XY hodoscope plane, then the calorimeter. The calorimeter has two parts, the preshower and the shower counters.

The HMS detector package is shown in Figure 2.22. The scattered particles guided by the optical system first enter a vacuum extension. Leaving the vacuum extension, the particles go through the pair of drift chambers. Then the particles pass through the other detectors in this order: aerogel Cerenkov, first XY hodoscope planes, Heavy Gas Cerenkov (HGC), a second set of XY hodoscope planes, then the calorimeter.
Similar to the SHMS, the HMS calorimeter also has two parts: preshower and shower. During the commissioning run period \(^3\) the aerogel Cherenkov was absent.

In the next few sections, the working principle of each spectrometer detector will be discussed in detail. Similar detectors from each of the spectrometers will be discussed together.

### 2.6.2.1 Drift Chambers

Each of the spectrometers (SHMS and HMS) has a similar pair of drift chambers which is used for the particle tracking. As shown in the Figure 2.23, each of the drift chambers consists of alternating wire planes (six in number) and cathode planes (eight in number). There is also a middle plane which separates a drift chamber in two parts. The whole structure is sandwiched between two aluminum plates which

---

\(^{3}\text{E12-10-002 is a part of four 12 GeV commissioning experiments in Hall C.}\)
provide the structural support as well as alignment. There are two drift chambers (DC1 and DC2) placed sequentially. The distance between the mid planes of the two chambers is 80 cm.

Figure 2.23. This cartoon figure of two drift chambers (DC1 and DC2). The +Z direction is the direction of the beam, hence the scattered particle first goes through the DC1 and then DC2. Each of the drift chambers consists of wire planes (U,U',X,X' and V,V') and cathode planes (shown in grey). There is also a mid plane (shown in black) which divides each of drift chambers in two parts.

For the SHMS, as shown in Figure 2.23, DC1 has the wire planes placed in the order U,U',X,X',V,V' and for DC2 the order of the wire planes is V',V,X,X',X',U',U. The diagram of Figure 2.24 shows a cross-section of a single drift chamber with the cathode planes, the wire planes, and the mid plane. The positions of the wires for each of the wire planes is shown relative to the mid plane. The mid plane in Figure 2.24 is at \( z = 0 \), where the \( z \) offset is arbitrary and nothing to do with the spectrometer co-ordinate system. In this figure the X, U, V planes are shown as X1, U1, V1.
Gas Windows & Aluminium plates are not shown.

All numbers are in inches.

Z=0 is set to the middle of the mid plane.

Z=0 just arbitrary and need to be changed for the parameter file in the hcana.

All positions are relative to Z=0.

SHMS DC parameter: positions of wire planes relative to the mid plane.

Diameter of the sense wire (Gold plated tungsten wire): 25 micron +/- 2%.

**Figure 2.24.** The cross-sectional diagram of a drift chamber and the positions of the wire planes and cathode planes relative to the mid plane of the chamber. The numbers shown in the picture are all in inches. The picture does not show the gas windows and the aluminum frame. The cartoon is not drawn to scale. The dimensions are in inches.

respectively; and the X', U', V' planes are shown as X2, U2, V2 respectively- to be consistent with the drift chamber labelling and software nomenclature.

**Wire Planes:** As described previously there are six wire planes per chamber. The wires were strung on 1/8 inch thick printed circuit boards (PCB). Two types of PCBs are used: one for X planes and another for U/V planes. All the planes have alternating field and sense wires. The wires were very carefully strung to the plane frame so that they are parallel to each other. The distance between the two consecutive field (or sense) wires is 1 cm. This distance is known as a cell width. For the SHMS, both U1 (or U2) and V1 (or V2) planes have 107 sense wires, and X1 (or
X2) planes have 79 sense wires. Each of the sense wires is made of 25 µm (±2%) gold plated tungsten, and each of the field wires is made of 80 µm copper plated beryllium.

A wire plane before chamber assembly is shown is Figure 2.27a.

Figure 2.25. The wire orientation for the U1,X1 and V1 plane for the DC1. In SHMS looking from the beam direction, the first drift chamber is named DC1. The white area in each of the wire plane is the active area. For SHMS the active area is 80 cm by 80 cm. The +Z direction is the beam direction and +X direction is the dispersive direction. Only a single wire for each plane is shown in dotted line. The U1 plane wires are at 30° angle with respect to the +X in anti-clockwise direction, whereas the V1 plane wires are at 30° angle with respect to the +X in clockwise direction. The X1 plane wires are at 90° with respect to the +X direction. The direction of the U2, V2 and X2 wires are parallel to U1, V1 and X1 plane wires, respectively, but shifted by half a cell width or 0.5 cm.

The wire orientations of the DC1 U1, X1 and V1 planes are shown in Figure 2.25. For the DC1, the U1 plane wires are at 30° (clockwise) with respect to the particle dispersive or +X direction. The DC1 X1 plane wires are at 90° with respect to the +X direction, and the DC1 V1 wires are at 30° (anticlockwise) with respect to the +X direction. The DC1 U2,X2,V2 wires have the same orientation as the DC1 U1,X1,V1 wires respectively, but shifted by half a cell width (0.5 cm).

If we take DC1 U1(or U2), X1(or X2), and V1(or V2) plane and rotate them by 180° about the +X direction, we get DC2 U1(or U2), X1(or X2), and V1(or V2)
Figure 2.26. The wire orientation for the U,X and V plane for the DC2. In SHMS looking from the beam direction the second drift chamber is named DC2. The white area in each of the wire plane is the active area. For SHMS the active area is 80 cm by 80 cm. The +Z direction is the beam direction and +X direction is the dispersive direction. Only a single wire for each plane is shown in dotted line. The V1 plane wires are at 30° angle with respect to the +X in anti-clockwise direction, whereas the U1 plane wires are at 30° angle with respect to the +X in clockwise direction. The X1 plane wires are at 90° with respect to the +X direction. The direction of the U2, V2 and X2 wires are parallel to U1, V1 and X1 plane wires, respectively, but shifted by half a cell width or 0.5 cm.

planes respectively. The wire orientations of the DC2 planes are shown in Figure 2.26.

**Cathode Planes:** The drift chambers consist of alternating wire and cathode planes, so that a wire plane is always between two cathode planes. The cathode planes are 5 mil thick copper plated Kapton stretched on the PCB frame. A cathode plane before chamber assembly is shown is Figure 2.27b.

**Mid Plane:** Each of the drift chambers consists of two identical half chambers separated by a fiberglass mid plane. The amplifier discriminator cards are mounted on this mid plane.
Figure 2.27. Figure (a) The SHMS X plane and figure (b) the SHMS cathode plane, while assembling the drift chambers inside a clean and moisture free room. The detector fabrication needed to be done in a clean room to prevent dust particles getting inside the chamber. Otherwise at the time of conditioning the chamber, electrical discharges of the dust particles may cause damage to the cathode foils or wires. As shown in Figure (a), reflection of light is used to show the alternating field and the sense wires.

**Ionization Gas:** The SHMS drift chambers are contained within a gas tight frame. During the data taking, a gas mixture of Argon:Ethane (50:50 by volume) flows through both the drift chambers.

**Electrical Design:** The field wires of all the planes and all the cathode planes are held at the same negative potential \( \sim 1940 \) volts. The sense wires are kept at zero potential, which is positive compared to the field wires and cathode planes. This produces an almost uniform electric field around the sense wires. The sense wires pick up the electric signal from the ionization of drift gas. A group of 16 sense wires are connected to the 16 inputs of a Nanometrics preamplifier/discriminator card or LeCroy Research Systems LRS 2735DC card. The inputs of the cards accept the negative signal from the sense wires, then amplify and discriminate them.

These Nanometric cards are powered by a \( \pm 5 \) volts of a low voltage power supply (LV). The low voltage is supplied from the two V6PH58AFHM power supplies from Accopian Technical Company. As shown in the Figure 2.28 the power goes to a
distribution box through 6AWG cables. From the distribution box, power is supplied to four copper LV rails attached to the mid plane through 10AWG cables. The amplifier cards are connected to the LV rails through the 22AWG jumper cables.

Figure 2.28. The SHMS Drift Chamber low voltage design. The figure was produced by Chuck Long.

The analog signals received by the Nanometric cards from the sense wires are amplified and converted to ECL (emitter coupled logic) signals. More detail about the mechanical and electrical designs can be found here [15]. The details of the drift chamber calibration and optimization of tracking parameters can be found in the section 3.2.1.
2.6.2.2 Hodoscope

The hodoscope in each spectrometer is a set of four planes (usually scintillator) grouped into two pairs separated by a distance of 2.2 meters. Each pair has one X plane and one Y plane. The X plane of the upstream pair (horizontal bars shown in blue in Figure 2.29) is a set of 13 horizontal plastic scintillators. The S1Y plane is a set of 13 sets plastic scintillators placed vertically, as shown in blue in Figure 2.29. The S2X and and S2Y planes are grouped together and placed ~2.2 m away from (S1X, S1Y). The S2X plane also has 14 plastic scintillators horizontally placed. The S2Y plane of SHMS has 21 quartz bars. To avoid any dead spot, in each plane the adjacent scintillator bars are overlapped by 5 mm. For the HMS, all the planes consist of plastic scintillators.

![Figure 2.29. Hodoscope scintillator planes.](image)
Each side of the scintillator paddle has a PMT attached to it. When a charged particle passes through the hodoscope, light is emitted from the paddle materials. Though this is the general process, light is emitted differently for plastic scintillators and quartz bars. In the plastic scintillators, the light is produced in the visible range in a process known as fluorescence. The photons produced inside the paddles propagate through the paddle and at the end of the scintillator are detected by a PMT. Due to the photoelectric effect, electrons are produced inside PMT and then they get multiplied by the dynodes of PMT, which generates a measurable analog signal. The signal is further processed in the Hall C electronics room. In the quartz bars, photons are generated by the Cherenkov effect. The Cherenkov effect will be discussed later in detail.

To minimize the light loss, the scintillator bars are wrapped with highly reflective aluminum foil. To ensure light tightness, over the aluminum foil the HMS scintillators are wrapped by multiple layers of Tedlar, and the SHMS scintillators are wrapped by multiple layers of electrical tape. More details about the hodoscopes and scintillators can be found in the References [43], [30].

2.6.2.3 Cherenkov

Both the spectrometers in Hall C are equipped with Cherenkov detectors for rejection of the pion background. The SHMS has a heavy gas Cherenkov (HGC), a noble gas Cherenkov (NGC), and an aerogel Cherenkov. The HMS has a heavy gas Cherenkov and an aerogel Cherenkov detector. The general working principle is same for any of these Cherenkovs and will be discussed next.

When a charged particle travels faster than light through a medium, it emits Cherenkov light. The Cherenkov radiation follows this equation

\[ \cos(\theta_c) = \frac{1}{n\beta}; \]  \hspace{1cm} (2.6.1)
where \( \beta \) is the ratio of the velocity of the charged particle to the velocity of light in vacuum, \( n \) is the index of refraction of the medium, and \( \theta_c \) is the Cherenkov angle. The condition for the Cherenkov radiation is \( \cos(\theta_c) < 1 \), which is equivalent to \( \beta > \frac{1}{n} \). If \( \beta \) for the electron and pion are respectively \( \beta_{e^-} \) and \( \beta_\pi \), then the condition on \( n \) to separate the electron from the pion is

\[
\frac{1}{\beta_{e^-\text{,min}}} < n < \frac{1}{\beta_{\pi^-\text{,max}}}. \tag{2.6.2}
\]

When the inequality of equation 2.6.2 is satisfied, all the electrons produce direct Cherenkov light but the pions do not produce any direct Cherenkov light.

For the SHMS data analysis, the noble gas Cherenkov is used. As shown in the Figure 2.30, four spherical mirrors are set to focus the Cherenkov light to four PMTs. All are kept inside a 2.5 meters long and 0.8 meters wide container. The PMTs
convert the light signals to electrical pulses so that the number of photoelectrons can be counted. An Argon/Neon gas mixture at $\sim 1$ atmospheric pressure is used, satisfying equation 2.6.2 for the momenta of interest.

For the HMS data analysis, a heavy gas Cherenkov is used. The HMS heavy gas Cherenkov is a 1.5 meters long cylindrical tank, which consists of two spherical mirrors to focus the light on two PMTs. For the $e/\pi$ discrimination, $C_4F_{10}$ gas is used.

2.6.2.4 Calorimeter

The electromagnetic calorimeters in both SHMS and HMS are used for particle identification and $e/\pi$ discrimination. The calorimeters are placed at the end of the detector stack. After passing through all other detectors, scattered electrons deposit the majority of their energy into the calorimeter.

When a charged particle passes through the calorimeter material, they get decelerated and emit bremsstrahlung photons. The bremsstrahlung photon produces $e^+e^-$ via pair production. These pairs produce cerenkov light and further produce more bremsstrahlung photons. This process continues until most of the initial energy is deposited into the calorimeter.

The SHMS calorimeters are divided longitudinally into two parts: the preshower (TF-1 lead glass block) and the shower (F-101 lead glass block) counter. The preshower section has two adjacent columns, each consisting of fourteen modules. A PMT is connected to one end of each module. The preshower is $3.6$ radiation length thick. One radiation length is defined as the mean distance of travel after which a charged particle loses all but $\frac{1}{e}$ of its initial energy. The shower modules are stacked in a fly’s eye configuration behind the preshower plane. There are 224 modules and a PMT is connected to the long end of each of them. The shower blocks are $\approx 18$ radiation
lengths thick. More information about the design and performance of the SHMS calorimeter can be found in [49].

*Figure 2.31.* SHMS calorimeter design. The figure is reprinted from [49].

The HMS calorimeters are made of four layers, and each layer has thirteen TF1 lead glass blocks. Each layer measures 3.65 radiation lengths along the particle trajectory (+z). As a whole the calorimeter has ~14.6 radiation lengths which absorbs almost all the energy of a scattered electron. For the first two layers, PMTs are attached to both the ends of the lead glass block. For the last two layers, PMTs are attached to one end of the lead glass block. More about the HMS calorimeter can be found in [37].
Figure 2.32. HMS calorimeter design. The figure is reprinted from [46], [37].
CHAPTER 3
DATA ANALYSIS

In this chapter I will discuss the steps of the data analysis needed before the final cross sections are extracted. This chapter will start with a detail description of the BCM calibration and then the different detector calibrations followed by the target density correction, ionization energy loss correction, and will conclude with the $\delta$ corrections for HMS and SHMS.

3.1 BCM Calibration

Using the global parameters for the for BCM4C and BCM4A all the runs were analyzed and $I_{BCM4A}/I_{BCM4C}$ is plotted for whole production run range. The figure 3.1 shows there is a disagreement between BCM4A and BCM4C. The whole production run period is divided into six periods depending on the ratio. For the first and the fifth periods the ratio is flat but the disagreement between them is almost 2%. Through the periods P2, P3 and P4 the ratio changes rapidly indicating a change in one or both of the BCMs. Hence the region of rapid change is divided into three periods. Then at end of the fifth period, the ratio jumps up to higher value and any runs after 3201 are considered in the sixth (P6) period. Then it was decided to do a BCM calibration run for each period separately, assuming within a single period the BCMs are stable or almost stable. From now on, the BCM calibration for each of the periods separately will be considered as a “local calibration”.

For the local calibration from each period, more than one production run was chosen with multiple beam off periods. The table 3.1 shows the list of runs chosen for each period for the local calibration. In the next few paragraphs I will discuss how the local calibration is done for the period P1.
Figure 3.1. The ratio $I_{BCM4A}/I_{BCM4C}$ plotted for the whole production run range. Depending on the variation in the ratio the production run range is divided into six regions e.g., P1 (Runs ≤ 2724), P2 (2724 > Runs ≤ 2745), P3 (2745 > Runs ≤ 2777), P4 (2777 > Runs ≤ 2838), P5 (2838 > Runs ≤ 3201), P6 (3201 > Runs).

Table 3.1

Production runs chosen for each period for local calibration

<table>
<thead>
<tr>
<th>Period Name</th>
<th>Run Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2518, 2675</td>
</tr>
<tr>
<td>P2</td>
<td>2732, 2735</td>
</tr>
<tr>
<td>P3</td>
<td>2758, 2760, 2761</td>
</tr>
<tr>
<td>P4</td>
<td>2793, 2795</td>
</tr>
<tr>
<td>P5</td>
<td>2878, 2926, 2941, 3058</td>
</tr>
<tr>
<td>P6</td>
<td>3203, 3223, 3215</td>
</tr>
</tbody>
</table>

Run 2518 is one of the production runs chosen for the P1 period. Figure 3.2 shows a three panel plot for the Unser rate in run 2518. The top plot shows the Unser rate (in Hz) vs time (in sec). Three beam off periods are fitted with polynomial zero fit and the fit line is shown in red horizontal lines. The seven beam on periods (which stayed...
Figure 3.2. The Unser rate vs average time for the SHMS run 2518 is shown in the top panel. The beam on periods are fitted with a straight line, shown in pink. The beam off periods are fitted with straight lines, shown in red. The middle plot shows the percent residual for the beam off and on periods. The two black lines show the ±1σ spread, for each period. The bottom panel shows the average Unser frequency vs time, along with the systematic error based on the spread of the frequency around the average value.

Relatively stable for a period of time) are also fitted with the polynomial zero and the fit lines are drawn in pink straight line. The Unser frequency \( f_{\text{Unser}} \) response to the beam current is calculated as

\[
f_{\text{Unser}} = f_{\text{Unser}}^{\text{ON}} - f_{\text{Unser}}^{\text{OFF Ave}}
\]  

(3.1.1)
where \( f_{\text{Unser}}^{\text{OFF ave}} \) is the average of the two off periods which are nearest to and at opposite sides of the on period. For a regular BCM calibration run there would be an off period between any two on periods. But for the production runs this is not available.

It was decided to apply an extra uncertainty due to possible unobserved drift of the Unser offset during the beam on periods. The middle panel of the figure 3.2 shows the percent residual vs the scaler time plot. The black horizontal lines show \( \pm 1\sigma \) for each of the off or on periods. This fluctuation of the percent residual were histogrammed and the standard deviation is chosen as an extra error. The third panel of Figure 3.2 shows the Unser off period frequencies in black point and the extra systematic error due to Unser fluctuation in red vertical line.

To get the Unser current \( (I_{\text{Unser}}) \), along with Unser frequency we also need the Unser gain \( (G_{\text{Unser}}) \). The Unser was calibrated separately by passing a known current through a wire which passes through the toroid. The Unser gain was calculated for four different times around the time period of the experiment.

As seen in the figure 3.3 the Unser gain is very stable and the average of the four measurement is used as the Unser gain for the BCM calibration. The Unser gain used is \( \text{Gain}_{\text{Unser}} = 0.0002491 \pm 2.594e - 07 \ \mu A/Hz \). Hence, \( I_{\text{Unser}} = f_{\text{Unser}} \times \text{Gain}_{\text{Unser}} \) is calculated for each of the beam on periods.

The BCM frequency \( (f_{\text{BCM}}) \) is also determined. The top panel of the figure 3.4 shows the BCM4A frequency vs scaler time plot. In the top plot here only the beam on periods were fitted with the polynomial 0 (pink horizontal lines). The fit parameter is chosen as the average BCM frequency for that particular on period.

This way the frequency were determined for each of the on period and for both the BCM4A and BCM4C. Then the \( f_{\text{BCM}} \) is plotted against the \( I_{\text{Unser}} \). In Figure 3.5 the \( f_{\text{BCM}} \) is plotted against the \( I_{\text{Unser}} \) for the BCM4C. There are three plots in total. The first plot shows the \( f_{\text{BCM}} \) vs \( I_{\text{Unser}} \). This is fitted with a straight line (2
Figure 3.3. The Unser gain vs run numbers. The Unser gain was calculated for the four different times during the production runs. The Unser gain was very stable during the production runs; and the Unser gain used for the BCM calibrations is the average of the four different measurements.

parameter fit). The parameter values are $p_0 = 2205 \pm 987.9$ and $p_1 = 6166 \pm 22.88$. Here $p_0$ is the offset ($\text{Offset}_{BCM}$) and $p_1$ is the gain ($\text{Gain}_{BCM}$) of the BCM4C. The left bottom panel shows the percent residual vs Unser current plot. The top right plot shows the residual vs Unser current plot. This way the BCM gain and offset were calculated for all the periods for BCM4A and BCM4C. BCM gains and offsets for all the periods for both the BCMs (4A and 4C) are shown in the tables 3.2 and 3.3.

The gains and offsets for all the six periods for both the BCMs are then updated in HCANA. Then all the runs are reanalyzed and the BCM4A and BCM4C currents ratio plotted vs run number.
Figure 3.4. The BCM rate vs time for the SHMS run 2518 is shown in the top panel. The bottom panel shows the Unser rate vs time.

Table 3.2

The gain and offset for BCM4A for different periods

<table>
<thead>
<tr>
<th>BCM4A</th>
<th>gain</th>
<th>$\text{Error}_{\text{gain}}$</th>
<th>offset</th>
<th>$\text{Error}_{\text{offset}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>12960</td>
<td>48.07</td>
<td>3969</td>
<td>2076</td>
</tr>
<tr>
<td>P2</td>
<td>13313.4</td>
<td>118.198</td>
<td>-19725.4</td>
<td>6413</td>
</tr>
<tr>
<td>P3</td>
<td>13520</td>
<td>37.35</td>
<td>1847</td>
<td>1197</td>
</tr>
<tr>
<td>P4</td>
<td>12772.4</td>
<td>63.541</td>
<td>10551.1</td>
<td>3750.87</td>
</tr>
<tr>
<td>P5</td>
<td>13223.8</td>
<td>71.3652</td>
<td>-3377.44</td>
<td>3836.28</td>
</tr>
<tr>
<td>P6</td>
<td>13128.4</td>
<td>110.657</td>
<td>-1982.1</td>
<td>5738.13</td>
</tr>
</tbody>
</table>
Figure 3.5. BCM calibration plots for the period P1, which includes data from SHMS runs 2518 and 2675. The top left panel shows the BCM4C frequency vs Unser current. The top right panel shows the residual vs Unser Current and the bottom left panel shows the % residual vs Unser Current.

### 3.1.1 Charge Noise Estimation

As discussed in section 3.1, the main two BCMs used for charge measurement were BCM4A and BCM4C. Study showed that these two BCMs disagreed with each other by $\sim 2\%$. But due to the absence of any other instrument for reliable independent charge measurement, the cause of disagreement couldn’t be pinned down to the “fault / drift” of a single BCM. Keeping that in mind, the charge error was calculated. In this section the process of calculating the charge error will be discussed.
Table 3.3

*The gain and offset for BCM4C for different periods*

<table>
<thead>
<tr>
<th>BCM4C</th>
<th>gain</th>
<th>Error$_{gain}$</th>
<th>offset</th>
<th>Error$_{offset}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>6165.56</td>
<td>22.88</td>
<td>2205.16</td>
<td>987.8</td>
</tr>
<tr>
<td>P2</td>
<td>6380</td>
<td>56.56</td>
<td>-8286</td>
<td>3069</td>
</tr>
<tr>
<td>P3</td>
<td>6522</td>
<td>18.01</td>
<td>1088</td>
<td>577.5</td>
</tr>
<tr>
<td>P4</td>
<td>6140.17</td>
<td>30.55</td>
<td>7733</td>
<td>1803</td>
</tr>
<tr>
<td>P5</td>
<td>6439.98</td>
<td>58.33</td>
<td>-1386.78</td>
<td>2811</td>
</tr>
<tr>
<td>P6</td>
<td>6238.48</td>
<td>52.6</td>
<td>-243.23</td>
<td>2727</td>
</tr>
</tbody>
</table>

For this experiment, the final quantity of interest was the scattering cross section. The charge normalized yield is proportional (CNY) to the scattering cross section. In principle, the CNY should be same for all the runs if the target and kinematics are same for all the runs. Considering that the beam energy, angles, efficiencies are known with reasonable confidence, any non-statistical fluctuations in CNY (from run to run for particular target and kinematics) should be attributed to the fluctuations in charge calculation.

Only the SHMS kinematics settings had enough runs for a significant statistical analysis are for angles 39 degrees and 33 degrees. The CNY was calculated and plotted vs run numbers for the kinematic settings: (39°, 2.5 GeV), (39°, 2.0 GeV), (33°, 3.2 GeV), and (33°, 2.6 GeV) for both the BCMs.

Figure 3.6 shows the CNY vs run numbers for the hydrogen target at the setting (39°, 2.5 GeV). The left plot corresponds to BCM4C and right plot corresponds to BCM4A. They are fitted with polynomial 0 fit to get the average CNY. The same method was applied for all the above mentioned kinematic settings to get the average CNY. The $\chi^2$/ndf for all the settings and two BCMs are listed in Table 3.4. The $\chi^2$/ndf is the measure of how much the CNY fluctuates from run to run in a single
setting. As shown in Table 3.4, BCM4A was fluctuating more than BCM4C for two settings, and reverse was true for the other two settings. This proved there was no preferred BCM as far as stability was concerned.

Table 3.4

$\chi^2/\text{ndf}$ corresponds to Figure 3.6 for different settings

<table>
<thead>
<tr>
<th>setting</th>
<th>$\chi^2/\text{ndf}$ (BCM4C)</th>
<th>$\chi^2/\text{ndf}$ (BCM4A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$39^\circ$, 2.5 GeV</td>
<td>50.69/31</td>
<td>51.64/31</td>
</tr>
<tr>
<td>$39^\circ$, 2.0 GeV</td>
<td>43.78/20</td>
<td>42.53/20</td>
</tr>
<tr>
<td>$39^\circ$, 3.2 GeV</td>
<td>28.85/14</td>
<td>32.94/14</td>
</tr>
<tr>
<td>$39^\circ$, 2.6 GeV</td>
<td>46.1/10</td>
<td>40.92/10</td>
</tr>
</tbody>
</table>

With earlier mentioned reasoning, the fluctuation of CNY can be attributed to the fluctuation in charge calculation. Hence in the next step it was checked, how much minimum charge error was needed, to get the $\chi^2/\text{ndf} \approx 1$ for a single setting.
Figure 3.7. The CNY vs run numbers for both the BCMs at exactly the same kinematics as in Figure 3.6. Only difference is 0.5% of error added to the charge for each run here. The $\chi^2$/ndf becomes $\sim 1$ after adding 0.5% error to the charge.

Figure 3.7 shows the CNY vs run number plots for BCM4A and BCM4C, for the kinematic setting ($39^\circ$, 2.5 GeV), with added error to charge. The minimum error for the charge were chosen in such a way, so that the $\chi^2$/ndf $\approx 1$. The same study was done for all the kinematic settings and for BCM4C, 0.5% of additional random error was needed to get $\chi^2$/ndf $\approx 1$. Hence the charge error for the analysis was selected as 0.5%.

3.2 Detector Calibration

In this section I will discuss the calibration of the spectrometer detectors. The calibrations of drift chamber, cerenkov and calorimeter will be discussed in this chapter.
3.2.1 Drift Chamber Calibration

The pair of drift chambers is the tracking detector in the spectrometer. The calibration of the drift chamber will be discussed in two parts. First, optimization of the tracking parameters will be discussed and then the calibration procedure will be discussed.

3.2.1.1 Optimization of Tracking Parameters

The SHMS drift chamber was a newly installed tracking device in Hall C. Hence it was important that the software and the hardware be consistent with each other. One important part of that was to update the tracking parameters in HCANA. For that it was important to understand the tracking algorithm in detail.

The drift chamber provides spatial information about the tracks using the drift times and known positions of the wires inside the chamber. The tracking algorithm first looks for pairs. A pair is a crossing of two wires in the same chamber. If two wires which got hits and the angle between them is more than 17.5°, they form a pair. By this condition \((x, x'), (u, u')\) or \((v, v')\) wires will not make a pair. In the figure 3.8 the \((x, y)\) positions of the pairs are shown in blue open circles for a particular event \((19^{th})\) of the run number 2484 for SHMS. In the next step the combos are formed; a combo is a pair of pairs. For example in Figure 3.8 the two pairs shown in the green open boxes form a combo. The centroid of that combo is shown by the blue diamond situated at the middle of the two green boxes. In HCANA these centroids of combos are actually read from a C++ list. The centroid of the first combo of the list is considered as the space point. The center of a space point is shown in the Figure 3.8 by the red circle. The space point criteria is set as 1.2 cm. That means that the centroid of any combo falling within 1.2 cm of the origin of the space point is considered: part of the same space point. In Figure 3.8 the circumference of the
Figure 3.8. The (x,y) positions of the different tracking parameters for the SHMS run number 2484 and event number 19. The open blue circles are the pairs. The green boxes shows a pair of pairs which form a combo. The green diamonds shows the centroids of all the combos formed for the event 19. The red circle is the center of the first space point for med in the event 19. The black curve shows the circumference of the space point. The red crosses are the centroids of those combos which are the part of the same space point.

The detail of the optimization study of space point criteria can be found here [17], [50]. The algorithm then loops over all the space points in single chamber and links
them to form tracks for that drift chamber, known as stub. To resolve the left right ambiguity the wires in similar kind of planes are shifted by 0.5 cm [2.6.2.1]. The stubs are fitted for all the possible wire combinations (wires which helps to form the space point), and the stub (track inside a single chamber) with best $\chi^2$ is chosen. The $\chi^2$ is calculated as

$$\chi^2 = \frac{DC_{WC} - DC_{tc}}{\sigma_{DC}^2},$$

where, $DC_{WC}$, $DC_{tc}$, and $\sigma_{DC}$ are wire coordinate, track coordinate, and wire chamber resolution for each plane.

The stubs from different chambers are linked to form the final tracks. The stubs from different chambers are linked if the positions ($X_{stub}$, $Y_{stub}$) and angles ($X_{p stub}$, $Y_{p stub}$) fall within a range. In general many tracks can be formed and are recorded with the corresponding $\chi^2$, then the track with the lowest $\chi^2$ is chosen as the final track.

Tracking efficiency is very sensitive to these stub fit parameters: $X_{stub}$, $X_{p stub}$, $Y_{stub}$, and $Y_{p stub}$. Hence stub parameters are need to be optimized. Apart from that I also optimized the parameter maximum hits per chamber. In the next paragraph I will discuss the parameterization process for the maximum hits per chamber.

The tracking algorithm will not continue with the track fitting unless the number of hits per chamber is less than the maximum hits per chamber criteria. Figure 3.9 shows the normalized Number of electrons / tracking efficiency vs maximum hits per chamber criteria for SHMS run 2548.

While doing this study all other parameters are kept constant at these values: $X_{stub} = 25$ cm, $X_{p stub} = 0.7$ rad, $Y_{stub} = 12.5$ cm, $Y_{p} = 0.2$ rad. The number of electrons are counted by total number of events passing the PID cuts. The green triangles are tracking efficiency normalized to 1. The blue triangles are the number of electrons/tracking efficiency normalized to 1. If the maximum hits per chamber criteria is set very low it is possible to loose the real tracks which is evident from the
Figure 3.9. The number of electrons/tracking efficiency vs maximum hits per chamber is shown in blue triangles. The green triangles shows the tracking efficiency vs maximum hits per chamber.

Figure 3.9 as tracking efficiency drops for very low value of the parameter. But if the criteria is set very high there is a high chance to include the noise or ghost tracks. Hence the parameter value is conservatively set right where the tracking efficiency is starting to reach its maximum value.

In figure 3.10 the number of electrons/ tracking efficiency is plotted against the X stub criteria. As discussed for the previous track parameter here also tracking efficiency drops for very low value of X stub as real tracks are eliminated. Where as the very high value of X stub gives the maximum tracking efficiency but noise / ghost tracks can be included with the real tracks. So the X stub criteria can be set right where the tracking efficiency is getting flat.

In the figure 3.11 the number of electrons/ tracking efficiency is plotted against the Xp stub criteria. Using the same logic used before Xp stub criteria can be set right where the tracking efficiency is getting flat.
Figure 3.10. The number of electrons/tracking efficiency vs X stub criterion is shown in blue triangles. The green triangles shows the relative tracking efficiency on the vertical axis.

Figure 3.11. The number of electrons/tracking efficiency vs Xp stub criterion is shown in blue triangles. The green triangles shows the relative tracking efficiency on the vertical axis.
Figure 3.12. The number of electrons/tracking efficiency vs Y stub criterion is shown in blue triangles. The green triangles shows relative tracking efficiency on the vertical axis.

The Y stub criteria is also parameterized in the same way as discussed above. The figure 3.12 shows the tracking efficiency vs Y stub criteria and the optimum value is chosen where the tracking efficiency is just reached the highest value.

In HCANA the tracking algorithm is written such a way that, the dependence of tracking efficiency on Yp can be turn on or off. During the data analysis the Yp dependence was turned off, and Yp stub criteria was fixed at 0.2 rad.

It is important to notice that these tracking parameter optimization study is valid for low rate runs, which is the case for this experiment. More about the rate dependent optimization study can be found here [51].

3.2.1.2 Drift Chamber Calibration Procedure

When a charged particle passes through the drift chamber, the drift gas gets ionized along the particle track. Due to the electric field, the electrons produced by the ionization drift towards the sense wires. As these electrons reach very near to the
Table 3.5

The optimized values of the tracking parameters

<table>
<thead>
<tr>
<th>tracking parameters</th>
<th>parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum hits per chamber</td>
<td>25</td>
</tr>
<tr>
<td>X stub criteria</td>
<td>25 cm</td>
</tr>
<tr>
<td>Xp stub criteria</td>
<td>0.7 rad</td>
</tr>
<tr>
<td>Y stub criteria</td>
<td>7 cm</td>
</tr>
<tr>
<td>Yp stub criteria</td>
<td>0.2 rad</td>
</tr>
</tbody>
</table>

sense wires, an avalanche takes place and induces a current signal in the sense wires. These current signals are amplified and discriminated by the discriminator cards and then sent to the TDCs (time to digital converter) by the ribbon cables. This time is delayed by the time needed for the signal to go through length of the sense wire and ribbon cable. For the calibration, we are interested in the quantity called drift time. The Drift time is defined as the time needed for the signal to travel from where the ionization takes place in the cell to the sense wire. The TDCs are look back TDCs, and a window open with the trigger and all times are measured within that time window, and all times are measured relative to the TDC reference ($t_R$) as a common stop. Hence the drift time can be expressed as,

$$t_D = (t_M - t_R) - [(t_W + t_C) - t_R],$$  \hspace{1cm} (3.2.2)

where $t_M$, $t_W$, and $t_C$ are time measured by TDC, travel time of the signal through the signal wire, and travel time for the signal through the ribbon cables, respectively.

Hence the first step of the drift chamber calibration is to fix the $t_0$ correction of the drift time distribution. As showed in Figure 3.13a, the uncorrected drift time distribution (SHMS, run 2528, plane 1U2, and card number 3) does not start from $t = 0$. The left rising part of the distribution is fitted with a straight line as showed in
The region which is fitted is determined as the 20% of maximum bin to ∼60% of the maximum bin. Then the red line is extrapolated to the Y=0 and the distance of the corresponding X position from X=0 is the $t_0$ correction. The assumption for this study is that all the cells are uniformly illuminated with a large number of events.

Figure 3.13. The (a) $t_0$ uncorrected drift time distribution and (b) $t_0$ corrected drift time distribution, for card 3 of plane 1U2 of the Drift Chamber for the SHMS run number 2528. The X axis shows the drift time in ns, and Y axis is the number of events/ns. The red line then extrapolated to Y=0. The $t_0$ correction is the distance between the X=0 position and the X position for Y=0 of the extrapolated red line.

Then whole distribution is shifted by the $t_0$ correction and so that the starting of the distribution starts from $t = 0$ as shown in Figure 3.13b. By careful observation we can see that the drift time distribution is spanned from 0 ns to ∼125 ns; where the $t = 0$ ns corresponds to the cell center or the sense wire position and $t = 125$ ns corresponds to the cell boundary or the nearest field wire position which is 0.5 cm apart. The average drift velocity of electron in the Argon-Ethane (50:50 by volume) is ∼50 µm/ns.

During the analysis, the drift time distribution is plotted for all the wires of a single plane, for a run of large number of events. The purpose of using a run with large number of events is to minimize the statistical error. After $t_0$ correction of the drift time, the next step is to generate a time to distance mapping. If $F(t)$ is the drift
time distribution, the time to distance map is given by

\[ D(t) = D_{\text{max}} \frac{\int_{t_{\text{min}}}^{t_{\text{max}}} F(t) dt}{\int_{t_{\text{min}}}^{t_{\text{max}}} F(t) dt}. \]  \hspace{1cm} (3.2.3)

Figure 3.14. Drift distance distribution.

Where \( D_{\text{max}} \) is the maximum possible distance a particle can come from to the sense wire or half a cell width (0.5 cm), \( t_{\text{max}} \) and \( t_{\text{min}} \) are the range of times included in the fit. As shown in Figure 3.14, the drift distance converted from drift time distribution is flat.

At high rate there are always chances of accidental events which cause the tracking inefficiency. To reduce these accidental events a timing window cut from -14000 to -10000 is applied, on the raw TDC time. For drift chamber 1 the timing cut window is shown in Figure 3.15.

3.2.1.3 Tracking Efficiency Error

In this section, I will describe the method of calculating the SHMS tracking efficiency error. As shown in Figure 3.16 (top panel) the tracking efficiency for all the production runs were plotted against the hodoscope’s S1X plane rate. The tracking
Figure 3.15. The timing cut for the DC raw TDC channels, for drift chamber 1. The different colors depict different planes of drift chamber 1. The conversion from TDC channel to time is $\sim 0.1 \text{ ns} / \text{channel}$.

efficiency varies with the rate. The variation was parameterized by fitting this plot with polynomial 1 (shown in red line in the top panel of Figure ). Then the percentage residuals were then calculated for all the runs; and histogrammed as shown in the bottom panel of Figure ?? . The standard deviation of the percentage residual plot was found to be 0.2%. Hence the SHMS tracking efficiency error was considered as 0.2 % as a global error for all the runs in this experiment.
Figure 3.16. The SHMS tracking efficiency error vs S1X rate, for all the production runs, is shown at the top plot. The red line is the polynomial 1 fit to the tracking efficiency. On the bottom panel the percentage residual is histogrammed and plotted.

3.2.2 Target Density Correction

The normalized scattered electron yield needs to be corrected for the liquid target density. In principle during the experiment the target density should be constant. In reality, when the electron beam passes through the target, the liquid heats up and hence the density changes locally. This heating effect is mitigated by the beam rastering system and the rapidly circulating fluid, but is not negligible. We assume
here the density change is directly proportional to the beam current. To estimate the target density change during the experiment, a luminosity scan was performed for the liquid hydrogen, liquid deuterium and carbon and the data was analyzed.

A luminosity scan is a set of runs performed at a particular kinematics (to eliminate any variation due to physics from run to run) but at different beam current values. For the experiment E12-10-002, several runs (eight runs for LH2, seven runs for LD2 and 3 runs for carbon) were taken at 25° and 4.4 GeV at the current values ranges from 10-77 µA. After all the corrections are applied, then the charge normalized yield will only vary with the target density. The Charge normalized yield is calculated three ways for this study: (1) the charge normalized yield for total number of HMS pre-trigger scaler counts, (2) the charge normalized yield without using tracking information, (3) the charge normalized yield using the tracking information.

The charge normalized yield is plotted against the average current. Then the plot is fitted with a straight line. The slope of the fit depicts the target boiling effect. The reasonable assumption is there is no boiling effect at 0 µA current, so whole plot can be normalized to the Y intercept (say $y_0$). Hence,

$$Y = M \cdot I + Y_0$$

or, $$\left(\frac{Y}{Y_0}\right) = \left(\frac{M}{Y_0}\right) \cdot I + 1.$$

So, the relative yield loss is given by $(m/y_0) \times I$, where $m$ is the slope. It is customary to express the relative yield loss as per 100µA, which is given by: $(100 \times m/y_0)$.

Figure 3.17 shows the target density plot for the C12 target. Carbon is a solid target, hence the target “boiling” effect is minimal. This is depicted in the very low value of the slope in the Figure 3.17. The boiling effect for the carbon target for
Figure 3.17. The charge normalized yield (Y axis) vs average current (X axis in $\mu$A), for carbon target. The blue line corresponds to the tracking yield. The red and black line corresponds to the non tracking and the scaler yield respectively. This figure is produced by W. Henry.

Tracking, non tracking, and scaler charged normalized yield are $(1.177 \pm 3.136)/100\mu$A, $(1.003 \pm 2.747)/100\mu$A, and $(1.908 \pm 2.553)/100\mu$A respectively.

Figure 3.18 shows the target boiling plot for the liquid hydrogen target. LH$_2$ is a liquid target, hence the density change is larger than for carbon. This is depicted in the larger value of the slope in the Figure 3.18 compared to carbon. The boiling effect for the LH$_2$ target for tracking, non tracking, and scaler charged normalized yield are $(3.84 \pm 0.7311)/100\mu$A, $(3.732 \pm 0.6492)/100\mu$A, and $(3.403 \pm 0.626)/100\mu$A respectively. The boiling estimated calculated in three different ways for LH$_2$ are same the within the uncertainties. For the cross section results, the tracking efficiency corrected slope is used.

The LD$_2$ runs can be separated into two periods. Before 22 March 2021, the target temperature was at 22 K; after 22 March 2021 it was at 22.4 K. The luminosity runs for the experiment were taken when the target temperature was at 22.4 K. The
Figure 3.18. The charge normalized yield (Y axis) vs BCM4B average current (X axis in $\mu A$), for liquid hydrogen target. The blue line corresponds to the tracking yield. The red and black line corresponds to the non tracking and the scaler yield respectively. This figure is produced by W. Henry.

plot for the LD$_2$ luminosity runs at 22.4 K is shown in Figure 3.19. The tracking, non tracking and scaler charge normalized yield slopes are $(4.295 \pm 0.8239)/100\mu A$, $(4.0 \pm 0.7365)/100\mu A$, and $(3.911 \pm 0.6941)/100\mu A$ respectively. In the second period (after 22 March 2021) for the production runs LD$_2$ also was 22.4 K, hence the tracking yield boiling correction used as shown in Figure 3.19. But what we were interested to know the yield at 22 K. The LD$_2$ density changes approximately as 1.5%/K. So ,for the 0.4 K change in temperature the density change is 0.6%. Hence during analysis for the second period the yield of LD$_2$ was increased by 0.6%.

But for the production runs before 22 March 2021, the target temperature was at 22 K. For this period D.Mack’s boiling correction study was used [29]. For this study he used the luminosity runs of 2018 Fall and for all those runs the target temperature was at 22 K. For this period the charge normalized yield for LD$_2$ was $(2.84 \pm 3.2%)/100\mu A$. 
Figure 3.19. The charge normalized yield (Y axis) vs BCM4B average current (X axis in $\mu$A), for liquid deuterium target. BCM4B is used because, for the high current values BCM4A can saturates sometimes. The blue line corresponds to the tracking yield. The red and black line corresponds to the non tracking and the scaler yield respectively. This figure is produced by W. Henry.

In this context it is important to discuss the error due to the target density. Table 5.1 shows the different factors which contributes to the total target density error for both the hydrogen and deuterium liquid target. The total percent error for the LH$_2$ is 0.60% and for the LD$_2$ is 0.63%.
### Table 3.6

**Target density error from different elements**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Error</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>19K</td>
<td>±182mK</td>
<td>0.27%</td>
</tr>
<tr>
<td>Pressure</td>
<td>25psia</td>
<td>±2psia</td>
<td>0.02%</td>
</tr>
<tr>
<td>Equation of state</td>
<td></td>
<td></td>
<td>0.1%</td>
</tr>
<tr>
<td>Target length</td>
<td>100mm</td>
<td>±0.26mm</td>
<td>0.26%</td>
</tr>
<tr>
<td>Precision of length measurement</td>
<td>100mm</td>
<td>±0.26mm</td>
<td>0.26%</td>
</tr>
<tr>
<td>Target Contraction</td>
<td>99.6%</td>
<td>±0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>beam Position</td>
<td>0</td>
<td>±3mm</td>
<td>0.2%</td>
</tr>
<tr>
<td>Average boiling correction LH2(LD2)</td>
<td></td>
<td></td>
<td>0.30%(0.36%)</td>
</tr>
</tbody>
</table>

**Total Error LH2(LD2)** 0.60%(0.63%)

### 3.2.3 Ionization Energy Loss

In this experiment when the beam electrons pass through the target, they lose their energy by ionization before they interact with target nucleus. This is an electromagnetic interaction between the beam electrons and the atomic electrons. This ionization energy loss causes a reduction in the beam energy from its value (10.602 GeV) measured at the Hall C arc. In this section we will estimate the energy loss of the beam electrons. The ionization energy loss effect is governed by the Bethe-Bloch formula

\[
-\frac{dE}{dX} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z z^2}{\beta^2} \left[ \ln \left( \frac{2m_e \gamma^2 \nu^2 W_{max}}{I^2} \right) - 2\beta^2 \right]
\]

\[(3.2.4)\]

where,

- \( N_a \): Avogadro’s Number = 6.022 \times 10^{23} \text{ mol}^{-1}
- \( r_e \): classical electron radius = 2.817 \times 10^{-13} \text{ cm}
- \( m_e \): electron mass
- \( c \): velocity of light
- \( \rho \): density of absorbing material
• $Z$: atomic number of absorbing material
• $A$: atomic weight of absorbing material
• $z$: charge of incident particle (in unit of e)
• $\beta$: $v/c$ of the incident particle
• $\gamma$: $\frac{1}{\sqrt{1-\beta^2}}$
• $I$: mean excitation potential
• $W_{\text{max}}$: maximum energy transfer in single collision

$W_{\text{max}} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e / M + (m_e / M)^2}$
• $M = $ mass of the incident particle

In practice, the mean excitation potential ($I$) is very hard to calculate as the oscillator strength is unknown for most of the materials. Hence experimentally $dE/dx$ is determined first and then $I$ is calculated by fitting the points with a semi-empirical formula for $I$ and $Z$. Mean excitation potential as adopted by ICRU (International Commission on Radiation Units and Measurements) are shown in the following figure 3.20. The Bethe-Bloch formula can be modified for the electrons. The formula is

![Figure 3.20. Mean excitation potential energy ($I$) divided by $Z$ plotted against $Z$ as adopted by ICRU. [22]](image-url)
given by

\[
\frac{dE}{dX} = \frac{1}{2} K \frac{Z}{A} \frac{m_e c^2 \beta^2 \gamma^2 \{m_e c^2 (\gamma - 1)/2\}}{I^2} + (1 - \beta^2) \frac{2\gamma - 1}{\gamma^2} \ln 2 + \frac{1}{8} \left(\frac{\gamma - 1}{\gamma}\right)^2 - \delta
\]

(3.2.5)

where \( K = 4\pi N_A r_e^2 m_e c^2 = 0.307075 \text{ MeV mol}^{-1} \text{ cm}^2 \), \( \delta \) = density effect correction to ionization energy loss. While implementing the energy loss in Monte Carlo, we assumed that the interaction occurs at the center of the target length. In reality, almost half of the scatterings will occur in the first half of the target length and the rest of the scatterings will occur in the last half. So we will underestimate the energy loss for half of the scatterings and overestimate it for the other half, although on average the mean energy loss will be correctly estimated.

![Figure 3.21. Ionization Energy Loss vs \( \delta E \). The blue curve represents the ionization energy loss for a particle heavier than the electron and red curve represents the same for the electron.](image)

The Bethe-Bloch formula described by equations 3.2.4 and 3.2.5 are for the mean energy loss. In reality the ionization energy loss is not a delta function, but rather a distributed around the mean energy loss. The reason behind this is the statistical
fluctuations in the number of collisions and the energy transferred in each collision. This distribution of energy depends on the thickness of the target material. The parameter which defines the range of applicability of different distribution functions is

\[ \kappa = \frac{\tilde{\Delta}}{W_{\text{max}}} \]  

(3.2.6)

where, \( \tilde{\Delta} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} (\frac{\tilde{\rho}}{\rho})^2 x \) = mean energy loss (an approximation of the Bethe-Bloch formula by ignoring the logarithmic term). \( z \) = charge of incident particle in the units of \( e \), \( x \) = thickness of the material.

To implement the correction, the energy loss is sampled \((f)\) for each of the events. For \( \kappa \leq 0.001 \) (thin absorbers) the energy loss distribution for the sample is a Landau distribution and can be approximately expressed as

\[ \delta E = Kx(f + 0.422784 + \beta^2 + \ln(Kx/W_{\text{max}})) + dE_{\text{mean}} \]  

(3.2.7)

For \( 0.001 < \kappa \leq 12.0 \) (thin absorbers) the energy loss distribution for the sample is Vavilov or Vavilov-Landau distribution and can be approximately expressed as

\[ \delta E = (Kx * ((f/\kappa) + 0.422784 + \beta^2)) + dE_{\text{mean}} \]  

(3.2.8)

For \( 0.001 < \kappa > 12.0 \) (thick absorbers) the energy loss distribution Landau distribution of the sample can be approximately expressed as

\[ \delta E = (f * \frac{Kx \sqrt{1 - \beta^2/2}}{\sqrt{\kappa}}) + dE_{\text{mean}} \]  

(3.2.9)
3.2.4 $\delta p/p$ Correction

3.2.4.1 HMS $\delta p/p$ Correction

The reconstructed variables for the data collected in this experiment were compared with MC to check the quality of the data. By studying the DATA-MC comparison plots for the HMS reconstructed quantity $\delta p/p$, we noticed a systematic variation. This systematic variation is observed in different angles, momenta and targets. The presence of the same variation in different kinematics for different length targets suggests that the effect is not due to physics but for wrong effective delta reconstruction. In this section I will discuss the method used to estimate the $\delta p/p$ correction.

**HMS normalized cross section ratio vs $\delta$**

*Figure 3.22.* The DATA/MC charge normalized yield ratio vs reconstructed $\delta p/p$ (in %). For each $\delta$ bin, averaged sum of runs from different kinematics and different targets (carbon, hydrogen and deuterium) was included. The red curve is the polynomial 3 fit to the red points with error. The assumption for the study is that there is no correction needed for the $\delta = 0$ bin. Hence the red curve is normalized in such a way that DATA/MC = 1 at $\delta = 0$, and we get the black curve.
First the \( DATA/MC \) charge normalized yield vs reconstructed \( \delta p/p \) is plotted, for all the kinematics and the targets together as shown in Figure 3.22. In this study the assumption is that at the bin \( \delta = 0 \), no correction is needed. Then the the first bin on the left (or right) of the central bin is checked. Let’s consider the \( DATA/MC = r \) at the center of \( n^{\text{th}} \) \( \delta \) bin. Also, \( n > 1 \) for the bins right to the \( \delta = 0 \) bin, and \( n < 1 \) for the bins left to the \( \delta = 0 \) bin. Then the outer edge of the first bin needs to be shifted outside by an amount

\[(1 - r) \times 100.0 \% \text{ for } r < 1 \text{ or,} \]

\[(r - 1) \times 100.0 \% \text{ for } r > 1.\]

Then the consecutive bins were checked and corrected by the same way. Hence by changing the outer bin edges the events were redistributed. The correction is mapped and then the correct \( \delta \) (\( \delta_{\text{real}} \)) is calculated from reconstructed \( \delta \) (\( \delta_{\text{reconstructed}} \)) by the formula

\[
\delta_{\text{real}} = \begin{cases} 
\delta_{\text{reconstructed}} - \left( \frac{\text{Yield}_{DATA}}{\text{Yield}_{MC}} - 1 \right) \text{ if } \frac{\text{Yield}_{DATA}}{\text{Yield}_{MC}} > 1 \\
\delta_{\text{reconstructed}} - (1 - \frac{\text{Yield}_{DATA}}{\text{Yield}_{MC}}) \text{ if } \frac{\text{Yield}_{DATA}}{\text{Yield}_{MC}} < 1 
\end{cases}
\]  

(3.2.10)

The \( \delta \) correction (\( \delta_{\text{corr}} \)) is mapped and fitted with a 5\(^{\text{th}}\) order polynomial which can be expressed as

\[
\delta_{\text{corr}} = 0 + (0.00013075950)\delta_{\text{recon}} + (-0.00052778790)\delta_{\text{recon}}^2 + \\
(5.981100e - 05)\delta_{\text{recon}}^3 + (8.6922000e - 06)\delta_{\text{recon}}^4 + \\
(-1.9570000e - 07)\delta_{\text{recon}}^5
\]  

(3.2.11)

Figure 3.23 shows the \( \delta \) correction plotted against the reconstructed \( \delta \). Figure 3.24 shows the DATA/MC for charged normalized yield vs true \( \delta p/p \) after the correction. In the lower \( \delta \) region, the DATA/MC ratio is almost flat whereas in the high \( \delta \) region
Figure 3.23. $\delta_{\text{correction}}(= \delta_{\text{reconstructed}} - \delta_{\text{true}})$ is plotted against $\delta_{\text{recon}}$ and fitted with a 5$^{th}$ order polynomial as shown in equation 3.2.11.

The ratio is within the $\pm 0.5\%$ of 1. This plot demonstrates that DATA is corrected within the $\pm 0.5\%$ of MC and all the delta dependence is removed.
3.2.4.2 SHMS $\delta p/p$ Correction

In this experiment, for each SHMS angular setting there are four momentum settings. Each central momentum chosen has a bite of -10% to +22% in $\delta$ acceptance. These central momenta were chosen in such a way that there is a kinematic overlap between two consecutive momenta settings. As for a particular kinematic and target, the cross-section should be same regardless of the momentum. But even after careful study, it is found that for the SHMS the cross-section from two different central momentum settings is not matching in the overlapping region.

The ratio of normalized $\sigma^{data}/\sigma^{model}$ for both the targets (hydrogen and deuterium) and for all the kinematics are plotted vs $\delta$ in the Figure 3.25. Then for each $\delta$ bin the averaged sum was calculated and plotted as shown in Figure 3.26.
Figure 3.25. The DATA/MC charged normalized yield ratio vs real $\delta p/p$ (in %). Runs from different kinematics and different targets was included.

It is clear from this figure that cross-sections corresponding to all central central momenta setting shows a general $\delta$ dependence behaviour. This general behaviour is parameterized by fitting it with a third degree polynomial. The polynomial function is given by

$$(\frac{\sigma_d}{\sigma_m})_{normalized} = 1.002 + (-0.002473)\delta_{recon} + (-1.546e-05)\delta_{recon}^2 + (6.64e-06)\delta_{recon}^3.$$  \hspace{1cm} (3.2.12)

Hence the correction factor is given by $1/(\frac{\sigma_d}{\sigma_m})_{normalized}$ and need to be multiplied with the extracted cross section. After the cross section was corrected the percentage residual was calculated and histogrammed as shown in the Figure 3.27. Based on the standard deviation of the histogram in Figure 3.27, a systematic error of 0.4% was estimated for the cross section due to the SHMS $\delta p/p$ correction.
**Figure 3.26.** The DATA/MC charged normalized yield ratio vs real \( \delta p/p \) (in %). For each \( \delta \) bin, averaged sum of runs from different kinematics and different targets (carbon, hydrogen and deuterium) was included.

**Figure 3.27.** After the \( dp/p \) correction SHMS, the quantity \( 100 \times (\text{Data-Fit})/\text{Fit} \) was histogrammed. As indicated by the standard deviation the data matches with the model within \( \sim 0.44\% \).
3.2.5 Systematic due to Forward Transport Matrix

In this section I will discuss about the estimation of the systematic error due to uncertainty in the SHMS forward transport matrix. As mentioned in section 2.6.1, SHMS optical system is consists of three quadruples (Q1, Q2, Q3) and two dipoles (D, HB). For this study the positions and strengths of the magnets were changed in the monte carlo and the cross section was calculated. The cross section for the nominal positions and strengths of the magnets was $\sigma_{\text{nominal}}$. Then the fractional change in the cross section compared $\sigma_{\text{nominal}}$ was calculated and parameterized.

Among all the magnets first the strength of the SHMS Q1 magnet was checked. The SHMS Q1 magnet strength in forward transport magnet is changed by 0.3\% \((1.003 \times B.dl_{\text{nom}}^{Q1})\) for the model. Then the cross section is calculated for both the nominal Q1 strength and then with the 0.3\% change in Q1 strength. As shown in Figure 3.28 fractional change in cross section \((\frac{\sigma_{1.003 \times B.dl_{\text{nom}}^{Q1}} - \sigma_{\text{nominal}}}{\sigma_{\text{nominal}}})\) is plotted vs $x$, for 21 degree and hydrogen target.

![Figure 3.28. The fractional change in cross section vs $x$ for hydrogen at 21 degree. Different color indicated the different central momentum setting.](image)
Figure 3.29. The fractional change in cross section vs $x$ for deuterium at 21 degree. Different color indicated the different central momentum setting.

Figure 3.30. The fractional change in cross section vs $x$ for carbon at 21 degree. Different color indicated the different central momentum setting.
Each of the central momentum setting in shown in different color. First it was tried to parameterized in $x$. So, the fractional change in cross section is fitted with parameter 2 function. The corresponding plots for the LD2 and carbon are shown in the Figures 3.29 and 3.30, respectively. The fractional change was also parameterized in cross section in $x$ for all other kinematic settings in SHMS, for all three targets. More details about the parameterization in $x$ can found here [10]. After careful inspection of the figures plotted vs $x$, it is understood that it would be easier to parameterize the fractional change in cross sections in $\delta$.

Figure 3.31. The fractional change in cross section vs $\delta$ (top panel), for all kinematics of LH2, LD2 together. The black line is the polynomial 5 fit the data. The bottom panel shows the residual (= fit-data) plot for the top panel plot.
So, in the next step fractional change in cross section is plotted against $\delta$ for all the kinematics and targets together, and for both the targets LH2 and LD2 together as shown in Figure 3.31. Then the fraction change in cross section vs $\delta$ plot is parameterized with a polynomial 5,

$$
\frac{\sigma_{1.003 \times B \cdot dl} - \sigma_{\text{nominal}}}{\sigma_{\text{nominal}}} = (0.0003619) - (6.701e - 05)\delta
+ (1.919e - 07)\delta^2 + (6.077e - 06)\delta^3 - (2.26e - 07)\delta^4.
$$

(3.2.13)

This $\pm \left( \frac{\sigma_{1.003 \times B \cdot dl} - \sigma_{\text{nominal}}}{\sigma_{\text{nominal}}} \right)$ in equation 3.2.13, are used as an extra systematic error in cross section for both LH2 and LD2.

The positions and strengths of the other magnets were also studied in the same way, but the fractional change in the cross section was very negligible compared to the total error estimation for the cross section.
CHAPTER 4
EXTRACTION OF DIFFERENTIAL SCATTERING CROSS-SECTIONS

The method used to extract the differential scattering cross-section in this analysis is known as **Monte-Carlo ratio method**. In this section I will discuss the Monte-Carlo ratio method. The number of electrons \( N_{e^-} \) in each bin of \( \Delta E', \Delta \Omega \) can be expressed as

\[
N_{e^-} = \left[ L \cdot \frac{d\sigma}{d\Omega dE'} (\Delta \Omega \Delta E') \cdot \epsilon_{tot} \cdot A(E', \theta) + BG \right] / PS. \quad (4.0.1)
\]

Where,

- \( L(\equiv \frac{Q}{e} \cdot \frac{N_A t}{A}) \) is the integrated luminosity with
  - \( Q \) is the total charge,
  - \( e \) is the electron charge,
  - \( N_A \) is the Avogadro’s Number,
  - \( \rho \) is the target density,
  - \( A \) is the atomic mass number, and
  - \( t \) is the target length.

- \( \frac{d\sigma}{d\Omega dE'} (\equiv d\sigma) \): Differential scattering cross-section,

- \( \epsilon_{tot} \): Total efficiency including detection and the PID cuts,

- \( A(E', \theta) \): Acceptance for the bin \((E', \theta)\), and

- \( BG \): Number of background events,

where \( PS \) is prescale factor. If the rate of the experiment is too high for the DAQ to handle, the DAQ does not record all the the events but every \( n^{th} \) event.
Hence, the efficiency corrected yield can be defined as

\[
Y(E', \theta) = \frac{P_S \times N_{e^-}}{\epsilon_{tot}} - BG = L \cdot \frac{d\sigma}{d\Omega dE'}(\Delta\Omega dE') \cdot A(E', \theta). \tag{4.0.2}
\]

The efficiency corrected yield can also be predicted based on a model of the cross section. Let’s consider \(Y_d\) and \(Y_{MC}\) as the efficiency corrected Born yield from data and model, respectively. Then \(Y_d\) and \(Y_{MC}\) can be expressed as

\[
Y_d = L \cdot \sigma_d(\Delta\Omega dE') \cdot A(E', \theta) \times RC_d \tag{4.0.3}
\]

and

\[
Y_{MC} = L \cdot \sigma_m(\Delta\Omega dE') \cdot A_{MC}(E', \theta) \times RC_{MC}, \tag{4.0.4}
\]

where \(RC\) is the radiative correction and will be discussed in detail in section 4.2. If the radiative correction model is accurate, \(RC_d = RC_{MC}\). Hence the ratio of the equations 4.0.3 and 4.0.4 is

\[
\frac{Y_d}{Y_{MC}} = \frac{L \cdot \sigma_d(\Delta\Omega dE') \cdot A(E', \theta)}{L \cdot \sigma_m(\Delta\Omega dE') \cdot A(E', \theta)} \cdot RC_{MC} \tag{4.0.5}
\]

To the extent that the spectrometer is accurately modeled then \(A(E', \theta) = A_{MC}(E', \theta)\), and if the luminosity(\(L\)) is same in both denominator and numerator of the equation 4.0.5, then

\[
d\sigma_d = d\sigma_m \times \frac{Y_d}{Y_{MC}}. \tag{4.0.6}
\]
4.1 Determination of backgrounds

As indicated in the equations 4.0.1 and 4.0.2, background events need to be subtracted from the number of electron events detected by the spectrometers. The main sources of the backgrounds in this experimental setting are:

- electrons scattered from the target window,
- electrons produced by charge symmetric processes, and
- misidentified \( \pi^- \) particles.

These background sources, and the elimination of each, will be discussed in the next three subsections.

4.1.1 Background from Al Target Windows

The Hall C cryogenic target is discussed in Section 2.5. When the beam hits the target, electrons scatter from the entrance and exit windows as well as from the LH\(_2\) or LD\(_2\). In principle window background events could be excluded by \( Y_{\text{targ}} \) cuts, but this is impractical for reasons of efficiency, offset shifts, limited resolution, and the need to precisely know the target length in equation 4.0.1. During the experiment in order to determine the fraction of events originating from the target window, data were taken with a dummy aluminum target at the same kinematic settings \((E, E', \theta)\) as the cryogenic target. The idea was to estimate the background yield from the target cell by scaling the dummy target yield to the same luminosity as the cryogenic data. The dummy target cell walls are almost 6 times thicker than the windows in the cryogenic target cell, to reduce the data taking time. The detailed geometry of the target cell was presented in section 2.5.

In general the luminosity, efficiency, prescale, radiative correction factor, etc. will be different for the dummy and cryo runs. So after selecting the electrons with PID cuts, the dummy run counts need to be scaled to the cryo run via the scale factor is
defined

\[ \text{scale factor} = \frac{PS \times N_{\text{end cap}}}{PS_{\text{dummy}} \times N_{\text{dummy}}}. \]  (4.1.1)

Writing the yield in terms of the luminosity \( (L) \), differential cross-section \( (d\sigma) \) and radiative correction factor \( (rc) \) as

\[ Y = L \times \frac{\sigma}{rc}, \]  (4.1.2)

and using equations 4.1.1, 4.1.2, the scale factor can be written as

\[ \text{scale factor} = \frac{L_{\text{end cap}}(\sigma_{\text{al}}/rc_{\text{end cap}})}{L_{\text{dummy}}(\sigma_{\text{al}}/rc_{\text{dummy}})}, \]  (4.1.3)

where \( L_{\text{end cap}}, L_{\text{dummy}}, rc_{\text{end cap}}, \) and \( rc_{\text{dummy}} \) are the width of the cryo cell window, width of the dummy window respective, radiative correction for cryo cell window, and radiative correction for the dummy window, respectively. As the cryo cell and dummy windows are made of same material only the external radiative correction is important here and \( rc_{\text{end cap}} \) and \( rc_{\text{dummy}} \) can be replaced by \( rc_{\text{end cap}}^{\text{ext}} \) and \( rc_{\text{dummy}}^{\text{ext}} \), respectively.

Using equation 4.0.2, and writing the luminosity in terms of \( Qe^- \), \( N_A, \rho, A, \) and \( t \), the scale factor can be written as

\[ \text{scale factor} = \frac{(Q \cdot \epsilon_{\text{tot}}/PS)_{\text{end cap}} t_{\text{end cap}} rc_{\text{end cap}}^{\text{ext}}}{(Q \cdot \epsilon_{\text{tot}}/PS)_{\text{dummy}} t_{\text{dummy}} rc_{\text{dummy}}^{\text{ext}}}. \]  (4.1.4)

To understand the effect due to the differences in external bremsstrahlung, the ratio \( (rc_{\text{dummy}}/rc_{\text{end cap}}) \) vs \( \delta \) was plotted for all kinematic settings. Figure 4.1 shows the plots for the typical kinematic setting: 21 degrees, 3.3 GeV. It was observed that \( (rc_{\text{dummy}}/rc_{\text{end cap}}) < 3.0\% \), for both upstream or downstream, and for all kinematics. It was also observed that the end cap contribution to the yield was \( \approx 10.33\% \) and \( 5.11\% \), for hydrogen and deuterium, respectively. Hence the maximum effect due to
this factor is 0.3% (~3% of 10.33%) and 0.15% (~3% of 5.11%), for hydrogen and deuterium respectively. Hence, for the $x$ region we are interested in, this factor can be ignored as the effect was much smaller than the final total error (2-3%).

Apart from the radiative effects, the proper treatment of the factor $t_{\text{end cap}}/t_{\text{dummy}}$ is important due to geometrical differences between the entrance and exit window. In the table 4.1, we summarize the target length ratio for both the entrance and exit window positions. The entrance and exit windows were separately dealt with rather than considering an average ratio.

### Table 4.1

<table>
<thead>
<tr>
<th>loop</th>
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<th>exit window ratio</th>
</tr>
</thead>
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<tr>
<td>2</td>
<td>$2.321 \times 10^{-1}$</td>
<td>$2.957 \times 10^{-1}$</td>
</tr>
<tr>
<td>3</td>
<td>$2.012 \times 10^{-1}$</td>
<td>$2.911 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

*Figure 4.1.* $r_{\text{dummy}}/r_{\text{end cap}}$ distribution vs $\delta$ for the SHMS kinematic setting; 21 degrees, 3.3 GeV. The left and right panel plot corresponds to upstream and downstream, respectively.
Figure 4.2. SHMS $Y_{\text{tar}}$ distribution at 21 degrees for the hydrogen target cell. Shown is the simulated yield from the cryogen (green histogram), the data yield before (red) and after (blue) subtraction of the backgrounds. Also shown is the dummy target yield (yellow) scaled to the luminosity seen by the end caps as discussed in the text.

To consider the entrance and exit windows separately, the cryogenic cell geometry is divided into three parts based on the $Y_{\text{tar}}$ quantity as shown in Figure 4.2. For the region $Y_{\text{tar}} \leq -0.5$ cm, the end cap to dummy thickness ratio was considered to be equal to entrance ratio where as for the region $Y_{\text{tar}} \geq 0.5$ cm , the Al target length ratio will be equal to exit ratio. This method will ensure the consideration of the target length difference for the entrance and exit windows. Now for the region $-0.5$ cm $> Y_{\text{tar}} < 0.5$ cm, linear interpolation between entrance and exit ratios is used.

### 4.1.2 Charge Symmetric Background

Another source of the background in this experiment is the processes leading to charge symmetric background (CSB). In this section, processes that result in equal number of electrons and positrons originating from target and the subtraction method will be discussed.

Process of external bremsstrahlung in the target material can produce a photon which can later convert into an electron-positron pair ($\gamma \rightarrow e^-e^+$) in the electric field of a nucleus. In addition, neutral pions are produced in the inelastic scattering process. These neutral pions can later produce the electron-positron pairs ($\pi^0 \rightarrow$...
\( \gamma \gamma^* \rightarrow \gamma e^- e^+ \). The spectrometer cannot distinguish between the electrons produced in pair production from the scattered electrons, hence they need to be estimated and subtracted from the total yield. Because these are charge symmetric processes the total number of CSB electrons is assumed the same as CSB positrons. By changing the polarity of the spectrometer and evaluating the positron yield for a particular kinematic setting, the number of CSB electrons can be estimated. Due to the time constraint, CSB data were collected for only three (21\(^\circ\), 29\(^\circ\), 39\(^\circ\)) of the five SHMS angles.

**Figure 4.3.** Measured cross section in \( nb/\text{sr}/\text{GeV} \) vs \( E' \) from analysis of the JMU group [40] for the three spectrometer angle settings (indicated by the different colors). Also shown is a fit to the cross section, as discussed in the text.

As indicated in Figure, 4.3 the positron cross-sections from 21\(^\circ\), 29\(^\circ\), and 39\(^\circ\) were parameterized with the formula

\[
\frac{d\sigma}{d\Omega dE'} = e^{p_0} (e^{p_1(E-E')} - 1). \tag{4.1.5}
\]

The values of \( p_0 \) and \( p_1 \) constants for each angle are shown in the legend of Figure 4.3. The \( \theta \) dependence of \( p_0 \) and \( p_1 \) was parameterized with a first order polynomial
as shown in the Figure 4.4 and Figure 4.5, e.g. $p_0(\theta) = -2.09\theta + 12.47$ and $p_1(\theta) = 0.2\theta - 0.6338$, providing a complete 2-dimensional parameterization in $E'$ and $\theta$.

\textit{Figure 4.4.} The $p_0$ values, extracted from Figure 4.3, fitted with a first order polynomial. (From the analysis of the JMU group [40].)

\textit{Figure 4.5.} The $p_1$ values, extracted from Figure 4.3, were fitted with a first order polynomial. (From the analysis of the JMU group [40].)

4.1.3 $\pi^-$ Background

In the data analysis, electrons were selected by the particle identification (PID) cuts, i.e. calorimeter and cerenkov cuts. For the SHMS Noble Gas Cerenkov detec-
tor the cut was \( N_{cer} > 2.0 \) and for the calorimeter cut was \( E_{calo}/E' > 0.7 \), where \( N_{cer} \) is the number of photoelectrons and \( E_{calo} \) is the total energy deposited in the calorimeter. Even after the PID cuts, some pions may sneak into the electron yield via the production of a knock-on electron (or delta ray) which fires the gas Cerenkov. Alternatively a \( \pi^- \) may undergo the charge exchange reaction, where it produces a neutral pions at the front of the calorimeter which then decays as \( \pi^0 \rightarrow \gamma + \gamma \) and results in an EM shower. If the \( \pi^- \) charge exchanges in the first few cm, nearly the total energy of the neutral pion can be deposited in the calorimeter. In general, the process to determine the pion background was:

1. Select the electrons from calorimeter spectrum with a cerenkov cut \( N_{cer} > 2.0 \) (Green histogram in the Figure 4.6),

2. Select pions with a cut \( N_{cer} = 0.0 \),

and then scale pion spectrum to match the pion contamination in the electron distribution near the average energy loss (around \( E_{calo}/E' = 0.5 \) at the kinematics shown

---

*Figure 4.6. Example histogram of normalized calorimeter energy. (From analysis of A.Sun.)*
in Figure 4.6) by charged pions through ionization. Following this method of calculating the pion background, it was found that the calorimeter energy distribution for $N_{cer} = 0$ (pion spectrum) showed a peak at $E_{calo}/E' = 1$ indicating some electrons did not fire the cerenkov and sneaked into the calorimeter spectrum. After a detailed study, the following cuts were chosen to exclude the electrons from the pion distribution:

$$0.9 < E_{calo}/E' < 1.2$$

(4.1.6)

$$E_{preshower} > 0.3$$

(4.1.7)

$$E_{calo}/E' - E_{preshower}/E' > 0.01$$

(4.1.8)

**Figure 4.7.** Fractional pion contamination determined and plotted vs $E'$ for hydrogen, deuterium, and aluminum respectively at $21^\circ$, by Abel Sun. From left to right the blue, orange and brown solid lines with an error band represent the fit to the data up to 4 GeV. The red solid line represents the systematic error.

Figure 4.6 shows, the pion distribution corresponding to before (blue histogram) and after (orange histogram) the cuts defined in Equations 4.1.6, 4.1.7, and 4.1.8. The pion contamination fraction is then given by the ratio of the areas under the orange and the green histograms, respectively for $E_{calo}/E' > 0.7$. As shown in Figure 4.7 the pion rejection factor is then plotted against $E'$ and fitted as

$$\frac{\pi}{e} = p_0 e^{p_1 E'}/E'^{p_2}.$$  

(4.1.9)
Table 4.2

Fit parameters for pion contamination for different targets and angles (from A. Sun).

<table>
<thead>
<tr>
<th>target</th>
<th>angle</th>
<th>( p_0 )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>hydrogen</td>
<td>21</td>
<td>4.49937e+00</td>
<td>1.75345e+00</td>
<td>1.17137e+01</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>5.74647e-01</td>
<td>-3.73983e-01</td>
<td>4.36376e+00</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>3.93917e-01</td>
<td>-4.71598e-01</td>
<td>4.18430e+00</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>8.55560e+00</td>
<td>-5.18789e+00</td>
<td>-4.72197e+00</td>
</tr>
<tr>
<td></td>
<td>39</td>
<td>1.66675e-01</td>
<td>2.99433e-01</td>
<td>5.98419e+00</td>
</tr>
<tr>
<td>deuterium</td>
<td>21</td>
<td>1.59403e+00</td>
<td>-1.38128e+00</td>
<td>1.59377e+00</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>1.50256e+00</td>
<td>-2.79117e-01</td>
<td>5.03933e+00</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>1.25565e+00</td>
<td>-1.93012e+00</td>
<td>7.61512e-01</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>2.64807e+00</td>
<td>-2.73395e+00</td>
<td>4.35966e-02</td>
</tr>
<tr>
<td></td>
<td>39</td>
<td>4.00041e-01</td>
<td>5.51476e-03</td>
<td>5.43935e+00</td>
</tr>
<tr>
<td>aluminum</td>
<td>21</td>
<td>2.91226e-01</td>
<td>-3.30604e-01</td>
<td>3.40186e+00</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>4.49401e-01</td>
<td>9.87751e-01</td>
<td>7.56522e+00</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>1.30350e-01</td>
<td>3.06715e-01</td>
<td>4.64607e+00</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>8.07092e-02</td>
<td>1.04335e+00</td>
<td>6.21548e+00</td>
</tr>
<tr>
<td></td>
<td>39</td>
<td>7.49856e-03</td>
<td>3.35537e+00</td>
<td>8.82817e+00</td>
</tr>
</tbody>
</table>

as shown in Figure 4.7, for hydrogen, deuterium, and aluminum at 21°. But the fitting is basically done separately for all different angular settings and targets. The fit parameters are listed in Table 4.2.

For the final analysis the yield was corrected for the pion contamination as

\[
\text{corrected yield} = \left[ \text{yield} \times \left( 1 - p_0 e^{p_1 E'} \frac{E'}{E_{\gamma 2}} \right) \right].
\]  

(4.1.10)
4.2 Radiative Correction

The born cross-section is the first order process in $\alpha$, where $\alpha$ is the fine structure constant. We are interested in calculating the born cross-section of the nucleon-electron interactions. In reality there are other higher order processes in $\alpha$ taking place as shown in Figure 4.8. The higher order processes in $\alpha$ includes two types of radiative processes- a) Internal Radiative process and b) External Radiative Process.

The internal radiative processes are calculable in QED and can be of these types listed below:

1. Vacuum Polarization : creation of electron-positron pair
2. Vertex correction : Emission and reabsorption of virtual photons
3. Bremsstrahlung : Emission of photons in the nuclear field during the scattering process

![Figure 4.8](image)
4. Multiphoton Emission:

Other than the internal radiative processes, there are processes that can occur before and after the actual scattering, e.g. in the target material, drift chamber windows, spectrometer windows etc. These are called the external processes. The external scattering processes include:

1. Low momentum transfer Bremsstrahlung

2. Ionization energy loss

Apart from these processes there is also the elastic tail that extends to the DIS region.

For this experiment the software used to calculate the radiative correction is in general known as rc-external. In this program the internal and external radiative corrections were calculated using Mo-Tsai formula and Bardin formalism respectively. The Born cross-section model were obtained from the program f1f221 by M.E. Christy. f1f221 is basically a fit to the previous resonance and DIS data. Each MC event was weighted by the radiated model for the true vertex kinematics. The weighting factor is defined as

\[
\frac{\sigma_{\text{mod}}}{\sigma_{\text{born}}} = \frac{\sigma_{\text{mod}}}{\sigma_{\text{rad}}}
\]

(4.2.1)

where, \(\sigma_{\text{mod}}\) is the radiative model cross-section which includes elastic, quasielastic and inelastic contributions.

\[
\sigma_{\text{rad}} = \sigma_{\text{rad-Elastic}} + \sigma_{\text{rad-QuasiElastic}} + \sigma_{\text{rad-Inelastic}}
\]

(4.2.2)

where, \(\sigma_{\text{born}}\) is the model born cross-section from the f1f221 program.
4.3 Application of Fiducial and PID cuts

The good electrons in the experiment are defined as the electrons scattered from the target. Sometimes there can be some unwanted electrons in the spectrometer acceptance due to the scattering from the magnets or the exit window. Also there are some regions in the spectrometers where the acceptance is not well determined. Hence to select only the good electrons, some common cuts were implemented in both data and MC. The cuts for the SHMS are shown in Table 4.3.

<p>| Table 4.3 |</p>
<table>
<thead>
<tr>
<th>List of Acceptance and PID cuts for SHMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fiducial Cuts for SHMS</strong></td>
</tr>
<tr>
<td>$-10.0 , cm &lt; y_{tar} &lt; 10.0 , cm$</td>
</tr>
<tr>
<td>$-0.1 , \text{radians} &lt; y_{tar} &lt; 0.1 , \text{radians}$</td>
</tr>
<tr>
<td>$-0.1 , \text{radians} &lt; x'_{tar} &lt; 0.1 , \text{radians}$</td>
</tr>
<tr>
<td>$-10.0 , % &lt; \delta &lt; 22.0 , %$</td>
</tr>
<tr>
<td><strong>PID Cuts for SHMS</strong></td>
</tr>
<tr>
<td>$N_{cer} &gt; 2.0 , \text{photo electrons}$</td>
</tr>
<tr>
<td>$E_{calo}/E' &gt; 0.7$</td>
</tr>
<tr>
<td><strong>Current Cut for SHMS</strong></td>
</tr>
<tr>
<td>$I_{BCM \ 4C} &gt; 5.0 , \mu A$</td>
</tr>
</tbody>
</table>

For the hydrogen target, to get rid of the elastic peak, an extra $W^2 > 1.2 \, \text{pion threshold cut}$ was implemented for the SHMS kinematic ($\theta = 21^\circ, E'_{central} = 5.1 \, GeV$) and HMS kinematic ($\theta = 21^\circ, E'_{central} = 5.7 \, GeV$). The cuts for the HMS are shown in Table 4.4. There is geometrical cut implemented on the acceptance by the octagonal collimator. During the experiment as the HMS cerenkov mirrors were partially broken, hence the $\delta$ cuts for HMS were reduced to (-6 to +9) from (-9 to +9).
Table 4.4

List of Fiducial and PID cuts for HMS

<table>
<thead>
<tr>
<th>Fiducial Cuts : HMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10.0 \text{ cm} &lt; y_{tar} &lt; 10.0 \text{ cm}$</td>
</tr>
<tr>
<td>$-0.1 \text{ radians} &lt; y_{tar} &lt; 0.1 \text{ radians}$</td>
</tr>
<tr>
<td>$-0.1 \text{ radians} &lt; x'_{tar} &lt; 0.1 \text{ radians}$</td>
</tr>
<tr>
<td>$-6.0 % &lt; \delta &lt; 9.0 %$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID Cuts : HMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{cer} &gt; 1.5 \text{ photo electrons}$</td>
</tr>
<tr>
<td>$E_{calo}/E' &gt; 0.7$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current Cut for HMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{BCM\ 4C} &gt; 5.0 \mu \text{A}$</td>
</tr>
</tbody>
</table>

4.3.1 PID Cut Efficiencies

In this experiment scattered electrons are detected and tracked by the spectrometer to determine the yield. But inside the target other negatively charged particles (e.g. muons and pions) can be produced. For this experiment other than electrons, mostly negative pions enter the spectrometer. Two detectors, a noble gas cerenkov and a calorimeter, are used to discriminate electrons from pions. To separate electrons from pions, two PID cuts are used $N_{cer} > 2.0$ (cerenkov cut) and $E_{calo}/E' > 0.7$ (calorimeter cut). While rejecting the pions, some of the good electrons may get rejected by these detectors, which causes an inefficiency in the detector and affect the yield. In this section this inefficiency will be discussed for cerenkov and calorimeter.

4.3.1.1 Cerenkov Cut Efficiency

The noble gas cerenkov is one of the two PID detectors in the SHMS. A $N_{cer} > 2.0$ (i.e. greater than 2 photo electrons) is used to get rid of the pions. But this cut may cause the rejection of some of the good electrons along with pions. To quantify this loss of good electrons, the cerenkov efficiency is calculated. The cerenkov efficiency
is defined as

$$
\epsilon_{\text{cer eff}} = \frac{N^e(\text{fiducial cuts} + E_{\text{calo}}/E' > 1.0 + N_{\text{cer}} > 2.0)}{N^e(\text{fiducial cuts} + E_{\text{calo}}/E' > 1.0)},
$$

(4.3.1)

where the fiducial cuts are defined in Table 4.3. In the equation 4.3.1, the denominator is the number of electrons chosen with fiducial cuts along with the tight calorimeter cut, $E_{\text{calo}}/E' > 1$. The numerator is the sample of electrons chosen by the same cuts as the denominator and $N_{\text{cer}} > 2.0$.

![Figure 4.9. Cerenkov efficiency vs $\delta$ fitted with a sixth order polynomial for cryo targets at 21°, 4.0 GeV central momentum. (From analysis of A.Karki.)](image)
As shown in Figure 4.9, $\epsilon_{\text{cer eff}}$ is plotted against $\delta$ at 21° and 4.0 GeV central momentum. For cryotargets the $\delta$ parameterization of the $\epsilon_{\text{cer eff}}$ at 21° worked well for the higher angular settings also. The yield was then corrected event by event for the cerenkov cut efficiency.

4.3.1.2 Calorimeter Cut Efficiency

In order to calculate the calorimeter cut efficiency, a “pure” sample of electrons needs to be selected without using the calorimeter. The calorimeter cut efficiency is defined as

$$
\epsilon_{\text{cal}} = \frac{N^{e^-}(\text{fiducial cuts} + N_{\text{cer}} > 10 + E_{\text{calo}}/E' > 0.7)}{N^{e^-}(\text{fiducial cuts} + N_{\text{cer}} > 10)},
$$

(4.3.2)

where the denominator of Equation 4.3.2 is for selecting the clean sample of electrons. The numerator counts how many electrons of the clean sample passes through the calorimeter cut. To get a very clean sample of the electrons, the $N_{\text{cer}}$ cut was raised up to $N_{\text{cer}} > 15$. But even for the very high $N_{\text{cer}}$ cuts, the $\epsilon_{\text{cal}}$ varied with $\delta$, indicating the electron sample selected in the denominator was not very clean. For more detail see Reference [25].

To overcome this problem, hydrogen elastic runs were used from another experiment KaonLT [2], leveraging the fact that hydrogen elastic peak is very well defined. For the elastic runs Equation 4.3.2 is reformulated as

$$
\epsilon_{\text{cal}} = \frac{N^{e^-}(\text{fiducial cuts} + N_{\text{cer}} > 10 + 0.9 < W^2 < 1.1 + E_{\text{calo}}/E' > 0.7)}{N^{e^-}(\text{fiducial cuts} + N_{\text{cer}} > 10 + 0.9 < W^2 < 1.1)},
$$

(4.3.3)

Figure 4.10 shows the calorimeter cut efficiency plotted vs $\delta$. To check the momentum dependence of the calorimeter cut efficiency, hydrogen elastic runs with different central momentum settings were chosen and the average efficiency was plotted, then parameterized against $E'$.

Figure 4.11 shows the calorimeter efficiency vs $E'$. 
Figure 4.10. Calorimeter cut efficiency plotted against $\delta$ for elastic runs.(From analysis of A.Karki.)

Figure 4.11. Calorimeter cut efficiency plotted against $E'$ for elastic runs.(From analysis of A.Karki.)
CHAPTER 5
RESULTS

In this chapter the differential scattering cross sections and $F_2$ structure function results for the proton and deuteron for the experiment E12-10-002 will be discussed. Then later on in this chapter, the duality averaging procedure results will described. The duality averaging procedure will be discussed for the world data as well as the data from E12-10-002.

5.1 Differential Cross Sections

In this section, the differential scattering cross section ($d\sigma/d\Omega/dE'$) results for the proton (p) and deuteron (d) will be discussed. Figure 5.1 shows differential cross section for the proton at SHMS angle 21.035°. The differential cross sections for the SHMS angles 24.980°, 28.990°, 32.975°, and 38.975° are shown in first, second, third, and fourth quadrant of Figures 5.2, respectively. For each angle there are two panels. The top panel shows the differential cross section (nb/sr/GeV) for the proton vs $x$. The red solid line is the F1F221 model, where the blue points are the data. The F1F221 model is a fit to the world data of inclusive cross sections, but does not include the data from this experiment. This model by M.E. Christy is valid for a wide range of nuclei for $W^2 < 30$ GeV$^2$ and $Q^2 < 30$ GeV$^2$.

The bottom panel for each angle, in each quadrant, shows the data to model ratio ($d\sigma_{\text{data}}/d\sigma_{\text{model}}$) vs $x$ along with the systematic error band for the ratio. For the SHMS, the LH$_2$ cross section data at $x = 0.2$ to 0.7 matches with the F1F221 model better than 3%. For the large $x$, the data does not match with the model very well. But this mismatch at the large $x$ is somewhat expected as not much data is available.
at large $x$ to constrain the model. This is one of the many reasons for this experiment.

**Figure 5.1.** Hydrogen differential cross section ($d\sigma/d\Omega/dE'$) plotted vs $x$, at the SHMS angle $\theta = 21.035^\circ$. In the top panel the red line is the F1F221 model and the blue points are the data. The bottom panel shows the data to model ratio ($d\sigma_{\text{data}}/d\sigma_{\text{model}}$) vs $x$ with the systematic error band for the ratio.

Figure 5.3 shows ($d\sigma/d\Omega/dE'$) for deuterium at SHMS angle 21.035°. Cross sections for the SHMS angles 24.980°, 28.990°, 32.975°, and 38.975° are shown in first,
Figure 5.2. Hydrogen differential scattering cross sections \( \frac{d\sigma}{d\Omega/dE'} \) plotted vs \( x \) for SHMS angles \( \theta = 24.98^\circ \), \( \theta = 28.99^\circ \), \( \theta = 32.975^\circ \), and \( \theta = 38.975^\circ \). In the top panel of each plot, the red line is the F1F221 model and the blue points are the data. The data to model ratio \( \frac{\sigma_{\text{data}}}{\sigma_{\text{model}}} \) vs \( x \) is plotted in the bottom panel along with the systematic error band for the ratio.

second, third, and fourth quadrant of Figures 5.4, respectively. For each quadrant plot there are two panels. The top panel shows the differential cross section for deuterium vs \( x \). The red solid line is the F1F221 model, while the blue points are the
data. The bottom panel shows the data to model ratio ($\sigma_{\text{data}}/\sigma_{\text{model}}$) ratio vs $x$, along with the systematic error band for the ratio.

Figure 5.3. Deuterium differential cross section ($d\sigma/d\Omega/dE'$) plotted vs $x$, at the SHMS angle $\theta = 21.035^\circ$. In the top panel the red line is the F1F221 model and the blue points are the data. The bottom panel shows the data to model ratio ($d\sigma_{\text{data}}/d\sigma_{\text{model}}$) vs $x$ with the systematic error band for the ratio.
Figure 5.4. Deuterium differential scattering cross sections \( (d\sigma/d\Omega/dE') \) plotted vs \( x \) for SHMS angles \( \theta = 24.98^\circ, \theta = 28.99^\circ, \theta = 32.975^\circ, \) and \( \theta = 38.975^\circ \). In the top panel of each plot, the red line is the F1F221 model and the blue points are the data. The data to model ratio \( (\sigma_{\text{data}}/\sigma_{\text{model}}) \) vs \( x \) is plotted in the bottom panel along with the systematic error band for the ratio.
5.2 Error Budget

In this section the error budget, which contributes to the total error of differential scattering cross section, for different quantities will be discussed. Table 5.1 shows the estimation of error for different quantities. As shown in the figures in the previous section there are two types of errors shown in the differential cross section plots. One is the point-to-point error shown along with the cross section points. Another is the systematic error shown at the bottom of the plot in black points. The different types of errors included in the point to point error are: 1) statistical error, 2) charge error, 3) target boiling, 4) live time error, 5) tracking efficiency error and 6) trigger efficiency error. The details of the charge error can be found in Section 3.1.1. The details of the tracking error can be found in section 3.2.1.3. The systematic error due to the trigger efficiency is found in the same way as tracking efficiency described in Section 3.2.1.3. The trigger efficiency error value is found to be 0.03%.

Table 5.1

<table>
<thead>
<tr>
<th>Systematic errors</th>
<th>hydrogen</th>
<th>deuterium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>0.75%</td>
<td>0.75%</td>
</tr>
<tr>
<td>Charge</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Calorimeter Eff</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Cerenkov Eff</td>
<td>0.3%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Pion contamination</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Kinematics</td>
<td>0.477-4.419%</td>
<td>0.507-3.226%</td>
</tr>
<tr>
<td>CSB</td>
<td>&lt;1.032%</td>
<td>&lt;1.183%</td>
</tr>
<tr>
<td>Acceptance</td>
<td>$1e^{-05}$-0.706 %</td>
<td>$1e^{-05}$-0.724 %</td>
</tr>
<tr>
<td>Rad Corr</td>
<td>1%-1.032%</td>
<td>1%-1.048%</td>
</tr>
<tr>
<td>Live Time</td>
<td>0.288-1.174 %</td>
<td>0.398-2.184 %</td>
</tr>
<tr>
<td>Tracking</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.03%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Total Error</td>
<td>1.482-4.75%</td>
<td>1.568-3.826%</td>
</tr>
</tbody>
</table>
The details of the target boiling error were already documented in Table 5.1. The total systematic errors due to the target boiling for LH$_2$ and LD$_2$ are 0.60% and 0.63% respectively. This is correlated between two different runs. But while calculating this, the average boiling correction error (first line of Table 5.1) is calculated corresponding to the global average current of all the production runs. The global average currents for all the LH$_2$ and LD$_2$ runs are 46.96 $\mu$A and 49.84 $\mu$A, respectively.

For this experiment the computer Live Time (CLT) for each individual run is calculated as

$$CLT = \frac{T \times PS}{S};$$  \hspace{1cm} (5.2.1)

where, PS is the prescale of the run, T is the total recorded triggers and S is the triggers recorded by the scaler. The Total Live Time (TLT) can be calculated from EDTM as shown above for CLT. But the same method couldn’t be used for the Electronic Live Time (ELT) because the statistical uncertainty associated with the ELT was very high. The EDTM rate at the time of the experiment was only 10 Hz. Hence TLT and CLT were determined first. Then the TLT/CLT was plotted against the hodoscope’s S1X plane rate for each run, and for each run the TLT/CLT vs S1X plane rate was fitted with a straight line (polynomial 1 fit). If the slope of the fit is $\tau$ the ELT is calculated as

$$ELT = 1.0 - (\tau \times \text{plane rate}).$$  \hspace{1cm} (5.2.2)

The $\tau$ value found for different runs for the SHMS is 37.1 $\pm$ 3.3 ns, and for HMS 23.8 $\pm$ 5.8 ns. It is found that even if the TLT/CLT vs S1X rate is fitted with polynomial 0 instead of polynomial 1, we can get a reasonable $\chi^2$/ndf with almost the same fit parameters. Hence 100% uncertainty was assigned to the ELT correction. This error is uncorrelated between one run to another. More detail about the error analysis can be found in Reference [24].
5.3 Extraction of Structure Function $F_2$

In this section the structure function $F_2$ results for hydrogen and deuterium will be discussed. Figure 5.5 shows the hydrogen $F_2$ for angle 21.035°. Hydrogen $F_2$ structure functions for SHMS angles 24.98°, 28.99°, 32.975°, and 38.975° are shown in first, second, third, and fourth quadrant of Figure 5.6, respectively.

![Hydrogen, SHMS, $\theta = 21.035^\circ$](image)

$3.39 \leq Q^2 \leq 8.25 \text{ GeV}^2$

- **model** (F1F221)
- **data**

*Figure 5.5. Hydrogen structure function $F_2$ plotted vs $x$, at the SHMS angle $\theta = 21.035^\circ$. In the top panel the red line is the F1F221 model and the blue points are the data. The bottom panel shows the data to model ratio ($F_{2\text{data}}/F_{2\text{model}}$) vs $x$ with the systematic error band for the ratio.*
Figure 5.6. Hydrogen structure function $F_2$ plotted vs $x$ for SHMS angles $\theta = 24.98^\circ$, $\theta = 28.99^\circ$, $\theta = 32.975^\circ$, and $\theta = 38.975^\circ$. In the top panel of each plot, the red line is the F1F221 model and the blue points are the data. The data to model ratio ($F_2^{\text{data}}/F_2^{\text{model}}$) vs $x$ is plotted in the bottom panel along with the systematic error band for the ratio.

Figure 5.7 shows the $F_2$ structure function for deuterium at SHMS angle 21.035$^\circ$. Deuterium $F_2$ for the other four SHMS angles 24.980$^\circ$, 28.990$^\circ$, 32.975$^\circ$, and 38.975$^\circ$ are shown in the Figure 5.8, respectively.
Figure 5.7. Deuterium structure function $F_2$ plotted vs $x$, at the SHMS angle $\theta = 21.035^\circ$. In the top panel the red line is the F1F221 model and the blue points are the data. The bottom panel shows the data to model ratio ($F_2^{\text{data}}/F_2^{\text{model}}$) vs $x$ with the systematic error band for the ratio.
Figure 5.8. Deuterium structure function $F_2$ plotted vs $x$ for SHMS angles $\theta = 24.98^\circ$, $\theta = 28.99^\circ$, $\theta = 32.975^\circ$, and $\theta = 38.975^\circ$. In the top panel of the each plot, the red line is the F1F221 model and the blue points are the data. The data to model ratio ($F_{2\text{data}} / F_{2\text{model}}$) vs $x$ is plotted in the bottom panel along with the systematic error band for the ratio.
5.4 Duality Average procedure

“Duality Averaging Procedure” is a new technique proposed by M. Eric Christy to investigate the quark-hadron duality. Previously this method was checked at the low $Q^2$ region. But now E12-10-002 experiment has enriched the world data by adding more data points at high $x$ and high $Q^2$ for both the DIS and the resonance regions. Hence the duality averaging process can be used to investigate the quark-hadron duality at even higher $Q^2$ with this method. In this chapter this process will be discussed and will it be shown how this sheds light on the quark-hadron duality.

The analysis of electron-nucleon scattering data from SLAC by Bloom and Gilman showed that the average of $F_2$ behaviour in the resonance region is the same as the $F_2$ structure function behaviour in the DIS region. The world data shows the average $Q^2$ dependence of the resonance region is the same as the $Q^2$ dependence of the DIS region. In Figure 5.9, the ratio of proton $F_2$ for the resonance curve over $F_2$ for the DIS curve ($F_{2\text{res}}/F_{2\text{DIS}}$) is plotted against $x$. Here proton data is used from different SLAC and JLab experiments. The three panels in Figure 5.9 correspond to three different $Q^2$: the top, middle and bottom panels correspond to $Q^2 = 0.5$ GeV$^2$, $0.875$ GeV$^2$ and $1.25$ GeV$^2$, respectively. We can see that the resonance structure wiggles around the $F_{2\text{res}}/F_{2\text{DIS}} = 1$ line, which indicates that the average $Q^2$ dependence of the resonance region is the same as the $Q^2$ dependence of the DIS region.

Another important observation from this plot is that a particular $x$ can see different peaks and dips of the resonance structures if we include enough different $Q^2$ values. For example in Figure 5.9, at $x = 0.4$ we see the left of the Delta peak for $Q^2 = 0.5$ GeV$^2$, right of the second resonance peak for $Q^2 = 0.875$ GeV$^2$ and dip between the second and third resonance peaks for $Q^2 = 1.25$ GeV$^2$. This indicates that for a particular $x$, if we average the resonance structure over enough $Q^2$ values, the DIS curve can be recovered “approximately”.
Figure 5.9. The ratio of the proton $F_2$ structure function from SLAC and JLab data to F2ALLM DIS fit plotted vs $x$. The resonance peaks and dips are seen to oscillate about unity, indicating that for a particular $x$ the average $Q^2$ dependence of the resonance region is the same as the $Q^2$ dependence of the DIS for the $Q^2$ values of 0.5 GeV$^2$ (top panel), 0.875 GeV$^2$ (middle panel), and 1.25 GeV$^2$ (bottom panel).

Before the duality averaging method is discussed in detail, it is important to check the kinematics coverage of the world data set for the DIS and resonance regions. Figure 5.10 shows the kinematic coverage of the world data set in ($Q^2$, $x$). The details of the experiments are written in the figure caption. The four black lines on the figure indicates the boundary of the different resonance regions.

The different resonance regions are defined by the $W^2$ values and are shown in Table 5.2. Depending on the availability of the resonance data, the entire $Q^2$ range of the world data set is divided into three sub ranges: (1) $1.5 \text{ GeV}^2 < Q^2 < 2.0 \text{ GeV}^2$ centered at $Q_c^2 = 1.25 \text{ GeV}^2$, (2) $2.0 \text{ GeV}^2 < Q^2 < 7.0 \text{ GeV}^2$ centered at $Q_c^2 = 4.5 \text{ GeV}^2$, and (3) $7.0 \text{ GeV}^2 < Q^2 < 14.0 \text{ GeV}^2$ centered at $Q_c^2 = 10.5 \text{ GeV}^2$. 
Figure 5.10. The kinematic coverage of the data utilized in the duality studies is shown in \((Q^2, x)\). Here the data sets include SLAC and JLAB experiments. The Whitlow reanalysis data sets which includes all different experiments from SLAC can be found here: www.slac.stanford.edu/exp/e140/. The experiments which are included in the JLAB data sets are: (a) E94-110, (b) E00-116, (c) E00-002, (d) E12-10-002. The old JLAB experiments’ data sets can be found here: https://hallcweb.jlab.org/resdata/database/. The black solid lines are the boundaries of the first, second, third, and fourth resonance regions. The \(W^2\) values which define the boundaries are given in Table 5.2.

For all of these three regions, the duality averaging procedure is checked. In the next subsections the procedure and results for the \(Q^2\) region (1) will be discussed in detail.
Table 5.2

$W^2$ (GeV$^2$) values defining the boundaries of the different resonance regions

<table>
<thead>
<tr>
<th>Resonance Regions defined by $W^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.3 &lt; $First Resonance$ &lt; 1.9$</td>
</tr>
<tr>
<td>$1.9 &lt; $Second Resonance$ &lt; 2.5$</td>
</tr>
<tr>
<td>$2.5 &lt; $Third Resonance$ &lt; 3.1$</td>
</tr>
<tr>
<td>$3.1 &lt; $Fourth Resonance$ &lt; 3.9$</td>
</tr>
</tbody>
</table>

5.4.1 Duality average for $0.5 \text{ GeV}^2 < Q^2 < 2.0 \text{ GeV}^2$

This region is the lowest $Q^2$ region among the three and has $Q_c = 1.25 \text{ GeV}^2$. The whole region is divided into 21 $Q^2$ bins with a bin width of 0.075 GeV$^2$. As described in Section 5.4, the average $Q^2$ dependence of the resonance region is the same as the $Q^2$ dependence of the DIS region. Hence for all 21 $Q^2$ bins, this $Q^2$ dependence in $F_2$ can be centered using the DIS model as

$$F_2^{\text{res}}(x, Q^2) = \frac{F_2^{\text{DIS}}(x, Q^2_c)}{F_2^{\text{DIS}}(x, Q^2)} \cdot F_2^{\text{res}}(x, Q^2), \quad (5.4.1)$$

where for the DIS model F2ALLM is used. This process takes the $Q^2$ dependence out of the resonance (and DIS) structure of $F_2$ for each $Q^2$ bin and bin centers them at the central $Q^2_c = 1.25 \text{ GeV}^2$. Figure 5.11 shows the proton structure function $F_2^p$ plotted against $x$. The green curve is the F2ALLM DIS curve for $Q^2_c = 1.25 \text{ GeV}^2$. Each of the black curves is the resonance model F1F221 for each of the 21 bins and bin centered to $Q^2_c = 1.25 \text{ GeV}^2$.

Careful investigation of Figure 5.11 reveals that for a particular $x$ value there are different dips and peaks from different resonance regions. Hence for a particular $x$ if we average over all the black curves we can approximately recover the green DIS curve. There is one important thing to investigate before that: for each $x$, what $W^2$
Figure 5.11. $F_2^P$ from model F1F221 plotted vs $x$ for $Q_c^2 = 1.25 \text{ GeV}^2$ with 21 bins between $Q^2$ of 0.5 and 2.0 GeV$^2$. The green line is the F2ALLM DIS curve for the proton $F_2$ for central $Q_c^2$. Each black curve is $F_2^{res}$ produced by the F1F221 model for each bin centered to $Q_c^2$ using Equation 5.4.1.

range is covered due to the contribution from different $Q^2$ bins? It is very intuitive that the larger the $W^2$ region is covered the more resonance structures are covered for a given $x$. The more dips and peaks of all the resonance structures are covered, the better chance of recovering the DIS curve after the averaging.

Until now only the model is discussed. But before the final duality averaging procedure is implemented, the model need to be corrected with the existing world data. Each of the black curves in Figure 5.11 needs to be corrected with the data which fall within the corresponding $Q^2$ bin. This correction procedure is shown in Figure 5.12. Figure 5.12 corresponds to the bin where the bin center is $Q_{bc}^2 = 1.17 \text{ GeV}^2$. The black line in the top plot is the F1F221 model for $Q_{bc}^2 = 1.17 \text{ GeV}^2$. Any
Figure 5.12. The bin centered $F_2^P$ data plotted vs $x$ corresponds to a particular bin of 21 bins (as described in Figure 5.11) with bin center $Q^2_{bc} = 1.17$ GeV$^2$. The solid line in the top plot shows the model F1F221 for $Q^2 = 1.17$ GeV$^2$.

Data lying in the bin $Q^2 = 1.17 \pm 0.075$ GeV$^2$ is then bin centered to $Q^2_{bc} = 1.17$ GeV$^2$ with the formula

$$F_2^{data}(Q^2_{bc}, x_{nom}) = F_2^{data}(Q^2_{data}, x_{data}) \times \frac{F_2^{res}(Q^2_{bc}, x_{nom})}{F_2^{res}(Q^2_{data}, x_{data})}, \quad (5.4.2)$$

where now $x_{nom}$ needs to be calculated from $W^2_{data}$ and $Q^2_{bc}$ as

$$x_{nom} = Q^2_{data} / (W^2_{data} + Q^2_{data} - M_P^2), \quad (5.4.3)$$
Figure 5.13. The data corrected model for $F_2^P$ plotted vs $x$ corresponds to one of the 21 bins in $Q^2$ (as described in Figure 5.11) with bin center $Q_{bc}^2 = 0.5$. The solid line shows the model F1F221 for $Q^2 = 0.5$ GeV$^2$ bin centered to the $Q_c^2 = 1.25$ GeV$^2$.

$M_P$ being the proton mass. The bottom plot of Figure 5.12 shows the percent residual defined as

$$\text{percent residual} = 100.0 \times \frac{F_2^{\text{data}}(Q_{bc}^2, x) - F_2^{\text{model}}(Q_{bc}^2, x)}{F_2^{\text{model}}(Q_{bc}^2, x)}. \quad (5.4.4)$$

For each of the $x$ bins the percent residual is fitted with a 0th order polynomial. If the fit parameter is $p0$, then the model at each $x$ bin is corrected by $p0$ as

$$\text{model corrected by data} = F_2^{\text{model}}(Q_{bc}^2, x) \times \left(1 + \frac{p0}{100.0}\right). \quad (5.4.5)$$
Figure 5.13 shows the F1F21 model for $Q^2 = 1.17$ GeV$^2$ in the black line, and the data corrected model for each of the $x$ bins. In this way the model $F_2$ is corrected by data for each of the 21 $Q^2$ bins. After correcting the model at each $x$ bin, the model $F_2$ can be bin centered at $Q^2_c = 1.25$ GeV$^2$ as described in Equation 5.4.1.

The next step is to average the $F^{res}_2$ for all the $Q^2$ for a particular $x$. Figure 5.14 shows the $F^{ave}_2$ points in red squares and the F2ALLM DIS curve in green. This plot needs to be discussed along with the information from the Figure 5.15.

Figure 5.14. The average $F^P_2$ plotted vs $x$. The red squares in the top panel show the per $x$ bin average of resonance data for different $Q^2$ corresponding to the black curves of Figure 5.11. The green line is the DIS curve for $Q^2_c$. The bottom plot shows the percent residual defined as $\frac{(F^{DIS}_2 - F^{ave}_2)}{F^{ave}_2} \times 100.0$. Where $F^{ave}_2$ is the average of the resonance data per $x$ bin.
Figure 5.15 shows the $W^2$ coverage for each $x$ bin. The four black lines show the resonance regions. This plot shows how effective the averaging procedure can be for a particular $x$. For example at $x = 0.4$, the data covers all the four resonance regions. At very low $x$, the data mostly covers the high $W^2$ and DIS region. As we go to higher and higher $x$, covering all the resonance regions requires a huge range in $Q^2$, as can be seen in Figure 5.10. As evident from Figure 5.15, at high $x$, the data does not cover all the resonance regions. Hence even after the averaging, at very large $x$ the accuracy in recovering the DIS curve could be poor. At the very low $x < 0.2$, the contribution is from only the DIS region and averaging $F_2$ there will of course recover the DIS curve.

Figure 5.15. The $W^2$ coverage plotted for the each $x$ bin. The horizontal black lines shows the first, second, third and fourth resonance regions defined by the $W^2$ values as shown in the table 5.2
Now going back to Figure 5.11, $F_2^{\text{ave}}$ is plotted for $0.2 \leq x \leq 0.6$. For $0.2 \leq x \leq 0.5$, the DIS curve is recovered better than 3% by averaging the resonance structures. For $x = 0.55$ and $x = 0.6$, as we can see from Figure 5.15 not enough resonances are covered and $W^2$ crosses the pion production threshold limit. Once the pion production threshold is crossed, duality is not expected to hold. Due to this effect the average $F_2$ drops below the DIS scaling curve as shown in the bottom panel of Figure 5.14.
5.4.2 Duality average for $2.0 \text{ GeV}^2 < Q^2 < 7.0 \text{ GeV}^2$

Figure 5.16. The average $F_2^p$ plotted vs $x$ corresponds to $Q_c^2 = 4.5 \text{ GeV}^2$. The red squares in the top plot shows the per $x$ bin average of resonance data for different $Q^2$. The green line is the DIS (F2ALLM) curve for $Q_c^2$. The bottom plot shows the percent residual defined as $\frac{(F_2^{DIS} - F_2^{ave})}{F_2^{ave}} \times 100.0$, where $F_2^{ave}$ is the average of the resonance data per $x$ bin.

The duality averaging procedure was already described in the 5.4.1. The results for the procedure for $2.0 < Q^2 < 7.0 \text{ GeV}^2$ will be discussed in this section. Figure 5.16 shows the duality averaging plots in the top panel, and the bottom panel shows the percent residual. As described earlier, this plot needs to be assessed along with the $W^2$ coverage plot shown in Figure 5.17.
Figure 5.17 shows the $W^2$ coverage for each $x$ bin for the full range of data $0.1 < x < 0.95$. For $0.1 \leq x \leq 0.4$, $W^2$ only covers the DIS region, whereas for $0.45 \leq x \leq 0.85$, at least two resonance regions are covered.

![Figure 5.17](image)

**Figure 5.17.** The $W^2$ coverage plotted for each $x$ bin, which corresponds to $Q^2_c = 4.5$ GeV$^2$. The horizontal black lines shows the first, second, third and fourth resonance regions defined by the $W^2$ values as shown in Table 5.2.

Now, going back to Figure 5.16, the $x$ coverage shown is $0.5 \leq x \leq 0.9$. For $0.5 \leq x \leq 0.85$, the DIS scaling curve is recovered to better than 5%. For $x = 0.9$ only the 1st resonance (dominated by the $\Delta$) is covered, and we can see the $\frac{F_{ave} - F_{DIS}}{F_{DIS}} \times 100$ drops down to -28%. That means for $x = 0.9$, where only the $\Delta$ resonance is covered, and crossing the pion production threshold, the DIS curve cannot be accurately recovered.
5.4.3 Duality average for $7.0 \text{ GeV}^2 < Q^2 < 14.0 \text{ GeV}^2$

Figure 5.18. The average $F_2^P$ plotted vs $x$ corresponds to $Q_c^2 = 10.5 \text{ GeV}^2$. The red squares in the top plot show the per $x$ bin average of resonance data for different $Q^2$. The green line is the DIS (F2ALLM) curve for $Q_c^2$. The bottom plot shows the percent residual defined as $\left(\frac{F_{2,\text{DIS}} - F_{2,\text{ave}}}{F_{2,\text{ave}}}\right) \times 100.0$. Here $F_{2,\text{ave}}$ is the average of the resonance data per $x$ bin.

In this section the duality averaging results for $0.7 \leq Q^2 \leq 14.0 \text{ GeV}^2$ will be discussed. Figure 5.18 shows the duality averaging results for this range, and Figure 5.19 shows the $W^2$ coverage for the whole $x$ range. As shown in Figure 5.19, below $x = 0.7$ only DIS region is covered. For $0.75 \leq x \leq 0.9$ at least two resonance regions are covered.

Figure 5.18 shows $x$ coverage from 0.6 to 0.9. For $0.6 \leq x \leq 0.8$, we can see the DIS matches with the average resonance curve better than 5%. Then for the last two
$x$ points of 0.85 and 0.9 the average of resonance curve is higher than that of the DIS. The reason is, in this kinematic region the F1F221 model is not constrained. The data from the current experiment is not included in the F1F221 model. From Figure 5.5, 5.6 it is evident that at very high $x$ the data is systematically higher than the model. Hence, when corrected by data model, the average resonance value becomes higher than the DIS scaling curve.

\[ W^2 \]

**Figure 5.19.** The $W^2$ coverage plotted for each $x$ bin, which corresponds to $Q^2_C = 10.5$ GeV$^2$. The horizontal black lines shows the first, second, third and fourth resonance regions defined by the $W^2$ values as shown in Table 5.2.
5.5 Summary

Inclusive electron-proton and electron-deuteron cross sections were measured by JLab Hall C experiment E12-10-002, for $3.8 < Q^2 < 16.0$ GeV$^2$ and for a large range of $x$ extending to $x > 0.9$. The data cover the resonance region as well as a large range of the DIS region, and extends the kinematic coverage for precise measurements of proton and deuteron cross sections and $F_2$ structure function, where few previous measurements were available.

A new method has been utilized to study quark-hadron duality in the proton for $Q^2$ up to 10.5 GeV$^2$. An open question in our understanding of this phenomenon is what is the scaling curve? Previous studies have shown that this includes more than just the perturbative physics. The method was designed to address this by determining the duality averaged structure function with fine binning in $x$, avoiding the typical pitfall of relying on local duality where one must choose the $W^2$ ranges carefully in order to ensure cancellation between resonance peaks and dips. The average $F_2$ at fixed $x$ was determined over a range of $Q^2$ which effectively averages over a range of $W^2$. Control over the $Q^2$ range allows inclusion of more than one resonance for a fixed $x$. This procedure allows the generation of pseudo DIS data in the resonance region, which could be checked within the pQCD (+TMC+HT) fit and if consistent together gives a scaling curve. This pseudo DIS data can then be utilized to constrain the PDFs at large $x$.

This work is a natural extension of previous duality studies based on JLab data taken at smaller $Q^2$ in the 6 GeV era by I. Niculescu [41], M. Osipenko [44, 45], Y. Liang [27], and S.P. Malace [32], with the latter two studies based on precision cross sections measured in Hall C. The most recent study prior to this experiment was done by S.P. Malace. This study showed that global duality holds to several percent for $0.4 \leq Q^2 \leq 6.5$ GeV$^2$ when F2ALLM, a phenomenological fit to the data in the region $W^2 > 3$ GeV$^2$, was used as the scaling curve. Additionally, the study
showed that when using a pQCD fit (the CTEQ6 PDF fit) the $F_2$ averaged over the entire resonance region was found to be 20% higher than this scaling curve for $3.5 \leq Q^2 \leq 6.5$ GeV$^2$, even after inclusion of the TMC. On the other hand this particular scaling curve did not include HT effects, which are intrinsically included in the F2ALLM fit. This seems to indicate that the duality curve is not purely perturbative. A common limitation of all scaling curves thus far considered is that they are extrapolations in the resonance region by virtue of the $W^2$ cuts utilized in the fitting procedure. Therefore the differences between the duality averaged data and this scaling curve cannot be strictly attributed to either missing physics (TMC+HT) or to the poorly constrained nature of the fit at large $x$. It is hoped that duality averaged data presented here could remedy this situation.

The current study extends the quark-hadron duality study for the proton to larger $Q^2$ up to 10.5 GeV$^2$. Here $F_2$ is averaged over a $Q^2$ range and not dictated by $W^2$ cuts. The $Q^2$ range is chosen to maximize the $W^2$ range averaged over for a particular $x$. However the largest $x$ is mostly dominated by the region of $\Delta$. In this study the $F_2$ averaged over a range of $Q^2$ is compared to F2ALLM. At $Q^2 = 1.25$ GeV$^2$ and $0.2 \leq x \leq 0.55$, the average of $F_2$ agrees with this scaling curve better than 5%, with at least two resonance regions covered in the averaging. At $x = 0.6$, the average $F_2$ is 10% lower than that of the scaling curve, however in this extreme kinematic the data is dominated by the pion production threshold. At $Q^2 = 4.5$ GeV$^2$ and $0.5 \leq x \leq 0.85$, the $F_2$ average over the resonance region agrees with the scaling curve better than 5%. Above this ($x = 0.9$) the average $F_2$ is 25% lower than that of the scaling curve. This region of the data is dominated by the $\Delta$ resonance and the pion production threshold and can be used to study local duality for the $\Delta$ with a pion production threshold cut in the future. At $Q^2 = 10.5$ GeV$^2$ and $0.65 \leq x \leq 0.8$, the average $F_2$ agrees with this scaling curve better than 5%. While for $x = 0.85$ and 0.9 the average $F_2$ is approximately 10% and 15% higher than that of the scaling
curve, respectively. Again the highest $x$ region can be used to study the local duality for the $\Delta$ peak.

In summary, the work described in this dissertation extends studies of quark-hadron duality up to $Q^2 = 10.5$ GeV$^2$ and shows that when covered enough resonances, the data driven $F_{2\text{ALLM}}$ scaling curve is in good agreement with the average of the resonance $F_2$ for the proton. The hope is that the duality averaged data can be used to determine the scaling curve at large $x$ from within global fitting efforts. This data driven procedure will ensure that all the physics (TMC+HT) are intrinsically included in the scaling curve. For example the duality averaged $F_2$ can be included in the large $x$ PDF fits such as the CTEQ-JLab (CJ) collaboration efforts, which includes higher $x$ non-perturbative physics (TMC+HT).

In future this study will be extended for deuterium and neutron for further investigation of the quark-hadron duality.
REFERENCES

[1] For a summary of the CTEQ Collaboration’s work on the topic, see http://www.phys.psu.edu/cteq/.


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Presentations


