Few body systems in lattice QCD

William Detmold
From quarks to nuclei
From quarks to nuclei

• Nuclear physics: an emergent phenomenon of the Standard Model
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- How do nuclei emerge from QCD?
From quarks to nuclei

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- Issues
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
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  - Issues
  - Recent progress
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- *How do nuclei emerge from QCD?*
  - Issues
  - Recent progress
  - Future directions
Quantum chromodynamics

- Lattice QCD: quarks and gluons
  1. Formulate problem as functional integral over gluonic degrees of freedom on $\mathbb{R}^4$
  2. Discretise and compactify system
  3. Integrate via importance sampling (average over important gluon configs)
  4. *Undo the harm done in previous steps*
- Major computational challenge ...
QCD Spectroscopy

- Measure correlator ($\chi = \text{object with q\# of hadron}$)

$$C_2(t) = \sum_x \langle 0 | \chi(x, t) \bar{\chi}(0, 0) | 0 \rangle$$

- Unitarity: $\sum_n |n\rangle\langle n| = 1$

$$= \sum_x \sum_n \langle 0 | \chi(x, t) | n \rangle \langle n | \bar{\chi}(0, 0) | 0 \rangle$$

- Hamiltonian evolution

$$= \sum_x \sum_n e^{-E_n t} e^{i \mathbf{p}_n \cdot \mathbf{x}} \langle 0 | \chi(0, 0) | n \rangle \langle n | \bar{\chi}(0, 0) | 0 \rangle$$

- Long times only ground state survives

$$\lim_{t \to \infty} e^{-E_0(0)t} |\langle 0; 0 | \bar{\chi}(\mathbf{x}_0, t) | 0 \rangle|^2 = Z e^{-E_0(0)t}$$

$$\chi(x) = \bar{u}(x) \gamma_5 d(x)$$
Effective mass

- Construct $M(t) = \ln \left[ \frac{C_2(t)}{C_2(t + 1)} \right]^{\frac{t}{t \to \infty}} M$

- Plateau corresponds to energy of ground state

- Fancier techniques able to resolve multiple eigenstates
Nuclear physics from LQCD
Nuclear physics from LQCD

- Can we compute the mass of $^{208}$Pb in QCD?
Nuclear physics from LQCD

• Can we compute the mass of $^{208}$Pb in QCD?
• Yes
Nuclear physics from LQCD

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$$\langle 0|Tq_1(t)\cdots q_{624}(t)\overline{q}_1(0)\cdots \overline{q}_{624}(0)|0\rangle$$
Nuclear physics from LQCD

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• Yes

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• Long time behaviour gives ground state energy up to EW effects

$$t \rightarrow \infty \quad \# \exp(-M_{Pb} t)$$
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$$\lim_{t \to \infty} \# \exp(-M_{\text{Pb}}t)$$

• But...
An \((\text{exponentially hard})^2\) problem?
An (exponentially hard)$^2$ problem?

- Complexity: number of Wick contractions $= (A+Z)!(2A-Z)!$
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- Dynamical range of scales (numerical precision)
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- Dynamical range of scales (numerical precision)

- Small energy splittings
An (exponentially hard)^2 problem?

- Complexity: number of Wick contractions = \((A+Z)!\cdot(2A-Z)!\)

- Dynamical range of scales (numerical precision)

- Small energy splittings

- Importance sampling: statistical noise exponentially increases with A
The trouble with baryons

- Importance sampling of QCD functional integrals
  - Correlators determined stochastically

- Variance in single nucleon correlator ($C$) determined by
  \[
  \sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2
  \]

- For nucleon:
  \[
  \frac{\text{signal}}{\text{noise}} \sim \exp \left[-(M_N - 3/2m_\pi)t\right]
  \]

- For nucleus $A$:
  \[
  \frac{\text{signal}}{\text{noise}} \sim \exp \left[-A(M_N - 3/2m_\pi)t\right]
  \]

[Lepage '89]
The trouble with baryons

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[Lepage '89]
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

@ $m_\pi = 390$ MeV
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

Golden window of time-slices where signal/noise const
**No? trouble with baryons**

High statistics study using anisotropic lattices (fine temporal resolution)

Phys. Rev D80, 074501, 2009

Golden window of time-slices where signal/noise const

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High statistics study using anisotropic lattices (fine temporal resolution)

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Interpolator choice can be optimised to suppress noise
Multi-baryon systems

- Scattering and **bound states**
- NB: Strong interaction bound states
- Dibaryons: H, deuteron, $\Xi\Xi$
- $^3\text{H}$, $^4\text{He}$ and more exotic: $^4\text{He}_\Lambda$, $^4\text{He}_{\Lambda\Lambda}$, ...
- Correlators for significantly larger A
- Caveat: at unphysical quark masses no electroweak interactions
Bound states at finite volume

• Two particle scattering amplitude in infinite volume

\[ A(p) = \frac{8\pi}{M} \frac{1}{p \cot \delta(p) - ip} \]

bound state at \( p^2 = -\gamma^2 \) when \( \cot \delta(i\gamma) = i \)

• Scattering amplitude in finite volume (Lüscher method)

\[ \cot \delta(i\kappa) = i - i \sum_{\tilde{m} \neq 0} e^{-|\tilde{m}|\kappa L} \frac{e^{-|\tilde{m}|\kappa L}}{|\tilde{m}|\kappa L} \]

\( \kappa \xrightarrow{L \to \infty} \gamma \)

• Need multiple volumes

• More complicated for \( n>2 \) body bound states
H-dibaryon

- Jaffe [1977]: chromo-magnetic interaction

\[
\langle H_m \rangle \sim \frac{1}{4}N(N - 10) + \frac{1}{3}S(S + 1) + \frac{1}{2}C_c^2 + C_f^2
\]

most attractive for spin, colour, flavour singlet

- H-dibaryon (uuddss) J=l=0, s=-2 most stable

\[
\Psi_H = \frac{1}{\sqrt{8}} \left( \Lambda \Lambda + \sqrt{3} \Sigma \Sigma + 2 \Xi N \right)
\]

- Bound in a many hadronic models

- Experimental searches

- Emulsion expts, heavy-ion, stopped kaons

- No conclusive evidence for or against

KEK-ps (2007)

K^- 12C \rightarrow K^+ \Lambda \Lambda \Lambda
H dibaryon in QCD

• Early quenched studies on small lattices: mixed results
  [Mackenzie et al. 85, Iwasaki et al. 89, Pochinsky et al. 99, Wetzorke & Karsch 03, Luo et al. 07, Loan 11]

• Semi-realistic calculations
  • “Evidence for a bound H dibaryon from lattice QCD”
    PRL 106, 162001 (2011)
    \(N_f=2+1, \; a_s=0.12 \text{ fm}, \; m_\pi=390 \text{ MeV}, \; L=2.0, 2.5, 3.0, 3.9 \text{ fm}\)

  • “Bound H dibaryon in flavor SU(3) limit of lattice QCD” *
    PRL 106, 162002 (2011)
    \(N_f=3, \; a_s=0.12 \text{ fm}, \; m_\pi=670, 830, 1015 \text{ MeV}, \; L=2.0, 3.0, 3.9 \text{ fm}\)

• NB: Quark masses unphysical, single lattice spacing

* use a somewhat different method
H dibaryon in QCD

- Extract energy eigenstates from large Euclidean time behaviour of two-point correlators

\[ C_\Lambda(t) = \sum_x \langle 0|\chi(x, t)\overline{\chi}(0)|0\rangle + \infty Z_\Lambda e^{-M_\Lambda t} \]

\[ C_\Lambda(t) = \sum_x \langle 0|\phi(x, t)\overline{\phi}(0)|0\rangle + \infty Z_{\Lambda\Lambda} e^{-E_{\Lambda\Lambda}t} \]

- Correlator ratio allows direct access to energy shift

![Graphs showing energy shift ΔE for 3.0 fm and 4.0 fm distances, with fit parameters and uncertainties.](image)
Simple extrapolations

- After volume extrapolation H bound at unphysical quark masses
- Quark mass extrapolation is uncertain and unconstrained
  \[ B^\text{quad}_H = +11.5 \pm 2.8 \pm 6.0 \text{ MeV} \]
  \[ B^\text{lin}_H = +4.9 \pm 4.0 \pm 8.3 \text{ MeV} \]
- Other extrapolations, see
  [Shanahan, Thomas & Young PRL. 107 (2011) 092004, Haidenbauer & Meissner 1109.3590]
- Suggests H is weakly bound or just unbound

* 230 MeV point preliminary (one volume)
Deuteron

- Deuteron also investigated
- NPLQCD
- PACS-CS
- More work needed at lighter masses

![Graph showing ΔE(3S1) vs. m²π]
Deuteron

- Deuteron also investigated
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- More work needed at lighter masses

![Graph showing $\Delta E(3S_1)$ vs. $m_{\pi}^2$ in GeV^2 with various data points and error bars.]

[Yamazaki et al. 1207.4277]
Many baryon systems

- New approach to many baryon correlator construction
- Interpolating fields – minimal expression as weighted sums

\[ \mathcal{N}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2,\ldots,a_{n_q})} \sum_{i} e^{i_1,i_2,\ldots,i_{n_q}} \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \cdots \bar{q}(a_{i_{n_q}}) \]

- Automated generation of weights (symbolic C++ code) for given quantum numbers
- Baryon blocks (partly contracted at sink)

\[ B_{b}^{a_1,a_2,a_3}(p,t;x_0) = \sum_{x} e^{ip\cdot x} \sum_{k=1}^{N_B(b)} \tilde{w}_b^{(c_1,c_2,c_3)} \sum_{i} e^{i_1,i_2,i_3} S(c_{i_1}, x; a_1, x_0) S(c_{i_2}, x; a_2, x_0) S(c_{i_3}, x; a_3, x_0) \]

WD, Kostas Orginos, 1207.1452
see also Doi & Endres 1205.0585
Many baryon systems

\[
\left[ \mathcal{N}_1^h(t) \mathcal{N}_2^h(0) \right]_U = \int \mathcal{D}q \mathcal{D}\bar{q} \ e^{-S_{QCD}[U]} \sum_{k' = 1}^{N'_w} \sum_{k = 1}^{N_w} \tilde{w}_h^{(a'_1, a'_2, \ldots, a'_{n_q}), k'} \tilde{w}_h^{(a_1, a_2, \ldots, a_{n_q}), k} \times \sum \sum \epsilon_{j_1, j_2, \ldots, j_{n_q}} \epsilon_{i_1, i_2, \ldots, i_{n_q}} q(a'_{j_{n_q}}) \cdots q(a'_{j_2}) q(a'_{j_1}) \times \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \cdots \bar{q}(a_{i_{n_q}})
\]
Many baryon systems

- Contractions

\[
\left[ \mathcal{N}_1^h(t)\mathcal{N}_2^h(0) \right]_U = \int \mathcal{D}q \mathcal{D}\bar{q} \, e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w} \sum_{k=1}^{N_w} \tilde{w}_{h}(a_1',a_2'\ldots,a'_{n_q}),k' \tilde{w}_{h}(a_1,a_2\ldots,a_{n_q}),k \times \\
\sum_j \sum_i \epsilon^{j_1,j_2,\ldots,j_{n_q}} \epsilon^{i_1,i_2,\ldots,i_{n_q}} q(a'_{j_{n_q}}) \cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2}) \cdots \bar{q}(a_{i_{n_q}})
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Many baryon systems

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\mathcal{N}_{1}^{h}(t)\mathcal{N}_{2}^{h}(0) = \int \mathcal{D}q \mathcal{D}\bar{q} \ e^{-S_{QCD}[U]} \sum_{k'=1}^{N_{w}'} \sum_{k=1}^{N_{w}} \tilde{w}_{h}(a_{1}',a_{2}',\ldots,a_{n_{q}}'),k' \tilde{w}_{h}(a_{1},a_{2},\ldots,a_{n_{q}}),k \times \sum_{j} \sum_{i} \epsilon_{j_{1},j_{2},\ldots,j_{n_{q}}} \epsilon_{i_{1},i_{2},\ldots,i_{n_{q}}} q(a_{j_{n_{q}}}') \cdots q(a_{j_{2}}')q(a_{j_{1}}') \times \bar{q}(a_{i_{n_{q}}})\bar{q}(a_{i_{2}})\cdots \bar{q}(a_{i_{n_{q}}})
\]

\[
= e^{-S_{eff}[U]} \sum_{k'=1}^{N_{w}'} \sum_{k=1}^{N_{w}} \tilde{w}_{h}(a_{1}',a_{2}',\ldots,a_{n_{q}}'),k' \tilde{w}_{h}(a_{1},a_{2},\ldots,a_{n_{q}}),k \times \sum_{j} \sum_{i} \epsilon_{j_{1},j_{2},\ldots,j_{n_{q}}} \epsilon_{i_{1},i_{2},\ldots,i_{n_{q}}} S(a_{j_{n_{q}}}' ; a_{i_{n_{q}}})S(a_{j_{2}}' ; a_{i_{2}})\cdots S(a_{j_{1}}' ; a_{i_{1}})
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Many baryon systems

- Contractions

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\sum_j \sum_i \epsilon^{j_1,j_2,\ldots,j_{n_q}} \epsilon^{i_1,i_2,\ldots,i_{n_q}} S(a'_{j_{n_q}};a_{i_{n_q}}) S(a'_{j_2};a_{i_2}) \cdots S(a'_{j_1};a_{i_1})
\]

- Express in terms of blocks (quark-hadron)
Many baryon systems

- Contractions

\[ \left[ \mathcal{N}_1^h(t) \tilde{\mathcal{N}}_2^h(0) \right]_U = \int \mathcal{D}q \mathcal{D}\bar{q} \ e^{-S_{QCD}[U]} \sum_{k'=1}^{N_w'} \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1', a_2' \cdots a_{n_q}')}, k' \tilde{w}_h^{(a_1 a_2 \cdots a_{n_q})}, k \times \sum_{j} \sum_{i} \epsilon^{j_1 j_2 \cdots j_{n_q}} \epsilon^{i_1 i_2 \cdots i_{n_q}} q(a_{j_{n_q}}') \cdots q(a_{j_2}') q(a_{j_1}') \times \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \cdots \bar{q}(a_{i_{n_q}}) \]

\[ = e^{-S_{eff}[U]} \sum_{k'=1}^{N_w'} \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1', a_2' \cdots a_{n_q}')}, k' \tilde{w}_h^{(a_1 a_2 \cdots a_{n_q})}, k \times \sum_{j} \sum_{i} \epsilon^{j_1 j_2 \cdots j_{n_q}} \epsilon^{i_1 i_2 \cdots i_{n_q}} S(a_{j_{n_q}}'; a_{i_1}) S(a_{j_2}'; a_{i_2}) \cdots S(a_{j_1}'; a_{i_{n_q}}) \]

- Express in terms of blocks (quark-hadron)

- Or write as determinant (quark-quark)

\[ \langle \mathcal{N}_1^h(t) \tilde{\mathcal{N}}_2^h(0) \rangle = \frac{1}{Z} \int \mathcal{D}U \ e^{-S_{eff}} \sum_{k'=1}^{N_w'} \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1', a_2' \cdots a_{n_q}')}, k' \tilde{w}_h^{(a_1 a_2 \cdots a_{n_q})}, k \times \det G(a'; a) \]

\[ G(a'; a)_{j,i} = \begin{cases} S(a_j'; a_i) & a_j' \in a' \text{ and } a_i \in a \\ \delta_{a_j', a_i} & \text{otherwise} \end{cases} \]
Nuclei

- Recent studies at SU(3) point (physical m_s)
- Isotropic clover lattices
- Single lattice spacing: 0.145 fm
- Multiple volumes: 3.4, 4.5, 6.7 fm
- High statistics

<table>
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<tr>
<th>Label</th>
<th>L/b</th>
<th>T/b</th>
<th>β</th>
<th>b m_q</th>
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<th>L [fm]</th>
<th>T [fm]</th>
<th>m_π [MeV]</th>
<th>m_π L</th>
<th>m_π T</th>
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<tr>
<td>A</td>
<td>24</td>
<td>48</td>
<td>6.1</td>
<td>-0.2450</td>
<td>0.145</td>
<td>3.4</td>
<td>6.7</td>
<td>806.5(0.3)(0)(8.9)</td>
<td>14.3</td>
<td>28.5</td>
<td>3822</td>
<td>48</td>
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<tr>
<td>B</td>
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<td>24</td>
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<td>C</td>
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<td>1212</td>
<td>32</td>
</tr>
</tbody>
</table>
Nuclei (A=2)

Quark-hadron contraction method

\[ \Lambda \Lambda \]

\[ n \Xi (\frac{3}{2} S_1) \]

\[ n p (\frac{3}{2} S_1) \]

\[ n \Sigma (\frac{3}{2} S_1) \]
Nuclei (A=2)

Quark-hadron contraction method

\[ \Delta E \text{ (MeV)} \]

-100
-80
-60
-40
-20
0

deuteron
nn
H–dib
nΛ (1s0)
nΛ (3s1)
nΣ (1s0)
nΣ (3s1)
nΞ (3s1)
pΞ (3s1)

L=24, |p|=0
L=32, |p|=0
L=32, |p|=1
L=32, |p|=2
L=48, |p|=0
L=48, |p|=1
L=48, |p|=2

NPLQCD arXiv:1206.5219
Nuclei \((A=2,3,4)\)

Quark-hadron contraction method
Nuclei \((A=2,3,4)\)

Quark-hadron contraction method
d, nn, $^3$He, $^4$He

- PACS-CS: bound d,nn, $^3$He, $^4$He
- Previous quenched work
- Recent unquenched study at $m_\pi=500$ MeV
- HALQCD
  - Extract an NN potential
- Strong enough to bind H, $^4$He at $m_{PS}=490$ MeV SU(3) pt
- d, nn not bound
Nuclei \((A=4,\ldots)\)

Quark-quark determinant contraction method

(low statistics, single volume)
Nuclei ($A=4,...$)

Quark-quark determinant contraction method

$^4$He (SP)

(low statistics, single volume)
Nuclei (A=4,...)  

Quark-quark determinant contraction method  

$^8\text{Be (SP)}$  

(low statistics, single volume)
Nuclei \((A=4,...)\)

Quark-quark determinant contraction method

\(12\text{C} (\text{SP})\)

(low statistics, single volume)

WD, Kostas Orginos, 1207.1452
Nuclei \((A=4,\ldots)\)

Quark-quark determinant contraction method

\[16^O\ (SP)\]

(logarithmic plot of \(C(t)\) vs. \(t/a\))

(low statistics, single volume)

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Nuclei ($A=4,\ldots$)

Quark-quark determinant contraction method

$^{28}$Si (SP)

(low statistics, single volume)

WD, Kostas Orginos, 1207.1452
The road ahead...
Hyperon-nucleon interactions
Hyperon-nucleon interactions

- Observation of 1.97 $M_\odot$ n-star [Demorest et al., Nature, 2010]
  
  "effectively rules out the presence of hyperons, bosons, or free quarks"
Hyperon-nucleon interactions

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  "effectively rules out the presence of hyperons, bosons, or free quarks"

- Relies significantly on poorly known hadronic interactions at high density
  - Hyperon-nucleon
  - nnn, ...

![](image)
Hyperon-nucleon interactions

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  “effectively rules out the presence of hyperons, bosons, or free quarks”

- Relies significantly on poorly known hadronic interactions at high density
  - Hyperon-nucleon
  - $nnn$, ...
  - Calculable in QCD
  - 30% measurements would have impact
Hypernuclear Spectroscopy
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- Table of nuclides very well determined experimentally
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- One plane in table of hypernuclei
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Hypernuclear Spectroscopy

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- All QCD eigenstates
- Significant experimental programs at JLab & KEK and soon at JPARC & FAIR
- Complementary QCD predictions for exotic systems
Nuclear properties
Nuclear properties

- Many phenomenologically important nuclear matrix elements
Nuclear properties

• Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   • pp fusion: “Calibrate the sun”
   • Muon capture: MuSun @ PSI
   • dν → n n e⁺ : SNO
Nuclear properties

- Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   - pp fusion: “Calibrate the sun”
   - Muon capture: MuSun @ PSI
   - $d\nu \rightarrow nn e^+$ : SNO

Diagram: Nuclear reactions and calculations for axial coupling and neutrino interactions.
Nuclear properties

- Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   - pp fusion: “Calibrate the sun”
   - Muon capture: MuSun @ PSI
   - \(d\nu \rightarrow nn e^+\) : SNO

2. Medium effects: eg EMC effect
   - Proof of principle (pion PDF in pion gas) [WD, HW Lin 1112.5682]
Nuclear properties

• Many phenomenologically important nuclear matrix elements

1. Axial coupling to NN system
   • pp fusion: “Calibrate the sun”
   • Muon capture: MuSun @ PSI
   • dν → nn e⁺ : SNO

2. Medium effects: eg EMC effect
   • Proof of principle (pion PDF in pion gas)
   • LQCD: not much harder than spectroscopy

[WD, HW Lin 1112.5682]
From quarks to nuclei
From quarks to nuclei

• QCD calculations of nuclei are possible
From quarks to nuclei

• *QCD calculations of nuclei are possible*
• More work needed to get to the physical quark masses
From quarks to nuclei

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• Need big computers and good ideas
From quarks to nuclei

• *QCD calculations of nuclei are possible*
  
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From quarks to nuclei

- **QCD calculations of nuclei are possible**
- More work needed to get to the physical quark masses
- Need big computers and good ideas
- Where is the field going?
From quarks to nuclei

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• Strong connections to experimental programs: hypernuclear spectroscopy at JLab, JPARC, FAIR
From quarks to nuclei

- QCD calculations of nuclei are possible
- More work needed to get to the physical quark masses
- Need big computers and good ideas
- Where is the field going?
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  - Answer questions that experiments have not and cannot: nnn, quark mass dependence
[FIN]

thanks to

NPLQCD
Hypernuclear Spectroscopy
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- Table of nuclides very well determined experimentally
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- All QCD eigenstates
- Significant experimental programs at JLab & KEK and soon at JPARC & FAIR
- Complementary QCD predictions for exotic systems
Hadron-hadron scattering
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- Old problem: interactions of two particles in a box changes quantization condition [Uhlenbeck 1930s, ...]
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Bound state
Scattering poles
Scattering amplitude at finite volume
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\[
E^{(n)} \equiv \sqrt{|\mathbf{q}(n)|^2 + m_A^2} + \sqrt{|\mathbf{q}(n)|^2 + m_B^2}
\]

\[
q(n) \cot \delta(q(n)) = \frac{1}{\pi \Lambda} S \left( \frac{q(n) L}{2\pi} \right)
\]

\[
S(\eta) = \lim_{\Lambda \to \infty} \left[ \sum_{\tilde{n} : |\tilde{n}| < \Lambda} \frac{1}{|\tilde{n}|^2 - \eta^2 - 4\pi \Lambda} \right]
\]

\( A \sim \ldots \)
Example: $l=2 \, \pi\pi$

- Study multiple energy levels of two pions in a box for multiple volumes and with multiple $P_{CM}$

@ $m_\pi = 390 \, \text{MeV}$
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- Allows phase shift to be extracted at multiple energies

![Graph showing phase shift vs. $k^2/m^2$ at $m_\pi = 390$ MeV for different $L/b_s$ values: 16, 20, 24, and 32.](Image)
Example: $l=2$ ̅̅̅̅ππ

- Combine with chiral perturbation theory to interpolate to physical pion mass

![Graph showing $\delta$ (degrees) vs. $k^2$ (GeV$^2$)]