Hadrons and Holography

Cite as: AIP Conference Proceedings 1374, 66 (2011); https://doi.org/10.1063/1.3647100
Published Online: 25 October 2011

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Abstract. Holographic QCD is an extra-dimensional approach to modeling QCD resonances and their interactions. Holographic models encode information about chiral symmetry breaking, Weinberg sum rules, vector meson dominance, and other phenomenological features of QCD. There are two complementary approaches to holographic model building: a top-down approach which begins with string-theory brane configurations, and a bottom-up approach which is more phenomenological. In this talk I will describe the techniques used to build holographic models of QCD and the tools used to calculate observables in those models.

Keywords: Holographic QCD, AdS/QCD

WHAT IS HOLOGRAPHIC QCD?

Holographic QCD, also known as AdS/QCD, is an extra-dimensional approach to modeling the light hadronic resonances in QCD, motivated by the AdS/CFT correspondence in string theory [1]. Holographic QCD models combine several features of previous approaches to modeling the spectrum and interactions of light hadrons, including chiral symmetry breaking, hidden local symmetry [2], large-$N$, and the Weinberg sum rules.

There are two complementary classes of AdS/QCD models: top-down models rooted in string theory, and bottom-up models which are more phenomenological. The benefit of top-down models is that both sides of the AdS/CFT duality are often well understood. The benefit of bottom-up models is that there is more freedom to build in properties of QCD.

Top-Down AdS/QCD

FIGURE 1. Brane configuration in the Sakai-Sugimoto model. The D4-branes create a spacetime horizon, forcing the D8 and D8-branes to intersect.

In the top-down approach, a brane configuration in string theory is engineered whose low-energy spectrum of open-string fluctuations has a known field-theoretic interpretation. According to Maldacena’s AdS/CFT correspondence, for certain brane configurations describing large-$N$ gauge theories with large ’t Hooft coupling $g^2 N$, a dual description exists in terms of supergravity on a fixed spacetime background [1]. The basic dictionary between the dual theories was mapped out independently by Witten [3] and by Gubser, Klebanov and Polyakov [4]. There now exists a large number of examples of field theories with gravity duals. Despite the fact that prototypical examples of the AdS/CFT correspondence describe supersymmetric conformal theories (“CFT” in AdS/CFT is short for conformal field theory), neither conformal invariance nor supersymmetry is essential for the existence of a gravitational dual. The field theory can be confining with chiral symmetry breaking, which in those respects is similar to QCD. Some examples of
confining theories with known supergravity duals are the $\mathcal{N}=1^*$ theory of Polchinski and Strassler [5], the Klebanov-Strassler cascading gauge theory [6], the D3-D7 system of Kruclenski et al. [7], and the D4-D8 system of Sakai and Sugimoto [8].

The Sakai-Sugimoto model is so far the example of the AdS/CFT correspondence most closely related to QCD. Ignoring the gravitational backreaction, the system includes a stack of $N$ D4-branes ($p$-branes extend in $p$ spatial dimensions) wrapped on a circle on which fermions satisfy antiperiodic boundary conditions which break supersymmetry. From an effective 3+1 dimensional point of view, the massless spectrum includes SU($N$) gauge fields, but the fermions and scalars are massive. This theory was analyzed by Witten shortly after the initial AdS/CFT conjecture [9]. Sakai and Sugimoto added $N_f$ D8-branes and $N_f$ $\overline{D}8$-branes transverse to the circle. The D8 and $\overline{D}8$-branes intersect the D4-branes on 3+1 dimensional manifolds at definite positions along the circle, as in Fig. 1. The massless fluctuations of open strings connecting the D4 and D8 or $\overline{D}8$-branes at their intersections describe 3+1 dimensional chiral fermions, with opposite chirality at the D8 and $\overline{D}8$-branes. This is the Sakai-Sugimoto model [8].

In the supergravity limit ($N \to \infty$, $g^2 N \gg 1$) with $N_f < N$, the D4-branes generate a horizon which effectively cuts off the spacetime geometry. The location of this horizon sets the mass scale of certain fluctuations in the 3+1 dimensional theory, including glueballs and some of their excitations. Hence, the location of the horizon is related to the confinement scale in the model. For the geometry to be smooth the location of the horizon is correlated with the size of the compact circle on which the D4-branes wrap [7, 9]. This has the implication that Kaluza-Klein modes around the circle, which do not correspond to QCD states, have masses comparable to the hadrons which we would like to model. Hence, the five-dimensional nature of the effective theory on the D4-branes becomes apparent at the same scale as the hadron masses we are interested in. This is an important distinction between this theory and QCD, and the extraneous states are normally ignored when predicting QCD observables from the model.

Because of the horizon, the D8 and $\overline{D}8$-branes intersect, which reflects the breaking of the SU($N_f$)×SU($N_f$) chiral symmetry in the theory. The perturbative massless spectrum is that of SU($N$) QCD with $N_f$ flavors of quarks. If we ignore the Kaluza-Klein modes around the circle, the fluctuations of 4+1 dimensional SU($N_f$)×SU($N_f$) gauge fields on the D8 and $\overline{D}8$-branes are identified with vector mesons, axial-vector mesons, and pions. The quantum numbers of the corresponding states can be identified with symmetries of the D-brane system. 3+1 dimensional parity, for example, is identified with a 4+1 dimensional parity, which also exchanges the two sets of SU($N_f$) gauge fields. Then the effective 3+1 dimensional action on the D8-branes describes the effective action for the light mesons, and easily allows for the calculation of decay constants ($f_{\pi}$, $F_{\pi}$, etc.) and couplings (e.g. $g_{\rho\pi\pi}$). Most results agree relatively well with experimental data (at around the 10-20% level). The light baryons in AdS/QCD have been identified with solitonic configurations of the 4+1 dimensional fields, which are closely analogous to the baryons of the Skyrme model [8].

**Bottom-Up AdS/QCD**

In the bottom-up approach, we begin with the observation that the Kaluza-Klein modes of fields in an extra dimension might be identified with the radial excitations of hadrons in a confining gauge theory. As in the top-down approach, the quantum numbers of those excitations depend on the transformations of the Kaluza-Klein modes under corresponding 5D symmetries. In the hard-wall model [10, 11], we begin with a 5D SU(2)×SU(2) gauge theory. To reproduce the discrete spectrum of hadronic excitations, the spacetime geometry must be such that the spectrum corresponding 5D symmetries. In the hard-wall model [10, 11], we begin with a 5D SU(2)×SU(2) gauge theory. To reproduce the discrete spectrum of hadronic excitations, the spacetime geometry must be such that the spectrum of Kaluza-Klein gauge fields is discrete. The metric of the five-dimensional Anti-de Sitter spacetime (AdS$_5$) can be written,

$$ds^2 = \frac{R^2}{z^2} (dx^\mu \eta^{\mu\nu} dx_\nu - dz^2),$$

where $\eta_{\mu\nu}$ is the 3+1 dimensional Minkowski metric with components diag(1,-1,-1,-1); $R$ is the AdS curvature, which we will normalize to 1. The isometries of AdS$_5$ were identified with the conformal symmetry of $\mathcal{N}=4$ supersymmetric Yang-Mills theory in the original AdS/CFT correspondence [1]. For simplicity, we take as the spacetime geometry a slice of AdS$_5$ between an ultraviolet cutoff scale $z = \epsilon$ and an infrared scale $z = z_m$ related to $\Lambda_{QCD}$. Other choices for the geometry may better match various aspects of QCD (such as running of the QCD coupling and the Regge spectrum [12]). In addition to its simplicity, the AdS spacetime has been motivated by appealing to the conformal invariance of the asymptotically free theory in the ultraviolet (UV), and also to hints of conformal behavior at low energies (e.g. [13]), based on measurements of the running QCD coupling at JLab [14].
The SU(2) × SU(2) gauge invariance is related to the approximate SU(2) × SU(2) chiral symmetry of the up and down quarks. (This may be extended to SU(3) × SU(3) in order to include the strange quark.) In order to break the chiral symmetry we introduce a scalar field that transforms in the bifundamental representation of the chiral symmetry, in analogy with the operator \( \bar{q}q \). If this field is arranged to have a nonvanishing background profile, the gauge invariance is spontaneously broken. The 4+1 dimensional action takes the form,

\[
S = \int d^5 x \sqrt{-g} \left( -\frac{1}{2g_5^2} \text{Tr} \left( L_{MN} L^{MN} + R_{MN} R^{MN} \right) + \text{Tr} \left( |D_M X|^2 + m_X^2 |X|^2 \right) \right),
\]

where \( L_{MN} \) and \( R_{MN} \) are the field strengths of the two sets of SU(2) gauge fields and \( m_X^2 \) is the squared mass of the bifundamental field \( X \); contractions of indices are by the AdS5 metric.

The scaling dimension of the operator \( \bar{q}q \) in the ultraviolet is three. By the AdS/CFT dictionary the mass of a field \( X \) with scaling dimension \( \Delta \) is given by \( m_X^2 = \Delta (\Delta - 4) \) in units of the AdS curvature, or in our case \( m_X^2 = -3 \). As a result of the curvature of Anti-de Sitter space, the negative squared mass does not lead to instability [15]. It is important to note that the choice \( m_X^2 = -3 \) is made here for definiteness, but it is not necessary to fix \( m_X^2 \) this way. The AdS/CFT correspondence in the classical limit is not valid for QCD with finite \( N \). Furthermore, reference to the scaling dimension in the ultraviolet ignores the effects of renormalization. However, for definiteness we may still choose to apply the AdS/CFT correspondence to fix parameters in the model.

The equations of motion for the \( z \)-dependent scalar field background, with the gauge fields turned off, are,

\[
\partial_z \left( \frac{1}{z} \partial_z X(z) \right) + \frac{3}{z^2} X(z) = 0.
\]

The solutions for the scalar field background are,

\[
X(z) = \left( \frac{m_q z}{2} + \frac{\sigma}{2} z^2 \right),
\]

where \( m_q \) and \( \sigma \) are arbitrary. By the AdS/CFT correspondence, the coefficient of the solution with divergent action has the interpretation of the source for the corresponding operator [3, 4]. The quark mass is a source for the operator \( \bar{q}q \), so \( m_q \) has the interpretation of the quark mass (which for simplicity is isospin-preserving in the model). Similarly, the coefficient of the finite-action (i.e. normalizable) solution has the interpretation of the expectation value of the corresponding operator, so \( \sigma \) has the interpretation of the chiral condensate \( \langle \bar{q}q \rangle \). The factors of \( 1/2 \) in (4) are chosen such that the product of coefficients, \( m_q \sigma / 4 \), is properly normalized. A more accurate scaling of the quark mass and chiral condensate with \( N \) can be incorporated by rescaling the model parameters \( m_q \) and \( \sigma \) appropriately [16]. In any case, since the hard-wall model is phenomenological and does not follow from a precise AdS/CFT correspondence, the physical identification of the parameters \( m_q \) and \( \sigma \) is not precise.

If we want, we can fix \( g_5 \) by comparison with perturbative QCD at large \(-q^2\), although the hard-wall model is not expected to be valid at high energies. The vector current two-point function at one-loop in SU(\( N \)) QCD with two flavors, valid for large \(-q^2\), is

\[
i \int d^4 x e^{i q \cdot x} \left( \langle J_{\mu}^a(x) J_{\nu}^a(0) \rangle \right) = \left( g_{\mu \nu} q_{\mu \nu} - g_{\mu \nu} q^2 \right) \delta^{ab} \frac{N}{24 \pi^2} \log(-q^2).
\]

The AdS/QCD prediction of the vector current two-point function is precisely of the perturbative form of Eq. (5) if we set \( g_5^2 = \frac{4\pi^2}{N} \) when \( N = 3 \) [10, 11]. It is not necessary to fix \( g_5 \) by matching to the ultraviolet, just as it was not necessary to fix the mass of the field \( X \) by matching to the conformal dimension of the operator \( \bar{q}q \). These choices are made for definiteness, but they may be relaxed. If we do fix \( m_X^2 \) and \( g_5 \) this way, the model has three remaining parameters. The masses of the Kaluza-Klein modes are identified with the masses of the hadrons with the same quantum numbers, and are determined by solutions to the eigenvalue problem specified by the linearized equations of motion with boundary conditions on the 5D fields and their derivatives. Decay constants are determined by the residues of poles in current-current two-point functions, which are determined by derivatives of the solutions to the linearized equations of motion [10, 11, 19]. Hadronic couplings are determined by the effective 4D theory after integrating the action over the extra dimension. A root-mean-squared fit of the remaining parameters to the central values of experimental and lattice data for seven observables gives \( z_m = 1/(346 \text{ MeV}) \), \( \sigma = (308 \text{ MeV})^3 \), \( m_q = 2.3 \text{ MeV} \) [10]. The hard-wall AdS/QCD predictions with these values for the parameters are given in Table 1.
A number of additional predictions in an extension of the model which includes a strange quark mass parameter are given in Table 2, from an analysis by Abidin and Carlson [18].

A relation between observables and the parameters \( m_q \) and \( \sigma \) can be derived which resembles the Gell-Mann–Oakes–Renner (GOR) relation [10],

\[ m_{\pi}^2 f_{\pi}^2 = 2m_q \sigma. \]  

(6)

The SU(3) extensions of the GOR relation (6) are satisfied in appropriate extensions of the 5D model [18].

**Soft-Wall AdS/QCD**

The AdS/QCD models described above are not expected to be valid much above the scale of the lightest vector resonances. For heavy resonances, the vector and axial-vector masses in the hard-wall model scale as \( m_n^2 \sim n^2 \). However, experimental data confirm the Regge behavior \( m_n^2 \sim n \). Misha Shifman and others have stressed this difference between AdS/QCD and QCD [20]. It is possible to reproduce the Regge spectrum by effectively modifying the AdS\(_5\) geometry [12]. One way to do this is to couple the fields in the AdS/QCD model to a background dilaton:

\[ S = \int d^5x \sqrt{-g} e^{-\Phi_0(z^2)}|\mathcal{L}|, \]

(7)

where the appropriate background for the dilaton is \( \Phi_0(z) \sim z^2 \). Low-energy predictions are comparable to, but not the same as, those of the hard-wall model.
Form Factors

It is straightforward to calculate form factors in AdS/QCD models, as the AdS/CFT correspondence teaches us that the contribution of a tower of resonances can be summed by use of the bulk-to-boundary propagator (see also Ref. [21]). Several authors have calculated various meson form factors. From these form factors were deduced moments of generalized parton distributions, charge radii and gravitational radii. For example, in the hard wall model with a particular choice of parameters [22, 23, 24]:

\[
\langle r_{\pi}^2 \rangle_{\text{charge}} = 0.33 \text{ fm}^2,
\langle r_{\pi}^2 \rangle_{\text{grav}} = 0.13 \text{ fm}^2
\]

\[
\langle r_{\rho}^2 \rangle_{\text{charge}} = 0.53 \text{ fm}^2,
\langle r_{\rho}^2 \rangle_{\text{grav}} = 0.21 \text{ fm}^2
\]

\[
\langle r_{a_1}^2 \rangle_{\text{charge}} = 0.39 \text{ fm}^2,
\langle r_{a_1}^2 \rangle_{\text{grav}} = 0.15 \text{ fm}^2
\]

Baryons

If one naively truncates an AdS/QCD model to the lightest modes, the 5D gauge kinetic terms include the 4D Skyrme term with coefficient that depends on the 5D dimensional gauge coupling and the spacetime geometry [8]. The light baryons have therefore been identified as Skyrmions in AdS/QCD. There has been some debate as to the stability of predictions for baryons in this approach, as in the original Skyrme model, based on the relative importance of higher-dimension operators that are neglected in this approach. A description of baryons in terms of 4+1 dimensional solitons, as opposed to the Skyrmions in the effective 3+1 dimensional theory, has been discussed by several authors, for example in Refs. [26, 27]. A comparison of these approaches appears in Ref. [28]. An alternative description of baryons modeled as fundamental fermions in the extra dimension has also been considered, and accurately reproduces the spectrum of excited Delta and nucleon resonances (see, for example, Refs. [29, 30]).

Light-front wave functions

Stan Brodsky and Guy de Teramond have noticed an intriguing relationship between the equations of motion for Kaluza-Klein modes of fields with general spin and scaling dimension in AdS/QCD models, and equations describing light-front wavefunctions of hadrons with general spin and orbital angular momentum (e.g. Ref. [31]). This is an interesting observation, and relates the radial direction of Anti-de Sitter space with partonic momenta inside the hadrons.

UNIVERSALITY IN ADS/QCD

Certain observables in AdS/CFT models at finite temperature have been found to be completely independent of the details of the model. A famous example is the ratio of shear viscosity \( \eta \) to entropy density \( s \), which in natural units is...
predicted to be [32],
\[ \frac{\eta}{s} = \frac{1}{4\pi}. \]  
(8)

A more recent example is the ratio of electrical conductivity \( \sigma \) to charge susceptibility \( \chi \), which depends only on the temperature and number of spacetime dimensions [33]. Universal predictions provide the strongest test of the AdS/CFT correspondence as applied to QCD. Despite the absence of complete universality of most AdS/QCD predictions, it is interesting that the various AdS/QCD models make comparable predictions for low-energy observables. It is worthwhile to better understand which observables are approximately independent of the details of the AdS/QCD model, and which details of the model are unimportant. For example, upon varying the mass of the field \( X \) by \( \pm 20\% \), as long as the remaining parameters are chosen so as to correctly reproduce a small number of observables \( (m_\pi, f_\pi \) and \( m_T) \) the remaining low-energy observables of Table 1 were found to vary by only a few percent, as in Fig. 2 [25]. Perhaps the surprising success of AdS/QCD models at low energies is the result of such universality in its predictions.

**ADDITIONAL APPLICATIONS OF ADS/CFT MODELS**

Models similar in spirit to AdS/QCD have been applied to other strongly-interacting dynamical systems. Holographic technicolor models [34] are based on AdS/QCD, and AdS/CFT methods allow for calculation of precision electroweak observables in these models. These models are examples of Higgsless models of electroweak symmetry breaking, in which unitarity of longitudinal W boson scattering is the result of interactions involving the massive Kaluza-Klein excitations of the electroweak gauge bosons.

An exciting recent application of the AdS/CFT correspondence is to condensed matter systems. An important development in this direction was the construction of a spacetime geometry whose isometries are the same as the nonrelativistic conformal group [35]. There are intriguing similarities of some models to high-temperature superconductors and systems of cold atoms [36].

**SUMMARY**

AdS/QCD models share the features of a number of earlier approaches to modeling QCD at low energies. AdS/QCD models generally predict low-energy observables at the 10-20% level, but do not fare as well at high energies. It is not yet clear why these models work as well as they do, but some predictions have been found to be universal as details of the model are varied.

**ACKNOWLEDGMENTS**

The author thanks Zainul Abidin, Stan Brodsky, Carl Carlson, Guy de Teramond, Hovhannes Grigoryan, Emanuel Katz, Henry Kwee, Richard Lebed, and Anatoly Radyushkin for discussions and for use of some of their results in this talk. The author’s work is supported by NSF grant PHY-0757481.

**REFERENCES**