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QCD angular momentum in $N \to \Delta$ transitions

June-Young Kim [a], Ho-Yeon Won [b], Jose L. Goity [c,a], Christian Weiss [a]

[a] Theory Center, Jefferson Lab, Newport News, VA 23606, USA
[b] Department of Physics, Inha University, Incheon 402-751, South Korea
[c] Department of Physics, Hampton University, Hampton, VA 23668, USA

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ABSTRACT

$N \to \Delta$ transitions offer new possibilities for exploring the isovector component of the QCD quark angular momentum (AM) operator causing the J^{u-d} flavor asymmetry in the nucleon. We extend the concept of QCD AM to transitions between baryon states, using light-front densities of the energy-momentum tensor in transversely localized states. We calculate the $N \to \Delta$ transition AM in the $1/N_c$ expansion, connect it with the J^{u-d} flavor asymmetry in the nucleon, and estimate the values using lattice QCD results. In the same setup we connect the transition AM to the transition GPDs sampled in hard exclusive electroproduction processes with $N \to \Delta$ transitions, enabling experimental study of the transition AM.

1. Introduction

Angular momentum (AM) has become an essential concept in hadron structure physics. The AM operator is derived from the QCD energy-momentum tensor (EMT) and represents the conserved current associated with rotational invariance. It measures the AM of chromodynamic field configurations, arising from their space-time dependence (orbital AM) and internal degrees of freedom (spin), and can be decomposed into quark and gluon contributions. Its formal properties have been discussed extensively and are now well understood; see Refs. [1,2] for reviews. Its experimental study becomes possible through the connection with the generalized parton distributions (GPDs) describing hadron structure as probed in high-momentum-transfer exclusive scattering processes; see Refs. [3–6] for reviews. Certain components of the EMT can be expressed as integrals of the GPDs (moments) and thus indirectly be related to observables measured in exclusive processes [7,8].

There is evidence of a large flavor asymmetry of the quark AM in the nucleon, J^{u-d}. The normalization of the Pauli form factor-type GPD E entering in the AM sum rules [7,8] is controlled by the nucleon anomalous magnetic moment, whose isovector component is much larger than the isoscalar, $\kappa^{p-n} = 3.7$ vs. $\kappa^{p+n} = -0.12$. Lattice QCD calculations of the quark AM show large flavor asymmetries [9–13]. GPD models consistent with present experimental data also suggest a large flavor asymmetry [3–6]. The question of "isovector AM" is central to the understanding of nucleon structure and nonperturbative dynamics and needs further study.

Like any local composite operator in QCD, the quark flavor components of the EMT have matrix elements not only between states of the same hadron (form factors) but also between states of different hadrons (transition form factors). This makes it possible to formulate a concept of AM for transitions between hadronic states. Of particular interest is the AM in $N \to \Delta$ transitions. Because the isospin difference is $\Delta I = 1$, the transition AM is a pure isovector and thus related to the J^{u-d} flavor asymmetry in the nucleon. Because the structure of the N and Δ baryons is closely connected, study of the transition AM can provide further insight into nucleon structure. The transition AM can be connected with the GPDs sampled in hard exclusive processes with $N \to \Delta$ transitions, enabling its experimental study [14–16].

The $1/N_c$ expansion of QCD is a powerful method for analyzing the spin-flavor structure of hadronic matrix elements of QCD operators such as the EMT and AM [17,18]. It establishes a hierarchy among the spin-flavor components of the $N \to N$ matrix elements of the EMT. It also connects the $N \to N$ and $N \to \Delta$ (and even $\Delta \to \Delta$) matrix elements of the EMT through the emergent spin-flavor symmetry in large-N_c limit [19–23]. The method is therefore uniquely suited for analyzing the flavor structure of QCD AM in the nucleon and exploring its extension to $N \to \Delta$ transitions [3].

In this letter we study the isovector QCD AM in $N \to \Delta$ transitions and its connection with the J^{u-d} flavor asymmetry in the nucleon. We formulate the concept of transition AM using light-front densities of the EMT in transversely localized baryon states. We calculate the $N \to \Delta$ transition AM in the $1/N_c$ expansion, connect it with the J^{u-d} flavor asymmetry in the nucleon, and estimate its numerical value using lattice QCD results. In the same

E-mail addresses: jykim@jlab.org (J.-Y. Kim), hoywon@inha.edu (H.-Y. Won), goity@jlab.org (J.L. Goity), weiss@jlab.org (C. Weiss).

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setup we connect the transition AM to the GPDs sampled in hard exclusive processes with $N \to \Delta$ transitions.

2. Transition angular momentum

The definition of the QCD AM operator and the interpretation of its $N \to N$ matrix elements have been discussed extensively in the literature [1,2]. The extension to transitions $B \to B'$ between baryon states with different mass and spin raises new questions that require fresh consideration. The definition of the nucleon AM of Ref. [7] uses the specific form of the $N \to N$ matrix element of the EMT and cannot immediately be extended to $B \to B'$ transitions. The definition of the nucleon AM density of Ref. [8] uses the Breit frame and assumes heavy nucleons (non-relativistic motion) and becomes ambiguous for transitions between states with different mass.

The formulation of AM as a transverse density at fixed light-front time [2] offers a natural framework for the extension to $B \to B'$ transitions. The light-front formulation is fully relativistic and can be extended to transitions between states with different mass and spin. It permits the preparation of transversely localized states independently of their mass, using the effectively non-relativistic kinematics in the transverse space. It contains a prescription for defining the hadron spin states through the light-front helicity, which enables consistent spin decomposition of the matrix elements. It also provides a simple mechanical picture of the longitudinal AM density as the cross product of transverse position and the momentum density measured by the EMT, which can be applied directly to $B \to B'$ transitions (see below). Transverse densities for $N \to \Delta$ transitions have been used successfully in the description of electromagnetic structure [24]. Here we employ this formulation to define the AM in $B \to B'$ transitions and discuss its properties.

In the following we use the symmetric (Belinfante-improved) version of the EMT, which gives rise to an AM operator measuring the total AM; the separation of spin and orbital AM is discussed below [1,2]. The operator describing the contribution of quarks with flavor f is

$$\hat{T}_f^{\alpha\beta}(x) = i\bar{\psi}_f(x)\gamma^{\{\alpha}\overleftrightarrow{\nabla}^{\beta\}}\psi_f(x), \tag{1}$$

where $\overleftrightarrow{\nabla}^{\mu} \equiv \frac{1}{2}(\overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu}) - igA^{\mu}$ is the covariant derivative and $\{\alpha\beta\} \equiv \frac{1}{2}(\alpha\beta + \beta\alpha)$; the operator for gluons is given in Ref. [2] and not needed here. We assume two quark flavors and define the isoscalar and isovector components as

$$(\hat{T}^{V,S})^{\alpha\beta} \equiv \hat{T}_u^{\alpha\beta} \pm \hat{T}_d^{\alpha\beta\alpha\beta}. \tag{2}$$

Note that these quark operators are not conserved currents; only the sum of the isoscalar quark and gluon EMT is a conserved current obtained from Noether's theorem. We consider the transition matrix elements of the operators Eq. (2) between general baryon states with masses m and m' and 4-momenta p and p',

$$\langle B', p'|\hat{T}^{\alpha\beta}(0)|B, p\rangle, \tag{3}$$

where $B \equiv \{S, S_3, I, I_3\}$ and $B' \equiv \{S', S_3', I', I_3'\}$ collectively denote the spin-isospin quantum numbers. The choice of spin states and the spin-isospin dependence of the matrix element are discussed below. The 4-momentum transfer is $\Delta \equiv p' - p$, the invariant momentum transfer $t \equiv \Delta^2$, and the average baryon 4-momentum is $P \equiv (p' + p)/2$. The 4-vectors and tensors are described by the light-front components $p^{\pm} \equiv p^0 \pm p^3$, $\boldsymbol{p}_T \equiv (p^1, p^2)$. We consider Eq. (3) in a class of frames where $\Delta^+ = 0$ and $\boldsymbol{P}_T = 0$ (generalized Drell-Yan-West frame). In these frames $t = -\boldsymbol{\Delta}_T^2 < 0$.[1] In the notation $p = [p^+, p^-, \boldsymbol{p}_T]$, the momentum components are given by

$$p = \left[p^+, \frac{m^2 + |\boldsymbol{\Delta}_T|^2/4}{p^+}, -\frac{\boldsymbol{\Delta}_T}{2}\right],$$
$$p' = \left[p^+, \frac{m'^2 + |\boldsymbol{\Delta}_T|^2/4}{p^+}, \frac{\boldsymbol{\Delta}_T}{2}\right],$$
$$\Delta = \left[0, \frac{m'^2 - m^2}{p^+}, \boldsymbol{\Delta}_T\right]. \tag{4}$$

p^+ remains undetermined, and its choice selects a particular frame in the class (boost parameter). The matrix element Eq. (3) becomes function of $\boldsymbol{\Delta}_T$. For constructing the AM, we take the $+i$ (i = 1, 2) component of the EMT

$$T^{+i}(\boldsymbol{\Delta}_T|B', B) \equiv \langle B', p'|\hat{T}^{+i}(0)|B, p\rangle, \tag{5}$$

and define a transverse coordinate density as

$$T^{+i}(\boldsymbol{b}|B', B) \equiv \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_T\boldsymbol{b}} T^{+i}(\boldsymbol{\Delta}_T|B', B). \tag{6}$$

This quantity can be interpreted as the transition matrix element of T^{+i} at the transverse position $\boldsymbol{b}$ between baryon states localized in transverse space at the origin [26–28]. The different masses $m' \neq m$ do not affect the preparation of the localized states because the description of the transverse motion in light-front quantization is independent of the mass, as in a non-relativistic system. We define the longitudinal transition AM as (the superscript z denotes the 3-component)

$$2S^z(S_3', S_3)\, J_{B\to B'} \equiv \frac{1}{2p^+}\int d^2b\, \left[\boldsymbol{b} \times \boldsymbol{T}^{+T}(\boldsymbol{b}|B', B)\right]^z, \tag{7}$$

where the factor $S^z(S_3', S_3)$ accounts for the kinematic spin dependence (to be specified below) and $J_{B\to B'}$ is independent of the spin projections S_3, S_3' (reduced matrix element). Equation (7) generalizes the light-front AM definition for diagonal transitions discussed in Refs. [2,29,30]. The integrand can be interpreted as the transverse coordinate space density of AM and gives rise to a simple mechanical picture [2,29,30]. In terms of the transverse momentum-dependent matrix element Eq. (5), the AM Eq. (7) is expressed as

$$2S^z(S_3', S_3)\, J_{B\to B'} = \frac{1}{2p^+}\left[-i\frac{\partial}{\partial\boldsymbol{\Delta}_T} \times \boldsymbol{T}^{+T}(\boldsymbol{\Delta}_T|B', B)\right]^z_{\boldsymbol{\Delta}_T=0}. \tag{8}$$

The baryon spin states in Eq. (5) are chosen as light-front helicity states. They are obtained by light-front boosts from rest-frame spin states with spins quantized in z-direction, and thus effectively depend on the rest-frame spins S, S' and their projections S_3, S_3'. The dependence of the matrix element Eq. (5) on the transverse direction of $\boldsymbol{\Delta}_T$ and on the spin projections S_3 and S_3' is kinematic and can be made explicit by performing a transverse multipole expansion. Showing only the dipole term (linear in $\boldsymbol{\Delta}_T$) that gives rise to the longitudinal AM, we write

[1] In the matrix element Eq. (3) the invariant momentum transfer can also attain timelike values $0 < \Delta^2 < (m' - m)^2$. The following definition of the transition AM and its density refers to the spacelike part of the form factors. This is consistent with the standard definition of the transition magnetic moment through the $t = 0$ magnetic transition form factor in electromagnetic processes [25]. In the $1/N_c$ expansion of N–Δ transition matrix elements, $(m_\Delta - m_N)^2 = \mathcal{O}(N_c^{-2})$ is strongly suppressed.

$$\boldsymbol{T}^{+T}(\boldsymbol{\Delta}_T|S_3', S_3) = 2p^+[i\boldsymbol{\Delta}_T \times \boldsymbol{e}_z S^z(S_3', S_3)]\, F_1(-\boldsymbol{\Delta}_T^2) + \ldots, \quad (9)$$

where $\boldsymbol{e}_z$ is the unit vector in the z-direction, $S^z(S_3', S_3)$ is the z-component of a spin 3-vector depending on the rest-frame spin projections S_3 and S_3' (in a form that is specific to the spins S and S'), and $F_1(t)$ is a form factor. For a $\frac{1}{2} \to \frac{1}{2}$ transition ($N \to N$), the z-component of the spin vector is

$$S^z(S_3', S_3) \equiv S_3\, \delta(S_3, S_3') = \pm\tfrac{1}{2}. \quad (10)$$

More generally, for any transition between states of the same spin $S \to S$ with $S = \frac{1}{2}, \frac{3}{2}, \ldots$ ($N \to N, \Delta \to \Delta, \ldots$), the z-component of the spin vector is

$$S^z(S_3', S_3) = \sqrt{S(S+1)}\, \langle S S_3, 10 | S S_3' \rangle, \quad (11)$$

where $\langle j_1 m_1 j_2 m_2 | J M \rangle$ are the vector coupling coefficients. For transitions between states with spins $|S' - S| = 0, 1$ such as $\frac{1}{2} \to \frac{3}{2}$ ($N \to \Delta$) we define the spin vector such that

$$S^z(S_3', S_3) = \sqrt{S(S+1)} \sqrt{\frac{2S+1}{2S'+1}}\, \langle S S_3, 10 | S' S_3' \rangle, \quad (12)$$

which reduces to Eq. (11) if $S' = S$ (with this definition the form factor is independent of S and S' in large-N_c limit; see below). In each case, the AM obtained from Eq. (9) with Eq. (8) is then given by the form factor at $t = 0$

$$J_{B \to B'} = F_1(0). \quad (13)$$

The normalization of $J_{B \to B'}$ adopted here is such that the spin sum rule for the nucleon, which involves the isoscalar quark and the gluon EMT, is [2,30]

$$J^S_{N \to N} + J^{\mathrm{glu}}_{N \to N} = \tfrac{1}{2}. \quad (14)$$

The isospin dependence of the matrix element Eq. (3) is governed by the usual selection rules. The isoscalar component of the EMT in Eq. (2) connects only states with $I' = I$, while the isovector component can connect states with $|I' - I| = 0$ or 1. In both cases the isospin projection is conserved, $I_3' = I_3$. More generally, the matrix element of the isovector operator in Eq. (2) is proportional to $\langle I I_3, 10 | I' I_3 \rangle$; for the transition $\frac{1}{2} \to \frac{3}{2}$ ($N \to \Delta$) this factor is $\langle \frac{1}{2} I_3, 10 | \frac{3}{2} I_3 \rangle = \sqrt{2/3}$ for both $I_3 = \pm\frac{1}{2}$. These isospin factors are included in the values of the transition AM defined in Eqs. (7) and (8).

3. $N \to \Delta$ transition angular momentum in $1/N_c$ expansion

In the $N_c \to \infty$ limit of QCD, the dynamics is characterized by the emergent SU($2N_f$) spin-flavor symmetry (here $N_f = 2$) [19–23]. The N and Δ baryons appear in the totally symmetric representation with spin/isospin $S = I = \frac{1}{2}, \frac{3}{2}, \ldots$. Transition matrix elements of QCD operators between these states are thus connected by the symmetry. A systematic expansion in $1/N_c$ can be performed, including subleading corrections [22,23]. The baryon masses are $m_{N,\Delta} = \mathcal{O}(N_c)$, and the mass splitting is $m_\Delta - m_N = \mathcal{O}(N_c^{-1})$. Here we apply this method to the transition matrix elements of the EMT and the transition AM.

The $1/N_c$ expansion of baryon transition matrix elements is performed in a class of frames where the baryons have 3-momenta $|\boldsymbol{p}|, |\boldsymbol{p}'| = \mathcal{O}(N_c^0)$, so that the velocities are parametrically small, $|\boldsymbol{p}|/m, |\boldsymbol{p}'|/m' = \mathcal{O}(N_c^{-1})$. The 3-momentum transfer is $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p} = \mathcal{O}(N_c^0)$, and the energy transfer for transitions within the same multiplet is $\Delta^0 = E' - E = \mathcal{O}(N_c^{-1})$. In particular, for the $1/N_c$ expansion of the EMT we choose the symmetric frame where the average baryon 3-momentum is zero, $\boldsymbol{P} = (\boldsymbol{p}' + \boldsymbol{p})/2 = 0$ (generalized Breit frame). In the notation $p = (p^0, \boldsymbol{p})$, the 4-momentum components are given by

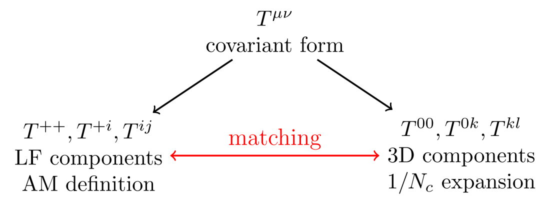

Fig. 1. Matching of light-front and 3D components of the EMT.

$$\begin{aligned} p &= (E, -\boldsymbol{\Delta}/2), & E &= \sqrt{m^2 + |\boldsymbol{\Delta}|^2/4}, \\ p' &= (E', \quad \boldsymbol{\Delta}/2), & E' &= \sqrt{m'^2 + |\boldsymbol{\Delta}|^2/4}, \\ \Delta &= (E' - E, \boldsymbol{\Delta}). \end{aligned} \quad (15)$$

In this frame the only 3-vector arising from the particle momenta is the momentum transfer $\boldsymbol{\Delta}$. The matrix elements of the tensor operator obey standard angular momentum selection rules, and a multipole expansion can be performed for the components

$$T^{00},\ T^{0k},\ T^{kl} \quad (k, l = 1, 2, 3). \quad (16)$$

The $1/N_c$ expansion of the light-front components of the EMT of Sec. 2 is obtained by matching the ordinary 4-vector components with the light-front components in the same frame (see Fig. 1). The symmetric frame Eq. (15) is contained in the class of $\Delta^+ = 0$ frames Eq. (4); namely, it is the frame with

$$p^+ = \sqrt{P^2} = \sqrt{(m^2 + m'^2)/2 - t/4}. \quad (17)$$

The light-front energy transfer in this frame is $\Delta^- = \mathcal{O}(N_c^{-1})$, see Eq. (4), and thus small and of the same order as the ordinary energy transfer Δ^0. The light-front components of the EMT are then calculated as

$$T^{+i} = T^{0i} + T^{3i} \quad (i = 1, 2) \quad \text{etc.} \quad (18)$$

The matching procedure performed here is unambiguous since one is dealing with on-shell matrix elements. Because the $1/N_c$ expansion of the 3D components of the EMT matrix element respects 3D rotational invariance, the matching procedure implements 3D rotational invariance for the light-front components of the matrix element; this property is not manifest in the light-front formulation and imposes conditions on the light-front matrix elements.[2]

We have computed the $1/N_c$ expansion of the 3-dimensional multipoles of the EMT in the symmetric frame Eq. (15) using a method based on the soliton picture of large-N_c baryons [3,35]; equivalently one can use methods based on the algebra of the spin-flavor symmetry group [22,23]. The full results will be presented elsewhere [36]; in the following we quote only the multipoles relevant to the AM. In leading order of $1/N_c$, the matrix elements of the isoscalar and isovector components [see Eq. (2)] of T^{0k} are of the form

[2] A similar procedure of matching light-front matrix elements with 3-dimensional Breit frame matrix elements is used in the construction of current operators in dynamical models of interacting few-body systems in light-front quantization (so-called angular conditions); see Refs. [31–34] and references therein. In our study here we do not construct an EMT operator in terms of constituent degrees of freedom but work directly with the matrix elements provided by the $1/N_c$ expansion.

$$\langle B', \mathbf{\Delta}/2|(\hat{T}^S)^{0k}|B, -\mathbf{\Delta}/2\rangle = 2m^2\langle S^i\rangle_{B'B}\left[i\epsilon^{kil}\frac{\Delta^l}{m}\mathcal{J}_1^S(t) + \ldots\right], \tag{19}$$

$$\langle B', \mathbf{\Delta}/2|(\hat{T}^V)^{0k}|B, -\mathbf{\Delta}/2\rangle = 2m^2\langle D^{3i}\rangle_{B'B}\left[i\epsilon^{kil}\frac{\Delta^l}{m}\mathcal{J}_1^V(t) + \ldots\right], \tag{20}$$

where we have omitted spin-independent terms $\propto \Delta^k$ that do not contribute to the AM. The spin/isospin dependence is contained in the structures (here $i = 0, \pm 1$ denote the spherical 3-vector components)

$$\langle S^i\rangle_{B'B} = \sqrt{S(S+1)}\,\langle SS_3, 1i|S'S_3'\rangle\,\delta_{S'S}\delta_{I'I}\delta_{I_3'I_3}, \tag{21}$$

$$\langle D^{3i}\rangle_{B'B} = -\sqrt{\frac{2S+1}{2S'+1}}\,\langle SS_3, 1i|S'S_3'\rangle\,\langle II_3, 10|I'I_3'\rangle. \tag{22}$$

S^i has only matrix elements between same spin/isospin, while D^{3i} can connect states with spin/isospin differing by one.[3] Thus $(\hat{T})^S$ in Eq. (19) contributes only to $N \to N$ and $\Delta \to \Delta$ transitions, while $N \to \Delta$ transitions arise only from $(\hat{T})^V$ in Eq. (20). $\mathcal{J}_1^{S,V}(t)$ in Eqs. (19) and (20) are the isoscalar and isovector dipole form factors. They are found to be of the order [36]

$$\mathcal{J}_1^S = \mathcal{O}(N_c^0), \qquad \mathcal{J}_1^V = \mathcal{O}(N_c). \tag{23}$$

The matrix elements of T^{3k} are suppressed by $1/N_c$ compared to those of T^{0k} in both the isoscalar and isovector sector. The light-front component T^{+i} is therefore given by T^{0k} in leading order of the $1/N_c$ expansion, and we can compute the AM Eq. (8) from Eqs. (20)–(23). We find:

(i) The isovector AM in the nucleon is leading in $1/N_c$; the isoscalar is subleading.

$$J^S_{N\to N} = \mathcal{J}_1^S(0) = \mathcal{O}(N_c^0), \qquad J^V_{p\to p} = -\frac{2}{3}\mathcal{J}_1^V(0) = \mathcal{O}(N_c). \tag{24}$$

This explains the observed large flavor asymmetry of the AM. Note that this scaling is consistent with that of the quark spin contribution to the nucleon spin as given by the axial coupling, $g_A^S = \mathcal{O}(N_c^0)$ and $g_A^V = \mathcal{O}(N_c^1)$.

(ii) The isoscalar component of the AM in the nucleon and Δ are related by

$$J^S_{N\to N} = J^S_{\Delta\to\Delta} = \mathcal{J}_1^S(0). \tag{25}$$

This provides insight into the spin structure of Δ resonance. Note that this relation is consistent with the spin sum rule for the Δ state.

(iii) The isovector AM in the nucleon, the AM in the $N \to \Delta$ transitions, and the isovector AM in the Δ are related by

$$J^V_{p\to p} = \frac{1}{\sqrt{2}}J^V_{p\to\Delta^+} = 5J^V_{\Delta^+\to\Delta^+} = -\frac{2}{3}\mathcal{J}_1^V(0). \tag{26}$$

This suggests that the $N \to \Delta$ transition AM is large and provides a way to probe the isovector nucleon AM with $N \to \Delta$ transition measurements.

Table 1
Estimates of the isoscalar and the isovector AM for $p \to p$, $p \to \Delta^+$ and $\Delta^+ \to \Delta^+$ obtained from lattice QCD data on $J^S_{p\to p}$ and $J^V_{p\to p}$ and the relations provided by the leading-order $1/N_c$ expansion. Here $S, V \equiv u \pm d$, and the nucleon matrix elements are normalized as in Eq. (14). Input values are marked by an asterisk *.

Lattice QCD	$J^S_{p\to p}$	$J^S_{\Delta^+\to\Delta^+}$	$J^V_{p\to p}$	$J^V_{p\to\Delta^+}$	$J^V_{\Delta^+\to\Delta^+}$
[9] $\mu^2 = 4\,\text{GeV}^2$	0.33*	0.33	0.41*	0.58	0.08
[10] $\mu^2 = 4\,\text{GeV}^2$	0.21*	0.21	0.22*	0.30	0.04
[11] $\mu^2 = 4\,\text{GeV}^2$	0.24*	0.24	0.23*	0.33	0.05
[12] $\mu^2 = 1\,\text{GeV}^2$	–	–	0.23*	0.33	0.05
[13] $\mu^2 = 4\,\text{GeV}^2$	–	–	0.17*	0.24	0.03

4. $N \to \Delta$ transition angular momentum from lattice QCD

We now evaluate the transition AM using the leading-order $1/N_c$ expansion relations together with lattice QCD results for the EMT matrix elements. This provides a numerical estimate of the transition AM and illustrates the dominance of the isovector component of the nucleon AM. Lattice QCD calculations of $N \to N$ matrix elements of the symmetric EMT Eq. (1) have been performed in various setups (fermion implementation, normalization scale, pion mass) [9–13]. Using these as input, we obtain the values listed in Table 1. One observes that a sizable isovector component of the nucleon AM is obtained in all lattice calculations (similar large values are obtained in the chiral quark-soliton model [37]). Note that the lattice results for the isoscalar nucleon AM in Refs. [9–11] are more uncertain than the isovector, as they involve disconnected diagrams and require careful treatment of the mixing of quark and gluon operators. Furthermore, when comparing the $1/N_c$ expansion with numerical values of matrix elements, one needs to keep in mind that it is a parametric expansion, and that the numerical values are determined not only by the power of $1/N_c$ but also coefficients of order unity.

5. $N \to \Delta$ transition angular momentum from GPDs

We now connect the $N \to \Delta$ transition AM with the transition GPDs measured in hard exclusive electroproduction processes such as DVCS $eN \to e'\gamma\Delta$ [14]. This opens the prospect of future experimental studies of the transition AM. QCD factorization at leading-twist accuracy expresses the amplitudes of hard exclusive processes in terms of matrix elements of quark light-ray (or partonic) operators of the type [3–5]

$$\hat{O}_f(z) = \bar{\psi}_f(-z/2)[-z/2, z/2]\not{z}\psi_f(z/2), \tag{27}$$

where z is a light-like 4-vector ($z^2 = 0$) and $[-z/2, z/2]$ denotes the gauge link operator. The non-local operator Eq. (27) can be represented as a power series in the distance z,

$$\hat{O}_f(z) = z^\alpha\bar{\psi}_f(0)\gamma_\alpha\psi_f(0) + z^\alpha z^\beta\bar{\psi}_f(0)\gamma_{\{\alpha}\overleftrightarrow{\nabla}_{\beta\}}\psi_f(0) + \ldots, \tag{28}$$

where the coefficients are local operators representing totally symmetric traceless tensors of spin $n \geq 1$ (twist-2 operators). The spin-2 operator coincides with the symmetric EMT Eq. (1). The light-like vector z is chosen such that it has light-front components along the "minus" direction, $z^- \neq 0, z^+ = \boldsymbol{z}_T = 0$. The expansion Eq. (28) thus involves the light-front component T^{++} of the EMT. In our approach based on the $1/N_c$ expansion, this light-front component

[3] The matrix elements Eq. (21) and (22) appear from the collective quantization of the soliton rotations [3,35]. In the formulation of the $1/N_c$ expansion based on the SU(4) spin-flavor symmetry [21–23], $\langle D^{ai}\rangle_{B'B}(i, a = 1, 2, 3)$ is related to the matrix element of the spin-flavor generator G^{ia}, namely $\langle D^{ai}\rangle_{B'B} = -4/(N_c+2)\langle G^{ia}\rangle_{B'B} + \mathcal{O}(N_c^{-2})$.

can be related to the 3D components T^{00}, T^{0i}, T^{ij}, and in this way be connected with the light-front component T^{+i} entering in the transition AM Eq. (7). This establishes a connection between the transition AM defined by Eq. (7) and the leading-twist partonic operators Eq. (27).

The transition matrix element of the isovector light-ray operator Eq. (27) between N and Δ states (here, between p and Δ^+ states) is parametrized covariantly through the spectral representation

$$\langle \Delta^+, p' | \hat{O}^V(z) | p, p \rangle = \sqrt{\tfrac{2}{3}} \sum_{I=M,E,C} \int_{-1}^{1} dx\, e^{-ixP\cdot z}\, H_I(x,\xi,t) \times \overline{u}^{\alpha}(p', S_3')\, (\mathcal{K}_I)_{\alpha\beta}\, z^{\beta}\, u(p, S_3). \tag{29}$$

u^{α} is the spin-$\frac{3}{2}$ Rarita-Schwinger vector-bispinor of the Δ, and u is the spin-$\frac{1}{2}$ bispinor of the nucleon. For the invariant bilinear forms in the decomposition in Eq. (29), various choices are possible (see also discussion below). Here we use the tensors as defined in Ref. [3]. The magnetic tensor (M) is

$$(\mathcal{K}_M)^{\alpha\beta} = \frac{3(m_\Delta + m_N)}{2m_N[(m_\Delta + m_N)^2 - t]}\, i\varepsilon^{\alpha\beta\gamma\delta} P_\gamma \Delta_\delta; \tag{30}$$

the other structures are given in Ref. [3]. The GPDs $H_I(x,\xi,t)$ in Eq. (29) depend on the spectral variable x, the light-cone momentum transfer $\xi \equiv -\Delta\cdot z/(2P\cdot z)$, and the invariant momentum transfer t. They are defined such that their first moments satisfy the relations (sum rules)

$$\int_{-1}^{1} dx\, H_{M,E,C}(x,\xi,t) = 2G^*_{M,E,C}(t), \tag{31}$$

where $G^*_{M,E,C}(t)$ are the $\gamma N\Delta$ transition form factors of Ref. [25], defined by multipole expansion of the decay $\Delta \to \gamma N$ in the Δ rest frame (magnetic dipole, electric quadrupole, and Coloumb quadrupole form factors).

In the context of the $1/N_c$ expansion we can now relate the N–Δ transition AM of Sec. 2 to the second moments of the GPDs of Eq. (29). The $1/N_c$ expansion of the GPDs is performed in the parametric regime where $x, \xi = \mathcal{O}(N_c^{-1})$ and $t = \mathcal{O}(N_c^0)$ [38,39] and can be implemented using the techniques described in Refs. [3,35]. The dominant $N \to \Delta$ GPD is the magnetic GPD H_M. In the large-N_c limit it scales as [3]

$$H_M(x,\xi,t) \sim N_c^3 \times \text{function}(N_c x, N_c \xi, t). \tag{32}$$

The power N_c^3 multiplying the scaling function can be inferred from the known N_c scaling of the N–Δ transition magnetic moment, which determines the first moment of H_M through Eq. (31),

$$\frac{\mu_{\Delta N}}{m} \equiv \frac{G^*_M(0)}{m} = \frac{1}{2m}\int_{-1}^{1} dx\, H_M(x,\xi,0) = \mathcal{O}(N_c), \tag{33}$$

where $m = \mathcal{O}(N_c)$ is the common baryon mass in the large-N_c limit. In leading order of $1/N_c$ we obtain

$$\int_{-1}^{1} dx\, x H_M(x,\xi,0) = 2J^V_{p\to\Delta^+} = -\frac{4\sqrt{2}}{3}\mathcal{J}^V_1(0), \tag{34}$$

which agrees with the N_c scaling established earlier, Eq. (24).[4] The derivation uses the covariant decomposition of the transition matrix elements of the EMT of Ref. [41]. This provides the desired connection between the $N \to \Delta$ transition AM as defined in Eq. (7) and the second moment of the transition GPDs.

In this study we have defined the transition AM through the T^{+i} component of the EMT, which can be interpreted as the cross product of momentum and distance in the transverse plane. The AM can be defined alternatively through the T^{++} component of the EMT, which can be understood as a dipole distortion of the two-dimensional momentum distribution when the baryon spins are polarized in the transverse direction. This definition of the transition AM will be explored elsewhere [36].

6. Discussion

In this work we have introduced the concept of transition AM and applied it to $N \to \Delta$ transitions in the context of the $1/N_c$ expansion. We want to discuss the significance and limitations of the present results and possible future extensions.

The present calculations are limited to the leading order of the $1/N_c$ expansion. At this level the N–Δ mass difference can be neglected, and the relation between the light-front components and the 3-dimensional multipoles of the EMT involve only a single structure. However, the method developed in Sec. 3 is general and permits also the calculation of subleading terms. They include "dynamical" corrections due to $1/N_c$ suppressed structures, and "kinematic" corrections due to the baryon motion and finite masses. Computing these corrections will be the objective of future work.

The present study uses the symmetric version of the EMT, which measures the total AM of field configurations in QCD [2]. Separation of orbital and spin AM in $B \to B'$ transitions would be possible by extending the definitions Eq. (7) et seq. to the non-symmetric EMT and the spin operator [2]. Our results show that the total AM and the quark spin (represented by the axial current) have the same $1/N_c$ scaling in the isoscalar and isovector sector, see Eq. (24), which is natural and required for consistent scaling of the isoscalar spin sum rule. The separation of spin and orbital AM is a question of dynamics and can be studied with dynamical models that are consistent with the $1/N_c$ expansion, such as the chiral quark-soliton model.

The operators describing the quark contributions to the EMT and the AM contributions derived from it are scale dependent; only the sum of isoscalar quark and gluon AM is protected by the spin sum rule and scale-independent; see e.g. Refs. [42,43]. An advantage of the isovector component is that the scale dependence is much weaker than that of the isoscalar (or of individual quark flavor components), as there is no mixing with gluon operators. The scale dependence can be taken into account in a more quantitative analysis.

Some comments are in order regarding the conservation of the EMT. Only the total EMT of QCD, given by the sum of the isoscalar quark and gluon tensors, is a conserved current as obtained from Noether's theorem, and its matrix elements Eq. (3) satisfy $\langle B'|(T^S)^{\mu\nu} + (T^g)^{\mu\nu}|B\rangle\, \Delta_\nu = 0$. The isovector quark EMT studied here is generally not conserved, and its matrix elements are not subject to a corresponding condition. This circumstance must be taken into account when performing a covariant decomposition of the transition matrix elements of the isovector quark EMT. The multipole analysis and $1/N_c$ expansion performed in the

[4] The coefficient in Eq. (34) agrees with the one in the large-N_c relation between the $N \to \Delta$ and $N \to N$ GPDs quoted in Ref. [40], but disagrees with the one quoted in Ref. [3].

present study rely only on 3-dimensional rotational invariance and discrete symmetries and do not assume any relation between the multipole structures.

The definition of the $N \to \Delta$ transition GPDs of Ref. [3] and other works refers to the $\gamma N \Delta$ transition form factors of Ref. [25], which are defined through a multipole expansion of the decay $\Delta \to \gamma N$ in the Δ rest frame. While this frame can be used in the entire physical region of $t < (m_\Delta - m_N)^2$, it does not appear natural for the definition of light-front transition matrix elements. It would be worth to revisit the definition of the transition GPDs using the class of frames introduced in Sec. 2.

In this work we have described a method for computing the $1/N_c$ expansion of light-front tensor operators by matching the light-front components with 3D components in a special frame. The procedure implements 3-dimensional rotational invariance of the light-front components (which is encoded in the matrix elements of the 3D components) order-by-order in $1/N_c$. The method is general and can be extended to other states and operators than those considered here. It can be applied to matrix elements of the EMT in hadronic states with higher spin. A particular advantage here is that it does not require the covariant decomposition of the matrix element in terms of invariant form factors, which becomes very cumbersome for higher spins. The method can also be applied to other tensor operators, such as the generalized form factors of twist-2 spin-n operators.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] E. Leader, C. Lorcé, The angular momentum controversy: what's it all about and does it matter?, Phys. Rep. 541 (3) (2014) 163–248, https://doi.org/10.1016/j.physrep.2014.02.010, arXiv:1309.4235.
[2] C. Lorcé, L. Mantovani, B. Pasquini, Spatial distribution of angular momentum inside the nucleon, Phys. Lett. B 776 (2018) 38–47, https://doi.org/10.1016/j.physletb.2017.11.018, arXiv:1704.08557.
[3] K. Goeke, M.V. Polyakov, M. Vanderhaeghen, Hard exclusive reactions and the structure of hadrons, Prog. Part. Nucl. Phys. 47 (2001) 401–515, https://doi.org/10.1016/S0146-6410(01)00158-2, arXiv:hep-ph/0106012.
[4] M. Diehl, Generalized parton distributions, Phys. Rep. 388 (2003) 41–277, https://doi.org/10.1016/j.physrep.2003.08.002, arXiv:hep-ph/0307382.
[5] A.V. Belitsky, A.V. Radyushkin, Unraveling hadron structure with generalized parton distributions, Phys. Rep. 418 (2005) 1–387, https://doi.org/10.1016/j.physrep.2005.06.002, arXiv:hep-ph/0504030.
[6] K. Kumerički, S. Liuti, H. Moutarde, GPD phenomenology and DVCS fitting: entering the high-precision era, Eur. Phys. J. A 52 (6) (2016) 157, https://doi.org/10.1140/epja/i2016-16157-3, arXiv:1602.02763.
[7] X.-D. Ji, Gauge-invariant decomposition of nucleon spin, Phys. Rev. Lett. 78 (1997) 610–613, https://doi.org/10.1103/PhysRevLett.78.610, arXiv:hep-ph/9603249.
[8] M.V. Polyakov, Generalized parton distributions and strong forces inside nucleons and nuclei, Phys. Lett. B 555 (2003) 57–62, https://doi.org/10.1016/S0370-2693(03)00036-4, arXiv:hep-ph/0210165.
[9] M. Göckeler, R. Horsley, D. Pleiter, P.E.L. Rakow, A. Schäfer, G. Schierholz, W. Schroers, Generalized parton distributions from lattice QCD, Phys. Rev. Lett. 92 (2004) 042002, https://doi.org/10.1103/PhysRevLett.92.042002, arXiv:hep-ph/0304249.
[10] P. Hägler, et al., Nucleon generalized parton distributions from full lattice QCD, Phys. Rev. D 77 (2008) 094502, https://doi.org/10.1103/PhysRevD.77.094502, arXiv:0705.4295.
[11] J.D. Bratt, et al., Nucleon structure from mixed action calculations using 2+1 flavors of asqtad sea and domain wall valence fermions, Phys. Rev. D 82 (2010) 094502, https://doi.org/10.1103/PhysRevD.82.094502, arXiv:1001.3620.
[12] G.S. Bali, S. Collins, M. Göckeler, R. Rödl, A. Schäfer, A. Sternbeck, Nucleon generalized form factors from two-flavor lattice QCD, Phys. Rev. D 100 (1) (2019) 014507, https://doi.org/10.1103/PhysRevD.100.014507, arXiv:1812.08256.
[13] C. Alexandrou, et al., Moments of nucleon generalized parton distributions from lattice QCD simulations at physical pion mass, Phys. Rev. D 101 (3) (2020) 034519, https://doi.org/10.1103/PhysRevD.101.034519, arXiv:1908.10706.
[14] K.M. Semenov-Tian-Shansky, M. Vanderhaeghen, Deeply-virtual compton process $e^- N \to e^- \gamma \pi N$ to study nucleon to resonance transitions, arXiv:2303.00119, 2023.
[15] P. Kroll, K. Passek-Kumerički, Transition GPDs and exclusive electroproduction of π-Δ(1232) final states, Phys. Rev. D 107 (5) (2023) 054009, https://doi.org/10.1103/PhysRevD.107.054009, arXiv:2211.09474.
[16] S. Diehl, et al., First measurement of hard exclusive $\pi^- \Delta^{++}$ electroproduction beam-spin asymmetries off the proton, arXiv:2303.11762, 2023.
[17] G. 't Hooft, A planar diagram theory for strong interactions, Nucl. Phys. B 72 (1974) 461, https://doi.org/10.1016/0550-3213(74)90154-0.
[18] E. Witten, Baryons in the $1/N$ expansion, Nucl. Phys. B 160 (1979) 57–115, https://doi.org/10.1016/0550-3213(79)90232-3.
[19] J.-L. Gervais, B. Sakita, Large-N QCD baryon dynamics: exact results from its relation to the static strong coupling theory, Phys. Rev. Lett. 52 (1984) 87, https://doi.org/10.1103/PhysRevLett.52.87.
[20] J.-L. Gervais, B. Sakita, Large-N baryonic soliton and quarks, Phys. Rev. D 30 (1984) 1795, https://doi.org/10.1103/PhysRevD.30.1795.
[21] R.F. Dashen, A.V. Manohar, Baryon-pion couplings from large-N_c QCD, Phys. Lett. B 315 (1993) 425–430, https://doi.org/10.1016/0370-2693(93)91635-Z, arXiv:hep-ph/9307241.
[22] R.F. Dashen, E.E. Jenkins, A.V. Manohar, $1/N_c$ expansion for baryons, Phys. Rev. D 49 (1994) 4713, https://doi.org/10.1103/PhysRevD.51.2489, Erratum: Phys. Rev. D 51 (1995) 2489, arXiv:hep-ph/9310379.
[23] R.F. Dashen, E.E. Jenkins, A.V. Manohar, Spin flavor structure of large N_c baryons, Phys. Rev. D 51 (1995) 3697–3727, https://doi.org/10.1103/PhysRevD.51.3697, arXiv:hep-ph/9411234.
[24] C.E. Carlson, M. Vanderhaeghen, Empirical transverse charge densities in the nucleon and the nucleon-to-Δ transition, Phys. Rev. Lett. 100 (2008) 032004, https://doi.org/10.1103/PhysRevLett.100.032004, arXiv:0710.0835.
[25] H.F. Jones, M.D. Scadron, Multipole γN–Δ form factors and resonant photoproduction and electroproduction, Ann. Phys. 81 (1973) 1–14, https://doi.org/10.1016/0003-4916(73)90476-4.
[26] M. Burkardt, Impact parameter dependent parton distributions and off forward parton distributions for $\zeta \to 0$, Phys. Rev. D 62 (2000) 071503, https://doi.org/10.1103/PhysRevD.62.071503, Erratum: Phys. Rev. D 66 (2002) 119903, arXiv:hep-ph/0005108.
[27] M. Burkardt, Impact parameter space interpretation for generalized parton distributions, Int. J. Mod. Phys. A 18 (2003) 173–208, https://doi.org/10.1142/S0217751X03012370, arXiv:hep-ph/0207047.
[28] C. Granados, C. Weiss, Chiral dynamics and peripheral transverse densities, J. High Energy Phys. 01 (2014) 092, https://doi.org/10.1007/JHEP01(2014)092, arXiv:1308.1634.
[29] L. Adhikari, M. Burkardt, Angular momentum distribution in the transverse plane, Phys. Rev. D 94 (11) (2016) 114021, https://doi.org/10.1103/PhysRevD.94.114021, arXiv:1609.07099.
[30] C. Granados, C. Weiss, Partonic angular momentum in the nucleon's chiral periphery, Phys. Lett. B 797 (2019) 134847, https://doi.org/10.1016/j.physletb.2019.134847, arXiv:1905.02742.
[31] L.A. Kondratyuk, M.I. Strikman, Relativistic correction to the deuteron magnetic moment and angular condition, Nucl. Phys. A 426 (1984) 575–598, https://doi.org/10.1016/0375-9474(84)90165-9.
[32] P.L. Chung, W.N. Polyzou, F. Coester, B.D. Keister, Hamiltonian light-front dynamics of elastic electron-deuteron scattering, Phys. Rev. C 37 (1988) 2000–2015, https://doi.org/10.1103/PhysRevC.37.2000.
[33] B.D. Keister, Rotational covariance and light front current matrix elements, Phys. Rev. D 49 (1994) 1500–1505, https://doi.org/10.1103/PhysRevD.49.1500, arXiv:hep-ph/9303264.
[34] F.M. Lev, E. Pace, G. Salmè, Electromagnetic and weak current operators for interacting systems within the front form dynamics, Nucl. Phys. A 641 (1998) 229–259, https://doi.org/10.1016/S0375-9474(98)00469-2, arXiv:hep-ph/9807255.
[35] P. Schweitzer, C. Weiss, Spin-flavor structure of chiral-odd generalized parton distributions in the large-N_c limit, Phys. Rev. C 94 (4) (2016) 045202, https://doi.org/10.1103/PhysRevC.94.045202, arXiv:1606.08388.

[36] J.-Y. Kim, H.-Y. Won, J.L. Goity, C. Weiss, 2023, in press.
[37] H.-Y. Won, H.-C. Kim, J.-Y. Kim, Nucleon cosmological constant term and flavor structure of the gravitational form factors, arXiv:2302.02974, 2023.
[38] D. Diakonov, V. Petrov, P. Pobylitsa, M.V. Polyakov, C. Weiss, Nucleon parton distributions at low normalization point in the large N_c limit, Nucl. Phys. B 480 (1996) 341–380, https://doi.org/10.1016/S0550-3213(96)00486-5, arXiv:hep-ph/9606314.
[39] V.Y. Petrov, P.V. Pobylitsa, M.V. Polyakov, I. Börnig, K. Goeke, C. Weiss, Off-forward quark distributions of the nucleon in the large N_c limit, Phys. Rev. D 57 (1998) 4325–4333, https://doi.org/10.1103/PhysRevD.57.4325, arXiv:hep-ph/9710270.
[40] V. Pascalutsa, M. Vanderhaeghen, New large-N_c relations among the nucleon and nucleon-to-Δ GPDs, arXiv:hep-ph/0611050, 2006.
[41] J.-Y. Kim, Parametrization of transition energy-momentum tensor form factors, Phys. Lett. B 834 (2022) 137442, https://doi.org/10.1016/j.physletb.2022.137442, arXiv:2206.10202.
[42] X.-D. Ji, J. Tang, P. Hoodbhoy, The spin structure of the nucleon in the asymptotic limit, Phys. Rev. Lett. 76 (1996) 740–743, https://doi.org/10.1103/PhysRevLett.76.740, arXiv:hep-ph/9510304.
[43] A.W. Thomas, Interplay of spin and orbital angular momentum in the proton, Phys. Rev. Lett. 101 (2008) 102003, https://doi.org/10.1103/PhysRevLett.101.102003, arXiv:0803.2775.