Phys. Lett. B 850 (2024) 138489

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

Flavor structure of the energy-momentum tensor form factors of the proton

Ho-Yeon Won [a,b], Hyun-Chul Kim [a,c,*], June-Young Kim [d,*]

[a] Department of Physics, Inha University, Incheon 402-751, South Korea
[b] CPHT, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France
[c] School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Republic of Korea
[d] Theory Center, Jefferson Lab, Newport News, VA 23606, USA

ARTICLE INFO

Editor: A. Schwenk

Keywords:
Energy-momentum tensor form factors of the proton
Flavor decomposition
$\bar{c}$ form factors
Pion mean-field approach

ABSTRACT

The energy-momentum tensor form factors furnish information on the mechanics of the proton. It is essential to compute the generalized isovector-vector form factors to examine the flavor structure of the energy-momentum tensor form factors. The flavor-decomposed form factors reveal the internal structure of the proton. The up quark dominates over the down quark for the mass and spin of the proton, whereas the down quark takes over the up quark for the D-term form factor. We investigate for the first time the isovector $\bar{c}(t)$ form factor of the proton and its physical implications. The flavor-decomposed $\bar{c}(t)$ form factors of the proton unveil how the up-quark contribution is exactly canceled by the down-quark contribution inside a proton within the framework of the pion mean-field approach. While the proton $\bar{c}(t)$ form factor does not contribute to the proton mass, its flavor structure sheds light on how the strong force fields due to the $\bar{c}(t)$ form factor characterize the stability of the proton.

1. Introduction

The cosmological constant term (CCT) in Einstein's equation in general relativity encodes the vacuum energy density of the universe, arising from the quantum fluctuations [1–3]. The cosmological constant (CC) is also known to be connected to the dark energy [4,5]. In non-perturbative quantum chromodynamics (QCD), the gluon condensate gives the energy of the QCD vacuum [6,7], which can be identified as the QCD CC. If we decompose the proton matrix element of the energy-momentum tensor (EMT) operator in terms of the EMT form factors, one term is proportional to the metric tensor. Its coefficient is called the proton $\bar{c}$ form factor, which has a similar structure as the CCT in Einstein's equation, $\Lambda g_{\mu\nu}$. The proton $\bar{c}$ form factor furnishes critical information on understanding the mechanics of the proton [8,9]. Since the $\bar{c}$ form factors arise only when EMT current is not conserved, they are naturally scale-dependent [10,11]. When both the quark and gluon degrees of freedom are considered, the proton $\bar{c}$ form factor disappears, because of conservation of the EMT current. However, the proton $\bar{c}$ form factor comes into play when the flavor structure of the proton energy-momentum tensor form factors (EMTFFs) as well as the proton mass decomposition is explored. The proton mass is decomposed in terms of πN sigma terms, the quark and gluon energies, and the trace anomaly [12,13]. On the other hand, the proton $\bar{c}$ form factor was recently interpreted as the isotropic pressure-volume work [14,15] by using the relation between the EMTFFs in the forward limit and the terminologies of perfect fluid in general relativity. It also gives a clue in understanding the partial internal energy inside a proton. When one investigates the flavor structure of the proton EMTFFs, the effects of the $\bar{c}$ form factor emerge. To carry out the flavor decomposition of the EMTFFs, one has to compute the generalized isovector-vector form factors (GIVFFs). Since there is no physical reason for conservation of the isovector EMT-like current, the isovector $\bar{c}$ form factor survives.

In this Letter, we investigate the proton $\bar{c}$ form factors that arise from the GIVFFs, which we currently have no empirical information about. To calculate the proton EMTFFs and GIVFFs, we use the pion mean-field approach, also known as the chiral quark-soliton model (χQSM) [16,17]. The χQSM is built on an effective chiral action that is solely composed of the quark degrees of freedom. This effective chiral action is obtained by integrating out the gluon degrees of freedom from the instanton vacuum, and established in Refs. [18,7]. This process can be broken down into several steps: first, after the gluon degrees of freedom have been integrated out, the quark-quark interaction with a $2N_f$ vertex is derived, where N_f denotes the number of flavors. This leads to the spontaneous breakdown of chiral symmetry, which results

* Corresponding authors.
E-mail addresses: hoyeon.won@polytechnique.edu (H.-Y. Won), hchkim@inha.ac.kr (H.-C. Kim), jykim@jlab.org (J.-Y. Kim).

https://doi.org/10.1016/j.physletb.2024.138489
Received 24 July 2023; Received in revised form 24 January 2024; Accepted 25 January 2024
Available online 1 February 2024

in the emergence of Nambu-Goldstone bosons and the dynamical quark mass. Next, the $2N_f$ interaction is bosonized by incorporating pseudo-Nambu-Goldstone fields. Then we have integrated over the dressed quark fields to obtain the one-loop effective chiral action. The influence of gluons is effectively accounted for through the dynamical quark mass in a renormalization sense. Consequently, the quark EMT current alone is conserved in this framework, resulting in a vanishing proton $\bar{c}$ form factor from the quark EMT. Interestingly, the zero value of the proton $\bar{c}$ form factor [19] is deeply linked to the von Laue condition for proton stability. Ref. [20] showed that both quark and gluon contributions were insignificant.

The large N_c behaviors of the $\bar{c}$ form factors in flavor SU(2) symmetry are rather subtle. While the isovector $\bar{c}$ form factor is proportional to N_c^{-1}, the isoscalar $\bar{c}$ form factor is of N_c^0 order. On the other hand, the conservation of the EMT current forces the isoscalar $\bar{c}$ form factors to vanish. It imposes a strong constraint on the isovector $\bar{c}$ form factor: The down-quark component should always be the same as the negative up-quark component in the present framework.

2. Energy-momentum tensor form factors of the proton

The matrix element of the bilocal quark and gluon vector operators on the light cone is parametrized in terms of the vector GPDs $H^{q,g}(x,\xi,t)$ and $E^{q,g}(x,\xi,t)$, where q and g denote the quarks and gluon degrees of freedom, respectively. They are given as functions of the longitudinal momentum fraction carried by partons x, the skewedness variable ξ, and the momentum transfer squared t. Here we consider quark contributions to them only. In the leading-twist accuracy, this matrix element can be expressed in terms of the unpolarized GPDs as follows [21]:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle p(p',J_3')\middle|\bar{\psi}_q\left(-\frac{\lambda n}{2}\right) \not{n}\psi_q\left(\frac{\lambda n}{2}\right)\middle|p(p,J_3)\right\rangle$$
$$= \bar{u}(p',J_3')\left[H^q(x,\xi,t)\not{n} + E^q(x,\xi,t)\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M_p}\right]u(p,J_3), \tag{1}$$

where ψ_q is the quark field with flavor q and M_p represents the proton mass. p and p' denote the initial and final momenta. Their average and difference are defined by $P=(p'+p)/2$ and $\Delta = p'-p$ with $\Delta^2=t$, respectively. n stands for a light-cone vector satisfying $n\cdot(p'+p)=2$. The longitudinal momentum fraction of a proton carried by a parton is denoted by x and the skewedness is expressed as ξ, which is defined as $n\cdot\Delta=-2\xi$. The first and second Mellin moments of the vector GPDs are identified as the electromagnetic (EM) form factors and EMTFFs, respectively. Note that the Mellin moments of GPDs must satisfy the polynomiality, of which the maximal order is given as $n+1$ due to Lorentz invariance [21,22]. Thus, the proton generalized form factors are defined by the $(n+1)$th Mellin moments of the GPDs as follows [23]:

$$\int_{-1}^{1} dx\, x^n H^q(x,\xi,t) = \sum_{i=0,\mathrm{even}}^{n} (2\xi)^i A^q_{n+1i}(t) + (2\xi)^{n+1}C^q_{n+10}(t)|_{n,\mathrm{odd}},$$
$$\int_{-1}^{1} dx\, x^n E^q(x,\xi,t) = \sum_{i=0,\mathrm{even}}^{n} (2\xi)^i B^q_{n+1i}(t) - (2\xi)^{n+1}C^q_{n+10}(t)|_{n,\mathrm{odd}}, \tag{2}$$

where A^q_{n+1i}, B^q_{n+1i} and C^q_{n+10} stand for the generalized form factors of the quark part in QCD. The first Mellin moments $A^q_{10}(t)$ and $B^q_{10}(t)$ are identified as the Dirac and Pauli form factors of the proton, $F_1^q(t)$ and $F_2^q(t)$, respectively.

The second Mellin moments are derived as

$$\int_{-1}^{1} dx\, x H^q(x,\xi,t) = A^q_{20}(t) + 4C^q_{20}(t)\xi^2,$$
$$\int_{-1}^{1} dx\, x E^q(x,\xi,t) = B^q_{20}(t) - 4C^q_{20}(t)\xi^2. \tag{3}$$

The EMTFFs are given by the linear combinations of the second Mellin moments. The matrix element of the symmetric EMT current $\hat{T}^{\mu\nu,q} = \frac{1}{4}\bar{\psi}_q i\overleftrightarrow{\mathcal{D}}^{\{\mu}\gamma^{\nu\}}\psi_q$ [24,25] with the covariant derivative $\overleftrightarrow{\mathcal{D}}^\nu = \overleftrightarrow{\partial}^\nu - 2igA^\nu$ and $\overleftrightarrow{\partial}^\nu = \overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu$ is parametrized in terms of the four different EMTFFs A^q, J^q, D^q, and $\bar{c}^q$:

$$\left\langle p(p',J_3')\middle|\hat{T}^q_{\mu\nu}(0)\middle|p(p,J_3)\right\rangle$$
$$= \bar{u}(p',J_3')\left[A^q(t)\frac{P_\mu P_\nu}{M_p} + J^q(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_p}\right.$$
$$\left. + D^q(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_p} + \bar{c}^q(t)M_p g_{\mu\nu}\right]u(p,J_3), \tag{4}$$

where A^q, J^q, D^q, and $\bar{c}^q$ are called the flavor-decomposed light-front (LF) momentum, spin, D-term, and $\bar{c}$ form factors, respectively. As mentioned above, the second Mellin moments of the vector GPDs are related to the EMTFFs as follows

$$A^q_{20}(t) = A^q(t), \quad \frac{1}{2}\left[A^q_{20}(t)+B^q_{20}(t)\right] = J^q(t),$$
$$4C^q_{20}(t) = D^q(t). \tag{5}$$

As we observe from Eqs. (3) and (5), the leading-twist GPDs do not provide the proton $\bar{c}$ form factors. Higher-twist GPDs are required to define them [26,24].

Note that the symmetric EMT current is conserved only when both the quark and gluon parts are considered:

$$\partial^\mu\hat{T}_{\mu\nu} = 0, \qquad \hat{T}_{\mu\nu} = \hat{T}^{u+d}_{\mu\nu} + \hat{T}^g_{\mu\nu}. \tag{6}$$

At the zero momentum transfer t, thus, the EMTFFs A^{u+d} and J^{u+d} (or proton EMTFFs) are normalized as $A^p = A^{u+d}(0) + A^g(0) = 1$ and $J^p = J^{u+d}(0) + J^g(0) = \frac{1}{2}$ together with the gluon contributions. However, there is no such constraint on the GIVFFs as well as the D-term. Note that the GIVFFs are derived from the isovector EMT-like current that is not conserved. The non-conserved isovector EMT-like current implies that $\bar{c}^q$ form factors with a specific flavor q do not need to vanish. Thus, the flavor-decomposed $\bar{c}$ form factors of the proton should be finite. In the current work, we will scrutinize the physical implications of the flavor-decomposed $\bar{c}^q$.

3. Pion mean-field approach

Since the χQSM has already been used for deriving the EMTFFs [19,27,28], we will mainly present the main results for the EMTFFs and GIVFFs within the framework of χQSM in flavor SU(2) symmetry. The χQSM is characterized by the low-energy QCD effective partition function in Euclidean space [18,16,17,7]

$$\mathcal{Z}_{\mathrm{eff}} = \int \mathcal{D}\pi^a \exp\left[-S_{\mathrm{eff}}(\pi^a)\right], \tag{7}$$

where π^a is the pseudo-Nambu-Goldstone boson fields and S_{eff} denotes the effective chiral action expressed as

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr}\log\left[i\not{\partial} + iMe^{i\gamma_5\pi^a\tau^a} + i\hat{m}\right]. \tag{8}$$

N_c designates the number of colors, M denotes the dynamical quark mass, and $\hat{m}$ is the current-quark mass matrix diag$(m_{\mathrm{u}}, m_{\mathrm{d}})$. The Dirac Hamiltonian $h(U)$ is defined by $h(U) = \gamma_4\gamma_i\partial_i + \gamma_4 Me^{i\gamma_5\pi^a\tau^a} + \gamma_4\bar{m}\mathbf{1}$ with the average value of the current-quark masses $\bar{m} = \left(m_u + m_d\right)/2$. We assume isospin symmetry ($m_{\rm u} = m_{\rm d}$). Introducing the hedgehog ansatz $\pi^a = P(r)n^a$, we can determine the profile function $P(r)$ by solving the classical equation of motion self-consistently. Since the pion-loop corrections are of $1/N_c$, we suppress them and carry out the functional integration over π^a, considering the rotational and translational zero modes, which is called the zero-mode quantization. Introducing the external tensor source field, we can compute the matrix element of the EMT current.

The proton matrix element of the symmetrized EMT current in Euclidean space can be calculated as follows:

$$\begin{aligned}&\left\langle p(p',J_3')\middle|\hat{T}^{\chi}_{\mu\nu}(0)\middle|p(p,J_3)\right\rangle\\&=\lim_{T\to\infty}\frac{1}{Z_{\rm eff}}\mathcal{N}^*(p')\mathcal{N}(p)e^{ip_4\frac{T}{2}-ip_4'\frac{T}{2}}\int d^3\boldsymbol{x}\,d^3\boldsymbol{y}\,e^{(-ip'\cdot y+ip\cdot x)}\\&\times\int\mathcal{D}U\int\mathcal{D}\psi\mathcal{D}\psi^\dagger J_p(\boldsymbol{y},T/2)\hat{T}^{\chi}_{\mu\nu}(0)J_p^\dagger(\boldsymbol{x},-T/2)\\&\times\exp\left[-S_{\rm eff}\right],\end{aligned}\tag{9}$$

where J_p represents the Ioffe-type current consisting of the N_c valence quarks [29] and $\hat{T}^{\chi}_{\mu\nu}(0)$ denotes the symmetrized EMT current derived from effective chiral theory in the Euclidean space. Note that the normalization factor $\mathcal{N}^*(p')\mathcal{N}(p')$ is reduced to the static normalization $2M_p$, and the proton state implicitly carries the spin and isospin quantum numbers, i.e., J, J_3, T, and T_3.

The temporal, mixed, and spatial components of the EMT current are expressed as

$$\begin{aligned}\hat{T}^{\chi}_{00}&=\frac{i}{2}\psi^\dagger\left(\gamma_4\overrightarrow{\partial}_4-\gamma_4\overleftarrow{\partial}_4\right)\tau^\chi\psi,\\\hat{T}^{\chi}_{0k}&=-\frac{1}{4}\psi^\dagger\left(\gamma_4\overrightarrow{\partial}_k+\gamma_k\overrightarrow{\partial}_4-\gamma_4\overleftarrow{\partial}_k-\gamma_k\overleftarrow{\partial}_4\right)\tau^\chi\psi,\\\hat{T}^{\chi}_{ij}&=-\frac{i}{4}\psi^\dagger\left(\gamma_i\overrightarrow{\partial}_j+\gamma_j\overrightarrow{\partial}_i-\gamma_i\overleftarrow{\partial}_j-\gamma_j\overleftarrow{\partial}_i\right)\tau^\chi\psi,\end{aligned}\tag{10}$$

where we introduce the superscripts $\chi = 0, 3$ that represent respectively the isoscalar $\chi = 0 = u + d$ and isovector $\chi = 3 = u - d$ components of the EMT current.

Defining the static symmetric EMT distributions in a Wigner sense [30,31]

$$T^{\chi}_{\mu\nu,p}(\boldsymbol{r})=\int\frac{d^3\Delta}{2M_p(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\left\langle p(p',J_3')\middle|\hat{T}^{\chi}_{\mu\nu}\middle|p\left(p,J_3\right)\right\rangle,\tag{11}$$

we obtain the expressions for the flavor-decomposed EMTFFs in the large N_c limit. The isoscalar components are expressed as

$$\begin{aligned}\left[A^{u+d}(t)-\frac{t}{4M_p^2}D^{u+d}(t)\right]\delta_{J_3'J_3}&=\frac{1}{M_p}\int d^3r\,j_0(r\sqrt{-t})\varepsilon_p^{u+d}(r),\\\frac{t}{6M_p^2}D^{u+d}(t)\delta_{J_3'J_3}&=\frac{1}{M_p}\int d^3r\,j_0(r\sqrt{-t})p_p^{u+d}(r),\\D^{u+d}(t)\delta_{J_3'J_3}&=4M_N\int d^3r\,\frac{j_2(r\sqrt{-t})}{t}s_p^{u+d}(r),\\2S^3_{J_3'J_3}J^{u+d}(t)&=3\int d^3r\,\frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}\rho_{J,p}^{u+d}(r),\end{aligned}\tag{12}$$

whereas the isovector components are written as

$$\begin{aligned}&\left[A^{u-d}(t)+\bar{c}^{u-d}(t)-\frac{t}{4M_p^2}\left(D^{u-d}(t)-2J^{u-d}(t)\right)\right]\delta_{J_3'J_3}\\&=\frac{1}{M_p}\int d^3r\,j_0(r\sqrt{-t})\varepsilon_p^{u-d}(r),\\&\left[\bar{c}^{u-d}(t)-\frac{t}{6M_p^2}D^{u-d}(t)\right]\delta_{J_3'J_3}=-\frac{1}{M_p}\int d^3r\,j_0(r\sqrt{-t})p_p^{u-d}(r),\\&D^{u-d}(t)\delta_{J_3'J_3}=4M_p\int d^3r\,\frac{j_2(r\sqrt{-t})}{t}s_p^{u-d}(r),\\&2S^3_{J_3'J_3}J^{u-d}(t)=3\int d^3r\,\frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}\rho_{J,p}^{u-d}(r),\end{aligned}\tag{13}$$

where ε_p, p_p, s_p, and $\rho_{J,p}$ denote the mass, pressure, shear force, and angular momentum distributions for the isoscalar and isovector components, respectively. For the explicit expressions for these EMT distributions are given in Appendix A. Note that they depend on the quantum numbers of the proton.

Before we proceed to compute the EMTFFs, it is worthwhile to mention about the polynomiality given in Eq. (2). Since the EMTFFs are regarded as the second Mellin moments, it is of great importance to examine whether this polynomiality satisfies within the framework of the χQSM. Noticeably, it was proven that the polynomiality of the GPDs in Eq. (2) is preserved within the χQSM [32–34].

Once we take the forward limit ($t \to 0$) and $J_3' = J_3 = 1/2$, we get the flavor-decomposed mass, spin, D-term, and $\bar{c}$ form factors as follows: For the isoscalar components, we have

$$\begin{aligned}A^{u+d}(0)&=\frac{1}{M_p}\int d^3r\,\varepsilon_p^{u+d}(r),\\D^{u+d}(0)&=-\frac{4M_p}{15}\int d^3r\,r^2 s_p^{u+d}(r),\\\bar{c}^{u+d}(0)&=-\frac{1}{M_p}\int d^3r\,p_p^{u+d}(r)=0,\\J^{u+d}(0)&=\int d^3r\,\rho_{J,p}^{u+d}(r),\end{aligned}\tag{14}$$

whereas for the isovector components, we obtain

$$\begin{aligned}A^{u-d}(0)+\bar{c}^{u-d}(0)&=\frac{1}{M_p}\int d^3r\,\varepsilon_p^{u-d}(r).\\D^{u-d}(0)&=-\frac{4M_p}{15}\int d^3r\,r^2 s_p^{u-d}(r),\\\bar{c}^{u-d}(0)&=-\frac{1}{M_p}\int d^3r\,p_p^{u-d}(r),\\J^{u-d}(0)&=\int d^3r\,\rho_{J,p}^{u-d}(r).\end{aligned}\tag{15}$$

The first and third relations in Eq. (14) resemble the thermodynamic potentials for the partial internal energy and isotropic pressure [14,15]. Thus, the flavor-decomposed $\bar{c}(0)$ form factors contribute to the decomposition of the proton mass.

Finally, we want to mention the N_c counting of the EMTFFs and GIVFFs, which are given as

$$\begin{aligned}&A^{u+d}(t)\sim O(N_c^0),\quad A^{u-d}(t)\sim O(N_c^{-1}),\\&J^{u+d}(t)\sim O(N_c^0),\quad J^{u-d}(t)\sim O(N_c^1),\\&D^{u+d}(t)\sim O(N_c^2),\quad D^{u-d}(t)\sim O(N_c^1),\\&\bar{c}^{u+d}(t)\sim O(N_c^0),\quad \bar{c}^{u-d}(t)\sim O(N_c^{-1}).\end{aligned}\tag{16}$$

We observe that A^{u+d} and A^{u-d} have the same N_c orders as the isoscalar and isovector $\bar{c}$ form factors, respectively.

4. Results and discussion

Concerning the fixing of the parameters for the χQSM, we refer to Refs. [17,28].

The solid curves in Fig. 1 present the numerical results for the EMTFFs, i.e. the $A(t)$ form factor, spin form factor, and the D-term form factor in the upper, middle, and lower panels, respectively. The results are the same as those in Refs. [19,28,35]. The GIVFFs A^{u-d}, J^{u-d}, and D^{u-d} can be considered as the isovector partners corresponding to the EMTFFs. Decomposing the EMTFFs into the up-quark and down-quark

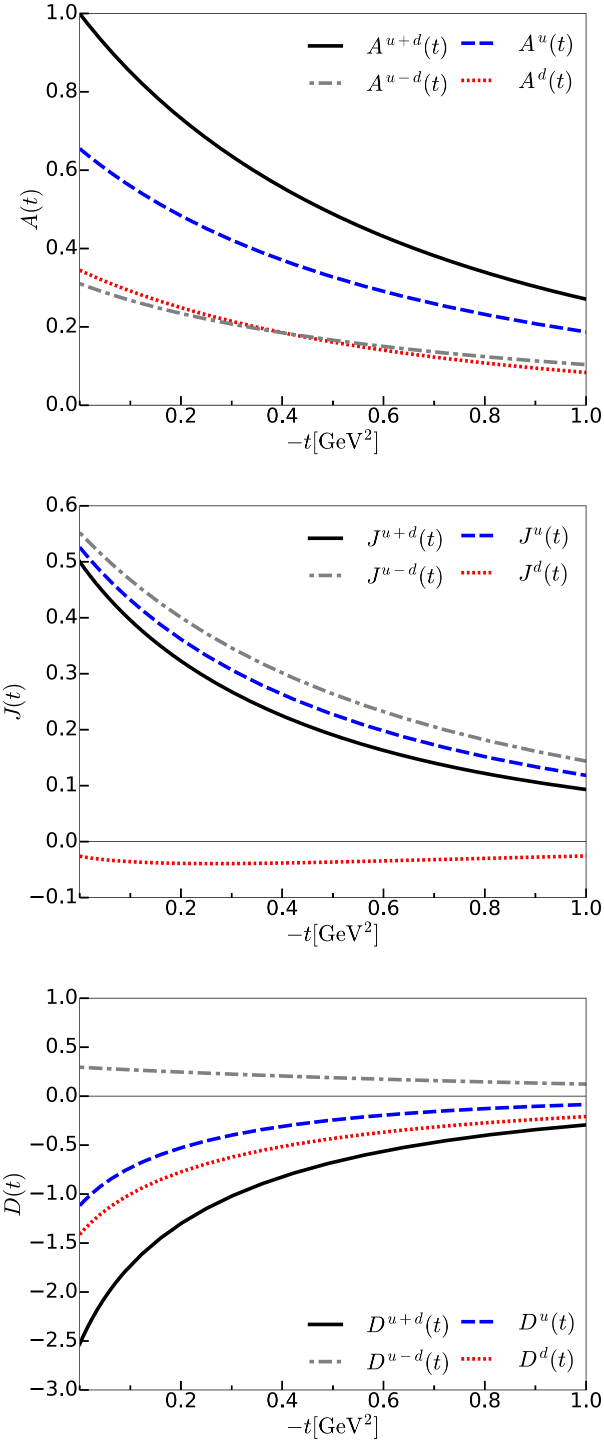

Fig. 1. The flavor decompositions of the LF momentum, spin, and D-term form factors are drawn in the upper, middle, and lower panels, respectively. The solid curves draw the corresponding EMTFFs and the dot-dashed ones depict the corresponding GIVFFs. The dashed and dotted ones represent the up-quark and down-quark contributions to the corresponding EMTFFs.

form factors, we find a very interesting feature. The up quarks dominate over the down quarks for the LF momentum and spin form factors. On the other hand, for the D-term form factor, the down-quark contribution turns out slightly larger than the up-quark one, both of which have negative values.

Fig. 2 depicts the flavor decomposition of the proton $\bar{c}$ form factor. As mentioned in the Introduction, the quark EMT current is conserved

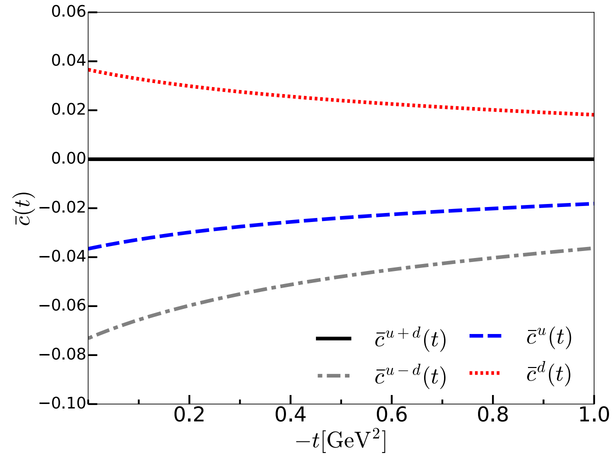

Fig. 2. The flavor decomposition of the proton $\bar{c}$ form factor is drawn. The solid curve draws proton $\bar{c}$ form factor and the dot-dashed ones depict $\bar{c}^{u-d}$. The dashed and dotted ones represent the up-quark and down-quark contributions to the proton $\bar{c}$ form factor.

in the present work. As expected, thus, $\bar{c}^{u+d}$ vanishes, which is also related to the Von Laue condition. However, the $\bar{c}^{u-d}$ remains finite because the isovector tensor current is in general not conserved. The solid and dot-dashed curves in Fig. 2 indicate this feature of the proton $\bar{c}$ form factor. When we decompose it into the up- and down-quark contributions, their magnitudes are exactly the same but their signs are opposite each other, so that they are canceled each other. So, while the flavor-decomposed proton $\bar{c}$ form factors do not contribute to the proton mass, they play a certain role in describing the isotropic pressure-volume work inside a proton [14,9]. In Table 1, we summarize the values of the flavor-decomposed EMTFFs at zero momentum transfer, comparing them with the lattice data [36,37]. Note that the normalization scale of the χQSM is determined to be around 0.6 GeV [38], whereas the lattice data [36,37] are derived at $\mu = 2$ GeV. The present work and Refs. [19,39] demonstrate that while the results for the isoscalar form factors are comparable to those obtained from lattice calculations, that for the $\bar{c}$ form factor is at variance with the lattice results [9]. When the χQSM was constructed from the instanton vacuum, the gluon degrees of freedom were integrated out via instantons. This means that we have only quark degrees of freedom and the effects of the gluons are effectively absorbed in the dynamical quark mass and pion mean fields. Thus we have correctly obtained the zero value of the $\bar{c}$ form factor, which effectively contains gluon contributions.

We also found that the results for the isovector mass and angular momentum form factors are in line with those from lattice calculations. The comparison of the current results with the lattice QCD requires an adjustment of the pion mass, and accordingly the associated low-energy constants will be varied. In addition, the scale evolution should be carried out. The comparison of the isoscalar component has been done in Ref. [39], and that of the isovector component will appear elsewhere.

The form factor A^q in the forward limit is equivalent to the momentum fraction of the proton: $A^q = \langle x \rangle_q$. While the total value of A^q becomes $\sum_q A^q = \sum_q \langle x \rangle_q = 1$, A^q can not be identified as the flavor-decomposed mass. The reason can be found in the fact that each flavor component of the $\bar{c}^q(0)$ form factors has a finite value, though the total $\bar{c}$ form factor vanishes. Thus, the flavor-decomposed mass can be expressed as

$$M_p = \sum_q M_p^q = \sum_q \left(A^q(0) + \bar{c}^q(0)\right) M_p \tag{17}$$

which indicates $\sum_q (A^q(0) + \bar{c}^q(0)) = 1$. Using Eq. (17), we find the following inequalities between M_p^q and $\langle x \rangle_q$:

$$M_p^u / M_p = 62\% < \langle x \rangle_u = 66\%$$

Table 1
We list the flavor-decomposed proton EMTFFs at $t = 0$ and compare them with results from lattice QCD [36,37], which are obtained from the chiral extrapolation at $t = 0$ and $m_{\pi,\mathrm{phys}}$.

	This work ($\mu \approx 0.6$ GeV)	Lattice QCD ($\mu = 2$ GeV) [36]	Lattice QCD ($\mu = 2$ GeV) [37]
$A^u(0)$	0.66	0.34	0.40
$A^d(0)$	0.34	0.18	0.15
$J^u(0)$	0.53	0.21	0.37
$J^d(0)$	-0.03	-0.00	-0.04
$D^u(0)$	-1.12	-0.57	-0.54
$D^d(0)$	-1.41	-0.50	-0.28
$\bar{c}^u(0)$	-0.04	–	–
$\bar{c}^d(0)$	0.04	–	–

$$M_p^d/M_p = 38\% > \langle x\rangle_d = 34\%. \tag{18}$$

The flavor-decomposed mass differs from the corresponding component of the momentum fraction by about 4% for both the up- and down-quark contributions. In addition, we obtain the following relations: if $\bar{c}^q(0) > 0$, then $M_p^q/M_p > \langle x\rangle_q$. If the flavor-decomposed $\bar{c}^q(0)$ form factor is zero, then $M_p^q/M_p = \langle x\rangle_q$.

Note that the isovector component has a much weaker scale dependence compared to the isoscalar or individual quark flavor components, since there is no gluon contribution to the isovector component. So, it implies that even though we consider the scale dependence of $\bar{c}$ form factor the results would not be much changed. Regarding the D-terms, we observed a significant difference between our predicted D-term form factor and the lattice result. However, the numerical uncertainties associated with both the isovector and isoscalar D-term form factors obtained from lattice QCD are substantial. These uncertainties can even affect the sign of the form factors.

As mentioned previously, the proton $\bar{c}$ form factor contributes to the isotropic pressure-volume work and force fields. As secured by the global stability condition for the proton [40], the pressure distribution of the proton is balanced between the level-quark and Dirac-continuum (pion-cloud) contributions [19] in the χQSM. When it comes to the proton $\bar{c}$ form factor, the cancelation takes place between the up- and down-quark contributions. To demonstrate it, we first derive the contributions of the flavor-decomposed $\bar{c}$ form factors by the Fourier transforms. In doing so, we parametrize $\bar{c}^{u-d}$ in terms of the dipole-type parametrization with the parameters $\Lambda^{u-d} = 1.5$ GeV. The conservation of the EMT current yields $\partial^i T_{ij}^{u+d} = f_j^u + f_j^d = 0$, which can be considered as an equilibrium equation for the internal forces between the u- and d-quark subsystems. A similar interpretation of the internal forces between the quark and gluon subsystem was conducted in Ref. [20]. The force-field vectors f_j^q and their magnitudes f^q are derived from the $\bar{c}$ form factors as follows:

$$\begin{aligned} f_j^q &= -M_p \frac{\partial}{\partial r^j}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\bar{c}^q(t), \\ f^q &= -M_p \frac{\partial}{\partial r}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\bar{c}^q(t), \end{aligned} \tag{19}$$

where the spherical symmetry is imposed as $f_j^q = \hat{n}_j f^q$. In Fig. 3, we illustrate $f^q(r)$. As already shown in Fig. 2, f^u is exactly canceled by f^d. Interestingly, the up-quark force field directs toward the center of the proton, whereas the down-quark on pushes outward. As a result, the force field from the $\bar{c}$ form factor vanishes.

In Fig. 4 we visualize the flavor-decomposed force fields from the proton $\bar{c}$ form factor, which portrays how $f^u(r)$ and $f^d(r)$ are distributed inside a proton. They are canceled each other at each point, so that the effects of the $\bar{c}$ form factor completely vanish for the proton.

5. Conclusions

The general isovector-vector form factors of the proton enable us to perform the flavor decomposition of the proton energy-momentum tensor form factors, which reveal novel features for the mechanical

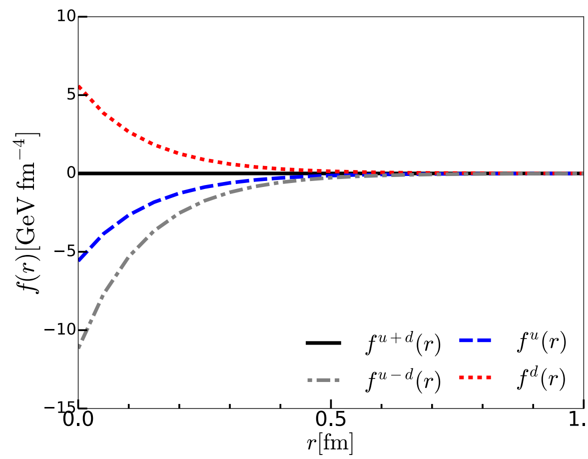

Fig. 3. Contribution of the $\bar{c}$ form factors to the internal force fields inside a proton. Notations are the same as in Fig. 2.

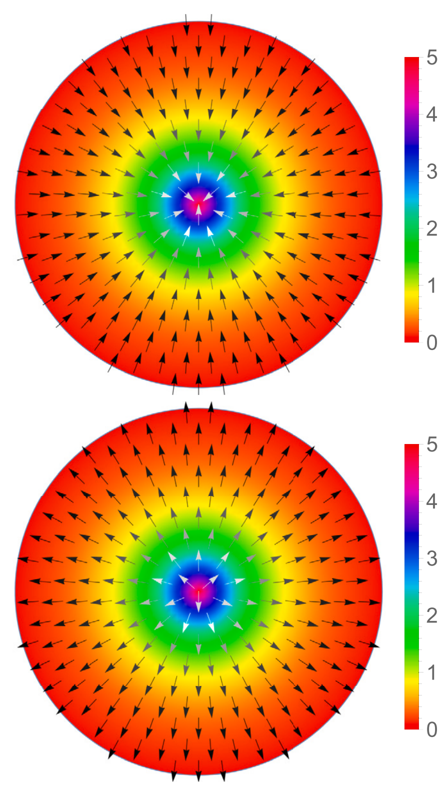

Fig. 4. Visualization of the flavor-decomposed force fields from the proton $\bar{c}$ form factor. In the upper (lower) panel, the up-quark (down-quark) force field is illustrated.

properties of the proton. While the up-quark contributions dominate the proton light-front momentum and spin form factors, the up- and down-quark contributions to the D term form factor are rather well balanced. However, in contrast to the light-front momentum and spin form factors, the magnitude of the down-quark contribution is slightly larger than that of the up-quark contribution. The proton isovector $\bar{c}$ form factor arises from the nonconservation of the isovector energy-momentum tensor-like current. This yields the flavor-decomposed the $\bar{c}$ form factors that do not vanish. They exhibit partial internal pressure and energy inside a proton, which are canceled each other. It results in vanishing the proton $\bar{c}$ form factor. In conclusion, the flavor-decomposed $\bar{c}$ form factors shed light on how the quarks describe the mechanics in the proton.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Hyun-Chul Kim reports financial support was provided by National Research Foundation of Korea. June-Young Kim reports financial support was provided by U.S. Department of Energy. Ho-Yeon Won reports financial support was provided by National Research Foundation of Korea.

Data availability

Data will be made available on request.

Acknowledgements

Authors want to express gratitude to Christian J. Weiss, Jose L. Goity, Cédric Lorcé, and Keh-Fei Liu for the invaluable comments and discussions. We are also grateful to Hyeon-Dong Son for the discussion about the gluon contribution to the proton $\bar{c}$ form factors. The work was supported by the Basic Science Research Program through the National Research Foundation of Korea funded by the Korean government (Ministry of Education, Science and Technology, MEST), Grant-No. 2021R1A2C2093368 and 2018R1A5A1025563. This work is also supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177 (JYK). HYW is supported by the France Excellence scholarship through Campus France funded by the French government (Ministère de l'Europe et des Affaires Étrangères), 141295X.

Appendix A. Expressions of the 3D EMT distributions in the χQSM

In this Appendix A, we compile the explicit expressions of the 3D EMT distributions. The Dirac Hamiltonian is diagonalized by the corresponding eigenenergies and eigenfunctions

$$h(U)\psi_n(r) = E_n\psi(r), \tag{A.1}$$

where E_n and ψ_n stand for the eigenenergies and eigenfunctions of the Dirac Hamiltonian $h(U)$, respectively. In the χQSM, the explicit expressions for the distributions are provided by

$$\begin{aligned}
\varepsilon_p^\chi(r) &= \mathcal{E}(r)\delta^{\chi 0} - \frac{2}{I_1}\left\langle D_{\chi i}J_i\right\rangle_p \mathcal{J}_1(r)\delta^{\chi 3},\\
\rho^\chi_{J,p}(r) &= \left\langle D_{\chi 3}\right\rangle_p\left(\mathcal{Q}_0(r) + \frac{1}{I_1}\mathcal{Q}_1(r)\right)\delta^{\chi 3} - \langle J_3\rangle_p \frac{1}{I_1}\mathcal{I}_1(r)\delta^{\chi 0},\\
s_p^\chi(r) &= \mathcal{N}_1(r)\delta^{\chi 0} - \frac{2}{I_1}\left\langle D_{\chi i}J_i\right\rangle_p \mathcal{J}_3(r)\delta^{\chi 3},\\
p_p^\chi(r) &= \mathcal{N}_3(r)\delta^{\chi 0} - \frac{2}{I_1}\left\langle D_{\chi i}J_i\right\rangle_p \mathcal{J}_5(r)\delta^{\chi 3},
\end{aligned}\tag{A.2}$$

where I_1 is the moment of inertia (see Ref. [17]). The $\langle\cdots\rangle_p$ represents the matrix element of the collective operators of the proton as follows:

$$\langle\cdots\rangle_p = \int dR\,\Psi^*_{(T'T'_3)(J'J'_3)}(R)\ldots\Psi_{(TT_3)(JJ_3)}(R). \tag{A.3}$$

The collective wave function of the proton $\Psi_{(TT_3)(JJ_3)}(R)$ is given by the SU(2) Wigner D function

$$\Psi_{(TT_3)(JJ_3)}(R) := \sqrt{2T+1}(-1)^{T+T_3}D^{J=T}_{-T_3,J_3}(R). \tag{A.4}$$

The distributions for the LF momentum form factors are expressed as

$$\begin{aligned}
\mathcal{E}(r) &= N_c\left[E_v\psi_v^\dagger(r)\psi_v(r) + \sum_n \psi_n^\dagger(r)\psi_n(r)R_{0n}\right],\\
\mathcal{J}_1(r) &= \frac{N_c}{4}\left[\sum_{n\neq v}\frac{E_n+E_v}{E_n-E_v}\langle n|\tau_3|v\rangle\,\psi_v^\dagger(r)\tau_3\psi_n(r)\right.\\
&\quad\left.+\frac{1}{2}\sum_{n,m}(E_n+E_m)\langle n|\tau_3|m\rangle\,\psi_m^\dagger(r)\tau_3\psi_n(r)R_{3nm}\right],
\end{aligned}\tag{A.5}$$

those for the spin form factors are given as

$$\begin{aligned}
\mathcal{Q}_0(r) &= \frac{N_c}{4}\left[\psi_v^\dagger(r)\Gamma^J_{vv3}\tau_3\psi_v(r)\right.\\
&\quad\left.-\frac{1}{2}\sum_n \mathrm{sign}(E_n)\psi_n^\dagger(r)\Gamma^J_{nn3}\tau_3\psi_n(r)\right],\\
\mathcal{Q}_1(r) &= \frac{N_c}{4}i\epsilon_{ij3}\left[\sum_{n\neq v}\frac{\mathrm{sign}(E_n)}{E_n-E_v}\langle n|\tau_i|v\rangle\,\psi_v^\dagger(r)\tau_j\Gamma^J_{vn3}\psi_n(r)\right.\\
&\quad\left.+\frac{1}{2}\sum_{n,m}\langle n|\tau_i|m\rangle\,\psi_m^\dagger(r)\tau_j\Gamma^J_{mn3}\psi_n(r)R_{6nm}\right],\\
\mathcal{I}_1(r) &= \frac{1}{4}\sum_{n\neq v}\frac{1}{E_n-E_v}\langle n|\tau_3|v\rangle\,\psi_v^\dagger(r)\Gamma^J_{nv3}\psi_n(r)\\
&\quad+\frac{1}{8}\sum_{\substack{n=\mathrm{all}\\ m=\mathrm{all}}}\langle n|\tau_3|m\rangle\,\psi_m^\dagger(r)\Gamma^J_{mn3}\psi_n(r)R_3(E_n,E_m),
\end{aligned}\tag{A.6}$$

those for the D-term form factors and $\bar{c}$ form factors are respectively written as

$$\begin{aligned}
\mathcal{N}_1(r) &= \frac{3}{2}N_c\left[\psi_v^\dagger(r)\Gamma^s\psi_v(r) + \sum_n\psi_n^\dagger(r)\Gamma^s\psi_n(r)R_{1n}\right],\\
\mathcal{J}_3(r) &= \frac{3}{4}N_c\left[\sum_{n\neq v}\frac{\langle n|\tau_3|v\rangle}{E_n-E_v}\psi_v^\dagger(r)\tau_3\Gamma^s\psi_n(r)\right.\\
&\quad\left.+\frac{1}{2}\sum_{n,m}\langle n|\tau_3|m\rangle\,\psi_m^\dagger(r)\tau_3\Gamma^s\psi_n(r)R_{5nm}\right],
\end{aligned}\tag{A.7}$$

and

$$\begin{aligned}
\mathcal{N}_3(r) &= \frac{N_c}{3}\left[\psi_v^\dagger(r)\Gamma^p\psi_v(r) + \sum_n\psi_n^\dagger(r)\Gamma^p\psi_n(r)R_{1n}\right],\\
\mathcal{J}_5(r) &= \frac{N_c}{6}\left[\sum_{n\neq v}\frac{\langle n|\tau_3|v\rangle}{E_n-E_v}\psi_v^\dagger(r)\tau_3\Gamma^p\psi_n(r)\right.\\
&\quad\left.+\frac{1}{2}\sum_{n,m}\langle n|\tau_3|m\rangle\,\psi_m^\dagger(r)\tau_3\Gamma^p\psi_n(r)R_{5nm}\right].
\end{aligned}\tag{A.8}$$

Note that $\psi_v(r) := \langle r|v\rangle$ and $\psi_n(r) := \langle r|n\rangle$. The total angular momentum Γ^J_{mn3}, shear force Γ^s, and pressure Γ^p operators are respectively expressed by

$$\begin{aligned}
\Gamma^J_{nm3} &= \left[2\hat{L}_3 + \left(E_n+E_m\right)\gamma_5(\boldsymbol{r}\times\boldsymbol{\sigma})_3\right],\\
\Gamma^s &= \gamma^0\left(\hat{\boldsymbol{n}}\cdot\boldsymbol{p}\right) - \frac{1}{3}\gamma^0\left(\boldsymbol{\gamma}\cdot\boldsymbol{p}\right),\\
\Gamma^p &= \gamma^0(\boldsymbol{\gamma}\cdot\boldsymbol{p}),
\end{aligned}\tag{A.9}$$

where the orbital angular momentum is defined by

$$\hat{L} = \left[r \times \frac{i}{2} \left(\overleftarrow{\nabla} - \overrightarrow{\nabla} \right) \right]. \tag{A.10}$$

The regularization functions for the distributions are expressed as

$$\begin{aligned}
R_{0n} &= \frac{1}{4\sqrt{\pi}} \int_{\Lambda^{-2}} \frac{du}{u^{3/2}} e^{-uE_n^2}, \\
R_{1n} &= -\frac{E_n}{2\sqrt{\pi}} \int_{\Lambda^{-2}} \frac{du}{\sqrt{u}} e^{-uE_n^2}, \\
R_{3nm} &= \frac{1}{2\sqrt{\pi}} \int_{\Lambda^{-2}} \frac{du}{\sqrt{u}} \left[\frac{1}{u} \frac{e^{-uE_n^2} - e^{-uE_m^2}}{E_m^2 - E_n^2} \right. \\
&\quad \left. - \frac{E_n e^{-uE_n^2} + E_m e^{-uE_m^2}}{E_n + E_m} \right], \\
R_{5nm} &= \frac{1}{2} \frac{\mathrm{sign}(E_n) - \mathrm{sign}(E_m)}{E_n - E_m}, \\
R_{6nm} &= \frac{1 - \mathrm{sign}(E_n)\mathrm{sign}(E_m)}{E_n - E_m}.
\end{aligned} \tag{A.11}$$

References

[1] A. Einstein, Cosmological considerations in the general theory of relativity, Sitz.ber. Preuss. Akad. Wiss. Berl. (Math. Phys.) 1917 (1917) 142–152.
[2] Y.B. Zeldovich, Cosmological constant and elementary particles, JETP Lett. 6 (1967) 316.
[3] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. 61 (1989) 1–23, https://doi.org/10.1103/RevModPhys.61.1.
[4] P.J.E. Peebles, B. Ratra, The cosmological constant and dark energy, Rev. Mod. Phys. 75 (2003) 559–606, https://doi.org/10.1103/RevModPhys.75.559, arXiv:astro-ph/0207347.
[5] J. Sola, Cosmological constant and vacuum energy: old and new ideas, J. Phys. Conf. Ser. 453 (2013) 012015, https://doi.org/10.1088/1742-6596/453/1/012015, arXiv:1306.1527.
[6] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, QCD and resonance physics. Theoretical foundations, Nucl. Phys. B 147 (1979) 385–447, https://doi.org/10.1016/0550-3213(79)90022-1.
[7] D. Diakonov, Instantons at work, Prog. Part. Nucl. Phys. 51 (2003) 173–222, https://doi.org/10.1016/S0146-6410(03)90014-7, arXiv:hep-ph/0212026.
[8] O.V. Teryaev, Gravitational form factors and nucleon spin structure, Front. Phys. (Beijing) 11 (5) (2016) 111207, https://doi.org/10.1007/s11467-016-0573-6.
[9] K.-F. Liu, Proton mass decomposition and hadron cosmological constant, Phys. Rev. D 104 (7) (2021) 076010, https://doi.org/10.1103/PhysRevD.104.076010, arXiv:2103.15768.
[10] Y. Hatta, A. Rajan, K. Tanaka, Quark and gluon contributions to the QCD trace anomaly, J. High Energy Phys. 12 (2018) 008, https://doi.org/10.1007/JHEP12(2018)008, arXiv:1810.05116.
[11] A. Metz, B. Pasquini, S. Rodini, Revisiting the proton mass decomposition, Phys. Rev. D 102 (2020) 114042, https://doi.org/10.1103/PhysRevD.102.114042, arXiv:2006.11171.
[12] X.-D. Ji, Breakup of hadron masses and energy - momentum tensor of QCD, Phys. Rev. D 52 (1995) 271–281, https://doi.org/10.1103/PhysRevD.52.271, arXiv:hep-ph/9502213.
[13] X. Ji, Proton mass decomposition: naturalness and interpretations, Front. Phys. (Beijing) 16 (6) (2021) 64601, https://doi.org/10.1007/s11467-021-1065-x, arXiv:2102.07830.
[14] C. Lorcé, On the hadron mass decomposition, Eur. Phys. J. C 78 (2) (2018) 120, https://doi.org/10.1140/epjc/s10052-018-5561-2, arXiv:1706.05853.
[15] C. Lorcé, H. Moutarde, A.P. Trawiński, Revisiting the mechanical properties of the nucleon, Eur. Phys. J. C 79 (1) (2019) 89, https://doi.org/10.1140/epjc/s10052-019-6572-3, arXiv:1810.09837.
[16] D. Diakonov, V.Y. Petrov, P.V. Pobylitsa, A chiral theory of nucleons, Nucl. Phys. B 306 (1988) 809, https://doi.org/10.1016/0550-3213(88)90443-9.
[17] C.V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, K. Goeke, Baryons as nontopological chiral solitons, Prog. Part. Nucl. Phys. 37 (1996) 91–191, https://doi.org/10.1016/0146-6410(96)00057-9, arXiv:hep-ph/9604441.
[18] D. Diakonov, V.Y. Petrov, A theory of light quarks in the instanton vacuum, Nucl. Phys. B 272 (1986) 457–489, https://doi.org/10.1016/0550-3213(86)90011-8.
[19] K. Goeke, J. Grabis, J. Ossmann, M.V. Polyakov, P. Schweitzer, A. Silva, D. Urbano, Nucleon form-factors of the energy momentum tensor in the chiral quark-soliton model, Phys. Rev. D 75 (2007) 094021, https://doi.org/10.1103/PhysRevD.75.094021, arXiv:hep-ph/0702030.
[20] M.V. Polyakov, H.-D. Son, Nucleon gravitational form factors from instantons: forces between quark and gluon subsystems, J. High Energy Phys. 09 (2018) 156, https://doi.org/10.1007/JHEP09(2018)156, arXiv:1808.00155.
[21] X.-D. Ji, Deeply virtual Compton scattering, Phys. Rev. D 55 (1997) 7114–7125, https://doi.org/10.1103/PhysRevD.55.7114, arXiv:hep-ph/9609381.
[22] X.-D. Ji, Gauge-invariant decomposition of nucleon spin, Phys. Rev. Lett. 78 (1997) 610–613, https://doi.org/10.1103/PhysRevLett.78.610, arXiv:hep-ph/9603249.
[23] M. Diehl, Generalized parton distributions, Phys. Rep. 388 (2003) 41–277, https://doi.org/10.1016/j.physrep.2003.08.002, arXiv:hep-ph/0307382.
[24] E. Leader, C. Lorcé, The angular momentum controversy: what's it all about and does it matter?, Phys. Rep. 541 (3) (2014) 163–248, https://doi.org/10.1016/j.physrep.2014.02.010, arXiv:1309.4235.
[25] C. Lorcé, L. Mantovani, B. Pasquini, Spatial distribution of angular momentum inside the nucleon, Phys. Lett. B 776 (2018) 38–47, https://doi.org/10.1016/j.physletb.2017.11.018, arXiv:1704.08557.
[26] E. Leader, A critical assessment of the angular momentum sum rules, Phys. Lett. B 720 (2013) 120–124, https://doi.org/10.1016/j.physletb.2013.01.050, Erratum: Phys. Lett. B 726 (2013) 927, arXiv:1211.3957.
[27] J.-Y. Kim, H.-C. Kim, M.V. Polyakov, H.-D. Son, Strong force fields and stabilities of the nucleon and singly heavy baryon Σ_c, Phys. Rev. D 103 (1) (2021) 014015, https://doi.org/10.1103/PhysRevD.103.014015, arXiv:2008.06652.
[28] H.-Y. Won, J.-Y. Kim, H.-C. Kim, Gravitational form factors of the baryon octet with flavor SU(3) symmetry breaking, Phys. Rev. D 106 (11) (2022) 114009, https://doi.org/10.1103/PhysRevD.106.114009, arXiv:2210.03320.
[29] B.L. Ioffe, Calculation of baryon masses in quantum chromodynamics, Nucl. Phys. B 188 (1981) 317–341, https://doi.org/10.1016/0550-3213(81)90259-5, Erratum: Nucl. Phys. B 191 (1981) 591–592.
[30] M.V. Polyakov, Generalized parton distributions and strong forces inside nucleons and nuclei, Phys. Lett. B 555 (2003) 57–62, https://doi.org/10.1016/S0370-2693(03)00036-4, arXiv:hep-ph/0210165.
[31] C. Lorcé, Charge distributions of moving nucleons, Phys. Rev. Lett. 125 (23) (2020) 232002, https://doi.org/10.1103/PhysRevLett.125.232002, arXiv:2007.05318.
[32] P. Schweitzer, S. Boffi, M. Radici, Polynomiality of unpolarized off forward distribution functions and the D term in the chiral quark soliton model, Phys. Rev. D 66 (2002) 114004, https://doi.org/10.1103/PhysRevD.66.114004, arXiv:hep-ph/0207230.
[33] P. Schweitzer, M. Colli, S. Boffi, Polynomiality of helicity off forward distribution functions in the chiral quark soliton model, Phys. Rev. D 67 (2003) 114022, https://doi.org/10.1103/PhysRevD.67.114022, arXiv:hep-ph/0303166.
[34] J. Ossmann, M.V. Polyakov, P. Schweitzer, D. Urbano, K. Goeke, The generalized parton distribution function (E**u + E**d) (x,xi,t) of the nucleon in the chiral quark soliton model, Phys. Rev. D 71 (2005) 034011, https://doi.org/10.1103/PhysRevD.71.034011, arXiv:hep-ph/0411172.
[35] M. Wakamatsu, On the D-term of the nucleon generalized parton distributions, Phys. Lett. B 648 (2007) 181–185, https://doi.org/10.1016/j.physletb.2007.03.013, arXiv:hep-ph/0701057.
[36] P. Hagler, et al., Nucleon generalized parton distributions from full lattice QCD, Phys. Rev. D 77 (2008) 094502, https://doi.org/10.1103/PhysRevD.77.094502, arXiv:0705.4295.
[37] M. Gockeler, R. Horsley, D. Pleiter, P.E.L. Rakow, A. Schafer, G. Schierholz, W. Schroers, Generalized parton distributions from lattice QCD, Phys. Rev. Lett. 92 (2004) 042002, https://doi.org/10.1103/PhysRevLett.92.042002, arXiv:hep-ph/0304249.
[38] H.-C. Kim, M.V. Polyakov, K. Goeke, Nucleon tensor charges in the SU(2) chiral quark - soliton model, Phys. Rev. D 53 (1996) 4715–4718, https://doi.org/10.1103/PhysRevD.53.R4715, arXiv:hep-ph/9509283.
[39] K. Goeke, J. Grabis, J. Ossmann, P. Schweitzer, A. Silva, D. Urbano, The pion mass dependence of the nucleon form-factors of the energy momentum tensor in the chiral quark-soliton model, Phys. Rev. C 75 (2007) 055207, https://doi.org/10.1103/PhysRevC.75.055207, arXiv:hep-ph/0702031.
[40] M.V. Polyakov, P. Schweitzer, Forces inside hadrons: pressure, surface tension, mechanical radius, and all that, Int. J. Mod. Phys. A 33 (26) (2018) 1830025, https://doi.org/10.1142/S0217751X18300259, arXiv:1805.06596.