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Azimuthal asymmetries in unpolarized SIDIS and Drell-Yan processes: A case study towards TMD factorization at subleading twist

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ABSTRACT

We consider the azimuthal distribution of the final observed hadron in semi-inclusive deep-inelastic scattering and the lepton pair in the Drell-Yan process. In particular, we focus on the $\cos\phi$ modulation of the unpolarized cross section and on its dependence upon transverse momentum. At low transverse momentum, for these observables we propose a factorized expression based on tree-level approach and conjecture that the same formula is valid in transverse-momentum dependent (TMD) factorization when written in terms of subtracted TMD parton distributions. Our formula correctly matches with the collinear factorization results at high transverse momentum, solves a long-standing problem and is a necessary step towards the extension of the TMD factorization theorems up to the subleading twist.

1. Introduction

The inclusive production of a system of one or more particles with a specific transverse momentum in lepton-hadron or hadron-hadron collisions is in general characterized by three different scales: the nonperturbative QCD scale Λ_{QCD}, the hard scale of the process Q, and the magnitude of the system's transverse momentum q_T. In both processes under study, namely semi-inclusive deep inelastic scattering (SIDIS), $\ell p \to \ell' h X$, and production of Drell–Yan lepton pairs (DY), $pp \to \ell^+ \ell^- X$, the hard scale Q is given by the virtuality of the gauge boson exchanged in the reaction.

Depending on the value of the transverse momentum q_T, two different frameworks are adopted for the description of these processes. At *high* q_T, namely $q_T \gg \Lambda_{\mathrm{QCD}}$, the transverse momentum in the final state is generated by the perturbative radiation and the cross section can be expressed in terms of collinear (*i.e.*, integrated over transverse momentum) parton distributions (PDFs) and fragmentation functions (FFs). Conversely, at *low* q_T, $q_T \ll Q$, transverse-momentum-dependent (TMD) factorization [1–3] can be applied and TMD PDFs and FFs (or TMDs for short) dependent on the transverse momentum, are used in the factorized expression. In principle, in the intermediate region $\Lambda_{\mathrm{QCD}} \ll q_T \ll Q$ both frameworks can be applied. In case they describe the same mechanism (characterized by the same power behavior), in this region they have to match. If, on the other hand, the two results describe competing mechanisms, they should be considered independently and added together (see Ref. [4]).

For the q_T-differential unpolarized cross section integrated over the azimuthal angle of the final particle, the matching of the TMD and the collinear factorization descriptions in the intermediate q_T region has been shown in, e.g., Refs. [5,6]. These results underpin all phenomenological studies of TMDs, even though some modifications of the original procedure are often needed [7,8].

Apart from the azimuthally independent cross sections, where unpolarized TMDs are important, various TMDs are involved in generating azimuthal modulations of unpolarized cross sections. Both in SIDIS and DY, neglecting parity-violating interactions, four structure functions are needed to parametrize the cross section that depends on the azimuthal angle and transverse momentum.

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Two of them are related to what is usually referred to as $\cos\phi$ and $\cos 2\phi$ modulations, where ϕ is the azimuthal angle between the leptonic and hadronic planes in a specific frame.

In this paper, we study $\cos\phi$ modulations: they involve twist-3 TMD PDFs and FFs and are suppressed by a factor $1/Q$ with respect to the leading (twist-2) terms. These modulations have been already studied, with somewhat contradictory results, in Refs. [9–11] for DY and in Ref. [4] for SIDIS. We remark that no factorization proof for TMD observables at twist-3 is available, although steps in this direction have recently been taken (see e.g. Refs. [12–17]). A yet unsolved problem is matching between the TMD and collinear descriptions in the intermediate q_T region, which might point to the conclusion that observables related to twist-3 TMDs cannot be properly factorized.

We work out here a solution to the problem of matching the TMD and collinear formulae. We suggest a TMD factorized formula modified with respect to the one used in Ref. [4]. We provide arguments in favor of this formulation and show that it leads to an agreement between the TMD and collinear results in SIDIS and in DY, in different reference frames. The leading logarithmic (LL) terms match also in the so-called Wandzura–Wilczek approximation [18]. Our results can be generalized to other observables involving twist-3 TMDs and they represent a necessary step in the direction of establishing TMD factorization at twist-3 level.

2. Azimuthal $\cos\phi$ asymmetry in SIDIS

We start from a detailed discussion of the semi-inclusive deep inelastic scattering process

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X(P_X), \tag{1}$$

where $\ell(\ell')$ is the incoming (outgoing) lepton with momentum $l(l')$, N is the nucleon with mass M and momentum P, and h is the detected hadron with mass M_h and momentum P_h. We introduce the SIDIS variables

$$x = \frac{Q^2}{2\,P\cdot q}\,, \quad y = \frac{P\cdot q}{P\cdot l}\,, \quad z = \frac{P\cdot P_h}{P\cdot q}\,, \quad \xi = -\frac{P_h\cdot q}{P\cdot P_h}\,, \tag{2}$$

where $q = l - l'$ and $Q^2 = -q^2$.

In the one-photon-exchange approximation, the differential cross section can be written as (see, e.g., Ref. [19])

$$\frac{d^5\sigma}{dx\,dy\,dz\,d^2\boldsymbol{P}_{hT}} = \frac{\pi\alpha^2}{2Q^4}\, y L_{\mu\nu} W^{\mu\nu}\,, \tag{3}$$

with α being the fine structure constant and $L_{\mu\nu}$ and $W^{\mu\nu}$ the leptonic and hadronic tensors, respectively.

The process is usually studied in a frame where $\boldsymbol{P}$ and $\boldsymbol{q}$ are collinear and taken to be along the z-axis, with the azimuthal angle ϕ_h of the final hadron defined w.r.t. the lepton plane according to the so-called Trento conventions [20]. We denote by P_{hT} the component of P_h transverse to the momenta P and q. Alternatively, one can choose $\boldsymbol{P}$ and $\boldsymbol{P}_h$ as the longitudinal directions: in this case, the photon will carry a transverse momentum q_T. We limit ourselves to a kinematic region where $Q^2 \gg \Lambda^2_{\rm QCD} \approx M^2$ at fixed values of x, y, z, and neglect corrections of order M^2/Q^2. Under these approximations, we can define

$$P^\mu_{hT} = P^\mu_h - z(2x - \xi)P^\mu - zq^\mu\,, \tag{4}$$

$$q^\mu_T = q^\mu + \xi P^\mu - \frac{P_h}{z} \approx -\frac{1}{z}\,P^\mu_{hT} - 2\rho^2 x P^\mu\,, \tag{5}$$

with $\rho^2 = q_T^2/Q^2$ and $q_T^2 = -q_T^\mu\, q_{T\mu} \equiv \boldsymbol{q}_T^2$. [1] The above definition needs to be slightly modified if mass corrections are taken into account (see, e.g., the discussion in Ref. [21]).

The cross section can be parametrized in terms of four structure functions that depend on x, z, Q^2 and P_{hT}^2 [22],

$$\begin{aligned}\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{hT}} &= \frac{\pi\alpha^2}{xQ^2}\,\frac{y}{1-\varepsilon}\Big\{F_{UU,T} + \varepsilon F_{UU,L} \\ &\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos 2\phi_h\,F_{UU}^{\cos 2\phi_h}\Big\}\,,\end{aligned} \tag{6}$$

where ε is the ratio of the longitudinal and transverse photon fluxes,

$$\varepsilon = \frac{1-y}{1-y+y^2/2}\,. \tag{7}$$

The first and second subscripts of the structure functions refer to the polarization of the initial lepton and proton, respectively, while the third one specifies the polarization of the virtual photon exchanged in the reaction.

We focus here on the structure functions $F_{UU,T}$ and $F_{UU}^{\cos\phi_h}$. At tree level, they can be written in terms of TMDs in the following way

$$F_{UU,T}\big(x,z,P^2_{hT},Q^2\big) = \sum_a e_a^2 x\mathcal{B}_0\Big[\widehat{f}_1^a\,\widehat{D}_1^a\Big] + \mathcal{O}\bigg(\frac{P^2_{hT}}{Q^2}\bigg)\,, \tag{8}$$

$$\begin{aligned}&F_{UU}^{\cos\phi_h}\big(x,z,P^2_{hT},Q^2\big) \\ &\quad = \sum_a e_a^2 x\frac{2MM_h}{Q}\,\mathcal{B}_1\bigg[x\widehat{h}^a\,\widehat{H}_1^{\perp a(1)} + \frac{M_h}{M}\,\widehat{f}_1^a\,\frac{\widehat{\tilde{D}}^{\perp a(1)}}{z} \\ &\qquad - \frac{M}{M_h}x\widehat{f}^{\perp a(1)}\,\widehat{D}_1^a - \widehat{h}_1^{\perp a(1)}\,\frac{\widehat{\tilde{H}}^a}{z}\bigg] + \mathcal{O}\bigg(\frac{P^2_{hT}}{Q^2}\bigg)\,,\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathcal{B}_n\big[\widehat{f}\,\widehat{D}\big] = 2\pi\int_0^\infty d\xi_T\,\xi_T^{n+1}\,J_n\bigg(\frac{\xi_T P_{hT}}{z}\bigg)\widehat{f}^a\big(x,\xi_T^2;Q^2\big) \\ \times \widehat{D}^a\big(z,\xi_T^2;Q^2\big)\,,\end{aligned} \tag{10}$$

the e_a is the electric charge of a parton with flavor a in units of the proton charge, and the Fourier transform of a generic TMD PDF, $f(x,k_\perp^2;Q^2)$, has been defined as

$$\begin{aligned}\widehat{f}\big(x,\xi_T^2;Q^2\big) &\equiv \frac{1}{2\pi}\int d^2\boldsymbol{k}_\perp e^{i\boldsymbol{\xi}_T\cdot\boldsymbol{k}_\perp} f\big(x,k_\perp^2;Q^2\big) \\ &= \int_0^\infty dk_\perp k_\perp\,J_0\big(\xi_T k_\perp\big)\,f\big(x,k_\perp^2;Q^2\big)\,.\end{aligned} \tag{11}$$

Furthermore, the ξ_T^2-derivatives of the TMDs are given by [23]

$$\widehat{f}^{(n)}(x,\xi_T^2;Q^2) = n!\bigg(-\frac{2}{M^2}\frac{\partial}{\partial\xi_T^2}\bigg)^n\widehat{f}(x,\xi_T^2;Q^2)$$

[1] If there is no ambiguity, in the following we will use the notation a_T for the modulus of the spatial vector $|\boldsymbol{a}_T|$.

$$= \frac{n!}{M^{2n}} \int_0^\infty dk_\perp k_\perp \left(\frac{k_\perp}{\xi_T}\right)^n J_n(\xi_T k_\perp)\, f(x, k_\perp^2; Q^2)\,. \tag{12}$$

For a generic TMD FF, $D(z, P_\perp^2/z^2; Q^2)$, the above formulas are identical, but with $k_\perp$ replaced by $P_\perp/z$.

For the structure function $F_{UU,T}$, and in general for twist-2 terms in the hadronic tensor, a general factorization proof can be given, see for example Ref. [1]. Soft gluon radiation to all orders is absorbed into an exponential Sudakov form factor which is partitioned between TMD PDFs and FFs, while all the remaining perturbative corrections are incorporated in the so-called hard factor $\mathcal{H}$. The final formula resembles very much the tree-level result:

$$\begin{aligned} &F_{UU,T}(x, z, P_{hT}^2, Q^2) \\ &= \mathcal{H}_{\text{SIDIS}}(Q^2, \mu^2) \sum_a e_a^2 x \mathcal{B}_0 \Big[\widehat{f}_1^a(x, \xi_T^2; \mu^2, \nu^2)\, \widehat{D}_1^a(z, \xi_T^2; \mu^2, \nu^2)\Big] \\ &\quad + \mathcal{O}\left(\frac{P_{hT}^2}{Q^2}\right), \end{aligned} \tag{13}$$

where the scales μ and ν arise as a consequence of regulating the ultraviolet and rapidity divergences of TMDs. These scales can be both set equal to Q in order to minimize logarithmic corrections. In the following, we will refer to these properly defined TMD functions as *subtracted* TMDs.

For what concerns the structure function $F_{UU}^{\cos\phi_h}$ (and in general twist-3 terms in the hadronic tensor), we *conjecture* that the correct formula can be constructed in the same way as for twist-2 terms, i.e., starting from the tree-level formula, adding the hard scattering function $\mathcal{H}$ and replacing TMDs with subtracted ones. We found no obvious way to prove this conjecture. However, what gives us confidence in its validity is that the resulting formula correctly matches the perturbative calculation at high transverse momentum, both in SIDIS and DY, thus fixing the mismatch observed in Sec. 8.3 of Ref. [4].

Our starting formula is therefore

$$\begin{aligned} &F_{UU}^{\cos\phi_h}(x, z, P_{hT}^2, Q^2) \\ &= \frac{2MM_h}{Q} \mathcal{H}'_{\text{SIDIS}} \sum_a e_a^2 x \mathcal{B}_1 \Bigg[x\widehat{h}^a\, \widehat{H}_1^{\perp a(1)} + \frac{M_h}{M} \widehat{f}_1^a\, \frac{\widehat{\tilde{D}}^{\perp a(1)}}{z} \\ &\quad - \frac{M}{M_h} x \widehat{f}^{\perp a(1)}\, \widehat{D}_1^a - \widehat{h}_1^{\perp a(1)}\, \frac{\widehat{\tilde{H}}^a}{z}\Bigg] + \mathcal{O}\left(\frac{P_{hT}^2}{Q^2}\right). \end{aligned} \tag{14}$$

This structure function has been analyzed by Cahn [24,25], who for the first time pointed out the presence of perturbative and non-perturbative contributions. Measurements of this azimuthal modulation are available in Refs. [26–32]. Phenomenological analyses that took into account various contributions separately have been reported in Ref. [33–35].

2.1. From high to intermediate transverse-momentum

In the high transverse-momentum region ($q_T \gg \Lambda_{\text{QCD}}$) structure functions can be expressed, using collinear factorization, in terms of convolutions of hard scattering coefficients with the usual collinear distribution and fragmentation functions, respectively denoted by f_1 and D_1 [4,36]. To the first order in the strong coupling α_s, this result can be further approximated in the intermediate transverse-momentum region ($\Lambda_{\text{QCD}} \ll q_T \ll Q$) as [4,36,37]

$$\begin{aligned} F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \Bigg\{ &\sum_a x e_a^2 \Bigg[f_1^a(x, Q^2)\, D_1^a(z, Q^2)\, L\left(\frac{Q^2}{q_T^2}\right) \\ &+ f_1^a(x, Q^2)\big(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z, Q^2) \\ &+ \big(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\big)(x, Q^2)\, D_1^a(z, Q^2)\Bigg] \\ &+ \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right) + \mathcal{O}\left(\frac{q_T}{Q}\right)\Bigg\}, \end{aligned} \tag{15}$$

$$\begin{aligned} F_{UU}^{\cos\phi_h} = -\frac{1}{Q\, q_T} \frac{\alpha_s}{2\pi^2 z^2} \Bigg\{ &\sum_a x e_a^2 \Bigg[f_1^a(x; Q^2)\, D_1^a(z; Q^2)\, L\left(\frac{Q^2}{q_T^2}\right) \\ &+ \sum_{i=a,g} \Big(f_1^a(x; Q^2)(D_1^i \otimes P'_{ia})(z; Q^2) \\ &+ (P'_{ai} \otimes f_1^i)(x; Q^2)\, D_1^a(z; Q^2)\Big)\Bigg] \\ &+ \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right) + \mathcal{O}\left(\frac{q_T}{Q}\right)\Bigg\}, \end{aligned} \tag{16}$$

where the factor L is defined as

$$L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln \frac{Q^2}{q_T^2} - 3C_F\,. \tag{17}$$

The convolutions are defined as

$$\begin{aligned} \big(C \otimes f\big)(x; Q^2) &= \int_x^1 \frac{d\hat{x}}{\hat{x}}\, C(\hat{x}; Q^2)\, f\left(\frac{x}{\hat{x}}; Q^2\right), \\ \big(D \otimes C\big)(z; Q^2) &= \int_z^1 \frac{d\hat{z}}{\hat{z}}\, D\left(\frac{z}{\hat{z}}; Q^2\right) C(\hat{z}; Q^2)\,, \end{aligned} \tag{18}$$

and the splitting functions are given by

$$\begin{aligned} P_{qq}(\hat{x}) &= C_F\left[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{3}{2}\delta(1-\hat{x})\right], \\ P_{qg}(\hat{x}) &= T_R\left[\hat{x}^2 + (1-\hat{x})^2\right], \\ P_{gq}(\hat{x}) &= C_F\, \frac{1+(1-\hat{x})^2}{\hat{x}}\,, \end{aligned} \tag{19}$$

$$\begin{aligned} P'_{qq}(\hat{x}) &= C_F\left[\frac{2\hat{x}^2}{(1-\hat{x})_+} + \frac{3}{2}\delta(1-\hat{x})\right], \\ P'_{qg}(\hat{x}) &= 2T_R\, \hat{x}\,(2\hat{x}-1)\,, \\ P'_{gq}(\hat{x}) &= -2C_F\,(1-\hat{x})\,, \end{aligned} \tag{20}$$

where $C_F = (N_c^2 - 1)/2N_c$, $N_c = 3$ being the number of colors, $T_R = 1/2$, and the plus-distribution is defined by

$$\int_z^1 dy\, \frac{G(y)}{(1-y)_+} = \int_z^1 dy\, \frac{G(y) - G(1)}{1-y} - G(1) \ln \frac{1}{1-z}\,. \tag{21}$$

2.2. From low to intermediate transverse-momentum

We use Eq. (14) as our starting point. The distribution functions $f^\perp$, h and the fragmentation functions $\tilde{D}^\perp$, $\tilde{H}$ are twist-3 TMDs. The QCD equations of motion (EOM) lead to the useful relations [22]

$$x f^{\perp} = x\tilde{f}^{\perp} + f_1, \qquad xh = x\tilde{h} + \frac{k_{\perp}^2}{M^2} h_1^{\perp}, \tag{22}$$

which allow us to separate the distributions in their twist-2 and *pure* twist-3 ($\tilde{f}^{\perp}$ and $\tilde{h}$) components. Similar equations [22] hold for the fragmentation functions

$$\frac{\tilde{D}^{\perp}}{z} = \frac{D^{\perp}}{z} - D_1, \qquad \frac{\tilde{H}}{z} = \frac{H}{z} + \frac{P_{\perp}^2}{M_h^2} H_1^{\perp}. \tag{23}$$

The perturbative results to first order in α_s for the (subtracted) functions f_1 and D_1 are well known (see, e.g., Refs. [2,38,39]):

$$\begin{aligned}&\widehat{f}_1^a(x, \xi_T^2; Q^2, Q^2)\\ &= \frac{1}{2\pi}\Bigg\{ f_1^a(x, \mu_b^2) + \frac{\alpha_s}{\pi}\Bigg[-\frac{1}{4} C_F \left(\ln^2 \frac{Q^2}{\mu_b^2} - 3 \ln \frac{Q^2}{\mu_b^2} \right) \\ &\qquad \times f_1^a(x, \mu_b^2) + \sum_{i=a,g} \left(C_{ai}^{(1)} \otimes f_1^i \right)(x, \mu_b^2) \Bigg]\Bigg\},\end{aligned} \tag{24}$$

$$\begin{aligned}&\widehat{D}_1^a(z, \xi_T^2; Q^2, Q^2)\\ &= \frac{1}{2\pi z^2}\Bigg\{ D_1^a(z, \mu_b^2) + \frac{\alpha_s}{\pi}\Bigg[-\frac{1}{4} C_F \left(\ln^2 \frac{Q^2}{\mu_b^2} - 3 \ln \frac{Q^2}{\mu_b^2} \right) \\ &\qquad \times D_1^a(z, \mu_b^2) + \sum_{i=a,g} \left(\hat{C}_{ai}^{(1)} \otimes D_1^i \right)(z, \mu_b^2) \Bigg]\Bigg\},\end{aligned} \tag{25}$$

where $\mu_b = 2e^{-\gamma_E}/\xi_T$, and γ_E is the Euler constant. With this choice, the first-order coefficient functions $C^{(1)}$ and $\hat{C}^{(1)}$ become ξ_T-independent.

We then apply the DGLAP equations to evolve $f_1^a(x, \mu^2)$ from the scale μ_b^2 to Q^2,

$$\begin{aligned} f_1^a(x, \mu_b^2) &= f_1^a(x; Q^2) - \frac{\alpha_s}{2\pi} \sum_{i=a,g} (P_{ai} \otimes f_1^i)(x, Q^2) \ln \frac{Q^2}{\mu_b^2} \\ &\quad + O(\alpha_s^2). \end{aligned} \tag{26}$$

From this point on, we work out the results only for *finite* $k_{\perp}$ and to first order in α_s. In this region, and at this order, we can neglect the contribution coming from the $C^{(1)}$ and $\hat{C}^{(1)}$ coefficient functions. Using Eqs. (A.1) and (A.2), we can Fourier-transform Eqs. (24) and (25) and obtain the corresponding results in transverse-momentum space:

$$\begin{aligned}& f_1^a(x, k_{\perp}^2; Q^2, Q^2)\Big|_{k_{\perp}\neq 0} \\ &= \frac{\alpha_s}{2\pi^2 k_{\perp}^2} \left\{ \frac{1}{2} L\left(\frac{Q^2}{k_{\perp}^2} \right) f_1^a(x, Q^2) + \sum_{i=a,g} (P_{ai} \otimes f_1^i)(x, Q^2) \right\},\end{aligned} \tag{27}$$

$$\begin{aligned}& D_1^a(z, P_{\perp}^2; Q^2, Q^2)\Big|_{P_{\perp}\neq 0} \\ &= \frac{\alpha_s}{2\pi^2 P_{\perp}^2} \left\{ \frac{1}{2} L\left(\frac{z^2 Q^2}{P_{\perp}^2} \right) D_1^a(z, Q^2) + \sum_{i=a,g} (D_1^i \otimes P_{ia})(z, Q^2) \right\}.\end{aligned} \tag{28}$$

The results of Eqs. (27) and (28) are in agreement with Eqs. (8.26) and (8.47) of Ref. [4], respectively, that were derived directly in momentum space. The only exceptions are the presence of the additional terms $-C_F f_1^a(x)$ and $-C_F D_1^a(x)$ in the formulas of Ref. [4], and the different arguments of the logs. Such a discrepancy is due to the different TMD definitions adopted in the two studies. In the present analysis, based on the formalism developed in Refs. [1–3], the subtracted TMD contains the soft factor that regulates the rapidity divergences. Conversely, in Ref. [4] (based on the original CSS formulation [5]) the soft factor is included in the structure function but not in the unsubtracted TMD itself. Including the (square root of the) soft factor in the definition of TMDs removes the rapidity divergences, which in practice reduces to the following replacements w.r.t. the formulas in Ref. [4]:

$$\begin{aligned} \frac{1}{2} L(\eta^{-1}) &\longrightarrow \frac{1}{2} L\left(\frac{Q^2}{k_{\perp}^2} \right) + C_F, \\ \frac{1}{2} L(\eta_h^{-1}) &\longrightarrow \frac{1}{2} L\left(\frac{z^2 Q^2}{P_{\perp}^2} \right) + C_F, \end{aligned} \tag{29}$$

where $\eta = k_{\perp}^2/x^2\zeta$, and ζ is a parameter that defines the gauge-fixing vector in the computation of the quark-quark correlator at twist 3 (see Eqs. (8.9) and (8.11) in Ref. [4]) and regulates the rapidity divergences. Similarly, $\eta_h = P_{\perp}^2/\zeta_h$ (see Eqs. (8.41) and (8.43) in Ref. [4]). We have verified that, as expected, the two formalisms lead to the same unpolarized structure function $F_{UU,T}$ in the intermediate q_T-region.

At high transverse momentum, the chiral-odd functions $h_1^{\perp}$ and $H_1^{\perp}$ are suppressed by factors of $M^2/k_{\perp}^2$ and $M^2/P_{\perp}^2$, respectively (see Eq. (5.45) of Ref. [4], Eq. (21) of Ref. [40], and Eq. (6) of Ref. [41]). We will therefore neglect the contributions to Eq. (14) that involve them.

We turn now to the twist-3 chiral-even TMDs, $f^{\perp}$ and $\tilde{D}^{\perp}$. The perturbative expansions of their unsubtracted analogues have been calculated at leading order in momentum space [4,11] (the presence of rapidity divergences in twist-3 TMDs was first pointed out in Ref. [42]). If we apply to these results the same recipe of Eq. (29) as for the unpolarized TMDs, then from Eq. (8.27) of Ref. [4] (and also Eq. (16) of Ref. [11]) we obtain the *subtracted* $f^{\perp}$:

$$\begin{aligned}& x f^{\perp a}(x, k_{\perp}^2; Q^2, Q^2)\Big|_{k_{\perp}\neq 0} \\ &= \frac{\alpha_s}{4\pi^2 k_{\perp}^2} \left\{ \frac{1}{2} L\left(\frac{Q^2}{k_{\perp}^2} \right) f_1^a(x, Q^2) + C_F f_1^a(x, Q^2) \right. \\ &\qquad \left. + \sum_{i=a,g} (P'_{ai} \otimes f_1^i)(x, Q^2) \right\}.\end{aligned} \tag{30}$$

Similarly, from Eq. (8.48) of Ref. [4], the *subtracted* $D^{\perp}$ is

$$\begin{aligned}& \frac{1}{z} D^{\perp a}(z, P_{\perp}^2; Q^2, Q^2)\Big|_{P_{\perp}\neq 0} \\ &= \frac{\alpha_s}{4\pi^2 P_{\perp}^2} \left\{ \frac{1}{2} L\left(\frac{z^2 Q^2}{P_{\perp}^2} \right) D_1^a(z, Q^2) \right. \\ &\qquad \left. + C_F D_1^a(z, Q^2) s + \sum_{i=a,g} (D_1^i \otimes (2P_{ia} - P'_{ia}))(z, Q^2) \right\}.\end{aligned} \tag{31}$$

Using the EOM relation in Eq. (23) we obtain

$$\begin{aligned}& \frac{1}{z} \tilde{D}^{\perp a}(z, P_{\perp}^2; Q^2, Q^2)\Big|_{P_{\perp}\neq 0} \\ &= -\frac{\alpha_s}{4\pi^2 P_{\perp}^2} \left\{ \frac{1}{2} L\left(\frac{z^2 Q^2}{P_{\perp}^2} \right) D_1^a(z, Q^2) - C_F D_1^a(z, Q^2) \right.\end{aligned}$$

$$+\sum_{i=a,g}(D_1^i\otimes P'_{ia})(z,Q^2)\Bigg\}\,. \tag{32}$$

Once again, this result differs by the term $-C_F D_1^a$ from Eq. (8.49) of Ref. [4] due to the use of subtracted versus unsubtracted TMDs.

In order to expand our starting formula in Eq. (14), we need the first derivatives of $f^\perp$ and $\tilde{D}^\perp$ defined in Eq. (12). Note that any contribution at vanishing transverse momentum in the original expressions in transverse-momentum space is irrelevant for these derivatives. Using the integrals in Eqs. (A.3) and (A.4) we find, up to first order in α_s,

$$\begin{aligned}&x\widehat{f}^{\perp(1)\,a}(x,\xi_T^2;Q^2,Q^2)\\&=\frac{1}{M^2\xi_T^2}\frac{\alpha_s}{4\pi^2}\left[\frac{1}{2}L\left(\frac{Q^2}{\mu_b^2}\right)f_1^a(x,Q^2)+C_F f_1^a(x,Q^2)\right.\\&\qquad\left.+\sum_i\left(P'_{ai}\otimes f_1^i\right)(x,Q^2)\right],\end{aligned} \tag{33}$$

$$\begin{aligned}&\frac{1}{z}\widehat{\tilde{D}}^{\perp(1)\,a}(z,\xi_T^2;Q^2,Q^2)\\&=-\frac{1}{z^2M_h^2\xi_T^2}\frac{\alpha_s}{4\pi^2}\left[\frac{1}{2}L\left(\frac{Q^2}{\mu_b^2}\right)D_1^a(z,Q^2)-C_F D_1^a(z,Q^2)\right.\\&\qquad\left.+\sum_i\left(D_1^i\otimes P'_{ai}\right)(z,Q^2)\right].\end{aligned} \tag{34}$$

If in Eq. (14) we are only interested in the high-q_T behavior of the structure functions and we work at first order in α_s, the $\mathcal{H}'_{\text{SIDIS}}$ factor can be set equal to 1. By further inserting the expressions of $\widehat{f}_1^a$ and $\widehat{D}_1^a$ from Eqs. (24), (25), respectively, the expressions of $\widehat{f}^{\perp(1)a}$ and $\widehat{\tilde{D}}^{\perp(1)a}$ from Eqs. (33), (34), respectively, and further neglecting the suppressed contributions from chiral-odd TMDs, we get

$$\begin{aligned}F_{UU}^{\cos\phi_h}&=-\frac{4\pi M^2}{Q}\left\{\sum_a e_a^2 x\int_0^\infty d\xi_T\,\xi_T^2\,J_1\left(\frac{\xi_T P_\perp}{z}\right)\right.\\&\qquad\times\left(-\frac{M_h^2}{M^2}\widehat{f}_1^a\frac{\widehat{\tilde{D}}^{\perp(1)a}}{z}+x\widehat{f}^{\perp(1)a}\widehat{D}_1^a\right)\\&\qquad\left.+\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right)+\mathcal{O}\left(\frac{q_T}{Q}\right)\right\}\\&=-\frac{1}{Q}\frac{\alpha_s}{2\pi^2z^2}\left\{\sum_a e_a^2x\int_0^\infty d\xi_T\,J_1\left(\frac{\xi_T P_\perp}{z}\right)\right.\\&\qquad\times\left[L\left(\frac{Q^2}{\mu_b^2}\right)f_1^a(x,Q^2)\,D_1^a(z,Q^2)\right.\\&\qquad+\sum_{i=a,g}\left(f_1^a(x;Q^2)(D_1^i\otimes P'_{ia})(z;Q^2)\right.\\&\qquad\left.\left.+(P'_{ai}\otimes f_1^i)(x;Q^2)\,D_1^a(z;Q^2)\right)\right]\\&\qquad\left.+\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right)+\mathcal{O}\left(\frac{q_T}{Q}\right)\right\}.\end{aligned} \tag{35}$$

Finally, using again the integrals of Eqs. (A.3) and (A.4) we recover the dominant term of $F_{UU}^{\cos\phi_h}$ in the intermediate momentum region,

$$\begin{aligned}F_{UU}^{\cos\phi_h}&=-\frac{1}{Q\,q_T}\frac{\alpha_s}{2\pi^2z^2}\left\{\sum_a xe_a^2\left[L\left(\frac{Q^2}{q_T^2}\right)f_1^a(x,Q^2)\,D_1^a(z,Q^2)\right.\right.\\&\qquad+\sum_{i=a,g}\left(f_1^a(x;Q^2)(D_1^i\otimes P'_{ia})(z;Q^2)\right.\\&\qquad\left.\left.+(P'_{ai}\otimes f_1^i)(x;Q^2)\,D_1^a(z;Q^2)\right)\right]\\&\qquad\left.+\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right)+\mathcal{O}\left(\frac{q_T}{Q}\right)\right\}.\end{aligned} \tag{36}$$

This result is identical to the one in Eq. (16) obtained in the collinear framework. On the contrary, in Ref. [4] a mismatch was found between the two descriptions at high- and low-q_T because of the extra term $-2C_F f_1^a D_1^a$ appearing only in the latter. We deduce that a systematic matching between the two descriptions (not only for $F_{UU,T}$ but also for $F_{UU}^{\cos\phi_h}$) is possible only by adopting the TMD definition of Refs. [1–3], that directly includes the (square root of the) soft factor. This applies to all the TMDs, not only to the unpolarized ones.

3. $\cos\phi$ asymmetry in Drell-Yan

Along the lines of the previous section, we now study the unpolarized DY process,

$$h_1(P_1)+h_2(P_2)\to\ell(l)+\bar{\ell}(l')+X\,, \tag{37}$$

where the momenta of the particles are within brackets. We consider only the electromagnetic interaction and denote by $q=l+l'$ the 4-momentum of the exchanged virtual photon with $Q^2=q^2$, being q_T its transverse component orthogonal to P_1 and P_2. The angular dependence of the cross section is conveniently written in the dilepton rest frame [43–45],

$$\begin{aligned}\frac{d\sigma}{d^4q\,d\Omega}=&\frac{\alpha^2}{2sQ^2}\Big\{(1+\cos^2\theta)F_{UU}^1+(1-\cos^2\theta)F_{UU}^2\\&+\sin 2\theta\cos\phi\,F_{UU}^{\cos\phi}+\sin^2\theta\cos 2\phi F_{UU}^{\cos 2\phi}\Big\}\,,\end{aligned} \tag{38}$$

where $d\Omega=d\cos\theta d\phi$ is the solid angle of the lepton ℓ and $s=(P_1+P_2)^2$. Measurements related to the above structure functions have been presented in Refs. [46–52]. Phenomenological analyses have been reported in, e.g., Refs. [53–57].

In the literature there are two common choices of reference frames, depending on the choice of the $\hat{z}$ axis: in the Collins-Soper frame (CS) [58], the $\hat{z}$ axis points in the direction that bisects the angle between $\boldsymbol{P}_1$ and $-\boldsymbol{P}_2$; in the Gottfried-Jackson frame (GJ) [59], it points in the direction of $\boldsymbol{P}_1$. A linear transformation connects the structure functions in the two frames [9],

$$\begin{pmatrix}F_{UU}^1\\F_{UU}^2\\F_{UU}^{\cos\phi}\\F_{UU}^{\cos 2\phi}\end{pmatrix}_{\text{GJ}}=\frac{1}{1+\rho^2}\begin{pmatrix}1+\frac{1}{2}\rho^2&\frac{1}{2}\rho^2&-\rho&\frac{1}{2}\rho^2\\\rho^2&1&2\rho&-\rho^2\\\rho&-\rho&1-\rho^2&-\rho\\\frac{1}{2}\rho^2&-\frac{1}{2}\rho^2&\rho&1+\frac{1}{2}\rho^2\end{pmatrix}\times\begin{pmatrix}F_{UU}^1\\F_{UU}^2\\F_{UU}^{\cos\phi}\\F_{UU}^{\cos 2\phi}\end{pmatrix}_{\text{CS}}, \tag{39}$$

where $\rho=q_T/Q$.

In collinear factorization, the structure functions F_{UU}^{1} and $F_{UU}^{\cos\phi}$ can be calculated to $\mathcal{O}(\alpha_s)$ in the intermediate transverse-momentum region ($\Lambda_{\rm QCD} \ll q_T \ll Q$) in both frames (see Eqs. (33,37) of Ref. [9])[2]:

$$
\begin{aligned}
F_{UU}^{1}\Big|_{\rm GJ} &= F_{UU}^{1}\Big|_{\rm CS} \\
&= \frac{\alpha_s}{2\pi^2 q_T^2}\Bigg\{\sum_a \frac{e_a^2}{N_c}\Bigg[L\Big(\frac{Q^2}{q_T^2}\Big) f_1^a(x_1) f_1^{\bar a}(x_2) \\
&\quad + \sum_{i=a,g}\Big((P_{ai}\otimes f_1^i)(x_1) f_1^{\bar a}(x_2) \\
&\quad + f_1^a(x_1)(P_{ai}\otimes f_1^i)(x_2)\Big)\Bigg] \\
&\quad + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{q_T}\Big) + \mathcal{O}\Big(\frac{q_T}{Q}\Big)\Bigg\}, \qquad (40)
\end{aligned}
$$

$$
\begin{aligned}
F_{UU}^{\cos\phi}\Big|_{\rm GJ} &= \frac{\alpha_s}{2\pi^2 Q\, q_T}\Bigg\{\sum_a \frac{e_a^2}{N_c}\Bigg[L\Big(\frac{Q^2}{q_T^2}\Big) f_1^a(x_1) f_1^{\bar a}(x_2) \\
&\quad + \sum_{i=a,g}\Big((P'_{ai}\otimes f_1^i)(x_1) f_1^{\bar a}(x_2) \\
&\quad + f_1^a(x_1)((2P_{ai}-P'_{ai})\otimes f_1^i)(x_2)\Big)\Bigg] \\
&\quad + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{q_T}\Big) + \mathcal{O}\Big(\frac{q_T}{Q}\Big)\Bigg\}, \qquad (41)
\end{aligned}
$$

$$
\begin{aligned}
F_{UU}^{\cos\phi}\Big|_{\rm CS} &= -\frac{\alpha_s}{2\pi^2 Q\, q_T}\Bigg\{\sum_a \frac{e_a^2}{N_c}\sum_{i=a,g}\Big[((P_{ai}-P'_{ai})\otimes f_1^i)(x_1) \\
&\quad \times f_1^{\bar a}(x_2) - f_1^a(x_1)((P_{ai}-P'_{ai})\otimes f_1^i)(x_2)\Big] \\
&\quad + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{q_T}\Big) + \mathcal{O}\Big(\frac{q_T}{Q}\Big)\Bigg\}, \qquad (42)
\end{aligned}
$$

where for simplicity we have suppressed the dependence of PDFs on the hard scale Q^2. Higher-order contributions in α_s have been implemented in Ref. [55,60]. Strikingly, $F_{UU}^{\cos\phi}$ have very different behaviors in CS and GJ frames, namely, the logarithmic terms are not present in CS frame. This fact was the reason to believe that resummation of $F_{UU}^{\cos\phi}$ is very different from CSS one (see Ref. [9]). The authors of Ref. [10] pointed out that the formalism based on collinear QCD factorization is not enough to reconcile these behaviors and derive the correct resummed formula. In order to obtain the complete resummation result, we will start from the TMD expression and show how it reduces to the correct collinear expressions in both frames.

In the low-q_T region, coherently with Eq. (14), we *assume* that the parton-model result for the $F_{UU}^{\cos\phi}$ structure function (see, e.g., Ref. [45]) can be generalized by including higher-order contributions to the hard scattering and replacing the TMDs with the subtracted ones. We obtain then

$$
F_{UU}^{1} = \mathcal{H}_{\rm DY}\sum_a \frac{e_a^2}{N_c}\mathcal{B}_0\Big[\widehat{f}_1^a \widehat{f}_1^{\bar a}\Big] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right), \qquad (43)
$$

$$
\begin{aligned}
F_{UU}^{\cos\phi} &= \frac{2M^2}{Q}\Bigg\{\mathcal{H}'_{\rm DY}\sum_a \frac{e_a^2}{N_c}\mathcal{B}_1\Bigg[\Big((1-c)x_1\widehat{f}^{\perp(1)\,a} \\
&\quad + c x_1 \widehat{\tilde{f}}^{\perp(1)\,a}\Big)\widehat{f}_1^{\bar a} - \widehat{f}_1^{a}\Big(c\,x_2\widehat{f}^{\perp(1)\,\bar a} + (1-c)\,x_2\widehat{\tilde{f}}^{\perp(1)\,\bar a}\Big) \\
&\quad + \frac{M_2}{M_1}\widehat{h}_1^{\perp(1)a}\Big(c\,x_2\widehat{h}^{\bar a} + (1-c)\,x_2\widehat{\tilde{h}}^{\bar a}\Big) \\
&\quad - \frac{M_1}{M_2}\Big(\Big((1-c)\,x_1\widehat{h}^{a} + c\,x_1\widehat{\tilde{h}}^{a}\Big)\widehat{h}_1^{\perp(1)\bar a}\Big)\Bigg] + \mathcal{O}\Big(\frac{q_T}{Q}\Big)\Bigg\}, \qquad (44)
\end{aligned}
$$

where $c=0$ for the GJ frame, $c=1/2$ for the CS frame, and M_i, with $i=1,2$, are the masses of the initial hadrons.

It is important to note that since our formula contains subtracted TMDs, we can guarantee the validity of the relations of Eq. (39) connecting different frames. In fact, the difference between the GJ and CS frames is, at first order in ρ,

$$
F_{UU}^{\cos\phi}\Big|_{\rm GJ} - F_{UU}^{\cos\phi}\Big|_{\rm CS} = \rho\Big(F_{UU}^{1} - F_{UU}^{\cos 2\phi}\Big)\Big|_{\rm CS} + \mathcal{O}(\rho^2). \qquad (45)
$$

The structure functions on the right-hand side (at this order, it is not relevant whether they are written in one frame or the other) contribute to the leading-twist part of the cross section. For the factorization theorem to hold, they must contain subtracted TMDs. We have directly checked that the difference of subleading-twist structure functions on the left-hand side matches the right-hand side only if subtracted TMDs are involved, as we conjecture in Eq. (44).

The dominant contribution of $F_{UU}^{\cos\phi}$ in the intermediate q_T region can now be calculated in a straightforward way, along the lines of the previous section. We neglect the (suppressed) chiral-odd terms. We replace $\widehat{f}_1$ with the expression in Eq. (24), $\widehat{f}^{\perp(1)}$ with Eq. (33), and we obtain $\widehat{\tilde{f}}^{\perp(1)}$ from the relation $x\widehat{\tilde{f}}^{\perp(1)} = x\widehat{f}^{\perp(1)} - \widehat{f}_1^{(1)}$, as in Eq. (22). We also use the following formula

$$
\begin{aligned}
&\widehat{f}_1^{(1)\,a}(x,\xi_T^2;Q^2,Q^2) \\
&\equiv -\frac{1}{M^2}\frac{1}{\xi_T}\frac{\partial}{\partial \xi_T}\widehat{f}_1^{a}(x,\xi_T^2;Q^2,Q^2) \\
&= \frac{1}{M^2\xi_T^2}\frac{\alpha_s}{4\pi^2}\Big[L\Big(\frac{Q^2}{\mu_b^2}\Big) f_1^a(x,Q^2) + 2\sum_{i=a,g}(P_{ai}\otimes f_1^i)(x,Q^2)\Big]. \qquad (46)
\end{aligned}
$$

By performing these substitutions, we obtain the general expression

$$
\begin{aligned}
F_{UU}^{\cos\phi} &= \frac{\alpha_s}{2\pi^2 Q\, q_T} \\
&\quad\times\Bigg\{\sum_a \frac{e_a^2}{N_c}\Bigg\{(1-2c)\,L\Big(\frac{Q^2}{q_T^2}\Big) f_1^a(x_1,Q^2) f_1^{\bar a}(x_2,Q^2) \\
&\quad + \sum_{i=a,g}\Big[\big[(-2cP_{ai}+P'_{ai})\otimes f_1^i\big](x_1,Q^2) f_1^{\bar a}(x_2,Q^2) \\
&\quad + f_1^a(x_1,Q^2)\big[((2-2c)P_{ai}-P'_{ai})\otimes f_1^i\big](x_2,Q^2)\Big]\Bigg\} \\
&\quad + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{q_T}\Big) + \mathcal{O}\Big(\frac{q_T}{Q}\Big)\Bigg\}. \qquad (47)
\end{aligned}
$$

It can be verified that the above expression is in agreement with Eq. (41) for $c=0$ and with Eq. (42) for $c=1/2$. We have reproduced the correct large-q_T results in both frames that were

[2] The prefactors in our formulas are different from Ref. [9] due to different definitions of the structure functions.

obtained in Refs. [9,10], and thus have solved the long standing problem of resummation of collinear results in large q_T region.

In the so-called Wandzura–Wilczek approximation, the pure twist-three components (functions with tilde) are neglected. In App. B we show that also in this case the leading logarithmic term proportional to $L(Q^2/q_T^2)$ matches in the two descriptions at low and high q_T, when using subtracted TMDs. The nonlogarithmic terms however cannot be correctly reproduced in Wandzura–Wilczek approximation.

4. Conclusions

In this work, we analyzed the $\cos\phi$ modulation of the unpolarized cross section in semi-inclusive DIS (SIDIS) and in the Drell-Yan process (DY). At high values of transverse momentum in the final-state system, this observable can be computed in a standard way in terms of collinear unpolarized PDF and FF. At low transverse momentum, it represents the simplest observable that can be written in terms of subleading-twist TMDs. However, no factorization proof for TMD observables at twist-3 is available and inconsistencies have been pointed out in the literature [4,9–11], casting a doubt on the possibility of achieving such a proof.

For the low transverse momentum region, in analogy with twist-2 observables, we propose a simple modification of the parton-model formula, with the replacement of TMDs with subtracted TMDs, according to Refs. [1–3]. We show that our formula correctly matches the collinear result at high transverse momentum for both SIDIS and DY, solving a problem first highligthed in Ref. [4].

As for DY, we further show that our formula guarantees the correct behavior under the change of frames of reference, which is a nontrivial feature when considering twist-3 contributions (see also Ref. [61]). We also solve the long standing problem of resummation of collinear QCD results in large-q_T region posed in Refs. [9,10].

Our conjecture provides a formula for the $\cos\phi$ modulation at low transverse momentum that is compatible with TMD factorization up to subleading twist. It can be readily applied to other modulations and to electron-positron annihilation [62]. We believe that this is an important step towards the full proof.

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Appendix A. Bessel integrals

Using the notation $b_0 = 2\exp(-\gamma_E)$, we can write the following integrals

$$\int_0^\infty dx\, x\, J_0(xy) \ln \frac{A^2x^2}{b_0^2}\bigg|_{y\neq 0} = -\frac{2}{y^2}, \tag{A.1}$$

$$\int_0^\infty dx\, x\, J_0(xy) \ln^2 \frac{A^2x^2}{b_0^2}\bigg|_{y\neq 0} = -\frac{4}{y^2}\ln\frac{A^2}{y^2}, \tag{A.2}$$

$$\int_0^\infty dx\, J_1(xy) \ln \frac{A^2x^2}{b_0^2} = \frac{1}{y}\ln\frac{A^2}{y^2}, \tag{A.3}$$

$$\int_0^\infty dx\, J_n(xy) = \frac{1}{y}. \tag{A.4}$$

Appendix B. Wandzura–Wilczek approximation

In the so-called Wandzura–Wilczek approximation, the pure twist-three components of TMDs (functions with tilde) are neglected. The EOM relations in Eqs. (22–23) reduce to

$$xf^\perp \approx f_1, \quad xh \approx \frac{k_\perp^2}{M^2}h_1^\perp, \quad D^\perp \approx zD_1, \quad H \approx -z\frac{P_\perp^2}{M_h^2}H_1^\perp. \tag{B.1}$$

The formula for the structure function $F_{UU}^{\cos\phi_h}$ in SIDIS, Eq. (14), considerably simplifies and we obtain (neglecting chiral-odd terms)

$$\begin{aligned} F_{UU}^{\cos\phi_h} &\overset{\text{WW}}{=} -\frac{2M^2}{Q}\mathcal{H}'_{\text{SIDIS}}\sum_a e_a^2 x\mathcal{B}_1\Big[\widehat{f}_1^{(1)a}\widehat{D}_1^a\Big] \\ &\approx -\frac{1}{Q\,q_T}\frac{\alpha_s}{2\pi^2 z^2}\Bigg\{\sum_a xe_a^2\Bigg[L\bigg(\frac{Q^2}{q_T^2}\bigg)f_1^a(x,Q^2)\,D_1^a(z,Q^2) \\ &\quad + 2\sum_{i=a,g}(P_{ai}\otimes f_1^i)(x,Q^2)\,D_1^a(z,Q^2)\Bigg] \\ &\quad + \mathcal{O}\bigg(\frac{\Lambda_{\text{QCD}}}{P_{hT}}\bigg) + \mathcal{O}\bigg(\frac{P_{hT}}{Q}\bigg)\Bigg\}. \end{aligned} \tag{B.2}$$

Comparing this result with Eq. (16), we see that the leading logarithmic term proportional to $L(Q^2/q_T^2)$ matches in the two descriptions at low and high q_T, when using subtracted TMDs. The nonlogarithmic terms are not correctly reproduced in the Wandzura–Wilczek approximation.

Similarly, for Drell–Yan we obtain

$$\begin{aligned} F_{UU}^{\cos\phi} &\overset{\text{WW}}{=} \frac{2M^2}{Q}\mathcal{H}'_{\text{DY}}\sum_a \frac{e_a^2}{N_c}\mathcal{B}_1\Big[(1-c)\widehat{f}_1^{(1)a}\widehat{f}_1^{\bar a} - c\widehat{f}_1^a\widehat{f}_1^{(1)\bar a}\Big] \\ &\approx \frac{\alpha_s}{2\pi^2 Q\,q_T}\Bigg\{\sum_a \frac{e_a^2}{N_c}\Bigg[(1-2c)L\bigg(\frac{Q^2}{q_T^2}\bigg) \\ &\quad \times f_1^a(x_1,Q^2)f_1^{\bar a}(x_2,Q^2) \\ &\quad + \sum_{i=a,g}\Big[2(1-c)(P_{ai}\otimes f_1^i)(x_1,Q^2)f_1^{\bar a}(x_2,Q^2) \\ &\quad - 2cf_1^a(x_1,Q^2)(P_{ai}\otimes f_1^i)(x_2,Q^2)\Big]\Bigg] \\ &\quad + \mathcal{O}\bigg(\frac{\Lambda_{\text{QCD}}}{q_T}\bigg) + \mathcal{O}\bigg(\frac{q_T}{Q}\bigg)\Bigg\}, \end{aligned} \tag{B.3}$$

where only the leading logarithmic term proportional to $L(Q^2/q_T^2)$ matches to Eq. (47).

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