THE $1/N_c$ EXPANSION IN BARYONS

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OUTLINE

- Introduction
- Baryon masses
- Partial widths
- Lattice and $1/N_c$
- Comments and outlook

INTRODUCTION

- PDG: 23 $N's$, 22 $\Delta's$, 18 $\Lambda's$, 28 $\Sigma's$, 12 $\Xi's$, 4 $\Omega's$;
Partial decay widths; Photocouplings; FF's
- Numerous experimental programmes: JLab, MAMI, ELSA, BES, ...
- Rapid progress in lattice QCD: LHPC, NPLQCD, PACS-CS, ...
- Organizing principles of baryon observables with QCD symmetries: J^P , T, Isospin, broken $SU(3)$
- Further organizing principle rooted in QCD: $1/N_c$ expansion
- We know how to implement $1/N_c$ expansion as effective theory at hadronic level
- It provides:
 - testable parameter free relations at given order in $1/N_c$
 - predictions

● Propaganda with the Meson/Glueball sector

- OZI suppression: $\Gamma_M^{OZI} = \mathcal{O}(1/N_c^3)$

$$\frac{\Gamma(\phi \rightarrow \rho\pi + 3\pi)}{\Gamma(\phi \rightarrow K\bar{K})} \sim 0.2$$

$$\frac{\Gamma(J/\psi \rightarrow \text{hadrons})}{\Gamma(J/\psi \rightarrow \ell^+\ell^-)} \sim 7.5 \quad \text{vs} \quad \frac{\Gamma(\rho \rightarrow \pi\pi)}{\Gamma(\rho \rightarrow \ell^+\ell^-)} \sim 10^4$$

- χPT : OZI suppression of Gasser-Leutwyler LECs:
 $2L_1/L_2 = 1 + \mathcal{O}(1/N_c), \quad L_4/L_5, \quad L_6/L_5 = \mathcal{O}(1/N_c)$

L_5	$0.97 \pm 0.11 \times 10^{-3}$
$2L_1/L_2$	1.17 ± 0.38
L_4	$-0.3 \pm 0.5 \times 10^{-3}$
L_6	$-0.2 \pm 0.3 \times 10^{-3}$

Predictions for $1/N_c$ scaling in Unitarized χPT [Peláez]

- Lattice gluodynamics: ratio $\frac{m_{GB}}{\sqrt{\sigma}} = \mathcal{O}(1) + \mathcal{O}(1/N_c^2)$ [Teper]

- Witten-Veneziano formula

$$M_{\eta'}^2 = \frac{4 N_f \chi_{YM}}{F_\pi^2}$$

$$\chi_{YM}|_{WV} \sim (160 MeV)^3$$

$$\chi_{YM}|_{latt} \sim (180 MeV)^3$$

[Panagopoulos & Vicari]

- **Baryons in $1/N_c$**

- Need for N_c valence quarks to form a color singlet
 - $M_B = \mathcal{O}(N_c)$, $r_B = \mathcal{O}(N_c^0)$
 - Hartree picture of baryons sufficient to figure out $1/N_c$ countings [Witten]
 - π -baryon couplings: $\frac{g_A}{F_\pi} \partial_i \pi_a G^{ia}$
- Axial currents G^{ia} have $\mathcal{O}(N_c)$ MEs $\implies g_{\pi BB} = \mathcal{O}(\sqrt{N_c})$
- $\Gamma_B = \mathcal{O}(N_c^0)$
 - Model realizations of all of the above: QM, Skyrme model

Contracted spin-flavor dynamical symmetry

The diagram shows two Feynman diagrams separated by a plus sign. The first diagram has a solid red horizontal line with an arrow pointing right. Two dashed lines branch off from this line: one goes up and to the left, the other goes up and to the right. The second diagram has a solid red horizontal line with an arrow pointing right. Two dashed lines branch off from this line: one goes up and to the left, the other goes down and to the right, crossing the first dashed line.

$$\propto \langle B' | [G^{jb}, G^{ia}] | B \rangle = \mathcal{O}(N_c^0)$$

Contracted spin-flavor symmetry [Gervais & Sakita; Dashen & Manohar]

$S^i, T^a, X^{ia} \equiv G^{ia}/N_c$: generators of *contracted* $SU(2N_f)_C$

- $SU(2N_f)$ group with generators S^i, T^a, G^{ia} can be used to build effective theory

- Justifies use of $SU(6)$ symmetry in large N_c

Approximate $O(3)$ symmetry

Baryon spectrum shows approximate $O(3)$ symmetry (small spin-orbit effects)

$O(3) \times SU(2N_f)$ most convenient classification scheme

Spin-flavor multiplets

Known states can be fitted into $SU(6)$ **56** and **70** plets

No definite evidence for **20**-plet nor pentaquark multiplets

Effective operators in the $1/N_c$ expansion

- Effective operators at baryon level

$$\hat{O}_{\text{QCD}} \Rightarrow \hat{O}_{\text{eff}} = \sum_i \left(\frac{1}{N_c} \right)^{\nu(i)} C_i \hat{O}_i$$

- operator basis $\{\hat{O}_j\}$ can be ordered in powers of $1/N_c$
- C_i effective constants or form factors: determined by the QCD dynamics; expanded also in $1/N_c$ and quark masses
- $\nu(i) = n_i - 1$ for n_i -body operator

- Matrix elements

$$\langle B'_{GS} | \hat{O}_j | B_{GS} \rangle = \mathcal{O}(N_c^{\kappa(j)}) ; \quad \langle B_{GS} | \hat{O}_j | B^* \rangle = \mathcal{O}(N_c^{-\frac{1}{2} + \kappa(j)})$$

$\kappa(j)$ coherence factor of operator

- General form of effective operators

$$\hat{O}_j = R_j \otimes \mathcal{G}_j$$

R_j : $O(3)$ tensor operator

$\mathcal{G}_j$: spin-flavor tensor operator made out of product of n_j generators of $SU(2N_f)$

$\kappa(\hat{O}_j) = \#G_{ia}$ or T^8 generators in $\mathcal{G}_j$

- Systematic way of building basis of $\mathcal{G}_j$ operators; numerous “reduction rules”

BARYON MASSES

• Ground state baryons

Most general mass operator after applying various reduction rules
Up to $\mathcal{O}(1/N_c)$ and $\mathcal{O}(m_s)$: old Gürsey-Radicati mass formula

$$M_{GS} = c_1 N_c + \frac{c_{HF}}{N_c} (S^2 - \frac{3}{4}N_c) - c_S \frac{m_s - m_{u,d}}{\Lambda} \mathcal{S}$$

Parameter free mass relations: deviations $\mathcal{O}(1/N_c^2; m_s^2/N_c)$ and also due to chiral loop $\mathcal{O}(m_s^{3/2})$ [Jenkins & Lebed]

GMO	$\Xi_8 - \Sigma_8 = \frac{1}{2}(3\Lambda - \Sigma_8) - N$	128 vs 141 MeV
ES	$\Sigma_{10} - \Delta = \Xi_{10} - \Sigma_{10}$	153 vs 145
"	$\Omega^- - \Xi_{10} = \Xi_{10} - \Sigma_{10}$	142 vs 145
8-10	$\Sigma_{10} - \Sigma_8 = \Xi_{10} - \Xi_8$	212 vs 195
"	$3\Lambda + \Sigma_8 - 2(N + \Xi_8) = \Xi_{10} + \Sigma_{10} - \Omega^- - \Delta$	26 vs 11

● **Excited baryons:** $[70, 1^-]$ [JLG; Carlson et al.; JLG, Schat & Scoccola]

$$\begin{array}{c}
 \text{core} \\
 \overbrace{\hspace{1.5cm}}^{N_c-1} \\
 \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} \\
 S_c^i, T_c^a, G_c^{ia}
 \end{array}
 \times
 \begin{array}{c}
 q^* \\
 \square \\
 s^i, t^a, g^{ia}
 \end{array}$$

$$O_{eff} = \sum \Lambda_c \times \lambda_{q^*} \times \lambda_\ell \qquad M_{70} = \underbrace{\sum_n c_n O_n}_{\text{SU(3) singlet}} + \underbrace{\sum_m d_m B_m}_{\text{SU(3) octet}}$$

● States in **70**-plet:

$$2 \mathbf{8}_{J=1/2}, \quad 2 \mathbf{8}_{3/2}, \quad 1 \mathbf{8}_{5/2}, \quad 1 \mathbf{10}_{1/2}, \quad 1 \mathbf{10}_{3/2}, \quad 1 \mathbf{1}_{1/2}, \quad 1 \mathbf{1}_{3/2}$$

● Operators:

$$\text{SU(3) singlets: } 1 \mathcal{O}(N_c) \text{ (SU(6) singlet), } 4 \mathcal{O}(N_c^0), 7 \mathcal{O}(\frac{1}{N_c})$$

$$\text{SU(3) octets: } 3 \mathcal{O}(m_s N_c^0)$$

Mass operators for 70-plets

$\mathcal{O}(N_c)$	$O_1 = N_c \mathbf{1}$	$449 \pm 2 \text{ MeV}$
$\mathcal{O}(N_c^0)$	$O_2 = \ell \cdot s$	52 ± 15
	$O_3 = \frac{3}{N_c} \ell_{ij}^{(2)} g^{ia} G_c^{ja}$	116 ± 44
	$O_4 = \frac{4}{N_c+1} \ell^i t^a G_c^{ia}$	110 ± 16
	$O_5 = \frac{1}{N_c} t^a T_c^a - \frac{1}{2\sqrt{3}N_c} O_1$	-81 ± 36
$\mathcal{O}(\frac{1}{N_c})$	$O_6 = \frac{1}{N_c} \ell \cdot S_c$	74 ± 30
	$O_7 = \frac{1}{N_c} S_c^2$	480 ± 15
	$O_8 = \frac{1}{N_c} s \cdot S_c$	-159 ± 50
	$O_9 = \frac{2}{N_c} \ell^{(2)ij} s^i S_c^j$	3 ± 55
	$O_{10} = \frac{3}{N_c^2} \ell^i g^{ja} \{S_c^j, G_c^{ia}\}$	71 ± 51
	$O_{11} = \frac{2}{N_c^2} t^a \{S_c^i, G_c^{ia}\}$	-84 ± 28
	$O_{12} = \frac{3}{N_c^2} \ell^i g^{ia} \{S_c^j, G_c^{ja}\}$	-44 ± 43
	$\bar{B}_1 = T_c^8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	-194 ± 17
$\mathcal{O}(m_s)$	$\bar{B}_2 = \frac{10}{N_c} d_{8ab} g^{ia} G_c^{ib} + \frac{5(N_c^2-9)}{8\sqrt{3}N_c^2(N_c-1)} O_1$ $+ \frac{5}{2\sqrt{3}(N_c-1)} O_6 + \frac{5}{6\sqrt{3}} O_7$	-15 ± 30
	$\bar{B}_3 = 3 \ell^i g^{i8} - \frac{\sqrt{3}}{2} O_2$	-27 ± 19

70-plet mass fit to NLO

State	Masses Expt	[MeV] $1/N_c$
$N_{1/2}$	1538 ± 18	1541
$\Lambda_{1/2}$	1670 ± 10	1667
$\Sigma_{1/2}$	(1620)	1637
$\Xi_{1/2}$	—	1779
$N_{3/2}$	1523 ± 8	1532
$\Lambda_{3/2}$	1690 ± 5	1676
$\Sigma_{3/2}$	1675 ± 10	1667
$\Xi_{3/2}$	1823 ± 5	1815
$N'_{1/2}$	1660 ± 20	1660
$\Lambda'_{1/2}$	1785 ± 65	1806
$\Sigma'_{1/2}$	1765 ± 35	1755
$\Xi'_{1/2}$	—	1927
$N'_{3/2}$	1700 ± 50	1699
$\Lambda'_{3/2}$	—	1864
$\Sigma'_{3/2}$	—	1769
$\Xi'_{3/2}$	—	1980

State	Masses Expt	[MeV] $1/N_c$
$N_{5/2}$	1678 ± 8	1671
$\Lambda_{5/2}$	1820 ± 10	1836
$\Sigma_{5/2}$	1775 ± 5	1784
$\Xi_{5/2}$	—	1974
$\Delta_{1/2}$	1645 ± 30	1645
$\Sigma''_{1/2}$	—	1784
$\Xi''_{1/2}$	—	1922
$\Omega_{1/2}$	—	2061
$\Delta_{3/2}$	1720 ± 50	1720
$\Sigma''_{3/2}$	—	1847
$\Xi''_{3/2}$	—	1973
$\Omega_{3/2}$	—	2100
$\Lambda''_{1/2}$	1407 ± 4	1407
$\Lambda''_{3/2}$	1520 ± 1	1520

- Parameter free mass relations: true to $\mathcal{O}(1/N_c^2; m_s^2/N_c)$
5 GMO, 4 ES, 6 new relations involving **8's**, **10's** and **1's**
Predict masses of several as yet unknown states in **70**-plet
- $1/N_c$ corrections natural (or smaller than natural) in size
- Hyperfine terms S_c^2 dominate spin-flavor breaking
- All $\mathcal{O}(N_c^0)$ operators in **70**-plets have small coefficients: three spin-orbit type and one of pure flavor type: **approximate $O(3)$ symmetry**
- All $\mathcal{O}(1/N_c)$ operators other than S_c^2 give small corrections
- $SU(3)$ breaking dominated by strangeness operator
- Mixing angles for N^* s

	$N_c \rightarrow \infty$	Masses	Strong decays	Helicity Amplitudes
θ_1	54.4°	$30 \pm 15^\circ$	$22 \pm 7^\circ$	-
θ_3	65.9°	$175 \pm 15^\circ$	$161 \text{ or } 136 \pm 7^\circ$	$161 \pm 7^\circ$

Mixing angle correlations in 70-plet [Pirjol & Schat] Two universal relations when only 2-body interactions are considered

- *Masses and decays to $O(1/N_c)$*
- *Flavor independent int*
- *Pirjol-Schat*
- *Isgur-Karl*

Baryon Regge trajectories: M_0^2 vs ℓ [JLG & Matagne]

Spin-flavor singlet component of baryon masses from analyses of all well and not so well known **56**-plets and **70**-plets:

$$[56, 0^+]_{\text{GS}}, [56, (2^+, 4^+, 6^+)], [70, (1^-, 2^+, 3^-, 5^-)]$$

- $M_0^2[56, \ell] = [(1.18 \pm 0.003) + (1.05 \pm 0.01) \ell] \text{ GeV}^2$
- $M_0^2[70, \ell] = [(1.13 \pm 0.02) + (1.18 \pm 0.02) \ell] \text{ GeV}^2$
- $(M_0[70, \ell] - M_0[56, \ell])^2 \simeq (5.7 + 4.2 \ell) \times 10^{-4} \text{ GeV}^2$
- Splitting $\mathcal{O}(N_c^0)$: exchange interaction in QM
Magnitude smaller than expected
- Regge trajectories with physical masses include contributions which violate linear behavior
- Strong indication of small **56-70** configuration mixings and good approximate $O(3)$ symmetry
- AdS/QCD gives separate 56-70 plet trajectories but of equal slope [de Teramond]

PARTIAL DECAY WIDTHS IN SU(6)

- Strong partial wave amplitudes

$$\mathcal{M}(\ell_\pi, I_\pi) = (-1)^{\ell_\pi} \sqrt{2M_{B^*}} \frac{\sqrt{N_c}}{F_\pi} \langle B_{GS} | \mathbf{B}^{\ell_\pi I_\pi} | B^* \rangle$$

- $1/N_c$ expansion

Expand $\mathbf{B}$ in effective operators

$$\begin{aligned} \mathbf{B}^{\ell_\pi I_\pi} &= \left(\frac{k_\pi}{\Lambda} \right)^{\ell_\pi} \sum_n C_n O_n^{\ell_\pi I_\pi} \\ O_n^{\ell_\pi I_\pi} &= [\xi^\ell \mathcal{G}_n^{j_n I_\pi}]^{[\ell_\pi, I_\pi]} \end{aligned}$$

$\mathcal{G}_n^{j_n I_\pi}$ are spin-flavor tensor operators

ℓ the $O(3)$ quantum number of excited baryon

C_n coefficients to be fitted

56-plet partial widths [JLG, Jayalath & Scoccola]

Spin-flavor operator basis to $\mathcal{O}(1/N_c; m_s)$

Operator	Order
$\mathcal{G}_1 \equiv \frac{1}{N_c} G$	$\mathcal{O}(N_c^0)$
$\mathcal{G}_2 \equiv \frac{1}{N_c^2} \{S, G\}^{[j=1]}$	$\mathcal{O}(1/N_c)$
$\mathcal{G}_3 \equiv \frac{1}{N_c^2} \{S, G\}^{[j=2]}$	$\mathcal{O}(1/N_c)$
$\mathcal{G}_{SB} \equiv \frac{1}{N_c} (d_{8ab} - \delta_{ab}/\sqrt{3}) G_{ib}$	$\mathcal{O}((m_s - m_{u,d})/\sqrt{N_c})$

[56, 0⁺] **Roper multiplet**

Fits to PDG B-W widths

	χ^2_{dof}	dof	C_1	C_2	B_1
LO	1.7	5	6.7(0.8)	—	4.2(3.2)
NLO	0.9	4	8.7(1.0)	-9.3(4.3)	8.2(3.8)

	$N_{1/2}(1440)$		$\Lambda_{1/2}(1600)$			$\Sigma_{1/2}(1660)$				$\Delta_{3/2}(1600)$	
	πN	$\pi \Delta$	$\bar{K} N$	$\pi \Sigma$	$\pi \Sigma^*$	$\bar{K} N$	$\pi \Lambda$	$\pi \Sigma$	$\pi \Sigma^*$	πN	$\pi \Delta$
LO	148	13	36	57	19	1.6	32	61	6.5	97	90
NLO	214	65	38	111	95	5	63	88	25	56	131
Error	48.8	17.3	28.3	25.2	27.3	3.9	14.3	20.1	7.1	43.9	29.8
Exp	211(88)	81(35)	34(25)	53(51)	—	24(20)	—	—	—	61(32)	193(76)

- LO:

$$\Gamma(N(1440) \rightarrow \pi N) / \Gamma(N(1440) \rightarrow \pi \Delta) = 7.5 \text{ vs } 2.6 \pm 2.1 \text{ Exp}$$

$$\Gamma(\Delta(1600) \rightarrow \pi N) / \Gamma(\Delta(1600) \rightarrow \pi \Delta) = 1.7 \text{ vs } 0.3 \pm 0.2 \text{ Exp}$$

$$\Gamma(N(1440) \rightarrow \pi N) / \Gamma(\Delta(1600) \rightarrow \pi \Delta) = 1.64 \text{ vs } 1.10 \pm 0.63 \text{ Exp}$$

valid also at NLO!

- NLO: 2-body operator is important but within natural size
- SU(3) breaking not clearly established: more decays involving strangeness need to be known

[56, 2⁺]: P and F waves

● Fits to P-waves

	χ^2_{dof}		dof	C_1	C_2	C_3	B_1
LO	0.7	5	2.2(0.3)	—	—	—	-1.2(1.6)
NLO	1.2	3	2.2(0.3)	0.2(1.0)	0.5(2.3)	—	-1.3(1.6)

	$N_{\frac{3}{2}}(1720)$					$\Lambda_{\frac{3}{2}}(1890)$					$N_{\frac{5}{2}}(1680)$	$\Lambda_{\frac{5}{2}}(1820)$	$\Sigma_{\frac{5}{2}}(1915)$	
	πN	ηN	$K\Sigma$	$K\Lambda$	$\pi\Delta$	KN	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$	$\pi\Sigma^*$	$\pi\Delta$	$\pi\Sigma^*$	$K\Delta$	$\pi\Sigma^*$
LO	36	1	0.1	3	2	38	16	4	0.3	1.6	9	6.6	9.6	2.5
NLO	27	2.4	0.2	2.9	2.5	38	16	9.8	1	1.8	10	6.1	7.2	1.8
Error	6.1	1.5	0.1	1.6	3.0	21.1	3.6	6.1	0.6	2.6	4.5	3.3	4.0	0.9
Exp	34(16)	—	—	18(17)	—	36(22)	8.5(6.4)	—	—	—	13(5.3)	—	—	—

	$\Delta_{\frac{1}{2}}(1910)$					$\Delta_{\frac{3}{2}}(1920)$					$\Delta_{\frac{5}{2}}(1905)$		
	πN	$K\Sigma$	$\pi\Delta$	$\eta_8\Delta$	$K\Sigma^*$	πN	$K\Sigma$	$\pi\Delta$	$\eta_8\Delta$	$K\Sigma^*$	$\pi\Delta$	$\eta_8\Delta$	$K\Sigma^*$
LO	35	13	6	0.4	0	19	9	23	2.2	0.8	26	1.7	0.1
NLO	45	9.9	5.7	1	0	22	6.5	19	5.2	0.6	18	4	0.1
Error	24.9	5.5	8.4	0.6	—	12.9	3.6	15.1	3.2	0.3	10.8	2.5	0.03
Exp	52(20)	—	—	—	—	28(19)	—	—	—	—	—	—	—

- **P-wave:**
LO: Consistent fit $\Rightarrow$ small NLO corrections
 Small SU(3) breaking effects
 Expect predictions for unknown P-wave widths to be reliable
- **F-wave:**
LO: $\chi^2_{dof} \sim 3.5$ (mostly due to $\Sigma_{5/2}(1915) \rightarrow \bar{K}N$) $\Rightarrow$ need for NLO corrections
NLO: both 2-body operators are important as well as the SU(3) breaking operator
- General observation: partial widths of non-strange 56-plet baryons into ηN or $\eta \Delta$ are $\mathcal{O}(1/N_c^2)$ (none observed in established 56-plets)
- Analysis of 70-plet decays in talk by C. Jayalath this afternoon

LATTICE BARYONS AND $1/N_c$ EXPANSION

- GS parameter free mass relations in lattice [Walker-Loud et al]
- Extract eff coeff's in $1/N_c$ mass formulas from lattice baryon masses
- Interesting to understand their quark mass dependence
- Example: GS baryons - LO and HF coefficients [Jayalath]

- Same for excited baryons [JLG & Jayalath in progress]

COMMENTS and OUTLOOK

- Other applications: photo-couplings [JLG, Matagne & Scoccola]
T-matrix [Cohen, Dakin & Nellore]
Configuration mixings [JLG]
Form factors [Buchmann & Lebed]
 $\pi - N$ scattering [Kwee & Lebed]
- $1/N_c$ expansion implements controlled framework to analyze breaking of $O(3) \times SU(6)$
- Numerous new parameter free relations and predictions (in particular masses)
- More (accurate) experimental and/or lattice QCD inputs needed for testing
- We need more insight into the physics hidden in the effective coefficients
- $1/N_c$ expansion is a fundamental expansion: does it work at $N_c = 3$?
Baryons can help clarify this question.
- Phenomenology of mesons and baryons seems to indicate consistency with the $1/N_c$ expansion, but further more accurate tests are needed