ANALYSIS OF SINGLE PION ELECTROPRODUCTION
DATA FROM CLAS

HOVANES EGIYAN, VOLKER D. BURKERT

Thomas Jefferson National Laboratory,
12000 Jefferson Avenue,
Newport News, VA 23606, USA

INNA AZNAURYAN

Yerevan Physics Institute,
2 Alikhanyan Brothers Street,
Yerevan, Armenia 375036

FOR THE CLAS COLLABORATION

The single pion electroproduction data in the first and second resonance regions from CLAS at $Q^2 = 0.4$ GeV$^2$ were analyzed using unitary isobar model and dispersion relations based frameworks. The combined data set included single beam asymmetries and differential cross sections in $ep \rightarrow e'\pi^0p$ and $ep \rightarrow e'\pi^+n$ reactions. The preliminary values for resonance photocoupling amplitudes for $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$ from the two approaches were compared with each other as well as with the results of previous analyses.

1. Introduction

Single pion electroproduction is one of the most suitable exclusive channels for studying the excitation of the resonances in the first and second resonance regions because of the large $\pi N$ coupling for these states. The detection of two out of three outgoing particles allows us to extract the amplitudes for the individual resonances by fitting the angular distributions of the pion in the hadronic center-of-mass frame.

The quantities of interest that can be derived from the study of the pion electroproduction in the second resonance region are photon coupling amplitudes for $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$. These three isospin $I = \frac{1}{2}$ states favor the decays into the $\pi^+n$ channel, and the measurement of the $ep \rightarrow e'\pi^+n$ channel is crucial for determining the photocoupling amplitudes for transitions into these excited states. Up to now the lack of high quality data in $\pi^+n$ has prevented a precise analysis for the states in the second resonance region.

2. Data

The present analysis was performed using $Q^2 = 0.4$ GeV$^2$ with the coverage of the $p\pi^0$ channel from 1.1 GeV to 1.6 GeV. The analysis and beam spin asymmetries for the $p\pi^0$ channel were obtained for $\Delta n^0$ channel were obtained for the proton. The spin asymmetries for $n\pi^+$ channel data from the 1999 run period for $p\pi^0$ channel were obtained from the advantage of the CLAS detector systems is the nearly complete reference frame for both $n\pi^+$ and $n\pi^0$ channels due to the physics analysis of single pion electroproduction formed on data with significant sample of differential cross sections $ep \rightarrow e'\pi^0p$ process are shown in the result of the fit based on the

![Figure 1](image1.png)

Figure 1. Sample plots showing single spin asymmetries versus $\phi$ angle in the case of the unitary isobar fit.
ELECTROPRODUCTION

CLAS

GER D. BURKERT
Lawrence Berkeley Laboratory,
100 1st Street, Berkeley, 94720, USA

YAN
Institute for Nuclear Studies,
3750006

LABORATION

first and second resonance regions
combined data set included single
processes for $e p \rightarrow e' + n p$ and $e p \rightarrow e' + \pi^+ n$

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\begin{itemize}
  \item most suitable exclusive chancinons in the first and second
  \item derived from the study of the
  \item $S_{11}(1535)$. These three isospin
  \item channel, and the measurement
  \item the photocoupling amplitudes
  \item to now the lack of high quality
\end{itemize}

data in $\pi^+ n$ has prevented a precise extraction of the electro-excitation amplitudes for the states in the second resonance region using coupled channel analysis techniques.

2. Data

The present analysis was performed on the data set from CLAS\textsuperscript{1} at $Q^2 = 0.4$ GeV\textsuperscript{2} with the coverage in invariant mass $W$ extending from 1.1 GeV to 1.6 GeV. The analyzed data included differential cross sections and beam spin asymmetries for single $\pi^+$ and $\pi^0$ electroproduction on the proton. The spin asymmetries for both neutral and charged channels and the cross sections for $n\pi^+$ channel were extracted from the analysis of CLAS data from the 1999 run period at 1.5 GeV. The differential cross sections for $p\pi^0$ channel were obtained from an earlier run period in 1998. The main advantage of the CLAS detector compared with the traditional two-arm detector systems is the nearly complete angular coverage in the center-of-mass reference frame for both $n\pi^+$ and $pm^0$ final states. This reduces the uncertainty due to the physics analysis model compared with previous analyses of single pion electroproduction data in the second resonance region performed on data with significantly less angular coverage for the $\pi^+$ channel. Sample plots of differential cross sections and beam spin asymmetries for $e p \rightarrow e' + n p$ process are shown in Fig. 1. The curves on these plots show the result of the fit based on the unitary isobar model described in the next section.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{example.png}
\caption{Sample plot showing single pion electroproduction cross sections and beam
spin asymmetries versus $\phi$ angle in the center-of-mass frame. The curves show the result
of the unitary isobar fit.}
\end{figure}
3. Analysis

3.1. Unitary Isobar Model

The unitary isobar model used in this analysis describes the observables from the electro- and photoproduction experimental data in the resonance region using s-channel resonance contributions parametrized in Breit-Wigner form with s-dependent width $\Gamma$, and fixed non-resonant background\textsuperscript{2}. The resonant multipoles, the masses and widths of the resonances are found by fitting the experimental data. The background terms in the first resonance region include nucleon pole terms in the s- and u-channels as well as the $t$-channel $\pi$, $\rho$ and $\omega$ exchanges. The background includes nucleon pole terms in the s- and u-channels as well as the $t$-channel $\pi$, $\rho$ and $\omega$ exchanges. To improve the description of the background contributions above the first resonance region this background with increasing energy is transformed into Regge-pole amplitudes\textsuperscript{3} according to\textsuperscript{2}:

$$M_{\text{bkg}} = M_{N,\pi,\rho,\omega} \frac{1}{1 + (s - s_0)^2} + ReM_{\text{Regge}} \frac{(s - s_0)^2}{1 + (s - s_0)^2},$$

(1)

where the value of the parameter $s_0 = 1.16$ GeV is found from the best description of photoproduction multipole amplitudes\textsuperscript{5} up to $W = 2$ GeV.

For each multipole amplitude the background is unitarized according to the $K$-matrix approximation:

$$M_{\text{bkg}}^{\text{unit}} = M_{\text{bkg}} (1 + h_{1N}^N),$$

(2)

where $h_{1N}^N$ is the $\pi N$ scattering amplitude.

3.2. Fixed-t Dispersion Relations

The dispersion relations analysis described in this work is based on the approach previously used to analyze pion photoproduction data\textsuperscript{2,4}. Using general principles of causality and analyticity of the amplitudes one can relate the real parts of the invariant amplitudes\textsuperscript{a} to the imaginary parts of these amplitudes\textsuperscript{4}:

$$Re B_i^{(\pm,0)}(s,t,Q^2) = R_i^{(v,s)}(Q^2) \left( \frac{1}{s - m_k^2} + \frac{\eta (\pm,0)}{u - m_N^2} \right)$$

$$+ \frac{P}{\pi} \int_{s_{th}}^{\infty} Im B_i(s',t,Q^2) \left( \frac{1}{s' - s} + \frac{\eta (\pm,0)}{s' - u} \right) ds',$$

(3)

where $R_i$ are the residues in the amplitudes are saturated by mass resonances. The resonant integral equations for multipole amplitudes other resonances are parametrized by the non-resonant contributions $E_{1+}$ were found using dispersive contributions from the integral Regge pole model\textsuperscript{3}. The strength fit to this parameter was low, yielded only slightly lower $\chi^2$.

4. Preliminary Results

To obtain the multipole amplitudes in the resonance region a combined fit was performed on photoproduction and the dispersion relations ap

<table>
<thead>
<tr>
<th>Process</th>
<th>$A_{1/2}^T$</th>
<th>$A_{1/2}^A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}(1440)$</td>
<td>$-19 \pm 1$</td>
<td>$-24 \pm 1$</td>
</tr>
<tr>
<td>$S_{11}(1535)$</td>
<td>$17 \pm 3$</td>
<td>$17 \pm 3$</td>
</tr>
<tr>
<td>$D_{13}(1520)$</td>
<td>$104 \pm 2$</td>
<td>$104 \pm 2$</td>
</tr>
<tr>
<td>UIM</td>
<td>$92 \pm 3$</td>
<td>$92 \pm 3$</td>
</tr>
<tr>
<td>DR</td>
<td>$-49 \pm 1$</td>
<td>$-50 \pm 1$</td>
</tr>
<tr>
<td>DR</td>
<td>$-46 \pm 2$</td>
<td>$-47 \pm 2$</td>
</tr>
<tr>
<td>DR</td>
<td>$71 \pm 3$</td>
<td>$72 \pm 3$</td>
</tr>
</tbody>
</table>

Table 1. Preliminary results: the amplitudes are given in units of $10^{-3}$ GeV$^{-1}$, and are in MeV.
where $R_i$ are the residues in the nucleon pole terms. The imaginary parts of the amplitudes are saturated with contributions from $\Delta(1232)$ and higher mass resonances. The resonant amplitudes for $\Delta(1232)$ are found by solving integral equations for multipoles analogous to Eq. 3. The amplitudes for the other resonances are parametrized in the Breit-Wigner form. In addition, the non-resonant contributions to imaginary parts of $E_{0+}$, $M_{1-}$, $M_{1+}$, and $E_{1+}$ were found using dispersion relations and the Watson theorem. The contributions from the integrands at $s > 3$ GeV were calculated using a Regge pole model. The strengths of the resonance amplitudes were found by fitting the experimental data. The masses and widths for $P_{11}(1440)$, $S_{11}(1535)$ and $D_{13}(1520)$ were allowed to vary during the fit. The width of the Roper resonance was fixed at 450 MeV because the sensitivity of the fit to this parameter was low, and unphysical higher values for the width yielded only slightly lower $\chi^2$ values.

4. Preliminary Results

To obtain the multipole amplitudes for the states in the second resonant region a combined fit was performed on differential cross sections and beam spin asymmetries measured in CLAS using both the unitary isobar model and the dispersion relations approach. Table 1 summarizes the parameters obtained from the fit, while Table 2 shows the values for masses, widths and $\pi N$ branching ratios for the resonances used to convert the multipoles into helicity amplitudes. The transverse photon coupling amplitudes for $S_{11}(1535)$ and $D_{13}(1520)$ obtained from isobar and dispersion relations based fits are in good agreement with each other. The differences can

\[
M_{Regge} \frac{(s - s_0)^2}{1 + (s - s_0)^2},
\]

\[
\chi^2 = \frac{1}{s - s_0} \left( \frac{\eta_{l} e^{i\phi}}{u - m_N^2} \right),
\]

Table 1. Preliminary results of the fit of the CLAS data. The amplitudes are given in units of $10^{-3}$ GeV$^{-\frac{3}{2}}$, the masses and widths of the states are in MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$A_{1/2}^p$</th>
<th>$S_{1/2}^p$</th>
<th>$W_R$</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}(1440)$</td>
<td>$-10 \pm 1$</td>
<td>$19 \pm 1$</td>
<td>$1426 \pm 4$</td>
<td>$340 \pm 27$</td>
</tr>
<tr>
<td>$S_{11}(1535)$</td>
<td>$-10 \pm 2$</td>
<td>$19 \pm 1$</td>
<td>$1426 \pm 4$</td>
<td>$340 \pm 27$</td>
</tr>
<tr>
<td>$D_{13}(1520)$</td>
<td>$92 \pm 3$</td>
<td>$-14 \pm 4$</td>
<td>$1519 \pm 5$</td>
<td>$138 \pm 26$</td>
</tr>
</tbody>
</table>
be considered as an estimate of the model dependent uncertainties of the multipole extraction procedures. On the other hand, the longitudinal amplitudes for these two states, and both $A_{1/2}$ and $S_{1/2}$ for $P_{11}(1440)$ appear to be model dependent. But in both methods the helicity amplitudes for the Roper resonance were found to be small. The background contribution to $M^1_{-2}$ and $S^1_{1/2}$ was dominant at $Q^2 = 0.4$ GeV$^2$, while in the case of the photoproduction it is small, and the dependence of $M^1_{-2}$ on $W$ at $Q^2 = 0$ has a clear resonant behavior. This observation can explain the differences in the results for $P_{11}(1440)$ obtained with isobar and dispersion relation fits. A more sophisticated coupled-channel analysis is required to obtain these amplitudes. Figs. 2 and 3 show the comparison of the photon couplings obtained in the present work with the results of previous analyses. The $A_{1/2}$ amplitude for $S_{11}(1535)$ is in reasonable agreement with the data from recent $\eta$-channel analysis from CLAS$^{11}$. The discrepancy may be due to uncertainty in the knowledge of the branching ratio of $S_{11}(1535)$ into $\pi N$ and $\eta N$ channels. The small value of $A_{1/2}$ obtained in both analyses of CLAS data for the Roper at $Q^2 = 0.4$ GeV$^2$ indicates a very fast fall-off of this amplitude with $Q^2$. The $D_{13}(1520)$ is consistent with indicates a smaller absolute value.

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**Table 2.** Masses, widths and $\pi N$ branching ratios for $P_{11}(1440)$, $S_{11}(1535)$ and $D_{13}(1520)$ used to obtain the photon coupling amplitudes.

<table>
<thead>
<tr>
<th>State</th>
<th>Mass</th>
<th>Full Width</th>
<th>$\pi N$ BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}(1440)$</td>
<td>1.440 GeV</td>
<td>0.350 GeV</td>
<td>0.7</td>
</tr>
<tr>
<td>$S_{11}(1535)$</td>
<td>1.535 GeV</td>
<td>0.150 GeV</td>
<td>0.45</td>
</tr>
<tr>
<td>$D_{13}(1520)$</td>
<td>1.520 GeV</td>
<td>0.110 GeV</td>
<td>0.6</td>
</tr>
</tbody>
</table>

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**Figure 2.** Preliminary results for $A_{1/2}$ photon coupling amplitudes for $P_{11}(1440)$ and $S_{11}(1535)$. Full circles are from the isobar fit, the full squares are from the dispersion relation fit. The diamonds and triangles are results from previous analyses$^{6-11}$. The solid, dashed, dash-dotted and dotted lines are from various theoretical calculations$^{7-9,10}$.

**Figure 3.** Preliminary results for $D_{13}(1520)$. The markers and line is from theoretical calculations$^{7-9,10}$.

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**Acknowledgments**

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**References**

6. V. D. Burkert, in Perspectives in Nuclear Physics, Plenum, 1981.
of this amplitude with $Q^2$. The obtained photocoupling amplitude $A_{3/2}$ for $D_{13}(1520)$ is consistent with previous analyses, while the CLAS analysis indicates a smaller absolute value of the $A_{1/2}$ amplitude.

![Graph showing preliminary results for $A_{1/2}$ and $A_{3/2}$ photon coupling amplitudes for $D_{13}(1520)$. The markers and lines are as in Fig. 2.](image)

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**References**