ISOSPIN MIXING IN THE ^4He BOUND STATE AND THE NUCLEON STRANGE FORM FACTOR

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The contribution of isospin admixtures in the ground state of the ^4He nucleus is studied using wave functions derived from the most modern nuclear interactions, including isospin symmetry breaking terms. The present calculations show that this contribution is larger than previous estimates had indicated. Its effect on parity violating elastic scattering of polarized electrons from ^4He is investigated. In particular, a simple analysis of the recently measured left-right asymmetry at low Q^2 shows that the contribution of these isospin admixtures is of comparable magnitude to that associated with strangeness components in the nucleon electric form factor.

Keywords: Isospin admixtures in complex nuclei, parity violating electron scattering

1. Introduction

Understanding the structure of the nucleon is one of the great challenges of modern hadron physics. In particular, an important issue is to determine, at a quantitative level, the role of the $q\bar{q}$ sea, especially of the strange quark. Parity-violating (PV) elastic scattering of polarized electrons from nucleons and nuclei offers the opportunity to investigate this issue experimentally. The PV asymmetry is due to the interference between the γ - and Z -induced amplitudes. The Z couples to the neutral current (NC) of the nucleon, which is sensitive to its flavor content in a different way than the electromagnetic current. Therefore, from measurements of the PV asymmetry it is possible

to disentangle the strange electric and magnetic form factors $G_E^{(s)}$ and $G_M^{(s)}$ of the nucleon, the analogs of the usual Sachs form factors G_E and G_M .

Indeed, new results of strangeness in the nucleon have been reported recently from the HAPPEX¹ and G0² Collaborations at Jlab, which complemented earlier work at MIT-Bates³ and Mainz.⁴ The experiments scatter longitudinally polarized electrons from an unpolarized target and measure the asymmetry $A_{LR} = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$, where σ_R (σ_L) is the cross section for right-handed (left-handed) electrons. The asymmetry is tiny, of the order of a few parts per million.

Most of these experiments study PV electron scattering from protons. They are sensitive to linear combinations of $G_E^{(s)}$ and $G_M^{(s)}$ (and also to the axial form factor G_A^Z).⁵ It would be desirable to have enough measurements to determine the contribution of the individual form factors. From this perspective, the study of PV electron scattering on ^4He is very interesting. This is because ^4He is a $J^\pi = 0^+$ state and therefore no contributions from magnetic and axial-vector currents are allowed.⁵ However, there are nuclear effects which could mask the strange quark content of the nucleon.

In a nuclear target, these effects have different origins. There are contributions from meson exchange currents (MEC). This problem was studied in Ref. 6 using a realistic ^4He wave function and a realistic model of MEC. The authors of that paper concluded that for $Q^2 < 0.15 \text{ GeV}^2$, where $Q^2 = q^2 - \omega^2$ denotes the square of the 4-momentum transfer (ω and q are the energy and the magnitude of the three-momentum transfer to the target), the MEC contributions are negligible. If, in addition to MEC, radiative corrections and a tiny relativistic contribution (associated with the spin-orbit term in the single-nucleon strange charge operator) are ignored, the PV asymmetry for electron scattering on a ^4He target is simply given by⁵

$$A_{LR} = \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \left[4\sin^2\theta_W + \frac{G_E^{(s)}}{(G_E^p + G_E^n)/2} - 2R_{T=1} \right], \quad (1)$$

where G_μ is the Fermi constant measured in muon decay, α the fine structure constant, and θ_W the weak mixing angle. The term $R_{T=1}$ is due to the presence of $T = 1$ components in the nuclear wave function. Since ^4He is to a very good approximation an isoscalar target, $R_{T=1}$ is usually neglected. The aim of this paper is to estimate the order of magnitude of $R_{T=1}$. The additional contribution arising from isospin admixtures in the *nucleon*⁷ is not shown explicitly in Eq. (1) and will be discussed elsewhere. In this paper we will consider the proton and neutron as an isospin doublet.

An approximate calculation of this quantity was performed more than 10 years ago,⁸ by taking into account only the isospin admixture induced by the Coulomb interaction and, furthermore, by generating the relevant $T = 1$ component in the ${}^4\text{He}$ wave function in first order perturbation theory. The calculated value for $R_{T=1}$ was found to be rather small, and it produced a less than 1% correction in Eq. (1)⁸ at low Q^2 , using the value for $G_E^{(s)}(Q^2)$ known at the time.

Three new facts have prompted us to re-analyze this issue. First, new refined measurements of A_{LR} have shown that $G_E^{(s)}(Q^2 \approx 0.1\text{GeV}^2)$ is considerably smaller than previously assumed.⁹

Second, there now exist several accurate models of nucleon-nucleon (NN) interactions^{10–14} which include explicit charge symmetry breaking (CSB) terms, in addition to the Coulomb repulsion between protons. These CSB terms have been determined by fitting both the pp and np scattering data sets, and therefore are tightly constrained by experiment. In order to study ${}^4\text{He}$, three-nucleon (NNN) interactions have also to be included in the Hamiltonian. These are still not well known, and the most commonly used models^{15–17} do not contain isospin-symmetry violating terms. However, the NNN interaction gives much smaller effects relative to the NN: for example, its expectation value in ${}^4\text{He}$ is around 5% than that of the NN interaction. Therefore, one would expect that NNN CSB terms to be very tiny indeed.

Third, several accurate methods have been developed to compute ${}^4\text{He}$ wave functions starting from a given realistic model of NN and NNN interactions.¹⁸ In these calculations, $T > 0$ isospin components are generated non-perturbatively. The $T = 1$ percentage in the ${}^4\text{He}$ wave function is typically found to be of the order of 10^{-3} %, systematically larger than estimated in Ref. 8 (see below).

For these reasons, we have reconsidered the problem of determining the magnitude of the term $R_{T=1}$. We have also studied the “model dependence” of this quantity, namely its dependence on the nuclear Hamiltonian used to generate the ${}^4\text{He}$ wave function.

In this paper, we use the Hyperspherical Harmonic (HH) expansion method to compute the ${}^4\text{He}$ wave function. The accuracy of this method is briefly discussed in Sec. 2, where some selected results obtained for ${}^4\text{He}$ observables are reported and compared with corresponding ones given in literature. The results for $R_{T=1}$ are presented and discussed in the last section.

Table 1. The α -particle binding energies B (MeV), the expectation values of the kinetic energy operator $\langle T \rangle$ (MeV), the mean square radii $\sqrt{\langle r^2 \rangle}$ (fm), and the P , D , $T = 1$ and $T = 2$ probabilities (%) for the four Hamiltonian models considered in this paper.

Interaction	Method	B	$\langle T \rangle$	$\sqrt{\langle r^2 \rangle}$	P_P	P_D	$P_{T=1}$	$P_{T=2}$
AV18	HH	24.210	97.84	1.512	0.347	13.74	0.0028	0.0052
	FY ²²	24.25	97.80	-	0.35	13.78	0.003	0.005
	FY ²⁶	24.223	97.77	1.516	-	-	-	-
	FY ²⁴	24.24	-	-	-	-	-	-
	EIHH ²⁹	24.268	-	1.518	-	-	-	-
AV18/UIX	HH	28.47	113.30	1.428	0.73	16.03	0.0025	0.0050
	FY ²²	28.50	113.21	-	0.75	16.03	-	-
	GFMC ²⁷	28.34(4)	110.7(7)	1.44	-	-	-	-
CDBonn/UIXb	HH	28.30	85.82	1.423	0.53	12.89	0.0020	0.0099
N3LO	HH	25.38	69.24	1.516	0.172	9.289	0.0035	0.0024
	FY ³⁰	25.37	69.20	-	0.172	9.293	0.0033	0.0024
	FY ²⁴	25.38	-	-	-	-	-	-
	NCSM ²⁵	25.36(4)	-	-	-	-	-	-

2. The Hyperspherical Harmonic Method

The HH method has been presented in detail in Refs. 19–21. Here, we shall limit ourselves to discuss the accuracy of the method. To this end, the fully converged values for the ^4He binding energies and other ground-state properties are reported in Table 1. The model Hamiltonians considered are: 1) the Argonne V18 NN interaction¹¹ (AV18 model); 2) the Argonne V18 NN + Urbana IX NNN interactions¹⁵ (AV18/UIX model); 3) the CD Bonn¹² NN + Urbana IXb NNN interactions (CDBonn/UIXb model); and 4) the chiral N3LO¹³ NN interaction (N3LO model). The Urbana UIXb NNN interaction is a slightly modified version of the Urbana UIX potential to be used together with the CD Bonn potential (in that model, the parameter U_0 in the central repulsive term has been multiplied by the factor 0.812). The CD Bonn and N3LO NN potentials are non-local and are given in momentum space. These results are compared with those obtained with other methods, including the Faddeev-Yakubovsky (FY) technique,^{22,24,26} No Core Shell Model (NCSM) approach,²⁵ Green's Function Monte Carlo (GFMC) method,²⁷ and Effective Interaction HH (EIHH) technique.^{28,29} There is good numerical agreement, at the level of 0.1 %, among the results obtained with these different techniques for all quantities considered in the Table.

We also report the calculated percentages of $T > 0$ components in the ^4He ground state wave function. In this case too, the results obtained with the HH method are in excellent agreement with those obtained using the FY technique. We find that, depending on the Hamiltonian model adopted, the

values of the $T = 1$ percentage vary little, while those of $T = 2$ percentage display significantly larger variation. The origin of these $T > 0$ components was discussed in detail in Ref. 20. We are confident to have full control of the $T > 0$ components in the ground state wave function.

3. Results and Conclusions

In ^4He the PV asymmetry reads (ignoring radiative corrections)

$$A_{LR} = \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \left[4\sin^2\theta_W + \frac{F^{(s)}(Q^2)}{F^{(T=0)}(Q^2)} - 2\frac{F^{(T=1)}(Q^2)}{F^{(T=0)}(Q^2)} \right], \quad (2)$$

where

$$F^{(a)}(Q^2) = \frac{1}{\sqrt{4\pi}} \langle \Psi | \rho_\mu^{(a)}(q_\mu) | \Psi \rangle, \quad (a) = (T=0), (T=1), (s), \quad (3)$$

are the various “charge form factors” of ^4He . In the equation above, Ψ is the ^4He ground state wave function and $\rho^{(a)}$ are charge operators. Their one-body components are given by

$$\rho^{(a)}(q_\mu) = \sum_{k=1}^A e^{i\mathbf{q}\cdot\mathbf{r}_k} \left[\frac{G_E^{(a)}(Q^2)}{\sqrt{1+\tau}} \right], \quad (a) = (T=0), (s), \quad (4)$$

$$\rho^{(T=1)}(q_\mu) = \sum_{k=1}^A e^{i\mathbf{q}\cdot\mathbf{r}_k} \tau_z(k) \left[\frac{G_E^{(T=1)}(Q^2)}{\sqrt{1+\tau}} \right], \quad (5)$$

where $\tau = Q^2/4m_N^2$ and spin-orbit relativistic corrections⁶ have been ignored. The isoscalar $G_E^{(T=0)}$ and isovector $G_E^{(T=1)}$ combinations of the electric nucleon form factors are defined as

$$G_E^{(T=0)}(Q^2) = \frac{1}{2} [G_E^p(Q^2) + G_E^n(Q^2)], \quad (6)$$

$$G_E^{(T=1)}(Q^2) = \frac{1}{2} [G_E^p(Q^2) - G_E^n(Q^2)]. \quad (7)$$

In this limit (*i.e.*, one-body charge operators), it follows that

$$\frac{F^{(s)}(Q^2)}{F^{(T=0)}(Q^2)} = \frac{G_E^{(s)}(Q^2)}{G_E^{(T=0)}(Q^2)}, \quad (8)$$

namely nuclear structure effects cancel out, and by comparison with Eq. (1), we identify

$$R_{T=1} \equiv \frac{F^{(T=1)}(Q^2)}{F^{(T=0)}(Q^2)}. \quad (9)$$

Fig. 1. The $F^{(T=0)}(Q^2)$ and $F^{(T=1)}(Q^2)$ form factors plotted as functions of Q for the Hamiltonian models considered in this paper. The $F^{(T=0)}(Q^2)$ is compared with the experimental ${}^4\text{He}$ charge form factor.^{31,32} Note that the calculated form factors reported here include also MEC contributions to the charge density operators. However, their effect is significant only for $Q > 2 \text{ fm}^{-1}$.

We have performed the calculation of the form factors $F^{(T=0)}$ and $F^{(T=1)}$ using Monte Carlo techniques⁶ and the ${}^4\text{He}$ wave functions discussed in the previous section. The calculated values of $F^{(T=0)}$ and $F^{(T=1)}$ are reported in Fig.1 for four Hamiltonian models (the AV18, AV18/UIX, CD-Bonn/UIXb and N3LO models). The theoretical predictions for $F^{(T=0)}$, *i.e.* the ${}^4\text{He}$ charge form factor, are compared with the experimental data of Refs. 31,32. As can be seen, the form factor $F^{(T=1)}$ is rather small and the various Hamiltonian models give predictions quite close to each other, the remaining differences reflect the different percentages of the $T = 1$ component in the ${}^4\text{He}$ wave function.

Let us now consider the effect of the present form factors predictions on PV electron scattering from ${}^4\text{He}$. The most accurate measurements have been recently reported at $Q^2 = 0.077 \text{ GeV}^2$, corresponding to $Q \approx 1.41 \text{ fm}^{-1}$. The experimental value for A_{LR} is⁹

$$A_{LR} = +6.40 \pm 0.23(\text{stat}) \pm 0.12(\text{syst}) \text{ ppm} , \quad (10)$$

Table 2. Calculated values of $F^{(T=0)}$, $F^{(T=1)}$, and $R_{T=1}$ for $Q^2 = 0.077 \text{ GeV}^2$ for Hamiltonian models considered in this paper. The values in parentheses have been obtained by including MEC in the charge operators.

Interaction	$F^{(T=0)}$	$F^{(T=1)}$	$R_{T=1}$
AV18	0.835	-0.00174	-0.00208 (-0.00211)
AV18/UIX	0.890	-0.00140	-0.00157 (-0.00160)
CDBonn/UIXb	0.897	-0.00141	-0.00157 (-0.00158)
N3LO	0.800	-0.00218	-0.00275 (-0.00274)

which, after inserting the values for G_μ , α , and θ_W , gives (see Eq. (1))

$$\frac{G_E^{(s)}(Q^2)}{G_E^{(T=0)}(Q^2)} - 2 R_{T=1} \equiv \Gamma = 0.0048 \pm 0.04, \quad Q^2 = 0.077 \text{ GeV}^2. \quad (11)$$

This result is consistent with zero. We report the calculated values of $R_{T=1}$ at $Q^2 = 0.077 \text{ GeV}^2$ in Table 2.

The $R_{T=1}$ term is found to be of the order of -0.002 . Note that the value previously estimated in Ref. 8 was $|R_{T=1}(Q^2 = 0.077 \text{ GeV}^2)| \approx 0.0003$, an order of magnitude less than the present result. The value obtained with N3LO is slightly larger than for other potentials, reflecting the larger percentage of $T = 1$ admixtures predicted by this potential. The inclusion of a NNN interaction tends to decrease the value of $R_{T=1}$ and the effect of MEC is confirmed to be, at this value of Q^2 , negligible.

Finally, the computed $R_{T=1}$ is of the same order of magnitude as the value of Γ in Eq. (11). Using in Eq. (11) the value $R_{T=1} \approx -0.00157$ obtained with the Hamiltonians including NNN forces, one would obtain $G_E^{(s)}(Q^2 = 0.077 \text{ GeV}^2) = 0.001 \pm 0.02$, thus suggesting that the value of Γ is almost entirely due to isospin admixtures in the ^4He wave function. Of course, the experimental error on Γ is still too large to allow us to draw a more definite conclusion.

In summary, we have estimated the value of $R_{T=1}$ at the Q^2 value relevant for the recent HAPPEX experiment,⁹ and have found it to be larger than previously estimated. A recent lattice QCD estimate of $G_E^{(s)}$ reports³³ $G_E^{(s)}(0.1 \text{ GeV}^2) = +0.001 \pm 0.004 \pm 0.003$. An increase of the experimental accuracy of one order of magnitude would be necessary to be sensitive to $G_E^{(s)}$ at low values of Q^2 . However, the present study shows that the effect of the $T = 1$ component needs to be taken into account. Indeed, if the present lattice QCD predictions are confirmed, the present data would suggest that the leading correction to the asymmetry is from $T = 1$ isospin admixtures in ^4He . As shown in Ref. 20, this component does

not only originate from the Coulomb repulsion between protons, but arises also from CSB terms in the nuclear interaction. The study of CSB is a very interesting problem, since it is linked to fundamental properties of the u and d quarks, namely their mass and charge differences.³⁴

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