Baryon Regge Trajectories and the $1/N_c$ Expansion

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Baryon Regge trajectories are discussed in the light of the $1/N_c$ expansion. The approximate dynamical symmetry $SU(6) \times O(3)$ is used to identify the spin-flavor singlet component of baryon masses. By fitting to the known baryon spectrum, this component is shown to produce distinct Regge trajectories for the $SU(6)$ **56**- and **70**-plets which are remarkably linear.

Keywords: $1/N_c$ expansion; baryons; Regge trajectories

1. Introduction

One of the best established and remarkable properties of meson and baryon spectra is the ordering of states into approximately linear trajectories in the Chew-Frautschi plot, also known as Regge trajectories. They are the most clear manifestation of the confining dynamics of QCD, supporting the picture of confinement as the result of string-like behavior of the interaction between color charges at long distances. In the light meson sector, the plot M^2 vs J shows very clearly an approximately linear law; for instance, non-strange mesons with masses up to about 2.5 GeV fall into several nearly linear trajectories: π -trajectory, $\rho - \omega$ -trajectory, f -trajectory, etc., with slopes in the range 1.1 to 1.4 GeV². One interesting aspect is the possibility of assigning an “orbital” quantum number ℓ to each meson, as one would do in a quark model when spin-orbit interactions are neglected. Plotting instead M^2 vs ℓ leads to a set of trajectories with similar slopes, and with intercepts that are similar (with exception of the π -trajectory, where

the Goldstone Boson nature of the pion forces a nearly vanishing intercept). One can proceed further and fit a single linear trajectory M^2 vs ℓ , which shows that all states are along this trajectory within a rather narrow spread, the only clear discrepancy resulting from the hyperfine splitting at $\ell = 0$ between the pion and ρ meson. There is, therefore, an important phenomenological manifestation of an approximate $O(3)$ orbital symmetry in the light mesons, which is also well known to be important in baryons, as we further discuss. In passing, we should mention that there are no established linear Regge trajectories in heavy mesons or heavy baryons. There is, however, an interesting observation one can make for D-mesons based on one assumption: the $D^*(2640)$ exists (only one group has claimed observation of this state¹) and has $\ell = 2$. In that case, a linear relation can be found: $(M_{D^*(\ell)} - M_{\bar{D}})^2 = 0.21 \ell \text{ GeV}^2$, where $M_{\bar{D}} = (M_D + 3M_{D^*})/4$ is the spin-flavor singlet component of the ground state D-mesons. This seems to be a plausible relation whose validity will be clarified with progress in establishing of the excited heavy meson spectrum. It may as well turn out to extend to heavy baryons.

For light baryons, the topic of this talk, Regge trajectories have been discussed for a long time, with a consensus that they are approximately linear and with similar slopes as those for mesons. The discussions are based on plotting trajectories for different types of baryons, say N , Δ , etc. Remarkably enough, baryon Regge trajectories in the framework presented in,^{2,3} which makes use of approximate dynamical symmetries, had not been discussed in the past. The motivation is based on the expectation that Regge trajectories are primarily determined by quark-spin independent components of the strong interactions. Thus, if one can separate in a clear manner the quark-spin independent component of baryon masses, a more pristine version of Regge trajectories should result for that mass component^a. In reference³ and in this talk, we show how to identify the quark-spin and flavor independent component of baryon masses in the framework of the $1/N_c$ expansion, and show that indeed a much clearer picture for the trajectories results.

The key to the implementation of the $1/N_c$ expansion in baryons is the emergent *contracted spin-flavor* dynamical symmetry in the large N_c limit of QCD.^{4,5} This dynamical symmetry results from the requirement of uni-

^aWe should emphasize that associating the terminology "Regge trajectory" with a piece of the baryon masses, in this case the spin-flavor singlet piece, and plotting it against ℓ instead of the baryon's J is not rigorous terminology from the point of view of Regge theory.

tarity of elastic pion-nucleon scattering at large N_c . The generators of the contracted group are: T^a , S^i and X^{ia} , where the first are the flavor generators, the second the spin generators and the last correspond to the axial currents at zero momentum scaled by a factor $1/N_c$. In the large N_c limit the generators X^{ia} become semiclassical, and this leads to the mentioned contracted symmetry.^{4,5} Those generators generate a contracted $SU(2N_f)$ group, where N_f is the number of light flavors. In general the irreducible representations of the contracted symmetry are infinite dimensional. There is an alternative and equally rigorous way of implementing the consistency requirements by using instead the compact Lie group $SU(2N_f)$ at finite N_c (whose generators are T^a , S^i and $G^{ia} = N_c X^{ia}$). The latter has the advantage that one works with finite dimensional representations and the implementation of the expansion is more straightforward.

On a less fundamental footing, and similarly to the case of mesons, one can phenomenologically identify an $O(3)$ approximate symmetry; excited baryons can be accommodated into multiplets of this orbital $O(3)$. Its origin is dynamical, without a known fundamental explanation from QCD, and there is no assurance that it will hold at large N_c , although this seems plausible. It is then natural to have a classification of baryons in multiplets of $SU(6) \times O(3)$, as it was realized back in the 1960's. This provides a framework for implementing the $1/N_c$ expansion, which in the particular case of baryon masses has been studied in various works.^{5,6,8}

2. Baryon masses

The effective mass operators for baryons are constructed as linear combinations of composite operators built with the generators of $SU(6) \times O(3)$. This is possible if one neglects the possibility of configuration mixings, i.e., mixings of states in different representations of $SU(6) \times O(3)$.⁷ For most phenomenological purposes this seems to be a reasonable approximation, which the conclusions of this work will further reaffirm. The $1/N_c$ power counting is determined by the n -bodyness of the effective operator: a suppression factor $1/N_c^{n-1}$ accompanies every n -body operator. In addition, matrix elements have N_c dependencies: so called coherent operators, such as the axial currents G^{ia} present $\mathcal{O}(N_c)$ matrix elements, while incoherent ones have $\mathcal{O}(1)$ or even $\mathcal{O}(1/N_c)$ matrix elements.

All baryons discussed here will be accommodated into the symmetric (**S**) or the mixed symmetric representations (**MS**) of $SU(6)$ which for $N_c = 3$ correspond to the **56**- and the **70**-plets respectively. For the excited baryons we distinguish one box in the Young tableaux of $SU(6)$, which in a quark

model picture one would identify with an excited quark. Coupling this box to the totally symmetric Young tableaux with $N_c - 1$ boxes (known as the core) one can build the representations of interest.

The ground state (GS) baryons belong into the $[56, 0^+]$ multiplet. The mass formula is the old and well known Gürsey-Radicati mass formula, which is the most general one to $\mathcal{O}(1/N_c)$ and $\mathcal{O}(\epsilon)$, where ϵ measures the breaking of flavor $SU(3)$.⁵

$$M_{GS} = c_1 N_c + \frac{1}{N_c} c_{HF} (\vec{S}^2 - \frac{3}{4} N_c) - c_S \mathcal{S} \quad (1)$$

The first term is the leading $\mathcal{O}(N_c)$ spin-flavor symmetric term, the second is the hyperfine term defined in such a way that in a quark model picture corresponds to the form $1/N_c \sum_{i \neq j} \vec{s}_i \cdot \vec{s}_j$. Different definitions of this term can be absorbed into changes $\mathcal{O}(1/N_c)$ to the first term. Finally, the third term gives the breaking of $SU(3)$ through the strangeness operator $\mathcal{S}$.

For excited baryons, the number of terms in the mass formulas proliferate. For instance, for the $[56, \ell > 0]$ multiplets there are in addition to the $\mathcal{O}(N_c)$ term, two $SU(3)$ symmetric $\mathcal{O}(1/N_c)$ terms (one of them a spin-orbit type term and the other the hyperfine type term), and three $SU(3)$ breaking terms, and for the $[70, \ell > 0]$ multiplets there are in addition to the $\mathcal{O}(N_c)$ term, four $\mathcal{O}(N_c^0)$ terms (three of them involving the coupling of the $O(3)$ generators, and thus called spin-orbit type), seven $SU(3)$ symmetric $\mathcal{O}(1/N_c)$ terms and three $SU(3)$ breaking terms. The various analyses of baryon masses⁶ have established several important facts: i) The breaking of spin-flavor symmetry is dominated by the hyperfine type interaction, which is $\mathcal{O}(1/N_c)$ in all multiplets. ii) The $\mathcal{O}(N_c^0)$ breaking of the $SU(6) \times O(3)$ symmetry (as it happens in the **MS** multiplets) is dynamically suppressed (this is the reason why there is approximate $O(3)$ symmetry to start with), and of smaller magnitude than the breaking due to the hyperfine interaction. In the strict large N_c limit one would eventually find a contracted spin-flavor symmetry and an approximate $O(3)$ symmetry as it was discussed in.^{9,10} iii) $SU(3)$ breaking is to a good first approximation given by the strangeness operator. Based on these observations, and for the present purpose, it is sufficient to keep only the three types of operators, namely leading spin-flavor singlet, hyperfine and strangeness. Thus, for excited baryons one can write down the mass formula:

$$M^* = c_1 N_c + \frac{1}{N_c} c_{HF} (\vec{S}_c^2 - \frac{3}{4} (N_c - 1)) - c_S \mathcal{S} \quad (2)$$

where $\vec{S}_c$ indicates the spin operator restricted to acting on the core of the baryon. Notice that these mass formulas are not dependent on a quark

model picture of baryons. Rather, the only assumptions that are being made concern the assignment of states to specific multiplets, which is typically what is done in effective theories. The only tuning in Eqn.(2) is in the choice of the hyperfine term: first, it has been shown in the analysis of the **70**-plet baryon masses that the $\vec{S}_c^2$ hyperfine is the only relevant one (the other one being $\vec{s} \cdot \vec{S}_c$ ($\vec{s}$ is the excited quark spin), which turns out to be almost entirely suppressed); it is natural that the same should occur for the **56**-plets, although in this case one cannot separate the two mentioned hyperfine terms explicitly because they are linearly dependent operators; second, we subtract the $\frac{3}{4}(N_c - 1)$ in analogy with what was done for the GS baryons. All this implies some $\mathcal{O}(1/N_c)$ arbitrariness for the leading terms, or equivalently $\mathcal{O}(1/N_c^2)$ corrections to c_1 . It is important to emphasize that with these very simple mass formulas the main features of the multiplets considered in here are well described. The finer features of the spectrum naturally requires the inclusion of further mass operators in the $1/N_c$ expanded basis.⁶

Explicit evaluation gives the following results for the excited baryon masses:

$$\begin{aligned}
 M'_{\mathbf{S}}(S) &= N_c c_1 + \frac{N_c - 2}{N_c^2} c_{\text{HF}} \left(S(S+1) - \frac{3}{4} N_c \right), \\
 M'_{\mathbf{MS}}(S=I) &= N_c c_1 + \frac{c_{\text{HF}}}{N_c} \left(\frac{N_c + 2}{N_c} S(S+1) - \frac{3}{4} N_c + \frac{1}{2} \right), \\
 M'_{\mathbf{MS}}(S=I-1) &= N_c c_1 + \frac{c_{\text{HF}}}{N_c} \left(S(S+2) - \frac{3}{4} (N_c - 2) \right), \\
 M'_{\mathbf{MS}}(S=I+1) &= N_c c_1 + \frac{c_{\text{HF}}}{N_c} \left(S^2 - \frac{3}{4} N_c + \frac{1}{2} \right), \tag{3}
 \end{aligned}$$

which for $N_c = 3$ read:

$$\begin{aligned}
 N_{\mathbf{GS}} &= 3 c_1 - \frac{1}{2} c_{\text{HF}}, \quad \Delta_{\mathbf{GS}} = 3 c_1 + \frac{1}{2} c_{\text{HF}}, \\
 N_{\mathbf{56}} &= 3 c_1 - \frac{1}{6} c_{\text{HF}}, \quad \Delta_{\mathbf{56}} = 3 c_1 + \frac{1}{6} c_{\text{HF}}, \\
 N_{\mathbf{70}} \left(S = \frac{1}{2} \right) &= 3 c_1 - \frac{1}{6} c_{\text{HF}}, \\
 N_{\mathbf{70}} \left(S = \frac{3}{2} \right) &= \Delta_{\mathbf{70}} \left(S = \frac{1}{2} \right) = 3 c_1 + \frac{1}{6} c_{\text{HF}}, \tag{4}
 \end{aligned}$$

where we denote $N \equiv M_N$, etc. Note that for the MS states we need to specify the total quark spin S . The case of strange baryons is obvious; one needs to add the SU(3) singlet Λ states in the **70**-plets, whose mass formula

becomes:

$$\Lambda_{70}^1 = 3c_1 - \frac{1}{2}c_{\text{HF}} + c_S \quad (5)$$

The effective coefficients c_1 , c_{HF} and c_S are obtained for each multiplet by fitting to the corresponding masses (we use here the Breit-Wigner masses as provided by the Particle Data Group (PDG)¹). The coefficients are $\mathcal{O}(N_c^0)$ and change by amounts $\mathcal{O}(1/N_c)$ as one goes from multiplet to multiplet. The following multiplets are considered for the analysis: i) the GS $[56, 0^+]$ where all states are empirically established, ii) the $[56, 2^+]$ where all non-strange states are established, but only a few strangeness -1 states are known, namely two Λ and two Σ states (one in an octet and one in a decuplet), iii) the $[56, 4^+]$ are sparsely known, with only one N , three Δ and one Λ states known, several of them with only one star status, iv) two states, which we assign to the $[56, 6^+]$, and which represent a very loose input in the fit, v) the $[70, 1^-]$ which is one of the best known excited multiplets, where some of the Σ states and none of the strangeness < -1 states are known, vi) the $[70, 2^+]$ where three N , two Δ and two Λ states are known, vii) the $[70, 3^-]$ with three N , one Δ and one Λ states known, and finally viii) the $[70, 5^-]$ where similarly to the case of the $[56, 6^+]$, the assignment is loose given that only one state is known. Clearly, the most critical aspect of the discussion is the assignment of states into the multiplets. This issue has been studied at length, and for the states in the second resonance region it seems to be clearly understood. In addition to the masses, the analyses of strong and electromagnetic transitions have been used for that purpose.

In the table the results of the fits using the mass formulas in Eqns. (4) are shown. We also include a fit where the masses of the non-strange states have been fitted using only the leading term, i.e., the coefficients c_1 . Clearly, the results for c_1 in the latter fit are close to the ones in the more accurate fit.

We plot the square of the spin-flavor singlet component of the baryon masses $N_c^2 c_1$ vs ℓ . The figure shows very distinctly two Regge trajectories, which are clearly determined up to $\ell = 5$, and distinguish between the **56** and **70** plets. As indicated earlier, the assignments to specific multiplets of the $\ell = 5, 6$ inputs are debatable and therefore these inputs should be taken with a grain of salt. The purpose to include them here is to argue that they seem to consistently fit into the linear trend established by the lower ℓ inputs.

Table 1. The coefficients c_1 , c_{HF} and c_S from fitting to the PDG masses.

Multiplet	c_1 [MeV]	c_{HF} [MeV]	c_S [MeV]
[56 , 0^+]	361 ± 1 361	269 ± 2	156 ± 1
[56 , 2^+]	603 ± 5 589	671 ± 58	126 ± 19
[56 , 4^+]	770 ± 20 361	398 ± 372	110 ± 94
[56 , 6^+]	954 ± 40 954	342 ± 720	—
[70 , 1^-]	529 ± 5 545	443 ± 19	148 ± 13
[70 , 2^+]	640 ± 16 671	~ 400 (input)	120 ± 86
[70 , 3^-]	731 ± 17 756	249 ± 315	30 ± 159
[70 , 5^-]	900 ± 20 (estimated) 900	—	—

Fig. 1. Plot $(N_c c_1)^2$ vs ℓ for the **56**-plets (+) and the **70**-plets (x). The linear fits to the respective trajectories are shown.

The linear fits to the trajectories give:

$$\begin{aligned}
 (3 c_1([\mathbf{56}, \ell]))^2 &= ((1.179 \pm 0.003) + (1.05 \pm 0.01) \ell) \text{ GeV}^2, \\
 (3 c_1([\mathbf{70}, \ell]))^2 &= ((1.34 \pm 0.02) + (1.18 \pm 0.02) \ell) \text{ GeV}^2.
 \end{aligned} \tag{6}$$

For a given multiplet, the coefficient c_1 has a dependence on ℓ which is $\mathcal{O}(1/N_c)$. Therefore, the slopes of the fitted trajectories are $\mathcal{O}(N_c)$. For comparison with the slopes between baryons and mesons it would be more reasonable to fit $N_c c_1^2$ instead. Thus, it seems in some sense fortuitous that in the real world, meson and baryon trajectories have similar slopes. In the large N_c limit a plot $N_c c_1$ vs ℓ would obviously be the appropriate one.

One important point to be emphasized here is that for a given ℓ , the splitting between the spin-flavor singlet pieces of the masses, namely $N_c \times (c_1(\mathbf{70}) - c_1(\mathbf{56}))$, is a quantity $\mathcal{O}(N_c^0)$. To ascertain this, one can simply consider a quark picture of the large N_c baryon in the Hartree approximation, and show that the exchange terms in the matrix elements of the spin-flavor independent piece of the quark-quark interaction give rise to mass contributions $\mathcal{O}(N_c^0)$ for $\mathbf{56}$ -plets and $\mathcal{O}(1/N_c)$ for $\mathbf{70}$ -plets. Therefore, one should expect on these grounds the two distinct trajectories we have found, and they should persist in a large N_c world. For the splitting between $\mathbf{56}$ - and $\mathbf{70}$ -plet, the following linear relation gives a fair approximation:

$$(c_1([\mathbf{56}, \ell]) - c_1([\mathbf{70}, \ell]))^2 = (5.3 + 4.4\ell) \times 10^{-4} \text{ GeV}^2. \quad (7)$$

This corresponds to a mass splitting that increases with ℓ , going from ~ 70 MeV at the $\ell = 0$ intersect to ~ 170 MeV at $\ell = 6$. The hyperfine effects are in general larger than the splitting between trajectories, and for this reason the two distinct trajectories cannot be distinguished when one plots using physical masses. This is again a QCD dynamical issue in which an effect leading in $1/N_c$ is smaller than a sub-leading one.

The validity of the $1/N_c$ expansion should translate into a dependence of c_{HF} and c_S on ℓ of $\mathcal{O}(1/N_c)$. One observes however a rather important dependence of c_{HF} (modulo the large error bars). This could be explained by the fact that the hyperfine interaction is more sensitive to the effective size of the core than the other terms in the mass formulas. In particular, in the quark-diquark picture of baryons, this sensitivity in the hyperfine effect indicates a reduction in the size of the diquark for $\ell > 0$.

Here we draw some conclusions which seem to result from our analysis.

i) The QCD string-like confinement dynamics giving rise to approximately linear Regge trajectories clearly resides in the $SU(6) \times O(3)$ singlet component of the baryon masses.

ii) The clear separation of the two trajectories corresponding to $\mathbf{56}$ and $\mathbf{70}$ -plets strongly suggest that configuration mixings are relatively small, thus emphasizing the correctness classification of multiplets of observed

excited baryons into multiplets of spin-flavor. This is also seen to be correct in the analyses of strong and electromagnetic transitions.

iii) Unlike mesons, the widths of excited baryons are $\mathcal{O}(N_c^0)$, and one may therefore expect some consequences for the behavior of Regge trajectories for highly excited states. Intuitively, in mesons the string breakup is suppressed in large N_c limit, and thus it is possible that linear trajectories extend out to highly excited states. In baryons, because the decays are not suppressed in large N_c , the string breakup should as well be unsuppressed, and therefore one would expect that the linear character of the trajectories should end at some finite excitation.

iv) Finally, in this meeting the framework of AdS/QCD was addressed in several talks. In this framework the spectrum of excited hadrons depends on the model used to introduce the QCD scale, which corresponds in the models discussed to modifying the AdS geometry in the 5-th dimension by introducing an IR cut-off in different ways. Regge trajectories impose, therefore, a selection criterion for these models. In this context, the results of this work provide useful constraints for AdS/QCD models of baryons.

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