Neutron and proton transition densities from $^{32,34}\text{S}(p, p')$ at $E_p = 318$ MeV. I. Isoscalar densities for ^{32}S

J. J. Kelly, M. A. Khandaker, P. Boberg, A. E. Feldman, B. S. Flanders,* S. Hyman,[†] and H. Seifert[‡]
Department of Physics, University of Maryland, College Park, Maryland 20742

P. Karen, B. E. Norum, and P. Welch
Department of Physics, University of Virginia, Charlottesville, Virginia 22901

S. Nanda and A. Saha
Continuous Electron Beam Accelerator Facility, Newport News, Virginia 23606
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Differential cross sections and analyzing powers for low-lying states of ^{32}S were measured using 318 MeV protons. The data were analyzed using an empirical effective interaction previously fitted to inelastic scattering data for ^{16}O and ^{40}Ca at the same energy. Transition densities for many states were fitted to the data using general expansions which permit evaluation of uncertainties due to statistical and normalization errors, penetrability and distortion, and incompleteness in momentum space. The accuracy of the procedure was tested by comparing isoscalar densities fitted to (p, p') data for ^{32}S with proton densities fitted to (e, e') data. The good agreement between these analyses supports the quantitative accuracy of densities fitted to (p, p') data. Isoscalar densities were also fitted to data for several states of ^{32}S for which no (e, e') data exist. We find that the experimental densities agree well with the shell model for the first 2^+ state, but that the neutron density for the second 2^+ state is distinctly different in shape. Good qualitative agreement between the data and the shell model is obtained for the first two 4^+ states of ^{32}S . Transition densities were also fitted to the data for the lowest 1^- , 3^- , and 5^- states in ^{32}S . The shape of the 1^- transition density is complicated, but the very small matrix element agrees with the approximate selection rule that suppresses isoscalar $E1$ moments.

I. INTRODUCTION

Neutron transition densities measured with proton scattering provide information complementary to the proton densities measured with electron scattering and can provide important new tests of structure models. Until recently, data for neutron contributions to normal-parity transitions have been largely limited to matrix elements deduced from lifetime measurements assuming isospin symmetry [1] or from analyses of hadron scattering based upon collective or scaling models [2]. While valuable, these techniques yield only a single moment dominated by small densities at large radii. More recently, it has been shown that proton scattering is capable of determining the neutron transition density with good radial sensitivity provided that the effective interaction is known [3–6].

We have developed methods for fitting transition densities to nucleon scattering data that are similar to the nearly model independent analysis long used to analyze electron scattering data [7–10]. Neutron transition densities may be extracted when the proton transition density is already known from electroexcitation data, or isoscalar transition densities may be extracted for self-conjugate targets without need of (e, e') data. The relevant density is expanded in a complete set of basis functions and the coefficients are fitted to the data, subject to physical constraints upon the density for large ra-

dii and the form factor for large momentum transfer. The error analysis includes contributions due to the penetrability of the probe, the statistical and normalization uncertainties in the data, and the limitation to a finite interval of momentum transfer. These uncertainties are represented by an error envelope which portrays the uncertainty in the fitted density as a function of radius. Our analysis of pseudodata demonstrates that radial densities may be obtained with good intrinsic sensitivity even in the interior of large nuclei [3]. However, the absolute accuracy of the fitted density is both more difficult and more important to determine. Residual errors in the reaction model can produce systematic errors in the density that cannot be represented by the error envelope.

It is well known that the components of the nucleon-nucleon interaction responsible for normal-parity transitions depend strongly upon local density for projectile energies in the range 0–500 MeV and probably higher [11–15]. Although nuclear matter theory provides a good qualitative description of the density dependence due to Pauli blocking and dispersion [16–18], the differences among the various models and between the models and the data remain too large for confident application to the extraction of transition densities [12,14]. Therefore, it has been necessary to develop an empirical effective interaction fitted to data for many states among several self-conjugate targets [13]. The parametrization of the empirical interaction is guided by nuclear matter

theory. The availability of accurate transition densities measured by electroexcitation of these states minimizes uncertainties due to nuclear structure. The requirement that the empirical interaction reproduce data for many states in several nuclei simultaneously, using self-consistent optical potentials, minimizes ambiguities among the parameters and ensures that the interaction is indeed a function of density rather than of target or state. Empirical interactions have been constructed for several energies [13–15] between 100 and 500 MeV and should hopefully be accurate enough for quantitative applications, such as extraction of neutron transition densities. However, although this procedure provides a good global fit to data for many states, effects omitted from the model may adversely affect predictions or fitted densities for particular transitions.

An important test of the method is the accuracy with which transition densities that are already known from (e, e') measurements can be extracted from (p, p') data independently. For self-conjugate targets, charge symmetry ensures that the neutron and proton transition densities for low-lying isoscalar excitations are very nearly equal. An empirical effective interaction has been fitted to data for the inelastic scattering of 318 MeV protons by ^{16}O and ^{40}Ca [14,19]. The accuracy of neutron transition densities may then be assessed by comparing densities fitted to proton scattering data for other self-conjugate targets with proton transition densities measured with electron scattering. This procedure, which we call the *self-conjugacy test*, has also been applied successfully to the data for ^{16}O and ^{40}Ca [20].

We have measured cross sections and analyzing powers for elastic and inelastic scattering of 318 MeV protons by ^{32}S , a self-conjugate target whose mass lies between those used to deduce the effective interaction. Isoscalar densities were extracted from these data and compared with proton transition densities fitted to (e, e') data [21,22] for the 2_1^+ and 3_1^- states as a test of the reaction model and a measure of systematic errors due to residual errors in the effective interaction. We find that very accurate results are obtained for those states whose densities are known independently provided that the empirical effective interaction is employed to analyze the data. Use of density-independent or theoretical density-dependent interactions instead seriously compromises the accuracy of the fitted densities.

This paper is the first of two presenting results for the scattering of 318 MeV protons from the sulfur isotopes. Having established in the present paper the quantitative accuracy of our analysis procedures using the data for ^{32}S , we report data for ^{34}S in the second paper [23]. Neutron transition densities were extracted for the lowest two 2^+ states. The second 2^+ state has attracted considerable interest [24–29] in that it is one of the only low-lying states for which the shell model predicts a relative negative sign between the neutron and proton matrix elements [24,30,31]. Bernstein *et al.* [25] reported (p, p') data at 650 MeV which they interpreted as supporting the shell-model prediction, but several subsequent experiments with a variety of probes [26–28] disputed this result and reported a relative positive sign between these matrix ele-

ments. However, none of those analyses considered the shape of the neutron density and the effect of shape differences upon the deduced ratio. We find the shape difference to be crucial to this controversy.

The experiment is described in Sec. II and the model in Sec. III. The accuracy tests are presented in Sec. IV. Transition densities for positive-parity states are compared with the shell model in Sec. V. Densities for several negative-parity states are presented in Sec. VI. Our conclusions are summarized in Sec. VII.

II. EXPERIMENT

The experiment was performed at the Los Alamos Meson Physics Facility (LAMPF) using polarized protons with nominal 318 MeV energy. The beam current was monitored by ionization chambers normalized by comparing elastic pp measurements made with a thin CH_2 target with phase-shift calculations [32]. The beam polarization was monitored by the continuous in-line polarimeter described in Ref. [33] and was typically in the range 0.7–0.8. Scattered protons were analyzed using the high-resolution spectrometer (HRS) and detected by the standard focal-plane array described in Ref. [34]. Using known spectrometer constants, the beam energy was found to lie in the range 317.7 ± 0.4 MeV.

Targets were made by pressing isotopically enriched sulfur powder between thin beryllium foils. Each target had rectangular dimensions of 2.5 cm $\times$ 5.1 cm and contained approximately 17 mg/cm² of sulfur between 1.0 mil beryllium foils. The target parameters are listed in Table I. The tabulated thicknesses are nominal values obtained from the weight and area of each sample. Data obtained for elastic and inelastic scattering to low-lying states of ^{32}S obtained with the nearly pure ^{32}S target agree very well with the calculations presented in Sec. IV based upon densities determined by electron scattering measurements. In particular, the data for elastic scattering from ^{32}S agree very well with calculations at low momentum transfer which are expected to be accurate to better than 10%. We also find agreement between (e, e') and (p, p') to better than 5% at the peak of the 2_1^+ and 3_1^- angular distributions. Therefore, we adopt the ^{32}S target as our normalization standard for the sulfur targets. Cross sections for these same states based upon the nominal thickness for the enriched ^{34}S target must be re-normalized by a factor of 0.85 to obtain agreement with the ^{32}S target. Because the isotopic analysis for the sulfur powder is believed to be more accurate than the nominal thickness of the fabricated target, we assume that this

TABLE I. Isotopic abundances and nominal thicknesses for the sulfur targets. Each target consisted of pressed sulfur powder between 1.0 mil beryllium foils.

Target	Nominal thickness (mg/cm ²)	Abundances (%)	
		^{32}S	^{34}S
^{32}S	16.6	99.89	0.09
^{34}S	19.1	31.55	67.92

FIG. 1. Representative fitted spectrum for the scattering of 318 MeV protons through 25° using a mixed isotope target containing sulfur powder pressed between beryllium foils. Selected peaks are labeled. Note the logarithmic vertical scale. Although relatively weak, the 2_2^+ peak for ^{34}S is clearly visible. The elastic peak for ^9Be is broadened by kinematic variation across the spectrometer acceptance.

same normalization factor applies to both ^{34}S and ^{32}S data obtained with that target. With this adjustment, cross section and analyzing power data for all states of ^{32}S which could be obtained with the mixed isotope target agree very well with the data for the ^{32}S target. Therefore, we estimate the normalization uncertainty for both ^{32}S and ^{34}S cross sections to be about $\pm 10\%$.

The spectra were analyzed with the ALLFIT program [35] and methods described in Refs. [12] and [14]. For most spectrometer settings, events were sorted into bins covering the entire $\pm 1.0^\circ \times 1.0^\circ$ angular acceptance. A typical fitted spectrum that covers the full acceptance is shown in Fig. 1. The resolution for the sulfur peaks was typically about 65 keV FWHM, but the beryllium peaks

are much wider due to kinematic broadening. Spectra for which the elastic scattering peak from ^9Be interfered with the 2_2^+ peak of ^{34}S were subdivided into three equally spaced bins of scattering angle, reducing the width of the beryllium peak and facilitating extraction of the peak of interest. The separations between close clusters of peaks or between weak peaks and neighboring strong peaks were constrained by the known energy levels compiled within Ref. [36].

Data were collected for laboratory angles between 5° and 39° , corresponding to center-of-mass momentum transfers between about 0.4 and 2.8 fm^{-1} . Measurements for alternate targets were made first for angles between 9.6° and 39° , followed by 9° , 7° , and 5° . Good agreement between cross sections obtained near the beginning and the end of the experiment indicates that target sublimation was negligible. Data tables are on deposit with the Physics Auxiliary Publication Service (PAPS) for the 10 states of ^{32}S and the 12 states of ^{34}S listed in Table II [37].

II. MODEL

Detailed presentations of our implementation of the folding model and local density approximation are available in Refs. [12] and [14]. The methods used to fit neutron transition densities to proton scattering data are described in Refs. [3] and [6] and several previous applications have been presented in Refs. [5] and [6]. In this section we briefly summarize those aspects of the model relevant to subsequent discussions both here and in the subsequent ^{34}S paper.

A. Folding model

Normal-parity transitions with $\Delta\pi = (-)^J$ are excited primarily by central (U^C) and spin-orbit (F^{LS}) scattering potentials of the form [14]

$$U_f^C(r) = \sum_{\lambda} \frac{2}{\pi} \int dq q^2 \eta t_{\lambda}^C(q, \rho) \rho_{J\lambda}(r), \quad (1a)$$

TABLE II. Energy levels for states of ^{32}S and ^{34}S for which (p, p') data at 318 MeV are reported. The comments columns indicate extracted quantities or describe data not discussed in the text. SM indicates that shell-model calculations have been performed.

^{32}S			^{34}S		
E (MeV)	J_{π}^{π}	Comments	E (MeV)	J_{π}^{π}	Comments
0.0	0_1^+		0.0	0_1^+	
2.230	2_1^+	ρ_0 , SM	2.127	2_1^+	ρ_n , SM
4.282	2_2^+	ρ_0 , SM	3.303	2_2^+	ρ_n , SM
4.459	4_1^+	ρ_0 , SM	4.114	2_3^+	SM
5.006	3_1^-	ρ_0	4.623	3_1^-	compared with ^{32}S
5.549	2_3^+	$q < 1.6 \text{ fm}^{-1}$	4.688	4_1^+	SM
5.798	1_1^-	ρ_0	4.889	2_4^+	SM
6.411	4_2^+	ρ_0 , SM	5.689	5_1^-	compared with ^{32}S
6.581	$(2^+ \text{ or } 3^-)$	sparse data, proposed assignment	5.753	1_1^-	compared with ^{32}S
6.762	(5_1^-)	ρ_0 , proposed assignment	6.174	3_2^-	$q > 1.6 \text{ fm}^{-1}$
			6.345	1_2^-	$q > 1.6 \text{ fm}^{-1}$
			6.864	5_2^-	sparse data

$$F_J^{LS}(r) = \sum_{\lambda} \frac{2}{\pi} \int dq q^2 \eta \tau_{\lambda}^{LS}(q, \rho) \rho_{J\lambda}(r), \quad (1b)$$

where the sum over $\lambda = (p, n)$ or $(0, 1)$ couples transition densities ($\rho_{J\lambda}$) with central (t^C) or spin-orbit (τ^{LS}) interactions in either proton-neutron or isospin representations. The factor η represents a Jacobian near unity [38]. Calculations based upon shell-model wave functions also include additional spin and current densities, but these contributions were found to be negligible for all normal-parity excitations of ^{32}S considered herein.

The dependence of the effective interaction upon the ground-state density ρ is most conveniently evaluated at the site of the projectile. A local approximation for exchange is incorporated within the interaction. The elastic optical potential is constructed, using similar formulas, from the same effective interaction as the inelastic scattering potential except for rearrangement contributions which enhance the density dependence of the inelastic interaction by a factor of $(1 + \rho \partial / \partial \rho)$ relative to the elastic interaction [39]. Ground-state proton densities were obtained by unfolding the proton form factor from the (e, e') measurements of charge densities tabulated in Ref. [40]. We assume $\rho_n \propto \rho_p$ for the ground state because distortion of inelastic scattering is quite insensitive to details of the ground-state neutron density.

To minimize distortions of fitted neutron densities due to residual errors in the reaction model, we have fitted an empirical model of the effective interaction to inelastic scattering data for both ^{16}O and ^{40}Ca [14,19]. The model was designed to describe any of the available theoretical interactions with a minimum number of parameters per component. The LR interaction [18], which is the only theoretical effective interaction specifically designed for energies above 300 MeV, was used to guide the parametrization and to provide an initial estimate of the parameters. The interaction at low density was based upon the Franey-Love t matrix [41], hereafter designated FL. A total of six parameters describing medium modifications to the isoscalar central and spin-orbit interactions was varied to fit (p, p') data for five states of ^{16}O and/or six states of ^{40}Ca . The parameters fitted to ^{16}O or ^{40}Ca , either independently or simultaneously, are virtually identical and the fit to the data achieved by the empirical effective interaction is much superior to the predictions of any theoretical interaction presently available [14,19].

Medium modifications to the isovector interaction were obtained from a parametrization of the LR interaction [18]. Fortunately, although there do not exist adequate data for calibration of the isovector components of the interaction, the results for both ^{32}S and ^{34}S are insensitive to medium modifications of the isovector interaction. This happy circumstance occurs because the relative weakness of the isovector interaction combined with the relative dominance of isoscalar densities for the states considered herein causes isovector contributions to be small enough that small medium corrections to the isovector interaction produce negligible changes in the calculations.

It is worth emphasizing that differences between neutron and proton densities may be deduced from scattering

data even if the effective interaction has a purely isoscalar character [4]. Although the strong charge dependence of pion scattering near the delta resonance enhances sensitivity to M_n/M_p , strong absorption limits sensitivity to shape differences. For proton scattering, on the other hand, an essentially isoscalar interaction gives equal weights to both neutron and proton densities and the relative transparency permits shape differences to be viewed with good sensitivity. Also note that given an independent measurement of ρ_p from (e, e') , measurement of ρ_n with (p, p') suffices to determine both the isoscalar and the isovector transition densities, even without a substantial isovector interaction.

B. Transition densities

A transition density $\rho_J(r)$ may be expanded in a complete set of radial basis functions:

$$\rho_J(r) = \sum_{n=0}^N a_n f_n^J(r). \quad (2)$$

It is customary to analyze electron scattering data using some form of the Fourier-Bessel expansion (FBE). For the FOUBES program used to analyze the $^{32}\text{S}(e, e')$ data [9,10], the FBE basis functions assume the form

$$f_n^J(r) = q_n j_J(q_n r), \quad (3)$$

where

$$j_{J-1}(q_n R) = 0, \quad (4)$$

and where the coefficients a_n are constrained so that the density vanishes at the truncation radius R . For proton scattering, we find it more convenient to employ the Laguerre-Gaussian expansion (LGE) for which

$$f_n^J(r) = x^J e^{-x^2} L_n^{J+1/2}(2x^2), \quad (5)$$

where $x = r/b$ is a radial variable scaled by the oscillator parameter b and L_n^β is a generalized Laguerre polynomial of order n . The expansion is complete for any b , but will usually be simplest when b is chosen near the value appropriate for the harmonic oscillator model. Following Brown *et al.* [30], we employ $b = 1.881$ fm for the sulfur isotopes.

A useful measure of the transition strength is provided by the multiple moment, or matrix element, defined by

$$M_J = \int dr r^{J+2} \rho_J(r). \quad (6)$$

Whenever the context is clear, the subscript J is omitted in favor of an isospin label $\tau = p(n)$ or $0(1)$ for proton (neutron) or isoscalar (isovector), where

$$M_0 = M_n + M_p, \quad \rho_0 = \rho_n + \rho_p, \quad (7a)$$

$$M_1 = M_n - M_p, \quad \rho_1 = \rho_n - \rho_p. \quad (7b)$$

It is also useful to define form factors

$$F_\tau(q) = \frac{\sqrt{4\pi}}{N_\tau} \int dr r^2 j_J(qr) \rho_\tau(r) = \frac{\sqrt{4\pi}}{N_\tau} \tilde{\rho}_\tau(q), \quad (8)$$

where $N_p = Z$ and $N_n = N$ are proton and neutron num-

bers, respectively. This normalization gives $F_n = F_p$ for the hydrodynamic model $\rho_n = (N/Z)\rho_p$. Similarly, the charge form factor observed in electron scattering is given by

$$F_{\text{ch}}(q) = \frac{\sqrt{4\pi}}{Z} \tilde{\rho}_{\text{ch}}(q) = \frac{\sqrt{4\pi}}{Z} [\tilde{\rho}_p(q)\tilde{f}_p(q) + \tilde{\rho}_n(q)\tilde{f}_n(q)] , \quad (9)$$

where $\tilde{f}_p$ and $\tilde{f}_n$ are proton and neutron charge form factors.

Assuming that transverse contributions may be neglected, the plane-wave approximation for electron scattering may be expressed in the form [42]

$$\frac{d\sigma}{d\Omega} = \eta Z^2 \sigma_M F_{\text{ch}}^2 = \sigma_{pt} F_{\text{ch}}^2 , \quad (10)$$

where η is the recoil factor and σ_M the Mott cross section. The effects of distortion can be represented by a shift of the momentum transfer

$$q \rightarrow q_{\text{eff}} = \left[1 + \frac{3}{2} \frac{Z\alpha}{R_0 E_i} \right] q \quad (11)$$

and a distortion factor which, for a given effective momentum transfer, depends upon energy. In Eq. (11), E_i is the incident energy and $R_0 = 1.12 A^{1/3}$ is the radius for a uniformly charged sphere. Hence, by scaling the data by the ratio of distortion factors, the data can be recalculated to the highest experimental energy. The ratio σ/σ_{pt} then depends only on q_{eff} and represents the distorted form factor [9,10].

C. Analysis procedures

The electron scattering data were analyzed using the distorted-wave code FOUBES [9,10] and the proton scattering data were analyzed using LEA [43]. The fitting procedures for both codes are similar, although the implementations of large- r and high- q constraints differ in detail. For our purposes it suffices to discuss the specifics of the (p, p') analysis and to refer to Refs. [9] and [10] for further details of the FOUBES code.

The large- r behavior of the fitted density is regulated by application of a tail bias which minimizes deviations from the asymptotic tail function

$$t(r) = s e^{-dr}/r^n . \quad (12)$$

In earlier work [6] we used $n=2$ and determined s and d by matching $t(r)$ to the fitted $\rho(r)$ at some match radius r_m . However, the stability of that procedure has been found somewhat less than satisfactory, in part because the data do not determine d very well. Therefore, we now prefer to use $n=0$ and to constrain d according to physical arguments.

For a single-particle transition between bound orbits h and p , the asymptotic tail should be proportional to e^{-dr} with $d = (2\mu\epsilon_p)^{1/2} + (2\mu\epsilon_h)^{1/2}$, where $\mu = (A-1)m/A$ is the reduced mass and ϵ_p (ϵ_h) is the particle (hole) separation energy [8]. For large radii, the least bound orbits with significant contributions to the transition can be ex-

pected to dominate the tail. If either orbit is unbound, the asymptotic slope will be smaller than this estimate. Alternatively, the Fermi model of the ground-state density suggests $d = a^{-1}$, where a represents the diffuseness and is dominated by the least bound particles. Therefore, we expect a value of d somewhat less than a^{-1} to give a reasonable estimate of the tail function.

Our analysis of the $^{32}\text{S}(p, p')$ data shows that the fits are relatively insensitive to d ; equivalent fits are obtained for a range at least as large as $1 < d < 4 \text{ fm}^{-1}$. We also find that the fits are insensitive to n and hence choose $n=0$ and $d=1.5 \text{ fm}^{-1}$. The match radius r_m was selected by examining the function $r^{J+2}\rho(r)$, which is the integrand for the moment calculation, and requiring that oscillations be suppressed for large radii without significant constraint upon the bulk of the fitted density. A value of $r_m = 6.0 \text{ fm}$ was found to be satisfactory for the sulfur isotopes.

The incompleteness error due to the limited range of momentum transfer for which data are available, or to which the reaction model is considered applicable, is estimated by evaluating the range of densities compatible with the high- q bias [7]

$$\rho_J(q) < \rho_J(q_m)(q_m/q)^4 . \quad (13)$$

We choose $q_m = 2.7 \text{ fm}^{-1}$ corresponding to about twice the Fermi momentum, beyond which form factors are expected to decrease rapidly with q . Further details may be found in Refs. [3] and [6]. Note that the FOUBES code employs an exponential high- q envelope instead [9,10].

Additional uncertainties of $\pm 5\%$ in cross section and ± 0.05 in A_y were folded into the experimental data before fitting to compensate in part for residual errors in the reaction model and to ensure that both observables are weighted similarly for all momentum transfers. These uncertainties affect the relative weights assigned to data points, and the error bands assigned to fitted densities, but are omitted from plots. A global normalization uncertainty of $\pm 10\%$ was also assumed for the (p, p') data. The contribution of the normalization uncertainty to the uncertainty in fitted densities was estimated by comparing fits made using renormalized cross-section data. The final error bands include statistical uncertainties (enhanced as described above), normalization uncertainties, the effects of penetrability and distortion, and incompleteness errors.

The error band for large radii is dominated by the normalization uncertainty through its effect upon the low- q data and the M_0 moment. The statistical uncertainties tend to dominate for the bulk of the transition density, for which the shape is determined by data for intermediate momentum transfer. Finally, the interior portion of the error band is dominated by the incompleteness error incurred by the absence of data for large momentum transfer.

The transition densities for all states we consider require only a few terms of the LGE to describe all of their essential features. Expansion coefficients for $n > 2$ are used primarily for construction of the error envelope. Although the constraints applied to the analysis produce

important correlations among the parameters, quoted parameter uncertainties are limited to diagonal elements of the error matrix. Fortunately, we find that the diagonal elements are nevertheless adequate to approximate the fully correlated error envelope for $r < r_m$. However, correlations due to the tail bias reduce $\delta\rho(r)$ for $r > r_m$ and consequently result in smaller values for δM_J than can be estimated from diagonal elements alone.

The calculations were folded over the finite spectrometer aperture using a method based upon the algorithm of Lyman, Hanson, and Scott [44]. These aperture corrections were found to be insignificant for the present experimental conditions.

IV. ACCURACY ANALYSIS

The self-conjugate target ^{32}S provides an opportunity for testing the accuracy with which transition densities can be fitted to proton scattering data. Electron scattering data for ^{32}S are available for the lowest 2^+ and 3^- states [21,22]. To the extent that charge symmetry between low-lying proton and neutron transition densities is valid, these data provide complete and unambiguous independent measurements of the structure of these transitions. Therefore, any deviations between calculations and data can be ascribed to the reaction model.

A. Transition charge densities from $^{32}\text{S}(e, e')$

Electron scattering data for both the 2_1^+ and 3_1^- states of ^{32}S are available from Li, Yearian, and Sick [21] and for the 2_1^+ state from Hultzsich [22]. The two data sets for the 2_1^+ state are consistent in their common range and together span momentum transfers between about 0.6 and 3.0 fm^{-1} . The data for the 3_1^- state are less complete, extending only to $q \lesssim 2.6 \text{ fm}^{-1}$, and appear to be contaminated by some additional contribution for $q \lesssim 1.0$

FIG. 2. Recalculated data for $^{32}\text{S}(e, e')$ are compared with DWBA fits, where the ratio σ/σ_{pt} represents the square of the form factor corrected for distortion.

fm^{-1} . Transition charge densities were fitted to the (e, e') data, assuming that transverse contributions are negligible, using the distorted-wave code FOUBES [9,10]. Appropriate aperture and multiple scattering corrections were included. The distorted form factors, σ/σ_{pt} , are shown in Fig. 2 and the FBE expansion coefficients are listed in Table III. Note that these coefficients describe ρ_{ch} ; to obtain proton densities ρ_p each coefficient a_n must be divided by the form factor $\hat{f}_p(q_n)$ evaluated for its characteristic momentum transfer q_n .

The fit describes the data for the 2_1^+ state very well

TABLE III. Proton transition charge density parameters for ^{32}S . FBE coefficients a_n are in units of fm^{-2} and are based upon a cutoff radius of 8 fm. The units of M_p are fm^J and the units of R_{ch} are fm.

n	2_1^+	3_1^-
0	$(1.448 \pm 0.02) \times 10^{-2}$	$(1.271 \pm 0.05) \times 10^{-2}$
1	$(2.690 \pm 0.02) \times 10^{-2}$	$(2.539 \pm 0.05) \times 10^{-2}$
2	$(1.213 \pm 0.01) \times 10^{-2}$	$(1.532 \pm 0.03) \times 10^{-2}$
3	$(-6.389 \pm 0.09) \times 10^{-3}$	$(-1.613 \pm 0.28) \times 10^{-3}$
4	$(-8.168 \pm 0.09) \times 10^{-3}$	$(-5.735 \pm 0.23) \times 10^{-3}$
5	$(-2.074 \pm 0.10) \times 10^{-3}$	$(-1.409 \pm 0.27) \times 10^{-3}$
6	$(3.907 \pm 1.74) \times 10^{-4}$	$(4.739 \pm 2.70) \times 10^{-4}$
7	$(0.035 \pm 1.91) \times 10^{-4}$	$(0.079 \pm 2.33) \times 10^{-4}$
8	$(-1.233 \pm 1.40) \times 10^{-4}$	$(-0.943 \pm 1.55) \times 10^{-4}$
9	$(7.463 \pm 8.21) \times 10^{-5}$	$(0.390 \pm 1.03) \times 10^{-4}$
10	$(-1.340 \pm 5.51) \times 10^{-5}$	$(-0.453 \pm 6.81) \times 10^{-5}$
11	$(-0.844 \pm 4.37) \times 10^{-5}$	$(-0.322 \pm 4.18) \times 10^{-5}$
12	$(0.381 \pm 4.09) \times 10^{-5}$	$(0.263 \pm 2.51) \times 10^{-5}$
13	$(0.548 \pm 1.45) \times 10^{-5}$	$(-0.128 \pm 1.41) \times 10^{-5}$
14	$(-3.090 \pm 6.05) \times 10^{-6}$	$(0.270 \pm 6.98) \times 10^{-6}$
M_p	7.47 ± 0.33	36.07 ± 4.33
R_{ch}	4.36 ± 0.03	4.67 ± 0.04

over the entire range of momentum transfer. The fit to the 3_1^- form factor describes the data well in the range $1.0 \lesssim q \lesssim 2.5 \text{ fm}^{-1}$, but falls below the data for $q \lesssim 1.0 \text{ fm}^{-1}$. Better agreement with low- q data for the 3_1^- state can be obtained by relaxing the constraint on the tail of the radial density, but the resulting bulge in the transition density for $r \gtrsim 6 \text{ fm}$ is considered unphysical. The density for $r \lesssim 6 \text{ fm}$ and the form factor for $q \gtrsim 1.0 \text{ fm}^{-1}$ are not appreciably affected by the choice of tail constraints and we prefer to suppress structures at large radii that can be induced by glitches in the data that are localized with respect to momentum transfer. Also note that the (p, p') analysis presented in Sec. IV B favors the smoother behavior for large radii.

B. Self-conjugacy tests

Proton scattering calculations based upon the empirical effective interaction and the assumption of charge symmetry are compared with the data for the 2_1^+ and 3_1^- states of ^{32}S in Fig. 3. For these states the dashed curves employ the proton densities fitted to the (e, e') data and assume $\rho_n = \rho_p$. These calculations describe the proton inelastic scattering data very well. A similar calculation for elastic scattering is also seen to describe the data very well for momentum transfers below about 2.5 fm^{-1} . The only significant discrepancies between the calculations and the data are found in the analyzing powers for $q \lesssim 1.0 \text{ fm}^{-1}$, but since the same types of deviations were

also observed in the analysis of the ^{16}O and ^{40}Ca data [14,19] this problem should be attributed to a residual defect in the model of the effective interaction that is common to all isoscalar normal-parity transitions.

The solid curves shown in Fig. 3 were obtained by fitting an isoscalar density directly to the (p, p') data without explicit use of the (e, e') results. The fitted LGE coefficients are listed in Table IV. These fits are practically indistinguishable from the *ab initio* predictions represented by the dashed curves. The most direct comparison between these two analyses may be made in terms of the form factors $[\tilde{\rho}_p(q)]^2$. For (e, e') this quantity is obtained by dividing the charge form factor by the proton form factor, whereas for (p, p') we assume $\rho_p = \rho_0/2$ for self-conjugate targets. The form factors from (e, e') and (p, p') , compared in Fig. 4, agree almost perfectly below their nodes and remain in very good agreement even for higher momentum transfer. The small discrepancy for the 2_1^+ form factor beyond its second maximum is correlated with the slight overprediction of the (p, p') data for $q \gtrsim 2 \text{ fm}^{-1}$ that results from using the (e, e') density and may represent a small residual error in the reaction model. The small discrepancy near the second peak of the 3_1^- form factor may be attributed to the paucity of (e, e') data in that region of momentum transfer.

The moments fitted to (e, e') and (p, p') data are compared in Table V and for both states agree within their estimated uncertainties. However, due to the r^{J+2} weighting, these quantities are most sensitive to small densities at large radii that are not strongly determined by the range of momentum transfer available in these data. Hence, the moments may be unduly influenced by the tail constraints and must be considered model dependent. The strength of $[\rho(q)]^2$ at its first peak, on the other hand, is strongly determined by the data and is much less

FIG. 3. Elastic and selected inelastic scattering data for $^{32}\text{S}(p, p')$ at 318 MeV are compared with calculations based upon the empirical effective interaction. The elastic calculation uses the proton density deduced from electron scattering measurements and is displayed relative to the Rutherford cross section σ_R to enhance detail. For inelastic scattering the dashed lines use (e, e') densities and the solid lines represent fits to the (p, p') data. Notice that differences between calculations based upon (e, e') or fitted densities are very small.

FIG. 4. Proton densities in momentum space extracted from (e, e') and (p, p') data are shown as dashed and solid lines, respectively.

TABLE IV. Neutron transition density parameters for ^{32}S . LGE coefficients a_n are in units of fm^{-3} and are based upon $b = 1.881$ fm. The units of M_n are fm^J and the units of R_n are fm. We assume $\rho_n = \rho_0/2$.

n	2_1^+	2_2^+	4_1^+	4_2^+
0	$(4.487 \pm 0.25) \times 10^{-2}$	$(3.903 \pm 0.21) \times 10^{-2}$	$(2.459 \pm 0.12) \times 10^{-2}$	$(1.598 \pm 0.08) \times 10^{-2}$
1	$(-3.456 \pm 0.17) \times 10^{-2}$	$(-1.489 \pm 0.08) \times 10^{-2}$	$(-6.988 \pm 1.21) \times 10^{-4}$	$(-1.368 \pm 0.11) \times 10^{-3}$
2	$(4.996 \pm 0.31) \times 10^{-3}$	$(-1.193 \pm 0.18) \times 10^{-3}$	$(-3.489 \pm 0.66) \times 10^{-4}$	$(-2.055 \pm 0.27) \times 10^{-4}$
3	$(1.245 \pm 0.13) \times 10^{-3}$	$(8.780 \pm 1.65) \times 10^{-4}$	$(-1.420 \pm 3.75) \times 10^{-5}$	$(5.685 \pm 1.36) \times 10^{-5}$
4	$(-4.602 \pm 0.80) \times 10^{-4}$	$(1.489 \pm 1.05) \times 10^{-4}$	$(4.977 \pm 2.09) \times 10^{-5}$	$(2.053 \pm 0.57) \times 10^{-5}$
5	$(-1.294 \pm 0.50) \times 10^{-4}$	$(-9.527 \pm 6.25) \times 10^{-5}$	$(-0.623 \pm 1.33) \times 10^{-5}$	$(-9.381 \pm 3.03) \times 10^{-6}$
6	$(8.125 \pm 3.40) \times 10^{-5}$	$(-4.122 \pm 4.16) \times 10^{-5}$	$(-6.203 \pm 8.11) \times 10^{-6}$	$(-1.815 \pm 1.77) \times 10^{-6}$
7	$(1.071 \pm 2.34) \times 10^{-5}$	$(0.645 \pm 2.97) \times 10^{-5}$	$(-0.471 \pm 5.35) \times 10^{-6}$	$(1.231 \pm 1.02) \times 10^{-6}$
8	$(-0.993 \pm 1.58) \times 10^{-5}$	$(1.195 \pm 1.64) \times 10^{-5}$	$(1.928 \pm 3.33) \times 10^{-6}$	$(5.834 \pm 6.57) \times 10^{-7}$
9	$(-0.433 \pm 1.13) \times 10^{-5}$	$(5.324 \pm 6.39) \times 10^{-6}$	$(0.169 \pm 2.49) \times 10^{-6}$	$(-3.191 \pm 4.31) \times 10^{-7}$
10	$(1.184 \pm 7.71) \times 10^{-6}$	$(1.247 \pm 1.65) \times 10^{-6}$	$(-0.295 \pm 1.46) \times 10^{-6}$	$(-1.935 \pm 3.13) \times 10^{-7}$
11	$(-0.255 \pm 2.39) \times 10^{-6}$	$(1.220 \pm 2.23) \times 10^{-7}$	$(-1.981 \pm 4.06) \times 10^{-7}$	$(-0.638 \pm 1.07) \times 10^{-7}$
M_n	7.20 ± 0.28	2.90 ± 0.14	61.50 ± 4.23	44.42 ± 2.85
R_n	4.23 ± 0.13	3.72 ± 0.15	4.88 ± 0.30	4.65 ± 0.25
n	1_1^-	3_1^-	5_1^-	
0	$(3.002 \pm 0.19) \times 10^{-2}$	$(3.803 \pm 0.20) \times 10^{-2}$	$(9.467 \pm 0.48) \times 10^{-3}$	
1	$(-1.985 \pm 0.11) \times 10^{-2}$	$(-1.962 \pm 0.10) \times 10^{-2}$	$(-8.345 \pm 0.62) \times 10^{-4}$	
2	$(-1.284 \pm 0.07) \times 10^{-2}$	$(7.464 \pm 2.53) \times 10^{-4}$	$(-7.879 \pm 2.17) \times 10^{-5}$	
3	$(4.207 \pm 0.42) \times 10^{-3}$	$(2.352 \pm 2.02) \times 10^{-4}$	$(3.012 \pm 1.11) \times 10^{-5}$	
4	$(3.044 \pm 3.16) \times 10^{-4}$	$(-0.400 \pm 1.24) \times 10^{-4}$	$(7.604 \pm 5.91) \times 10^{-6}$	
5	$(-0.859 \pm 2.31) \times 10^{-4}$	$(0.881 \pm 8.11) \times 10^{-5}$	$(-5.206 \pm 3.47) \times 10^{-6}$	
6	$(-3.699 \pm 1.89) \times 10^{-4}$	$(4.191 \pm 5.96) \times 10^{-5}$	$(-0.119 \pm 2.00) \times 10^{-6}$	
7	$(1.586 \pm 1.49) \times 10^{-4}$	$(-1.368 \pm 3.91) \times 10^{-5}$	$(0.505 \pm 1.27) \times 10^{-6}$	
8	$(0.604 \pm 1.13) \times 10^{-4}$	$(-0.833 \pm 2.80) \times 10^{-5}$	$(1.782 \pm 7.65) \times 10^{-7}$	
9	$(-1.766 \pm 8.49) \times 10^{-5}$	$(-0.028 \pm 2.05) \times 10^{-5}$	$(-1.566 \pm 5.85) \times 10^{-7}$	
10	$(-2.812 \pm 6.34) \times 10^{-5}$	$(0.180 \pm 1.13) \times 10^{-5}$	$(-0.852 \pm 3.52) \times 10^{-7}$	
11	$(0.667 \pm 2.30) \times 10^{-5}$	$(-0.198 \pm 2.98) \times 10^{-6}$	$(-0.352 \pm 1.01) \times 10^{-7}$	
M_n	-0.07 ± 0.04	34.42 ± 1.39	313.00 ± 21.8	
R_n	12.96 ± 4.33	4.73 ± 0.16	5.29 ± 0.32	

dependent upon the model. For the 2_1^+ state the agreement between the two analyses for that quantity is better than 2% and for the 3_1^- state is better than 4%. This level of agreement justifies our selection of the ^{32}S target as the standard against which the data for the mixed isotope target is normalized.

The small deviations between the data and calculations based on (e, e') densities can be translated by the fitting procedure into modest density differences. To the extent

that charge symmetry is valid and the measured ρ_{ch} is accurate, these differences can be interpreted as the degree of distortion caused by residual errors in the reaction model. The isoscalar densities fitted to (ρ, ρ') are compared with the charge densities fitted to (e, e') in Fig. 5. For the purposes of this comparison, densities of the form $\rho_0/2$ were folded with the proton form factor to produce transition charge densities corresponding to the isoscalar densities deduced from (p, p') data. Error en-

TABLE V. Matrix elements for ^{32}S obtained from fits to (e, e') data or fits to (p, p') data using the several interactions described in the text. The empirical interaction is denoted by EI. The matrix elements are in units of fm^J and the uncertainty in the last significant figure is within parentheses. Results from (e, e') are labeled M_p , results from isoscalar fits are labeled $M_0/2$, and fits to ρ_n made with fixed ρ_p are labeled M_n .

State	M_p	EI	LR	PH	NL	FL	Type
2_1^+	7.47(33)	7.20(28) 7.25(56)	7.66(28) 7.55(58)	6.88(26) 6.46(48)	8.20(32) 8.51(62)	7.16(31) 6.94(63)	$M_0/2$ M_n
3_1^-	36.1(43)	34.4(14) 33.8(28)	36.6(15) 36.9(29)	33.1(13) 31.6(24)	39.1(16) 42.0(30)	35.7(17) 35.3(33)	$M_0/2$ M_n

FIG. 5. Isoscalar densities fitted to (p,p') data for the 2_1^+ and 3_1^- states of ^{32}S were converted to charge densities and are shown as solid lines. The bands show the charge densities fitted to (e,e') with their uncertainties. Note that the error bands for (p,p') are wider but were omitted to reduce clutter. Thus, the two analyses agree within their estimated uncertainties.

velopes were omitted from the (p,p') densities to reduce clutter in the figure; the error envelopes for (p,p') are somewhat wider than the bands shown for the densities fitted to (e,e') data. This comparison demonstrates that densities extracted from $^{32}\text{S}(p,p')$ data agree with elec-

tron scattering densities within the combined estimated uncertainties for both analyses. Therefore, the accuracy of isoscalar densities fitted to (p,p') is commensurate with the estimated uncertainties.

C. Sensitivity to models of the effective interaction

In this section we assess the sensitivity of the (p,p') analysis to the choice of effective interaction. Perhaps the simplest choice would be the density-independent t matrix due to Franey and Love (FL) [41]. Alternatively, several theoretical models of medium corrections to the effective interaction are available [16–18] and have been compared with each other and with data for this energy in Refs. [12–15]. These are the Paris-Hamburg interaction (PH) [16], the Nakayama-Love interaction (NL) [17], and the Ray interaction (LR) [18]. In Figs. 6 and 7 we compare calculations based upon the (e,e') densities and fits made using these various interactions. For each interaction the *ab initio* calculations for ^{32}S deviate from the data in much the same manner that the corresponding calculations for ^{16}O and ^{40}Ca deviate from those data [14,19].

Fits to the data made using these various interactions are shown in Fig. 7 and the corresponding isoscalar densities are compared in Fig. 8. Although variation of the density permits fits to be made to the cross-section data, none of the fits achieves significant improvement of the low- q analyzing power calculations. The discrepancies between the densities obtained using the empirical interaction, represented by bands, and the densities that emerge using the NL interaction are unacceptably large. Even worse results are obtained using the impulse approximation based upon the FL t matrix. The discrepan-

FIG. 6. *Ab initio* calculations using (e,e') densities and the PH (solid), LR (dotted), NL (short dashes), or FL (long dashes) interactions are compared with the data for the 2_1^+ and 3_1^- states of ^{32}S .

FIG. 7. Fits to the data for the 2_1^+ and 3_1^- states of ^{32}S using the PH (solid), LR (dotted), NL (short dashes), or FL (long dashes) interactions.

cies for the PH and LR interactions are smaller and similar, but still substantially larger than the width of the band. Among these choices, the LR interaction appears to give the most accurate results, justifying our choice of the LR interaction as the model upon which the empirical effective interaction for this energy was based. Therefore, although fits based upon interactions incorporating theoretical estimates of medium modifications yield densities with qualitatively correct features, an empirical interaction is required to achieve quantitative accuracy commensurate with the intrinsic sensitivity of the reaction.

Fits of the transition density may be made in two ways. First, for self-conjugate targets we may fit ρ_0 directly to the data without need of (e, e') information. Second, for any target with N not necessarily equal to Z , we may use measured proton densities ρ_p as fixed input and fit the neutron density ρ_n to the data. In principle, these two methods should give very similar results for self-conjugate targets except that the error bands and the distortions due to interaction errors are larger using the second method because only half of the structure infor-

mation is allowed to vary. Even though the first method can be expected to give better results for self-conjugate targets, the second gives a more realistic measure of the accuracy than can be expected for neutron densities fitted to data for targets with $N > Z$. In fact, even larger errors would be incurred whenever the neutron density for a particular state happens to make a smaller contribution to the data than the proton density.

Neutron densities fitted to the data, subject to fixed proton densities, are compared for the various interactions in Fig. 9. As expected, the error band shown for the empirical interaction is roughly twice as wide when fitting ρ_n as when fitting ρ_0 , but is still sufficiently narrow to challenge models of nuclear structure. In fact, the error band over the peak of the density is dominated by the normalization uncertainty of $\pm 10\%$, which may be overly generous considering the excellent agreement between the (e, e') and (p, p') normalizations. However, the discrepancies between the true density and the densities obtained using the other interactions are magnified when the entire deviation between theory and experiment must be borne by the neutron density alone.

FIG. 8. Isoscalar densities for the 2_1^+ and 3_1^- states of ^{32}S based upon fits using the PH (solid), LR (dotted), NL (short dashes), or FL (long dashes) interactions. Densities fitted using the empirical effective interaction are shown as bands. The dotted lines lie close to or within the bands and hence are somewhat difficult to distinguish. Note that fitted densities for each of the interactions have similar error bands, but only one can be shown without confusion.

FIG. 9. Neutron densities for the 2_1^+ and 3_1^- states of ^{32}S based upon fits using the PH (solid), LR (dotted), NL (short dashes), or FL (long dashes) interactions. Densities fitted using the empirical effective interaction are shown as bands. For these fits the proton transition density from (e, e') was supplied as fixed input. Note that the dotted line for the 3_1^- density is mostly within the band and hence is somewhat difficult to distinguish.

Most of the qualitative differences between various fitted densities can be readily interpreted in terms of the q dependence of the defects in effective interactions that are revealed by the calculations based upon (e, e') densities. In folding models of the $t(q)\rho(q)$ type, an error in $t(q)$ can be compensated by a corresponding change to $\rho(q)$. For example, Fig. 6 shows that the FL $t(q)$ is too weak for large momentum transfer so that fits to data give too much strength to $\rho(q)$ for large q . This enhanced high- q structure manifests itself in the radial density by a shift toward smaller radii and a stronger oscillatory structure. Because the low- q strength of the FL interaction is approximately correct, the overall strength of $\rho(r)$ is increased such that the integral of $r^{J+2}\rho(r)$ is preserved and the moment M_n agrees well with M_p .

The moments obtained from fits using these several interactions are compared with the results from (e, e') and from the empirical effective interaction in Table V. We find that variations among the fitted moments are strongly correlated with the peak cross sections predicted from the (e, e') densities using these several interactions. For example, the NL interaction strength for $q \sim 1 \text{ fm}^{-1}$ is smaller than any of the other interactions [14], so that fits to the data give the largest moments using the NL interaction. Similar results are obtained by fitting ρ_0 or ρ_n , but δM_n is approximately twice $\delta M_0/2$ so that the same δM_0 is obtained by either method.

We also performed fits using variations of the empirical effective interaction based upon fits to data for ^{16}O separately, ^{40}Ca separately, or the combined data set. Finally, fits were made using empirical effective interactions based upon the PH interaction rather than the FL t matrix as the low-density limit. All of these variations yield virtually indistinguishable results, showing that ambiguities in the precise form of the empirical interaction have little impact upon extraction of neutron transition densities from data. Nor is there any indication of mass dependence in the effective interaction as interactions fit to data for ^{16}O or ^{40}Ca describe data for self-conjugate targets of intermediate mass equally well.

Therefore, we conclude that quite accurate transition densities can be extracted from proton scattering data, but that it is important to use an empirical effective interaction calibrated upon experimental data for states with known transition densities. The empirical effective interaction we use is based upon nuclear matter theory, but the details of the density dependence must be adjusted to reproduce data. This calibration procedure minimizes uncertainties in the reaction model and permits de-

tailed and quantitative analyses of the nuclear-structure content of proton scattering data to be made.

V. COMPARISON BETWEEN SHELL MODEL AND FITTED DENSITIES

Shell-model calculations for positive-parity states of ^{32}S and ^{34}S have been performed using the *universal sd-shell* (USD) interaction of Brown and Wildenthal [31] in the OXBASH shell-model code [45]. Transition densities were constructed using the prescription of Brown, Radhi, and Wildenthal [30]. Further details may be found in our subsequent paper on the structure of ^{34}S [23]. The calculated energy levels and moments are compared with experimental results in Table VI. In this table, the shell-model moments are subdivided into valence and core contributions, M_0^v and M_0^c , according to the conventions of Ref. [23].

Calculations based upon shell-model densities for the lowest two 2^+ and 4^+ states of ^{32}S are shown as dashed lines and compared with the data in Figs. 10 and 11. The solid lines show the results of fitting isoscalar transition densities to these data. The fitted densities are compared with shell-model densities in Fig. 12. The shell-model calculation for the 2_1^+ state of ^{32}S agrees well with both the (p, p') data and with the fitted density. The fitted density is slightly stronger at the surface and somewhat damped in the interior relative to the shell-model density.

The (p, p') data for the 2_2^+ state, a state for which no (e, e') data have been published, have also been fitted with an isoscalar density. Although the shell model predicts the cross section for low momentum transfer, the outward shift required to reproduce higher-momentum-transfer data clearly reveals a significant difference between the shapes of the shell model and fitted densities. The shift toward larger q is accomplished by shifting the density toward smaller r , while increasing the amplitude to maintain approximately constant M_0 and low- q cross section. The accuracy of the present analyses for states with known densities suggests that shape differences of this magnitude can be confidently extracted from (p, p') data for self-conjugate targets even in the absence of (e, e') data.

Finally, isoscalar fits to the data for the 4_1^+ and 4_2^+ states are also compared with shell-model predictions in Figs. 11 and 12. We find that the shell model describes the data and the deduced densities relatively well. The

TABLE VI. Shell-model energy levels (in units of MeV) and matrix elements (in units of fm^J) for ^{32}S .

State J_n^π :	2_1^+	2_2^+	2_3^+	2_4^+	4_1^+	4_2^+	4_3^+	4_4^+
Energy (expt.)	2.230	4.282	5.549	6.666	4.459	6.411	6.852	7.964
Energy (theor.)	2.148	4.353	5.490	6.695	4.698	6.265	6.866	8.131
$M_0^v/2$	3.93	1.97	0.69	0.17	37.30	24.82	6.41	0.89
$M_0^c/2$	2.75	1.38	0.48	0.12	37.30	24.82	6.41	0.89
$M_0/2$	6.68	3.35	1.17	0.28	74.60	49.65	12.81	1.78

FIG. 10. Data for the 2_1^+ and 2_2^+ states of ^{32}S are compared with shell-model calculations, shown as dashed lines, and fits to the isoscalar transition densities, shown as solid lines.

success of the shell model for 4^+ states of ^{32}S can be compared with the situation for the 4_2^+ state of ^{30}Si , for which the shell model describes ρ_p well but predicts (p, p') cross sections more than a factor of 2 below the data [6]. Although there are no (e, e') data for the 4_1^+ state of ^{30}Si ,

FIG. 12. Comparison between shell-model predictions (dashed) and fitted isoscalar densities (solid) are shown for positive-parity states of ^{32}S . Shell-model densities include both valence and core contributions. Error bands are omitted to reduce clutter.

we note that shell-model predictions fall well below the (p, p') cross-section data for that state also. These results suggest that the Brown, Radhi, and Wildenthal model is much more accurate for the isoscalar than for isovector $C4$ densities in this mass region.

VI. TRANSITION DENSITIES FOR NEGATIVE-PARITY STATES OF ^{32}S

Isospin symmetry ensures that the dipole matrix element for 1^- excitations of self-conjugate targets, which would vanish in the absence of isospin violation, is highly suppressed [42,46]. Indeed, we find that when an isoscalar transition density is fitted to the (p, p') data for the 1_1^- state of ^{32}S , the fitted matrix element is consistent with zero. Constraining the matrix element to zero or to the value $|M_p| = 0.031(6)$ deduced from electromagnetic lifetime data [47] has no perceptible effect upon either the fit to the data or the fitted density. The fit and the density are shown in Figs. 13 and 14, respectively.

Finally, we observe that the angular distribution for the state at 6.76 MeV, whose multipolarity is unknown, suggests a 5^- assignment. Collective model calculations strongly favor that assignment. A fit to the data and the density deduced under this assumption are also shown in Figs. 13 and 14. A simple surface-peaked density characteristic of collective 5^- excitations emerges naturally from this analysis, strongly supporting the suggested assignment.

FIG. 11. Data for the 4_1^+ and 4_2^+ states of ^{32}S are compared with shell-model calculations, shown as dashed lines, and fits to the isoscalar transition densities, shown as solid lines.

FIG. 13. The data for the lowest 1^- and 5^- states of ^{32}S are compared with fits of isoscalar transition densities. The state at 6.76 MeV is identified as 5_1^- based upon this angular distribution and the deduced transition density.

FIG. 14. Fitted densities for the lowest 1^- and 5^- states of ^{32}S .

VII. SUMMARY AND CONCLUSIONS

We have obtained cross section and analyzing power data for $^{32}\text{S}(p,p')$ at 318 MeV and have demonstrated that accurate transition densities can be extracted from proton scattering data for normal-parity transitions driven primarily by the matter interaction. The analysis depends upon an empirical effective interaction calibrated upon inelastic scattering data for self-conjugate targets, in this case ^{16}O and ^{40}Ca , using states for which accurate transition densities measured with electron scattering minimizes ambiguities due to nuclear structure. The procedure was tested by comparing isoscalar densities fitted to $^{32}\text{S}(p,p')$ data with proton transition densities fitted to (e,e') data. Using the empirical effective interaction we find that the densities fitted to (p,p') agree with (e,e') densities within their estimated uncertainties. On the other hand, use of density-independent or theoretical density-dependent interactions that are not calibrated to proton-nucleus scattering data gives results that are definitely inferior to those obtained with the empirical effective interaction. Therefore, we conclude that quantitatively accurate transition densities can be extracted from (p,p') data using an effective interaction whose density dependence, although based upon nuclear matter theory, is empirically calibrated.

We applied this technique to fit isoscalar densities to $^{32}\text{S}(p,p')$ data for several states for which corresponding electron scattering data are unavailable. The results for positive-parity states were compared to full $0\hbar\omega$ shell-model calculations based upon the USD interaction. Relatively good agreement is obtained for the 2_1^+ , 4_1^+ , and 4_2^+ states, but the shape of the transition density for the 2_2^+ state differs significantly from the shell-model prediction. The structure of 2_2^+ states for nuclei near the middle of the sd shell has proven somewhat difficult to understand with the shell model. In particular, the USD model predicts that M_n/M_p is negative for the 2_2^+ state of ^{34}S and this prediction is tested in the following paper reporting ρ_n measurements for ^{34}S .

Isoscalar transition densities were also fitted to the lowest 1^- , 3^- , and 5^- states of ^{32}S . The analysis for the 1^- transition is consistent with the approximate selection rule that hinders isoscalar dipole matrix elements. We also find that angular distributions for excitation of corresponding negative-parity states of ^{34}S and ^{32}S are quite similar, differing only by scale factors near unity.

Using complementary (e,e') and (p,p') experiments it is now possible to measure both proton and neutron transition densities for normal-parity excitations with good accuracy. We have also shown that it is possible to obtain accurate isoscalar transition densities for normal-parity excitations of self-conjugate targets from proton scattering data alone. Use of a properly calibrated empirical effective interaction ensures that the error bands for densities fitted to (p,p') data represent realistic appraisals of their accuracy. These radial densities often provide much more insight into the structure of a transition than can be obtained from the less discriminating information provided by scaling or collective model analyses of data for strongly absorbed probes.

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- *Present address: Department of Physics, American University, Washington, D.C. 20016.
 †Present address: NIH, Bethesda, MD 20892.
 ‡Present address: NASA–Goddard Space Flight Center, Greenbelt, MD 20771.
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