Can one observe the spin monopole resonance in ^{208}Pb?

P. M. Boucher
Continuous Electron Beam Accelerator Facility, 12 000 Jefferson Avenue, Newport News, Virginia 23606

B. Castel
Department of Physics, Queen's University, Kingston, Canada K7L 3N6
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We investigate the feasibility of identifying the spin monopole isovector resonance from the quadrupole contribution occurring at the same energy. In that context, we present the result of a continuum random-phase-approximation calculation which clearly indicates that because of very different momentum transfer patterns for $L=0$ and $L=2$ resonances, the spin monopole should be easily identifiable in reactions like ^{208}Pb(n,p) at forward angles or low momentum transfer.

The last ten years have seen increased interest in giant resonances in which spin degrees of freedom are involved.[1,2] These resonances have contributed important information on the spin-isospin part of the nuclear interaction. In that context, the spin isovector monopole resonance (SIMR), presumably because of its connection to the well known giant monopole resonance, is also beginning to receive the attention it deserves. Macroscopically, the SIMR can be thought of as a breathing mode where the neutron and proton distributions interpenetrate, oscillating rapidly out of phase. Microscopically, the relevant operator for the SIMR is $Q_{01}=\sum_i r_i^2\sigma_i\tau_\mu$ with $\mu=\pm1$ and 0. Because of the $L=0$ and $L=2$ coupling to the spin operator σ, the strength of the spin monopole resonance (with $J^\pi=1^+$) can arise from both $L=0$ and $L=2$ $2\hbar\omega$ contributions. From a theoretical standpoint, the SIMR has been investigated by Auerbach who included the spin monopole resonance in his recent study of isovector spin excitations.[2,3] On the experimental side, there have been preliminary investigations of the SIMR using nucleon charge-exchange reactions on Pb and Zr as well as with strongly absorbed projectiles.[3-5] Certainly the (n,p) reaction in the 100–400 MeV range is particularly well suited for these types of studies because of the relatively strong $T=S=1$ component of the nucleon-nucleon interaction in this energy range and because, in heavy nuclei, the large neutron excess results in stronger Pauli blocking of the $0\hbar\omega$ (Gamow-Teller) and $1\hbar\omega$ dipole excitations than of $2\hbar\omega$ excitations.

The purpose of this Brief Report is to present the result of a continuum random-phase-approximation (RPA) calculation of spin monopole and quadrupole resonances in ^{208}Pb and derive conclusions on how to best differentiate between $L=0$ and $L=2$ contributions to the SIMR using the ^{208}Pb$(n,p)^{208}$Tl reaction as an example. We first derive briefly the continuum RPA formalism used.

The continuum RPA calculation we have performed uses Hartree-Fock wave functions as a basis. It is similar to the one described in detail by Bertsch and Tsai[6] and is based on a coordinate space representation of the Green functions. The method has been widely adopted since it is easily amenable to the use of the Skyrme interactions.

The first step is a calculation of the unperturbed Green's function as a function of excitation energy, ω, from the Hartree-Fock wave functions ϕ_h,

$$G^{(0)}(\mathbf{r},\mathbf{r}';\omega)=-\sum_h\phi_h(\mathbf{r})\left[\frac{1}{H_0-\epsilon_h-\omega}+\frac{1}{H_0-\epsilon_h+\omega}\right]\phi_h(r')\ . \quad (1)$$

In this equation, H_0 is the free Hamiltonian and ϵ_h is the single-particle energy associated with the state defined by ϕ_h. This relation can be used to generate the RPA Green's function,

$$G^{\mathrm{RPA}}=G^{(0)}\left[1-\frac{\delta V}{\delta\rho}G^{(0)}\right]^{-1}, \quad (2)$$

where V is the interaction expressed as a functional of the density, ρ. The approach is particularly well suited to the use of a Skyrme interaction in which the functional derivative is straightforward to use. We have focused on the SIII interaction since it is well known to describe nuclei in the Pb region quite adequately.[7] Note that the sum in Eq. (1) is over the occupied states and $(H_0-\omega)^{-1}$ is the Hartree-Fock Green's function for a particle propagated from $\mathbf{r}$ to $\mathbf{r}'$. For H_0 with a discrete spectrum, the single-particle Green's function has the coordinate space evaluation

$$\frac{1}{H_0-\omega}=\sum_p\phi_p(\mathbf{r})\frac{1}{\epsilon_p-\omega}\phi_p(\mathbf{r}')\ , \quad (3)$$

where the sum is here restricted to occupied states. The continuum is treated using the single-particle Green's function,

$$g(\mathbf{r},\mathbf{r}';\omega)=\left\langle \mathbf{r}\left|\frac{1}{H_0-\omega}\right|\mathbf{r}'\right\rangle$$
$$=\frac{2m}{\hbar^2}\frac{v(r_>)w(r_<)}{W(v,w)}\ , \tag{4}$$

where v and w are the regular and irregular solutions for the Hartree-Fock Hamiltonian; W is the Wronskian of the two solutions. The single-particle Green's function in Eq. (4) contains the entire spectrum of H_0. The form of Eq. (1) forces the numerical cancellation of all Pauli-violating transitions within the RPA. Finally, the strength distribution was evaluated directly from the RPA Green's function

$$S(\omega)=\int d\mathbf{r}\, f^*(\mathbf{r}')G(\mathbf{r},\mathbf{r}';\omega)f(\mathbf{r})\ . \tag{5}$$

We now focus on the monopole and quadrupole contributions to the SIMR. The relevant operators for the $L=0$ and $L=2$ components of the $J^\pi=1^+$ resonances in ^{208}Pb are, respectively, $f(\mathbf{r})=j_0(qr)Y_0(\hat{\mathbf{r}})\sigma\tau_+$ and $f(\mathbf{r})=j_2(qr)Y_2(\hat{\mathbf{r}})\sigma\tau_+$. The form of these operators allows the dependence of the strength on the momentum transfer, q, or equivalently the scattering angle, θ, to be readily examined. At low values of the momentum transfer, the monopole contribution is expected to be larger than the quadrupole component. This is evident from the asymptotic expansion of the Bessel functions in the $q\to 0$ limit,

$$j_0(qr)\to 1-\tfrac{1}{6}(qr)^2 \quad \text{vs} \quad j_2(qr)\to \tfrac{1}{15}(qr)^2\ . \tag{6}$$

For low q values, the relative strength of the quadrupole contribution will thus be

$$\frac{S_2(\omega)}{S_0(\omega)}\approx(\tfrac{6}{5})^2=\tfrac{4}{25}\ .$$

We now turn to a specific discussion of the monopole and quadrupole strength distribution resulting from (n,p) reactions on ^{208}Pb, where in addition to the escape width calculated above, the spreading width resulting from the 2p-2h coupling to the continuum has been included. This calculation is similar to the one described by Smith and Wambach[8] who relate the coupling to 2p-2h states with the imaginary part of an optical potential using empirical information from the decay widths of particle and hole excitation. In Fig. 1 we display our calculated strength distribution in the limited energy range 11–16 MeV, where practically all the monopole and quadrupole strengths are concentrated. As we can see, although the monopole and quadrupole resonances occur at nearly the same energy, very pronounced differences are seen for $L=0$ and $L=2$ distributions when momentum transfer is made to vary. First of all, as is apparent, the strength of the monopole state is concentrated in a single peak around 13.2 MeV. As q increases, this peak becomes more pronounced before decreasing beyond $q\approx 0.5$ fm^{-1}. Conversely, the quadrupole strength, very weak at low q, shows a more dramatic increase at larger momentum transfer. Further, before the spreading width is introduced, the quadrupole strength is fragmented into three main peaks at 12.3, 13.0, and 14.2 MeV, so that the quadrupole scale at a given energy corresponds roughly to $\frac{1}{3}(\frac{4}{25})=0.05$ compared with the monopole one *at low values of momentum transfer.* In our opinion, this should provide for a good opportunity to observe the monopole strength at forward angles. Since the monopole component will interfere with the quadrupole one in the relevant energy range, there will be potentially some difficulty identifying it at higher momentum transfer especially because a realistic tensor force will always mix

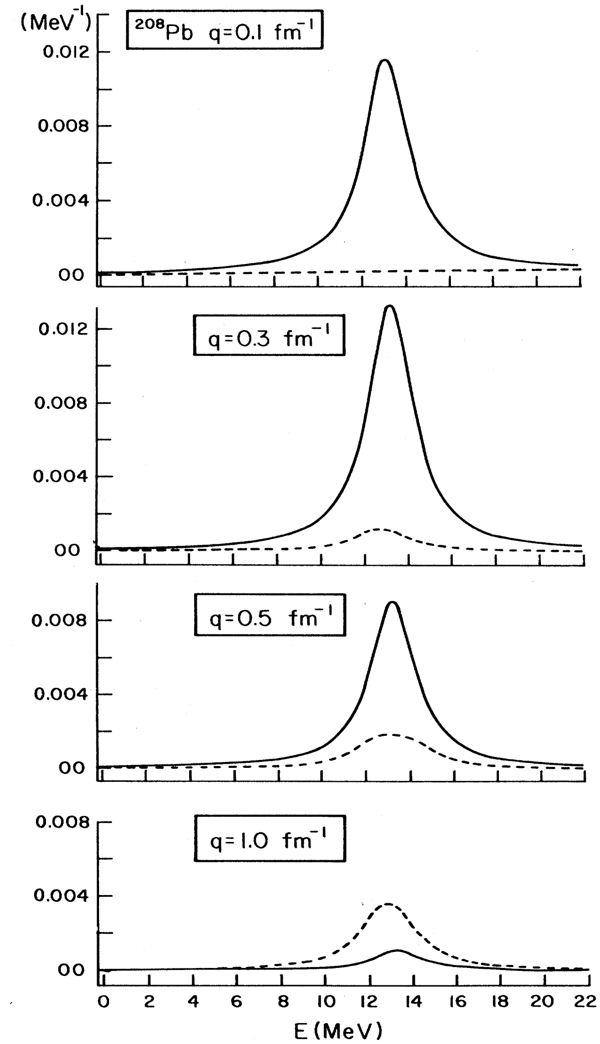

FIG. 1. The spin isovector monopole and quadrupole strength for (n,p) reactions on ^{208}Pb. The excitation energy dependence of the strength, S (MeV^{-1}), arising from collective excitation operators of the form $j_0(qr)Y(\hat{\mathbf{r}})\sigma\tau_+$ and $j_2(qr)Y(\hat{\mathbf{r}})\sigma\tau_+$ has been plotted for a range of momentum transfer values. The monopole strength, strong at low momentum transfer with a peak at $q\simeq 0.5$ fm^{-1} suffers a strong reduction as q increases, whereas the opposite is true for the $L=2$ component.

to some extent the $L=0$ and $L=2$ $2\hbar\omega$ modes.[8,9] This mixing is not expected to be severe, however,[5,9] so we suggest that because of the widely different momentum transfer patterns, the $L=0$ component of the SIMR should be easily identifiable in reactions like ^{208}Pb$(n,p)^{208}$Tl at forward angles.

Although we have concentrated on nucleon charge-exchange reactions. Auerbach *et al.*[5] proposed recently that strongly absorbed projectiles would also be excellent probes to detect the SIMR, citing the ^{90}Zr$(^3$He$,t)$ reaction as a good example. Although we generally concur that pions and other strongly absorbed projectiles provide a good method of investigating the SIMR because of the large surface contribution of the spin monopole transition densities, we find that the ^{208}Pb case is less attractive than the one in ^{90}Zr. Indeed, although our investigations of the SIMR radial transition density (Fig. 2) show characteristic nodes as expected from the integral,

$$4\pi\int dr\, r^2\rho_M(r)=0\ , \tag{7}$$

we find the surface contribution to be smaller in ^{208}Pb than Auerbach *et al.* found for ^{90}Zr. We thus still believe in our conclusion that nucleon charge-exchange reactions provide an experimental tool well suited to isolate the monopole component, especially if the experimental facility like the one at TRIUMF is able to provide good resolution at small angles.

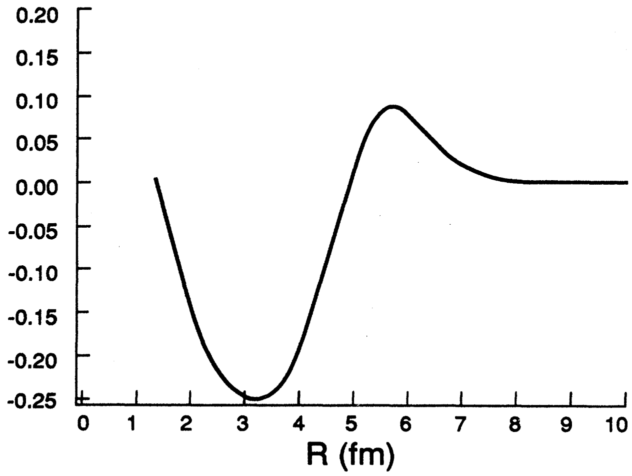

FIG. 2. The spin isovector monopole transition density for (n,p) reactions on ^{208}Pb. The transition density has been plotted for the SIMR in ^{208}Pb. Although the general features are similar to those for monopole transition densities in lighter nuclei, the surface peak is not as pronounced in ^{208}Pb and the possibility of observing the SIMR in charge-exchange reactions with strongly absorbed projectiles is less likely.

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