Nucleon $g_2$ Structure Function and Quark-Gluon Correlations

Zein-Eddine Meziani
Temple University
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CIPANP-2009
Outline

- Quark-gluon correlations: Average Color Lorentz force, Color polarizabilities
  - Nucleon $g_2$ and $g_1$ spin structure functions
  - Twist-3 $d_2$ and twist-4 $f_2$ matrix elements
  - $d_2^n$ (E06-114 in Hall A) and SANE (E07-003 in Hall C) experiments at Jefferson Lab
  - Future prospects
Spin Structure Functions

- **Unpolarized structure functions** $F_1(x, Q^2)$ and $F_2(x, Q^2)$

  \[
  \frac{d^2 \sigma}{dE'd\Omega}(\downarrow \uparrow + \uparrow \uparrow) = \frac{8\alpha^2 \cos^2(\theta/2)}{Q^4} \left[ \frac{F_2(x, Q^2)}{\nu} + \frac{2F_1(x, Q^2)}{M} \tan^2(\theta/2) \right]
  \]

- **Polarized structure functions** $g_1(x, Q^2)$ and $g_2(x, Q^2)$

  \[
  \frac{d^2 \sigma}{dE'd\Omega}(\downarrow \uparrow - \uparrow \uparrow) = \frac{4\alpha^2}{MQ^2 \nu E} \left[ (E + E' \cos \theta)g_1(x, Q^2) - \frac{Q^2}{\nu}g_2(x, Q^2) \right]
  \]

  \[
  \frac{d^2 \sigma}{dE'd\Omega}(\downarrow \Rightarrow - \uparrow \Rightarrow) = \frac{4\alpha^2 \sin \theta}{MQ^2 \nu^2 E} \left[ \nu g_1(x, Q^2) + 2E g_2(x, Q^2) \right]
  \]

- **Variables:**
  - $Q^2$: Four-momentum transfer
  - $x$: Bjorken variable
  - $\nu$: Energy transfer
  - $M$: Nucleon mass
  - $W$: Final state hadrons mass
Example of a standard setup in Hall A

- **Polarized beam**
  - **Energy**: 0.86-5.1 GeV
  - **Polarization**: > 70%
  - **Average Current**: 5 to 15 µA

- **Hall A polarized $^3$He target**
  - **Pressure**: ~ 10 atm
  - **Polarization average**: 35%
  - **Length**: 40 cm with 100 µm thickness

- **Highest polarized luminosity**: $\sim 10^{36} \text{cm}^{-2}\text{s}^{-1}$

**Electron beam**

- Measurement of helicity dependent $^3$He cross sections
- Extract $g_1$ and $g_2$ spin structure functions of $^3$He
- Extract moments of spin structure functions of $^3$He and Neutron

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$g_2(x, Q^2) = g_{2WW}(x, Q^2) + \bar{g}_2(x, Q^2)$

- a twist-2 term (Wandzura & Wilczek, 1977):

$$g_{2WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(x, Q^2) \frac{dy}{y}$$

- a twist-3 term with a suppressed twist-2 piece (Cortes, Pire & Ralston, 1992):

$$\bar{g}_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y} \left[ \frac{m_q}{M} h_T(y, Q^2) + \xi(y, Q^2) \right] \frac{dy}{y}$$

Transversity

$q-g$ correlations
Quark-Gluon Correlations

How did we investigate them?
- By direct comparison of $g_2$ to $g_2^{WW}$ to observe deviations
- As for any other nucleon structure function it is hard to have ab-initio calculation of $g_2$
- Quark models are can be useful to gain insight

Second moment of $g_2$ offers another avenue of investigation
- Moments of $\overline{g}_2$ can be calculated using lattice QCD since they correspond to specific matrix elements of quark and gluon field operators
- These matrix elements have a physical interpretation
  - Average Color Lorentz Force (M. Burkhardt)
  - “Color polarizabilities” (X. Ji, Stein et al.)
Nucleon world results of $g_2$

- SLAC E155x (proton and deuteron)
- JLab E99-117 (helium-3), $A_1^n$ in DIS
- Jlab E97-103 (helium 3) DIS, $Q^2$ dependence mainly below 1.4 GeV$^2$

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Q^2 dependence below 1 GeV^2

JLab E97-103 (helium 3) DIS, Q^2 dependence mainly below 1.4 GeV^2

Spokespersons: T. Averett and W. Korsch
Student: K. Kramer

$x^2 g_2^3$He at constant $Q^2$

JLab Experiment E01-012

Spokespersons: J.P Chen, Seonho Choi and N. Liyanage

Thesis student: P. Solvignon
Moments of Structure Functions

\[ d_2(Q^2) = 3 \int_0^1 x^2 \left( g_2(x, Q^2) - g_2^{WW}(x, Q^2) \right) dx \]

\[ d_2 S_{[\mu P {\nu]} P} P^\lambda = \frac{1}{8} \sum_q \langle P, S | \overline{\psi}_q \ g F_{\{\mu \nu \gamma}^{\lambda\}} \psi_q | P, S \rangle \]

\[ d_2 (Q^2) \rightarrow \text{dynamical twist-3 matrix element} \]

\[ d_2(Q^2) = \int_0^1 dx \ x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right] \]
“Color Polarizabilities”

X. Ji 95, E. Stein et al. 95

How does the gluon field respond when a nucleon is polarized?

Define color magnetic and electric polarizabilities (in nucleon rest frame):

\[ \chi_{B,E} 2M^2 \vec{S} = \langle PS | \vec{O}_{B,E} | PS \rangle \]

where

\[ \vec{O}_B = \psi^\dagger g \vec{B} \psi \]

\[ \vec{O}_E = \psi^\dagger \vec{\alpha} \times g \vec{E} \psi \]

\[ d_2 = (\chi_E + 2\chi_B)/8 \]

\[ f_2 = (\chi_E - \chi_B)/2 \]

\[ d_2 \text{ and } f_2 \text{ represent the response of the color } \vec{B} \text{ & } \vec{E} \text{ fields to the nucleon polarization} \]
Average Color Lorentz Force (M. Burkardt)

\[ \int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6 M P^2 S^x} \langle P, S | \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) | P, S \rangle \]

\[ \rightarrow \quad d_2 \text{ a measure for the color Lorentz force acting on the struck quark in SIDIS in the instant after being hit by the virtual photon} \]

\[ \langle F^y(0) \rangle = -M^2 d_2 \quad \text{(rest frame; } S^x = 1) \]
Interpretation of $d_2$ with the transverse FSI force in DIS also consistent with $\langle k_{\perp}^y \rangle \equiv \int_0^1 dx \int d^2k_\perp \ k_\perp^2 f_{1T}^1(x, k_\perp^2)$ in SIDIS (Qiu, Sterman)

$$\langle k_{\perp}^y \rangle = -\frac{1}{2p^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dx^- gG^{+y}(x^-) \gamma^+ q(0) \right| P, S \right\rangle$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity
Models and Lattice evaluations of $d_2$

**Quark Bag Models**

**Chiral Soliton Model**
H.Weigel and L.Gamberg,

**Lattice QCD**
M.Gockeler et al.,
$Q^2$ evolution of the neutron $d_2$
Moments of Structure Functions (continued)

\[ \tau = 2 \]

single quark scattering

\[ \Gamma_1(Q^2) = \int_0^1 dx \ g_1(x, Q^2) \]

\[ = \Gamma_1^{\text{twist-2}}(Q^2) + \frac{M_N^2}{9 Q^2} \left[ a_2(Q^2) + 4 d_2(Q^2) + 4 f_2(Q^2) \right] + O \left( \frac{M_N^4}{Q^4} \right) \]

\[ \tau > 2 \]

qq and qg correlations
Moments of Structure Functions (continued)

\[ a_2(Q^2) \equiv 2 \int_0^1 dx \, x^2 \, g_1^{\text{twist}-2}(x, Q^2) \]  \text{target mass correction term}

\[ d_2(Q^2) \rightarrow \text{dynamical twist-3 matrix element} \]

\[ d_2(Q^2) = \int_0^1 dx \, x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right] \]

\[ f_2(Q^2) \rightarrow \text{dynamical twist-4 matrix element} \]

\[ f_2(Q^2) = \frac{1}{2} \int_0^1 dx \, x^2 \left[ 7g_1(x, Q^2) + 12g_2(x, Q^2) - 9g_3(x, Q^2) \right] \]

\[ f_2 M^2 S^\mu = \frac{1}{2} \sum_q e_q^2 < N | \bar{\psi}_q g \tilde{F}_{\mu\nu}^\ast \gamma^\nu \psi_q | N > \]
Adding $1/Q^6$ term gives the same $f_2$ and $\mu_6$ with $\mu_8 = (0.00 \pm 0.03)M^2$
Determination of \( f_2 \) for the proton

World data + EG1a data: R. Fatemi et al., PRL, 91 22200 (2003)


\[
\frac{f_2}{\mu_6/M^4} = 0.039^{+0.037}_{-0.043}
\]

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Hall A $d_2^n$ and Hall C SANE experiments
Neutron and Proton

Spokespeople:
B. Sawatzky, S. Choi, X. Jiang and Z.-E.M

Students:
D. Flay, D. Parno, M. Posik

and the Hall A collaboration

Spokespeople:
O. Rondon, S. Choi, M. Jones, Z.-E. M

Students:
W. Armstrong, H. Kang, A. Liyanage, J. Maxwell, J. Mulholland

and the Hall C collaboration
Two beam energies 4.6 and 5.7 GeV (4 pass, 5 pass)  
BigBite fixed at single scattering angle (\(\theta = 45^\circ\))  
(data divided into 10 bins during analysis)  

Experiment ran Jan.-Mar. 09  
May-30, 2009  
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At large $Q^2$, $d_2$ coincides with the reduced twist-3 matrix element of gluon and quark operators.

At low $Q^2$, $d_2$ is related to the spin polarizabilities.
SANE experiment in Hall C

Two beam energies:
- 6.0 GeV (black)
- 4.8 GeV (green)

Target
- UVa NH3 target
- 5 T field

Beamline
- Chicanes
- SEM
- He Bag

Electron Arm
- BETA

Background Studies
- HMS

CEBAF polarized beam
- 85 nA
- 75% beam polarization

Experiment Ran January-March 09
Three subsystems:
- Lead glass calorimeter BigCal: Energy Measurement
- Gas Cherenkov: e- identification
- Lucite hodoscope: tracking
- Front tracker: tracking

Target field sweeps low E background

Characteristics
- Effective solid angle (with cuts) = 0.194 sr
- Energy resolution $5%/\sqrt{E(\text{GeV})}$
- Angular resolution = 2°
- 1000:1 pion rejection
SANE experiment $g_2, g_1$ projected errors
$d_2^p$ RSS and SANE $d_2^p$ projection in Hall C

RSS spokesperons: M. Jones, O. Rondon
SANE spokespersons: S. Choi, M. Jones, O. Rondon, Z.-E. M

![Graph](image-url)

Lattice QCD - Goeckeler et al.
SANE $d_2$ expected statistical errors
RSS $d_2$
Elastic contribution
SLAC
$d_2 = d_2(RSS) \sqrt{Q^2_{RSS}/Q^2}$
SANE combined $Q^2$ 3.5 to 6.5 GeV$^2$
$d_2$ pQCD (NP B201:141)

Preliminary RSS inelastic $d_2$
$g_2$ at JLab with 11 GeV

CLAS 12

Hall C HMS/SHMS

$\chi^2 g_{2p}$

$\chi^2 g_n$

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Past experiments on the neutron and proton suggest that the twist-3 and twist-4 are small but finite.

Precision measurements of $g_1$ and $g_2$ in the range $1 < Q^2 < 4 \text{ GeV}^2$ are crucial for an improved extraction of the

- Average color Lorentz force
- “Color polarizabilities”

In the next year or two we will have results from two recently ran experiments at Jefferson Lab, SANE in Hall C (proton) and E06-14 in Hall A (neutron).

The non-singlet combination $(d_2^p - d_2^n)$ should provide a benchmark test for present lattice QCD calculations since no disconnected diagrams are needed.

This program will be pursued at JLab 11 GeV for higher precision and greater $Q^2$ and $x$ coverage.
QCDSF Collaboration Lattice Calculations

hep-lat/0506017

M. Gockeler, R. Horsley, D. Pleiter, P.E.L. Rakow, A. Schafer, G. Schierholz, H. Stueben, J.M. Zanotti

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E94-010 B-C sum rule; results

Neutron

\[ \Gamma_2 \]

\[ Q^2 (\text{GeV}^2) \]

\[ M. \text{Amarian et al., Phys. Rev. Lett. 92, 022301 (2004)} \]

\[ \text{3He} \]

\[ \Gamma_2 \]

\[ Q^2 (\text{GeV}^2) \]

\[ \text{Preliminary} \]

\[ M. \text{Amarian et al., in preparation} \]
75-80% polarized beam at 15µA

35-40% polarized target in beam
Proton $g_2$ and $A_1$

- DIS data up to $x = 0.6$; Resonances measured down to $W = 1.38$ GeV
- $g_2$ measured in region of most sensitivity for $d_2$