Analysis of 56-plet Positive Parity Baryon Decays in the $1/N_c$ Expansion

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Abstract

The partial decay widths of positive parity baryons belonging to **56**-plets of $SU(6)$ are analyzed in the framework of the $1/N_c$ expansion. The channels considered are those with emission of a single π , K or $\bar{K}$ mesons, and the analysis is carried out to subleading order in $1/N_c$ and to first order in $SU(3)$ symmetry breaking. The results for the multiplet $[\mathbf{56}, 0^+]$, to which the Roper resonance belongs, indicate a poor description at leading order, requiring important next to leading order corrections. For the multiplet $[\mathbf{56}, 2^+]$, the P-wave decays in the non-strange sector are well described at leading order, while the F-wave decays require the next to leading order corrections, which turn out to be of natural magnitude. $SU(3)$ breaking effects are poorly determined because only few decays with K meson in final state are established, and their widths are not known with sufficient accuracy.

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I. INTRODUCTION

One of the most important objectives in hadronic physics is the increasingly accurate determination of the properties of baryon resonances, the search for predicted and yet unobserved resonances, and the theoretical description and understanding of the resonances' observables. The study of baryons, which is complementary to that of mesons, plays indeed an important role in exposing non-perturbative aspects of QCD, such as the ordering of states into approximately linear Regge trajectories, the various strong and electromagnetic transition observables, and the remarkable identification of spin-flavor multiplets in the known spectrum of baryons. This latter property was identified back in the 1960s [1], and in QCD it can be explained in terms of the $1/N_c$ expansion: in the large N_c limit, baryons must fill multiplets of the contracted spin-flavor symmetry group $SU^c(6)$ [2, 3]. In the real world with $N_c = 3$, we do have evidence of an $O(3) \times SU(6)$ multiplet structure, in particular thanks to the well established ground state octet and decuplet states which are accommodated in a $[56, \ell^P = 0^+]$, and the convincingly established $[56, 2^+]$ and $[70, 1^-]$ multiplets in the second resonance region, which, although incomplete, have enough known states for a good identification of them. For other multiplets, the identification needs further scrutiny, based on mass formulas in particular. One such a multiplet is the one containing the Roper resonance $N(1440)$, which is assumed to be a $[56, 0^+]$. Only a few of the resonances which would be assigned to that multiplet are established.

While in the large N_c limit there must be a contracted $SU^c(6)$ symmetry in the baryon sector, its breaking happening at $\mathcal{O}(1/N_c)$, there is no justification of principle for the observed approximate $O(3)$ symmetry. Within a given $O(3) \times SU(6)$ multiplet, the $O(3)$ symmetry is recovered in the large N_c limit if the multiplet corresponds to the symmetric spin-flavor representation, but it is broken at $\mathcal{O}(N_c^0)$ for a mixed-symmetry multiplet. Thus for the states we discuss in this paper, and provided we neglect configuration mixings [4], i.e., mixings with for instance a mixed-symmetry multiplet, the breaking of the $O(3)$ symmetry is a sub-leading effect in $1/N_c$.

Based on the $O(3) \times SU(6)$ symmetry scheme, it is possible to implement a $1/N_c$ expansion in baryons as an effective theory built in terms of effective operators associated with the observable to be analyzed. This framework has been utilized in the analysis of baryon masses [5, 6], strong decays [7–10], magnetic moments [11], electromagnetic helicity amplitudes [12],

etc. From these works it transpires that the ordering of effects according to the $1/N_c$ power counting is remarkably well manifested for most observables that have been studied, with some exception in the case of the $[\mathbf{56}, 0^+]$ Roper multiplet, which is part of the present analysis.

This work extends the analysis of the strong decays of positive parity baryons studied within $SU(4)$ [10] to the $\mathbf{56}$ -plet of $SU(6)$. The analysis is carried out to subleading $\mathcal{O}(1/N_c)$ and to first order in $SU(3)$ symmetry breaking, and the framework follows a similar implementation as in the case of decays in $SU(4)$ [10]. An earlier analysis of the decays of the $[\mathbf{56}, 0^+]$ baryons in the $1/N_c$ expansion was made in Ref. [8]. The present work presents a full fledged analysis to the orders mentioned for that multiplet as well as for the $[\mathbf{56}, 2^+]$.

The inputs utilized for the analysis are the partial decay widths as given in terms of fractions of the Breit-Wigner widths by the Particle Data Group [13]. Most of these inputs have changed only slightly over the last few editions of the particle listings, and we refer to the latest edition for references to the most recent partial wave analyses.

The work is organized as follows: in section II we outline the implementation of the $1/N_c$ expansion for the decays, in section III we give the results for the analyses of the $[\mathbf{56}, 0^+]$ and the $[\mathbf{56}, 2^+]$ baryon decays, followed by the conclusions in section IV. For completeness, an appendix provides details on the $SU(3)$ isoscalar factors needed in the calculations.

II. THEORETICAL BACKGROUND

A. Baryon states in $O(3) \times SU(6)$

The excited baryon states in the large N_c limit must fill towers of states, which correspond to irreducible representations of a contracted spin-flavor symmetry group $SU^c(2N_f)$. The states of angular momentum of order unity in these towers have mass splittings $\mathcal{O}(1/N_c)$. As already mentioned, in the real world with $N_c = 3$, it is observed that there is in addition an approximate $O(3)$ symmetry, which does not follow from the $1/N_c$ expansion but is rather from phenomenological observation. This symmetry is most clearly displayed in the spectrum of the known excited baryons. Thus, the classification of the excited baryons in terms of the symmetry group $O(3) \times SU(2N_f)$ is the proper approach. In large N_c the contracted symmetry group will emerge as a subgroup of that larger group [14].

For three light flavors we need the group $SU(6)$, which has thirty five generators, namely $\{S_i, T_a, G_{ia}\}$, with $i = 1, 2, 3$ and $a = 1, \dots, 8$, where the first three are the generators of the spin $SU(2)$, the second eight are the generators of flavor $SU(3)$, and the last twenty four can be identified as an octet of axial-vector currents in the limit of zero momentum transfer. The algebra of $SU(6)$ has the following commutation relations that fix the normalizations of the generators:

$$\begin{aligned} [S_i, S_j] &= i \epsilon_{ijk} S_k, & [T_a, T_b] &= i f_{abc} T_c, \\ [S_i, G_{ja}] &= i \epsilon_{ijk} G_{ka}, & [T_a, G_{ib}] &= i f_{abc} G_{ic}, \\ [G_{ia}, G_{jb}] &= i \delta_{ij} f_{abc} T_c + i \epsilon_{ijk} (\delta_{ab} S_k + d_{abc} G_{ck}), \end{aligned} \quad (1)$$

where d_{abc} and f_{abc} are the $SU(3)$ symmetric and antisymmetric invariant tensors, respectively.

The states of interest in this work belong to the totally symmetric irreducible representation **S**, which is given by a Young tableau consisting of a single row of N_c boxes. These states correspond at $N_c = 3$ to the **56**-plet of $SU(6)$. Another important representation is the mixed symmetric **MS** which consists of a row with $N_c - 1$ boxes and a second row with a single box, which for $N_c = 3$ corresponds to the **70**-plet of $SU(6)$. We denote by ℓ the $O(3)$ quantum number of the states. The states belonging to the $[\mathbf{56}, \ell]$ multiplet are then given by

$$|(\ell, S) J J_3; R = (p, q), Y, I I_3 \rangle_{\mathbf{S}} = \sum_m \langle \ell m, S S_3 | J J_3 \rangle | S S_3; R = (p, q), Y, I I_3 \rangle_{\mathbf{S}} | \ell m \rangle, \quad (2)$$

where the label **S** indicates that the state belongs to the symmetric spin-flavor representation, S is the spin quantum number associated with the spin subgroup of $SU(6)$, J is the total angular momentum of the baryon, $R = (p, q)$ indicates the $SU(3)$ irreducible representation given in terms of the usual labeling of a Young tableau with $p + 2q = N_c$, and Y and I are respectively the hypercharge and the isospin.

From the decomposition of the **S** representation of $SU(6)$ as a sum of direct products of irreducible representations of $SU(2) \otimes SU(3)$, it results that $p = 2S$. This latter relation is a consequence of the fact that for the **S** representations the two factors in the direct products involved in the decomposition have the same Young tableau (p, q) . The relation then follows from the relation $p = 2S$ in $SU(2)$. The $p = 2S$ relation is a generalization of the $I = S$ relation for the **S** representations of $SU(4)$.

The correspondence of multiplets between generic $N_c > 3$ and $N_c = 3$ is as follows: i) $(p = 1, q = \frac{N_c-1}{2}) \rightarrow (\mathbf{8}, S = \frac{1}{2})$ and ii) $(p = 3, q = \frac{N_c-3}{2}) \rightarrow (\mathbf{10}, S = \frac{3}{2})$. Clearly, all this holds for odd N_c .

For $\ell = 0$, which also includes the ground state (GS) baryons, we have the $J = S = 1/2$ octet and the $J = S = 3/2$ decuplet. For $\ell = 2$ we have octets with $J = 3/2$ and $5/2$, and decuplets with $J = 1/2$ through $7/2$.

B. $O(3) \times SU(6)$ framework for decays

In the following, the framework for implementing the calculation of decays via emission of a single pseudoscalar meson is described. We first give the partial decay widths in terms of reduced matrix elements of the effective baryonic operators, which describe the transition between the initial excited baryon and the final ground state baryon. If we altogether neglect $SU(3)$ symmetry breaking, the partial widths could be expressed solely in terms of $SU(3)$ reduced matrix elements of the effective operators. However, $SU(3)$ symmetry breaking is noticeable in the decays and must be included. This is done with effective operators generated by the octet component of the quark masses (i.e., by the term $\frac{1}{2\sqrt{3}}(m_u + m_d - 2m_s)\lambda_8$ in the quark mass matrix), and thus the partial widths are rather given in terms of reduced matrix elements of the isospin $SU(2)$. We start from the general expression for a partial decay width of an initial excited baryon state $|(\ell, S^*)J^*, J_3^*; R^*, Y^*, I^*, I_3^*\rangle$ to a final state $|S, S_3; R, Y, I, I_3\rangle$ emitting a pseudoscalar meson in the state $|\tilde{\ell}, \tilde{\ell}_3; \tilde{R}, \tilde{Y}, \tilde{I}, \tilde{I}_3\rangle$. The effective operator must carry the same quantum numbers of the meson. The partial decay width is then given by:

$$\Gamma = \frac{k}{8\pi^2} \frac{M_B}{M_{B^*}} \frac{1}{(\hat{I}^* \hat{J}^*)^2} \times \sum_{J_3^*, I_3^*, S_3, I_3, \tilde{\ell}_3, \tilde{I}_3} |\langle S, S_3; R, Y, I, I_3 | B_{[\tilde{\ell}_3, \tilde{Y}, \tilde{I}, \tilde{I}_3]}^{[\tilde{\ell}, \tilde{R}]} | (\ell, S^*)J^*, J_3^*; R^*, Y^*, I^*, I_3^* \rangle|^2, \quad (3)$$

where M_{B^*} and M_B are the masses of the excited and final baryon respectively, k is the meson momentum, and the notation $\hat{j} \equiv \sqrt{2j+1}$ is used. The baryonic transition operator is denoted by $B_{[\tilde{\ell}_3, \tilde{Y}, \tilde{I}, \tilde{I}_3]}^{[\tilde{\ell}, \tilde{R}]}$, where the upper labels display the angular momentum of the operator and its $SU(3)$ representation, and the lower labels the corresponding projections. The operator is built as a linear combination in a basis of effective operators which is ordered

in powers of $1/N_c$:

$$B^{[\tilde{\ell}, \tilde{R}]} = \left(\frac{k}{\Lambda}\right)^{\tilde{\ell}} \sum_n C_n^{[\tilde{\ell}, \tilde{R}]}(k) B_n^{[\tilde{\ell}, \tilde{R}]}, \quad (4)$$

where the B_n are the operators in the basis, and the coefficients C_n encode the dynamics of the decay amplitudes. In this work they are determined by fitting to the empirical partial decay widths. A centrifugal factor is included, which is expected to carry the chief momentum dependence of the transition amplitude, with the arbitrary scale Λ to be chosen to be 200 MeV in what follows.

The effective operators in the basis can be expressed in terms of spin-flavor operators in the following general form:

$$B_{n \ [j_3, \alpha]}^{[\tilde{\ell}, \tilde{R}]} = \sum_{m, j_3} \langle \ell m, j_n j_3 \mid \tilde{\ell} \tilde{\ell}_3 \rangle \xi_m^\ell \mathcal{G}_{n \ [j_3, \alpha]}^{[j_n, \tilde{R}]}, \quad (5)$$

where ξ^ℓ is the tensor operator which gives the transition between the $O(3)$ state of the excited baryon and the ground state baryon, and $\mathcal{G}_n^{[j_n, \tilde{R}]}$ is a spin-flavor operator. Without any loss of generality, ξ^ℓ is normalized such that $\langle 0 \mid \xi_{m'}^\ell \mid \ell m \rangle = (-1)^{\ell-m} \delta_{m-m'}$. The spin-flavor operator, which is a tensor with angular momentum j_n , can be built as tensor products of the $SU(6)$ generators.

We can express the partial widths in terms of the RMEs of the basis operators as follows:

$$\begin{aligned} \Gamma &= \frac{k}{8\pi^2} \left(\frac{k}{\Lambda}\right)^{2\tilde{\ell}} \frac{M_B}{M_B^*} \frac{\hat{I}^2}{(\hat{I}^* \hat{J}^*)^2} \\ &\times \left| \sum_n C_n^{[\tilde{\ell}, \tilde{R}]}(k) \mathbf{B}_n(\{S, R, Y, I\}, \{(\ell, S^*)J^*, R^*, Y^*, I^*\}, \{\tilde{\ell}, \tilde{R}, \tilde{Y}, \tilde{I}\}) \right|^2, \end{aligned} \quad (6)$$

where the $\mathbf{B}_n$ are the RMEs. These can also be expressed in terms of RMEs of spin-flavor operators. For operators B_n which do not involve $SU(3)$ symmetry breaking, one obtains:

$$\begin{aligned} &\mathbf{B}_n(\{S, R, Y, I\}, \{(\ell, S^*)J^*, R^*, Y^*, I^*\}, \{\tilde{\ell}, \tilde{R}, \tilde{Y}, \tilde{I}\}) \\ &= (-1)^{j_n + J^* + \ell + S} \frac{\hat{j}^* \hat{\ell}}{\sqrt{\dim R}} \left\{ \begin{matrix} J^* & S^* & \ell \\ j_n & \tilde{\ell} & S \end{matrix} \right\} \sum_\gamma \left(\begin{matrix} R^* & \tilde{R} \\ Y^* & I^* \end{matrix} \parallel \begin{matrix} R \\ Y & I \end{matrix} \right)_\gamma \\ &\times \langle S, R \parallel \mathcal{G}_n^{[j_n, \tilde{R}]} \parallel S^*, R^* \rangle_\gamma, \end{aligned} \quad (7)$$

where, with obvious notation, there appear a $SU(2)$ 6-j symbol, $SU(3)$ isoscalar factors, and the reduced matrix element of the corresponding spin-flavor operator. γ labels the possible multiplicities for coupling the product of representations $R^* \otimes \tilde{R}$ to R in $SU(3)$.

Throughout, the $SU(2)$ conventions are those of Edmonds [15], and the $SU(3)$ conventions are those established in the article by Hecht [16].

In the case of $SU(3)$ symmetry breaking operators we proceed as follows. The symmetry breaking is due to the mass difference between the s quark and the u and d quarks. To first order in the quark masses, this symmetry breaking is implemented at the level of the spin-flavor operators according to:

$$\mathcal{G}_{n,\gamma_n}^{[j_n,\tilde{R}]} = \frac{1}{\Lambda} [\mathcal{M}_q^8 \mathcal{G}_n^{[j_n,R_n]}]_{\gamma_n}^{\tilde{R}}, \quad (8)$$

where $\mathcal{M}_q^8$ is the octet component of the quark mass matrix, Λ an arbitrary scale to render operators dimensionless, and γ_n indicates the particular coupling $\mathbf{8} \otimes R_n$ to $\tilde{R}$. For the case of interest here, $\tilde{R} = \mathbf{8}$, and therefore R_n can be $\mathbf{1}$, $\mathbf{8}$, $\mathbf{10}$, $\bar{\mathbf{10}}$ or $\mathbf{27}$. These possibilities give rise to a large proliferation of $SU(3)$ breaking operators at 2-body level. Fortunately for the decays of 56-plet baryons, those 2-body operators are higher order in $1/N_c$, and thus contribute corrections to the leading order decay amplitudes of order $(m_s - m_{u,d})/N_c$, which are beyond the accuracy of the present analysis. This is quite different in the case of the $\mathbf{70}$ -plet, as it will be discussed elsewhere. The RME of an $SU(3)$ breaking operator will then be given by:

$$\begin{aligned} & \mathbf{B}_n(\{S, R, Y, I\}, \{(\ell, S^*)J^*, R^*, Y^*, I^*\}, \{\tilde{\ell}, \tilde{R}, \tilde{Y}, \tilde{I}\}) \\ &= (-1)^{j_n+J^*+\ell+S} \frac{\hat{j}^* \hat{\ell}}{\sqrt{\dim R}} \left\{ \begin{matrix} J^* & S^* & \ell \\ j_n & \tilde{\ell} & S \end{matrix} \right\} \left(\begin{matrix} \mathbf{8} & R_n \\ 0 & 0 \end{matrix} \parallel \begin{matrix} \tilde{R} \\ \tilde{Y} & \tilde{I} \end{matrix} \right)_{\gamma_n} \sum_{\gamma} \left(\begin{matrix} R^* & R_n \\ Y^* & I^* \end{matrix} \parallel \begin{matrix} R \\ Y & I \end{matrix} \right)_{\gamma} \\ & \times \langle S, R \| \mathcal{G}_n^{[j_n,R_n]} \| S^*, R^* \rangle_{\gamma}, \end{aligned} \quad (9)$$

where γ_n indicates the $SU(3)$ recoupling corresponding to the operator $\mathbf{B}_n$. With the definition of effective operators used in this work, all coefficients $C_n^{[\ell,\tilde{R}]}(k)$ in Eqn. (4) are of zeroth order in N_c . In addition, we will normalize the operators in such a way that the *natural* size of all coefficients would be the same. The leading order of the decay amplitude is in fact N_c^0 [4]. At this point it is important to comment on the momentum dependence of the coefficients. The spin-flavor breakings in the masses, of both excited and ground state baryons, give rise to different values of the momenta k . In the 56-plets the mass splittings are however $\mathcal{O}(1/N_c)$ or order $m_s - m_{u,d}$, and therefore those effects on k are taken into account automatically in the expansion we are performing. Thus, we can ignore any momentum dependence of the coefficients $C_n^{[\ell,\tilde{R}]}$ as such effects are absorbed into the operators.

C. Operator basis

The construction of a basis of spin-flavor operators follows similar lines as in previous works on baryon decays [10]. The main difference between the $SU(4)$ and $SU(6)$ cases is that in the first case the matrix elements of a given spin-flavor operator are all of the same order in $1/N_c$, while in the latter case they are not. For instance, the emission of K mesons is suppressed by a factor $1/\sqrt{N_c}$ with respect to the emission of pions. This is due to the fact that in the $1/N_c$ counting baryons are considered to have strangeness $\mathcal{O}(N_c^0)$. Note that this does not represent any $SU(3)$ symmetry breaking. Due to this issue, we classify operators which are $SU(3)$ preserving according to the order in $1/N_c$ at which they contribute in pion emission. The order at which an n -body operator contributes to the transition amplitude is given by $\nu = n - \kappa$, where κ is its degree of coherence factor determined by the number of coherent generators that appear in the product building the operator. Details on the derivation of this power counting can be found in [4].

In the decays of the positive parity baryons we have odd partial waves, and we analyze the P and F waves. The $SU(3)$ preserving operators for the emission of the pseudoscalar octet are as follows. There is only one 1-body operator, namely G_{ia}/N_c . This operator is leading order and gives contributions to the decay widths into pions at $\mathcal{O}(N_c^0)$. Next we have 2-body operators, which are built by multiplying a pair of generators such that one can couple them to $j = 1$ or 2 and to $R = 8$. One can construct six such products, which upon using the $SU(6)$ reduction formulas [17] and keeping only up to operators $\mathcal{O}(1/N_c)$, only two operators are left, namely $1/N_c^2 \{S, G\}^{[j=1,8]}$ and $1/N_c^2 \{S, G\}^{[j=2,8]}$. One can show that 3-body operators contribute to amplitudes at $\mathcal{O}(1/N_c^2)$, which is beyond the order of this work.

To the order we are working, $SU(3)$ breaking is described by two 1-body operators, namely $f_{8ab}G_{ib}/N_c$ and $d_{8ab}G_{ib}/N_c$. The first operator does not contribute to decays involving a π meson, and the second one can be redefined by adding to it a non-breaking piece in such a way that it does not contribute to matrix elements involving a π meson. Due to the small number of empirically known decay channels into K -mesons, we will not be able to fit the effects of both operators. For this reason only the operator $d_{8ab}G_{ib}/N_c$ will be utilized.

The $1/N_c$ counting for the reduced matrix elements of operators is finally given as follows: i) for $SU(3)$ preserving LO operators the amplitudes $B^* \rightarrow \pi B$ are $\mathcal{O}(N_c^0)$ and $B^* \rightarrow KB$ or

$B^* \rightarrow \bar{K}B$ are $\mathcal{O}(N_c^{-1/2})$; NLO amplitudes are simply an extra factor $1/N_c$ in all cases. ii) for the $SU(3)$ breaking operator, once re-defined to have vanishing contributions to amplitudes with pion emission, it contributes at $\mathcal{O}(m_s N_c^{-1/2})$.

Table I summarizes the set of spin-flavor operators relevant to the decays of the **56**-plet baryons as needed in this work.

TABLE I: Spin-flavor operators

Operator	Order
$\mathcal{G}_1 \equiv \frac{1}{N_c} G$	$\mathcal{O}(N_c^0)$
$\mathcal{G}_2 \equiv \frac{1}{N_c^2} \{S, G\}^{[j=1]}$	$\mathcal{O}(1/N_c)$
$\mathcal{G}_3 \equiv \frac{1}{N_c^2} \{S, G\}^{[j=2]}$	$\mathcal{O}(1/N_c)$
$\mathcal{G}_{SB} \equiv \frac{1}{N_c} (d_{8ab} - \delta_{ab}/\sqrt{3}) G_{ib}$	$\mathcal{O}((m_s - m_{u,d})/\sqrt{N_c})$

The RMEs of the spin-flavor operators between the **56**-plets can be expressed in terms of the $SU(3)$ reduced matrix elements of G and of $\{S, G\}$. The RMEs of G are given in Table II, and the ones of $\{S, G\}^{[j]}$ are related to those of G by the following formula:

$$\begin{aligned} \langle R' || \{S, G\}^{[j, \mathbf{8}]} || R \rangle_\gamma &= (-1)^{S+S'} \hat{j} \langle R' || G || R \rangle_\gamma \\ &\times \left((-1)^j \hat{S}' \sqrt{S'(S'+1)} \begin{Bmatrix} S' & S' & 1 \\ 1 & j & S \end{Bmatrix} + \hat{S} \sqrt{S(S+1)} \begin{Bmatrix} S & S & 1 \\ 1 & j & S' \end{Bmatrix} \right), \end{aligned} \quad (10)$$

where, as mentioned earlier, S and S' in the **56**-plet are determined by the respective $SU(3)$ representation R and R' . In this work we only need the cases $j = 1, 2$.

Notice that the $1/N_c$ power counting is not given solely by the reduced matrix elements of the baryonic operator, but also involves the N_c dependence of the isoscalar factors. For this reason, we give in an appendix the expressions for general N_c of those isoscalar factors as needed in the present calculations.

TABLE II: $SU(3)$ RMEs of the spin-flavor operator G between $SU(3)$ multiplets in the $SU(6)$ **56**-plet. $\langle \mathbf{10} \| G \| \mathbf{8} \rangle = \langle \mathbf{8} \| G \| \mathbf{10} \rangle$, and $\langle \mathbf{10} \| G \| \mathbf{10} \rangle_{\gamma=2} = 0$ when $N_c = 3$.

RME
$\langle \mathbf{8} \ G \ \mathbf{8} \rangle_{\gamma=1} = \sqrt{\frac{(N_c+1)(N_c+5)}{2}}$
$\langle \mathbf{8} \ G \ \mathbf{8} \rangle_{\gamma=2} = -\sqrt{\frac{(N_c^2-1)(N_c+5)(N_c+7)}{32}}$
$\langle \mathbf{8} \ G \ \mathbf{10} \rangle = -\frac{1}{4} \sqrt{(N_c^2-1)(N_c+5)(N_c+7)}$
$\langle \mathbf{10} \ G \ \mathbf{10} \rangle_{\gamma=1} = (N_c+3) \sqrt{\frac{10(N_c-1)(N_c+7)}{(45+N_c(N_c+6))}}$
$\langle \mathbf{10} \ G \ \mathbf{10} \rangle_{\gamma=2} = -\frac{1}{\sqrt{8}} \sqrt{\frac{(N_c^2-1)(N_c-3)(N_c+5)(N_c+7)(N_c+9)}{(45+N_c(N_c+6))}}$

III. RESULTS

In this section we present and analyze the fits to partial widths. In general, the LO fits correspond to the results one would obtain from coupling the pseudoscalar meson to the excited quark, similar to the framework of the chiral quark model, and also give results which are similar to those in quark models in general [18]. Our analysis below shows the shortcomings of the LO approximation, and the need for the NLO contributions which are given by 2-body effects in the decay amplitudes.

A. Results for the $[\mathbf{56}, 0^+]$ decays

The basis of operators in this case contains the LO $\mathcal{G}_1$ operator, the 2-body NLO $\mathcal{G}_2$ operator and the $SU(3)$ breaking $\mathcal{G}_{SB}$ operator. We normalize these operators according to $\mathcal{G}_n \rightarrow \alpha_n \mathcal{G}_n$, with $\alpha_1 = 6/5$, $\alpha_2 = 1/\sqrt{2}$, $\alpha_{SB} = 2\sqrt{3}/5$, so that the corresponding matrix elements have natural magnitude. In this multiplet, the experimentally established states to be used in the analysis along with their respective star rating by the Particle Data Group are: the Roper resonance $N(1440)$ (****), the $\Lambda(1600)$ (***) and the $\Sigma(1660)$ (***), all of them in the **8**, and only the $\Delta(1600)$ (***) in the **10**.

The results of the fits are shown in Table III.

TABLE III: Fit parameters and partial widths in MeV for P-wave decays of the $[56, 0^+]$ baryons. Listed are only the decay widths of those states whose mass is currently experimentally known. The Error row indicates the theoretical error of the NLO fit.

	χ^2_{dof} dof			C_1	C_2	B_1
LO	2.2	6		8.8(1.3)	—	—
LO*	1.7	5		6.7(0.8)	—	4.2(3.2)
NLO	0.8	4		8.1(0.9)	-8.0(2.6)	2.7(4.6)
NLO*	0.9	4		8.7(1.0)	-9.3(4.3)	8.2(3.8)

	$N_{\frac{1}{2}}(1440)$		$\Lambda_{\frac{1}{2}}(1600)$			$\Sigma_{\frac{1}{2}}(1660)$				$\Delta_{\frac{3}{2}}(1600)$	
	πN	$\pi \Delta$	$\bar{K} N$	$\pi \Sigma$	$\pi \Sigma^*$	$\bar{K} N$	$\pi \Lambda$	$\pi \Sigma$	$\pi \Sigma^*$	πN	$\pi \Delta$
LO	90	12	0	73	24	0	41	58	6	93	55
LO*	148	13	36	57	19	1.6	32	61	6.5	97	90
NLO	214	65	38	111	95	5	63	88	25	56	131
NLO*	250	45	36	97	66	1.6	55	103	23	51	153
Error	49	17	28	25	27	4	14	20	7	44	30
Exp	211(88)	81(35)	34(25)	53(51)	—	24(20)	—	—	—	61(32)	193(76)

In all the leading order fits considered here denoted as LO, the decays with K or $\bar{K}$ mesons in final state are not included because their widths start at $\mathcal{O}(1/N_c)$. The LO fit involves only one operator, and has been carried out by assigning error bars to the input partial widths, obtained from the PDG, which are 30% or the experimental value if it is larger than 30%. This is to test whether or not the LO is consistent. In the case of the $[56, 0^+]$ decays, it turns out that the LO fit is not consistent. This was well known from previous work in $SU(4)$ [10], where the non-strange channels were shown to be poorly described. The problem is best illustrated by the two ratios which are parameter-free at LO : i) $\Gamma(N(1440) \rightarrow \pi N)/\Gamma(N(1440) \rightarrow \pi \Delta) = 7.5$ (Th) vs 2.6 ± 2.1 (Exp), and ii)

$\Gamma(\Delta(1600) \rightarrow \pi N)/\Gamma(\Delta(1600) \rightarrow \pi\Delta) = 1.7$ (Th) vs 0.3 ± 0.2 (Exp). Thus, spin-flavor symmetry is badly broken in these decays; this is the most notable inconsistency of the $1/N_c$ expansion, and points to the particular nature of the Roper multiplet. At LO, the decay rate $\Lambda(1600) \rightarrow \pi\Sigma$ is remarkably close to the observed one, but not so well described at NLO. As in the other fits in this work, we include a fit denoted by LO*, in which we do not expand the matrix elements of the LO operator and also include the $SU(3)$ breaking operator. The reason for doing this is that some of the reduced matrix elements and isoscalar factors are poorly represented by the first term in the expansion. This fit gives significant improvement over the LO one, shows that the $SU(3)$ breaking operator is nearly irrelevant, and it further exposes the problematic decay channels, which are the $\pi\Delta$ channels. Therefore, the NLO corrections must be very important, and they are provided by the single 2-body operator. The inclusion of these NLO corrections leads to a consistent fit. One issue is that the $\Lambda(1600) \rightarrow \pi\Sigma$ channel is not as well described as at LO. The main NLO effect is to enhance the $N(1440) \rightarrow \pi\Delta$ width to become consistent with the experimental one. One point to be noted is that the coefficient of the NLO operator is of natural size, although this gives NLO effects in some of the widths which are large. In the NLO* we do not expand the matrix elements. The main effect is to change the coefficients C_2 and B_1 , and the reason is that the NLO operator does not contribute to the decays into K mesons in the NLO fit, while such contributions do occur in NLO*.

For the $SU(3)$ symmetry breaking we find, for all the fits in this work, two consistent solutions. In all cases, one of the solutions has a coefficient which is unnaturally large, and we assume that such a solution is unacceptable. The only way to confirm this assumption is by determining empirically other widths sensitive to the symmetry breaking. In the case of the $[56, 0^+]$, the role of the $SU(3)$ breaking operator cannot be established: it is irrelevant in the NLO fit and relevant in NLO* fit. The reason for this ambiguity is that only two decay widths are involved, and both have large error bars. Finally, the fits permit us to give rough predictions for the unknown channels.

B. Results for the $[56, 2^+]$ decays

The basis of operators in this case involves the LO $\mathcal{G}_1$ operator, the NLO $\mathcal{G}_2$ and $\mathcal{G}_3$ 2-body operators and the $SU(3)$ breaking $\mathcal{G}_{SB}$ operator. In this case the normalization

factors are taken to be as follows: i) for the P-wave decays $\alpha_1 = 4/\sqrt{7}$, $\alpha_2 = 1$, $\alpha_3 = \sqrt{3}/2$, $\alpha_{SB} = 4/\sqrt{21}$, and ii) for F-wave decays $\alpha_1 = 3/\sqrt{8}$, $\alpha_2 = 1/2$, $\alpha_3 = \sqrt{3/10}$, $\alpha_{SB} = \sqrt{3/8}$.

The experimentally established states in the $[\mathbf{56}, 2^+]$ to be used in the analysis are the following ones. In the octets: i) $J = 3/2$: $N(1720)$ (****) and $\Lambda(1890)$ (****), ii) $J = 5/2$: $N(1680)$ (****), $\Lambda(1820)$ (****) and $\Sigma(1915)$ (****). In the decuplets: i) $J = 1/2$: $\Delta(1910)$ (****), ii) $J = 3/2$: $\Delta(1920)$ (***), iii) $J = 5/2$: $\Delta(1905)$ (****), iv) $J = 7/2$: $\Delta(1950)$ (****) and $\Sigma_{10}(2030)$ (****).

Let us discuss the results for the P- and F-wave decays separately.

- P-wave decays: All states with $J < 7/2$ have a P-wave decay channel. At LO there is only one operator and the fit is consistent as previously observed [10], giving $\chi^2_{dof} \sim 1.6$. The main inconsistency appears in the decay $\Lambda(1890) \rightarrow \pi\Sigma$, and this problem gets improved but not solved at NLO. We also show a LO* fit as described earlier. This fit gives further improvement over the LO one, and shows the $SU(3)$ breaking operator to be insignificant. Since at LO* one already has a very good description, one expects the NLO corrections to be small, which is indeed the case. As it occurs for the $[\mathbf{56}, 0^+]$, the breaking of $SU(3)$ cannot be accurately determined because there are only two widths with large error bars which are affected. We note some rearrangement in going from NLO to NLO*, in the first case the breaking being rather large, and in the second one is marginally relevant. Thus, the $1/N_c$ expansion works particularly well in P-wave decays, but the $SU(3)$ breaking cannot be clearly determined.

We have included predictions for the decay rates into η mesons. Although these widths are suppressed, starting at $\mathcal{O}(1/N_c^2)$, it is interesting to have an idea of their magnitude. The fact that no such decays for states assigned to $\mathbf{56}$ -plets are experimentally observed is a strong indication of the correctness of such assignments. For $\mathbf{70}$ -plet baryons the η modes are not suppressed and start at $\mathcal{O}(N_c^0)$ [9].

- F-wave decays: All states with $J > 1/2$ have F-wave decays. Here the LO results are problematic as shown by its $\chi^2_{dof} = 3.7$. This is a one-parameter fit to twelve data. The main issue is that the widths for $N(1680) \rightarrow \pi N$ and $\Delta(1905) \rightarrow \pi N$ resulting from the fit are too small. The situation for these decays at LO is substantially worse than in the case of the analysis in $SU(4)$ [10]. This is due to the Λ and Σ decays that are

TABLE IV: Fit parameters and partial widths in MeV for P-wave decays of the $[56, 2^+]$ baryons. Listed are only the decay widths of those states whose mass is currently experimentally known.

	χ^2_{dof}	dof	C_1	C_2	C_3	B_1
LO	1.6	6	2.8(0.4)	—	—	—
LO*	0.7	5	2.2(0.3)	—	—	-1.2(1.6)
NLO	0.9	3	1.9(0.2)	0.2(1.2)	0.4(2.8)	-4.2(2.2)
NLO*	1.2	3	2.2(0.3)	0.2(1.0)	0.5(2.3)	-1.3(1.6)

	$N_{\frac{3}{2}}(1720)$					$\Lambda_{\frac{3}{2}}(1890)$					$N_{\frac{5}{2}}(1680)$	$\Lambda_{\frac{5}{2}}(1820)$	$\Sigma_{\frac{5}{2}}(1915)$	
	πN	ηN	$K\Sigma$	$K\Lambda$	$\pi\Delta$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$	$\pi\Sigma^*$	$\pi\Delta$	$\pi\Sigma^*$	$\bar{K}\Delta$	$\pi\Sigma^*$
LO	21	0	0	0	1.7	0	19	0	0	1.8	8.1	7.7	0	2.2
LO*	36	1	0.1	3	2	38	16	4	0.3	1.6	9.2	6.6	9.6	2.5
NLO	27	2.4	0.2	2.9	2.5	38	16	9.8	1	1.8	10	6.1	7.2	1.8
NLO*	36	1	0.1	2.9	2.4	38	16	4	0.3	2	8.1	6	8.8	2.2
Error	6	2	0.1	2	3	21	4	6	0.6	3	5	3	4	1
Exp	34(16)	—	—	18(17)	—	36(22)	8.5(6.4)	—	—	—	13(5.3)	—	—	—

	$\Delta_{\frac{1}{2}}(1910)$					$\Delta_{\frac{3}{2}}(1920)$					$\Delta_{\frac{5}{2}}(1905)$		
	πN	$K\Sigma$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$	πN	$K\Sigma$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$
LO	31	0	3.5	0	0	17	0	13	0	0	14.5	0	0
LO*	35	13	6	0.4	0	19	8.6	23	2.2	0.8	26	1.7	0.1
NLO	45	9.9	5.7	1	0	22	6.5	19	5.2	0.6	18	4	0.1
NLO*	41	15	9	0.6	0.1	18	8.3	28	2.5	0.9	22	1.5	0.1
Error	25	6	8	0.6	—	13	4	15	3	0.3	11	3	0.03
Exp	52(20)	—	—	—	—	28(19)	—	—	—	—	—	—	—

included in the fits of the present work. The LO* fit, for which we use the experimental errors, is substantially better. This is because not expanding the matrix elements leads to an important enhancement of the rates $N(1680) \rightarrow \pi N$ and $\Delta(1905) \rightarrow \pi N$. We note that these channels are the main source of the large χ^2 at LO. Clearly the NLO corrections must be important. The NLO fit has a minor problem of consistency with the bound $\Gamma_{\text{F-wave}}(N(1680) \rightarrow \pi\Delta) < 2.6 \text{ MeV}$, violating the bound by a relatively small amount. The main problem is that the decays $\Lambda(1820) \rightarrow KN$ and $\pi\Sigma$ cannot be simultaneously fitted. The fit gives $\Gamma(\Lambda(1820) \rightarrow KN) < \Gamma(\Lambda(1820) \rightarrow \pi\Sigma)$, which is opposite to the empirical ordering. The problem is partly resolved in the fit NLO*, for which the inconsistency remains in the decay $\Lambda(1820) \rightarrow \pi\Sigma$. We have therefore carried out the fits without this channel, because its inclusion gives a large χ^2 , while the fit is consistent without it. Since there are no Λ baryons in the mass proximity of $\Lambda(1820)$ with which it could mix, the inconsistency should be resolved either by higher order terms in the $1/N_c$ expansion, or by a better empirical value of the width, whose measurement dates back to the 1970s. The $SU(3)$ breaking effects are determined by three widths, and turn out to be within the expectations. Note that the experimental errors for the F-wave widths are significantly smaller than for the P-waves, and thus the predictions for the unknown widths should be better than for the P-wave decays.

IV. CONCLUSIONS

This work implemented the $1/N_c$ expansion for the decays of positive parity **56**-plet baryons, and analyzed the empirically known partial decay widths to $\mathcal{O}(1/N_c)$ and first order in $SU(3)$ symmetry breaking. As it had been well established in previous work within $SU(4)$, the decays of the $[\mathbf{56}, 0^+]$ baryons are poorly described at LO, requiring some large NLO corrections by a 2-body operator, which is an indication of the special nature of this multiplet. The coefficient of the NLO operator is, however, within natural magnitude. On the other hand, the decays of the $[\mathbf{56}, 2^+]$ baryons are well described, showing for both P- and F-wave decays natural size contributions by the NLO operators. We have noticed that

TABLE V: Fit parameters and partial widths in MeV for F-wave decays of the $[56, 2^+]$ baryons. Listed are only the decay widths of those states whose mass is currently experimentally known.

	χ^2_{dof}	dof	C_1	C_2	C_3	B_1
LO	3.7	11	0.44(0.03)	—	—	—
LO*	3.4	10	0.41(0.02)	—	—	-0.20(0.07)
NLO	0.7	6	0.52(0.02)	-0.76(0.10)	0.42(0.18)	0.26(0.08)
NLO*	0.83	7	0.51(0.02)	-0.32(0.06)	0.40(0.13)	-0.11(0.07)

	$N_{\frac{3}{2}}(1720)$	$\Lambda_{\frac{3}{2}}(1890)$	$\Delta_{\frac{3}{2}}(1920)$			$N_{\frac{5}{2}}(1680)$				$\Lambda_{\frac{5}{2}}(1820)$				
	$\pi\Delta$	$\pi\Sigma^*$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$	πN	$\pi\Delta$	ηN	$K\Lambda$	$\bar{K}N$	$K\Xi$	$\pi\Sigma$	$\eta\Lambda$	$\pi\Sigma^*$
LO	4.5	5.6	8.7	0	0	22	1.1	0	0	0	0	12	0	1
LO*	8	7	22	0.3	0	55	1.8	0.2	0.1	42	0	14	0.6	1.3
NLO	27	22	59	0.1	0	84	6	0.1	0	12	0	29	0.2	4
NLO*	13	12	77	0.8	0	83	2.8	0.3	0.1	52	0	21	0.7	2
Error	3	3	13	0.04	—	6	0.8	0.03	—	3	—	2	0.1	0.6
Exp	—	—	—	—	—	88(8)	—	—	—	48(7)	—	9(3)	—	—

	$\Sigma_{\frac{5}{2}}(1915)$							$\Delta_{\frac{5}{2}}(1905)$				
	$\bar{K}N$	$\pi\Sigma$	$\eta\Sigma$	$K\Xi$	$\pi\Lambda$	$\bar{K}\Delta$	$\pi\Sigma^*$	πN	$\eta\Delta$	$\pi\Delta$	$K\Sigma$	$K\Sigma^*$
LO	0	18	0	0	15	0	0.6	13	0	12	0	0
LO*	3.6	29	1	0.5	18	2	1	20	0.2	30	0.7	0
NLO	3	44	0.4	0.1	36	0.4	2	45	0.1	58	0.1	0
NLO*	4.4	43	1.1	0.7	27	2.6	1.6	44	0.3	65	1.2	0
Error	0.7	3	0.1	0.03	3	0.1	0.4	11	0.02	7	0.03	—
Exp	12(7)	—	—	—	—	—	—	40(13)	—	—	—	—

	$\Delta_{\frac{7}{2}}(1950)$					$\Sigma_{\frac{7}{2}}(2030)$							
	πN	$\eta\Delta$	$\pi\Delta$	$K\Sigma$	$K\Sigma^*$	$\bar{K}N$	$\pi\Lambda$	$\pi\Sigma$	$\pi\Sigma^*$	$\bar{K}\Delta$	$\eta\Sigma$	$K\Xi^*$	$K\Xi$
LO	72	0	18	0	0	0	41	14	11	0	0	0	0
LO*	115	0.6	45	6	0	43	50	22	18	16	8	0	1.8
NLO	123	0.2	48	0.9	0	36	35	12	17	3	3	0	0.5
NLO*	92	0.4	36	4.5	0	31	40	18	14	11	8	0	1.2
Error	21	0.1	9	0.2	—	8	8	3	4	0.8	1	—	0.1
Exp	114(25)	—	—	—	—	35(7)	35(7)	13(5)	18(9)	—	—	—	—

for a considerable number of matrix elements better fits result if they are not expanded in $1/N_c$, in particular in the case of the the F-waves. For this reason we included fits LO* and NLO* to see that effect.

One point to note is that in general the empirical widths have rather large error bars, considerably larger than the 10% which would allow for an accurate determination of the NLO effects. This impacts on the predictions one can deduce from the present analysis; in the case of the $[\mathbf{56}, 0^+]$ decays, the predictions should be taken with caution, as in this case the $1/N_c$ expansion seems to be poorly converging. In the case of the $[\mathbf{56}, 2^+]$ decays, the predictions for P-wave widths are within errors of the order of 50%, and for F-waves are more accurate and typically with an accuracy of 20%. Using the elements given in the paper, it is straightforward to make predictions for the widths of the missing states in the multiplets.

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V. APPENDIX: $SU(3)$ ISOSCALAR FACTORS

This appendix gives the isoscalar factors which appear in the matrix elements needed in this work, which correspond to the emission of mesons belonging to an octet of $SU(3)$. They are given for the irreducible representations of $SU(3)$ as needed in this work for generic N_c . The representations are given in terms of the two labels defining the Young tableau, namely (p, q) , where $p + 2q = N_c$. The isoscalar factors needed for the decays into the octet of mesons are denoted by:

$$\left(\begin{array}{cc|cc} (p, q) & (1, 1) & & (p', q') \\ Y_1 & I_1 & Y_2 & I_2 \end{array} \parallel \begin{array}{cc} Y & I \end{array} \right)_\gamma. \quad (11)$$

Here (p', q') is the representation of the final ground state baryon and (p, q) of the initial excited baryon. For baryons, the correspondences between multiplets for generic odd N_c and $N_c = 3$ are as follows: $(p = 1, q = \frac{N_c-1}{2}) \rightarrow \mathbf{8}$, and $(p = 3, q = \frac{N_c-3}{2}) \rightarrow \mathbf{10}$. Table VI displays these correspondences more explicitly.

TABLE VI: Representation correspondences for arbitrary odd N_c .

8 Baryons		10 Baryons		Mesons	
$(p, q) = (1, \frac{N_c-1}{2})$		$(p, q) = (3, \frac{N_c-3}{2})$		$(p, q) = (1, 1)$	
State	(Y, I)	State	(Y, I)	State	(Y, I)
N	$(\frac{N_c}{3}, \frac{1}{2})$	Δ	$(\frac{N_c}{3}, \frac{3}{2})$	π	$(0, 1)$
Σ	$(\frac{N_c-3}{3}, 1)$	Σ^*	$(\frac{N_c-3}{3}, 1)$	η	$(0, 0)$
Λ	$(\frac{N_c-3}{3}, 0)$	Ξ^*	$(\frac{N_c-6}{3}, \frac{1}{2})$	K	$(1, \frac{1}{2})$
Ξ	$(\frac{N_c-6}{3}, \frac{1}{2})$	Ω	$(\frac{N_c-9}{3}, 0)$	$\bar{K}$	$(-1, \frac{1}{2})$

In the following we give the isoscalar factors as needed for the calculations carried out in this work.

TABLE VII: Isoscalar factors for $\mathbf{8} \rightarrow \mathbf{8}$ decays. The listed values should be multiplied by $f_1 = \frac{1}{(N_c+3)}$ and $f_2 = \frac{1}{(N_c+3)}\sqrt{\frac{(N_c-1)}{(N_c+7)}}$ to obtain the actual isoscalar factors for $\gamma = 1$ and $\gamma = 2$, respectively.

		N				Λ			
		ηN	πN	$K\Sigma$	$K\Lambda$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\gamma = 1$	N_c	3	$\sqrt{3(N_c-1)}$	$\sqrt{3(N_c+3)}$	$-\sqrt{\frac{3(N_c+3)}{2}}$	0	N_c-3	$3\sqrt{\frac{N_c-1}{2}}$	
$\gamma = 2$	3	$-(N_c+6)$	$\frac{N_c+15}{\sqrt{3(N_c-1)}}$	$-\sqrt{3(N_c+3)}$	$\sqrt{\frac{3(N_c+3)}{2}}$	$-\sqrt{\frac{(N_c+3)^3}{3(N_c-1)}}$	6	$\frac{9-N_c}{\sqrt{2(N_c-1)}}$	

		Σ				
		$\bar{K}N$	$\eta\Sigma$	$\pi\Sigma$	$\pi\Lambda$	$K\Xi$
$\gamma = 1$	$3\sqrt{\frac{N_c-1}{2}}$	N_c-3	$2\sqrt{6}$	0	$\sqrt{\frac{3(N_c+3)}{2}}$	
$\gamma = 2$	$\frac{N_c+15}{\sqrt{2(N_c-1)}}$	$\frac{2(N_c-9)}{N_c-1}$	$-\sqrt{\frac{2}{3}}\frac{(N_c-3)(N_c+7)}{N_c-1}$	$\sqrt{\frac{(N_c+3)^3}{N_c-1}}$	$-\frac{5N_c+3}{N_c-1}\sqrt{\frac{N_c+3}{6}}$	

		Ξ			
		$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\eta\Xi$	$\pi\Xi$
$\gamma = 1$		$-\sqrt{N_c+3}$	$3\sqrt{N_c-1}$	N_c-6	3
$\gamma = 2$		$\frac{5N_c+3}{3(N_c-1)}\sqrt{N_c+3}$	$\frac{9-N_c}{\sqrt{N_c-1}}$	$\frac{7N_c-15}{N_c-1}$	$\frac{N_c^2+3N_c+36}{3(N_c-1)}$

TABLE VIII: Isoscalar factors for $\mathbf{10} \rightarrow \mathbf{10}$ decays. The listed values should be multiplied by $f_1 = \frac{1}{\sqrt{45+N_c(N_c+6)}}$ and $f_2 = \sqrt{\frac{5(N_c-3)(N_c+5)}{(N_c+1)(N_c+9)(45+N_c(N_c+6))}}$ to obtain the actual isoscalar factors for $\gamma = 1$ and $\gamma = 2$, respectively.

Δ				Σ^*			
$\eta\Delta$	$\pi\Delta$	$K\Sigma^*$	$\bar{K}\Delta$	$\eta\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$	
$\gamma = 1$	N_c	$3\sqrt{5}$	$\sqrt{3(N_c+5)}$	$-\frac{3\sqrt{N_c+5}}{2}$	N_c-3	$2\sqrt{6}$	$\sqrt{6(N_c+3)}$
$\gamma = 2$	3	$-\frac{N_c+6}{\sqrt{5}}$	$\frac{3-N_c}{\sqrt{3(N_c+5)}}$	$\frac{N_c-3}{2\sqrt{N_c+5}}$	$\frac{4(N_c+3)}{N_c+5}$	$-\frac{N_c^2+10N_c+33}{\sqrt{6(N_c+5)}}$	$\frac{3-N_c}{N_c+5}\sqrt{\frac{2(N_c+3)}{3}}$
Ξ^*				Ω			
$\bar{K}\Sigma^*$	$\eta\Xi^*$	$\pi\Xi^*$	$K\Omega$	$\bar{K}\Xi^*$	$\eta\Omega$		
$\gamma = 1$	$-2\sqrt{N_c+3}$	N_c-6	3	$3\sqrt{N_c+1}$	$-3\sqrt{\frac{N_c+1}{2}}$	N_c-9	
$\gamma = 2$	$\frac{2(N_c-3)\sqrt{N_c+3}}{3(N_c+5)}$	$\frac{5N_c+9}{N_c+5}$	$-\frac{N_c^2+9N_c+36}{3(N_c+5)}$	$\frac{(3-N_c)\sqrt{N_c+1}}{N_c+5}$	$\frac{N_c-3}{N_c+5}\sqrt{\frac{N_c+1}{2}}$	$\frac{6(N_c+1)}{N_c+5}$	

TABLE IX: Isoscalar factors for $\mathbf{8} \rightarrow \mathbf{10}$ decays. The listed values should be multiplied by $f = \frac{\sqrt{2}}{\sqrt{(N_c+1)(N_c+5)}}$ to obtain the actual isoscalar factors.

N				Λ			
$\pi\Delta$		$K\Sigma^*$		$\pi\Sigma^*$		$K\Xi^*$	
$-\sqrt{\frac{(N_c-1)(N_c+5)}{2}}$		$-2\sqrt{\frac{N_c-1}{3}}$		$-\sqrt{\frac{(N_c+3)(N_c-1)}{3}}$		$-\sqrt{2(N_c-1)}$	

Σ				Ξ			
$\bar{K}\Delta$	$\eta\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$	$\eta\Xi^*$	$\pi\Xi^*$	$\bar{K}\Sigma^*$	$K\Omega$
$\sqrt{N_c+5}$	2	$\frac{N_c+1}{\sqrt{6}}$	$\sqrt{\frac{2(N_c+3)}{3}}$	2	$\frac{2N_c}{3}$	$\frac{2\sqrt{N_c+3}}{3}$	$2\sqrt{N_c+1}$

TABLE X: Isoscalar factors for $\mathbf{10} \rightarrow \mathbf{8}$ decays. The listed values should be multiplied by $f = ((N_c+7)(N_c-1))^{-1/2}$ to obtain the actual isoscalar factors.

Δ				Ξ^*			
πN		$K\Sigma$		$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\eta\Xi$	$\pi\Xi$
$-\sqrt{(N_c-1)(N_c+5)}$		$2\sqrt{\frac{N_c+5}{3}}$		$\frac{2\sqrt{N_c+3}}{3}$	$2\sqrt{N_c-1}$	-2	$-\frac{2N_c}{3}$

Σ^*					Ω
$\eta\Sigma$	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$K\Xi$	$\bar{K}\Xi$
-2	$\sqrt{2(N_c-1)}$	$-\frac{N_c+1}{\sqrt{6}}$	$-\sqrt{(N_c+3)(N_c-1)}$	$\sqrt{\frac{2(N_c+3)}{3}}$	$\sqrt{2(N_c+1)}$

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