π^0 → γγ TO NLO IN CHPT

José L. Goity

Department of Physics, Hampton University, Hampton, VA 23668, and Thomas Jefferson National Accelerator Facility, Newport News, VA 23606.

The π^0 → γγ width is determined to next to leading order in the combined chiral and 1/N_c expansions. It is shown that corrections driven by chiral symmetry breaking produce an enhancement of about 4.5% with respect to the width calculated in terms of the chiral-limit amplitude leading to Γ_{π^0 → γγ} = 8.10 ± 0.08 MeV. This theoretical prediction will be tested via π^0 Primakoff production by the PRIMEX experiment at Jefferson Lab.

1 Introduction

In QCD there are predictions whose character is fundamental: one of them is the π^0 → γγ decay width. In the limit of exact SU_L(2) × SU_R(2) chiral symmetry, i.e. when the u- and d-quark masses vanish, the decay amplitude is predicted by the chiral anomaly induced by the EM interaction on the axial current associated with π^0 Goldstone mode. This amplitude results in the width Γ_{π^0 → γγ} = (g_{γγ}^2(M_{π^0})^3 = 7.73 eV. Although this prediction agrees well with the world averaged experimental value, this has a generous error of about 7% that prevents the observation of deviations from the chiral limit prediction resulting from explicit chiral symmetry breaking by the quark masses. This situation will change with the PRIMEX experiment at Jefferson Lab that aims at a more precise measurement in the 1 to 2% range. This talk reports on the theoretical prediction for such deviations including next to leading order corrections in the low energy expansion, and shows that indeed these deviations would be observable in that experiment.

As shown in 2, the non-vanishing u- and d-quark masses induce corrections to the π^0 → γγ amplitude that can be predicted in the framework outlined below. These corrections are of two types: i) Mixing corrections induced by isospin breaking implying that the physical π^0 is not a pure isospin eigenstate having a projection on the pure U(3) states associated with the η and the η'. They are controlled by the ratios (m_u - m_d)/m_s and N_c(m_u - m_d)/Λ_X (Λ_X is the chiral expansion scale, and N_c the number of colors). ii) corrections controlled by the ratio m/Λ_X, where m = (m_u + m_d)/2.

Since the first type of corrections turn out to be dominant, it is natural to work in a framework where the η and η' are included as active degrees of freedom. Such a framework is indeed available and consists in chiral pertur-
bation theory (ChPT) with three light flavors supplemented with the $1/N_c$ expansion\textsuperscript{3,4}. In this framework, the corrections i) start at leading order (LO), while the corrections ii) are of next-to-leading order (NLO). In this talk the calculation of the rate $\Gamma_{\pi^0 \to \gamma\gamma}$ including NLO corrections\textsuperscript{2} is outlined and the results discussed.

2 The decay amplitude

The two-photon decay amplitudes of the self-conjugate pseudoscalars are obtained from the Ward identity satisfied by the associated axial currents:

$$\partial^\mu A^a_\mu = \frac{\alpha N_c}{4\pi} (\lambda^a Q^2) F \bar{F} + \frac{\alpha_s}{4\pi} (\lambda^a) G \bar{G} + \frac{i}{2\pi} \gamma_5 \{\lambda^a, \mathcal{M}_q\} q + \cdots,$$

(1)

where $\mathcal{M}_q$ is the quark mass matrix, $eQ$ is the electric charge operator, $F \bar{F} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$, and similarly $G \bar{G} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma}$. Considering the matrix elements of Eqn. (1) between the vacuum and two-photon states and selecting the pole terms generated by the physical $\pi^0$, $\eta$ and $\eta'$, it is possible to extract the two-photon transition amplitudes for these states. At NLO these matrix elements require a NLO calculation of the masses and the decay constants, as well as contributions due to excited states and continuum that are of type ii) and thus of NLO. The NLO masses and decay constants are calculated in the mentioned framework of ChPT and $1/N_c$ expansion, while the contributions due to excited states are represented by an $\mathcal{O}(p^6)$ unnatural parity Lagrangian, the $\mathcal{O}(p^6)$ Wess-Zumino (WZ) term.

With the low energy counting in which $1/N_c$ is a quantity of order $p^2$, the NLO evaluation of masses and decay constants requires the chiral Lagrangian up to order $p^4$, which with standard notation reads:

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \cdots$$

$$\mathcal{L}^{(2)} = \frac{1}{4} F_0^2 (D_\mu U D^\mu U^\dagger) + \frac{1}{4} F_0^2 (\chi U^\dagger + \chi^\dagger U) - \frac{1}{2} M_0^2 \pi_0^2$$

$$\mathcal{L}^{(4)} = L_5 (D_\mu U^\dagger D^\mu U (\chi^\dagger U + U^\dagger \chi)) + L_8 (\chi U^\dagger \chi U^\dagger + h.c.)$$

$$+ \frac{\Lambda_1}{2} D_\mu \pi_0 D_\mu \pi_0 - \frac{iF_0 h_2}{2\sqrt{6}} \pi_0 (\chi U^\dagger - \chi^\dagger U),$$

(2)

where only terms relevant for masses and decay constants are kept. Here $U$ is the $U(3)$ matrix parametrized by the pseudoscalar nonet, where $\pi_0$ is the singlet member whose mass in the chiral limit is $M_0$ when $1/N_c$ corrections are disregarded. Note that terms such as the $L_4$ and others are not included because they are $\mathcal{O}(p^4 \times 1/N_c) \equiv \mathcal{O}(p^6)$, and thus of NNLO. The terms $\Lambda_{1,2}$ represent $1/N_c$ corrections. Since one-loop corrections with $\mathcal{L}^{(2)}$ are of order $\mathcal{O}(p^4 \times 1/N_c) \equiv \mathcal{O}(p^6)$, they ought to be neglected as well. The masses and decay constants are then extracted by calculating the two-point function of axial currents, and are given by:

$$F_{\pi^0} = F_0 + \frac{4L_5 B_0}{F_0} (m_\pi + m_\eta); \quad F_{\eta^0} = F_0 + \frac{4L_5 B_0}{F_0} (m_\eta + m_\eta); \quad F_{\eta^+} = F_0; \quad F_{\eta^-} = F_0; \quad F_{\eta^{*0}} = F_0;$$
supplemented with the \(1/N_c\) corrections. In this talk the corrections are outlined.

The electric charge operator, \(\rho_{\mu} \bar{C}^{\mu\nu\rho} C^{\mu\nu\rho}\). Considering the two-photon states and selecting 2) and thus of NLO. The NLO mentioned framework of ChPT excited states are represented. The Wess-Zumino (WZ) term.

A quantity of order \(p^2\), the NLO e chiral Lagrangian up to order

\[
\chi^T(U) - \frac{1}{2} M_0^2 \pi_0^2
\]

\[
\bar{\psi}(\chi U^\dagger \chi U^\dagger + h.c.) + \chi^T(U),
\]

\(\bar{\psi}(\chi U^\dagger \chi U^\dagger + h.c.) + \chi^T(U)\),

\[
\bar{\psi}(\chi U^\dagger \chi U^\dagger + h.c.)
\]

\(\bar{\psi}(\chi U^\dagger \chi U^\dagger + h.c.)\),

\[
\bar{\psi}(\chi U^\dagger \chi U^\dagger + h.c.)
\]

\[
F_{\delta} = F_0 + \frac{4L_5 B_0}{3F_0}(m_u + m_d + 4m_s); F_{\delta 0} = F_0(1 + \frac{\Lambda_1}{2}) + \frac{8L_5 B_0}{3F_0}(m_u + m_d + m_s);
\]

\[
F_{\delta 3} = \frac{4L_5 B_0}{3F_0}(m_u - m_d); F_{\delta 30} = \sqrt{2} F_{\delta 3}; F_{\delta 30} = \frac{\sqrt{3}L_5 B_0}{3F_0}(m_u + m_d - 2m_s);
\]

\[
M_{\delta +}^2 = 2B_0 m + \frac{32L_{5\delta}}{F_0^2} B_0^2 m^2; M_{\delta K^+}^2 = B_0 (m_u + m_s) + \frac{8L_{5\delta}}{F_0^2} B_0^2 (m_u + m_s)^2;
\]

\[
M_{\delta K^0}^2 = B_0 (m_d + m_s) + \frac{8L_{5\delta}}{F_0^2} B_0^2 (m_d + m_s)^2; M_{\delta 3}^2 = 2B_0 m + \frac{16L_{5\delta}}{F_0^2} B_0^2 (m_u + m_d)^2;
\]

\[
M_{\delta 3}^2 = \frac{2}{3} B_0 (m + 2m_s) + \frac{16L_{5\delta}}{F_0^2} B_0^2 (m_u + m_s)^2 + 4m_s^2;
\]

\[
M_{\delta 0}^2 = M_0^2 (1 - \Lambda_1) + \frac{32L_{5\delta}}{3F_0^2} B_0^2 (m_u + m_d + m_s)^2 + \frac{2}{3}(1 + \rho)B_0 (m_u + m_d + m_s);
\]

\[
M_{\delta 3}^2 = \frac{1}{3} B_0 (m_u - m_d) + \frac{16L_{5\delta}}{\sqrt{3}F_0^2} B_0^2 (m_u - m_d)^2;
\]

\[
M_{\delta 30} = -\frac{\sqrt{2}}{3}(1 + \frac{\rho}{2})B_0 (m_d - m_s) + \frac{16L_{5\delta}}{F_0^2} \sqrt{\frac{2}{3}} B_0^2 (m_u - m_d)^2;
\]

\[
M_{\delta 30} = \frac{\sqrt{2}}{3}(1 + \frac{\rho}{2})B_0 (m_u + m_d - 2m_s) + \frac{16L_{5\delta}}{F_0^2} \frac{\sqrt{2}}{3} B_0^2 (m_u^2 + m_d^2 - 2m_s^2),
\]

where \(L_{5\delta} \equiv 2L_{5} - L_{3} \) and \(\rho \equiv -\Lambda_1 + 2\Lambda_2 - 8L_{5}\).

The \(O(p^6)\) WZ term relevant here is determined in terms of a single low energy constant \(t_1\), and reads:

\[
\mathcal{L}^{(6)WZ} = -i\pi\alpha t_1 (\chi^\dagger Q^2) F \tilde{F}; u = \sqrt{U}, \chi_\pm = u^\dagger \chi u^\dagger - u^{-1} \chi u^\dagger u.
\]

The low energy constants are determined by fitting to the masses in the nonet, to \(F_{\delta}^1 + F_{K^+}^1\), and to the two-photon decay widths of the \(\eta\) and \(\eta'\). Corrections to Dashen's theorem are included for the extraction of isospin breaking by the quark masses. Unfortunately, \(t_1\) cannot be determined from the fit and its estimate via QCD sum rules is used: \(t_1 \simeq \frac{1}{m_W^2}(F_0^2 + \frac{m_W^2}{M_{\pi^0}^2})\), where \(m_V \simeq m_\pi\). The term proportional to \(t_1\) involving the excited pion pole is small and can be neglected.

3 Results and conclusions

The two-photon amplitudes of the physical states for \(N_c = 3\) finally reads:

\[
(\gamma \gamma | \pi_\delta) = -i\pi\left(1 + \frac{1}{4\pi} C_{F_{\delta}}^{-1} + \pi \frac{B_0}{F_0} t_1 \Theta_{8\delta} ((\lambda^\dagger, M_{\delta}) Q^2)\right) (\gamma \gamma | F \tilde{F} | 0),
\]

where \(\pi_3 = \pi^0, \pi_3 = \eta, \pi_0 = \eta', \) and \(C_2 = 1, C_8 = 1/\sqrt{3} \) and \(C_8 = \sqrt{8/3}, \Theta\) is the mixing matrix determined from the mass matrix in the \(\pi^\dagger - \eta - \eta'\) subspace. The decay constant matrix in this subspace is defined by: \(\langle \pi_3, p | A_\mu^\dagger | 0 \rangle = -i p_\mu F_{\pi 3}\).
where \( F_{aa} = \Theta_{aa} F_{aa} \), with \( F_{ab} \) the decay constant matrix in the basis of pure \( U(3) \) states as given in Eqn.(3). At LO only the first term in Eqn.(5) is left with \( F_{aa} = F_0 \Theta_{aa} \), where \( \Theta \) is determined from the LO mass formulas. At LO the \( \pi^0 \) width is enhanced as a result of the mixing by 4.5% from 7.73 to 8.08 eV, an effect first noted in Ref. 5. The mixings with the \( \eta \) and \( \eta' \) give similar contributions that add up; this is the reason why an analysis where the \( \eta' \) is explicitly included is important for understanding the effect. The NLO result for the width is almost identical to the LO one: 8.10 eV. This stability is however non-trivial. The mixing matrix \( \Theta \) is affected in the entries involving the \( \pi^0 \) by corrections of the order of 20%, but at the same time the decay constant matrix in the \( U(3) \) basis receives corrections that tend to undo those leaving the relevant decay constant matrix elements \( F_{aa} \) almost unchanged. In the absence of the \( O(p^6) \) WZ term these NLO corrections amount to a 0.5% further increase in the width, which is then largely compensated by the estimated contribution of the said WZ term. There are several sources of theoretical errors. The most important one is the uncertainty in the ratio \( R = m_s/(m_d - m_u) \) that largely determines the corrections due to mixing; this ratio is determined from \( M_{K^0} - M_{K^+} \) after removing the EM contribution determined using the corrected Dashen theorem; this leads to \( R = 37 \pm 5 \) and an uncertainty in the \( \pi^0 \) width of 0.6%. Other uncertainties such as EM corrections and NNLO corrections are estimated to be in the 0.2-0.3% range. In all, it is expected that the overall uncertainty is below 1%. In conclusion, the theoretical prediction \( \Gamma_{\pi^0 \rightarrow \gamma\gamma} = 8.10 \text{ eV} \) obtained in the framework of ChPT\#1/\( N_c \) indicates a substantial enhancement of about 4.5% over the prediction based on the chiral limit decay amplitude. The magnitude of this enhancement is such that it should be observed in the forthcoming \( \pi^0 \) lifetime measurement by the PRIMEX collaboration 1 at Jefferson Lab where a measurement with a precision better than 2% is expected.

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References

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