Negative Parity Baryon Decays in the $1/N_c$ Expansion

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Abstract. The $1/N_c$ expansion of QCD provides a useful framework for phenomenological studies of both ground state and excited baryons. Here, we focus on its application to the excited baryon strong decays via emission of single pseudoscalar meson. The S- and D-wave decay amplitudes of the negative parity baryons in the $[70, 1^-]$ of $SU(6)$ are analyzed to subleading order in $1/N_c$ and to first order in $SU(3)$ symmetry breaking. In particular, the $SU(3)$ symmetry breaking is studied in detail through $SU(3)$ breaking decay operators and state mixing.

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INTRODUCTION

The increasingly accurate determination of the properties of baryon resonances is one of the most important objectives in hadron physics, as they play an important role for exploring non-perturbative aspects of QCD. The $1/N_c$ expansion in QCD [1], where N_c is the number of colors, has proved to be a useful framework for sorting out the physics of baryons, both in ground state and excited states. In the large N_c limit, the baryon sector of QCD has an exact contracted spin-flavor symmetry $SU^c(6)$ [2, 3], which follows from the consistency conditions on baryon-meson scattering amplitudes. That symmetry is broken by $\mathcal{O}(1/N_c)$ effects, and it provides the foundation for implementing the $1/N_c$ expansion.

In the baryon sector, it is observed that there is an additional approximate $O(3)$ symmetry, which does not follow from $1/N_c$ expansion, but is a phenomenological fact. This approximate $O(3)$ symmetry is manifested in the observed $SU(6) \times O(3)$ multiplet structure of baryons, while the contracted symmetry emerges as a subgroup of this large group [6]. The $O(3)$ symmetry is actually recovered in the large N_c limit only for the symmetric (S) representation, which at $N_c = 3$ is the 56-plet of $SU(6)$, whereas it is broken at $\mathcal{O}(N_c^0)$ for mixed-symmetric (MS) representation, which is the 70-plet of $SU(6)$. Configuration mixing is possible, however the effects of such a mixing are expected to be small for dynamical reasons [4, 5]. Thus, it is a good approximation to disregard the configuration mixing for the practical purpose of phenomenology. Based on the $SU(6) \times O(3)$ symmetry scheme, the $1/N_c$ expansion is implemented as an effective theory built in terms of effective operators associated with the observable to be analyzed.

BARYON STATES IN $SU(6) \times O(3)$

The excited baryon states in the large N_c limit fill towers of states, corresponding to irreducible representations of a contracted spin-flavor symmetry group $SU^c(6)$, which up to subleading effects taken into account in the expansion, can be replaced by $SU(6)$. For the given $O(3)$ angular momentum ℓ , the states can be written as follows:

$$\left| (\ell, S) JJ_3, \begin{smallmatrix} R \\ Y II_3 \end{smallmatrix} \right\rangle_{\alpha} = \sum_{m, S_3} \langle \ell m, S S_3 | JJ_3 \rangle \left| S S_3, \begin{smallmatrix} R \\ Y II_3 \end{smallmatrix} \right\rangle_{\alpha} |\ell m\rangle \quad (1)$$

where α indicates the spin-flavor representation of the states, and $R = [p, q]$ indicates the $SU(3)$ irreducible representation given in terms of the usual labeling of a Young tableau with $p + 2q = N_c$. The rule $p = 2S$ holds for the symmetric

¹ Presenter

representation. Y, S and I are the hypercharge, spin and isospin quantum numbers respectively. The spin-flavor state can be built in general starting from a symmetric core of $(N_c - 1)$ boxes in Young tableau, and coupling to it one extra box forming either S or MS representation. The spin-flavor states are then defined given by:

$$\left| SS_3, \begin{smallmatrix} R \\ Y, II_3 \end{smallmatrix} \right\rangle_\alpha = \sum_{\eta=\pm\frac{1}{2}} C_\alpha(R, S, \eta) \left| SS_3, \begin{smallmatrix} R \\ Y II_3 \end{smallmatrix}; S^c = (S + \eta), R^c \right\rangle \quad (2)$$

where the coefficients C_α distinguish the two different representations S and MS and are given by [7, 8]

$$C_S(S, \eta = \pm 1/2) = \sqrt{\frac{(2S+1 \pm 1)[N_c+1 \mp (2S+1)]}{2N_c(2S+1)}} \quad (3)$$

$$C_{MS}(R, S, \eta = \pm 1/2) = \begin{cases} 1 & \text{if } I = S \pm 1 \\ 0 & \text{if } I = S \mp 1 \\ \pm \sqrt{\frac{(2S+1 \mp 1)[N_c+1 \pm (2S+1)]}{2N_c(2S+1)}} & \text{if } I = S \end{cases}$$

where I is the isospin of the non-strange partner within the multiplet R , and R^c depends on S^c , such that $R^c = [p^c, q^c] = [2S^c, (N_c - 2S^c - 1)/2]$.

DECAYS IN THE $SU(6) \times O(3)$ FRAMEWORK

We describe briefly the implementation of the framework for the decay of excited baryons via emission of a single pseudoscalar meson. The decay amplitudes are expanded in powers of $1/N_c$ by means of a basis of effective operators. The partial decay widths can then be expressed in terms of the reduced matrix elements (RMEs) of those operators.

The general expression for a partial decay width of an initial excited baryon state in the spin-flavor representation α decaying into a final ground state and a pseudoscalar meson state $|\ell^\pi \ell_3^\pi; R^\pi = \mathbf{8}, Y^\pi I^\pi I_3^\pi\rangle$ is given by [7, 8]

$$\Gamma = \frac{k}{8\pi^2} \frac{M_B}{M_B^*} \frac{1}{(\hat{f}^* \hat{f}^*)^2} \sum_{\substack{J_3^*, I_3^*, S_3, \\ I_3, \ell_3^\pi, I_3^\pi}} \left| \left\langle (\ell, S) JJ_3, \begin{smallmatrix} R \\ Y II_3 \end{smallmatrix} \left| \mathcal{B}_{GS}^{[\ell^\pi, R^\pi]} \right| (\ell, S^*) J^* J_3^*, \begin{smallmatrix} R^* \\ Y^* I^* I_3^* \end{smallmatrix} \right\rangle_\alpha \right|^2 \quad (4)$$

where M_B, M_B^* are the excited and ground state baryon masses respectively, and k is the outgoing meson momentum. The notation $\hat{f} = \sqrt{2J+1}$ is used for simplicity. The effective baryon transition operator, denoted by $\mathcal{B}^{[\ell^\pi, R^\pi]}$ must carry the same quantum numbers of the meson, and it can be built as a linear combination in the basis of effective operators, which are expanded in $1/N_c$:

$$\mathcal{B}^{[\ell^\pi, R^\pi]} = \left(\frac{k}{\Lambda} \right)^{\ell^\pi} \sum_n C_n^{[\ell^\pi, R^\pi]}(k) B_n^{[\ell^\pi, R^\pi]} \quad (5)$$

The dynamics of the decay amplitudes are encoded in the coefficients of $\mathcal{O}(N_c^0)$, denoted by $C_n^{[\ell^\pi, R^\pi]}$, which are to be determined by fitting to the empirical partial decay widths. The centrifugal factor $(k/\Lambda)^{\ell^\pi}$, is included to take into account the chief meson momentum dependence of the corresponding partial wave. The arbitrary scale Λ is chosen in what follows to be 200 MeV. The effective operators in the basis can be expressed in terms of spin-flavor operators in the general form

$$B_n^{[\ell^\pi, R^\pi]} = \sum_{m, j_n^3} \langle \ell m, j^n j_3^n | \ell^\pi \ell_3^\pi \rangle \xi_m^\ell \mathcal{G}_n^{[j^n, R^\pi]} \quad (6)$$

where ξ^ℓ is an $O(3)$ tensor operator, which drives the orbital transition from excited to ground state baryon. Without loss of generality, this operator is normalized such that $\langle 0 | \xi_{m'}^\ell | \ell m \rangle = (-1)^{\ell-m} \delta_{m, (-m')}$. The spin-flavor operator $\mathcal{G}_n^{[j^n, R^\pi]}$, of spin j_n , can be built as tensor products of the $SU(6)$ generators.

Finally, by averaging over the initial and summing over the final baryon spins and isospins, the partial decay width for ℓ^π partial wave in terms of the RMEs of the basis operators is given by:

$$\Gamma^{[\ell^\pi]} = \underbrace{\frac{k^{(1+2\ell^\pi)}}{8\pi^2 \Lambda^{2\ell^\pi}} \frac{M_B}{M_B^*}}_{\text{phase space}} \frac{\hat{f}^2}{(\hat{f}^* \hat{f}^*)^2} \left| \sum_n C_n^{[\ell^\pi, R^\pi]}(k) \overbrace{\mathbf{B}_n(\{S, R, Y, I\}, \{(\ell, S^*)^{J^*}, R^*, Y^*, I^*\}, \{\ell^\pi, R^\pi, Y^\pi, I^\pi\})}^{RME} \right|^2 \quad (7)$$

NEGATIVE PARITY BARYON DECAYS

S and D wave decays of the $[70, 1^-]$ baryons are analyzed, by implementing the operators upto order $1/N_c$ and first order in SU(3) symmetry breaking. The effective operators, built by the spin-flavor generators, must connect the mixed-symmetric with a symmetric representation. Generators of the general spin-flavor group $SU(6)$ do not produce such connections. Thus, one has to implement the spin-flavor operator as a product of spin-flavor generators restricted to act either on the excited quark or on the core states. In order to build a basis of spin-flavor operators, the following types should be considered:

$$\text{1-body: } \lambda \quad \quad \quad \text{2-body: } \frac{1}{N_c} \lambda \times \Lambda^c \quad (8)$$

where λ represents the spin-flavor generators acting on the excited quark $\{s, t, g\}$, while Λ^c represents those of core states $\{S^c, T^c, G^c\}$. The order in $1/N_c$ of an n-body operator is given by $\nu = n - \kappa - 1$, where κ is the coherence factor determined by the number of coherent generators, which appear in the operator product. More details of this power counting can be found in [9, 5]. The spin-flavor operator basis for the negative parity decays consists of nine SU(3) symmetric operators and two 1-body SU(3) symmetry breaking operator of the form

$$\begin{aligned} \mathcal{O}_0 &= g^{ia}, \quad \mathcal{O}_1 = \frac{1}{N_c} (sT^c), \quad \mathcal{O}_2 = \frac{1}{N_c} (tS^c), \quad \mathcal{O}_3 = \frac{1}{N_c} (sG^c)^{(j=1)}, \quad \mathcal{O}_4 = \frac{1}{N_c} (gS^c)^{(j=1)} \\ \mathcal{O}_5 &= \frac{1}{N_c} (f^{abe} g_b T_e^c), \quad \mathcal{O}_6 = \frac{1}{N_c} (d^{abe} g_b T_e^c), \quad \mathcal{O}_7 = \frac{1}{N_c} (sG^c)^{(j=2)}, \quad \mathcal{O}_8 = \frac{1}{N_c} (gS^c)^{(j=2)} \\ \mathcal{O}_9 &= \frac{1}{N_c} (f^{abe} g_b G_e^c)^{(j=2)}, \quad \mathcal{O}_{SB}^1 = \left(d^{8ab} - \frac{\delta^{ab}}{\sqrt{3}} \right) g^{ib}, \quad \mathcal{O}_{SB}^2 = f^{8ab} g^{ib} \end{aligned} \quad (9)$$

However with the available empirical decay widths involving strangeness, one can not distinguish the effects of the two SU(3) breaking operators, and therefore only one of them will be used. At 2-body level, one has a proliferation of SU(3) breaking operators, which we neglect because of the mentioned lack of data on decays involving strangeness. All the operators in the above basis do not contribute for the different partial decays. For the S-wave decays, four symmetric operators, namely $\mathcal{O}_0 - \mathcal{O}_4$, and seven symmetric operators for the D-wave decays $\mathcal{O}_0 - \mathcal{O}_2$, $\mathcal{O}_4 - \mathcal{O}_5$, $\mathcal{O}_7 - \mathcal{O}_8$, and only one symmetry breaking operator $\mathcal{O}_{SB}^1$ for both S and D decays enter in the analysis. In negative parity decays, π and η meson decay widths contribute at $\mathcal{O}(1)$, whereas $K(\bar{K})$ meson decays are suppressed by $1/N_c$, except for the singlet Λ decays.

The $[70, 1^-]$ contains two pairs of octets with $J=1/2$ and $3/2$. Each of these pairs has two different spin S values, $S=1/2$ and $3/2$, therefore, state mixings are possible. In general, the physical states are given by [10, 11]

$$\begin{pmatrix} 8_j^* \\ 8_j'^* \end{pmatrix} = \begin{pmatrix} \cos\theta_{2j} & \sin\theta_{2j} \\ -\sin\theta_{2j} & \cos\theta_{2j} \end{pmatrix} \begin{pmatrix} 28_j \\ 48_j \end{pmatrix} \quad (10)$$

There are two possible mixings in exact SU(3) limit, however SU(3) symmetry breaking produces different mixing angles for N, Λ and Σ . Therefore, six mixing angles are considered in this analysis. The mixing angles can be constrained to be in the interval $[0, \pi)$.

Using the different empirical S and D wave decay widths, provided by the PDG [13], preliminary fit results for our current work in progress [14] are given in Table 1. Both S and D wave decay widths have to be simultaneously fitted, because there are common mixing angles for both partial widths. The LO fit involves the operators with matrix elements of $\mathcal{O}(1)$. Only the π and η meson decays contribute at this order. The $K(\bar{K})$ meson decays, except for the singlet Λ , are always subleading. At this order, it is possible to derive some parameter free relations. For instance, the

relations for S and D wave decays given below can be used to extract the mixing angles at LO.

$$\begin{aligned}
\text{S-wave decays:} \quad & \frac{\tilde{\Gamma}({}^2N_{\frac{1}{2}} \rightarrow N\pi) - \tilde{\Gamma}({}^4N_{\frac{1}{2}} \rightarrow N\pi)}{\tilde{\Gamma}({}^2N_{\frac{1}{2}} \rightarrow N\pi) + \tilde{\Gamma}({}^4N_{\frac{1}{2}} \rightarrow N\pi)} = \frac{1}{5}(3\text{Cos}2\theta_1^N - 4\text{Sin}2\theta_1^N) \\
& \frac{\tilde{\Gamma}({}^2N_{\frac{1}{2}} \rightarrow N\eta) - \tilde{\Gamma}({}^4N_{\frac{1}{2}} \rightarrow N\eta)}{\tilde{\Gamma}({}^2N_{\frac{1}{2}} \rightarrow N\eta) + \tilde{\Gamma}({}^4N_{\frac{1}{2}} \rightarrow N\eta)} = \text{Sin}2\theta_1^N \quad \frac{\tilde{\Gamma}({}^2\Lambda_{\frac{1}{2}} \rightarrow N\bar{K}) - \tilde{\Gamma}({}^4\Lambda_{\frac{1}{2}} \rightarrow N\bar{K})}{\tilde{\Gamma}({}^2\Lambda_{\frac{1}{2}} \rightarrow N\bar{K}) + \tilde{\Gamma}({}^4\Lambda_{\frac{1}{2}} \rightarrow N\bar{K})} = \text{Cos}2\theta_1^\Lambda \\
\text{D-wave decays:} \quad & \frac{\tilde{\Gamma}({}^2N_{\frac{3}{2}} \rightarrow N\pi) - \tilde{\Gamma}({}^4N_{\frac{3}{2}} \rightarrow N\pi)}{\tilde{\Gamma}({}^2N_{\frac{3}{2}} \rightarrow N\pi) + \tilde{\Gamma}({}^4N_{\frac{3}{2}} \rightarrow N\pi)} = \frac{1}{41}(39 \text{Cos}2\theta_3^N + 4\sqrt{10} \text{Sin}2\theta_3^N)
\end{aligned} \tag{11}$$

The relations are written conveniently in terms of the reduced widths $\tilde{\Gamma}$, where the phase space factor in Eqn.(7) has been removed. Using the empirical values, the mixing angles resulting from above relations are $\theta_1^N = 0.46$ or 1.76 , $\theta_1^N = 0.51$ and $\theta_1^\Lambda = 1.25$ respectively from S-wave and $\theta_3^N = 3.08$ or 0.38 from D-wave. The angle θ_1^N extracted from both π and η decay relations turns out to be close to the angle obtained in the NLO fit, while the angles θ_1^Λ and θ_3^N are very different for the NLO results.

TABLE 1. Fit parameters for S- and D-wave decays of the $[70, 1^-]$ baryons. χ_{dof}^2 is 1.0 for LO fit and 0.75 for NLO fit. Ci are the coefficients of SU(3) preserving operators and B1 is the coefficient of SU(3) violating operator. Mixing angles are given in radian. (Preliminary results)

coef	S-wave		D-wave		Angle	Mixing	
	LO	NLO	LO	NLO		LO	NLO
C1	12.7(1.1)	12.6(0.8)	1.9(0.1)	1.9(0.1)	θ_1^N	0.49(0.07)	0.50(0.06)
C2	—	7.4(2.9)	—	-1.6(0.2)	θ_3^N	2.77(0.04)	2.77(0.05)
C3	—	-7.5(2.0)	—	0.6(0.5)	θ_1^Λ	0.95(0.08)	0.69(0.07)
C4	7.0(2.6)	4.6(2.0)	—	—	θ_3^Λ	2.57(0.52)	2.50(0.61)
C5	—	-7.7(2.6)	—	1.9(0.3)	θ_1^Σ	0.82(0.68)	0.92(0.12)
C6	—	—	—	-0.2(0.6)	θ_3^Σ	2.57(0.83)	2.92(0.45)
C7	—	—	—	—			
C8	—	—	1.2(0.1)	1.2(0.1)			
C9	—	—	—	-0.4(0.4)			
C10	—	—	—	—			
B1	-17.2(4.5)	-10.6(3.2)	—	-2.5(0.3)			

FIGURE 1. Empirical decay widths and evaluated widths from LO and NLO fits for S and D wave decays.

The LO fit is carried out by assigning the errors of the empirical widths, unless they are smaller than 30% in which case the error is set at 30%. This is to check the consistency of the LO approximation. In this fit, we have included only π and η decay modes. At this order, the fit is a consistent with $\chi_{dof} = 1$. Not only 1-body operators, but also 2-body operators contribute at LO. In S and D wave decays, two SU(3) preserving operators in the π channel, while only one SU(3) preserving and one SU(3) breaking operators in the η channel are involved in this fit. All the π and η decays are fitted well in S-wave, and there are some inconsistencies in the D-wave π decay channels at LO. There are no experimentally seen η decays so far for the D-waves [13]. There is a twofold ambiguity for the angle θ_1^Λ , which is 0.95(0.08) or 0.25(0.79), but this ambiguity disappears in the NLO fit, resulting in a single value for the angle equal to 0.69(0.07). The Σ state mixing θ_1^Σ is extracted from only one input channel $\Sigma(1750) \rightarrow \eta N$, which has a large relative experimental error.

In the NLO fit, we do not include singlet Λ decay channels, and try to reconstruct them from the fitting coefficients. At NLO, there are 28 input channels available to fit 20 parameters: 14 coefficients and 6 angles. However at this order, we have a consistent fit with $\chi_{dof} \sim 0.8$. The empirical decay channels with their corresponding partial widths and reconstructed partial widths from both LO and NLO fits are plotted in Figure1. One can see that, most of the empirical channels are consistent at NLO. The biggest discrepancy coming from the S-wave channel $\Lambda(1670) \rightarrow \bar{K}N$ and D-wave channel $\Lambda(1690) \rightarrow \bar{K}N$. One possible explanation of the discrepancy may be that we have used only one SU(3) breaking operator, and may be resolved by other SU(3) breaking effects included in operators that have been disregarded. The partial decay widths of the singlet Λ decay channels, evaluated from the NLO fit results, agree with the empirical values within their uncertainties. Going from LO to NLO fit, the LO coefficients do not change much, and

there is no real need for NLO improvements of the S-wave π and η modes. However, this is not the case in D-waves. There are some significant improvements of π modes at NLO, for instance in the decay channels $\Sigma(1670) \rightarrow \pi\Sigma$ and $\Lambda(1830) \rightarrow \pi\Sigma$. The mixing angles, obtained in this analysis are similar to those of obtained in the $[70, 1^-]$ mass analysis [10], where they are more precisely determined than in the decays. However, the NLO values of the mixing angles θ_1^N and θ_3^N are somewhat different from the values $\theta_1^N = 0.61$ and $\theta_3^N = 3.04$ obtained from an early analyses [11, 12], while the rest of the angles are very different.

CONCLUSIONS

We implemented the $1/N_c$ expansion for the decays of negative parity 70-plet baryons, and analyzed the empirically known widths to $\mathcal{O}(1/N_c)$ and first order in SU(3) symmetry breaking, assuming negligible configuration mixing. The NLO corrections are of natural size, even though some of the $1/N_c$ corrections are poorly determined due to the magnitude of the errors in the empirical widths. The role of SU(3) symmetry breaking operator cannot be well established due to the lack of more complete experimental information. One important observation is the ordering of the decay widths for different mesons in $1/N_c$. Both π and η mesons decays are LO, while $K(\bar{K})$ meson decays are suppressed by factor $1/N_c$. The S-wave π and η decays are well described by the LO operators. The D-wave π decays are improved by the NLO corrections, thus the NLO effects are significant in D-waves than in S-waves. The NLO results describe well the empirical singlet Λ decay channels, $\Lambda(1405) \rightarrow \pi\Sigma$, $\Lambda(1520) \rightarrow \pi\Sigma$ and $\bar{K}N$. The mixing angles, resulting at NLO are similar to the ones determined at LO, and also to the angles determined from the baryon mass analysis. The predictions for the unseen decay widths can be deduced from the results of our analysis. The same framework has been implemented for the positive parity baryon decays, in the Ref[7, 5].

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