G4BEAMLINE Simulations of Parametric Resonance Ionization Cooling of Muon Beams^{*}

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Abstract. The technique of using a parametric resonance to allow better ionization cooling is being developed to create small emittance beams so that high collider luminosity can be achieved with fewer muons. While parametric resonance ionization cooling (PIC) of muons has been shown to work in matrix-based simulations using `OptiM` [1] when the system is properly tuned, doing the same using a much more detailed GEANT-based `g4beamline` [2] simulation has been more difficult.

INTRODUCTION

The starting point for this work is a the linear channel; a half integer resonance is induced such that the normal elliptical motion of particles in x - x' phase space becomes hyperbolic, with particles moving to smaller x and larger x' as they pass down the channel. Thin absorbers placed at the focal points of the channel cool the angular divergence of the beam by the ionization cooling mechanism where each absorber is followed by RF cavities. Thus the phase space of the beam is compressed in transverse position by the dynamics of the resonance and its angular divergence is compressed by the ionization cooling mechanism.

The concepts and basic equations for PIC have been described previously [3]. An essential practical question for PIC is how well detuning effects can be compensated. Chromatic aberration, the detuning effect we first considered, is where the momentum-dependent betatron frequency causes off-momentum particles to be out of resonance with the focusing lattice. Analytic studies [4] and matrix based simulations [5] using the `OptiM` program showed that by choosing suitable synchrotron motion parameters, the resonance condition can be maintained. This paper reports the extension of that work using a much more detailed GEANT-based simulation program `g4beamline`.

SOLENOID TRIPLET CELL

Our studies of optimum lattice configurations indicated that an ideal cell for PIC consists of an alternating solenoid triplet. This provides strong focusing, so that the horizontal and vertical betatron phases advance by 3π across the cell and by

π between the absorbers. This configuration provides a periodic half-integer and integer resonant lattice in a very compact cell.

For $p = 286.8$ MeV/c, the length of the cell is 7.2 m. While *OptiM* uses the three “soft edge” solenoids with fields of $B_0 = -34.1, 32.4,$ and -34.1 kG, *g4beamline* models the solenoids with cylindrical current sheets. These currents were adjusted empirically, giving fields of $B_0 = -33.1, 34.8,$ and -33.1 kG. In both cases the solenoid lengths are $L=80, 130,$ and 80 cm, respectively, and radius $a=20$ cm.

The periodic solution of the cell is illustrated in terms of Twiss functions below:

FIGURE 1. Beta functions for the solenoid triplet cell. Thin absorbers are placed at the two central focal points. In the *OptiM* simulations for FIG 2 the lost energy is simply replaced at the absorbers.

FIGURE 2. The phase advance over a single cell; 400 MHz RF cavities at each end provide synchrotron motion.

FIGURE 3. *g4beamline* view of one cell; the Be absorbers are shown in yellow, RF cavities as dark red.

THIN ABSORBER AND RF

The *OptiM* model considered the ionization cooling as due to energy loss ($-\Delta p$) in a thin absorber followed by immediate re-acceleration (Δp):

$$\Delta\theta_z = -\theta_z \frac{\Delta p}{p}$$

The corresponding canonical transfer matrix can be written as

$$M_{abs} = K \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - \frac{\Delta p}{p} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 - \frac{\Delta p}{p} \end{bmatrix} K^{-1}$$

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k/2 & 0 \\ 0 & 0 & 1 & 0 \\ k/2 & 0 & 0 & 1 \end{bmatrix} \quad \hat{\mathbf{x}} \equiv \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} \quad \mathbf{x} \equiv \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}$$

where $k = eB_z / pc$ and $\hat{\mathbf{x}} = K\mathbf{x}$.

TABLE 1. Initial parameters for simulation studies.

normalized emittance: $\varepsilon_x/\varepsilon_y$	mm	30
longitudinal emittance: ε_l ($\varepsilon_l = \sigma_{\Delta p} \sigma_z / m_{\mu} c$)	mm	0.8
momentum spread: $\sigma_{\Delta p}/p$		0.01
bunch length: σ_z	mm	30
momentum	MeV/c	286.8

In contrast, the `g4beamline` model placed a 400 MHz pillbox RF cavity just after the (not necessarily thin) absorber; this cavity provides on-crest acceleration and some RF focusing. The time offset was simply set as the time-of-flight at the nominal energy. The energy loss compensating voltage, V_{acc} , was iteratively adjusted to replace the energy loss of the absorber (57.3 MV/m for a 32.8mm Be absorber).

To induce synchrotron oscillation into the channel dynamics two 400 MHz RF cavities at zero crossing were added symmetrically to each cell. Their time offset was also set by a the nominal time-of-flight, and their voltage (V_{syn}) iteratively adjusted to give the smallest final 6D emittance, which for a 32.8mm Be absorber V_{syn} was 13.08 MV/m. Since the RF cavities in the simulations provide some focusing, it was necessary to readjust the solenoids.

SIMULATIONS

A single text input file described the model for `g4beamline`. Typical runs used from 100 to 1000 particles and the same starting distribution as used in the `Optim` model.

It was quite difficult to analytically determine the correct operating parameters for the `g4beamline` simulation. A program, `kmimf`, using the CERNLIB-based MINUIT package, ran an iterative search in parameter space to minimize a specified quantity (such as the final r_{RMS} or ε_{6D}); that quantity was extracted from the analysis of each `g4beamline` run. This was very helpful in tuning the system. Another

utility program, `retrack`, was used to gather statistics, convert output formats, and manipulate the output of `g4beamline`.

The first goal was to compare with the previous `OptiM` simulations. `kmimf` found a reasonable tune within a few iterations. The emittance was calculated using a slightly modified `ecal9` [6] for the cases with the synchrotron cavities turned off ($V_{\text{syn}}=0$) and with an optimum value ($V_{\text{syn}}=13.08$ MV/m). To compare to `OptiM` in these simulations, all stochastic processes (multiple scattering, energy straggling, muon decay, etc.) were disabled.

FIGURE 4. The transverse (ϵ_T [m-rad]), longitudinal (ϵ_L [m]), and 6-dimensional emittance (ϵ_{6D} [m³]) in the absence of all stochastic processes is plotted as a function of distance along the channel for the cases without and with synchrotron motion. In this example $L_{Be}=32.8$ mm, $V_{acc}=57.29$ MV/m, and $V_{syn}=0$ or 13.08 MV/m. In each case, the curve ending higher is the case without synchrotron motion.

The next step was to turn on all the stochastic processes to see what would happen. It was quite clear that multiple scattering and energy straggling are very important effects (FIG 5.). Optimization, not shown here, using `kmimf` resulted in an additional 10x reduction in the final 6-dimensional emittance.

FIGURE 5. The transverse (ϵ_T [m-rad]), longitudinal (ϵ_L [m]), and 6-dimensional emittance (ϵ_{6D} [m³]) is plotted as a function of distance along the channel for the cases without and with stochastic processes enabled. In this case, $L_{Be}=32.8$ mm, $V_{acc}=57.29$ MV/m, and $V_{syn}=13.08$ MV/m. In each case, the curve ending higher is the case with stochastic processes enabled.

CONCLUSIONS

To realistically explore compensation schemes for PIC, much more realistic g4beamline simulations, which many import effects lacking in the earlier OptiM simulations, are being conducted.. While both programs show the importance of synchrotron motion, the new simulations make it clear that, in addition to optimization, new methods should be considered to make PIC work in the presence of physical stochastic processes.

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