A Clean Measurement of the Neutron Skin of $^{208}$Pb Through Parity Violating Electron Scattering

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Abstract. The difference between the neutron radius $R_n$ of a heavy nucleus and the proton radius $R_p$ is believed to be on the order of several percent. This qualitative feature of nuclei, which is essentially a neutron skin, has proven to be elusive to pin down experimentally in a rigorous fashion. A new Jefferson Lab experiment will measure the parity-violating electroweak asymmetry in the elastic scattering of polarized electrons from $^{208}$Pb. Since the $Z$-boson couples mainly to neutrons, this asymmetry provides a measure of the size of $R_n$ that can be interpreted with as much confidence as the traditional electron scattering data. The projected experimental precision corresponds to a 1% determination of $R_n$, which will have a big impact on nuclear theory and its application to neutron rich matter such as neutron stars.

INTRODUCTION

The size of a heavy nucleus is one of its most basic properties. However, because of a neutron skin of uncertain thickness, the size does not follow from measured charge radii and is relatively poorly known. For example, the root mean square neutron radius in $^{208}$Pb, $R_n$, is thought to be about 0.25 fm larger than the proton radius $R_p \approx 5.45$ fm. An accurate measurement of $R_n$ would provide the first clean observation of the neutron skin. This is thought to be an important feature of all heavy nuclei.

Donnelly, Dubach and Sick [1] suggested that parity-violating electron scattering can measure neutron densities. This is because the $Z$-boson couples primarily to the neutron at low $Q^2$. Therefore, one can deduce the weak-charge density and the closely related neutron density from measurements of the parity-violating asymmetry in polarized elastic scattering $A = (\sigma_R - \sigma_L) / (\sigma_R + \sigma_L)$. In PWIA, the relationship between the asymmetry and the neutron form factor is given by:

$$A = -\frac{G_F Q^2}{4\pi\alpha \sqrt{2}} \left[ 1 - 4 \sin^2 \theta_W \right] \frac{F_n(Q^2)}{F_p(Q^2)} ,$$

where $G_F$ is the Fermi constant, $\alpha = \frac{1}{137}$ is the fine structure constant, $\sin^2 \theta_W$ is the Weinberg angle, and $F_n(Q^2)$ and $F_p(Q^2)$ are the neutron and proton form factor of

1 For the Jefferson Lab Hall A Collaboration.
the nucleus. This is similar to how the charge and proton densities are deduced from unpolarized cross sections.

**PHYSICS IMPACT AND INTERPRETATION**

A single measurement of $R_n$ with 1% accuracy can have an impact on several areas of physics, including nuclear theory, atomic parity violation, and neutron star structure.

In a recent analysis of neutron radii in nuclear mean field models, Furnstahl [2] showed that the variable range of $R_n$ allowed by a large set of viable nuclear models was associated primarily with the density dependence of the nuclear symmetry energy. This is an energy cost associated with having unequal numbers of neutrons and protons. A measurement of $R_n$ would pin down this one parameter and could potentially demonstrate that an entire class of models is less likely than another. For example, relativistic mean field models tend to favor larger neutron skins than non-relativistic models because of a larger symmetry energy.

The impact of an accurate $R_n$ measurement on atomic parity violation experiments has been analyzed in [3]. Knowledge of $R_n$ at the 1% level is needed for interpreting atomic physics measurements of the Weinberg angle at the level of the Standard Model weak radiative corrections.

Measurements of the equation of state of neutron rich matter are important for calculating the structure of neutron stars [4]. The radius of a neutron star is deduced from optical and X-ray observations. To find out possible exotic phases of dense matter one needs to combine the high density measurements of neutron stars with low density precision measurements of $R_n$ in nuclei. As a second example of application, the proton fraction of neutron rich matter in beta equilibrium depends on the symmetry energy, which is calibrated by $R_n$. A large symmetry energy favors more protons, and if the proton fraction is high enough then the following “URCA” process can cool neutron stars $n \rightarrow p + e + \bar{\nu} ; \ p + e \rightarrow n + \nu$ where the $\nu, \bar{\nu}$ carry off energy. URCA cooling might explain recent Chandra observations of the neutron star 3C58, a remnant of the supernova seen in the year 1186 that appears to be unexpectedly cold [5]. A neutron skin larger than about 0.2 fm may imply that URCA cooling is possible, while a smaller skin implies it is probably not possible.

The physics interpretation of the experiment can be summarized as follows. From the measured asymmetry one may deduce the weak form factor, which is the Fourier transform of the weak-charge density at the momentum transfer of the experiment. One must correct for Coulomb distortions, which has been done accurately by Horowitz [6]. The weak-charge density can be compared directly to theoretical calculations and this will constrain the density dependence of the symmetry energy. The weak density can be directly applied to atomic PNC because the observables have approximately the same dependence on nuclear shape. From the weak-charge density one can also deduce a neutron density at one $Q^2$ by making small corrections for known nucleon form factors. The uncertainty in these corrections for a realistic experiment have been estimated and are small [3]. The corrections considered were Coulomb distortions (which was by far the biggest), strangeness and the neutron electric form factor, parity admixtures, dis-
persion corrections, meson exchange currents, isospin admixtures, radiative corrections, and possible contamination from excited states and target impurities.

Finally from a low $Q^2$ measurement of the point neutron density one can deduce $R_n$. This requires knowledge of the surface thickness to about 25% to extract $R_n$ to 1%. The spread in surface thickness among successful mean field models is much less than 25%, hence we can extract $R_n$ with the desired accuracy. In summary, the physics results of the experiment are the weak-charge density, the point neutron density, and $R_n$.

**EXPERIMENT**

**Experimental Overview**

The $^{208}$Pb experiment [7] will take place in Hall A at Jefferson Lab (see Fig. 1). The two Hall A 3.7 msr spectrometer systems supplemented by septum magnets focus elastically scattered electrons onto total-absorption detectors in their focal planes. Separate studies at lower rates are required to measure backgrounds, acceptance, and $Q^2$. The experimental conditions are listed in Tab. 1.

**Choice of Nucleus and Kinematics**

The target nucleus was chosen to be $^{208}$Pb. The advantages of Pb are that it is very well known and has a simple doubly magic shell structure, and most importantly that it has the largest separation to the first excited state (2.6 MeV) of any heavy nucleus, thus permitting flux integration of the elastically scattered electrons.

The choice of kinematics (incident energy and scattering angle) is guided by the objective of minimizing the running time required for a 1% accuracy in $R_n$. The three
TABLE 1. Summary of the experimental conditions.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Asymmetry ($p_eA$)</td>
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</tr>
<tr>
<td>Beam Energy</td>
<td>850 MeV</td>
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<tr>
<td>Beam Current</td>
<td>50 µA</td>
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<tr>
<td>Beam Polarization</td>
<td>80%</td>
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<td>Target</td>
<td>10% r.l. Pb</td>
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<tr>
<td>Scattering Angle</td>
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<td>Required Statistical Accuracy</td>
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<td>Energy Cut (due to detector)</td>
<td>4 MeV</td>
</tr>
<tr>
<td>Detected Rate (each Spectrometer)</td>
<td>860 MHz</td>
</tr>
<tr>
<td>Running Time</td>
<td>30 days</td>
</tr>
</tbody>
</table>

FIGURE 2. Cross section, parity violating asymmetry, and sensitivity to $R_n$ for $^{208}$Pb elastic scattering at 0.85 GeV. The fourth plot shows the variation of $\text{FOM} \times \epsilon^2$ with energy and angle, showing an optimum at 0.85 GeV for a 6° scattering angle which corresponds to $q = 0.45 \text{ fm}^{-1}$.

The ingredients which enter into this optimization are: the cross section $d\sigma/d\Omega$, the parity violating asymmetry $A$, and the sensitivity to the neutron radius $\epsilon = dA/A = (A1 - A)/A$ where $A$ is the asymmetry computed from a mean field theory (MFT) calculation [6] and $A1$ is the asymmetry for the same MFT in which the neutron radius is increased by 1%. These three ingredients, which each vary with energy and angle, are plotted in Fig. 2 for a beam energy of 0.85 GeV which turns out to be optimum energy. The minimum running time is equivalent to maximum product in $\text{FOM} \times \epsilon^2 = R \times A^2 \times \epsilon^2$, where $R$ is the detected rate and is proportional to $d\sigma/d\Omega$ and “FOM” is the conventionally defined figure of merit for parity experiments, $\text{FOM} = R \times A^2$. Note that rather than only maximizing the conventional FOM, parity violating neutron density measurements take into account the sensitivity ($\epsilon$) to $R_n$ which varies with kinematics. Figure 2 shows the product $\text{FOM} \times \epsilon^2$ for $^{208}$Pb. The running time needed to reach a 1% accuracy in $R_n$ is 30 days.

A novel feature of the $^{208}$Pb experiment is a high powered lead target which will
withstand 40 Watt for a 50 μA beam. Improving the thermal properties of the target is necessary since lead has a low melting temperature. A 0.5 mm foil of lead will be sandwiched between two 0.15 mm sheets of diamond, which is pure $^{12}$C. This assembly is clamped in a spring loaded copper block assembly which is cooled by liquid helium. The copper block has a hole to allow the beam to pass through; the beam only sees $^{208}$Pb and $^{12}$C. The diamond has an extremely high thermal conductivity, and calculations show this target should be stable up to 100 μA.

The target thickness required to maximize the rate in the momentum bite defined by the detector is 0.5 mm (10% r.l.). A thicker target suffers more radiative loss and hence less rate. By integrating the rate up to a cutoff of 4 MeV, we reduce the running time by 25% compared to a cutoff that would exclude the first excited state. The resulting contamination from inelastic scattering constitutes a fraction 0.5% of our signal. The systematic errors from inelastics and from $^{12}$C background are tolerable [3].

**Systematics**

Measuring a tiny asymmetry of 0.5 ppm to 3% absolute accuracy is a major challenge involving the following considerations:

1. The experimental systematic error must be much smaller than the statistical goal ($1.5 \times 10^{-8}$), hence a goal of $\leq 10^{-9}$. The main issues here are with the control of false asymmetries associated with helicity correlated beam parameters such as intensity, energy, and position. This will require good setup and feedback loops in the source, as well as betatron matching in the accelerator. The betatron matching will ensure maximal dampening of helicity correlated beam positions on target.

To measure the beam parameters accurately we are installing microwave cavity beam position and current monitors. The position monitors supplement the existing stripline monitors, which provides a complementary method with presumably different systematics to help unfold beam fluctuation noise from instrumentation noise. This experiment requires beam position differences to be less than 1 nm with an accuracy of 0.1 nm averaged over the whole experiment. The charge asymmetry must be maintained to less than 100 ppb with an accuracy of 10 ppb.

2. Because of the high rates (860 MHz per spectrometer), the statistical error in each 30 msec window will be 140 ppm. All other noises, e.g. instrumental noises, must be kept well below this.

3. The normalization of the asymmetry must be better than 3%. There are two main issues: the $Q^2$ measurement and the beam polarization. We expect to be able to measure $Q^2$ to 0.3%. The polarization must be measured to 1% preferably, or at least 2%. With a polarization accuracy of 1% (2%) we can extract $R_n$ to 1% (1.2%) respectively. To achieve this the Hall A Compton Polarimeter must be upgraded to use a green laser and new photon and electron detectors must be installed.

4. Since we must integrate our detected signal, the backgrounds must be measured separately; in addition, pedestals and nonlinearities need to be controlled at the few tenths of percent level.
FIGURE 3. The difference between neutron radii $R_n = r_n$ and proton radii $R_p = r_p$ for several nuclei of different mass number $A$. The filled symbols are for the relativistic mean field NL1 interaction while the open symbols are for the non-relativistic zero range Skyrme skiii interaction. This figure is taken from calculations of Ring et. al. [8]. A possible 1% measurement in $^{208}$Pb is indicated by the error bar which has been arbitrarily placed at $R_n - R_p = 0$.

CONCLUSION

The neutron skin is a qualitative feature of heavy nuclei which has never been cleanly observed. Measurement of $R_n$ will have a fundamental impact on nuclear physics. This experiment requires state-of-the-art control of systematic errors and very accurate polarimetry. Figure 3 shows the projected error bar compared to two different theoretical calculations.

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REFERENCES