Study of the ${}^7_{\Lambda}\text{He}$ hypernucleus in formalism of the Faddeev equations

Igor Filikhin*, Vladimir M. Suslov*,[†] and Branislav Vlahovic*,**

*North Carolina Central University, Durham, NC 27707, USA

[†]Fock Institute of Physics, Sankt-Petersburg State University, 198504, Petrodvorets, Russia

**Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

Abstract. P -shell $A = 7$ hypernuclei are considered in the cluster ${}^5_{\Lambda}\text{He} + N + N$ model. The folding procedure are applied to construct the ${}^5_{\Lambda}\text{He} - N$ interaction. We use the OBE simulating NSC97f potential for ΛN interaction and various phenomenological potentials for $\alpha\Lambda$ interaction. Configuration space Faddeev calculations are performed for hyperon binding energy of the ${}^7_{\Lambda}\text{He}(\frac{1}{2}^+)$ and ${}^7_{\Lambda}\text{Li}(\frac{1}{2}^+$ and $\frac{3}{2}^+$, $T=0$) nuclei. Predicted value for $B_{\Lambda}({}^7_{\Lambda}\text{He})$ is 5.35 MeV. This value is obtained when the ${}^6_{\Lambda}\text{He}(2^-)$ excitation energy is equal 0.26 MeV by the adjustment of pair ${}^5_{\Lambda}\text{He} - N$ effective potential to reproduce the experimental value of the ${}^7_{\Lambda}\text{Li}(\frac{3}{2}^+)$ excitation energy. Our results are compared with those of E. Hiyama et al. *Phys. Rev. C* **53**, 2075-2085 (1996).

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INTRODUCTION

P -shell $A = 7$ hypernuclei are considered as appropriate objects for studying the ΛN interaction and its spin-dependence, particularly. The ${}^7_{\Lambda}\text{Li}$ and ${}^7_{\Lambda}\text{Be}$ nuclei have been experimentally studied quiet well [1, 2, 3], when there exists no experimental data for ${}^7_{\Lambda}\text{He}$ [4] at the present time. Theoretical studies of $A = 7$ hypernuclei in the different approaches have been undertaken a long time ago by many authors [5, 6]. Recently the microscopic variational calculations [7] were performed on the basis of the three-body model ${}^5_{\Lambda}\text{He} + N + N$, and within the framework of the four-body model $(\alpha + \Lambda + N + N)$ [8, 9]. In this paper we report results of the study based on the configuration space Faddeev formalism [10] for the P -shell $A = 7$ hypernuclei which are considered as the cluster system ${}^5_{\Lambda}\text{He} + N + N$. We calculate the hyperon binding energy of ${}^7_{\Lambda}\text{He}$ in the $\frac{1}{2}^+$ state, and ${}^7_{\Lambda}\text{Li}$ with $T=0$ in the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ (excitation energy) states. In the framework of this model a new prediction of 5.35 MeV has been obtained for the hyperon energy of the ground state of ${}^7_{\Lambda}\text{He}$. This value corresponds to the excitation energy of the 2^- state of ${}^6_{\Lambda}\text{He}$ of 0.26 MeV. The calculated hyperon energy of ${}^7_{\Lambda}\text{He}$ is close to the one reported by E. Hiyama et al. [7], which was obtained using another value of the excitation energy equal to 0.36 MeV. This distinction disappears after an correction for method of the hyperon energy extracting from the calculated three-body energy of the ${}^5_{\Lambda}\text{He} + n + n$ system. After such correction in the result of E. Hiyama et al. a new value 5.14 MeV is obtained which is in good agreement with the present calculation.

MODEL DESCRIPTION

The system ${}^5_\Lambda\text{He} + \text{N} + \text{N}$ can be studied in three-body model as bound cluster $\text{Core} = \alpha + \Lambda$ (the ${}^5_\Lambda\text{He}$ nucleus). To construct the ${}^5_\Lambda\text{He} - \text{N}$ pair potential we take into account four-body nature of ${}^7_\Lambda\text{He}$ and consider two subsystem which are ΛNN and αNN . Spin-isospin variables of our ${}^5_\Lambda\text{He} + \text{N} + \text{N}$ system are defined by the ΛNN subsystem when angular variables are defined by the αNN subsystem. The $\text{Core} + \text{N} + \text{N}$ system is considered in the spin-isospin state with $S=1/2$ and $T=0$. The potential model used in our calculation for this system includes a semirealistic potential for NN interaction and an effective ${}^5_\Lambda\text{He} - \text{N}$ potential. The last being given by a sum of an αN potential and a ΛN potential averaged over the Λ density function of the $\alpha\Lambda$ cluster (folding procedure). A phenomenological local potential was used to describe the $\text{N}\alpha$ interaction. For the ΛN potential an OBE simulating NSC97 model was applied. The ${}^5_\Lambda\text{He} - \text{N}$ folding potential that we thus obtain has s and p -wave components. The p -wave component is used for adjusting the parameters of the potential to reproduce the experimental values for the excitation energy of the 2^- states of the ${}^6_\Lambda\text{He}$ hypernucleus and the $\frac{3}{2}^+$ state of ${}^7_\Lambda\text{Li}$.

POTENTIALS

The nuclear αN interaction is given by the phenomenological potential SBB [11] having s and p -wave components including $(\vec{l}\vec{s})$ coupling, as modified in [12]. We use the ΛN potential simulating the NSC97 model [13] for barion-barion interaction. This s -wave potential is the three-range Gaussian [14] corrected in [15]. Parameters of the potential was chosen to reproduce the scattering length and the effective range for a given model [13]. For the triplet and singlet NN interaction we used the semirealistic Malfliet-Tjon potential MT-I-III [16]. Both s and p -wave components of the ${}^5_\Lambda\text{He}-\text{N}$ ($\text{Core} - \text{N}$) folding potential have the form

$$l = 0: V_{\text{Core}-\text{N}}^{l,t(s)} = V_{\alpha\text{N}}^l + V_{\Lambda\text{N}}^{f,t(s)}; \quad l = 1: V_{\text{Core}-\text{N}}^{l,t(s)} = V_{\alpha\text{N}}^l + \eta^{t(s)} V_{\Lambda\text{N}}^{f,t(s)}, \quad (1)$$

where $\eta^{t(s)}$ is the spin-dependent coefficient. $V_{\Lambda\text{N}}^{f,t(s)}$ is the triplet(singlet) component of the folding ΛN potential:

$$V_{\Lambda\text{N}}^{f,t(s)}(|\vec{r}|) = \int d\vec{\xi} \rho(|\vec{\xi}|) V_{\Lambda\text{N}}^{s(t)}(|\vec{r} - \vec{\xi}|), \quad (2)$$

where $\rho(|\vec{r}|)$ is the Λ -particle density function for the ${}^5_\Lambda\text{He}$ hypernucleus ($\int d\vec{r} \rho(|\vec{r}|) = 1$). $V_{\Lambda\text{N}}^{s(t)}$ and $V_{\text{NN}}^{s(t)}$ are the s -wave ΛN and NN potentials, respectively. $V_{\alpha\text{N}}^l$ is the partial component (s or p -wave) of the αN potential (spin-orbit coupling is neglected). The folding potentials averaged on spins of the $\text{Core} + \text{N}$ pair are as follows:

$$\begin{aligned} V_{\text{Core}-\text{N}}^l &= \frac{3}{4} V_{\text{Core}-\text{N}}^{l,t} + \frac{1}{4} V_{\text{Core}-\text{N}}^{l,s}, \quad \text{for } {}^7_\Lambda\text{He}(\frac{1}{2}^+, T=1), \\ V_{\text{Core}-\text{N}}^l &= \frac{3}{4} V_{\text{Core}-\text{N}}^{l,s} + \frac{1}{4} V_{\text{Core}-\text{N}}^{l,t}, \quad \text{for } {}^7_\Lambda\text{Li}(\frac{1}{2}^+, T=0), \\ V_{\text{Core}-\text{N}}^l &= V_{\text{Core}-\text{N}}^{l,t} \quad \text{for } {}^7_\Lambda, \quad \text{for } \text{Li}(\frac{3}{2}^+, T=0). \end{aligned}$$

TABLE 1. Hyperon binding energy B_Λ (in MeV) of $A=7$ hypernuclei calculated with various $\Lambda\alpha$ models of the ${}^5_\Lambda\text{He} - \text{N}$ folding potential. The excitation energy E_x (in MeV) of the $(\frac{3}{2}^+)$ state of ${}^7_\Lambda\text{Li}$ is also shown. In our calculation the Coulomb force is not taken into account and the excitation energy of ${}^6_\Lambda\text{He}(2^-)$ was chosen equal to 0.36 MeV[7].

	$\alpha\Lambda$	${}^7_\Lambda\text{He}(\frac{1}{2}^+)$	${}^7_\Lambda\text{Li}(\frac{1}{2}^+, T=0)$	$E_x({}^7_\Lambda\text{Li}(\frac{3}{2}^+, T=0))$
	TH	5.17	6.05	0.98
	Gibson	5.16	5.99	0.98
	MSA	5.10	5.88	0.98
	Isle	5.10	5.85	0.98
[7]	–	5.44	5.87	0.88
our correction of [7]	–	5.14	–	–
exp. [1],[3]		–	5.58	0.69

The hyperon density function $\rho(|\vec{r}|)$ of ${}^5_\Lambda\text{He}$ used in equation (2) was calculated for a number of diverse phenomenological $\Lambda\alpha$ potentials (for instant see [17]).

RESULTS

The coupled configuration space Faddeev equations for a $\text{CoreN} + \text{N}$ system was numerically solved, applying the method given in [18]. For the αNN system the following combinations of relative momenta $(\ell_{NN}, \lambda_{(\text{NN})-\alpha}) = (0, 0)$ and $(\ell_{\alpha\text{N}}, \lambda_{(\alpha\text{N})-\text{N}}) = (0, 0), (1, 1)$, were taken into account. Using the potential model with the averaged total momentum of the pair we obtained the value -1.19 MeV for the binding energy of ${}^6_\Lambda\text{He}$. This value is smaller than the value of -1.0 MeV from [12] where the spin-orbital interaction was taken into account. For better agreement with the experimental data we made a slight correction in dept parameter of the p -wave component of the αN potential. After this correction the calculated binding energy $E_B({}^6_\Lambda\text{He})$ was -0.98 MeV, which coincides with the experimental value $E_B^{\text{exp}}({}^6_\Lambda\text{He}) = -0.98$ MeV [19]. For the ${}^6_\Lambda\text{Li}$ nucleus our calculation without the Coulomb interaction gives $E_B({}^6_\Lambda\text{Li}) = -6.68$ MeV, which is smaller than the Coulomb-adjusted experimental value -4.53 MeV [20, 21] by 2 MeV approximately. In Equation (1) there are two free parameters η^s and η^t to determinate the spin dependence of the effective folding ${}^5_\Lambda\text{He} + \text{N}$ potential. The singlet coefficient η^s was chosen to reproduce the experimental value for the binding energy of the 1^- state of ${}^6_\Lambda\text{He}$ which is located below the ${}^5_\Lambda\text{He} + n$ threshold by -0.17 MeV [22]. The experimental data for the 2^- state of ${}^6_\Lambda\text{He}$ does not still exist. Therefore one can use some theoretical prediction from [6, 7, 8] to adjust the triplet coefficient η^t . In initial calculation we chose an excitation energy for the 2^- state equal to 0.36 MeV as in [7] (in [6] the value 0.42 MeV was used). The hyperon binding energy B_Λ of ${}^7_\Lambda\text{He}$ (${}^7_\Lambda\text{Li}$) calculated with diverse $\Lambda\alpha$ models in the ${}^5_\Lambda\text{He} - \text{N}$ folding potential is presented in Table 1. Note that the Coulomb force was not taken into account. The hyperon binding energy was calculated using the formula

$$B_\Lambda({}^7_\Lambda Z) = E_B({}^6 Z) - (E_B({}^7_\Lambda Z) + E_B({}^5_\Lambda\text{He})). \quad (3)$$

FIGURE 1. $^7_\Lambda\text{He}(\frac{1}{2}^-)$ hyperon binding energy B_Λ and the $^7_\Lambda\text{Li}(\frac{3}{2}^-, T=0)$ excitation energy E_x (open triangles) with variations of the $^6_\Lambda\text{He}(2^-)$ excitation energy. The Isle $\Lambda\alpha$ potential has been used to construct the $^5_\Lambda\text{He} - \text{N}$ folding potential. In our calculations the Coulomb force is not taken into account. Results of the tree-body calculation from [7] and [6] are pointed by the solid triangles and circle, respectively. Our correction of $B_\Lambda(^7\text{He})$ for calculation of [7] is denoted by the open "triangle in box". The solid square corresponds to the result of the Hiyama et al. four-body calculation [8].

For $E_B(^5_\Lambda\text{He})$ the experimental value -3.12 MeV was chosen. The excitation energy of $^7_\Lambda\text{Li}(\frac{3}{2}^+)$ was computed as $E_x(\frac{3}{2}^+) = E_B(\frac{3}{2}^+) - E_B(\frac{1}{2}^+)$. One can see from Table 1 that some disagreement with results of [7] occurs for the $^7_\Lambda\text{He}$ hypernucleus while for $^7_\Lambda\text{Li}$ there is satisfactory agreement. The discrepancy for the $^7_\Lambda\text{He}$ hyperon energy does not come from the three-body calculation. It comes from the way the hyperon energy $B_\Lambda(^7\text{He})$ is extracted from the calculated tree-body energy. In [7] the value of -0.68 MeV for the ^6He binding energy, calculated in their $\alpha + \text{N} + \text{N}$ model, was used for the hyperon energy calculation with Equation (3). However, the three-body system $^5_\Lambda\text{He} + \text{N} + \text{N}$ does not include the αNN system as a subsystem explicitly. One has to take into account that in [7] an adjustment of the ΛN potential was also made. This adjustment is similar to our procedure for choosing the $\eta^{t(s)}$ coefficients for the $^5_\Lambda\text{He} - \text{N}$ potential (1). But as follows from Equation (1) there exists the possibility of varying the parameters of the p -wave component of the αN potential within $^5_\Lambda\text{He} + \text{N} + \text{N}$, the three-body model, since any change of the p -wave parameters can be compensated by a adjustment of the $\eta^{t(s)}$ coefficients. After the correction of the $E_B(^6\text{He})$ value from [7] by using the experimental data for $E_B(^6\text{He})$ (-0.98 MeV) we obtain the value 5.14 MeV for $B_\Lambda(^7\text{He})$ (see Table 1). This value is close to the results of our calculations with $E_x(^6\text{He})=0.36$ MeV. We considered the dependence of the excitation energy of $^7_\Lambda\text{Li}$ on the excitation of $^6_\Lambda\text{He}$. To evaluate $B_\Lambda(^7\text{He})$, we take into account this dependence together with the analogous one for $B_\Lambda(^7\text{Li})$. The illustration of this evaluation is presented in Fig. 1. The value 0.26 MeV for E_x of $^6_\Lambda\text{He}(2^-)$ corresponding to the observed excitation energy of $^7_\Lambda\text{Li}(\frac{3}{2}^+)$ (the horizontal dot-dashed line in Fig 1) was used

to calculate the hyperon binding energy of ${}^7_{\Lambda}\text{He}$. From this figure follows that calculated value is equal to 5.35 MeV (the vertical dashed line). This is a new prediction for the hyperon energy of the ${}^7_{\Lambda}\text{He}$ ground state obtained in the framework of a three-body model. One can see in Fig. 1 that the results of [6, 7, 8] do not contradict results obtained in our model. In presented model which does not include the Coulomb force the ${}^7_{\Lambda}\text{He}$ and ${}^7_{\Lambda}\text{Be}$ nuclei are identical, therefore the hyperon binding energy of ${}^7_{\Lambda}\text{Be}$ is also equal to 5.35 MeV. The experimental value is 5.09 ± 0.11 MeV [1]. The difference of $B_{\Lambda}^{cal.} > B_{\Lambda}^{exp.}$ can be caused by the Coulomb effect. It agrees with the calculations for the ${}^7_{\Lambda}\text{Li}$ nucleus. The calculated value for the hyperon energy of ${}^7_{\Lambda}\text{Li}$ is bigger than the experimental value when is clear from Table 1.

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