EXCITED BARYON SPECTRUM FROM LATTICE QCD *

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We report lattice calculations of the low-lying baryon mass spectrum, with an emphasis on the introduction of two new techniques. One is the constrained fitting method based on Bayesian statistics which allows reliable extraction of excited states. The other is the use of the overlap fermion action which preserves exact chiral symmetry of QCD on the lattice and allows us to reach deep down into the chiral regime (our pion mass is as low as 180 MeV) where interesting chiral dynamics are exposed. One example is the elusive Roper state $N(1440)_{\frac{1}{2}}^{+}$ which shows up naturally as the 1st-excited state of the nucleon from the standard interpolating field. Together with other baryons, our preliminary results indicate that the level-ordering of the low-lying baryon states on the lattice is largely consistent with experiment and with that expected from meson-exchange quark models.

The rich structure of the excited baryon spectrum, as documented by the particle data group ¹, provides a fertile ground for exploring the nature of quark-quark interactions. One outstanding example is the ordering of the lowest-lying states which has the order of positive and negative-parity excitations inverted between N , Δ and Λ channels. Conventional quark models have difficulty explaining the ordering in a consistent manner. There are two contrasting views. One is from the constituent quark model ^{2,3} which has the interaction dominated by one-gluon-change type, *i.e.*, color-spin $\lambda_1^c \cdot \lambda_2^c \vec{\sigma}_1 \cdot \vec{\sigma}_2$. The other is based on Goldstone-boson-exchange ^{4,5} which has flavor-color $\lambda_1^f \cdot \lambda_2^f \vec{\sigma}_1 \cdot \vec{\sigma}_2$ as the dominant part. Even though evidence from valence QCD ⁶ supports the flavor-color picture, the challenge of reproducing the ordering still faces lattice QCD.

There exist a number of lattice studies of the excited baryon spectrum using a variety of actions ^{7,8,9,10,11}. The nucleon channel is the most-

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Figure 1. Ordering of some of the low-lying observed excited states. The values are taken from PDG ¹.

studied, focusing on two independent local fields:

$$\chi_1 = \epsilon^{abc} (u^{Ta} C \gamma_5 d^b) u^c, \quad (1)$$

$$\chi_2 = \epsilon^{abc} (u^{Ta} C d^b) \gamma_5 u^c. \quad (2)$$

χ_1 is the standard nucleon operator, while χ_2 , which has a vanishing non-relativistic limit, is sometimes referred to as the 'bad' nucleon operator. Note that baryon interpolating fields couple to both positive and negative-parity states, which can be separated by well-established parity-projection techniques. The consensus so far appears to be that, first, the negative-parity splitting of $N(\frac{1}{2}^-)$ is largely established and consistent with experiment. Secondly, the Roper state $N'(\frac{1}{2}^+)$ as the 1st-excited state of the nucleon is still elusive. Since χ_2 couples little to the nucleon ground state, there was initial speculation that it couples to the Roper state. But that identification has been mostly abandoned since the mass extracted from χ_2 is consistently too high. What χ_2 couples to remains an open question.

The lattice size we use is $16^3 \times 28$ with the scale of $a = 0.202(1)$ fm set from f_π , which is our preferred choice for scale ¹². We consider a wide range of quark masses: 26 masses with the lowest $m_\pi = 180$ MeV (or $m_\pi/m_\rho = 0.248$), very close to the physical limit, and with 18 masses below

Figure 2. Solid symbols denote $N(\frac{1}{2}^+)$ states: ground ($\bullet$) and 1st-excited ($\star$). Empty symbols denote $N(\frac{1}{2}^-)$ states: lowest (Δ) and 2nd lowest ($\square$). The experimental points ($\star$) are taken from PDG¹.

Figure 3. Similar to Fig. 2, but for $\Lambda(\frac{1}{2}^+)$ states (left) and $\Sigma(\frac{1}{2}^+)$ states (right).

the strange quark mass. We analyzed 80 configurations. Details of the simulation can be found in¹³.

We adapted the constrained curve-fitting method advocated in^{14,15}, adhering to the following guidelines: a) fit as many time slices in the correlation function $G_{\text{data}}(t)$ and as many terms in $G_{\text{theory}}(t)$ as possible; b) use prior knowledge, such as $A > 0$ and $E_n - E_{n-1} > 0$; c) seek guide for priors from a subset of data (empirical Bayes method); d) un-constrain the

Figure 4. Similar to Fig. 2, but for $\Xi(\frac{1}{2}^{\pm})$ states (left) and $\Delta(\frac{3}{2}^{\pm})$ states (right).

term of interest in $G_{\text{theory}}(t)$ to have conservative error bars. The details of the implementation are discussed in ¹⁶.

Fig. 2 to Fig. 4 show the results in various channels. At large quark masses, the level ordering in the various channels is consistent with quark-model expectations. As the quark mass decreases, however, the states exhibit a strong dependence on the quark mass. In the case of the nucleon channel, there is a dramatic crossover causing the ordering among $N(938)\frac{1}{2}^+$, $N'(1440)\frac{1}{2}^+$, $N(1535)\frac{1}{2}^-$, to be consistent with experiment. This pattern of $++-$ at small quark mass and $+ - +$ at large quark mass is also repeated in the Δ channel. In the Λ channel, near the physical limit, our data strongly hint at a cross-over, giving rise to the inverted ordering of $+ - +$ as compared to the nucleon channel, consistent with the observed spectrum. Note the negative-parity Λ state here is the flavor-singlet $\Lambda_s(1405)\frac{1}{2}^-$. For heavy quarks, on the other hand, the flavor-spin-dominant picture of the quark model ⁵ suggests that this flavor-dependence should not matter and the conventional ordering of $+ - +$, just like other baryons, should emerge. Our lattice data correctly produces this trend at large quark masses. These preliminary results, taken together, demonstrate the importance of covering the small quark mass region below $m_\pi \sim 300$ for baryon spectrum on the lattice. Conventional extrapolation from the relatively large quark masses (around the strange quark mass) is likely to fail.

Further study is under way to assess possible systematic errors, the most important of which is the effect of $N \rightarrow \eta' N$ which is an artifact of the quenched approximation. It makes a negative contribution to the

correlation function and is momentum-dependent. We are working on two ways to evaluate the effects. One is to incorporate in the constrained fit an explicit negative term based on lattice perturbation theory in a manner similar to the treatment of the scalar meson in Ref. ¹⁷. The other method is to do a finite-volume check exploiting the momentum-dependence of these two-particle η N states. It is expected that by shrinking the box size (or 'squeezing' the baryon states), the artifacts will show a strong sensitivity, while the true states survive largely unscathed. To this end, we have repeated the entire calculation on a smaller lattice of $12^3 \times 28$ with all other parameters fixed. The lowest m_π on this lattice is about 250 MeV, still small enough to probe the crossover region. The results of this study will be reported elsewhere.

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