Experimental Constraints on Polarizability Corrections to Hydrogen Hyperfine Structure

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We present a state-of-the-art evaluation of the polarizability corrections—the inelastic nucleon corrections—to the hydrogen ground-state hyperfine splitting using analytic fits to the most recent data. We find a value $\Delta_{\rm pol} = 1.3 \pm 0.3$ ppm. This is 1–2 ppm smaller than the value of $\Delta_{\rm pol}$ deduced using hyperfine splitting data and elastic nucleon corrections obtained from modern form factor fits.

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Hyperfine splitting in the hydrogen ground state is measured to 13 significant figures [1],

$$E_{\rm hfs}(e^- p) = 1420.405\,751\,766\,7(9)\ \text{MHz}. \qquad (1)$$

Theoretical understanding of hydrogen hyperfine splitting is far less accurate, being at about the part-per-million (ppm) level. The main theoretical uncertainty lies in proton structure corrections, which are not presently calculable from fundamental theory. However, proton structure corrections can be calculated as functionals of quantities measurable in other experiments, specifically as integrals [2] over proton form factors measured in elastic electron-proton scattering plus integrals [3–7] over structure functions measured in inelastic polarized electron-proton scattering. The quality of the data for the latter has improved greatly in recent times, especially in the lower momentum transfer region which is important for proton hyperfine corrections. In this article, we present a state-of-the-art evaluation of the polarizability corrections using analytic fits to the most recent data, in particular, using lower momentum transfer data [8–10] from Jefferson Lab.

The proton structure corrections, Δ_S below, can be isolated by taking the experimental values for the hyperfine splitting (hfs) and subtracting the other corrections, which include QED corrections $\Delta_{\rm QED}$, recoil corrections Δ_R^p, as well as some smaller terms due to hadronic vacuum polarization $\Delta_{\rm hvp}^p$, muonic vacuum polarization $\Delta_{\mu\rm vp}^p$, and weak interactions $\Delta_{\rm weak}^p$. One has [11–13]

$$E_{\rm hfs}(e^- p) = (1 + \Delta_{\rm QED} + \Delta_R^p + \Delta_{\rm hvp}^p + \Delta_{\mu\rm vp}^p + \Delta_{\rm weak}^p + \Delta_S)E_F^p, \qquad (2)$$

where the scale is set by the Fermi energy, given by

$$E_F^p = \frac{8}{3\pi}\alpha^3 \mu_B \mu_p \frac{m_e^3 m_p^3}{(m_p + m_e)^3}, \qquad (3)$$

for an electron of mass m_e bound to a proton of mass m_p, magnetic moment $\mu_p = (g_p/2)(e/2m_p)$, Landé g-factor g_p, and fine structure constant α. By convention, the exact magnetic moment μ_p is used for the proton, but only the lowest order term, the Bohr magneton μ_B, is inserted for the e^-.

The proton structure correction inferred from atomic hyperfine splitting data by removing other corrections is

$$\Delta_S = \Delta_Z + \Delta_{\rm pol} = -38.58(16)\ \text{ppm}. \qquad (4)$$

The uncertainty comes mainly from Δ_R^p; we used 5.84 ± 0.15 ppm. The central value [12] uses the CODATA [14] charge radius and the dipole magnetic radius. The uncertainly encompasses a spread due to other choices of these radii. A discussion is in [15]. Other quantities were taken from [12]. Eliminating the large QED corrections by using both hydrogen and muonium hfs [15] leads to a similar result, with an uncertainty limit 0.18 ppm.

The structure-dependent correction Δ_S is conventionally split into two terms, Δ_Z and $\Delta_{\rm pol}$. The bulk of the first term was calculated by Zemach [2] long ago, and the modern expression is

$$\Delta_Z = -2\alpha m_e r_Z (1 + \delta_Z^{\rm rad}), \qquad (5)$$

where the Zemach radius is

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa_p} - 1\right]; \qquad (6)$$

G_E and G_M are the electric and magnetic form factors of the proton, normalized so that $G_E(0) = G_M(0)/(1 + \kappa_p) = 1$, $\kappa_p = (g_p - 2)/2$. The radiative correction $\delta_Z^{\rm rad}$ is estimated in [11] and calculated in [16] for form factors that are represented by dipole forms: $\delta_Z^{\rm rad} = (\alpha/3\pi) \times [2\ln(\Lambda^2/m_e^2) - 4111/420]$. For $\Lambda^2 = 0.71\ \text{GeV}^2$, one finds $\delta_Z^{\rm rad} = 0.0153$; for a Λ^2 corresponding to the relatively large charge radius in [17], one would instead get 0.0150, or a 0.01 ppm change in the hyperfine splitting.

The second term, $\Delta_{\rm pol}$, involves contributions where the proton is excited [3–7]. In the limit where the electron mass is neglected except for one overall factor,

$$\Delta_{\rm pol} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2), \qquad (7)$$

(the prefactor is 0.2265 ppm) with

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\},$$
$$\Delta_2 = -24m_p^2\int_0^\infty \frac{dQ^2}{Q^4}B_2(Q^2). \tag{8}$$

Here F_2 is the Pauli form factor of the proton,

$$B_1 = \int_0^{x_{\rm th}} dx\beta(\tau)g_1(x, Q^2),$$
$$B_2 = \int_0^{x_{\rm th}} dx\beta_2(\tau)g_2(x, Q^2), \tag{9}$$

$$\beta(\tau) = \frac{4}{9}[-3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}]$$

and $\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$, (10)

in which $\tau = \nu^2/Q^2$, ν is the lab-frame energy transfer, Q^2 is the squared four-momentum transfer, $x_{\rm th} = Q^2/(2m_p m_\pi + m_\pi^2 + Q^2)$, m_π is the pion mass, and g_1 and g_2 are the spin-dependent structure functions, measured in doubly polarized electron-proton inelastic scattering.

Reference [7] was to our knowledge the first to use $g_{1,2}$ data to obtain results not consistent with zero for the polarizability corrections. However, Δ_1 and Δ_2 are sensitive to the behavior of the structure functions at low Q^2. Hence with new low Q^2 data available [8,10], there is a significant possibility that the numerical results for $\Delta_{\rm pol}$ could require noticeable revision.

The integrands of Eq. (8) converge because there are factors that cancel the poles at $Q^2 = 0$. The function β_2 has limiting behavior

$$\beta_2(\tau) = \begin{cases} 1/4\tau = Q^2/4\nu^2 & \tau \to \infty, Q^2 \to 0 \\ 1 & \tau \to 0, Q^2 \to \infty. \end{cases} \tag{11}$$

Given that ν is never zero for inelastic processes, even for $Q^2 \to 0$, and noting that the width of the integration region for the B_i is proportional to Q^2 for small Q^2, one sees that the integral for Δ_2 is well behaved for finite g_2.

For the integral Δ_1 to be finite, given

$$\beta(\tau) = \begin{cases} 1 - 5/18\tau & \tau \to \infty \\ 0 & \tau \to 0 \end{cases}, \tag{12}$$

one needs a cancellation that follows from the Gerasimov-Drell-Hearn (GDH) [18,19] sum rule,

$$\lim_{Q^2\to 0}\frac{8m_p^2}{Q^2}\int_0^{x_{\rm th}} dx g_1(x, Q^2) = -\kappa_p^2; \tag{13}$$

$\kappa_p = F_2(0)$ is the proton anomalous magnetic moment.

Further, regarding Δ_2, there is little g_2 data for the proton—there is some from SLAC E155 at higher Q^2 [20] and there is some Jefferson Lab Hall C data at $Q^2 = 1.3$ GeV^2 under analysis [21]. Hence, the g_2 results rely on models. However, the g_2 contributions to the polarizability corrections are small because the weighting factor β_2 in the Δ_2 integral is generally small within the integral. The weighting factor β by contrast is on average close to 1.

Our main results for $\Delta_{\rm pol}$ are detailed in Table I. We evaluated $\Delta_{\rm pol}$ using two different fits to g_1 and g_2 and several different parameterizations of $F_2(Q^2)$.

The current best and most up-to-date parameterization is the one developed by CLAS EG1 [8,9]. This fit begins with the most recent published data [8], which has $Q^2 \gtrsim 0.15$ GeV^2, and is based on AO [22] and MAID [23] parameterizations of the resonances, the E155 fit [24] in the deep-inelastic scattering (DIS) region, and the Wandzura-Wilczek [25] form $g_2^{\rm WW} = -g_1 + \int_x^1 g(y)dy/y$ for g_2 in the DIS region. This fit also gives a good account of the new data [10], which has Q^2 down to 0.05 GeV^2. The other structure function fit is that of Simula *et al.* [26], which is based on data available through the year 2001. Our results using this fit are shown in the last column of Table I, for one $F_2(Q^2)$.

For F_2, we show results using the parameterizations of Kelly [27] and of Sick [17], and include the dipole $F_2(Q^2)$ as a benchmark. Although the dipole fit is a traditional standard, it does not fit modern data well and results obtained from it are not reliable. The Kelly parameterization fits form factor data well overall, and for G_E elects to fit the polarization transfer results [28]. (There has been a discrepancy between the Rosenbluth and polarization transfer determinations of the proton F_2. Theoretical analysis [29] is resolving this by suggesting that two-photon corrections to the Rosenbluth determination at nonzero Q^2 will give agreement with the polarization method.) The continued fraction parameterization of [17] concentrates on the low Q^2 data, and is valid for $0 \le Q^2 \le 0.62$ GeV^2. Beyond this, the $F_2(Q^2)$ contributions to the integrals are small and we substituted the dipole form. This procedure was also used by Friar and Sick [30] in their analysis of the Zemach radius. Substituting the Kelly parameterization instead made no difference on a scale set by our uncertainty limits.

We show our numerical results in Table I. We have split the Q^2 integration into segments [0–0.05] GeV^2 (where no data exist), [0.05–20] GeV^2, and >20 GeV^2, and show contributions from these regions separately. We also separate, except for the lowest Q^2 region for Δ_1, the contributions from F_2, g_1, and g_2.

Three columns in Table I use the CLAS EG1 model [8] for g_1 and g_2. Errors were assigned to the F_2 contribution to Δ_1 using the parameter uncertainties that coherently gave the largest error. For $Q^2 < 0.05$ GeV^2 this error was added in quadrature with a 10% overall systematic uncertainty as a guess about the absolute accuracy of the data at low Q^2. The contribution from g_1 was given a 10% error for the deep-inelastic region ($Q^2 > 20$ GeV^2), and a 50% error ($0.05 < Q^2 < 20$) dominated by the uncertainties in the preliminary CLAS data near $Q^2 = 0.05$ GeV^2. The

TABLE I. Contributions to $\Delta_{\rm pol}$ using various models.

Term	Q^2 (GeV^2)	From	CLAS EG1 Kelly's F_2	CLAS EG1 Sick's F_2	CLAS EG1 Dipole	Using Simula *et al.* g_1, g_2 fit Kelly's F_2
Δ_1	[0, 0.05]	F_2 and g_1	0.45 ± 0.30	0.49 ± 0.30	0.60 ± 0.28	-1.78 ± 0.6
	[0.05, 20]	F_2	7.01 ± 0.22	6.86 ± 0.27	7.12	7.01 ± 0.22
		g_1	-1.10 ± 0.55	-1.10 ± 0.55	-1.10 ± 0.55	-1.78 ± 1.86
	$[20, \infty]$	F_2	0.00	0.00	0.00	0.00
		g_1	0.12 ± 0.01	0.12 ± 0.01	0.12 ± 0.01	0.10 ± 0.01
total Δ_1			6.48 ± 0.89	6.38 ± 0.92	6.74 ± 0.84	3.55 ± 2.48
Δ_2	[0, 0.05]	g_2	-0.24 ± 0.24	-0.24 ± 0.24	-0.24 ± 0.24	-0.72 ± 0.14
	[0.05, 20]	g_2	-0.33 ± 0.33	-0.33 ± 0.33	-0.33 ± 0.33	-1.14 ± 0.23
	$[20, \infty]$	g_2	0.00	0.00	0.00	0.00
total Δ_2			-0.57 ± 0.57	-0.57 ± 0.57	-0.57 ± 0.57	-1.86 ± 0.37
$\Delta_1 + \Delta_2$			5.91 ± 1.06	5.81 ± 1.08	6.18 ± 1.02	1.69 ± 2.51
$\Delta_{\rm pol}$			1.34 ± 0.24 ppm	1.32 ± 0.24 ppm	1.40 ± 0.23 ppm	0.38 ± 0.57 ppm

errors on Δ_2 were taken to be a conservative 100%, since there are no significant systematic measurements of g_2 at low Q^2. To combine errors, we grouped the errors due to F_2, to g_1, and to g_2 into three sets, combined the errors within each set coherently, and then combined the errors from the three sets in quadrature.

When doing the $Q^2 < 0.05$ GeV^2 part of the Δ_1 integral with the EG1 parameterization, we ensured the GDH cancellation by doing a Taylor expansion in Q^2. In terms of the moments

$$\Gamma_{1,2}^{(N)}(Q^2) = \int_0^{x_{\rm th}} x^N g_{1,2}(x, Q^2) dx. \tag{14}$$

[which, at low Q^2, satisfy $\Gamma_{1,2}^{(N)} \sim (Q^2)^{N+1}$] one can expand Eq. (9) and obtain $B_1 = \Gamma_1^{(0)} - 10 m_p^2 \Gamma_1^{(2)}/(9Q^2) + \cdots$. From the generalized forward spin polarizability [31],

$$\gamma_0(Q^2) = \frac{16\alpha m_p^2}{Q^6} \int_0^{x_{\rm th}} x^2 \left(g_1 - \frac{4 m_p^2 x^2}{Q^2} g_2 \right) dx, \tag{15}$$

one obtains $\Gamma_1^{(2)} \to \gamma_0 Q^6/(16\alpha m_p^2)$ as $Q^2 \to 0$, and from experiment [32] $\gamma_0 = [-1.01 \pm 0.08(\text{stat}) \pm 0.10(\text{syst})] \times 10^{-4}$ fm^4. For the lowest moment, we let $\Gamma_1^{(0)} = -\kappa_p^2 Q^2/(8m_p^2) + c_1 Q^4 + \cdots$ at low Q^2, and integrate the low Q^2 part of Δ_1 keeping terms to $\mathcal{O}(Q^4)$. This gives

$$\Delta_1[0, Q_1^2] = \left\{ -\frac{3}{4} r_P^2 \kappa_p^2 + 18 m_p^2 c_1 - \frac{5 m_p^2}{4\alpha} \gamma_0 \right\} Q_1^2, \tag{16}$$

where r_P is the Pauli radius of the proton. The constant in front of the Q^2 term in $\Gamma_1^{(0)}$ reflects the GDH sum rule. Calculations in chiral perturbation theory [33] give $c_1 = 3.89$ GeV^{-4}, whereas a fit of order Q^6 to $\Gamma_1^{(0)}$ using the latest CLAS data [10] in the range [0.05–0.30] GeV^2 yields $c_1 = 2.95 \pm 0.11(\text{stat})$ GeV^{-4}. We used the experimental value 2.95 GeV^{-4} for Table I.

Regarding Δ_2, if we assume $g_2 = g_2^{WW}$ we find that $\Gamma_2^{(N)} = -N\Gamma_1^{(N)}/(N+1)$. Therefore, if we naively extrapolate this relation to low Q^2, we get $\Delta_2[0, 0.05] = -0.40 \pm 0.05$ using γ_0 from above. From Ref. [31], keeping terms to $\mathcal{O}(Q^4)$, we find that $\Delta_2[0, Q_1^2] = 3 m_p^2 Q_1^2 (\gamma_0 - \delta_{LT})/2\alpha$. Using the MAID π-channel estimate [31] $\delta_{LT} = 1.35 \times 10^{-4}$ fm^{-4}, we obtain $\Delta_2[0, 0.05] = -1.4$. Both of these values would lower the final $\Delta_{\rm pol}$. We quote the CLAS EG1 model ($-$ 0.24 $\pm$ 0.24) in Table I.

For the low Q^2 contributions in the Simula *et al.* [26] fit, we straightforwardly integrated the analytic form. The numerical differences from the EG1 result arise because a Taylor expansion of $\Gamma_1^{(0)}(Q^2)$ from Ref. [26] leads to a small curvature parameter c_1.

For all Q^2, the g_1 and g_2 fits of [26] have available error bands for the g_i. To evaluate uncertainties in the polarizability corrections due to uncertainties in the g_i, we recalculated the Δ_i using consistently the largest g_i and smallest g_i. This gives an uncertainty estimate for Δ_2 from g_2 of ± 0.37 units, as quoted in Table I. The uncertainty estimate for Δ_1 due to g_1 and F_2 is ± 2.48 units.

The uncertainty limits involving g_2 on the proton may appear remarkable considering the amount of existing data. However, one expects that much of g_2 is due to the Wilczek-Wandzura term, which is gotten directly from g_1. This can be verified from the much larger body of data for g_2 on the neutron (using polarized ^{3}He targets).

Table II shows Δ_Z and the Zemach radius calculated from the form factor sets used above. Table II also gives $\Delta_S(\text{data}) - \Delta_Z(\text{calc})$, i.e., the value of $\Delta_{\rm pol}$ "desired" for consistency between measurement and proton structure corrections calculated using a given form factor set.

In summary, we quote our best value as

$$\Delta_{\rm pol} = (1.3 \pm 0.3) \text{ ppm}. \tag{17}$$

The earlier value of Faustov and Martynenko [7] was

TABLE II. Pauli and Zemach radii and Δ_Z including $\delta_Z^{\rm rad}$ for several form factors. The column "residual" is Δ_S(from hfs data) $-$ Δ_Z, i.e., the value of $\Delta_{\rm pol}$ that would make theory and data consistent. In this Letter, we obtained $\Delta_{\rm pol} = (1.3 \pm 0.3)$ ppm using inelastic polarized electron-proton scattering data. All uncertainties are calculated from the fits except for Sick's r_Z and Δ_Z, which come from [30].

Form factor	r_P (fm)	r_Z (fm)	Δ_Z (ppm)	Residual (ppm)
Kelly [27]	0.878(15)	1.069(13)	−41.01(49)	2.43(52)
Sick [17]	0.871(35)	1.086(12)	−41.67(46)	3.09(49)
Dipole	0.851	1.025	−39.32	0.74

(1.4 ± 0.6) ppm. It is remarkable that this value, based on few data, agrees with the determination using fits to the extensive CLAS data set. We thus corroborate extractions of hadronic quantities from data as done in [12,13].

We should now focus on modern form factor parameterizations represented by [17] or [27], which fit low Q^2 data well. Accepting these and Table II could lead one to desire a larger $\Delta_{\rm pol}$ to reconcile theory and experiment for hydrogen hfs. The experimentally determined $\Delta_{\rm pol}$ differs from the value deduced from the measured hyperfine splittings by over 4 (6) standard deviations of $\Delta_{\rm pol}$ for Kelly (Sick).

We cannot, in our opinion, anticipate that new proton g_1 or g_2 data will change the evaluation of $\Delta_{\rm pol}$ by enough to reconcile the proton structure corrections with the measured hydrogen hyperfine splittings. For example, to get the requisite $\Delta_{\rm pol}$ by changing the curvature parameter we called c_1, would require making it about 5 times larger than the value we used. This is essentially unthinkable given data already available. A further look at elastic form factor fits that could give a more suitable Zemach radius r_Z could well be warranted.

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