Two-Photon Decays of Charmonia from Lattice QCD

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We make the first calculation in lattice QCD of two-photon decays of mesons. Working in the charmonium sector, using the Lehmann-Symanzik-Zimmermann reduction to relate a photon to a sum of hadronic vector eigenstates, we compute form factors in both the spacelike and timelike domains for the transitions $\eta_c \rightarrow \gamma^* \gamma^*$ and $\chi_{c0} \rightarrow \gamma^* \gamma^*$. At the on-shell point, we find approximate agreement with experimental world-average values.

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The two-photon widths of charmonia have attracted the attention of a range of a theoretical and experimental techniques. Within perturbative QCD, the two-photon branching fraction is believed to give access to the strong coupling constant at the charmonium scale though cancellation of nonperturbative factors, while in quark models and the effective field theory nonrelativistic quantum chromodynamics (NRQCD), two-photon widths have been proposed as a sensitive test of the corrections to the nonrelativistic approximation. Experimental measurements are diverse, coming from, on the one hand, two-photon fusion at e^+e^- machines with subsequent reconstruction of the charmonium in light hadrons and, on the other, $p\bar{p}$ annihilation to charmonium with decay to real $\gamma\gamma$ pairs being detected. Improvements in extracted branching fractions from both methods will soon arrive via more accurate measurements from the Belle, BABAR, and CLEO-c Collaborations of exclusive hadronic η_c decays.

Despite the considerable theoretical and experimental interest, there has not yet been an *ab initio* estimation of two-photon widths in charmonia. In this Letter, we address this issue using a novel application of lattice QCD.

Conventionally, lattice QCD calculations involve the evaluation of the matrix element of an operator between

hadron states. The hadrons are induced using interpolating fields (some combination of quark and gluon fields) with the appropriate hadron quantum numbers. However, these fields will typically have overlap with many excited states which have the same external quantum numbers so that, in order to isolate the ground states, the interpolating fields are taken out to Euclidean times far from the operator (and each other). Such an approach will clearly not work if the initial or final state contains no hadron, as is the case for two-photon decays since the photon is not an eigenstate of QCD. In this case, we can adopt a slightly more sophisticated approach, using the formal relationship between the S matrix and field-theoretic N -point functions and the accurate perturbative expansion in the photon-quark coupling to express the photon as a superposition of QCD eigenstates.

The concept and realization is lucidly presented by Ji and Jung in Refs. [1,2], where they consider the hadronic structure of the photon using lattice three-point functions. We will outline the method as applied to meson two-photon decays.

The amplitude for the two-photon decay of a meson M can be expressed in terms of a photon two-point function in Minkowski space by means of the Lehmann-Symanzik-Zimmermann reduction

$$\langle \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) | M(p) \rangle = - \lim_{\substack{q'_1 \rightarrow q_1 \\ q'_2 \rightarrow q_2}} \epsilon_\mu^*(q_1, \lambda_1) \epsilon_\nu^*(q_2, \lambda_2) q_1^\mu q_2^\nu \int d^4x d^4y e^{iq'_1 \cdot y + iq'_2 \cdot x} \langle 0 | T \{ A^\mu(y) A^\nu(x) \} | M(p) \rangle,$$

up to photon renormalization factors. The explicit photon fields prevent direct computation of this quantity in lattice QCD; however, we can utilize the perturbative expansion of the photon-quark coupling to approximately integrate them out (the path integral over gluon fields is suppressed):

$$\int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_{\text{QED}}[A, \bar{\psi}, \psi]} A^\mu(y) A^\nu(x) = \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_0[A, \bar{\psi}, \psi]} \left(\dots + \frac{e^2}{2} \int d^4z d^4w [\bar{\psi}(z) \gamma^\rho \psi(z) A_\rho(z)] \right. \\ \left. \times [\bar{\psi}(w) \gamma^\sigma \psi(w) A_\sigma(w)] + \dots \right) A^\mu(y) A^\nu(x).$$

The integration over the photon field can be carried out by Wick contracting the fields into propagator products, so that, neglecting disconnected pieces,

$$\langle \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) | M(p) \rangle = (-e^2) \lim_{\substack{q'_1 \rightarrow q_1 \\ q'_2 \rightarrow q_2}} \epsilon_\mu^*(q_1, \lambda_1) \epsilon_\nu^*(q_2, \lambda_2) q_1^\mu q_2^\nu \int d^4x d^4w d^4z e^{iq'_1 \cdot x} D^{\mu\rho}(0, z) D^{\nu\sigma}(x, w) \\ \times \langle 0 | T \{ j_\rho(z) j_\sigma(w) \} | M(p) \rangle.$$

The photon propagator can be written $D^{\mu\nu}(0, z) = -ig^{\mu\nu} \int [d^4k/(2\pi)^4] [e^{ik \cdot z}/(k^2 + i\epsilon)]$, cancelling the inverse propagators outside the integral.

As explained in Ref. [1], the resulting expression can be analytically continued from Minkowski to Euclidean space-time provided the photon virtualities $Q_1^2 = |\vec{q}_1|^2 - \omega_1^2$ and $Q_2^2 = |\vec{q}_2|^2 - \omega_2^2$ are not sufficiently timelike that they can produce on-shell hadrons. In charmonium [3], this limits us to $Q^2 > -m_{J/\psi}^2$. Using a suitable QCD interpolating field to produce M and reversing the operator time ordering for convenience, we have

$$\begin{aligned} \langle M(p) | \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) \rangle = & \lim_{t_f \rightarrow \infty} e^2 \frac{\epsilon_\mu(q_1, \lambda_1) \epsilon_\nu(q_2, \lambda_2)}{\frac{Z_M(p)}{2E_M(p)} e^{-E_M(p)(t_f-t)}} \int dt_i e^{-\omega_1(t_i-t)} \langle 0 | T \left\{ \int d^3\vec{x} e^{-i\vec{p} \cdot \vec{x}} \varphi_M(\vec{x}, t_f) \right. \\ & \times \left. \int d^3\vec{y} e^{i\vec{q}_2 \cdot \vec{y}} j^\nu(\vec{y}, t) j^\mu(\vec{0}, t_i) \right\} | 0 \rangle. \end{aligned} \quad (1)$$

It is clear from the previous discussion that obtaining two-photon widths is a natural extension to the study of radiative transitions carried out in Ref. [4]—there we computed three-point functions involving vector currents with the source (t_i) and sink (t_f) positions fixed and varied the vector current “insertion” (t) across the temporal direction to plot out a plateau. For two-photon widths, we repeat this but with a varying sink [or, in the case of Eq. (1), source] position which will be integrated over with an exponential weighting.

Our calculation was performed in the quenched truncation of QCD using the CHROMA software system. We employed a $24^3 \times 48$ lattice generated using a Wilson gauge action with coupling $\beta = 6.5$ and lattice spacing $a \approx 0.047$ fm (determined from the static quark potential in Ref. [5]). Quark propagators were computed using a nonperturbatively improved Clover action [6] with Dirichlet boundaries in the temporal direction, with the charm quark mass set using the spin-averaged η_c , J/ψ mass, accurate to within 3%. Since we adopted only smeared local fermion bilinears as meson interpolating fields, we had access to three-point functions involving η_c and χ_{c0} mesons. In Ref. [4], typical radii of charmonium states were extracted and found to be much smaller than the ~ 1.2 fm box used here. We adopt the conserved (“point-split”) vector current at the insertion—our experience in radiative transitions is that including the improvement term [7] has an effect below 5% near $Q^2 = 0$.

We applied two different methods to calculate (1). The first (using 174 configurations) was to place the meson state at a fixed sink position $t_f = 37$. As in Ref. [4], the sink was used as a sequential source for a backward propagator inversion, meaning that its properties were fixed for each computation while we were able, without further cost, to vary the direction and momentum of the insertion and the direction of the source field. We then computed with all possible source positions t_i , which, while costly in computing time, allowed us to freely vary the value of ω_1 and, hence, Q_1^2 and, in addition, to view the subsequent integrand. In Fig. 1(a), we display the integrand for insertion positions $t = 4, 16, 32$, $\vec{p}_f = (000)$, and $\vec{q}_1 = (100)$ with an η_c at the sink.

It is clear that, provided the insertion is not placed too close to the Dirichlet wall (i.e., $t \gtrsim 7$), we will be able to

capture the full integral by summing time slices t_i . The integral as a function of insertion position t is shown in Fig. 1(b) for a selection of Q_2^2 with $Q_1^2 = 0$ ($\omega_1 = |\vec{q}_1|$) where we observe plateaus with the deviation from plateau behavior at larger t coming from excited η_c contributions, both of which are fitted simultaneously. Extracting the plateau values for a range of Q_1^2 (which we are free to choose continuously) and Q_2^2 (which is fixed for a given set of $\omega_1, \vec{q}_2, \vec{p}$), we find the dependence displayed in Fig. 2. We plot dimensionless F defined by $\langle \eta_c | \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) \rangle = 2(\frac{2}{3}e)^2 m_{\eta_c}^{-1} F(Q_1^2, Q_2^2) \times \epsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu q_1^\rho q_2^\sigma$, where the on-shell decay width is $\Gamma(\eta_c \rightarrow \gamma\gamma) = \pi \alpha_{em}^2 \frac{16}{81} m_{\eta_c} |F(0, 0)|^2$.

FIG. 1 (color online). (a) Integrator in (1) at three values of vector current insertion time ($t = 4, 16, 32$) with a pseudoscalar sequential source at sink position $t_f = 37$. (b) Pseudoscalar two-photon form factor as a function of time slice t from (1). The first six time slices are ghosted out due to the Dirichlet wall truncating the integral. Constant plus single exponential fits are shown as continuous curves.

In order to more clearly express the voluminous data in Fig. 2, we adopt a simple one-pole parametrization to fit the data for each value of Q_1^2 :

$$F(Q_1^2, Q_2^2) = \frac{\hat{F}(Q_1^2, 0)}{1 + \frac{Q_2^2}{\mu^2(Q_1^2)}}. \quad (2)$$

The curves in Fig. 2 are fits of this form. In the insets in Fig. 2, we display the fit parameters $\hat{F}(Q_1^2, 0)$, $\mu(Q_1^2)$. $\hat{F}(Q_1^2, 0)$ is itself then fitted with a one-pole form with $\hat{F}(0, 0) = 0.168(4)$, $\hat{\mu} = 3.040(20)$ GeV, which is compatible with $\mu(0) = 3.008(16)$ GeV, indicating the reasonableness of the one-pole parametrization in this Q^2 region. The both-photons on-shell point is 3 standard deviations smaller than the value extracted from the Particle Data Group (PDG) [8] decay width $0.274(43)$.

The error quoted on our result is statistical only and must be augmented by an uncertainty due to scaling from our fixed lattice spacing to the continuum and one related to the lack of light-quark loops within the quenched truncation. Since we compute at only one lattice spacing, we cannot accurately determine a scaling uncertainty; however, we expect the nonperturbatively tuned Clover action to have small $\mathcal{O}(a)$ scaling errors. On the other hand, it is possible that we have non-negligible $\mathcal{O}(m_c a)$ scaling which we conservatively estimate to be at the 15% level.

In the NRQCD factorized approach, the decay amplitude is proportional to the η_c wave function at the origin with relativistic corrections. In the quenched theory utilized here, the incorrect running of the coupling down to short distances with the scale set at long distances does typically lead to a depletion of the wave function at the ori-

gin. On the lattice used here, we find [9] $f_\psi = 386(6)$ MeV in comparison to the experimental $411(7)$ MeV; however, we cannot determine to what degree the quenched depletion is offset by $\mathcal{O}(m_c a)$ scaling effects, and as such we assign an estimated 10% error due to quenching.

The similarity of $\mu(0)$ to the J/ψ mass in our calculation [3006(5) MeV] suggests an approximate vector meson dominance description of the form factor. This is at first sight somewhat surprising, given that vector meson dominance (VMD) was demonstrated not to hold for the η_c charge “form factor” in Ref. [4]. This was explained as being due to the presence of very many closely spaced vector poles (the J/ψ , ψ' ...) which must be summed—in contrast to the light-quark sector where the $\rho(1450)$ is rather distant in comparison to the $\rho(770)$ and where VMD tends to be a good approximation.

What we suspect is happening here is that, while the excited ψ poles are not negligible by their remoteness, they are almost negligible by their small residues. Consider the case $Q_1^2 = 0$ and the time ordering in which photon 1 is emitted from the η_c first; then the time-dependent perturbation theory amplitude is proportional to

$$\sum_N \frac{\langle \eta_c | j_\mu(0) | \psi^{(N)} \rangle \langle \psi^{(N)} | j_\nu(0) | 0 \rangle}{m(\eta_c) - E(\psi^{(N)})}. \quad (3)$$

The numerator is essentially the residue of the N th excited ψ pole and is seen to be proportional to the product of the $M1$ transition matrix element between η_c and $\psi^{(N)}$ and the decay constant of the $\psi^{(N)}$. The decay constants fall slowly with increasing N as observed from the experimental e^+e^- widths, while the $M1$ transition amplitudes are expected to fall rather rapidly—they are “hindered” transitions that are proportional to the overlap of orthogonal wave functions with a small correction for recoil. Because of this, only the J/ψ term in the sum has a considerable residue (since the wave function overlap with the η_c is close to 1) and we observe something like VMD. A hint of this behavior was observed in the L3 experiment [10] and a model realization demonstrated in Ref. [11].

The fact that μ varies with Q_1^2 suggests that as this photon goes off-shell the higher ψ resonances get a larger residue (through a larger recoil correction to the hindered transition) and contribute more to the sum over poles moving the effective pole position $\mu(Q_1^2)$.

The method of computing three-point functions with all possible source positions ($t_i = 0 \dots t_f$) is extremely costly in computing resources. With the penalty of losing the ability to freely vary Q_1^2 , we can reduce the computing time by a factor of $\mathcal{O}(L_t)$ by putting the meson interpolating field at the source and using

$$\int dt e^{i\omega_1 t} \int d^3\vec{z} e^{i\vec{q}_1 \cdot \vec{z}} \bar{\psi}(\vec{z}, t) \gamma^\mu \psi(\vec{z}, t) \quad (4)$$

in the sequential source for the backward propagator inversion. It is then necessary to fix ω_1 and $\vec{q}_1$ in advance, and one cannot view the integrand since the integration is

FIG. 2 (color online). η_c two-photon form factor $F(Q_1^2, Q_2^2)$. Points are lattice QCD data; fits are monopole forms as described in the text. The lower left shows fitted amplitudes at $Q_2^2 = 0$ and the lower right the fitted pole masses $\mu(Q_1^2)$.

FIG. 3 (color online). (a) $\eta_c \rightarrow \gamma\gamma^*$ amplitude. (b) $\chi_{c0} \rightarrow \gamma\gamma^*$ amplitude. Fits are monopole forms as described in the text.

being performed “on the fly” within the sequential source. We computed 300 configurations with $Q_1^2 = 0$ in this way, and the results for η_c and χ_{c0} are displayed in Fig. 3. Note that the one-pole fit to the η_c data yields $F(0, 0) = 0.160(8)$ and $\mu = 2.899(71)$, values that are in agreement with those extracted from the method described above.

We define the χ_{c0} two-photon form factor by

$$\begin{aligned} \langle \chi_{c0} | \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) \rangle &= 2 \left(\frac{2}{3} e \right)^2 m_{\chi_{c0}}^{-1} G(Q_1^2, Q_2^2) \\ &\times (\epsilon_1 \cdot \epsilon_2 q_1 \cdot q_2 \\ &- \epsilon_2 \cdot q_1 \epsilon_1 \cdot q_1). \end{aligned} \quad (5)$$

The PDG [8] average of experimental measurements for the two-photon width corresponds to $|G(0, 0)| = 0.150(13)$, with which we are in excellent agreement—our single-pole fit to lattice data returns $|G(0, 0)| = 0.146(16)$ and $\mu = 2.976(81)$ GeV.

The logic used to justify the approximate J/ψ VMD observed for the η_c does not so obviously apply to the χ_{c0} . The radiative transitions to virtual vector meson states are now $E1$ and are not hindered for the excited states to the same extent as $M1$ (this can be seen in, for example, the quark model study [12]). Since the residues of excited state poles are not falling so quickly, they can contribute to the sum and we will not necessarily see dominance of the J/ψ .

Unfortunately, in this case the two nearest poles, the $\psi(3686)$ and the $\psi(3770)$, can conspire to cancel each other’s effect. Using a combination of experimental data

and quark model predictions [12], one estimates that the residues of these two poles are approximately equal (they have a value of about 1/3 the J/ψ residue). If these residues happen to have opposite sign, then the J/ψ pole will dominate with small contributions from the more highly excited ψ resonances. Our data are insufficiently precise to distinguish any deviation from a one-pole behavior.

In summary, we have demonstrated the feasibility of using a suitable sum over lattice time slices to simulate a photon in an external state, using the phenomenologically interesting case of the two-photon decays of charmonia. Our results $\Gamma(\eta_c \rightarrow \gamma\gamma) = 2.65(26)_{\text{stat}}(80)_{\text{scal.}}(53)_{\text{quen.}}^{\text{sys}}$ keV and $\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 2.41(58)_{\text{stat}}(72)_{\text{scal.}}(48)_{\text{quen.}}^{\text{sys}}$ keV are in reasonable agreement with experiment. We believe them to be systematics dominated, in particular, owing to the neglect of light-quark loops and discretization effects related to the heavy-quark mass. The systematic error can be reduced with further computations, which are now warranted given the feasibility demonstrated herein. This lattice technique has the advantage over nonrelativistic models that it can be applied to the light-quark sector without fundamental change.

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