Resonance-DIS transition and low Q^2 phenomena

W. Melnitchouk^a

^aJefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

Recent analyses of electromagnetic structure functions in the resonance region suggest that duality-violating higher twists are small above $Q^2 \sim 1 \text{ GeV}^2$. We review duality at the resonance-scaling transition, both phenomenologically and in the context of a simple quark model. While most studies have focussed on electromagnetic probes, we also discuss expectations for the workings of duality in neutrino scattering. Finally, we describe the transition of the structure functions to the photoproduction limit at $Q^2 = 0$.

1. Introduction

Hadronic cross sections, averaged over appropriate energy intervals, often exhibit the scaling behavior expected from perturbative QCD calculations in terms of quark and gluon degrees of freedom. A remarkable example of such quark-hadron duality is seen in inclusive electron-nucleon scattering, first noted by Bloom and Gilman [1]. Here the inclusive nucleon structure function measured in the region dominated by low-mass nucleon resonance excitations is observed to follow a global scaling function describing the high energy region, to which the resonance structure function averages. In recent years high-precision data from Jefferson Lab, HERMES, and elsewhere on spin-averaged [2] and spin-dependent [3,4] structure functions has enabled the global and local aspects of this ‘‘Bloom-Gilman (BG) duality’’ to be quantified, including its flavor, spin and nuclear medium dependence [5].

2. Duality and QCD

Before the advent of QCD, BG duality was interpreted in the context of finite-energy sum rules, in analogy with the s and t channel duality in hadron-hadron scattering. In QCD, BG duality can be reformulated in the language of the operator product expansion (OPE), where for large Q^2 the moments of structure functions are expanded in inverse powers of the hard momentum

scale Q^2 , with the expansion coefficients related to matrix elements of operators of a specific twist.

For a generic structure function $F(x, Q^2)$, the n -th moment $M^{(n)}$ can be written as

$$M^{(n)}(Q^2) \equiv \int_0^1 dx x^{n-2} F(x, Q^2) \quad (1)$$

$$= \sum_{i=2,4,\dots} \frac{A_i^{(n)}}{Q^{i-2}}, \quad (2)$$

where the expansion coefficients $A_i^{(n)}$ are matrix elements of operators with a specific twist (dimension – spin). The leading twist (twist-2) term, $A_2^{(n)}$, corresponds to scattering from free partons, and is responsible for the scaling (modulo QCD logarithmic corrections) of the structure functions. The higher twist terms $A_{i>2}^{(n)}$ involve multi-quark and mixed quark-gluon operators, and contain information on long-range, non-perturbative correlations between partons.

As pointed out by De Rújula et al. [6], in the OPE language the approximate independence on Q^2 of the moments is naturally attributed to the dominance of the twist-2 term, and suppression (or cancellation) of the higher twist contributions. This observation reveals two important practical applications of duality: if one finds empirically that higher twists are small down to some scale $Q_{\min}^2$, then one can extend leading twist analyses of structure function data to $Q_{\min}^2$. For the unpolarized F_2 structure function of the proton, $Q_{\min}^2$ has been found to be around 1 GeV^2 [2], which is lower than that used in standard parton

distribution analyses. If, on the other hand, duality is found to be violated, and if the violations are not overwhelming, then the data can be used to extract matrix elements of higher twist operators. This provides unique information about the strength of nonperturbative, multi-parton interaction effects in the nucleon.

3. Higher twist matrix elements

In the context of global analyses of parton distributions, higher twist effects are often seen as unwelcome complications. On the other hand, higher twists contain valuable information on the nonperturbative structure of the nucleon, and can therefore provide important insights into quark confinement.

An example of fundamental information on the structure of the nucleon which can be extracted from higher twist matrix elements are the so-called “color polarizabilities” of the nucleon. These describe how the background color electric and magnetic fields respond to the polarization of the nucleon, and are defined as

$$\chi_E \vec{S} = \frac{1}{2M^2} \langle N | \vec{j}_a \times \vec{E}_a | N \rangle, \quad (3)$$

$$\chi_B \vec{S} = \frac{1}{2M^2} \langle N | j_a^0 \vec{B}_a | N \rangle, \quad (4)$$

respectively, where $j_a^\mu = -g\bar{\psi}\gamma^\mu t_a\psi$ is the quark current, t_a are color SU(3) matrices, and $\vec{E}_a$ and $\vec{B}_a$ are the color electric and magnetic fields. The color polarizabilities can be expressed in terms of the twist-3 matrix element d_2 , which is related to the x^2 -weighted moment of the transverse g_2 structure function, and the twist-4 matrix element f_2 , defined as the matrix element of the operator $\bar{\psi}\tilde{G}^{\mu\nu}\gamma_\nu\psi_q$, where $\tilde{G}^{\mu\nu}$ is the dual gluon field strength tensor.

Recently the world data on the proton and neutron (in practice ^3He) spin dependent g_1 structure functions have been reanalyzed [7,8] in order to consistently extract the lowest moment $\Gamma_1(Q^2) = \int_0^1 dx g_1(x, Q^2)$. The order $1/Q^2$ correction to Γ_1 is directly proportional to f_2 , once target mass effects and the g_2 contribution are subtracted. From the values of f_2 obtained in the global analyses [7,8], and the results for $d_2^{p,n}$

from measurements of g_2 , one finds for the color polarizabilities in the proton and neutron:

$$\begin{aligned} \chi_E^p &= 0.026 \pm 0.028, \\ \chi_B^p &= -0.013 \mp 0.014, \end{aligned}$$

$$\begin{aligned} \chi_E^n &= 0.033 \pm 0.029, \\ \chi_B^n &= -0.001 \pm 0.016, \end{aligned}$$

where the error includes statistical and (the more dominant) systematic uncertainties, as well as from the $x \rightarrow 0$ extrapolation and uncertainty in α_s at low Q^2 . These results indicate that both the color electric and magnetic polarizabilities in the proton and neutron are relatively small, with the central values of the color electric polarizabilities being positive, and the color magnetic zero or slightly negative.

The small values of the higher-twist corrections suggest that the long-range, nonperturbative interactions between quarks and gluons in the nucleon are not as dominant at $Q^2 > 1 \text{ GeV}^2$ as one may have expected [9]. For the polarized neutron moment [8], they may even play a minor role down to $Q^2 \approx 0.5 \text{ GeV}^2$. This means that there are strong cancellations between nucleon resonances resulting in the dominance of the leading twist contribution to the moments. To see how such cancellations can take place, we examine a simple model in which the resonance transitions can be evaluated exactly and the degree to which duality holds quantified.

4. Local duality in a simple quark model

Duality for structure function moments can be analyzed in terms of the OPE, however no such simple interpretation exists for the x dependence of the functions themselves, or for integrals over restricted regions of x – known as “local duality”. To understand the emergence of a scaling function out of resonances, one must address the question of how coherent contributions (“squares of sums of quark charges”) can yield results consistent with incoherent scattering (“sums of squares of quark charges”).

Close and Isgur [10] elucidated this problem by establishing the conditions necessary for duality

to occur within the spin-flavor symmetric quark model. They found that for duality to hold at least one complete set of even and odd parity resonances must be summed over. The specific model used was based on spin-flavor SU(6) symmetry, where one can explicitly calculate all of the relative strengths of the contributions to the proton and neutron spin averaged and spin dependent structure functions from the $N \rightarrow N^*$ transition matrix elements, for the lowest even parity 56^+ and odd parity 70^- representations of SU(6).

Summing over all of the states in the 56^+ and 70^- multiplets then “miraculously” gives rise to a neutron to proton ratio $R^{np} \equiv F_2^{en}/F_2^{ep} = 2/3$, and polarization asymmetries $A_1^p \equiv g_1^{ep}/F_1^{ep} = 5/9$ and $g_1^{en}/F_1^{en} = 0$, just as in the quark-parton model in which the structure functions are calculated in terms of partonic (rather than N^*) degrees of freedom. In particular, for the case of g_1^{en} one sees that the cancellations leading to duality already occur *within* each multiplet.

The SU(6) predictions for the structure functions hold approximately at $x \sim 1/3$, however significant deviations are observed at larger x . One can identify several mechanisms of symmetry breaking which lead to results for the large- x structure function that are consistent in the quark and hadron representations. Among these, a suppression of the symmetric configuration at large x gives rise to a suppressed d quark distribution relative to u , which in turn leads to the famous neutron to proton ratio $R^{np} \rightarrow 1/4$ [12]. This is natural if one considers that if the mass difference between the nucleon and Δ is attributed to spin dependent forces, the energy associated with the symmetric part of the wave function will be larger than that of the antisymmetric component.

On the other hand, at large Q^2 one expects transition form factors, which determine the large- x behavior of structure functions, to be constrained by perturbative QCD, which predicts that photons predominantly couple to quarks with the same helicity as the nucleon. In this case the helicity-3/2 cross section will be suppressed relative to the helicity-1/2 cross section, leading to $A_1 \rightarrow 1$ for both protons and neutrons, and that the neutron to proton ratio $R^{np} \rightarrow 3/7$ —

exactly as obtained in the parton level calculation on the basis of perturbative QCD [13]. Whether the $x \rightarrow 1$ behavior of structure functions follows the symmetric wave function suppression scenario or is governed by helicity conservation will be addressed experimentally at Jefferson Lab, where the 12 GeV energy upgrade will allow definitive tests of the properties of structure functions at large x .

5. Neutrino scattering

The prospect of high-intensity neutrino beams at Fermilab [14], as well as in Japan and Europe, offers a valuable complement to the study of duality and resonance transitions. In particular, neutrino-induced reactions can provide important consistency checks on the validity of duality. Such studies will also be crucial for neutrino oscillation experiments, whose interpretation will rely heavily on understanding neutrino–nucleon interactions at low Q^2 .

The main difference between electron and neutrino scattering reactions can be most easily understood considering specific resonance transitions. While a neutrino beam can convert a neutron into a proton, it cannot convert a proton into a neutron, for example (and *vice versa* for an antineutrino beam). Similarly, there are dramatic differences between inelastic production rates in the Δ resonance region [15] — because of charge conservation, only transitions to isospin-3/2 states from the proton are allowed.

A particularly relevant measurement would be of the ratio of neutron to proton neutrino structure functions at large x . Here, similar valence quark dynamics as in charged lepton scattering are probed, but with different sensitivity to quark flavors. At the hadronic level, quark model studies reveal rather distinct patterns of resonance transitions to the lowest-lying positive and negative parity SU(6) multiplets [10,11,15].

The contributions of the $N \rightarrow N^*$ transition matrix elements to the neutrino F_1 and g_1 structure functions of the proton and neutron in the SU(6) quark model were also calculated in Refs. [10,11]. Summation over the $N \rightarrow N^*$ transitions (for the case of equal symmetric and an-

tisymmetric contributions to the wave function) yields the expected SU(6) quark-parton model results, providing an explicit confirmation of duality. On the other hand, some modes of spin-flavor symmetry breaking yield neutrino structure function ratios which at the parton level are in obvious conflict with those obtained from electroproduction [11]. Neutrino structure function data can therefore provide valuable checks on the appearance of duality and its consistency between electromagnetic and weak probes.

Recently a study of duality in neutrino scattering has been undertaken [16] using phenomenologically determined weak transition form factors for both $J = 3/2$ and $J = 1/2$ resonances [17]. Currently there are only rudimentary data on resonances beyond the Δ region, and with more accurate data expected, a precise comparison will be possible in the near future.

6. Transition to low Q^2

As Q^2 decreases from the perturbative regime, eventually a description of the structure function in terms of scattering from a few parton constituents becomes unreliable. For $Q^2 \sim \Lambda_{\text{QCD}}^2$ a perturbative expansion in α_s ceases to be meaningful, as obviously does an expansion in terms of powers of $1/Q^2$. Nevertheless, as one traverses through the intermediate- Q^2 region, where neither the perturbative high-energy nor effective low-energy expansion schemes are applicable, physical quantities should still remain smooth functions of Q^2 , and some sort of duality may still hold in terms of variables other than Q^2 .

There are indeed some empirical indications which suggest the existence of a duality between resonances and continuum cross sections even at the real photon point, $Q^2 = 0$. This may at first sight be surprising since the physics at $Q^2 = 0$ is in some ways qualitatively different from that applicable for deep inelastic scattering at large Q^2 . The former limit is dominated by purely coherent processes, with cross sections generically proportional to the squares of sums of charges of the interacting quarks, $(\sum_q e_q)^2$. Processes at large Q^2 , on the other hand, are dominated by incoherent scattering from individual quarks in the nucleon,

with a strength proportional to sums of squares of the individual quark charges, $\sum_q e_q^2$. The difference between these is then a sum over the nondiagonal contributions, $\sum_{q \neq q'} e_q e_{q'}$, which in general can be as large as the terms diagonal in e_q . Despite this, for real photons the nondiffractive contribution to the ratio of the total neutron to proton cross sections is empirically found to be

$$\frac{\sigma^{\gamma n}}{\sigma^{\gamma p}} \sim \frac{2}{3} \equiv \frac{2e_d^2 + e_u^2}{2e_u^2 + e_d^2}. \quad (5)$$

The ratio thus behaves as if it were given by the squares of constituent quark charges, even though there is no reason for the dominance of the incoherent terms when $Q^2 = 0$.

The behavior of the structure functions in the $Q^2 \rightarrow 0$ limit is to some extent constrained by global symmetries, in particular, by electromagnetic current conservation. The requirement that there be no kinematical singularities at $Q^2 = 0$ implies that for fixed ν , the F_1 and F_2 structure functions at small Q^2 must behave as

$$F_2 = \mathcal{O}(Q^2), \quad (6)$$

$$F_1 + \frac{p \cdot q}{q^2} F_2 = \mathcal{O}(Q^2). \quad (7)$$

Since at fixed ν one has $x \sim Q^2$, this implies that the longitudinal structure function F_L is suppressed by an additional power of Q^2 compared with F_2 in the $Q^2 \rightarrow 0$ limit.

At $Q^2 = 0$ the total (transverse) cross section remains finite, which implies that the F_2 structure function must vanish linearly with Q^2 ,

$$F_2(x, Q^2) \rightarrow Q^2 \text{ as } Q^2 \rightarrow 0, \quad (8)$$

while the F_L structure function vanishes as

$$F_L(x, Q^2) \rightarrow Q^4 \text{ as } Q^2 \rightarrow 0, \quad (9)$$

and therefore the longitudinal to transverse ratio

$$R(x, Q^2) = \frac{\sigma_L}{\sigma_T} \rightarrow Q^2 \text{ as } Q^2 \rightarrow 0. \quad (10)$$

Note that each of these kinematical constraints is a direct consequence of electromagnetic gauge invariance. In the case of neutrino scattering, where there are no such constraints on the axial vector current, the neutrino structure function

F_2^ν does not vanish in the $Q^2 \rightarrow 0$ limit, but by PCAC is proportional to the total πp cross section, $F_2^\nu \sim \sigma^{\pi p} \rightarrow \text{constant}$.

An important question at low Q^2 is whether gauge invariance is the only physics which underlies the transition to $Q^2 = 0$, or whether there is additional dynamics which drives the transition. Some clues to this may be provided by the low- Q^2 data on $F_2^p(x, Q^2)$ measured recently at Jefferson Lab [2]. These experiments found that the F_2^p structure function at low Q^2 does not follow the linear $\sim Q^2$ behavior that would be expected from gauge invariance constraints alone, but instead behaves as $F_2 \sim Q^{0.5}$ down to $Q^2 \sim 0.3 \text{ GeV}^2$. This indicates that these values of Q^2 are still transitional, and that lower Q^2 is needed before the behavior becomes driven by gauge invariance.

Further progress in revealing the dynamics of the $Q^2 \rightarrow 0$ transition can be made with additional data on spin-dependent structure functions at low Q^2 . For the g_1 and g_2 structure functions there are no gauge invariance constraints as for F_2 and F_L , although sum rules such as the Gerasimov-Drell-Hearn and Burkardt-Cottingham sum rules provide some constraints on their moments at $Q^2 = 0$ (see Ref. [5]).

7. Outlook

The discussion of duality in the simple quark model provides a clear illustration of how parton model results can be replicated by explicit sums over nucleon resonances. An important challenge in the future will be to consistently include the effects of both resonances and the nonresonant background in the same framework.

Future experimental exploration of duality will focus on determining its flavor, spin and target dependence, as well as its workings when probed by the weak interaction, in neutrino-nucleon scattering. Another important avenue will be to explore duality in semi-inclusive reactions. Confirmation of duality here would open the way to an enormously rich semi-inclusive program in the preasymptotic regime, allowing unprecedented access to the spin and flavor distributions of the nucleon, especially at large x .

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