THE $Q^2$ EVOLUTION OF THE GDH SUM RULE 
(ON $^3$HE AND THE NEUTRON)

GORDON D. CATES
(FOR THE JLAB E94-010 COLLABORATION)
Department of Physics, University of Virginia, Charlottesville, VA 22903

We discuss the extension of the Gerasimov-Drell-Hearn (GDH) sum rule, which pertains to real photons, to include scattering due to virtual photons. We present data from Jefferson Laboratory experiment E94-010 which measured the inclusive scattering of polarized electrons from a polarized $^3$He target over the quasielastic and resonance regions. From these data we extract the transverse-transverse interference cross section $\sigma_{TT}$, and compute the $Q^2$ dependent extended GDH integral.

1 Introduction

The study of spin structure has proven to be a valuable tool for understanding quantum chromodynamics (QCD). It is natural, therefore, to turn to spin structure to understand the complex transition from parton-like behavior in deep inelastic scattering to hadronic-like behavior that is observed at lower energies. It is thus useful to identify observables that might reveal the relevant dynamics in this transition. One example is the extended GDH integral $I(Q^2)$, a quantity that is intimately connected to both high and low energy sum rules.

In this paper, we begin by discussing the extended GDH sum rule, its connection to other sum rules, and what can be learned through its study. We then go on to present data on the process $^3$He $(e,e')$ taken at Jefferson Laboratory during E94-010, the first experiment to use a polarized $^3$He target at JLab. From this data we extract the transverse-transverse interference cross section $\sigma_{TT}$, and compute the extended GDH integral $I(Q^2)$ for the range $0.1 \text{GeV}^2 < Q^2 < 1.0 \text{GeV}^2$.

2 The extended GDH sum rule

An important sum rule is that due to Gerasimov, Drell, and Hearn (GDH) that relates a sum over the total spin-dependent photoabsorption cross sections of the nucleon to the square of the anomalous magnetic moment $\kappa$ of the
nucleon\(^2\). The sum rule can be written
\[
\int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[ \sigma_{1/2}(\nu) - \sigma_{3/2}(\nu) \right] = -\frac{2\pi^2\alpha}{M^2\kappa^2},
\]
where \(\sigma_{1/2}(\nu)\) (\(\sigma_{3/2}(\nu)\)) is the cross section for absorption when the total spin of the photon and nucleon, projected onto the photon momentum direction, is 1/2 (3/2). The integral is performed over the photon energy \(\nu\), beginning at the pion production threshold \(\nu_0\). Here \(\alpha\) is the fine structure constant, and \(M\) is the mass of the nucleon.

The GDH integral can be generalized to include not just the absorption of real photons, but the exchange of virtual photons as occurs in electron scattering. This corresponds to values of the four-momentum transfer squared \(Q^2 > 0\), in contrast to the absorption of real photons for which \(Q^2 = 0\). Perhaps the most natural way to make this extension is to convert the photoabsorption cross sections of the GDH sum rule into electroproduction cross sections:
\[
I(Q^2) = \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[ \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) \right] = 2 \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \sigma'_T(\nu, Q^2)
\]
where \(\sigma_{1/2}(\nu, Q^2)\) and \(\sigma_{3/2}(\nu, Q^2)\) are spin-dependent total virtual photoabsorption cross sections on the nucleon. The generalized GDH integral is constrained at two limits:
\[
\text{As } Q^2 \to 0 \quad I(Q^2) \to -\frac{2\pi^2\alpha}{M^2\kappa^2}
\]
\[
\text{As } Q^2 \to \infty \quad I(Q^2) \to \frac{16\pi^2\alpha}{Q^2}\Gamma_1
\]
where \(\Gamma_1 = \int_0^1 g_1 dx\) is the first moment of the spin structure function \(g_1\). The first relation follows from the GDH sum rule (for \(Q^2 = 0\)), and the second relation follows from the definition of \(I(Q^2)\), and the asymptotic form of \(\sigma'_T\).

It is certainly desirable to be able to equate \(I(Q^2)\) to some quantity throughout all values of \(Q^2\), and thus have a true sum rule. It has recently been shown by Ji and Osborne that
\[
I(Q^2) = 2\pi^2\alpha S_1(0, Q^2)
\]
where \(S_1(0, Q^2)\) is the forward virtual Compton amplitude. To the extent that we consider \(S_1(0, Q^2)\) well defined everywhere, this expression gives us a true extended GDH sum rule. Using chiral perturbation theory, \(S_1(0, Q^2)\) can be computed for small values of \(Q^2\), probably up to values of around 0.1 GeV\(^2\), or perhaps even larger.

At large values of \(Q^2\), one can also make definite statements about the Compton amplitudes. For instance, beginning with the operator product
expansion for the Compton amplitudes for the proton and the neutron, it can be shown that

\[ S_1^p(0, Q^2) - S_1^n(0, Q^2) = \frac{4}{3Q^2} g_A , \] (6)

where the superscripts \( p \) and \( n \) refer to the proton and the neutron. This relation, in combination with equation (4), brings us to the equation

\[ \Gamma_1^p - \Gamma_1^n = \frac{1}{6} g_A \] (7)

which we recognize as the Bjorken Sum Rule\(^8\). Similarly, it follows that

\[ I_1^p(Q^2) - I_1^n(Q^2) = \frac{8 \pi^2 \alpha}{3Q^2} g_A \] (8)

showing that at the very least the extended GDH sum rule for the difference of the proton and the neutron is well defined for \( Q^2 \to \infty \). In fact, relations for the proton and neutron taken individually also exist\(^9\). For the sake of brevity, however, we will stop here.

There are also important experimental constraints on \( I(Q^2) \). At \( Q^2 = 0 \), there are recent measurements of the GDH sum rule on the proton\(^10\), although the neutron has yet to be investigated. At high \( Q^2 \), there is also an extensive body of data on both \( \Gamma_1^p \) and \( \Gamma_1^n \),\(^11\) although most of the data is in the deep inelastic regime where \( Q^2 > 1 \text{ GeV}^2 \).

In summary, there are both theoretical and experimental constraints on \( I(Q^2) \) over a wide range of values of \( Q^2 \), spanning the region where the nucleon is governed by hadronic degrees of freedom and non-perturbative QCD, to the region where the nucleon is governed by quark degrees of freedom and perturbative QCD. At low \( Q^2 \) chiral perturbation theory provides theoretical guidance, while at high \( Q^2 \) techniques involving the operator product expansion are useful. In the important intermediate transition regime there is at present little theoretical guidance. Since we are concerned with the first moments of the spin structure functions, however, rather than the spin structure functions themselves, it is believed that lattice QCD will be an effective tool\(^12\).

3 Experimental measurement \( I^n(Q^2) \)

To investigate \( I^n(Q^2) \), and more generally the low \( Q^2 \) spin structure of the neutron, we investigated the process \( ^3\text{He} (e, e') \) in Hall A of Jefferson Laboratory. We took data at six energies: 5.06, 4.24, 3.38, 2.58, 1.72, and 0.86 GeV, all at a nominal scattering angle of 15.5°. The measurements covered values
of the invariant mass $W$ from the quasielastic peak through the resonance region, into the deep inelastic regime, as is shown in Fig. 1.

![Figure 1](image.png)

Figure 1. Shown are the regions in the space spanned by $Q^2$ and $W$ covered by our kinematics. For each energy, each block of space represents a separate spectrometer setting.

Because we were studying inclusive scattering, we had the opportunity to use both of the Hall A high resolution spectrometers for the detection of electrons. Particle identification was accomplished using Čerenkov detectors and Pb-glass shower counters. Momentum analysis was accomplished using drift chambers.

Our experiment was the first at Jefferson Laboratory to utilize the Hall A polarized $^3$He target$^{1,13,14}$. In fact, the experiment was one of the initial motivations for the target's construction. The target utilizes the technique of spin-exchange optical pumping in which rubidium (Rb) vapor is polarized using lasers, and the $^3$He is polarized during subsequent collisions$^{15}$. The $^3$He is contained in sealed glass "target cells" at a pressure of roughly 10 atmosphere. An example of a target cell is shown in Fig. 2. Also contained in the cell is a few droplets of Rb metal and about 70 Torr of nitrogen ($N_2$). The $N_2$ is present to "quench" the excited states of Rb without the emission of a photon. Otherwise, the emitted photons, which in general do not have the desired state of polarization, would tend to depolarize the Rb vapor. The target cells are comprised of two chambers: an upper pumping chamber, in which the optical pumping and spin exchange take place, and a lower target chamber, through which the electron beam passes. The temperature of the
pumping chamber is regulated in order to control the number density of the Rb vapor. It is also irradiated with a 90 W laser system with a wavelength centered at 795 nm, corresponding to the D₁ line of Rb. The lower chamber is approximately 40 cm in length and has thin “end windows” about 140–150 μm in thickness.

![Pumping Chamber](image)

**Figure 2.** Shown is one of the E94-010 polarized ³He target cells.

The ³He target system included two pairs of Helmholtz coils that produced a static magnetic field of approximately 25 G. The polarization was monitored using the NMR technique of adiabatic fast passage (AFP). The NMR system was calibrated in two ways. In one approach, a glass cell with a geometry closely approaching that of the target cell was filled with water and placed in the apparatus. The small but well defined polarization of the water due to the thermal Boltzman distribution was detected and the resulting water signals were compared with the ³He signals. In another approach, an oscillator was locked to the electron paramagnetic resonance (EPR) frequency corresponding to the Zeeman splitting between two $m_F$ levels of the $^{87}$Rb. A small shift in the EPR frequency occurred because of an effective magnetic field caused by the presence of the polarized ³He. The relationship between the shift and the absolute polarization of the ³He has been well characterized in separate experiments. By measuring the EPR shift, which yields an absolute polarization measurement, in close proximity in time to measuring ³He AFP signals, a second calibration of the NMR system resulted.
4 Extracting spin observables

We made measurements with the target polarization in both the longitudinal and transverse orientations. The quantities we measured experimentally are related to $\sigma'_{TT}$ and $\sigma'_{LT}$ by the relations

$$\frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega dE'} = B (\sigma'_{TT} + \eta \sigma'_{LT})$$

(9)

and

$$\frac{d^2\sigma^{\downarrow\uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega dE'} = B \sqrt{\frac{2\epsilon}{1 + \epsilon}} (\sigma'_{LT} - \zeta \sigma'_{TT})$$

(10)

where the left side of eq. (9) (eq. (10)) corresponds to the spin asymmetries measured for parallel (transverse) target orientations. Also $B = -2(\alpha/4\pi^2)(K/Q^2)(E'/E)(2/(1 - \epsilon))(1 - E'/E)$, $E$ and $E'$ are the initial and final energies of the electron, $\epsilon^{-1} = 1 + 2[1 + Q^2/4M^2x^2] \tan^2(\theta/2)$, $\theta$ is the scattering angle in the laboratory frame, $\eta = \epsilon \sqrt{Q^2/(E - E'\epsilon)}$, and $\zeta = \eta(1 + \epsilon)/2\epsilon$. The quantity $K$ represents the virtual photon flux and is convention dependent. We use the convention $K = \nu - Q^2/2M$, due to Hand

The virtual photoabsorption cross sections $\sigma'_{TT}$ and $\sigma'_{LT}$ correspond to the Born Approximation in which scattering is due to a single virtual photon. In practice, the measured cross sections include higher order radiative processes, and it is necessary that these be accounted for when extracting the Born cross sections. The procedure used was that due to Mo and Tsai, which in their first treatment, dealt strictly with unpolarized scattering. For polarized scattering, we used the program POLRAD, which we modified in a number of ways. In particular, we incorporated our data for the quasielastic and resonance regions.

5 Preliminary results

The data, with radiative corrections, are shown in Fig. 3a, where $\sigma'_{TT}$ is plotted as a function of invariant mass $W$ for each of the six energies measured. The error bars indicate the uncertainties due to statistics, and the bands indicate the errors due to systematic effects, which include uncertainty in the beam and target polarizations, uncertainty due to radiative corrections, and uncertainty in the measured cross sections.

Because we are interested in the $Q^2$ evolution of the GDH integral, we have determined $\sigma'_{TT}$ for various values of constant $Q^2$. This was done by
Figure 3. Preliminary data from JLab E94-010. Shown in (a) are measured values of $Q^2$. In (b) $\sigma_{\gamma\gamma}$ is plotted as a function of energy loss $\nu$ for each of six constant values of $Q^2$. Solid circles indicate interpolated values and open circles indicate extrapolated values.

Considerable care was taken to ensure a smooth interpolation, including accounting as much as possible for the physics determining $\sigma_{\gamma\gamma}$ at each point. Several different methods of interpolation/extrapolation were tried to gain a
sense of the systematic uncertainty of the process. Typically the error incurred was roughly 5% of \( \sigma'_{TT} \), although it was somewhat worse in the neighborhood of the delta. The additional error due to interpolation/extrapolation is folded into the systematic errors mentioned earlier in the plotted error bands.

We have computed the extended GDH integral \( I(Q^2) \) using equation (2). For each \( Q^2 \), we have computed the integral from the pion threshold up to a \( \nu \) corresponding to a \( W \) of 2 GeV. The results are shown in Fig. 4 as open circles, where the error bars represent the uncertainty due to systematics only. The error associated with systematic effects is shown with a dark band. Because our measurements were made using a \(^3\)He target, we must apply a correction to account for the fact that the neutron was imbedded in the nucleus. Here we have used the prescription of C. Ciofi degli Atti and S. Scopetta \(^{20}\), whose technique is essentially an impulse approximation that takes into account the relative polarizations of the protons and neutrons in the \(^3\)He nucleus. The authors estimate that for the extended GDH integral this approach should be accurate at roughly the 5% level. The results for \( I(Q^2) \) for the neutron are shown with open squares. The last correction we consider is an estimate off the extended GDH integral for the integrating range of \( 4 \text{ GeV}^2 < W^2 < 1000 \text{ GeV}^2 \). For this we use the parameterization due to Bianchi and Thomas\(^{21}\), and the results are shown with the closed squares.

Ideally one would like the GDH integral to provide insight into the transition from hadronic behavior at low \( Q^2 \) to partonic behavior at high \( Q^2 \). Toward this end, it is desirable to compare with theoretical predictions that contain as much physical content as possible. At the \( Q^2 = 0 \) point, the GDH sum rule provides a solid prediction, which is shown on Fig. 4 with an eight-pointed star. Chiral perturbation theory can also be used to extend this prediction to small values of \( Q^2 \). Here, a calculation due to Ji, Kao, and Osborne is shown with a dotted line. At high values of \( Q^2 \), operator product expansion (OPE) techniques can be used to expand the Compton amplitudes in a power series of \( \alpha_S \) and \( 1/Q^2 \). Our results can either be used to check the results of OPE calculations, or to determine some of the coefficients that appear. For reference, we also show in Fig. 4 a calculation due to Drechsel, Kamalov, and Tiator\(^{22}\) that is based largely on the phenomenological model MAID. While the basic shape is in agreement with our observations, the overall magnitude is clearly smaller than our data.

In conclusion, we believe that the study of the \( Q^2 \) evolution of the GDH integral offers a valuable opportunity to study a single observable over the important transition from hadronic to partonic behavior. Our data is the first such data on the neutron, and provides a valuable glimpse of the current state of agreement between experiment and theory. Further developments,
References

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Institute in both the experimental and theoretical fronts, should provide important

an estimate of the unmeasured portion of the integral.

open squares above a correction to account for nuclear effects, and the closed squares include
no correction for the fact that the neutron is embossed in a finite nucleus. This
data with open circles represent our

Figure 4. Preliminary results from JLAB E94-010 for \( Q^2 \). The open circles represent our

\( Q^2 (GeV^2) \)

\( 0 \)

\( 0.1 \)

\( 0.2 \)

\( 0.3 \)

\( 0.4 \)

\( 0.5 \)

\( 0.6 \)

\( 0.7 \)

\( 0.8 \)

\( 0.9 \)

\( 1 \)

\( 1.1 \)

\( 1.2 \)

\( 1.3 \)

\( 1.4 \)

\( 1.5 \)

\( 1.6 \)

\( 1.7 \)

\( 1.8 \)

\( 1.9 \)

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\( 2.5 \)

\( 3 \)

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\( 9 \)

\( 9.5 \)

\( 10 \)

\( 0 \)

\( 50 \)

\( 100 \)

\( 150 \)

\( 200 \)

\( 250 \)

\( 300 \)

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\( 165 \)

\( 170 \)

\( 175 \)

\( 180 \)

\( 185 \)

\( 190 \)

\( 195 \)

\( 200 \)

\( \text{GDH Integral (arb.)} \)
22. D. Drechsel, S.S. Kamalov, and L. Tiator, Phys. Rev. D 63, 114010 (2001). Note, model $I_C$ is plotted in Fig. 3.