Isospin Breaking in the Goldberger-Treiman Discrepancies

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Abstract

Effects of isospin breaking at the level of the Goldberger-Treiman discrepancies involving the neutral isotriplet axial and pion-nucleon couplings are analyzed to leading non-trivial order in chiral perturbation theory.

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In the baryon sector, the most direct manifestation of the explicit breaking of chiral symmetry by the quark masses is in the deviations from the Goldberger-Treiman relations (GTRs)[1]. The GTRs are exact in the chiral limit and relate the baryon's axial-vector couplings to the corresponding couplings of the Goldstone Bosons (GBs) (π , K , η) to baryons. The deviations produced by explicit chiral symmetry breaking, known as the GT discrepancies (GTDs), are defined by:

$$\frac{m_b + m_c}{2} g_A^{abc} = (1 - \Delta^{abc}) g_{abc} F_a, \quad (1)$$

where $a, b, \dots = 1, \dots, 8$, m are masses of the baryon octet, g_A are axial-vector couplings, g are baryon-GB couplings and F are decay constants. The GTDs Δ^{abc} are quantities of $\mathcal{O}(p^2)$. It was first shown in[2] that the chiral corrections of $\mathcal{O}(p^3)$ to the GTDs are absent. This result implies that relations among the GTDs, such as the Dashen-Weinstein relation, that are exact at LO, will show rather small deviations. The GTDs at LO are determined entirely by terms of $\mathcal{O}(p^3)$ in the baryon chiral Lagrangian[3]. In standard notation of HBChPT (see for instance[4]), there are three such terms, namely:

$$\begin{aligned} \mathcal{L}_{GTD}^{(3)} = & -i F_{19} \langle \bar{B}_v S_v^\mu [\nabla_\mu \chi_-, B_v] \rangle - i D_{19} \langle \bar{B}_v S_v^\mu \{ \nabla_\mu \chi_-, B_v \} \rangle \\ & - i b_{20} \langle \bar{B}_v S_v^\mu B_v \rangle \langle \nabla_\mu \chi_- \rangle. \end{aligned} \quad (2)$$

Neglecting possible electromagnetic contributions, the GTDs are given by:

$$\begin{aligned} \Delta^{\pi^0 p \bar{p}} &= \frac{2B_0 m_0}{g_{\pi^0 p \bar{p}} F_\pi} ((F_{19} + D_{19}) m_u + b_{20} (m_u - m_d)) \\ \Delta^{\pi^0 n \bar{n}} &= -\frac{2B_0 m_0}{g_{\pi^0 n \bar{n}} F_\pi} ((F_{19} + D_{19}) m_d + b_{20} (m_d - m_u)) \\ \Delta^{\pi^- p \bar{n}} &= \frac{\sqrt{2} B_0 m_0}{g_{\pi^- p \bar{n}} F_\pi} (F_{19} + D_{19}) (m_u + m_d) \\ \Delta^{K \Lambda \bar{N}} &= -\frac{B_0 m_0}{\sqrt{3} g_{K \Lambda \bar{N}} F_K} (3F_{19} + D_{19}) (m_s + \hat{m}) \\ \Delta^{K^+ \Sigma^- \bar{N}} &= \frac{\sqrt{2} B_0 m_0}{g_{K^+ \Sigma^- \bar{N}} F_K} (F_{19} - D_{19}) (m_s + \hat{m}), \end{aligned} \quad (3)$$

where m_0 is the baryon octet mass in the chiral limit and B_0 is proportional to the chiral condensate. Isospin breaking has been kept only in the non-strange sector. A best fit in the isospin limit using the couplings $g_{\pi N \bar{N}}$, $g_{K \Lambda \bar{N}}$, and $g_{K \Sigma \bar{N}}$ from available analyses[5], leads to[2]:

$$F_{19} + D_{19} = 0.58 \pm 0.17 \text{ GeV}^{-2}, \quad D_{19} = 0.37 \pm 0.15 \text{ GeV}^{-2} \quad (4)$$

FIG. 1: The cross indicates the insertion of $\bar{q}\gamma_5 q$, and the solid lines are baryons.

In this talk the isospin breaking between the first two GTDs in Eqn. (3) is addressed. A complete analysis requires the low energy constant b_{20} to be known as well as the possible EM contributions.

- b_{20} : it can be obtained from the matrix elements in the baryon octet of the singlet SU(3) pseudoscalar current. This is used to build a model with the assumption that the dominant contribution to the matrix elements is given by the coupling of the current to the η' meson as shown in Fig. 1. In large N_c this dominance is a consequence of the behavior of the η' mass in the chiral limit, namely $M_{\eta'}^2 = \mathcal{O}(1/N_c)$. This model implies that:

$$2 D_{19} + 3 b_{20} \sim \sqrt{\frac{3}{8}} \frac{F_{\eta'}}{m_0 M_{\eta'}^2} g_{\eta' N \bar{N}}. \quad (5)$$

The only quantity that is not well known on the RHS of Eqn. (5) is $g_{\eta' N \bar{N}}$. It is sufficient to consider the chiral limit where it satisfies the $U_A(1)$ GTR:

$$g_{\eta' N \bar{N}} = \frac{m_0}{F_{\eta'}} (g_A^0 + \sqrt{6} \tilde{b}), \quad (6)$$

where g_A^0 is the singlet axial coupling that is determined from polarized DIS: $g_A^0|_{\text{pDIS}} = 0.20 - 0.35$ [6], and $\tilde{b}$ is related to the short distance contribution of the $U_A(1)$ anomaly matrix element between nucleons:

$$\tilde{b} m_0 \bar{N} \gamma_5 N = \frac{\alpha_s}{4\pi} \langle N | G \tilde{G} | N \rangle_{\text{contact}} \quad (7)$$

Besides this relation, it is known that $\tilde{b}$ is of order $1/N_c$ while g_A^0 is of zeroth order. Consistent with the model in large N_c , $\tilde{b}$ could be neglected. Then Eqn. (6) implies that $g_{\eta' N \bar{N}} \sim 3$.

On the other hand, it turns out that in the same limit:

$$g_{\eta' N \bar{N}} \sim \sqrt{\frac{8}{3}} \frac{m_0}{F_{\eta'}} D, \quad (8)$$

where D is the usual SU(3) axial coupling whose value is approximately 0.6, implying that $g_{\eta' N \bar{N}} \sim 9$. The large N_c limit is not very good here as the two different ways of extracting the coupling lead to disparate results. This is of course related to the small contribution by the quark spins to the spin of the proton. It is reasonable to take these disparate values as bounds on $g_{\eta' N \bar{N}}$. With this and the values of the low energy constants in Eqn. (4), b_{20} is in the range:

$$-0.45 \text{ GeV}^{-2} \leq b_{20} \leq -0.30 \text{ GeV}^{-2} \quad (9)$$

One good measure of the isospin breaking is given by the asymmetry:

$$A_I = 2 \frac{\Delta^{\pi^0 p \bar{p}} - \Delta^{\pi^0 n \bar{n}}}{\Delta^{\pi^0 p \bar{p}} + \Delta^{\pi^0 n \bar{n}}} = \frac{m_u - m_d}{\hat{m}} \left(1 - \frac{b_{20}}{F_{19} + D_{19}}\right). \quad (10)$$

The values of b_{20} and $F_{19} + D_{19}$ obtained above lead to

$$A_I = \frac{m_u - m_d}{\hat{m}} (0.36 \pm 0.25). \quad (11)$$

Since the isospin breaking quark-mass ratio involved is about -0.57 the asymmetry is expected to be in the range -6% to -34% . The important conclusion at this point is that the contributions proportional to the low energy constant b_{20} tend to reduce the isospin breaking in the GTDs considered above.

• **EM contributions:** A fortunate aspect of the GTDs considered above is that the EM contributions vanish at $\mathcal{O}(p^2)$. Deferring the details to a forthcoming publication[7], it can be shown that after including electromagnetism, the only possible diagrams that can contribute to the GTDs studied above are of the type shown in Fig. 2. Explicit calculation shows that the contributions to the GTDs are proportional to $\alpha_{\text{EM}} M_\pi^2$. In the low energy counting where α_{EM} is taken to be $\mathcal{O}(p^2)$ these EM contributions are NLO, *i.e.* $\mathcal{O}(p^4)$. A more extensive analysis of EM corrections including the third GTD, namely $\Delta^{\pi^- p \bar{n}}$, will be presented elsewhere[7].

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FIG. 2: One loop EM corrections that contribute to GTDs.

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