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NUCLEAR PHYSICS

THE ELECTRIC FORM FACTOR OF THE NEUTRON AT
 $Q^2 = 1.17$ AND 1.47 (GeV/c)^2 FROM THE ${}^2H(\vec{e}, e'\vec{n}){}^1H$ REACTION (159 pp.)

Directors of Dissertation: Bryon D. Anderson and Richard Madey

The Jefferson Laboratory E93-038 collaboration measured the ratio of the electric form factor to the magnetic form factor of the neutron, $g \equiv G_E^n/G_M^n$, via recoil polarimetry from the quasielastic ${}^2H(\vec{e}, e'\vec{n}){}^1H$ reaction at $Q^2 = 0.45, 1.17$ and 1.47 (GeV/c)^2 in Hall C of the Thomas Jefferson National Accelerator Facility. A polarimeter designed specifically for E93-038 was used to measure the up-down scattering asymmetry from the transverse component of the recoil neutron's polarization vector, and a dipole magnet located in front of the polarimeter was used to precess the polarization vector in the scattering plane through an angle of χ . Sequential measurements of the scattering asymmetry with the polarization vector precessed through angles $\chi = 0^\circ$ and $\chi = \pm 90^\circ$ for $Q^2 = 1.47 \text{ (GeV/c)}^2$ were made during January 2001 and through angles $\chi = \pm 40^\circ$ for $Q^2 = 1.17 \text{ (GeV/c)}^2$ during April 2001 and will be reported on in this dissertation. This ratio method removes the need to know the analyzing power of the polarimeter and the incident electron beam polarization.

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CHAPTER 1

Introduction

This dissertation will describe a polarization-transfer measurement of the electric form factor of the neutron G_E^n conducted at the Thomas Jefferson National Accelerator Facility (TJNAF) in Newport News, Virginia. The experiment utilized the longitudinally polarized continuous electron beam produced by the superconducting accelerator available at Jefferson Laboratory to measure G_E^n from the reaction $^2\text{H}(\vec{e}, e'\vec{n})^1\text{H}$. Measurements were made at three central squared four-momentum transfer, Q^2 , values of 0.45, 1.15, and 1.47 $(\text{GeV}/c)^2$. This dissertation is concerned primarily with the measurements at 1.17 and 1.47 $(\text{GeV}/c)^2$ and will be concentrated on that particular result although the experimental setup and overview is valid for all cases. The technique utilized in this experiment was of particular interest because the final results are insensitive to meson-exchange currents (MEC), final-state interactions (FSI), isobar configurations(IC), and the choice of deuteron wave function[1]. The experiment that pioneered the use of this technique was conducted at the MIT-Bates Linear Accelerator Center[2, 3, 4] and this experiment is a derivative of that one.

This experiment used the process of polarization-transfer from the longitudinally polarized electrons to the neutrons in a liquid deuterium target from the $^2\text{H}(\vec{e}, e'\vec{n})^1\text{H}$ reaction. The polarimeter designed for this experiment[5] was sensitive only to the transverse component of the polarization of the recoil neutron. In order to employ the ratio method discussed within this dissertation, measurements of two different

projections of the neutron's polarization vector in the transverse direction were performed. This was accomplished by placing a dipole magnet between the target and the polarimeter to precess the polarization vector through an angle χ . The ratio of electric to magnetic form factors $g \equiv G_E^n/G_M^n$ is proportional to the ratio of the transverse and longitudinal polarization components. It should be noted that the analyzing power of the polarimeter cancels in the ratio. Knowledge of the analyzing power of the polarimeter was a major unknown in the experiment and is difficult to determine to a high precision. The cancellation of the analyzing power allows this experiment to produce a precise measurement of the electric form factor without the need to run another difficult experiment to measure the analyzing power.

This dissertation is broken down into the following chapters. Chapter 1 contains introductory material including descriptions of past G_E^n experiments and a discussion of the electron-deuteron scattering process employed in this experiment. The experimental arrangement is covered in Chapter 2. This chapter covers details on the electron accelerator, the Møller polarimeter, cryogenic targets, the electron spectrometer, and the dipole precession magnet. The details of the setup and calibration of the neutron polarimeter (NPOL) are covered in Chapter 3. A brief overview of neutron polarimetry is given along with discussions on scintillator detectors and the polarimeter calibration. Data analysis details are discussed in Chapter 4. The choice of event cuts is discussed along with the evaluation of the effects of background events on the data. An overview of the G_E^n calculation from the experimental asymmetry is also covered. The final chapter will discuss results and conclusions from the experimental data and detailed systematic uncertainties and corrections to the final G_E^n result.

1.1 Historical Overview

Work began on the study of nucleon structure in the 1940's. Dirac's relativistic theory of the electron[6] and Mott's consideration of the scattering of electrons from point-like nucleons[7] formed the central basis for the studies that ensued after World War II. Two papers in 1947 described the attempt to interpret the results of experiments that scattered thermal neutrons from atomic electrons in bound atomic orbits. Havens, Rabi, and Rainwater[8] scattered thermal neutrons from lead, and Fermi and Marshall[9] used the noble gas xenon as their target. This work was attempting to separate the cross sections for scattering from the bound electrons from the very much larger scattering from the nucleus. Far from being concrete proof, the results of these experiments were suggestive that meson theories of the neutron deserved further consideration and experimentation. Hence, a step toward the idea that the neutron (and the proton) have intrinsic structure was advanced.

Rosenbluth[10] in 1950 produced an equation for the elastic scattering process between an electron and a proton. This formulation can be extended to the neutron as described in Schiff's paper[11]. The cross section for electron scattering from a free nucleon can be calculated from the Rosenbluth formula,

$$(1.1) \quad \sigma = \sigma_o \left\{ F_1^2 + \left(\frac{q^2}{4M^2} \right) [2(F_1 + \kappa F_2)^2 \tan^2 \frac{1}{2}\theta + (\kappa F_2)^2] \right\}$$

$$(1.2) \quad \sigma_o = \frac{e^4 \cos^2 \frac{1}{2}\theta}{4E^2 [1 + (2E/M) \sin^2 \frac{1}{2}\theta] \sin^4 \frac{1}{2}\theta}$$

$$(1.3) \quad q^2 = \frac{4E^2 \sin^2 \frac{1}{2}\theta}{1 + (2E/M) \sin^2 \frac{1}{2}\theta}.$$

Here σ_o is the Mott scattering of electrons from a point-like nucleon, κ is the anomalous magnetic moment of the nucleon, θ is the scattering angle of the electron, M is the nucleon mass, q^2 is the magnitude of the four-momentum transfer, and the charge and magnetic form factors are F_1 and F_2 respectively. This particular form of the Rosenbluth formula was taken from Schiff[11]. This work by Rosenbluth was an attempt to describe the process of electron scattering from a nucleon and produced a basis by which future high energy experiments could interpret their results in terms of Quantum Electrodynamics (QED).

It was during the 1950's that the first attempts to study the electromagnetic structure of the nucleons were put forward. Hofstadter and collaborators at the new Stanford high-energy physics laboratory performed electron scattering experiments on various targets to measure the magnetic and charge distributions of the proton and the neutron[12, 13, 14, 15]. A review article[16] written in 1958 summarized all the work up to that point pertaining to the electromagnetic structure of the proton and neutron. The basic conclusion was that the cross sections for electron scattering differed from the point-like theories significantly; therefore, the proton and neutron (nucleons) could not be considered point-like. Theoretical development was still needed in order to account for these results. Quantum Chromodynamics (QCD) is a theory developed to explain the structure of the nucleon and mesons. QCD states that the nucleon is the bound state of three constituent partons called quarks. These bound states have internal degrees of freedom (spin, angular momentum, color, etc.) that can be excited to form other states. Each of these three constituent quarks have a fractional electronic charge and their position in the nucleon determines the distribution of the electric charge which is measured by the electric form factor. The

motion of these constituent quarks forms the magnetic moment distribution which is measured by the magnetic form factor.

1.2 Probing Nucleon Structure with the Electron

The electron has been used to probe the electromagnetic structure of the neutron for over sixty years. Beginning with the experiments by Havens, Rabi, and Rainwater[8] and Fermi and Marshall[9], the electron has been used to probe the characteristics of the neutron. By the 1950's, it was realized that both the neutron and the proton had internal structure and attempts both theoretical and experimental were being made to determine the exact characteristics of this internal structure. A natural question arises about this electron-nucleon interaction — why is it so useful? First, because the electron does not interact through the strong interaction, the process of scattering electrons from nucleon targets is governed by Quantum Electrodynamics (QED). QED is a well established theory and can calculate, in principle, processes to a high precision. The second reason for the electron's usefulness is the weak coupling, on the order of the fine structure constant, of the interaction to the structure of the nucleon. This means the process of measuring the nucleon structure can be made without disturbing the internal structure of the nucleon.

The following subsections will go into more detail about nucleon structure and the process of electron scattering for this experiment.

1.2.1 Nucleon Structure

The electromagnetic structure of the neutron can be described as the distribution of charge, electric form factor, and the distribution of current, magnetic form factor, within the neutron. It is of interest to the scientific community to understand how the charge and current is distributed within the neutron. One would like to map this out directly but it is impossible to control the impact parameter of a colliding electron with the precision sought by experimentalists. Instead, it is possible to control the momentum transferred to the neutron by choosing the desired kinematic setup of the detectors. Actually, it is the four-momentum transferred squared, Q^2 , that is controlled because it is a Lorentz invariant. Through the measurement of cross sections, it is possible to “map out” the distribution of charge or current density in momentum space as a function of q^2 . The appropriate transformation from momentum space to position space will give the functional form of the charge or the current distribution in position space.

In order to get a basic understanding, it is necessary to do a brief overview of the QED theory leading up to the form factors. Halzen and Martin[17] present a nice development of the form factors and I will follow that development here. Halzen and Martin’s presentation is focused on the proton, but this picture can be generalized to include neutrons as shown by Foldy[18] and more generally by Salzman[19]. Figure 1.1 shows the lowest-order Feynman diagram of electron-nucleon scattering.

The transition amplitude for this scattering is given by

$$(1.4) \quad T_{fi} = -i \int j_\mu \left(-\frac{1}{q^2} \right) J^\mu d^4x.$$

Figure 1.1: First-order Feynman diagram for electron-nucleon scattering. The p , p' , k , and k' values are the four-momentum of the electron and nucleon respectively. The four-momentum transferred is represented by q and equals $p' - p$. The square of the four-momentum transfer is a Lorentz invariant.

Here $q = p' - p$ and the transition currents for the electron and the nucleon are

$$(1.5) \quad j^\mu = -e\bar{u}(p')\gamma^\mu u(p) \exp^{i(p'-p)\cdot x}$$

$$(1.6) \quad J^\mu = e\bar{u}(k') \left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M}F_2(q^2)i\sigma^{\mu\nu}q_\nu \right] u(k) \exp^{i(k'-k)\cdot x},$$

respectively. Notice that in the limit $q^2 \rightarrow 0$ the wavelength of the virtual photon that is probing the nucleon becomes too long to probe the internal structure. The photon effectively sees the whole particle so limits can be placed on the values of the F_1 and F_2 factors.

$$(1.7) \quad F_1^p(q^2 = 0) = 1, \quad F_2^p(q^2 = 0) = 1$$

$$(1.8) \quad F_1^n(q^2 = 0) = 0, \quad F_2^n(q^2 = 0) = 1$$

Calculating the differential cross section for this process using Eqs. 1.5 and 1.6 results in the Rosenbluth formula Eq. 1.1. At this point it should be noted that F_1 and F_2 are not true measures of the charge and current distributions within the nucleon. Investigations by Yennie et al.[20] and Ernst et al.[21] showed linear combinations of these two factors produce a cleaner interpretation of the charge and current distributions.

$$(1.9) \quad G_E \equiv F_1 - \frac{\kappa Q^2}{4M^2} F_2$$

$$(1.10) \quad G_M \equiv F_1 + \kappa F_2$$

These are referred to as the Sachs form factors after the original work done by R.G. Sachs and collaborators and shown rigorously by Sachs in 1962[22].

Once an expression for the form factors of the nucleon is obtained, the charge and current distributions as a function of position space can be found by performing a Fourier transform on the form factors in the appropriate reference frame. Using the Breit frame (sometimes called the “brick-wall” frame), it is possible to show that the form factor is the Fourier transform for the charge and current distributions (see Sachs 1962 paper[22]). The Breit frame is defined as $\vec{k} = -\vec{k}'$ and no energy is transferred to the target nucleon.

1.2.2 Electron-Deuteron Scattering

The advancement of technologies for producing polarized targets and polarized beams required the development of theoretical methods for understanding the scattering of polarized leptons. Dombey[23] formulated a density matrix for the virtual photon initiating the reaction between a lepton and a nucleon. The main result that

Dombey claimed is that the density matrix presents a virtual photon in the pure polarization state. Dombey went on to interpret this as meaning the process of polarized lepton scattering is equivalent to the scattering of a polarized photon [viz., a lepton with a polarization that is the superposition of a transverse elliptic component and a longitudinal component]. Dombey then went on to calculate the elastic scattering formula and showed that an interference term, $G_E G_M$, in the cross section could determine the sign on G_E^n by flipping the sign on the target polarization.

Arnold, Carlson, and Gross[24] reported in a paper on elastic electron-deuteron scattering that the form factors of the nucleons have a large effect on extracting the deuteron structure functions $A(Q^2)$ and $B(Q^2)$. Namely the choice in the neutron electric form factor's functional form has strong effects on the structure functions. This trio in a 1981 paper[25] proposed three polarization-transfer experiments to better determine the deuteron charge and quadrupole form factors and the electric form factors of the neutron and proton. For the neutron, they proposed that the neutron electric form factor could be measured to higher precision by measuring the polarization of the recoil neutron from the quasielastic ${}^2H(\vec{e}, e'\vec{n}){}^1H$ reaction. This reaction is a double scattering experiment because the recoil neutron's polarization must be detected in a polarimeter which requires that the recoil neutron be scattered by an analyzer to measure the scattering asymmetry.

The cross section for this double scattering process is as follows:

$$(1.11) \quad \frac{d^2\sigma}{d\Omega d\Omega_2} = \frac{d^2\sigma}{d\Omega d\Omega_2} \Big|_0 (1 + ap_x A_y \sin \phi_2)$$

where a is the electron polarization, $\frac{d^2\sigma}{d\Omega d\Omega_2} \Big|_0$ is the double scattering cross section with unpolarized electrons, and ϕ_2 is the angle between the secondary scattering plane and the primary scattering plane (Fig. 1.2). The only nonzero polarizations

Figure 1.2: This definition of the electron-nucleon scattering plane and the second scattering plane was adopted from Arnold, Carlson, and Gross[25]. The electron scattering plane is defined to be the xz coordinate plane where z is parallel to the direction of travel of the recoil nucleon and x is perpendicular. The angle θ is the electron scattering angle, θ_2 is the secondary scattering angle in the secondary scattering plane, and ϕ_2 is the angle between the electron scattering plane and the secondary scattering plane.

are represented by the following equations adopted from Arnold et al.[25]:

$$(1.12) \quad I_0 P_x = -(G_E^n G_M^n) K_S,$$

$$(1.13) \quad I_0 P_z = (G_M^n)^2 K_L,$$

$$(1.14) \quad I_0 = (G_E^n)^2 + (G_M^n)^2 K_0,$$

where P_x and P_z are the recoil neutron's polarization components. Here the kinematic factors are

$$(1.15) \quad K_S = 2\sqrt{\tau(1+\tau)} \tan(\theta/2),$$

$$(1.16) \quad K_L = 2\tau\sqrt{(1+\tau)(1+\tau\sin^2(\theta/2))} \frac{\sin(\theta/2)}{\cos^2(\theta/2)},$$

$$(1.17) \quad K_0 = \tau[1 + 2(1+\tau)\tan^2(\theta/2)],$$

$$(1.18) \quad \tau = \frac{Q^2}{4M^2},$$

θ is the electron scattering angle (Fig. 1.2), and M is the neutron mass. Dividing Eq. 1.12 by Eq. 1.13 yields

$$(1.19) \quad \frac{P_x}{P_z} = -\frac{K_S}{K_L} \frac{G_E^n}{G_M^n}$$

which can be rearranged to

$$(1.20) \quad \frac{G_E^n}{G_M^n} = -\frac{K_L}{K_S} \frac{P_x}{P_z}.$$

The ratio of electric to magnetic form factors is defined as $g \equiv G_E^n/G_M^n$ so the following is true (replacing $x = S'$ and $z = L'$)

$$(1.21) \quad g \equiv \frac{G_E}{G_M} = -\frac{K_L}{K_S} \frac{P_{S'}}{P_{L'}}.$$

Thus, if a measurement of the transverse(S') and longitudinal(L') components of the recoil neutron's polarization can be made, then the ratio g can be extracted. More discussion on how this is done using the measured asymmetry of the polarimeter will

be presented in Chapter 4.

1.3 Experimental Basis

Some of the originating experimental work was mentioned in an earlier section of the history of nucleon structure. Since the first experiments on the electron-neutron interaction[8, 9], many attempts have been made to assess quantitatively the nucleon form factors. The proton being a charged object as a whole was somewhat easier to measure because its form factors are larger. It also helps that a free proton target exists in the form of hydrogen. Neutron form factors are a bit more difficult to measure because the value of the form factor is smaller and there are no free neutron targets. One has to use a composite target such as deuterium or helium-3 that requires corrections to be made for the presence of other nucleons in the nuclear medium. The following two sections are an attempt to cover some(definitely not all) of the experiments that have been performed since 1950 to measure the neutron form factors.

1.3.1 Prior Experiments

Experiments of the 1950's at Stanford's high-energy physics laboratory began the study of the internal structure of the proton and the neutron[11, 12, 13, 14, 15, 16]. The first of these experiments focused on investigating the structure of the proton, but interest in the neutron was alive as well. Schiff[11] pointed out that the magnetic moment measurements made by previous experiments were also dependent on the electric form factors. That stems from the fact that a magnetic moment is a measure of the current distribution which depends on the location of the charge.

Budnitz et al. in the late 1960's reported on work done with quasielastic electron-deuteron scattering to measure the electric form factor of the neutron[26, 27]. This work reported values for the neutron-to-proton cross section ratio σ_n/σ_p that would not fit the two possibilities at the time:

$$(1.22) \quad G_E^n(q^2) = 0$$

$$(1.23) \quad G_E^n(q^2) = -\tau G_M^n(q^2).$$

Here τ is equal to $q^2/4M^2$. The first case Eq. 1.22 leads to the wrong slope for G_E^n at lower q^2 values, while the second case Eq. 1.23 predicts ratios of σ_n/σ_p to be much larger than the measured values. Budnitz et al. then proposed that an ad hoc parameterization could be chosen that has the slope of Eq. 1.23 at low q^2 and then goes over to Eq. 1.22 at higher q^2 values. This ad hoc analytic form is

$$(1.24) \quad G_E^n(q^2) = \left(\frac{-\tau}{1 + p\tau} \right) G_M^n(q^2).$$

where $p = 4$. It should be noted that there was, and still is, no reason to assume this analytic form has anything to do with the physical world. The authors were attempting to find an analytical form which could be used to empirically fit the data known at that time.

In 1971, Galster et al.[28] in Hamburg, Germany performed an elastic electron-deuteron scattering experiment to measure the $A(q^2)$ value of the deuteron. G_E^n for the neutron was also extracted from the data. A fit to the data was calculated using the ad hoc relationship Eq. 1.24. This time the fit produced a value of $p = 5.6$ as the best value for this parameterization. This became known as the Galster parametrization after the groups lead author S. Galster[28]. Although Eq. 1.24 has no connection with a physical theory, it has, like a lot of empirical relationships, earned a place in the

scientific literature because of its simple form and its ability to give at least a rough estimate of where to expect future G_E^n measurements to fall. The work by Galster and colleagues was good at the time and would stand for some time to come, but the G_E^n results were plagued by strong model dependencies. In order to extract G_E^n from the $A(q^2)$ data, it was necessary to assume a model of the deuteron to produce the deuteron wave functions for the extraction process.

A more recent $A(q^2)$ deuteron structure function experiment was performed in Saclay, France. This experiment took more data over a wider range of q^2 values, with better statistical errors than the previous experiments. Platchkov et al.[29] analyzed these data for the neutron form factor as well. They compared the results to previous data and along with doing a comparison on how the results depend on the choice of nucleon-nucleon (NN) model. The Platchkov results showed that the impulse approximation predictions are not sufficient to explain the $A(q^2)$ data. The addition of meson-exchange currents (MEC) and relativistic corrections (RC) improve the agreement between theory and data. The G_E^n results from this experiment were more precise than from previous experiments. The uncertainties were small enough to distinguish between different NN potential models. These results for G_E^n showed also reasonable agreement with the Galster parameterization mentioned earlier[28]. One major point that the author made in the conclusion was that the $A(q^2)$ data becomes increasingly dependent on MEC and RC at higher Q^2 values, and so does the difference between different NN potentials. This makes it increasingly difficult to extract a reliable value for G_E^n from this method. They point out that above about 1 $(\text{GeV}/c)^2$ it would become necessary to employ a different experimental approach to determining G_E^n , such as recoil polarization or polarized electron on polarized target

measurements.

Another method of determining the form factors is called Rosenbluth separation. Chen et al.[30] used this method to measure the electromagnetic form factors of the proton at high momentum transfers. This is done by measuring the cross section for electron-proton scattering at the same Q^2 but different angles by changing the incident energy. A plot of the ratio of the measured cross section to the Mott cross section versus $\tan^2(\theta/2)$ (Eq. 1.25) can be fitted with a straight line[31]. The slope of the line is $2\tau(G_M^p)^2$ and the intercept at $\theta = 0$ is $[(G_E^p)^2 + \tau(G_M^p)^2]/(1 + \tau)$.

$$(1.25) \quad R = \frac{(d\sigma/d\Omega)_{meas}}{(d\sigma/d\Omega)_{Mott}} = 2\tau(G_M^p)^2 \tan^2(\theta/2) + \frac{(G_E^p)^2 + \tau(G_M^p)^2}{(1 + \tau)}$$

This method works nicely for unbound particles such as protons but it is a different story for neutrons. There are no free neutron targets, so a deuterium target is used in its place.

A bit of theoretical work and some algebra is needed in order to refashion the Rosenbluth formula to deal with the bound state of the deuteron. A group at SLAC[32] performed the necessary steps. The resulting “reduced” cross section for the quasielastic electron-deuteron scattering is related to the electric and magnetic form factors of both the proton and the neutron all multiplied by an over all factor[3]. This multiplicative factor is an integral over the deuteron S-state and D-state wave functions. One has to assume a NN potential model in order to calculate this integral over S-state and D-state wave functions. It should be noted that the final state interactions (FSI) and meson-exchange currents (MEC) were not included in the Lung et al.[32] analysis and could contribute significantly to the final result. Lung et al.[32] reported G_M^n results consistent with the dipole form and G_E^n results consistent with zero.

1.3.2 Recent Experiments Utilizing Polarization

More recently, polarized electron beams and polarized targets have become regularly available. The 1990's have seen a shower of experiments aimed at determining the electric form factor of the neutron to higher precision than before. The model dependencies of the experiments which do not use spin polarization are left behind (at least to a certain extent) by the employment of experiments with spin polarization included.

Many experiments employing inclusive $[^3\vec{H}e(\vec{e}, e')]$ and exclusive $[^3\vec{H}e(\vec{e}, e'n)]$ reactions on $^3\vec{H}e$ targets have been performed. Woodward et al.[33, 34, 35], Thompson et al.[36], and Kievsky et al.[37] performed inclusive $^3\vec{H}e$ experiments in the 1990's. The principle behind these experiments is that the scattering asymmetry of polarized electrons from $^3\vec{H}e$ is due principally to the unpaired neutron in the $^3\vec{H}e$ target nucleus[38]. Polarization of the target results in the spins of the two protons being approximately canceled (anti-aligned) and the neutron spin is nearly aligned with the $^3\vec{H}e$ spin. The doubly differential cross section for scattering polarized electrons from $^3\vec{H}e$ is

$$(1.26) \quad \frac{d^2\sigma}{d\Omega dE} = \Sigma + h\Delta(\theta^*, \phi^*)$$

where the electron helicity $h = \pm 1$. This equational form was taken from Donnelly and Raskin[39]. The angles θ^* and ϕ^* define the direction of the target polarization. The asymmetry is the ratio between the spin-dependent and the spin-independent cross sections, as written in Donnelly and Raskin[39],

$$(1.27) \quad A = \frac{\Delta(\theta^*, \phi^*)}{\Sigma} = -\frac{\cos \theta^* \nu_{T'} R_{T'} + 2 \sin \theta^* \cos \phi^* \nu_{TL'} R_{TL'}}{\nu_L R_L + \nu_T R_T}.$$

The $\nu_{T'}$, $\nu_{TL'}$, ν_L , and ν_T are kinematic factors. R_L and R_T factors stem from the unpolarized cross section and are the longitudinal(transverse) response functions. These two functions are functions of both energy loss and four-momentum squared. The $R_{T'}$ ($R_{TL'}$) response functions do not contribute to the unpolarized cross sections. According to theoretical work done by Blankleider and Woloshyn[38], the $R_{T'}$ response function is sensitive to the square of the neutron magnetic form factor, $(G_M^n)^2$, and $R_{TL'}$ is sensitive to the product of the neutron form factors, $G_E^n G_M^n$. It should be noted that the experimental asymmetry is related to this asymmetry by the electron polarization p_e and the target polarization p_T .

$$(1.28) \quad A_{exp} = p_e p_T A$$

The results of these inclusive experiments have gone a long way to increase the understanding of the electric form factor. The earliest experiments suffered from lack of statistics to produce a result with small enough error bars to contribute to the understanding of G_E^n . These experiments did show that the method for determining the electric form factor was feasible. The contribution from final state interactions (FSI) and meson-exchange currents (MEC) were not included in most of the early work. These contributions have been considered to be small up to this point but further study is necessary.

A group at the Mainz Microtron (MAMI) in Mainz, Germany performed the exclusive quasielastic scattering experiment of polarized electrons from polarized ^3He [40]. The exclusive process requires the detection of the recoil neutron in coincidence with the scattered electron. This method is powerful because the coincidence measurement reduces the amount of background events by a considerable amount. The measurement at Mainz was made for a Q^2 value of $0.31 \text{ (GeV}/c)^2$.

An attempt to extend the knowledge to higher momentum transfers using electron-deuteron elastic scattering data was performed by Schiavilla and Sick[41]. The power of this method lies in the exploitation of the quadrupole deuteron form factor, $F_{C2}(q)$, instead of the deuteron structure function, $A(Q)$, which is a combination of the monopole and quadrupole form factors. They reanalyzed the world's data for elastic electron-deuteron scattering to extract parameters for the three deuteron form factors $F_{C0}(q)$, $F_{M1}(q)$, and $F_{C2}(q)$. Then using theoretical models they extract G_E^n by bringing the theoretical models for $F_{C2}(q)$ into agreement with the experimental value. This method resulted in values for G_E^n in the large Q^2 region with significantly smaller error bars than Lung et al.[32].

Up to this point the discussion has been limited to polarized electron scattering from polarized ^3He . It is also possible to measure neutron form factors using the reactions $^2\vec{H}(\vec{e}, e'n)^1H$ and $^2H(\vec{e}, e'\vec{n})^1H$. The former was used in the Jefferson Laboratory experiment E93-026 at $Q^2 = 0.5 \text{ (GeV/c)}^2$ [42] and more data were taken in the fall of 2001 for a higher Q^2 point of $\sim 1.0 \text{ (GeV/c)}^2$ as of yet unpublished. This method is similar to the previous method for extracting form factors from a polarized ^3He target.

The second reaction $^2H(\vec{e}, e'\vec{n})^1H$ requires the use of a polarimeter to measure the polarization transferred from the incident electron to the neutron in the deuterium target. This reaction is the focus of this experiment and is described in more detail in the rest of this dissertation. A previous experiment using this method was conducted in the early 1990's at the MIT Bates electron accelerator at $Q^2 = 0.255 \text{ (GeV/c)}^2$. Eden et al. reported a value of $G_E^n = 0.066 \pm 0.036 \pm 0.009$ and thus proved the feasibility of this method[4].

1.4 Motivations

The electromagnetic structure of the nucleons is very important to nuclear physics and models of the nucleon. Besides its fundamental interest to the physicist, precise knowledge of the electromagnetic structure of the proton and neutron is necessary to distinguish between different models for the nucleon. Because the form factor describes the location and movement of charge within the nucleon, these values provide tests for any model that describes subnucleonic degrees of freedom, such as the quark model. Presently, proton form factors have been well established experimentally. The magnetic form factor of the neutron has been established to the 10% level for some time now. The electric form factor is less well-known and is the subject of this work.

The lack of free neutron targets makes the measurements difficult and has presented a tough challenge to experimentalists. Experimental methods that take advantage of the advances in polarized electron accelerators and polarized targets have greatly improved the situation but challenges still exist. The extraction of form factor information from data is still difficult because of the nature of the neutron. The electric form factor is a small quantity and to produce a result with 10-15% relative errors requires large amounts of beam time, and because beam time is limited, experiments are limited as well. There is much discussion among scientists about the effects of the nuclear medium and the effects of final state interactions (FSI), meson-exchange currents (MEC), and isobar configurations (IC).

It is clear that a better measurement of the neutron form factor and measurements at higher Q^2 values is important. This experiment and this dissertation are dedicated to this pursuit.

CHAPTER 2

Experimental Arrangement

This chapter will discuss the equipment utilized to measure the quantities of interest for determining G_E^n . The experiment was setup in Hall C of the Thomas Jefferson National Accelerator Facility (from now on called Jlab) during the summer of 2000. Data were collected from September 2000 through April 2001. The following sections will describe in greater detail the different parts of the experimental arrangement. A separate chapter will be devoted to the neutron polarimeter because it was the primary contribution by the Kent State collaboration to the experiment and was not a permanent piece of equipment in Hall C.

Jlab is located in Newport News, Virginia and is managed by the Southeastern Universities Research Association (SURA) for the U.S. Department of Energy. Jlab employs approximately 550 scientists, technicians, administrators, and support staff to maintain and operate a 6 GeV superconducting polarized electron accelerator and three experimental halls (Hall A, Hall B, and Hall C). Jlab is also the home of other technologies and areas of research which the curious reader can look up at <http://www.jlab.org>.

2.1 Electron Accelerator

The electron accelerator consisted of three main parts. The polarized electron source, the accelerator oval, and the experimental Hall C. I will discuss each of these in the next three subsections in greater detail. The accelerator had many subsystems

which were essential to accelerator operations but not necessary to my discussion. It should be noted that the purpose of this discussion is to present a general description of the delivery of polarized electrons from the electron source to the target. This will not be an attempt to present a technical discussion on accelerator operations. For that I refer the interested reader to Jefferson Laboratory for more information.

2.1.1 Polarized Electron Injector

Many of the experiments conducted at Jlab require high beam polarization and high average beam currents. These requirements put heavy demands on the injector group to develop and maintain a state-of-the-art polarized injector. The details of the polarized injector are best left to those in charge of its operation. In this section, I will attempt to highlight the important details of the polarized source and injector.

Polarized sources have been under study for many years. The crystal of choice at Jlab is the GaAs crystal. The left side of Fig. 2.1 shows the band-level diagram for the GaAs crystal. The maximum in the valence band (VB) is aligned at the Γ point in momentum space with the minimum in the conduction band (CB). This alignment in momentum space allows optical transitions between energy bands that follow the angular momentum selection rules for optical transitions in atoms [43, 44]. Optical transitions to the conduction band occur when the crystal is illuminated with circularly polarized light with energy between E_{CB} and $E_{CB} + E_{VB}$. Right circularly polarized light results in transitions shown by the right side of Fig. 2.1 with the solid lines. Transitions caused by left circularly polarized light are indicated by the dashed lines. Using the relevant Clebsch-Gordan coefficients it can be shown that the relative transitions from the $P_{3/2}$ state to the $S_{1/2}$ state is one $P_{3/2}$, $m_j =$

Figure 2.1: Energy level and transition diagram for a GaAs cathode.

$+1/2$ electron going to the $S_{1/2}$, $m_j = -1/2$ state for every three $P_{3/2}$, $m_j = +3/2$ electrons going to the $S_{1/2}$, $m_j = +1/2$ state. This results in a theoretical limit to the polarization of 50% in the conduction band from the two-fold degeneracy of the $P_{3/2}$ state. Note that increasing the incident photon energy to near the $E_{CB} + E_{VB}$ value results in transitions from the $P_{1/2}$ state to the $S_{1/2}$ state resulting in a decreasing polarization[43].

A way to separate this degeneracy is to add a strain to the crystal in the direction of crystal growth. This is done by growing a thin layer of GaAs crystal on top of a layer of bulk crystal with a slightly different lattice constant. As the GaAs crystal grows on top of the bulk crystal with a different lattice constant, this introduces a strain in the direction perpendicular to the crystal's surface [43, 45]. This strain results in a band gap between the two degenerate states in the $P_{3/2}$ band (Fig. 2.2). If the strain

is sufficient, the photon energy can then be adjusted to a value between E_{CB} and $E_{CB} + E_{strain}$ and only the $P_{3/2}, m_j = +3/2$ to $S_{1/2}, m_j = +1/2$ transitions occur. This results in a theoretical limit to the polarization of 100%. In practice polarizations above 80% have been routinely achieved and maintained for long periods of time at Jlab.

Figure 2.2: Energy level and transition diagram for a strained GaAs cathode.

One of the biggest problems with the delivery of polarized beam is the small quantum efficiencies of strained GaAs crystals. Quantum efficiencies are usually less than 1% which requires high laser powers in order to produce the high beam currents demanded by experiments at Jlab. As an example this experiment, E93-038, often ran at 50 to 75 μ A. Combining the low initial quantum efficiencies (QE) with the processes that degrade the crystal's QE over time, we found that it can be nearly impossible to maintain these high beam currents for long periods of time. The injector group solved this problem by switching E93-038 over to the high-power Ti-Sapphire laser in place of the diode laser system. The Ti-Sapphire laser improved crystal lifetime by

delivering more power to the crystal versus the older system that used diode lasers. This increased the time between intrusive procedures that are needed to restore the crystal QE [46, 47].

Once the polarized electrons are produced, they must go through a relatively complicated procedure of bunching and chopping the beams. It should be noted that the accelerator accelerates not one but three interlaced electron beams simultaneously; one for each experimental hall. The bunching and chopping section of the accelerator is often called the 100 keV line since it accelerates the electrons to 100 keV. A non-superconducting standing wave accelerator section captures the 100 keV beam and accelerates it to 0.54 MeV. A short version of the standard JLab cryomodules is used to finish bunching the beam and preaccelerating it to 5.15 MeV. The beam then proceeds through a 6-meter long diagnostics area used mostly for beam tuning. The electron beam then proceeds through two standard Jlab cryounits that accelerate the beam to 45 MeV for injection into the north linac. The electron accelerator is a recirculating accelerator so it is necessary to accelerate the beam to relativistic energies before injecting the beam into the north linac so as to be match the recirculating beams already in the accelerator [48].

2.1.2 Superconducting Accelerator

The electron beam enters the accelerator oval after being accelerated to approximately 45 MeV in the injector. The accelerator resembles a race track oval with two long straight sections and two semicircular arcs (Fig. 2.3). The long straight sections contain the superconducting radio-frequency (SRF) linear accelerator modules and the arcs contain magnets for steering the beam through a 180° turn into the next

linac.

There are two modes in which RF accelerators can run, continuous-wave (CW) and pulsed-wave (PW) mode. Most non-superconducting accelerators like the BATES linear accelerator produce pulses of electrons by turning on and off the accelerator. The percent of time the accelerator is on is the duty factor of the accelerator. These accelerators operate in this manner because of joule heating of the RF cavities from the movement of electrons in the cavity metal. There are a few non-superconducting accelerators that can operate in CW mode but the peak beam current is limited by the joule heating. Superconducting accelerators like the one at Jefferson lab can run with continuous power on the RF cavities because the super-cooled cavities of niobium do not produce joule heating due to their superconductivity. It is possible for superconducting accelerators to produce a continuous-wave in the cavities for electron acceleration.

Initially Jlab was called the continuous electron beam accelerator facility (CEBAF). This terminology can be a bit misleading to the untrained observer because the linacs are RF accelerators. The facility doesn't deliver a true continuous beam of electrons. Instead it delivers so-called "buckets" of electrons. The accelerator operates at a frequency of 1497 MHz and each hall receives buckets of electrons at a frequency of 499 MHz. The time between arrival of each bucket is short enough that the sensitive spectrometers in the experimental halls effectively see a continuous stream of electron-scattering data.

The electron beam enters the north linac from the injector and gains energy as specified by the settings of the linac. The beam then goes around the east arc via bending magnets and enters the south linac to be accelerated by an equal amount as

Figure 2.3: A schematic of the Jefferson Laboratory electron accelerator. (NOT DRAWN TO SCALE)

the north linac. Upon exiting the south linac, a room temperature RF cavity either kicks the beam to the beam switch yard or allows it to enter the west arc to be recirculated to the north linac for further acceleration. The accelerator contains nine recirculation arcs, five at the east arc and four at the west arc, that allow the beam to pass around the accelerator oval up to five times. The beam can be removed from the accelerator at the end of any of the five “passes” through the accelerator and sent to the experimental halls[49].

An unique feature of the Jlab accelerator is its ability to deliver a different beam, in terms of beam current and energy, to each of the three experimental halls simultaneously. This is done by dividing up the accelerator frequency of 1497 MHz into three parts each operating at 499 MHz. Basically, this means that every third “bucket” of electrons in the accelerator is designated for a specific hall. The lasers in the injector are timed to the accelerator frequency to fill every third bucket with electrons for

the appropriate hall. Tuning each lasers' intensity in turn controls the beam current each hall receives. Thus, each hall can have a different beam current, up to some set limits, at the same time.

On the subject of beam energy, lets consider an example. Suppose the accelerator is setup with each linac at 500 MeV and the injector at 45 MeV. The rough estimate of the energy can be found from the following equation.

$$(2.1) \quad E = E_{injector} + nE_{linac} \quad n = 1, 2, 3, 4, 5$$

where $E_{injector}$ is the energy gained from the injector and E_{linac} is the sum of the two linac energies. This means there are only five possible energies the experimental areas can use: 1.045 GeV, 2.045 GeV, 3.045 GeV, 4.045 GeV, 5.045 GeV. The beam switch yard's equipment can not deliver all energies to all halls at the same time. Only one hall can have the highest energy and the other two halls can have either separate energies of the remaining four possibilities or they can have the same energy delivered to them. It should be noted that this energy calculation is simplified and a true calculation is complicated by the polarization of the electron. The electron's spin-polarization vector precesses when passing through the magnetic fields of the steering magnets. The full calculation needs to take this fact into account as well when calculating the possible energies for accelerator operations.

2.1.3 Hall C Beam Line

The Hall C beam line consists of three main sections. The first section is the Hall C arc. This portion of the beam line steers the electron beam from the beam switch yard into Hall C. It contains several beam diagnostic devices including beam position

monitors (BPM), beam current monitors (BCM), and superharps for measuring beam profile and position to high accuracy.

The second part is the upstream beam line. This section contains beam diagnostic equipment (BPM's and superharps) and small corrector magnets for final tuning of the beam on target. This part of the beam line also contains the fast raster and the Møller polarimeter. The fast raster is a magnet system that produces a controlled steering of the beam on the target to spread the heat load of the electron beam over a larger area of the target. This experiment used a 2 mm by 2 mm square raster. The Møller polarimeter was used to measure the electron beam polarization and will be discussed more in the following section.

The last section of the beam line is the down stream beam line. The down stream beam line consists of a large section of aluminum pipe filled with helium gas and separated from the scattering chamber vacuum by a beryllium oxide window. The down stream beam line starts as a two-inch diameter pipe at the scattering chamber and slowly grows to a 36-inch diameter pipe 25 meters later as it enters the beam dump tunnel. The down stream beam line grows bigger because there are no steering or focusing magnets between the scattering chamber and the beam dump tunnel. Helium is used in this section of the beam line because maintaining a vacuum in such a large beam line is very expensive and the beam is unusable anyway. The use of a low- Z material such as helium reduces the amount of background radiation created by electrons transversing this portion of the beam line.

2.2 Møller Polarimeter

One of the requirements to make this experiment successful was that the polarization of the electron beam needed to be stable and consistent from one data run to the next data run. Large changes in the polarization would have produced a large systematic uncertainty that would have needed to be folded into the final uncertainty in G_E^n . As it turned out this systematic uncertainty was one of the largest and that will be discussed in more detail in Chapter 5. The Møller polarimeter installed on the Hall C beam line was utilized by E93-038 to measure the electron beam polarization to the 1% statistical level.

2.2.1 Møller Polarimetry

Møller polarimeters have become the device of choice when measuring electron polarizations due to their simple, relatively inexpensive construction and their proven reliability over a wide range of incident electron energies. Møller polarimeters were operated in the GeV range at SLAC in the 1970's[50], and later in the late 1980's and 1990's Møller polarimeters were designed to operate in the MAMI accelerator[51] and the MIT-Bates accelerator[52] at intermediate energies. Møller polarimetry has the added benefit that the physical process by which the electrons were scattered is a purely Quantum Electrodynamics (QED) process and can be calculated to high precision.

The theory of Møller scattering is discussed in detail in [51]. It suffices for this discussion to present only a brief overview of Møller scattering. The cross section for

elastic-polarized electron-electron scattering is

$$(2.2) \quad \frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[1 + \sum a_{ij} P_i^B P_j^T \right] \quad i, j = x, y, z$$

where $(d\sigma_0/d\Omega)$ is the unpolarized cross section, a_{ij} is the asymmetry matrix, P_i^B is the polarization of the incident electron beam, and P_j^T is the polarization of the target electrons. The coordinate system is defined as the z-axis pointing along the direction of the incident electrons and the y-axis is perpendicular to the scattering plane. In this system, conservation of parity demands that $a_{xy} = a_{yx} = a_{zy} = a_{yz} = 0$ and in the high-energy limit the remaining two off diagonal terms, $a_{xz} = a_{zx}$, vanish as well. This leaves only a_{xx} , a_{yy} , and a_{zz} as the remaining terms in the asymmetry matrix. Figure 2.4 shows the relative strengths of the three remaining components of the asymmetry matrix. The maximum analyzing power is at a center-of-mass angle of 90° .

The electron beam is longitudinally polarized; thus, the matrix element of interest is a_{zz} . Replacing the i and j in eq. 2.2 with z yields

$$(2.3) \quad \frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[1 + a_{zz} P_z^B P_z^T \right]$$

which contains the measured asymmetry ϵ in the polarimeter from Møller scattering in the term $a_{zz} P_z^B P_z^T$. The asymmetry is calculated by the difference in the number of coincidence events for beam helicity positive and number of events for beam helicity negative divided by the sum of the events. Note the number of counts are normalized to the integrated beam charge.

$$(2.4) \quad \epsilon = \frac{N_+ - N_-}{N_+ + N_-}$$

Figure 2.4: Plot of relative Møller analyzing powers versus center-of-mass scattering angle for the three non-zero components of the analyzing power matrix. Notice that the maximum in the analyzing power occurs at center-of-mass angle of 90° .

Therefore, if the target polarization is known and the asymmetry is measured the beam polarization can be extracted from the equation

$$(2.5) \quad P_z^B = \frac{\epsilon}{P_z^T a_{zz}}$$

The minimization of systematic errors is necessary in order to obtain the desired 1% accuracy. Because the Hall C Møller polarimeter is a coincident polarimeter the systematic uncertainties due to Mott-scattered events is reduced below statistical levels and did not contribute to the overall uncertainty. The knowledge of the analyzing power is controlled by the polarimeter's kinematic setup. In order to maximize the counting efficiency of the polarimeter, the polarimeter arrangement is such as to place

the detectors at the 90° center-of-mass scattering angle. In the laboratory frame, this places the detectors very close to the downstream beam line. It is also important to define the acceptance of the detectors to keep the scattering angles that pass acceptance cuts (physical or software) close to the peak in the analyzing power (see Fig.2.4).

2.2.2 Design

The Møller polarimeter is located in the alcove on the Hall C Beam line upstream from the target chamber. The Møller polarimeter has a design similar to the polarimeters used at other electron beam facilities [51, 52]. A schematic of the polarimeter arrangement is shown in Fig. 2.5 [54, 55]. The polarimeter contains six primary components. The target is a thin iron foil that is on a mobile target ladder that can be inserted and retracted from the beam path. The iron foil is in the center of a superconducting Helmholtz coil which produces a 4 Tesla magnetic field parallel to the direction of the beam. This field aligns the outer shell electrons of the iron atoms parallel or anti-parallel to the beam direction.

After passing through the foil, the primary electron beam continues through the quadrupole magnets, the collimator, and on down to the beam dump. The Møller electrons (those electrons that scattered from atomic electrons) are momentum analyzed by the magnets. The scattered electrons first pass through a quadrupole that continues to defocus the beam onto the collimators. The collimator box actually consists of seven individually controlled Densimet (tungsten alloy) collimators that define the physical acceptance of the apparatus[55]. The second quadrupole magnet helps to momentum analyze the remainder of the Møller electrons.

The detector package consist of two arms each containing a bank of 16 hodoscope

Figure 2.5: This top view schematic showed the Hall C Møller polarimeter used to measure the polarization of the electron beam delivered to Hall C.

segments and a lead-glass shower counter. The hodoscopes consists of 14 vertical plastic scintillator detectors with channels 3 and 12 being split vertically to supply some vertical position information (the hodoscopes are not mean-timed) [55]. The definition of a Møller scattering event is the detection in coincidence[53] of an electron pair between the two lead-glass shower counters. Using a narrow window in the coincidence timing (about 5 ns) helps to reduce the accidental background rate.

2.2.3 Data Collection and Analysis

This experiment required high electron beam polarizations to obtain the statistical uncertainties desired by the experimenters. The use of a Møller polarimeter to measure the incident electron polarization was fundamentally sound but had a few major drawbacks. First, the measurement was completely destructive to the beam. The Møller apparatus was located upstream from the cryogenic target and required

the insertion of an iron foil into the beam path. This meant any beam that did not scatter into the Møller polarimeter was unusable as G_E^n beam. Therefore, the beam polarization measurement was not made simultaneously with the data acquisition for G_E^n .

Secondly, the Møller polarimeter was designed to handle only up to $3\ \mu\text{A}$ of beam current with optimum counting rates in the 0.5 to $1.0\ \mu\text{A}$ range. The G_E^n measurements were made using beam currents up to $50\ \mu\text{A}$ with the Charybdis magnet off and up to $70\ \mu\text{A}$ with the Charybdis magnet on. There was some concern that beam polarization may be correlated to beam current. The assumption here is that the polarization measured at low currents is valid at the high currents used in the measurements of G_E^n . A Compton polarimeter installed in Hall A has confirmed this assumption and a Compton polarimeter is to be installed in Hall C at some time in the future. For this experiment, the above assumption was used.

Lastly, the polarization measurements were limited in how often they could be made due to the time it takes to make the measurements. To maximize the amount of beam time for G_E^n measurements, the polarization measurements were made only every few days. It was necessary early on to build convincing data to support this minimization of polarization measurements. It was found that unless a major change to beam delivery was made, the beam polarization remained constant. Changes to beam delivery could include change in energy per pass, Wien filter angles, laser spot on the GaAs crystal, or re-Cesium of the crystal surface. If no changes prompted a polarization measurement during a particular time frame, then a polarization measurement was made every time the half-wave plate was changed.

Each new kinematic setting for the experiment required a Møller tune-up period

where the counting rates were maximized in the polarimeter by adjusting the collimators. This was done prior to the taking of G_E^n data. Once the Møller Polarimeter had been optimized by the experts, Møller data could be collected by a few persons trained to take the Møller data runs. Enough statistics were taken for each Møller run to keep the relative statistical error below 1%. Multiple data runs were taken at each Møller opportunity and averaged together to assure confidence in the quality of the data and the operation of the polarimeter. Table 2.1 contains averages of Møller asymmetries taken during January 2001 for $Q^2 = 1.47 \text{ (GeV/c)}^2$ and Table 2.2 contains Møller data for $Q^2 = 1.17 \text{ (GeV/c)}^2$ taken during April 2001.

The weighted average of the values in Table 2.1 was $75.7 \pm 0.08 \%$ and in Table

Date	Average Polarization (%)
January 5, 2001	74.1 ± 0.2
January 7, 2001	75.7 ± 0.2
January 8, 2001	75.6 ± 0.4
January 10, 2001	76.3 ± 0.2
January 12, 2001	76.0 ± 0.4
January 17, 2001	76.2 ± 0.2
January 20, 2001	75.8 ± 0.2
January 22, 2001	76.2 ± 0.4
January 25, 2001	76.5 ± 0.4
Average	75.7 ± 0.08

Table 2.1: Average polarizations of the Møller data runs taken on the indicated date for $Q^2 = 1.47 \text{ (GeV/c)}^2$.

2.2 was $83.6 \pm 0.19 \%$. The G_E^n data were corrected for the polarization of the incident electron beam based on blocks of time after a particular Møller measurement was taken up to the next Møller measurement for the $Q^2 = 1.47 \text{ (GeV/c)}^2$ data. Because the $Q^2 = 1.17 \text{ (GeV/c)}^2$ data collection period was so short, only one correction factor was used and that is the average polarization from Table 2.2.

Date	Average Polarization (%)
April 19, 2001	84.0 ± 0.35
April 22, 2001	83.3 ± 0.28
April 26, 2001	83.9 ± 0.35
Average	83.6 ± 0.19

Table 2.2: Average polarizations of the Møller data runs taken on the indicated date for $Q^2 = 1.17 \text{ (GeV}/c)^2$.

2.3 Hall C Cryogenic Target

The Hall C cryogenic target was located on top of the pivot in Hall C (Fig. 2.7). The pivot was the point at which the HMS and SOS spectrometers rotate around the target. The targets were arranged on movable ladders in the scattering chamber. There was one ladder for the cryogenic targets and one for the solid (foil) targets. Because E93-038 did not require a lot of time on the solid targets, the solid target ladder was removed and five solid targets were attached to the bottom of the cryogenic ladder for diagnostic purposes. The scattering chamber was made of aluminum and houses the target ladders, sensors, and all the control devices for moving individual targets into the path of the beam. A vacuum was maintained in the scattering chamber and the cryogenic targets were cooled with 15 Kelvin helium from the central helium liquefier (CHL).

The targets were controlled and monitored from a dedicated target control computer in the control room. Records were kept on the operation of the target including heat loading, average beam current, temperature, pressure, and various other quantities. A very simple schematic of the cryogenic target ladder can be seen in Fig. 2.6. The target ladder consisted of three cryogenic loops mounted on a cell block. Each

Figure 2.6: This side view of the target ladder showed the three cryo loops with two target cans in each loop. The cans were 4 cm and 15 cm long respectively. The solid targets attached to the bottom of the cell block were used for optics checks on the experimental setup.

loop had two target cans. The shorter of the two presented 4-cm of liquid cryogen to the path of the beam while the longer one presented 15-cm of liquid cryogen to the beam. The thickness of the entrance and exit windows of the cryogenic targets was measured by the Hall C target group. The target group measured the thickness of the entrance and exit windows at several different places and Table 2.3 gives the average values of these measurements. Loop 1 of the target ladder contained liquid deuterium and loop 2 contained liquid hydrogen. Loop 3 was filled with helium gas and kept as a reserve loop in case one of the other loops began leaking uncontrollably. The solid targets used for checking spectrometer optics and simulating a target cell without the liquid cyrogen were mounted on the bottom side of the cyro target ladder.

Description	Average thickness (mils)
Loop 1, 4 cm, entrance	4.43
Loop 1, 4 cm, exit	5.48
Loop 1, 15 cm, entrance	4.41
Loop 1, 15 cm, exit	5.95
Loop 2, 4 cm, entrance	5.72
Loop 2, 4 cm, exit	6.32
Loop 2, 15 cm, entrance	5.00
Loop 2, 15 cm, exit	5.86

Table 2.3: This table contains average values of the window thickness for the cryogenic targets of E93-038. It should be noted that the measurements were in mils (1mil = 1/1000 of an inch).

Four centimeter and fifteen centimeter “dummy” targets were used for simulating the empty target cell system. Data were collected with the 15-cm dummy target, which consisted of two aluminum foils placed 15-cm apart, to determine the contribution of neutron events to the G_E^n data from the cyro target cell windows.

2.4 High Momentum Spectrometer

To obtain the best possible timing resolution for this experiment, it was necessary to detect the scattered electron from the ${}^2\text{H}(\vec{e}, e'\vec{n}){}^1\text{H}$ reaction. The detector used for detecting the electrons was Hall C’s High Momentum Spectrometer (HMS). Figure 2.7 shows a basic schematic layout of the HMS device. The HMS is a superconducting QQQD spectrometer. It has three quadrupole magnets for focusing the scattered electrons and then a dipole magnet for momentum analyzing the electrons. The specifications of the the magnets were found from the Hall C web site[56] and are given in tables 2.4 and 2.5.

Figure 2.7 shows the basic layout of the HMS. The scattering chamber is located

Figure 2.7: This side view of the HMS spectrometer shows the relative positions of the major components. Note that the schematic is not drawn to scale.

on top of the pivot which is the point about which the spectrometers pivot. The three quadrupole magnets are then arranged in a row just prior to the dipole magnet. A vacuum line is maintained between the magnets and the detector shield house that lies on top of the carriage platform. Vacuum tight seals are difficult to maintain and even more difficult if the seal is at a pivot point. Therefore, there is an air gap between the scattering chamber window and the window on the the first quadrupole.

	Quad 1	Quad 2/Quad 3
Gradient	605 G/cm	445 G/cm
“Good Field” radius	22 cm	30 cm
Pole Tip Field	1.5 T	1.56 T
Radius of Pole	25 cm	35 cm
Magnetic Length	189 cm	210 cm
Stored Energy (10^6 J)	0.25	10.5
Weight	20 tons	30 tons

Table 2.4: Specifications for the quadrupoles on the Hall C High Momentum Spectrometer. Notice that quads 2 and 3 are identical.

	Dipole
Gap	421 cm
“Good Field” Width	± 30 cm
Bend angle	25°
Max. Pole Tip Field	1.66 T
Dynamic Field Range	10:1
Field Uniformity B/B	0.001
Pole Face Rotations	$6^\circ, -6^\circ$
Effective Length	5.26 m
Coil Cooling	Thermal Siphon 4.3 K Liquid He
Weight	470 tons

Table 2.5: Specifications for the dipole magnet on the Hall C High Momentum Spectrometer.

The detectors in the HMS shield house are used for momentum information and particle identification. The package includes two wire drift chambers for position tracking, a layer of scintillator hodoscopes for timing, a gas Cerenkov counter for particle identification, and a segmented lead-glass shower counter for energy determination. Signals detected in the HMS detectors are processed by the HMS electronics in the counting house. A trigger signal generated by those electronics is then sent to the data acquisition electronics for creation of the coincidence trigger.

Hall C performance information[56] has the maximum central momentum of the spectrometer to be 7.5 GeV/ c , a momentum bite of 18%, and a momentum resolution of $< 0.1\%$. The solid angle acceptance is greater than 6 milliradians and the minimum central scattering angle is 12.5° . The maximum central scattering angle is $\sim 90^\circ$. E93-038 did not even come close to testing the limits of the spectrometer’s performance. The luminosity of the experiment is limited by the count rates in the front neutron detectors of NPOL and the rate that the data acquisition computer

could collect events without significant dead times. The HMS central scattering angles for all Q^2 values is reported in Table 4.1.

2.5 Neutron Precession Dipole Magnet

The neutron polarimeter (NPOL) described in more detail in chapter 3 was sensitive only to the transverse portion of the neutron's polarization vector, i.e., the component transverse to the neutron's momentum vector. In order to make the ratio method work for measuring G_E^n , the longitudinal portion of the neutron's polarization vector also had to be measured. This was achieved by placing a dipole magnet between the scattering chamber and the collimator of the neutron polarimeter. This dipole magnet could be used to precess the neutron's polarization vector through the desired amount in order to precess the longitudinal component to the transverse direction. In the case of data collected at $Q^2 = 1.47 \text{ (GeV}/c)^2$, the precession angles used were 0° and $\pm 90^\circ$, in order to measure the transverse and the longitudinal components of the neutron's polarization vector, respectively. The $Q^2 = 1.17 \text{ (GeV}/c)^2$ data used precession angles of $\pm 40^\circ$.

The dipole magnet, named Charybdis, was a water-cooled normal state magnet with an iron core. Due to the proximity to the scattering chamber, field clamps consisting of approximately two inches of iron plate were placed on the ends of the dipole magnet to limit the range of the fringe fields. Each field clamp had a section of the plate cut out to match the 8.25 inch gap between the pole faces on the dipole magnet so that the acceptance was not affected by the clamps.

2.6 Neutron Polarimeter

The neutron polarimeter (NPOL) was designed specifically for the measurement of the electric form factor of the neutron. Because the polarimeter was the major contribution in equipment to the experiment by the collaboration, it will be discussed in greater detail in Chapter 3. NPOL was a device designed to measure the polarization of the recoil neutron from the ${}^2\text{H}(\vec{e}, e'\vec{n}){}^1\text{H}$ reaction in the target via the recoil polarimetry method. In relation to other Hall C equipment, NPOL was situated to the left of the beam line at a central angle of 46.0° (Fig. 2.8). The polarimeter was a fixed device in that, once it was set in place, it was not moved for the duration of the experiment. The Q^2 points for the experiment were changed by changing the position of HMS and the energy of the incident electron beam. The center of the front array in the polarimeter was 7 meters from the target.

NPOL installation began in June of 2000 with concurrent work on the detector installation in the unistrut frame and building of the platform and the shielding house. Safety of personnel and equipment required that the platform and first layer of shield blocks needed to be installed before the frame holding the detectors was to be lifted onto the platform. Once this was completed the rest of the detectors were installed and cabling was completed. Installation was completed by the end of August and pulse-height calibration of the detectors was performed just prior to the taking of beam in September. Data collection continued through April 2001 with various minor delays and scheduled maintenance down time.

Figure 2.8: Top view schematic of the neutron polarimeter and shielding house in relation to the target and downstream beam line. The dashed line shows the direct path of neutrons proceeding from the target through the center of the shielding enclosure. The center of target to center of front array distance was 7 meters.

CHAPTER 3

Neutron Polarimeter (NPOL)

The neutron polarimeter (NPOL) was a specially designed apparatus for the measurement of the polarization vector of neutrons via the recoil polarimetry method. Figure 3.1 shows a cartoon view of NPOL with the HMS and target for location reference. The figure shows that the polarimeter contains several different parts which make up the whole apparatus. Each of these will be discussed in a later section along with the concept of recoil polarimetry and polarimeter calibration.

Figure 3.1: A schematic of the Neutron Polarimeter and Hall C High Momentum Spectrometer (HMS).

3.1 Neutron Polarimetry

The purpose of this section is to give a brief overview of neutron polarimetry and the evolution of the detector called NPOL. These two topics are broken into two subsections that follow.

3.1.1 The Principle behind the Detector

Neutron polarimetry is the measurement of the average value $\langle \vec{P}_{S'} \rangle$ of the transverse component of the neutron polarization which lies in the scattering plane and is transverse to the neutron direction of travel. The entire bases for neutron polarimetry is the asymmetry $\xi(\theta, \phi)$ in the scattering of polarized neutrons from unpolarized hydrogen nuclei or from unpolarized protons in a carbon nucleus in the scintillator material of the front array[2, 3]. The asymmetry for this scattering is

$$(3.1) \quad \langle \xi_{S'}(\theta, \phi) \rangle = \langle \vec{P}_{S'} \cdot \vec{A}_y(\theta) \rangle = \langle \vec{P}_{S'} \rangle \cdot \langle \vec{A}_y \rangle = \langle \vec{P}_{S'} \rangle A_y(\theta) \cos \phi$$

where $\langle \vec{P}_{S'} \rangle$ is the average transverse neutron polarization component and $\vec{A}_y$ is the analyzing power of n-p scattering at the angle θ between the incident neutron momentum vector and the scattered neutron momentum vector. By measuring the average asymmetry $\langle \xi_{S'} \rangle$ via n-p scattering in the polarimeter and knowing the analyzing power of the polarimeter it is possible to extract the average transverse polarization of the neutron.

As pointed out earlier, the polarimeter was sensitive only to the transverse component of the neutron polarization vector. If the neutron polarization vector were precessed by 90° with a dipole magnet, a measurement of the longitudinal component of the neutron's polarization vector could be made because the precession results in

the longitudinal component having been rotated to the transverse direction. This asymmetry measurement is related to the longitudinal component of the neutron polarization by

$$(3.2) \quad \langle \xi_{L'}(\theta, \phi) \rangle = \langle \vec{P}_{L'} \cdot \vec{A}_y(\theta) \rangle = \langle \vec{P}_{L'} \rangle \cdot \langle \vec{A}_y \rangle = \langle \vec{P}_{L'} \rangle \cdot A_y(\theta) \cos \phi$$

If the ratio η of these two asymmetry measurements is made, it can be seen that the analyzing power of the polarimeter cancels to first order and the ratio of the asymmetries is related to the ratio of the transverse and the longitudinal components of the polarization vector.

$$(3.3) \quad \eta = \frac{\langle \xi_{S'}(\theta, \phi) \rangle}{\langle \xi_{L'}(\theta, \phi) \rangle} = \frac{\langle P_{S'} \rangle}{\langle P_{L'} \rangle}$$

It is possible to connect this ratio η to the ratio of electric to magnetic form factor $g = G_E^n/G_M^n$ and thus extract the value of G_E^n for a given value of G_M^n at a particular Q^2 value. More on this will be discussed in the chapter on data analysis.

3.1.2 A Brief History of NPOL

A neutron polarimeter was specifically designed and calibrated[57, 2, 58] in the 1980's for measuring medium-energy neutrons from $(\vec{p}, \vec{n})$ reactions at the Indiana University Cyclotron Facility (IUCF). A few years later, a polarimeter was used at the MIT-Bates linear accelerator to measure G_E^n from the reaction $^2H(\vec{e}, e'\vec{n})^1H$ at $Q^2 = 0.255 \text{ (GeV/c)}^2$ [4] and the magnetic form factor of the neutron G_M^n at Q^2 values of 0.109, 0.176, and 0.255 (GeV/c)^2 [59]. In order to calculate the neutron magnetic form factor, it was necessary to determine the neutron detection efficiency of the polarimeter as well[60].

A change in the design of the polarimeter was proposed by Madey[5] to double the height of the front array without exposing the rear neutron detectors to the direct flux of particles coming from the target (Fig. 3.2). This doubling of the height results in a doubling of the neutron solid angle and an increase in the electron solid angle for a coincidence experiment. This new configuration increases the neutron counting rates and has a comparable analyzing power. The proposed behind this change was to take full advantage of the newest generation of polarized electron accelerators which feature high duty factors, high average beam currents, larger acceptance magnetic spectrometers, and electron polarizations which are routinely measured at 80%. It was this basic design that was calibrated at IUCF[61] and optimized to run at Jlab in E93-038.

3.2 Scintillator Detectors

NPOL used large volume plastic scintillator detectors[62]. NPOL contained 70 mean-timed plastic scintillator detectors of four different sizes. Twenty detectors in the front array act as the active analyzers, and 24 detectors in two back arrays counted neutrons scattered from the front array to the rear arrays (Fig. 3.1). The other 26 detectors were used for charged particle identification at the entrance and the exit sides of the front array. Ten were split into two layers in front of the front array and 16 were split into two layers behind the front array. There were several general features to these detectors which will be discussed in this section with more specific details saved for the subsections on each major part of the polarimeter.

The detectors were made of Bicron-400 fast plastic scintillator material (NE 102A equivalent). Bicron-400 plastic has a wavelength of maximum emission of 423 nm

Figure 3.2: A side view schematic of the NPOL detector enclosure with the collimator, dipole magnet, and target for reference. The optical ray lines for the center and the maximum up and down angles from the target through the collimator are shown as broken lines. (NOT DRAWN TO SCALE)

and a main component decay constant of 2.4 ns [63]. Leo[64] states that the principle applications for Bicron-400 plastic was gamma, alpha, beta, and fast neutron detection. Because NPOL's purpose was to detect neutrons with energies up to 800 MeV, Bicron-400 provides good timing characteristics along with good neutron detection efficiency for the experiment.

Each detector had an appropriate light guide attached with optical cement to the long ends of the scintillator to guide the light from the large cross section of the detector to the smaller cross section of the attached photomultiplier (PM) tubes. The light guides were made of Plexiglas and were designed such that much of the light incident on the sides of the guides will reflect back into the Plexiglas and continue to

the PM tube. Plexiglas was also desirable since its index of refraction was very near that of Bicron-400 plastic making the coupling an excellent transmitter of light and transparent to the wavelengths of interest.

Attached to each light pipe was a PM tube. Three different PM tubes were used in this experiment and they will be discussed in later subsections. For now it is sufficient to know that they were coupled to the ends of the light guides with small bolts via a Plexiglas coupling plate which was optically cemented to the face of the PM tube. This coupling plate gave the means to quickly and easily change a PM tube, in case one failed during the experiment, without having to break the optical cement joint.

Each PM tube had its own magnetic shield. Magnetic shields are cylinders of metal with a high value of magnetic permeability that deflect the earth's magnetic field and any stray magnetic fields away from the PM tubes. If magnetic shields were not used on the PM tubes, the level of sensitivity to light would have been affected because electrons traveling through the PM tube dynode chain would be deflected and this would effectively reduce the electron multiplication process.

The detectors are wrapped in at least one layer of white paper and then covered with two or more layers of black electrical tape. The purpose of this layering of material was to prevent the intrusion of ambient light into the detection system and thus produce false signals. These false signals in the PM tube output are referred to as "light leaks" by the detector community.

3.2.1 Front Neutron Detectors

NPOL contained twenty mean-timed Bicron-400 plastic detectors in the front array (Figs. 3.1 and 3.2). Each detector had dimensions of 10 cm x 10 cm x 100 cm and had an appropriate light pipe cemented to the ends of the detector to channel the 10 cm x 10 cm cross section down to a circular 51-mm cross section. The 51-mm cross section matched the 51-mm diameter of the Hamamatsu R1828-01, 12-stage, bi-alkali photocathode head-on type photomultiplier tubes chosen for these detectors. These PM tubes had a spectral response range of 300 to 650 nm with a peak response at 420 nm. This was very close to the Bicron-400's maximum wavelength of emission of 423 nm. The rated gain for this model of PM tube was 2.0×10^7 [65].

The PM tubes were powered by high voltage dividers, called bases, designed and built by Alan Baldwin at Kent State University. The R1828-01 PM tubes were rated to a maximum of 3000 Vdc. Each PM tube had a base connected to the end of the tube via 20 pins. Each of these pins was in turn connected to a different internal part of the PM tube specified on the technical sheet issued by Hamamatsu. Appropriate design of the base results in a high degree of linearity in the output signal and a high level of gain from the PM tube. By using a fairly large current for the dynode resistors and capacitors, the bases also provide good high count-rate ability.

The front array of the polarimeter was called the active scattering layer. As described in the section on recoil polarimetry, the front array “analyzed” the polarization of the incident neutrons through the spin-orbit coupling with the proton in hydrogen atoms of the scintillator. The interaction resulted in a scattering of neutrons to the back array depending on the polarization of the incident neutron. The front detectors also provided “hit” information. As an active scattering layer, the detectors

not only functioned as the analyzer for determining the polarization of the neutron but they also provided timing and pulse height information about the interaction in the detectors themselves. This information allowed us to reconstruct time-of-flight information and aided in particle identification. This was very important because a free neutron target doesn't exist, so a liquid deuterium target was used as a supply of neutrons.

The long dimension of the detectors was placed horizontal and perpendicular to the central momentum vector for neutrons from the target incident on the front array (Fig. 3.1). The front array was arranged in four vertical layers with five detectors in each layer. The center of the front array was surveyed to be $7 \text{ m} \pm 0.5 \text{ cm}$ from the center of the target. The center of each layer was set to be at beam height. Beam height was the distance between the main floor of Hall C and the center of the electron beam as it passed through the center of the target. This height was 396 cm and was marked on the wall of the shielding enclosure. The vertical center of the front array was then adjusted using jacks mounted on the detector frame until it matched the beam height.

3.2.2 Rear Neutron Detectors

Figure 3.1 shows that NPOL has two rear arrays. They were called the “top” rear array and the “bottom” rear array and each array contained 12 detectors split into three layers. Each layer contained two $10 \text{ cm} \times 25 \text{ cm} \times 100 \text{ cm}$ and two $10 \text{ cm} \times 50 \text{ cm} \times 100 \text{ cm}$ detectors each. The two 25-cm wide detectors were placed side-by-side in between the two 50-cm wide detectors. Figure 3.2 shows a side view of NPOL and gives the measurements for the rear layers referenced to the center of the front

array and beam height. In order to maximize the counting rates, a simulation was performed which found the optimum position at which to place the rear detectors. This resulted in the staggering of the layers seen in Fig. 3.2. The inner and outer layer in each array was 252-cm from the center of the front array to the center of the back layer. The middle layers are 257-cm from the center of the front array to the center of the back layer. The closest side to the beam height line (dashed line through center of Fig. 3.2) for the inner layers was 51.6-cm from beam height. The next layers were 68.1-cm from the beam height line, and the outer layers were 84.6 cm from the beam height line.

The larger size of the rear detectors required they have larger PM tubes attached to them. Both sizes of detectors in the rear arrays had Hamamatsu R1250, 127-mm, bi-alkali photocathode, 14-stage, head-on type PM tubes coupled to each end of the detector. Much like the detectors in the front array, the PM tubes were coupled to the light guides via two coupling plates and small bolts. This made changing the PM tubes easy and less time consuming. The R1250 PM tube had a wavelength of maximum response of 420 nm with an useful range of 300 to 650 nm. The typical gain of a R1250 PM tube, according to the specifications sheet, was 1.4×10^7 [66].

The R1250 PM tubes were powered by the same bases as the R1828-01 on the front detectors. This ability to use a standard base between the front and rear detectors even though they are slightly different PM tubes, saved a lot of time when building the bases. This fact also allowed for exchanging bases more quickly when one failed.

3.2.3 Charged Particle Detectors

Four layers of thin plastic scintillators, two at the entrance and two at the exit of the front array, were used as charged particle detectors. The veto detectors were borrowed from University of Virginia and they were originally designed and used for E93-026. The detectors had dimensions of 0.5 cm x 10 cm x 100 cm in the active scintillating volume. Figures 3.1 and 3.2 show two slightly different views of the charged particle detectors. Two layers of 5 detectors each were placed in front of the front array and two layers of 8 detectors each were placed behind the front array. The long dimension was placed horizontal and the 10-cm wide direction was placed facing toward the target and perpendicular to the direction of travel of the incident particle flux. Thus, the incident particle traveled through the thinnest portion of the detector. It was the thinness of the detector which made them good at identifying charged particles. Most neutrons passed through the detector without interacting. Charged particles interacted by ionizing atoms as they past through the scintillator material. This resulted in a burst of light which was detected by the PM tube.

The charged particle detectors were coupled to their PM tubes via an optical cement connection. Twelve stage, 51-mm, bi-alkali, XP2262 PM tubes were used on these detectors. The XP2262 had a wavelength of maximum sensitivity of 400 ± 30 nm and a typical gain of 3×10^7 [67]. The PM tubes were powered by standard Phillips bases supplied with the PM tubes and had a maximum operating voltage of 2500 Vdc.

3.3 Steel Collimator and Lead Curtain

The active surface area of the polarimeter was defined by the physical dimensions of the front array's active scattering material. Each detector was 100-cm long and 10-cm high. There were 5 detectors stacked on top of each other for a total height of 50-cm. The physical acceptance, therefore, was 100-cm wide by 50-cm high. The platform supporting the detector stand was enclosed in a concrete and steel enclosure to limit the number of accidental coincidences. The details of this shielding will be discussed in the next section. In the front wall of the shielding, an opening was left to permit the flux of particles from the target to enter the enclosure and illuminate the front face of the front array.

The acceptance of particles was defined by a tapered stainless-steel collimator whose dimensions are shown in Fig. 3.3. The line drawn down the center indicates the central path for particles incident on the center of the front array coming from the target. The coordinates for the measurements were centered at the target with the z coordinate pointed toward the polarimeter along the central path, the x coordinate pointed to the left and horizontal to the floor, and the y coordinate pointed vertically upward. The collimator was set in the front wall of the shielding house and centered on the central ray by the survey crew to an accuracy of ± 2 mm.

A significant amount of electromagnetic radiation made it to the front array from the target. To stop this electromagnetic radiation, a curtain of lead was placed in front of the collimator entrance. The curtain consisted of 40 sheets of lead each 0.259-cm thick resulting in a total of 10.36-cm of lead in the curtain. The Pb curtain thickness reduced the singles rates in the front detectors by a factor of 3 over that with only 20 sheets of lead.

Figure 3.3: Side and Top cut away views of the NPOL collimator. Particles enter from the left and exit from the right. A lead curtain of 10.36 cm was placed in front of the collimator to attenuate the flux of electromagnetic radiation. (NOT DRAWN TO SCALE)

3.4 Shielding

One of the major concerns in this experiment was the accidental coincidence events between the front and rear arrays in the polarimeter. A large shielding enclosure was built around the NPOL detector platform to cut down on the number of accidentals in each detector. The front wall was four-feet thick and was made of steel blocks. The only opening in the front wall was the collimator. It was set into the wall and surveyed into place by the Jlab survey crew. The back wall of the enclosure was also

a four-foot thick wall of steel blocks.

The side walls of the enclosure consisted 48-inch thick reinforced concrete blocks stacked in two layers. The roof of the enclosure consisted of 21-inch thick reinforced concrete beams. They were laid across the structure to shield the detectors from so-called “sky shine”, particles and radiation reflected from the atmosphere and ceiling randomly traveling around the experimental area while beam is being delivered to the target. Because the roof beams were not as thick as the side walls, two layers were used to increase the shielding. There were not enough beams to cover the entire enclosure with two complete layers, so the second layer was limited to covering the section directly over the NPOL detectors (Fig. 3.2).

3.5 Neutron Polarimeter Electronics

The electronics used to process the signals from the 140 PM tubes were split into two different parts. One part was placed in the shielding enclosure under the detector platform. The second part was in the counting house electronics room at ground level next to the experimental control room. The counting house contains all the Short Orbit Spectrometer (SOS) electronics and the High Momentum Spectrometer (HMS) electronics as well as the Read Out Controllers (ROCs) for data acquisition and other control modules. Through previous neutron time-of-flight experiments with these scintillator detectors, it has been concluded that the closer one could place the timing electronics to the PM tubes the better the timing signal created from the analog signal received from the PM tubes. A minor consideration was the amount of radiation the electronics could be exposed to during beam operations. Because the electronics in the hall were placed in the shield enclosure and below the detector

platform the amount of radiation they received was minimal.

3.5.1 Detector Hut Electronics

The detector hut electronics were housed under the detector platform inside the shielding enclosure. All the electronics were mounted in three large racks and include four NIM power crates for the discriminators and various other NIM modules. Figure 3.4 shows a schematic of the detector hut electronics for the 44 neutron detectors (20 front detectors and 24 rear detectors). The pulse height signals from each PM tube was processed in the same way in the detector hut for each of the 44 neutron detectors. As mentioned earlier, the main purpose of the detector hut electronics was to form the timing signal as soon as possible after the signal comes out of the PM tube to take advantage of the best timing characteristics. Each detector had two identical PM tubes which were handled with identical methods. From Fig. 3.4 we see that the PM tubes were powered remotely by a 64 channel high voltage CAEN mainframe crate located on the second floor of the counting house. The PM tube anode signal, called the pulse-height signal, was first amplified by a fast amplifier with a gain of eight, which were designed and constructed at Kent State by Alan Baldwin. The purpose for amplifying the anode signal was to reduce the operational high voltage of the PM tubes. This resulted in less current being drawn by the tubes during high counting operations and therefore extended the operational lifetime of the PM tubes.

The amplified anode signal then proceeded to the LED driver and pulse-height monitoring box. This device operated the LED system and monitored the pulse height response from each PM tube. Abbreviated PHM for pulse-height monitor, this device flashed individual blue LED's on each end of the detector. These LED's

were mounted in the light guides. The PM tube produced a response to the flash of light that was integrated by the PHM device and plotted on a 1024 channel analyzer. This signal was used to monitor any PH drift of the detector. A specific response to energy was not needed in this case. The value of interest was the pulse height response of the PM tube to the LED over time. Therefore, the energy calibration, as described in a later section, was completed before the LED's were calibrated and set.

The response of the LED was Gaussian so a fit to the peak was done and

Figure 3.4: This is a schematic of the electronics used to produce the timing and pulse height signals for the 44 neutron detectors in NPOL. All electronics were installed in the NPOL detector enclosure under the detector platform.

the central channel was calculated. This value was then recorded and used as our base for future checks on PM tube gain. After the detectors had been exposed to particle flux for a period of time, the central values of the LED peak was checked

against the original values. If the central channel value was lower than the original value then the gain of the PM tube can be said to have dropped and the gain was then brought back by raising the high voltage on the PM tube slightly. A TCL (Tool Command Language) script was written to periodically check the central values stored in the PHM device and compare them to the original values and calculate adjustment voltages. A second TCL program was then used to change the high voltage values in the CAEN mainframes to make the gain adjustment.

The anode signal then proceeds from the PHM device to a 50/50 resistive splitter. One half of the signal was then diverted to a bundle of passive delay cables made of RG-58 cable. Passive delay was chosen over active delay modules because the rates in each PM tube was expected to be above 300 kHz and in reality were as high as 700 kHz. These signals were then sent via a patch panel to the counting house for processing by an analog-to-digital converter (ADC) and read out by the data acquisition computer for data storage. The other half of the anode signal entered an ORTEC 934 constant-fraction discriminator (CFD) which produced the timing pulse for each PM tube. These signals were then sent via the Hall C patch panel to the counting house for formation of the NPOL trigger and processing by the time-to-digital converter (TDC). The TDC values were read out by the data acquisition computer and placed on data storage. The details of counting house electronics will follow in the next section.

The charged particle detectors used a different electronics setup (Fig. 3.5). The charged particle detectors didn't have LED's for gain monitoring so they didn't need the PHM device in the circuit. There was no preamplifier in the circuit either. The anode signal went straight from the PM tube to the 50/50 splitter which sent half

the signal through the 400 ns delay cable and the other half of the signal to a LeCroy P710 leading-edge discriminator. These discriminators formed the timing pulse and sent the signal via the Hall C patch panel to the counting house along with the pulse height signals.

Figure 3.5: This schematic shows the electronics used to produce the timing and pulse height signals for the 26 charged particle detectors in NPOL. All electronics were installed in the NPOL detector enclosure under the detector platform.

3.5.2 Counting House Electronics

The counting house electronics room was divided into two arms. The NPOL arm and the HMS arm. The HMS was designed and installed as a stand alone spectrometer which has the versatility to be put into coincidence with the Short Orbit Spectrometer (SOS) and so-called third arm detectors. This experiment utilized this

versatility to reduce background contributions and increase momentum resolution. HMS was maintained by JLab so my discussions of counting house electronics will be limited to the NPOL side of the electronics and any discussion of HMS electronics will be limited to the coincidence timing.

The timing and analog signals from the detector hut electronics (Figs. 3.4 and 3.5) were sent to the counting house via permanently installed low-loss coaxial signal cables (Fig. 3.6). Once in the counting house, the signals passed through a passive filter/transformer to remove any DC ground noise in the signals. The analog signals then proceeded to the FastBus crate ADC channels and the timing signals proceeded to a discriminator that regenerated the timing signals and rejected any signals below a specified threshold. These discriminators had dual outputs. One output proceeded to the timing circuit. The timing circuit consisted of a logic module, a pretrigger circuit, and the LeCroy 2365 logic module. The trigger was then sent to the data trigger supervisor (TS) which was part of the data-acquisition system. The second output from the discriminator was sent through 400 ns of delay cable to bring it in time with the timing circuit and then it went to a module that regenerated the degraded timing signal and splits the output again. One signal was sent to the FastBus crate TDC channels and the other signal was sent to the CAMAC crate scaler channels. The FastBus and CAMAC crates were read out by the data-acquisition system.

3.6 Polarimeter Calibration

Each detector in the neutron polarimeter provided information about a particle's energy deposition in the detector, the time the interaction occurred relative to a common start, and the logical yes or no if a particle passed through the detector. NPOL

Figure 3.6: This schematic shows the NPOL side of the counting house electronics.

consisted of 70 detectors with two photomultiplier tubes each along with hundreds of cables of various lengths. In order to get useful values for energy deposition and timing from the detectors, each detector was energy and time calibrated. The energy calibration required measuring the ADC pulse height for two or more known energies of a detected particle. The timing calibration required measuring time constants for each detector in reference to a particular fiducial time.

3.6.1 Pulse Height Calibration

The energy deposited in a detector ionizes atoms in the scintillator. As the electrons fall back to their initial orbitals, electromagnetic radiation in the visible wavelengths is emitted. These photons then traveled down the scintillator to the PM tubes where they emitted electrons from the surface of the photocathode via the photoelectric effect. These electrons then produced a cascade in the PM tube's dynode chain resulting in a pulse at the anode that could be read out with an analog-to-digital converter (ADC). The ADC's used in this experiment were charge-integrating converters that basically means the amount of charge on the ADC was proportional to the amount of energy deposited in the detector. The full range of the ADC was divided into 4096 channels. When the charge on the ADC was integrated, the process assigned a channel number to the amount of charge integrated. This channel number can then be converted into energy units if one knows the channel to energy conversion ratio (channel/energy unit) and assumed that this holds over the operating range of the ADC. The method by which the energy calibration and the results of those calibrations will be the subject of the rest of this section.

To calibrate the ADC's, two peaks resulting from detection of energy depositions of known values was necessary[3, 68]. It was also possible to get the energy calibration parameters if one knows the pedestal value and the channel of a known energy deposition. A pedestal value was the amount of charge recorded on the ADC when there was no signal from the detectors. This value represents the zero point for energy measurements and gave one point for a two point calibration of the ADCs. All of this required that the ADC had a linear response over the entire range of operation. The LeCroy FastBus ADC modules that were used in this experiment were designed

to be highly linear over a large dynamic range.

The energy calibration method chosen for the neutron detectors used the Compton scattering of the 2.61 MeV gamma ray from the radio nuclide ^{228}Th as the source of known energy deposition. Compton scattering is the scattering of a photon on a free electron[64]. If the photon's energy is large in comparison to the binding energy of the electron in orbit around an atom, the electron can be considered to be free. It is relatively easy to use conservation of energy and momentum to calculate the recoil energy, T , of the electron Compton scattered.

$$(3.4) \quad T = h\nu \frac{\gamma(1 - \cos \theta)}{1 + \gamma(1 - \cos \theta)}$$

$$(3.5) \quad \gamma = \frac{h\nu}{m_e c^2}$$

The differential (in energy) cross section for Compton scattering as calculated from quantum electrodynamics is called the Klein-Nishina formula. One could substitute into the Klein-Nishina formula and obtain the following formula for the energy distributions of the Compton recoil electrons. The cross section then becomes

$$(3.6) \quad \frac{d\sigma}{dT} = \frac{\pi r_e^2}{m_e c^2 \gamma^2} \left[2 + \frac{s^2}{\gamma^2(1-s)^2} + \frac{s}{1-s} \left(s - \frac{2}{\gamma} \right) \right]$$

where $s = \frac{T}{h\nu}$ and the maximum electron recoil energy, called the Compton edge, is $T_{max} = h\nu(\frac{2\gamma}{1+2\gamma})$. This formula produces the theoretical Compton spectra with its characteristic Compton edge at T_{max} (Fig. 3.7).

This description works well in describing the theoretical process which transfers energy from the photon to the electron, but it doesn't give the best description of energy deposited in a detector. "Real-world" detectors have finite resolutions that

Figure 3.7: Energy distribution of Compton recoil electrons. The relative intensities for four different photon energies, including the 2.61 MeV gamma of ^{228}Th , as a function of the electron energy. The Compton edge that represents the maximum amount of energy that can be transferred to the recoil electron can be clearly seen.

result in a “smearing” of the Compton energy spectrum. Figure 3.8 shows a typical energy spectrum taken from a neutron detector with a ^{228}Th source placed at the center of the detector. The nice, clean Compton edge of Fig. 3.7 becomes the Compton peak of Fig. 3.8 with its broad width and energy tail. The real spectrum displays on the left a characteristic background found in all real detectors and at the very far left a small, narrow peak that was the so called ADC pedestal peak.

Using this intuition about the real spectra, the Compton theoretical formulation could be modified by folding a Gaussian function into the equation and adding an exponential function to the result to simulate the background. This result could

Figure 3.8: This spectrum was the ADC output from the left PM tube of detector 1 in the front array with the ^{228}Th source in middle of the detector. The spectrum shows the characteristic pedestal events on the left, the decaying exponential background, and the Compton peak on the right.

then be worked into a function which could then be called by a fitting routine. The function had several adjustable parameters that were adjusted by the fitting program until the best fit to the spectrum was obtained. In the case of the function used in the E93-038 calibration, the function contained five adjustable parameters, one of which happened to be the MeV/channel conversion value that was of the most interest. The function had the following form

$$(3.7) \quad Compton = abs(P_3) \exp^{-\left(\frac{t_0-t}{\sigma}\right)^2} K N_t + abs(P_4) \exp^{P_5 t}$$

where $t = P_1(x - ped)$ and $\sigma = P_2\sqrt{t}$. Here x is the channel number fed into the function by the fitting program, ped is the value of the pedestal for that histogram (often called the zero point for the ADC channel), and t_0 is the energy value integrated over by the fitting function.

The function has two components. The first one is the “smearing” of the Klein-Nishina formula by a Gaussian distribution function. The term had the value of the Klein-Nishina formula, KN_t , evaluated at the initial energy value, t , multiplied by a Gaussian function with a resolution factor σ in it. The second term in the function was the exponential background that consisted of an exponential function to the t power that is related to the position on the graph x being fitted. The five adjustable parameters were the P_i . Parameters P_3 and P_4 were just scaling parameters for the overall contribution of the Gaussian “smearing” function and exponential background function, respectively. Parameter P_2 controlled the width of the Gaussian function in σ and P_5 controlled the slope of the exponential background function. The P_1

parameter was the conversion value for converting channels to MeVes' ¹. This was the value of interest in the pulse-height fitting procedure.

Calibration data were taken for the 44 neutron detectors with the ^{228}Th button source placed at the center of the outside surface. Data were not taken for the 26 tagger detectors because it was not important for the pulse height data to be known in MeVee for charged particle identification. The neutron energies for this experiment were such that a nuclear scattering with a proton in the scintillator resulted in a recoil proton that deposited energies greater than 10 MeVee; often greater than 20 MeVee. The ^{228}Th gamma ray source only emitted a 2.61 MeV gamma ray that upon Compton scattering would produce a recoil electron with energies ranging from 0 to 2.38 MeV. There was a significant difference between the calibration energy and the neutron energies of interest; the natural question was can a calibration at such low energies be trusted at higher energies? To make matters worse, the thresholds on the constant fraction discriminators (CFD) in the timing circuit were set at 4(10) MeVee for the front(rear) neutron detectors. This meant that any signal from the Compton scattering was too small to trigger the CFD and thus the trigger circuit at the FastBus/CAMAC crates resulting in no data.

These problems were remedied by inserting just before the CFDs a linear amplifier designed and constructed by Alan Baldwin at Kent State with a gain of ten and with a precision attenuator with four settings. The settings were 0.1x, 0.5x, 0.2x, and 1.0x resulting in a total amplification of the anode signal of 1x, 2x, 5x, and 10x. Therefore, using the highest possible setting resulted in a 10 times amplification of the signal coming from the anode of the detectors and effectively a calibration of 23.8 MeV of

¹millions of electron volts electron equivalent

the ADCs was performed without actually having a source which produced gammas with that energy.

The procedure for taking the calibration data was simple. The ^{228}Th source was placed at the center of the detector of interest and the 10x amplifier was placed in the circuit after the pulse height monitor and before the 50/50 splitter in the detector hut electronics. Next a minimum of 200,000 events was taken with the Hall C data-acquisition computer (DAQ) in the NPOL singles trigger setting. This trigger setting simply means that any signal from the 44 neutron detectors in NPOL which has a coincidence between its two PM tubes resulted in a trigger being sent to the DAQ to download all channels of data. The information was then saved to a temporary data directory for later storage to the tape silos and analysis.

The data were then analyzed to produce the pulse-height histograms from all detectors. There were 88 histograms in all; one for each of the PM tubes on the 44 neutron detectors. The histograms were then loaded in the Physics Analysis Workstation (PAW) and fitted with the histogram fitting routine that called a user defined function containing the modified Klein-Nishina formula discussed earlier. This formula (Eq. 3.7) had five adjustable parameters that the fitting routine adjusted to make the best fit to the data. The routine then returned the best fit values with statistical errors to the user. One of the fit parameters was the conversion value from channel number to MeVee. Therefore, once all the histograms have been fitted, a table of calibration values was created including the MeVee/channel conversion parameters and the pedestal values. This table was then fed into the G_E^n analysis programs to calibrate the pulse-height spectrum.

There were three pulse-height calibrations taken on October 10, 2000, January 28-29, 2001, and April 29-30, 2001 for all 44 neutron detectors. The three calibrations were used for three different periods of time during the data collection. There was concern that even with the pulse-height monitoring system, one would not be totally successful in maintaining constant pulse-height gain from the PM tubes; so a comparison between the calibration values determined from each of the three calibrations was performed. The three comparisons were between the October-January, October-April, and January-April calibrations[69]. The results from the comparisons can be found in Table 3.6.1. It was concluded from these results that an overall uncertainty of 10% could be assigned to the energy calibration of the detectors.

	Conversion Parameter Relative Difference	Pedestal Parameter Relative Difference
October-January	less than 15%	less than 15%
January-April	less than 10%	less than 10%
October-April	less than 10%	less than 2%

Table 3.1: The relative differences between the three pulse-height calibrations performed on the 44 neutron detectors of the NPOL apparatus.

3.6.2 Timing Calibration

In order to optimize track reconstruction and background rejection, it was necessary to produce accurate calibrations of the timing relationships between detector elements and between NPOL and HMS. The detectors in NPOL had many different attributes which gave them different timing characteristics. The primary reasons for a timing calibration were (1) the LeCroy TDC modules used for data collection were set at 50 ps/channel which is much smaller than the transit times of electrons in the

photomultiplier tubes, and (2) the transit time for signals in individual channels could be different by several microseconds. There was considerable room for improvement in the timing relationship between the nucleon(NPOL) and electron (HMS) arms of the experiment as well. Therefore, in order to optimize the track reconstruction and background rejection, it was necessary to create a program that produced a file of time calibration parameters which could be used to calibrate the data.

There were two quantities that can be calculated from the TDC information for each detector[70]. The hit position along the long axis of the detector, $xd(id)$, was expressed in terms of the TDC information from the right and left PM tubes as

$$(3.8) \quad xd(id) = x_{slope}(id) * (TDC_R(id) - TDC_L(id)) + x_{offset}(id)$$

where the x_{slope} was related to the velocity and path length of light in the scintillator and x_{offset} was related to the delays in the PM tubes, cables and electronics. The event hit time relative to the NPOL trigger was

$$(3.9) \quad td(id) = t_{slope}(id) * \frac{(TDC_R(id) + TDC_L(id))}{2} + t_{offset}(id)$$

where t_{slope} was the TDC slope value. The LeCroy TDCs used in this experiment were set such that the TDC slope value was 0.05 ns/channel. The t_{offset} parameter was used to center the mean-time peak on zero for all detectors.

A program was created by E93-038 collaborators that uses data to determine the timing calibration parameters. The program was split into several parts each with a specific task. Each part of the timing calibration program does the same two things before proceeding to the actual calibration step it was specifically designed to perform. Each event was filtered from a chain of ntuples through a list of hard-coded cuts that filled the needed histograms for that calibration procedure with the desired

events. The choice of cuts was different for each step in the calibration so it is necessary to filter the events from the ntuple chain through a different set of cuts for each procedure.

The timing algorithms were programed into the timing program as several different steps. An input file contained a list of run numbers for each procedure in the timing calibration program. Up to ten runs could be used for each calibration procedure in order to increase the statistics of events that passed the cuts for the particular procedure. The position and tagger calibration algorithms required runs with the precession magnet Charybdis turned off so that charged particles would fully illuminate the front detectors and tagger arrays. The other algorithms required that the events be single-hit events with no hit in the front tagger array. A single-hit event was any event in the front detectors which doesn't produce a second interaction in another detector of the front array. This event selection was important to the calibration algorithms to produce the best parameters possible. The timing program used raw data taken from ntuples for each run. Each run was a chain of ntuples that was read into the program and searched event by event for events which pass the particular algorithms' cuts. Once a timing calibration had been completed and checked it could be used to calibrate many data runs around the set of runs used in the timing calibration via a second program. This program read in the timing calibration parameters of interest, corrected the timing signals, and rewrote the corrected variables along with a few new variables containing detector hit information and kinematic values to new ntuples for storage to the tape silo. Three timing calibrations of the data for $Q^2 = 1.47 \text{ (GeV/c)}^2$ were performed, breaking the data collection period into three different periods. The data for $Q^2 = 1.17 \text{ (GeV/c)}^2$ was time calibrated

with one calibration file because all data were collected in eight days; too short for multiple time calibrations to have any effect. Once this was done the data could then be analyzed with the asymmetry program which will be discussed in the chapter on data analysis.

CHAPTER 4

Data Analysis

The data analysis for E93-038 was carried out during the months following the last data being collected for G_E^n . The following chapter puts together the process by which data were collected and analyzed. The procedures will be covered in several sections including data collection and analysis. This chapter will be devoted to the basic and important aspects of the data collection and analysis process.

4.1 Data Collection

The collection of data was to begin in September of 2000 but a catastrophic failure of the scattering chamber vacuum caused the destruction of the cryogenic targets. Data collection was then delayed to late October when the target was finally repaired and continued to the end of April 2001. Data collection was originally planned for three different kinematic settings and using the precession magnet at settings for 0° , $\pm 90^\circ$ precession. It was decided part way through data collection that using a different set of precession angles would improve the efficiency of data collection. Table 4.1 shows the different data sets that were collected during E93-038.

The change in precession angles from 0° , $\pm 90^\circ$ to $\pm 40^\circ$ was made to increase the efficiency of the detection processes. By using precession angles which had the magnet on all the time, most of the charged particles coming from the target were then deflected from the physical acceptance of the polarimeter. This allowed the experiment to operate at higher average beam currents all the time. The measurements with

Q^2 (GeV/c) ²	Precession Angles (degrees)	Beam Energy (GeV)	HMS angle (degrees)	HMS Momentum (MeV/c)	Nucleon Momentum (MeV/c)	Nucleon Energy (MeV)
0.447	$\pm 40^\circ$	0.884	52.65	643	711	239
1.136	$0^\circ, \pm 90^\circ$	2.326	30.93	1718	1227	606
1.169	$\pm 40^\circ$	2.415	30.15	1789	1249	624
1.474	$0^\circ, \pm 90^\circ$	3.395	23.55	2606	1448	786
1.474	$\pm 40^\circ$	3.395	23.55	2606	1448	786

Table 4.1: Kinematic settings for the different data collection sets. The NPOL central angle was at 46.0° . It should be noted that data were collected at the highest Q^2 value and near the middle Q^2 a second time with different precession angle settings.

precession angles of $0^\circ, \pm 90^\circ$ had the disadvantage of a high flux of charged particles in the polarimeter for the 0° data. This high flux of charged particles resulted in extremely high count rates in the front detectors of the polarimeter which translates to more multiple hit events in the polarimeter that are harder to analyze. The front detector count rates were limited to about 500 kHz with preferred count rates in the 200 kHz range.

Data collection began when the trigger supervisor (TSO) received the signal from the coincidence circuit that a coincidence which fit the hardware timing constraints was detected. A G_E^n coincidence was defined as the triple coincidence between the detection of an electron in the HMS, a particle in the front detectors, and a particle in the rear detectors of NPOL. The charge of the particles in NPOL was determined by the layers of charged particle detectors in front and behind the front array but was not included in the trigger circuit. It was decided to keep all events in which there was a triple coincidence and later filter out the ones involving charged particles using software cuts. When the appropriate signal from the TSO was received, the DAQ computer sent signals to download the data from the read-out-controllers (ROCs)

and saved the information to hard disk space on the computer. Later, an automatic script transferred the data from the hard disks to the tape silos on the other side of the facility. Each data run took up to two million DAQ triggers which was a sum of the HMS single hit events, NPOL single hit events, G_E^n events, G_M^n events, and operational information events. In a normal two-million event run, 1.7 to 1.8 million of those events would fit the hardware requirements to be a G_E^n event. The rest fit into the other categories.

4.2 Raw Data Replay and Calibration

The raw data was stored in a compact format developed and used at Jlab. This format has little use in the real world. It was necessary to replay the data using a program often referred to as “Engine”. Engine took in the raw data from the tape silos and processed it through a series of routines to calculate HMS information, beam information, and raw neutron polarimeter information. The processed data was then stored in “ntuples,” a format for data storage used by Physics Analysis Workstation (PAW) and accessible to Fortran routines. The raw neutron polarimeter data included timing information, pulse-height information, and detector information.

Once this first replay of the data was completed, a second procedure was performed on the data. This was the calibration procedure as discussed in section 3.6 of Chapter 3. The calibration procedure took a calibration file created by the method discussed in section 3.6 and then corrected the raw timing data. The calibration procedure then wrote the corrected data into new “ntuples” along with several other “physics” variables that were calculated from the corrected data. These included particle momentum, hit position in the detectors, particle charge identification, and

energy deposition. This information then was used to calculate the asymmetries of the experiment and set appropriate cuts on the data.

4.3 Event Analysis

As mentioned previously, the coincidence between an event in the front array, an event in the rear array, and an electron in the HMS detectors forms the G_E^n trigger. The different trigger signals arrived at the LeCroy 8LM unit with the relative timing shown in Fig. 4.1[71]. The rear neutron detector trigger circuit defined the coinci-

Figure 4.1: The relative timing signals from the four components of the coincidence timing circuit as they arrive at the LeCroy 8LM. Note that the horizontal scale is 10ns/division and the vertical scale is in volts.

dence window and produced a 100 ns wide signal that must fit the HMS trigger and the front neutron detector trigger signal within this 100-ns. This gave a 100-ns “window” for a coincidence to occur in the hardware trigger circuit. The trigger signal

from the charged particle detectors was stored only in the data stream and not used in the actual coincidence trigger circuit. All particles entering the front array and scattering to the rear array and fulfilling the coincidence requirements were saved in the G_E^n data stream. In the analysis of the data, a particle ID cut was applied to filter out the incident charged particles from the G_E^n data and fill the histograms with only neutral particle events.

The raw data were analyzed by several programs as discussed earlier. The final analysis was the asymmetry calculation, but before this could be performed the data cuts had to be set to produce the cleanest data possible. That part of the analysis will be discussed in the following two subsections.

4.3.1 HMS Analysis

The HMS device contained several detectors that gave several pieces of information about the scattered electron to the data stream. The descriptions of the individual detectors were discussed in Chapter 2. These data were analyzed to determine the best cuts for the G_E^n events in the asymmetry analysis. The major aspects looked for in analyzing HMS data were the Cerenkov detector, the lead-glass calorimeter, the invariant mass, and the electron momentum.

The Cerenkov detector is a particle identification device that gave out data in number of photo-electrons. Figure 4.2 shows the typical output seen from the Cerenkov detector. The horizontal axis is the number of photo-electrons detected and the vertical axis is the number of particles with this number of photo-electrons. Typically, an electron passing through this detector produces two or more photo-electrons while other, more massive, negative particles produced less than two photo-electrons. A

cut of two or more photo-electrons per event was set as the requirement for electron identification in the Cerenkov detector. This is shown on the spectrum with a vertical dashed line at two photo-electrons.

The vertex reconstruction for the HMS in the point-to-point setup was quoted by the Hall C group to be ~ 1 mm and the parallel-to-point setup was quoted as being 3.4 cm[56]. A thin-foil target could be surveyed into position in the beam line with a higher precision than target reconstruction could ever achieve so the parallel-to-point setup would be used in this case. This experiment used a 15-cm liquid target that required a much better reconstruction accuracy. The center of the target was surveyed into position to a high degree of precision, but the overall vertex reconstruction had to be good in order to determine if the electron-neutron interaction occurred within the target cell length, so point-to-point reconstruction was used. Figure 4.3 shows a typical spectrum for target reconstruction for data at $Q^2 = 1.47$ (GeV/c)². The vertical dashed lines shows the placement of the software cuts in the final analysis. It should be noted that the center of the target was at the origin and the z axis was defined as being along the direction of the incident electron beam. The cuts were set at ± 7.0 cm for the final analysis to remove the possibility of accepting events that occurred from electron interactions in the target cell windows. With a reconstruction accuracy on the order of a few millimeters, it was reasonable to cut the acceptance of events from the first and last 0.5 cm of the target. The spectrum shows that the loss of good events from this cut was acceptable.

The next cut pertains to the electron velocity measured in the HMS spectrometer. The HMS routines calculated the electron velocity and the beta ratio, $\beta = v/c$.

Figure 4.4 shows a typical spectrum from one data run for this variable. It was noticed that the average beta value comes out to be a bit above $\beta = 1$ which is the expected value. Because this variable was of minor significance to the experiment, the cuts that were used on this variable were set rather liberally as shown by the dashed lines in Fig. 4.4. The cuts were set to make sure the entire peak was bracketed within the cut region. This particular variable would not change event selection in terms of rejecting pion events so it was decided to use it only to reject those few events that had passed all other cuts but had a beta value far off from the expected value.

A cut on the so-called HMS “delta” spectrum was performed in order to select the quasi-elastic events from the data. Figure 4.5 is a typical spectrum displaying the HMS delta quantity. The value displayed in Fig. 4.5 is the difference between HMS central momentum and the measured momentum of the detected electron divided by the central momentum of the HMS. It is often called the dp/p spectrum. This value is then multiplied by 100 to convert it to a percent. This gave a momentum spectrum that contained the distribution of measured electron momenta around the central momentum. The central momentum was the electron momentum that the HMS spectrometer was set to accept down the center of its physical acceptance.

The reaction used in this experiment, ${}^2\text{H}(\vec{e}, e'\vec{n}){}^1\text{H}$, was a quasi-elastic reaction due to the composite nature of the deuteron. Fermi motion of the two nucleons around each other in the nuclei adds several MeV of energy to the system that results in a broadening of the quasi-elastic peak. Because of the energy transfer in the experiment, pion production was possible. The “shoulder” in the spectrum to the left of the quasi-elastic peak was the pion contribution being added into the spectrum. It was cut off on the left side by the finite acceptance of the HMS device. A cut of

-3, +5% was applied to the delta spectrum to select electrons from the quasi-elastic events and to eliminate electrons from pion production events. The chosen cuts are displayed in the spectrum by the vertical dashed lines and the accepted events are in the shaded area.

The next cut made on HMS data concerned the lead-glass calorimeter. An electron traveling through the detector package in the HMS detector hut encounters the lead-glass shower counter last. The lead-glass shower counter uses the principles of energy loss in heavy materials to create an *electromagnetic shower* as long as the energy of the detected electron was greater than the energy below which ionization energy loss becomes dominant. A more detailed explanation can be found in Hodgson's book[72]. This electromagnetic shower is then detected by the photomultiplier tubes and a value of the energy deposition is calculated from the detected photons. The calorimeter spectrum was calibrated to optimize the electron energy resolution. Figure 4.6 shows the detected electron energy in the calorimeter normalized on the electron momentum. This normalization was done by dividing the reconstructed calorimeter value by the central electron momentum and by one plus the HMS delta value.

$$(4.1) \quad E_e = \left(\frac{HMS_{showerenergy}}{2.606 \text{ GeV}/c} \right) \left(\frac{1}{1 + \left(\frac{HMS_{delta}}{100} \right)} \right)$$

The normalized electron energy spectrum then has the typical shape of Fig. 4.6. Because the normalization was on the central electron momentum for the HMS, the central value of the spectrum was 1 with some distribution about this value. Cuts of 0.7 and 1.4, shown as vertical dashed lines, were set to remove events which had energies far from the expected electron energy from quasi-elastic collisions in the target.

This spectrum was formed using the cuts that the event had to be an electron identified by the Cerenkov detector. Note that the accepted events from the calorimeter cut are shown between the two vertical dashed lines as the shaded region.

4.3.2 NPOL Analysis

Much like the analysis of the HMS data, several different spectra from the NPOL data needed to be analyzed and appropriate cuts set on the data. The most important of these cuts were the ones on the so-called corrected time-of-flight (cTOF) and corrected delta time-of-flight spectra ($r\Delta\text{TOF}$). The cuts on these two spectra were used to eliminate a large portion of the background events in the spectra and to select the events that were best described by quasi-elastic kinematics.

Figure 4.7 is a typical corrected time-of-flight spectrum. This spectrum was created by taking the difference between the calibrated TDC time in the front detectors of the neutron polarimeter and the calibrated TDC time in the HMS for the detected electron. The time difference was then corrected by subtracting the calculated time-of-flight from quasi-elastic kinematics to form the final corrected time-of-flight spectrum as shown in Fig. 4.7. This has the effect of “zeroing” the cTOF peak. From this spectrum it was decided to use a very narrow cut of ± 1 ns to eliminate the background due to random events in the large volume detectors of NPOL. This cut also eliminated some of the undesirable events coming from protons converted in the lead curtain to neutrons. These events produced a false asymmetry in the polarimeter that had to be studied in order to assess their contribution to the overall asymmetry. It was found that some of these unwanted events could be eliminated by cutting off the tail on the right side of the cTOF peak.

The second timing histogram used to remove background events from the data was the delta time-of-flight (ΔTOF) spectrum. This spectrum was formed by the difference in the calibrated TDC time in the rear detector arrays and the calibrated TDC time in the front array. A calculation of the delta time-of-flight made from the elastic scattering of a nucleon from a proton in the front scintillator material was subtracted from the measured value to produce a corrected delta time-of-flight which centered the peak on zero. This time-of-flight for the nucleon from the front array to the rear arrays was used to eliminate background events from the asymmetry calculation. Figure 4.8 displays a typical ΔTOF spectrum. The spectrum was formed from the events that passed the corrected time-of-flight cut in the previous section. The shaded area between the two vertical dashed lines (-1, +8 ns) were the events that were kept after the cut to the ΔTOF spectrum was applied.

Investigation of the ΔTOF spectrum showed that the main peak consisted of neutral events in the front array scattering to the rear array in an acceptable amount of kinematic time. The leading edge of the main peak on the left corresponds to fast particles arriving at the rear detectors sooner than expected by kinematic calculations. For example, events in the lead curtain sometimes produced neutral pions that then decayed to two gamma rays. Because these gamma rays are neutral they can pass our neutral/charged particle identification as neutral particles and scatter in the front array to the rear array producing a type of background event. Because the gamma rays propagated at the speed of light, they arrived in the rear detectors slightly sooner than the true neutron events producing that distinctive “shoulder” or peak on the left side of the the main peak in the spectrum. These events were removed from the asymmetry calculation by applying a cut to the data between the

two peaks at -1 ns.

The high side cut was set at +8 ns. Events which arrive later in the spectrum come from neutral events which scatter in the front array to positions in the rear array away from the central scattering angle. The ability of the polarimeter to analyze the polarization of the incident particle was dependent on the angle through which the particle was scattered. This affects the measured asymmetry by diluting the asymmetry with events with poor analyzing powers thereby increasing the number of events which pass cuts but adding no additional events to the data which have a usable asymmetry. The cut of +8 ns was chosen by comparing the measured quantities for different values of the right side cut and determining where the events produced no additional information about the asymmetry.

When a particle interacts within the scintillator material of the detectors, it deposits energy in the material that causes the material to scintillate. This light was detected by the photomultiplier tubes and was, to first order, proportional to the amount of energy deposited within the scintillator. A more detailed explanation of the detectors and calibration process was given in Chapter 3. Figure 4.9 shows two spectra that are termed the pulse-height spectra. The top histogram in Fig. 4.9 was the pulse-height spectrum for all the front array detectors summed together. Namely detectors 1 through 20. A software threshold of 8 MeVee was used for the front detectors to discriminate between low energy events and desired nucleon scattering events. The peak at just over 15 MeVee represented the typical energy loss experienced by a proton passing through the 4 cm of plastic scintillator. This value would have been different for a different thickness of plastic scintillator.

The bottom histogram of Fig. 4.9 is the pulse-height spectrum for all the rear array detectors which were detectors 21 through 44. A software threshold of 20 MeVee was used on the rear detectors to remove low energy background events that came from random particles moving through the experimental device. Because the scattering of the nucleons in the front array reduced the energy of the nucleon and the path through the rear detectors was larger than 4 cm, a large energy deposition was expected and therefore a higher threshold was set.

The recoil momentum of the detected nucleon is plotted in Fig. 4.10. This particular spectrum was formed by accepting the events that passed the cTOF cut of ± 1 ns, were identified as being neutral, and the event had an associated particle in the HMS which was identified as being an electron from the Cerenkov detector. In the collision between the electron and the neutron, the recoil momentum of the neutron could have a fairly wide spread. The acceptance of the HMS detector and NPOL did allow events that included pion production into the data. As discussed earlier in the section on HMS cuts, pion production was reduced in several ways including the cut on the HMS delta spectrum. Another way to eliminate pion events was to cut on this recoil momentum spectrum. A cut of $0.100 \text{ GeV}/c$ was assigned to this spectrum and all events below this in recoil momentum were kept. Those events are depicted by the shaded region of Fig. 4.10.

The last spectrum to be considered in the NPOL analysis was the invariant mass spectrum. Figure 4.11 is a typical invariant mass spectrum from the data. The tall peak at just over $0.9 \text{ GeV}/c^2$ corresponds to the good events from the data. The question about where to set the cut is answered by the physics of the spectrum. An invariant mass value was expected to be around $0.938 \text{ GeV}/c^2$, which is the mass of

the neutron. Values above $1.04 \text{ GeV}/c^2$ were from events which represent invariant masses larger than expected. This means these events must be from pion production. The event was reconstructed as being a collision from a target nucleus that was heavier than $1.04 \text{ GeV}/c^2$ with a neutron detected in the polarimeter. This means the difference in the mass of the neutron and the measured invariant mass must have gone to the pion production process. The vertical dashed lines show where the cut was set and the shaded region indicates the accepted events.

4.4 Background Events

There were a number of ways for background events to get into the data stream and it was necessary to understand where these events originated so the software could be written to deal with these events. First consider the definition of a G_E^n event by the hardware trigger, as described earlier. The triple-coincidence was defined as the detection of an event in the HMS detectors, the front array of NPOL, and the rear array of NPOL within the coincidence time setup with the coincidence electronics (see section 4.3). Now a real event would have been the collision of an electron with a neutron in the target resulting in a signal in the HMS from that scattered electron and the recoil neutron producing a signal in the front array and the the rear array of the NPOL detectors. These events make up the desired G_E^n data. Starting with this picture, it is possible to image background events that could and did come into the data.

4.4.1 Random Background Events

The first background event to consider was the correlated electron-neutron front detector event with an uncorrelated event in the rear detectors. Because these events had a correlated electron-neutron-front coincidence, they fit the requirements for being in the cTOF peak (Fig. 4.7) but the rear detector signal was random. This was depicted by the flat background in the corrected delta time-of-flight spectrum (Fig. 4.8). This background could not be removed from the spectra but its contribution to the Δ TOF peak could be estimated by averaging the flat background to the left side of the main peak. This could then be subtracted from the calculated yield of the peak when the asymmetry calculation was performed.

The next type of background event was the random coincidence of an electron in the HMS with a correlated coincidence of a neutron in the front and rear arrays of NPOL. These events populate the random coincidence area of the cTOF spectrum as shown in Fig. 4.7, but at the same time fall under the Δ TOF peak since they were correlated neutron events in the front and rear arrays. A lot of these events were removed from the delta time-of-flight spectra by doing a tight cut on the cTOF peak of ± 1 ns.

The final background event possibility stemming from random signals in detectors comes from the triple random coincidence of three uncorrelated particles in the HMS, neutron-front array, and the neutron-rear array. This type of event was characterized by appearing in the random background of both the cTOF and Δ TOF spectrum. These events were removed by the cuts on both cTOF and Δ TOF spectra.

4.4.2 Background Events from Charge-Exchange Reactions

There was an event that passed all cuts and particle identification requirements but still falls in the background category. This event formed a proper triple-coincidence signal in the selected regions of the cTOF and Δ TOF spectra and was identified in the front array as a neutral particle. This event did not, though, originate from a neutron in the target. Instead this event was an electron-proton collision in the target that then underwent charge-exchange during passage through the lead curtain in front of the polarimeter shield house. This reaction was often called the “two-step process” which is written as $H^2(\vec{e}, e'\vec{p})^1H + {}^{208}\text{Pb}(\vec{p}, \vec{n})$. The lead curtain was 10.36-cm thick and with recoil energies of 786 MeV, the protons easily exited the lead. The passage of the protons through the lead curtain resulted in a degradation of the proton energy and therefore a loss in velocity which translates into a slightly longer time-of-flight between the target and the front array. This means the neutrons that came from the charge-exchange process in the lead curtain would populate the tail of the cTOF peak. A tight cut on the cTOF peak helped to eliminate, or at the least, reduce, the contribution of a false asymmetry due to this charge-exchange process.

An experiment at the MAMI accelerator in Mainz, Germany used the quasi-elastic reaction ${}^3\vec{H}e(\vec{e}, e'n)pp$ to measure G_E^n . The setup required the use of a 5-cm lead wall to shield the neutron detectors from low energy target radiation and the collaboration reported a relative contribution from these contamination events of less than 3% [40]. A previous G_E^n experiment, E85-05, at the BATES accelerator used the same technique as E93-038 and reported a contribution to the events under the $(\vec{e}, e'\vec{n})$ coincidence peak to be less than 1% [2]. It was expected that data taken at the lowest Q^2 of $0.45 \text{ (GeV}/c)^2$ in E93-038 would also have a negligible contribution from

this two-step process. The contribution from the two-step process to the data at higher Q^2 values was relatively unknown. An analysis of the data to determine the contribution to the asymmetry from these events needed to be done.

To evaluate this contribution, data were taken with a liquid hydrogen target (LH_2) in the normal G_E^n coincidence mode. It was desired to analyze the LH_2 data with the same cuts and requirements as the LD_2 data so that a conclusion about the number of charge-exchange events under the coincidence peak could be reached. If the fraction of events in the G_E^n calculation was significant, a correction factor would have to be applied to the final answer. If the fraction of events was small enough, the correction would then be dropped. The details of the false asymmetry analysis calculations can be found in Appendix A for the interested reader. Because the measured asymmetries from the LH_2 target are small, it is difficult to determine if there is polarization being transferred from the proton to the neutron in the lead. The conclusion from the initial analysis using the previous method done by Madey, Tireman, and Plaster[74] was that because the measured asymmetries were consistent with zero, then the polarization transfer must also be consistent with zero. This conclusion, though, is plagued by low statistics and has large statistical error bars associated with it.

A more recent estimation of the charge-exchange systematic uncertainty was made by Brad Plaster[73]. From Appendix A, the asymmetry measured by the polarimeter, ξ_m , can be written as

$$(4.2) \quad \xi_m = f_R \xi_R + f_B \xi_B$$

where f_B is the contamination fraction from the two-step process, $f_R = 1 - f_B$, ξ_R is the asymmetry from the ${}^2\text{H}(\vec{e}, e'\vec{n})$, and ξ_B is the asymmetry from the two-step

process. In general, the asymmetry from the two-step process can be written as

$$(4.3) \quad \xi_B = P_L(D_{LS}^p \cos \chi_p + D_{LL}^p \sin \chi_p)D_{SS'}^{Pb}A_y.$$

Here P_L is the electron beam polarization, D_{LS}^p and D_{LL}^p are the proton polarization transfer coefficients in the LH₂ target, χ_p is the proton spin precession angle, $D_{SS'}^{Pb}$ is the proton-neutron transfer coefficient in the lead, and A_y is the analyzing power of the polarimeter. Combining Eqs. 4.2 and 4.3 yields a relationship for the real asymmetry, ξ_R , that takes into account the uncertainties from f_B , χ_p , $D_{SS'}^{Pb}$, and A_y .

$$(4.4) \quad \xi_R = \frac{\xi_m - f_B P_L(D_{LS}^p \cos \chi_p + D_{LL}^p \sin \chi_p)D_{SS'}^{Pb}A_y}{1 - f_B}$$

From this equation, it is possible to calculate ξ_R for three possible values of $D_{SS'}^{Pb}$: -1, 0, +1. With these values of ξ_R , the values of g can be extracted for each of the three values of $D_{SS'}^{Pb}$. The magnitude of the charge-exchange systematic uncertainty is then estimated by Plaster[73] with the relation

$$(4.5) \quad \left(\frac{\Delta g}{g}\right) \sim \frac{\frac{1}{2}[g(D_{SS'}^{Pb} = +1) - g(D_{SS'}^{Pb} = -1)]}{g(D_{SS'}^{Pb} = 0)}.$$

The results of this method of estimating the charge-exchange systematic uncertainty will be discussed in Chapter 5 along with the results of the previous method.

4.4.3 Background Events from Interactions in the Target Cell Windows

Another source of background events that fits the timing requirements of the data analysis are from the target cell end caps and walls. Unfortunately, because of safety concerns, it was not possible to take data on an equivalent target cell with the liquid cryogen removed; the so-called empty target configuration. Instead, the experiment

used a set of aluminum foils spaced 15 cm apart (the same distance as the length of the real target) to simulate the end caps of the target cell. The aluminum foils were also made thicker to compensate for the low event rate. The idea was that the rate of events scales linearly with the thickness of the aluminum foils. The target group reported that the aluminum foils were ~ 8 times thicker than the ends of the target cell.

There were 360.4 mC of data taken on the 15-cm dummy target. As in the case of the two-step process, the data were analyzed using the same event requirements and calibrations as the normal LD₂ data. The resulting histograms from the asymmetry program were nearly empty. The conclusion was that the event cuts removed essentially all the events that come from the ends of the target cells. Indeed, a look at the target reconstruction spectra shows that the target cut of ± 7 cm cuts nearly all the events from the dummy target runs (Fig. 4.12).

4.5 G_E^n Calculation

4.5.1 Calculation of the Experimental Asymmetry

The polarimeter was designed to be up/down (U/D) symmetric in terms of physical acceptance and threshold detection. Errors in positioning the detectors and differences in detector characteristics and calibration can cause instrumental asymmetries which will be discussed later. A very small asymmetry in the number of left/right (L/R) helicity electrons was maintained. The ratio between L/R helicity electrons was never allowed to rise above $1 \cdot 10^{-3}$. This leaves the measured scattering asymmetry in the polarimeter as being the cross-ratio of the four possible scattering events. The total number of neutrons scattering to the upper-rear(lower-rear) array

from left helicity electrons was denoted as $N_U^L(N_D^L)$, and the total number of neutrons scattering to the upper-rear(lower-rear) array from right helicity electrons was denoted as $N_U^R(N_D^R)$. The cross-ratio is defined as

$$(4.6) \quad r \equiv \left[\frac{N_U^L N_D^R}{N_D^L N_U^R} \right]^{1/2}$$

and the asymmetry is

$$(4.7) \quad \xi_m \equiv \frac{r - 1}{r + 1}.$$

It is also possible to form the geometric ratio r_g

$$(4.8) \quad r_g \equiv \left[\frac{N_D^L N_D^R}{N_U^L N_U^R} \right]^{1/2}$$

and the beam ratio r_b

$$(4.9) \quad r_b \equiv \left[\frac{N_U^L N_D^L}{N_U^R N_D^R} \right]^{1/2}$$

by rearranging the components in Eq. 4.6. These two ratios can be used to calculate the instrumental asymmetry, ξ_{inst} , and the beam asymmetry, ξ_{beam} , and determine their contribution to the physical asymmetry Eq. 4.7. Asymmetries for two different projections of the transverse polarization of the neutron were measured in this experiment. As will be seen in the next section, the ratio of these two asymmetry measurements forms a very powerful method for extracting the ratio $g \equiv G_E^n / G_M^n$ that eliminates most of the instrumental asymmetries and the need to know the analyzing power of the polarimeter.

4.5.2 Calculating G_E^n from the Asymmetries

In Chapter 1 it was shown that the ratio of electric to magnetic form factors is related to the ratio of the transverse and longitudinal polarizations of the recoil

neutron (Eq. 1.21). Defining $K_R \equiv K_L/K_S$ reduces Eq. 1.21 to

$$(4.10) \quad g = -K_R \frac{P_{S'}}{P_{L'}}$$

By measuring the scattering asymmetry for both the transverse and longitudinal components of the recoil neutron, a measurement of the ratio g could be made. In general the scattering asymmetry in the polarimeter for a particular negative precession angle is

$$(4.11) \quad \xi_- = P_{S'} \cos(-\chi) + P_{L'} \sin(-\chi)$$

and for a positive precession angle it is

$$(4.12) \quad \xi_+ = P_{S'} \cos(\chi) + P_{L'} \sin(\chi).$$

Here $P_{S'}$ ($P_{L'}$) are the transverse(longitudinal) components of the neutrons polarization before they are precessed by the magnetic field. This experiment used two different precession methods in its data collection and each case will be considered separately next.

In the case of the $0^\circ, \pm 90^\circ$ precession method, the replacement of $-\chi$ with 0° and χ with 90° yields the following results.

$$(4.13) \quad \xi_{S'} = P_{S'}$$

$$(4.14) \quad \xi_{L'} = P_{L'}$$

Taking the ratio of these above results and defining it as η yields

$$(4.15) \quad \eta_{SL} \equiv \frac{\xi_{S'}}{\xi_{L'}} = \frac{P_{S'}}{P_{L'}}.$$

Combining Eq. 4.10 with Eq. 4.15 yields

$$(4.16) \quad g = -K_R \frac{\xi_{S'}}{\xi_{L'}} = -K_R \eta_{SL}$$

The $\pm\chi^\circ$ case comes out a bit different. Recall that the polarimeter was setup to be sensitive only to the transverse component of the recoil neutron's polarization. Going back to Eqs. 4.11 and 4.12 and taking the ratio results in the following

$$(4.17) \quad \eta_{\pm} \equiv \frac{\xi_-}{\xi_+} = \frac{P_{S'} \cos(\chi) - P_{L'} \sin(\chi)}{P_{S'} \cos(\chi) + P_{L'} \sin(\chi)}.$$

It is then possible to solve this equation for $P_{S'}/P_{L'}$ in terms of the angle χ and $\eta_{\pm}$.

$$(4.18) \quad \frac{P_{S'}}{P_{L'}} = \frac{1 + \eta_{\pm}}{1 - \eta_{\pm}} \tan \chi$$

This result can then be substituted into Eq. 4.10 to get g in terms of the asymmetry ratio $\eta_{\pm}$

$$(4.19) \quad g = -K_R \frac{1 + \eta_{\pm}}{1 - \eta_{\pm}} \tan \chi.$$

The relative uncertainty in g for the two cases can be found from propagation of errors through Eqs. 4.16 and 4.19, respectfully. The relative error for g in the $0^\circ, \pm 90^\circ$ method is

$$(4.20) \quad \left(\frac{\Delta g}{g} \right)_{0^\circ, \pm 90^\circ}^{1/2} = \left(\frac{\Delta \xi_{S'}}{\xi_{S'}} \right)^{1/2} + \left(\frac{\Delta \xi_{L'}}{\xi_{L'}} \right)^{1/2}$$

and for g for the $\pm\chi$ method is

$$(4.21) \quad \left(\frac{\Delta g}{g} \right)_{\pm\chi}^{1/2} = \frac{2\eta_{\pm}}{((\eta_{\pm})^2 - 1)} \left(\frac{\Delta \xi_-}{\xi_-} \right)^{1/2} + \left(\frac{\Delta \xi_+}{\xi_+} \right)^{1/2}.$$

Going back to the ratio of the electric to magnetic form factor, $g = G_E^n/G_M^n$, this can be solved for the electric form factor and propagation of errors results in an expression for the relative error.

$$(4.22) \quad G_E^n = g G_M^n$$

$$(4.23) \quad \left(\frac{\Delta G_E^n}{G_E^n} \right)^{1/2} = \left(\frac{\Delta g}{g} \right)^{1/2} + \left(\frac{\Delta G_M^n}{G_M^n} \right)^{1/2}$$

With this expression and a value of G_M^n , it is now possible to calculate the experimental value of G_E^n from the data. The value of G_M^n used in the final calculations is calculated from a fit performed by Jim Kelly on known G_M^n data and is discussed in Chapter 5.

This completes the necessary equations for calculating G_E^n values from asymmetry measurements in the polarimeter. It is now possible to calculate an experimental value for G_E^n given the weighted-average of all the asymmetries from all the data runs for the two measurements of the transverse projection of the neutron's polarization.

Figure 4.2: Typical output from the Cerenkov detector in the HMS detector package. The detector was used for electron particle identification. It was accepted that any particle producing a response of 2 photo-electrons or more was an electron and this is shown on the spectrum by the dashed vertical line.

Figure 4.3: HMS target reconstruction along the z axis in the target coordinate system. The z axis points in the direction of the incident electron beam momentum vector. The origin of the coordinate system is the center of the target. The two vertical dashed lines shows the placement of the software cuts in the final analysis.

Figure 4.4: The distribution of beta values calculated for the electron events in the HMS. These electrons were well within the relativistic range so a beta value of 1 was expected. The dashed lines shows the cuts chosen for this particular variable.

Figure 4.5: The delta spectrum displays the dp/p information in a percent. The value was calculated by taking the difference between the measured electron momentum and the central momentum of the spectrometer (dp) and dividing it by the central momentum. This value was then multiplied by 100 to convert it to a percent. This spectrum shows the quasi-elastic peak from the ${}^2\text{H}(\vec{e}, e'\vec{n}){}^1\text{H}$ reaction and the pion production shoulder to the left of the main peak. A cut depicted by the vertical dashed lines of $-3, +5\%$ was used to select electrons from the quasi-elastic events and suppress pion production events.

Figure 4.6: The HMS lead-glass calorimeter energy spectrum. Note the spectrum is normalized on the electron momentum by dividing the calorimeter energy by the central electron momentum and 1 plus the delta value. This normalization results in a spectrum centered around the value of 1. The cuts used in the data analysis for the HMS calorimeter spectrum are shown by the vertical dashed lines and all events found between these two lines (shaded region) are accepted as good events.

Figure 4.7: Time-of-flight (TOF) between the HMS and the front detectors in NPOL. This was a measure of the flight time from the target to the front NPOL detectors. The time-of-flight spectrum was centered around a value of zero by subtracting the expected time-of-flight from quasi-elastic kinematic calculations from the measured time-of-flight. Note the FWHM was about 1.4 ns. The cut was set at ± 1 ns (as depicted by the vertical dashed lines) to eliminate the background events from the data.

Figure 4.8: The difference between the TDC time in the rear detectors and the TDC time in the front detectors. Also known as the delta time-of-flight spectrum (ΔTOF), this spectrum represents on a event-by-event basis the time it takes for a particle in the front array to travel to the rear array minus the calculated time-of-flight from elastic proton scattering from the center of the front array to the center of the rear array. The shaded region between the dashed vertical lines shows the events that were kept after cuts.

Figure 4.9: The top spectrum represents the sum of all the pulse-height responses in the front array of NPOL, namely detectors 1 through 20. The shaded region represents the events that were kept after a software threshold of 8 MeVee was set. The bottom spectrum is a composite of all the pulse-height responses in the rear detectors which are detectors 21 through 44. Once again the shaded region represents the events that were kept after a software threshold of 20 MeVee was set.

Figure 4.10: The distribution of the nucleon recoil momenta. A cut of 0.100 GeV/ c was chosen to suppress pion production events. The shaded region showed the accepted events.

Figure 4.11: This invariant mass spectrum was created from the reconstruction of the electron-neutron collision event. The invariant mass should be centered around the value of $0.938 \text{ GeV}/c^2$, the mass of the neutron. A generous resolution was given and a cut of $1.04 \text{ GeV}/c^2$ was chosen to cut off events that stem from pion production. The shaded region below the vertical dashed line indicates the events that were kept.

Figure 4.12: The reconstructed target vertex spectrum for the 15-cm dummy target. The cuts at ± 7 cm eliminate nearly all of the neutron events stemming from the entrance and exit windows of the target cell.

CHAPTER 5

Results and Conclusions

This final chapter will cover the G_E^n extraction procedure and results. The contributions to the uncertainty in the resulting G_E^n values will be considered in the sections on statistical and systematic uncertainties. A section is devoted to analyzing the instrumental asymmetry and its effect on the physical asymmetry. Acceptance-averaging and nuclear corrections to G_E^n will be handled in a separate section. Finally, the chapter (and the dissertation) will be finished with a section on conclusions and comments.

5.1 Extracting G_E^n from the Data

The extraction equations were covered in the last section of Chapter 4. Here the results for G_E^n from all the data collected at $Q^2 = 1.17$ and 1.47 $(\text{GeV}/c)^2$ will be presented along with a brief discussion on how this was accomplished. First, a discussion about the cutting of data runs with particular problems will be covered. Following that will be the discussion of the G_E^n result and the error analysis.

5.1.1 Quality Data Cuts

It is important to first describe the data selection process that was followed in the data analysis. Data were collected in portions called data runs. Each run could be as short as a few thousand data acquisition (DAQ) triggers and as many as three million

triggers. The data runs were to be three million DAQ events, but equipment failures often forced the early termination of a data run. It was necessary to establish a list of “quality” data cuts to eliminate any contamination of the data from known problems with the equipment. Tables 5.1 and 5.2 list the total amount of data analyzed and the amount of data that was rejected for $Q^2 = 1.47 \text{ (GeV}/c)^2$. Data runs that were rejected fell into the seven categories listed in Table 5.2. Runs with less than 500,000

Data Group	Amount Before Quality Cuts (mC)	Amount After Quality Cuts (mC)
Charybdis off	32219	27378
Charybdis on	10182	8195
Amount of charge	42401	35573

Table 5.1: Summary of the amount data collected at $Q^2 = 1.47 \text{ (GeV}/c)^2$ in units of electron charge (mC).

Rejection reason	Amount (mC)	% of total
low statistics	1672	3.94
HMS problems	2689	6.34
DAQ problems	1308	3.08
MCC problems	359	0.85
Target Problems	411	0.96
NPOL/Charybdis	21	0.05
Other problems	368	0.87
total	6828	16.10

Table 5.2: The quality data cuts that were applied to the data at $Q^2 = 1.47 \text{ (GeV}/c)^2$ are listed here with the amount of data each rejected in units of charge (mC) and as a percentage of the total data analyzed.

total events were rejected for low statistics and this accounted for about 3.9% of the data. Any runs forced to end early for problems with the electron spectrometer HMS were rejected and this accounted for another 6.3% of the data. About 3% of the data

was rejected for crashes with the data acquisition computer and the associated read out controllers at the electronics racks. Another 0.85% was lost due to problems at the machine control center (MCC) resulting in excessive RF trips and poor beam delivery. Data lost to target issues accounted for 0.96% and NPOL and Charybdis magnet failures accounted for only 0.05% of the data. Anything that didn't fall into one of the above categories was put in the other category and this accounted for 0.87% of the data rejected. When it is all added up, 16.1% of the data was rejected for reasons of data quality.

The $Q^2 = 1.17 \text{ (GeV}/c)^2$ data are summarized in Tables 5.3 and 5.4. A total of 16681.2 mC of data was collected and 3322.4 mC was rejected for reasons listed in Table 5.4. This left a total of 13358.6 mC of data in the final analysis. Most of the data rejected, approximately 13.2%, was for HMS problems. The second largest contributor to rejected data was DAQ problems at 3.4% while low statistics came in at 1.4% and MCC problems at 1.3%. No data were rejected for NPOL/Charybdis failures and the target accounted for less than 1% of the data rejected. Non-specific problems labeled simply as "Other Problems" also contributed to less than 1% of the rejected data.

Data Group	Amount Before Quality Cuts (mC)	Amount After Quality Cuts (mC)
Charybdis +226.13 A	9098.6	7008.2
Charybdis -226.13 A	7582.6	6350.4
Amount of charge	16681.2	13358.6

Table 5.3: Summary of the amount data collected at $Q^2 = 1.17 \text{ (GeV}/c)^2$ in units of electron charge (mC).

Rejection reason	Amount (mC)	% of total
low statistics	234.6	1.41
HMS problems	2196.0	13.16
DAQ problems	563.4	3.38
MCC problems	220.0	1.33
Target Problems	28.5	0.17
NPOL/Charybdis	0	0
Other problems	80.1	0.48
total	3322.4	19.92

Table 5.4: The quality data cuts that were applied to the data at $Q^2 = 1.17 \text{ (GeV/c)}^2$ are listed here with the amount of data each rejected in units of charge (mC) and as a percentage of the total data analyzed.

5.1.2 Point Extracted G_E^n for $Q^2 = 1.17$ and 1.47 (GeV/c)^2

Extracting the value of G_E^n from the experimental asymmetry required knowledge of the magnetic form factor of the neutron G_M^n because this method measures the ratio of the electric to magnetic form factors. Jim Kelly[75] computed G_M^n values in the Q^2 region of this experiment using a fit to present data. Table 5.5 contains the G_M^n values used in this analysis where $\mu_n = -1.913$ and $G_D = (1 + Q^2/\Lambda^2)^{-2}$ where $\Lambda^2 = 0.71 \text{ (GeV/c)}^2$. Note that the values for Q^2 are the acceptance-averaged results for the experiment and will be discussed in more detail in a later section. Equation

$Q^2 \text{ (GeV/c)}^2$	$G_M^n/\mu_n G_D$	G_M^n	$\Delta G_M^n/G_M^n \text{ (\%)}$
0.447	1.003 ± 0.006	-0.7225 ± 0.0043	0.6
1.132	1.057 ± 0.017	-0.3004 ± 0.0048	1.6
1.450	1.044 ± 0.024	-0.2158 ± 0.0050	2.3

Table 5.5: G_M^n values calculated by Jim Kelly for this analysis.

4.16 shows that the ratio of the measured asymmetries is related to the g value which is defined in Eq. 4.22 as being the ratio of the electric to magnetic form factors of

the neutron. Therefore, using the measured asymmetries to calculate g and using the values in Table 5.5 for the magnetic form factor one can extract the value of G_E^n .

There are two different ways one can measure the scattering asymmetry in the polarimeter. Because the recoil neutron energy is 786 MeV, the scattering process in the front detectors of NPOL results in recoil protons with sufficient energy to escape the front detectors altogether. Using the charged particle detectors on the exit side of the front array (see Fig. 3.1), it is possible to detect these recoil protons and track their angle to the rear arrays. This makes it possible to use either the neutron or the recoil proton from the scattering in the front array to determine the scattering asymmetry in the polarimeter. The analysis code allows for the determination of the asymmetry from either one of these two processes that are referred to as (n,n) analysis and (n,p) analysis. The (n,n) analysis refers to the detection of a neutral in the front detectors and a neutral in the rear detectors. The (n,p) analysis refers to detecting a neutral in the front detectors and a charged particle in the rear detectors. Notice that the detected particles are referred to as being neutral or charged. The detectors supply only timing and pulse height information that can be used to determine if the event was caused by a neutral particle or a charged particle but it can not identify the type of particle (neutron, proton, pion, electron, photon, etc.).

The results for the (n,n) and (n,p) analyses separately and the weighted average are presented in Table 5.6 for $Q^2 = 1.17 \text{ (GeV}/c)^2$ and in Table 5.7 for $Q^2 = 1.47 \text{ (GeV}/c)^2$. A few things should be noted. First, the weighted average was performed with the value of g which, of course, is related to the ratio of the scattering asymmetries. Secondly, the errors in this table are statistical only. Systematic errors need to be accounted for as well and that will be covered in following sections. Lastly, it

	(n,n) analysis	(n,p) analysis	weighted average
$\xi_{S'} (\%)$	$+2.92 \pm 0.29$	$+1.42 \pm 0.22$	——
$\xi_{L'} (\%)$	-4.75 ± 0.31	-2.76 ± 0.25	——
g	-0.121 ± 0.013	-0.162 ± 0.024	-0.130 ± 0.011
G_E^n	$+0.036 \pm 0.0039$	$+0.048 \pm 0.0073$	$+0.039 \pm 0.0034$

Table 5.6: Results for G_E^n at $Q^2 = 1.17 \text{ (GeV}/c)^2$ without any corrections or acceptance averaging performed on the data. The results were calculated for the raw asymmetry measured in the polarimeter and include statistical errors only.

	(n,n) analysis	(n,p) analysis	weighted average
$\xi_{S'} (\%)$	-1.26 ± 0.19	-0.60 ± 0.16	——
$\xi_{L'} (\%)$	-4.85 ± 0.34	-2.53 ± 0.33	——
g	-0.173 ± 0.029	-0.158 ± 0.047	-0.169 ± 0.025
G_E^n	$+0.037 \pm 0.0063$	$+0.034 \pm 0.010$	$+0.036 \pm 0.0055$

Table 5.7: Results for G_E^n at $Q^2 = 1.47 \text{ (GeV}/c)^2$ without any corrections or acceptance averaging performed on the data. The results were calculated for the raw asymmetry measured in the polarimeter and include statistical errors only.

should be noted that this G_E^n extraction was performed at the central Q^2 value, the so-called point analysis. The experimental acceptance has a finite value that covers a range of Q^2 values, so the final value of G_E^n will need to be extracted using an acceptance-averaging technique. That will be discussed in a later section as well.

5.2 Statistical Uncertainty

The statistical uncertainty in the asymmetry is found by first propagating the errors through the cross ratio in Eq. 4.6 and then the asymmetry Eq. 4.7. This results in an error expression for the cross ratio that has the following form.

$$(5.1) \quad dr = \frac{1}{2}r \left[\left(\frac{dN_U^L}{N_U^L} \right)^2 + \left(\frac{dN_D^R}{N_D^R} \right)^2 + \left(\frac{dN_D^L}{N_D^L} \right)^2 + \left(\frac{dN_U^R}{N_U^R} \right)^2 \right]^{1/2}$$

The propagation of errors for the asymmetry equation yields

$$(5.2) \quad d\xi_m = \frac{2}{(r+1)^2} dr.$$

Recall from Chapter 4 that the values N_j^i are the yields for the four possible combinations of left(right) electron helicity and up(down) scattering in the polarimeter. The statistical errors in these values, dN_j^i , are the square root of the yields. One can insert Eq. 5.1 into Eq. 5.2 to obtain an expression for the statistical error in ξ_m in terms of the cross ratio r and relative uncertainties in the yields.

$$(5.3) \quad d\xi_m = \frac{r}{(r+1)^2} \left[\left(\frac{dN_U^L}{N_U^L} \right)^2 + \left(\frac{dN_D^R}{N_D^R} \right)^2 + \left(\frac{dN_D^L}{N_D^L} \right)^2 + \left(\frac{dN_U^R}{N_U^R} \right)^2 \right]^{1/2}$$

5.3 Systematic Uncertainties

The statistical uncertainty is not all that can be said about the error associated with the experimental value of G_E^n . Any experiment has opportunity for systematic errors to affect the results of the measurements. In the following subsections, the systematic errors of this experiment will be outlined and their contribution to the overall error will be discussed.

5.3.1 Beam Polarization

The beam polarization measured by the Møller polarimeter was performed to better than 1% statistical error for each individual polarization measurement. These beam polarization measurements were made at discrete times during the experiment. One of the requirements for this method of G_E^n extraction to work was that the electron beam polarization needed to be stable over the time the data were collected.

Fluctuations in the beam polarization between sequential measurements of the transverse projection of the neutron's polarization would introduce a systematic error into the ratio of electric to magnetic form factors, $g \equiv G_E^n/G_M^n$. The contribution of this systematic error to the overall error was evaluated by Plaster and Madey[76] in an internal report to the E93-038 collaboration. That report is used here to support this work. It uses the polarization data reported in the master's thesis of W. Seo[55].

The scattering asymmetry, $\xi_{t'}$, in the polarimeter can be written in terms of the transverse projection of the neutron's polarization vector, $P_{t'}$, as

$$(5.4) \quad \xi_{t'} = P_{t'} A_y$$

where A_y is the analyzing power of the polarimeter. For the precession angle of 0° and $\pm 90^\circ$ case, the subscript t' will be replaced by S' and L' , respectfully. The ratio g is defined as

$$(5.5) \quad g = -\frac{K_L D_{LS'}}{K_S D_{LL'}}.$$

The factors K_L and K_S are kinematic factors in Q^2 and the electron scattering angle θ_e . The other two factors are polarization transfer coefficients that have the following form

$$(5.6) \quad D_{LS'} = \frac{\xi_{S'}}{A_y P_L}$$

$$(5.7) \quad D_{LL'} = \frac{\xi_{L'}}{A_y P_L}$$

where P_L is the polarization of the incident electron. Substituting these values into Eq. 5.5, yields the following expression

$$(5.8) \quad g = -\frac{K_L}{K_S} \frac{\xi_{S'}}{A_y P_L} \frac{A_y P_L}{\xi_{L'}} = -\frac{K_L}{K_S} \frac{\xi_{S'}}{\xi_{L'}}.$$

The power of this ratio method comes to its fullest form if the electron polarization, P_L , remains constant between the two measurements of the transverse projection of the neutron's polarization. If this is true, then the electron polarization cancels and the ratio g is now related to the ratio of the two measured asymmetries by a kinematic factor (Eq. 4.16). In general, the analyzing power is constant between the two measurements of the transverse polarization projection but uncertainties in the knowledge of the electron polarization require a look at their effect on the knowledge of g .

To get a handle on the systematic uncertainty from beam polarization in the 0° , $\pm 90^\circ$ case, let's consider the following equation where the measured electron beam polarization for $\xi_{S'}$ is $P_L^{S'}$ and for $\xi_{L'}$ it is $P_L^{L'}$.

$$(5.9) \quad g = -\frac{K_L}{K_S} \frac{\xi_{S'}}{P_L^{S'}} \frac{P_L^{L'}}{\xi_{L'}}$$

It can then easily be shown that the systematic uncertainty in g due to the uncertainty in beam polarization is given by the following equation.

$$(5.10) \quad \left(\frac{\Delta g}{g} \right)_{pol}^2 = \left(\frac{\Delta P_L^{S'}}{P_L^{S'}} \right)^2 + \left(\frac{\Delta P_L^{L'}}{P_L^{L'}} \right)^2$$

The reality of the experimental data collection is that the beam polarization was taken at times when polarization sensitive changes were made at the electron injector. This means a measurement of the beam polarization was not made to coincide with a change in the transverse projection that was being measured. One can make the replacements $P_L^{S'} = P_L^{L'} \equiv P_L$ and $\Delta P_L^{S'} = \Delta P_L^{L'} \equiv \Delta P_L$ into Eq. 5.10 and reduce the equation down to the following form.

$$(5.11) \quad \left(\frac{\Delta g}{g} \right)_{pol}^2 = 2 \left(\frac{\Delta P_L}{P_L} \right)^2$$

The $\pm\chi$ case turns out a bit different. Starting with Eq. 4.19 the same methodology can be employed by inserting P_L^+ for the ξ_+ measurements and P_L^- for the ξ_- measurements resulting in the following equation:

$$(5.12) \quad g = K_R \tan \chi \left[\frac{\eta \frac{P_L^+}{P_L^-} + 1}{\eta \frac{P_L^+}{P_L^-} - 1} \right]$$

where $\eta \equiv \xi_+/\xi_-$. It can then be shown that by doing a simple calculation and employing the definitions $P_L^+ = P_L^- \equiv P_L$ and $\Delta P_L^+ = \Delta P_L^- \equiv \Delta P_L$ the result

$$(5.13) \quad \left(\frac{\Delta g}{g} \right)_{pol}^2 = \frac{4\eta^2}{(\eta^2 - 1)^2} \left[2 \left(\frac{\Delta P_L}{P_L} \right)^2 \right]$$

is found. This allows the calculation of the systematic error from beam polarization for the $\pm\chi$ method.

The assessment of the uncertainty $\Delta P_L/P_L$ was done initially by Seo[55] in a masters thesis. Clusters of polarization measurements for a particular Q^2 value and precession technique under similar conditions at the polarized injector were identified. Offsets were added to the individual measured values to bring the average of the clusters to 80% and the individual errors were scaled by the ratio of the measured polarization to the mean polarization of 80%. Once this centering around 80% was done for each cluster, the statistical error for the entire Q^2 was calculated by means of a weighted average. It is necessary to account for a possible underestimation of the error by multiplying the relative error calculated by this process by the square root of the χ^2 of the weighted average. Therefore, the relative uncertainty, $\Delta P_L/P_L$, can be estimated by

$$(5.14) \quad \frac{\Delta P_L}{P_L} = \frac{\delta \overline{P_L} \sqrt{\chi^2}}{\overline{P_L}}$$

where $\overline{P_L}$ is the mean polarization and $\delta \overline{P_L}$ is the statistical error in $\overline{P_L}$.

These two methods were used to obtain the values in Table 5.8 which was adopted

from[76]. The conclusion is that the systematic uncertainty in g from the uncertainty

Q^2 (GeV/c) ²	Precession Angle (deg)	$\delta P_L/P_L$ (%)	χ^2	$\Delta P_L/P_L$ (%)	$\Delta g/g$ (%)
0.45	± 40	0.13	13.1	0.47	1.4
1.15	0, ± 90	0.13	5.5	0.30	0.4
1.15	± 40	0.25	1.1	0.26	0.8
1.47	0, ± 90	0.13	2.9	0.22	0.3
1.47	± 40	0.13	18.1	0.55	1.7

Table 5.8: The beam polarization systematic errors as calculated by the method described by Plaster and Madey[76].

in the beam polarization is less than 2%.

5.3.2 Traceback and Positioning

The equation for the electric to magnetic form factor, Eq. 4.16, contains a kinematic factor out front which is the ratio of two kinematic factors, $K_R = K_L/K_S$. These factors are called kinematic factors because they depend only on the electron scattering angle and the square of the four-momentum transferred. To this point in this dissertation, this kinematic factor is considered to be perfectly known. This is not true, of course, and a systematic error in g needs to be estimated from knowledge of the error in the measured angle θ_e . Brad Plaster and Richard Madey[77] evaluated this systematic uncertainty and reported on this contribution to the overall uncertainty. This report will be used as the source of information for this section.

The contribution of the uncertainty in the electron angle to the systematic error comes from two areas. The first is an error in positioning the spectrometer in the physical setup, $(\Delta\theta_e)_{pos}$. A second contribution comes from the uncertainty in the

reconstruction of the electron scattering angle from the detected position in the focal plane, $(\Delta\theta_e)_{tra}$. To obtain an expression relating these uncertainties to the systematic uncertainty in g , one must evaluate the following expression

$$(5.15) \quad (\Delta g)_{\theta_e}^2 \equiv \left(\frac{\partial g}{\partial \theta_e} \right)^2 (\Delta \theta_e)^2$$

where

$$(5.16) \quad \frac{\partial g}{\partial \theta_e} = \frac{\partial g}{\partial K_L} \frac{dK_L}{d\theta_e} + \frac{\partial g}{\partial K_S} \frac{dK_S}{d\theta_e}.$$

The curious reader can view the evaluation of these partial and total derivatives in Appendix B. Here it suffices to say that the relative systematic error due to the electron angle, $(\Delta g/g)_{\theta_e}^2$, is independent of g which is important.

The results of this analysis are reported in Table 5.9. A value for $(\Delta\theta_e)_{pos}$ of 1.2 mrad is quoted by Plaster and Madey[77]. Their information stems from an analysis performed by Christy[78] for Hall C. The traceback uncertainty was estimated by Jones and Aghalaryan[79] to be $(\Delta\theta_e)_{tra} = 1.3 \text{ mrad}$. The final uncer-

Q^2 (GeV/c) ²	Precession Angle (deg)	$(\Delta\theta_e)_{pos}$ (mrad)	$(\Delta\theta_e)_{tra}$ (mrad)	$(\Delta g/g)_{\theta_e, pos}$ (%)	$(\Delta g/g)_{\theta_e, tra}$ (%)	$(\Delta g/g)_{\theta_e}$ (%)
0.45	± 40	1.2	1.3	0.12	0.13	0.18
1.15	0, ± 90	1.2	1.3	0.19	0.20	0.27
1.15	± 40	1.2	1.3	0.19	0.21	0.28
1.47	0, ± 90	1.2	1.3	0.24	0.26	0.36
1.47	± 40	1.2	1.3	0.24	0.26	0.36

Table 5.9: The systematic uncertainties for all Q^2 values for traceback and positioning uncertainty in the electron scattering angle, θ_e .

tainty value, $(\Delta g/g)_{\theta_e}$, was calculated by combining the individual uncertainties, $(\Delta g/g)_{\theta_e, pos}$ and $(\Delta g/g)_{\theta_e, tra}$, in quadrature. This is an over estimation of the error because $(\Delta g/g)_{\theta_e, pos}$ and $(\Delta g/g)_{\theta_e, tra}$ are correlated. The conclusion is that the values

of $(\Delta g/g)_{\theta_e}$ in Table 5.9 are upper bounds and they are small; ranging from 0.2 to 0.4 %. This adds very little to the final uncertainty in g .

5.3.3 Two-Step Process [$^2H(\vec{e}, e'\vec{p}) + ^{208}Pb(\vec{p}, \vec{n})^{208}Bi$]

A brief but lively history encompasses the estimation of the charge-exchange contribution. During the planning stages for E93-038, estimations of the level of contamination were made to satisfy planning requirements and it was at this time that the first attempts at formulating the necessary equations was made. Later, after the data were collected and analysis was in full swing, a renewed effort to assess the contamination from this two-step process was initiated. The equations were reworked and cast in a notation to fit the analysis (and the different neutron precession angles) and this discussion can be found in Section 4.42 of Chapter 4. Estimations were made by taking the values obtained by analyzing the liquid hydrogen data. The values for events per millicoulomb and asymmetries were obtained by running the LH₂ data through the same cuts and asymmetry program as the LD₂ data which uses a fitting routine to calculate the yields in the peaks.

More recently, the asymmetry program was modified to also calculate the yields in the peaks by simple counting of the events within the cuts rather than fitting the peaks. This counting algorithm was used in the analysis performed by Plaster[73] and resulted in lower estimations of the charge-exchange contribution. The final estimation is reported in Table 5.10. Along with the systematic error, a correction of $1.3 \pm 0.1\%$ was applied to the $Q^2 = 1.47 \text{ (GeV}/c)^2$ $0^\circ, \pm 90^\circ$ data to correct for a dilution of the asymmetry from the charge-exchange process. The contamination to the other data sets was negligible so no correction was applied. The contribution of

Q^2 (GeV/c) ²	Precession Angle $\chi(^{\circ})$	$\Delta g/g$ (%)
0.45	± 40	<0.01
1.14	0, ± 90	0.02
1.17	± 40	0.05
1.47	± 40	<0.01
1.47	0, ± 90	0.2

Table 5.10: Relative systematic errors from the charge-exchange process are presented for each data set.

this two-step process to the systematic uncertainty in g is small except for the data at $Q^2 = 1.45$ (GeV/c)² with precession angles of $0^{\circ}, \pm 90^{\circ}$.

5.3.4 Depolarization

Neutrons passing through the lead curtain can undergo nuclear scattering that still results in a neutron being detected in the polarimeter. This scattering works to depolarize the flux of neutrons entering the polarimeter. An evaluation of this effect was performed by Brad Plaster[80] using the GENGEN simulation program developed under the leadership of Jim Kelly. Plaster[80] added a spin-dependent multiple-scattering algorithm to the simulation code that employed quasifree scattering from a Fermi gas. The results of this analysis are displayed in Table 5.11. The

Q^2 (GeV/c) ²	Precession Angle $\chi(^{\circ})$	$\Delta g/g$ (%)
0.45	± 40	<0.1
1.14	0, ± 90	<0.1
1.17	± 40	0.2
1.47	± 40	0.1
1.47	0, ± 90	0.6

Table 5.11: Estimated systematic uncertainties from neutron depolarization in the lead curtain.

highest systematic uncertainty occurs with the $Q^2 = 1.47 \text{ (GeV}/c)^2$ $0^\circ, \pm 90^\circ$ data which comes in at 0.6%. The others are even smaller at 0.2% for the $Q^2 = 1.17 \text{ (GeV}/c)^2 \pm 40^\circ$ data and $< 0.1\%$ for $Q^2 = 0.45$ and $1.14 \text{ (GeV}/c)^2$ data.

5.3.5 Precession Angle

The process to evaluate the precession angle of the neutron's spin polarization vector and the associated contribution to the systematic uncertainty in g was performed by Simon Taylor. Taylor showed in [81] that the systematic uncertainty, $\Delta g/g$, due to the precession angle of the neutron to be

$$(5.17) \quad \left| \frac{\Delta g}{g} \right| = \left| \left(\frac{\cos \chi - \eta}{1 - \eta \cos \chi} \right) \frac{\delta \chi}{\sin \chi} \right|$$

for the $0^\circ, \pm 90^\circ$ method. When $\chi = 90^\circ$, then Eq. 5.17 becomes

$$(5.18) \quad \left| \frac{\Delta g}{g} \right| = |\eta \delta \chi|.$$

The case for the precession angle being $\pm \chi$ results in a relative systematic uncertainty in g being

$$(5.19) \quad \frac{\Delta g}{g} = \sec \chi \csc \chi \delta \chi.$$

These two equations, Eq. 5.18 and 5.19, provide the means by which the contribution to the systematic uncertainty in g due to the precession angle of the neutron can be evaluated assuming the precession angle and uncertainties can be determined.

The precession angle, χ , of the neutron's polarization vector was assumed during initial analysis to be $0^\circ, \pm 90^\circ$ and $\pm 40^\circ$ as the nominal values. These nominal values were based on the assumptions that the neutron traversed the center of the dipole magnet aperture and the TOSCA maps of the Charybdis magnet could be scaled

linearly from the maps at 200, 300, 500, and 600 amps. Later, Simon Taylor performed two separate yet better analyses[82, 83] to more correctly determine the mean precession angle of the neutrons.

In the first study, Taylor[82] used the simulation program GENGEN developed for this experiment to produce “events” that were then processed by the spin precession code. A program available at Jlab called TOSCA was used to calculate the magnetic field in one octant of the Charybdis dipole magnet on a grid starting at the center of the magnet. The magnet was assumed to be symmetric about the center so only one octant is needed to complete the magnet map. This magnetic field map was calculated only for settings of 200, 300, 500, and 600 amps in the dipole magnetic. The actual experimental values usually fell in between these values. A scale factor was used to scale the field values on the grid to the current in the magnet. The lower current settings used a linear relationship between the current settings and the field map. The higher current settings move away from this linear relationship so a scale factor determined by the relationship between the field integral and the current in the magnet was used[81]. The results of this study are listed in Table 5.12. Taylor notes

Q^2 (GeV/c) ²	Nominal β	Precession Angle $\chi(^{\circ})$	$\Delta g/g$ (%)
0.45	0.603430	$39.97 \pm 0.51 \pm 0.30$	2.1
1.14	0.794677	$89.9 \pm 0.8 \pm 0.04$	0.27
1.17	0.799134	$39.89 \pm 0.40 \pm 0.08$	1.4
1.47	0.838876	$40.03 \pm 0.66 \pm 0.14$	2.4
		$89.87 \pm 1.0 \pm 0.09$	0.33

Table 5.12: These values of mean neutron precession angles for the different Q^2 were calculated by Simon Taylor[82] using a spin precession code and events generated by the GENGEN simulation code[85].

that these values are most likely overestimates of the actual systematic uncertainties

due to the precession angle.

The second method for determining the systematic uncertainty in g due to the precession angle involves the use of actual data instead of a simulation program[83]. The initial neutron momenta incident on the face (entrance) of Charybdis were taken from the data. The spin precession was calculated in the same manner as the previous method (see Ref. [82]) and the initial spin was generated uniformly on a unit sphere. The TOSCA field maps used in this method were the same as the previous method so a scale factor was employed to scale the field maps to the appropriate current setting. The results of this analysis are reported in Table 5.13.

Taylor comments that the strength of this method lies in the fact that the same

Q^2 (GeV/c) ²	Nominal β	Precession Angle $\chi(^{\circ})$	$\Delta g/g$ (%)
0.45	0.603430	$40.17 \pm 0.02 \pm 0.30$	1.1
1.14	0.794677	$89.99 \pm 0.22 \pm 0.04$	0.08
1.17	0.799134	$40.19 \pm 0.02 \pm 0.08$	0.29
1.47	0.838876	$40.26 \pm 0.03 \pm 0.14$	0.51
		$90.34 \pm 0.22 \pm 0.09$	0.08

Table 5.13: These values of mean neutron precession angles for the different Q^2 were calculated by Simon Taylor[83] using a spin precession code and events taken from data runs.

cuts that are applied to the G_E^n analysis are applied to this precession angle analysis. Therefore, the neutrons that pass event analysis cuts are the only ones being considered for neutron precession determination. Furthermore, the uncertainties tabulated in Table 5.13 should be considered lower bounds to the uncertainty in the precession angle because they were calculated by either a measurement of the difference between the results at the two different polarities or differences arising from differences in the

field shape for different currents[84]. These values for $\Delta g/g$ from Table 5.13 are considered the systematic uncertainties for this data analysis due to the precession angle uncertainty.

5.3.6 Time Calibration

The timing calibration procedure, as outlined in Chapter 3, involves the use of real data to calibrate the timing relationships between individual detectors within the polarimeter. There were a few constraints on the choice of data runs used in this calibration procedure. One constraint was at least one run had to be a run with Charybdis current being set to zero. This was necessary to calibrate the charged-particle detectors. The other constraint was that all other calibration procedures needed the Charybdis magnet on in order to minimize multiple hit processes from charged particles. The program allows the use of multiple runs for each calibration procedure which gives the flexibility to achieve the necessary statistics. The $Q^2 = 1.47$ (GeV/c)² $0^\circ, \pm 90^\circ$ data was broken into three subsets and each subset has its own time calibration file. Runs for that time calibration file were chosen out of each particular subset. The $Q^2 = 1.17$ (GeV/c)² $\pm 40^\circ$ data has so few runs (and was taken in a relatively short period of time) that only one time calibration file was necessary.

Because time calibrations were created using only user selected runs, it is natural to assume that a systematic error, $(\Delta g/g)_{time}$, could be assigned to the final value of g due solely to the choice of runs in the time calibration. To estimate this contribution to the systematic error, each graduate student was assigned a “secondary” Q^2 point to analyze independently from the student assigned that Q^2 point as their primary data. A comparison between the two analyses was then made and an estimated systematic

error was computed. An example of this comparison method for the $Q^2 = 1.17$ $(\text{GeV}/c)^2 \pm 40^\circ$ was described in an internal report[86]. The following paragraphs will briefly describe that procedure.

The procedure for this comparison is quite simple. The data were analyzed by the second student using his own judgment on data quality and choice of runs to be used in the time calibration procedure. Asymmetries were obtained using the same cuts as the primary student used in his analysis. G_E^n was then extracted using these asymmetries and a comparison between the primary and secondary student's results was performed as follows:

$$(5.20) \quad \delta g = g(\text{secondary}) - g(\text{primary}).$$

The systematic uncertainty then was estimated as being one-half the difference between the g values.

$$(5.21) \quad \left(\frac{\Delta g}{g} \right)_{\text{time}} = \frac{1}{2} \left[\frac{g(\text{secondary}) - g(\text{primary})}{\bar{g}} \right]$$

The results of the comparison for $Q^2 = 1.17$ $(\text{GeV}/c)^2 \pm 40^\circ$ data was $\pm 2\%$ [86]. An analysis performed by S. Tajima on $Q^2 = 1.13$ $(\text{GeV}/c)^2$ $0^\circ, \pm 90^\circ$ at Duke University using code developed at Duke University produced excellent agreement with an analysis done by Brad Plaster at Jlab. The result of the comparison is that a systematic uncertainty of $\pm 1\%$ can be assigned to this data point.

5.3.7 Radiative Corrections

Radiative corrections for this analysis were calculated by Afanasev et al.[87] to the polarization-transfer coefficients, $P_{S'}/P_L$ and $P_{L'}/P_L$. Using the corrections supplied

by Afanasev et al.[87], the effect on g was found to be small. The strongest effect is the depolarization of the electron that results in an increase in the polarization-transfer coefficients of 1.9%, 3.7%, and 4.4% for $Q^2 = 0.45, 1.13$, and $1.45 \text{ (GeV}/c)^2$, respectively. Because this effect cancels in the polarization ratio, the resulting systematic uncertainty in g is small (Table 5.14).

$Q^2 \text{ (GeV}/c)^2$	Precession Angle $\chi(^{\circ})$	$\Delta g/g \text{ (\%)}$
0.45	± 40	0.7
1.13	0, ± 90	0.1
1.13	± 40	0.1
1.45	± 40	0.05
1.45	0, ± 90	0.05

Table 5.14: Systematic uncertainties in the radiative corrections for the acceptance-averaged Q^2 points.

5.4 Instrumental Asymmetry

The instrumental asymmetry was calculated from Eq. 4.7 using the relationship in Eq. 4.8. The resulting expression is

$$(5.22) \quad \xi_{instr} = \frac{r_g - 1}{r_g + 1}$$

where

$$(5.23) \quad r_g = \left[\frac{N_D^L N_D^R}{N_U^L N_U^R} \right]^{1/2}.$$

Notice that the detection efficiencies cancel in the r_g ratio, so it was expected that there would be little or no instrumental asymmetry from variations in the threshold settings on the individual detectors.

Plaster and Madey[88] detailed the process by which the effects of the instrumental asymmetry on the experimental asymmetry were studied. The process involved changing the software threshold on the top-rear array while maintaining the original threshold setting on the bottom-rear array. A plot of the instrumental asymmetry versus top-rear threshold was produced and the points were fitted with a straight line to determine the threshold at which the instrumental asymmetry went to zero. Concurrently, a plot of the experimental asymmetry versus top-rear threshold was made as well to determine if the experimental asymmetry (sometimes called the physical asymmetry) changed with changes in the top-rear thresholds.

For this analysis, the effect of the instrumental asymmetry on the experimental asymmetry was analyzed for two runs. One for the magnet off pertaining to the neutron precession angle of 0° and another run for the magnet on pertaining to a precession angle of 90° . Figures 5.1, 5.2, 5.3, and 5.4 show the instrumental asymmetry and experimental asymmetry as the top-rear threshold is varied between 10 and 22 MeVee.

In Fig. 5.1, the data points are fitted with a straight line using a least squares method. This fit is displayed on the plot as a $y = mx + b$ equation. Solving this equation for x with $y = 0$ results in 5.51 MeVee. This is the threshold for the top-rear array needed to bring the instrumental asymmetry to zero. Figure 5.2 displays the physical asymmetry versus the top-rear threshold over the same range. A straight-line fit was applied and the resulting equation yields a very small slope that shows the change in the experimental asymmetry over a wide range of top-rear thresholds was small.

The next two figures, Figs. 5.3 and 5.4, represent the analysis performed on one

Figure 5.1: Instrumental asymmetry versus top-rear threshold for the LD₂ target and the Charybdis magnet off (0° neutron precession).

run with the Charybdis magnet set to 90°. Figure 5.3 shows the change in the instrumental asymmetry with changes in top-rear threshold. The straight-line fit is plotted on the graph and solving this for the $y = 0$ crossing point yields an x value of 6.24 MeVee. The experimental asymmetry versus top-rear threshold is shown in Fig. 5.4. Once again a straight line fit returns a small value for the slope which is interpreted as meaning the experimental asymmetry is very stable over a large range of top-rear thresholds.

The results of this study are that the instrumental asymmetry does not effect the

Figure 5.2: Physical asymmetry versus top-rear threshold for the LD₂ target and the Charybdis magnet off (0° neutron precession).

experimental asymmetry. It can be concluded as well that the experimental asymmetry is not affected by large differences in the polarimeter's operational performance. In this case, differences in threshold settings between the top and bottom rear arrays in the polarimeter have little effect on the measured experimental asymmetry.

5.5 Acceptance Averaging

The experimental apparatus like all instruments has a finite acceptance. Most of the early analyses treated the data as though it came from an instrument with

Figure 5.3: Instrumental asymmetry versus top-rear threshold for the LD₂ target and the Charybdis magnet on (90° neutron precession).

an infinitesimally small acceptance. Namely, the analysis was performed with the central Q^2 value being considered to be the Q^2 value for all events, the so-called point analysis. The asymmetry could then be extracted with the equation

$$(5.24) \quad \xi = A_y P_x$$

where A_y is the analyzing power and P_x is the transverse component of the neutrons polarization vector after precession through any setting of the magnetic field. The reality is different. The front acceptance of the polarimeter was defined by the collimator which was optimized to best illuminate the front array while minimizing

Figure 5.4: Physical asymmetry versus top-rear threshold for the LD₂ target and the Charybdis magnet on (90° neutron precession).

background radiation in the shield enclosure (see Chapter 3 for details). This finite physical acceptance represents a range of Q^2 values that were accepted by the polarimeter and this finite acceptance must be accounted for in the final analysis. Equation 5.24 becomes

$$(5.25) \quad \xi = \langle A_y P_x^{Aren} \rangle$$

where the left/right angle brackets denote averaging over all kinematic variables. It should be noted that Arenhövel's formalism[91, 92] was used as well and that is why the polarization component takes on the form P_x^{Aren} . The analyzing power of the polarimeter can be considered to be stable over the range of our kinematic variables

and can be factorized out of the equation resulting in the following

$$(5.26) \quad \xi = A_y < P_x^{Aren} > .$$

To handle this, an acceptance averaging procedure needed to be developed and implemented for the final analysis.

The E93-038 collaborators tackled this problem by creating two programs to evaluate the acceptance average and to account for effects from final state interactions (FSI), meson exchange currents (MEC), and isobar configurations (IC). Reports to the collaboration were made and are used here as sources[89, 90]. The major difference between the two programs is how they derive their kinematic information from $^2H(\vec{e}, e'\vec{n})^1H$ events. The first method[90] employed the Monte Carlo simulation program, GENGEN, developed for this experiment to produce the events and included detailed models of the high momentum spectrometer and the neutron polarimeter. The second method[89] obtained its kinematic information from reconstructed quasielastic events from data. This method has the advantage of reducing the sensitivity to models of the experimental acceptance and detection efficiencies.

The first method uses the E93-038 simulation program, GENGEN, to produce events from the differential cross sections computed from Arenhövel's structure functions. Each event contained not only kinematic information but also polarization information because Arenhövel's structure functions contain this information as well. GENGEN contains realistic models of the HMS and NPOL detectors in order to successfully reproduce the conditions of the experiment. Each event was then transported through the Charybdis magnet with a subroutine written to precess the spins of the neutron and detection in the polarimeter was simulated from which the value of $< P_x^{Aren} >$ could be calculated.

The other method uses kinematic information from real data to provide the events for the acceptance averaging. Each event was assigned a polarization value based upon Arenhövel's structure functions for the kinematics of that reconstructed event. Each event was then transported through the spin precession code and the average value of the transverse polarization vector was calculated. As mentioned earlier, this method doesn't need to pay close attention to the experimental setup because by the nature of it being real data, this has been taken into account. Of course, every advantage usually has a disadvantage, and for this method it means that the uncertainties in the kinematic variables are propagated through to the polarization vector.

The nuclear physics effects such as final state interactions (FSI), meson exchange currents (MEC), and isobaric configurations (IC) were accounted for by both methods by averaging Arenhövel's theoretical ${}^2H(\vec{e}, e'\vec{n})^1H$ calculations[91, 92] over the experimental acceptance. Polarizations of the recoil neutrons were calculated over a kinematic grid where multidimensional interpolation between grid elements was performed. The first iteration of the averaging procedure assumed the Galster parameterization multiplied by five scale factors (0.50, 0.75, 1.00, 1.25, and 1.50) for the Arenhövel theoretical calculations of the neutron polarization vectors and performed the averaging procedure for each scale factor. The average transverse polarization component was determined by simulation for each setting of the magnetic field for each Q^2 point and each scale factor. The optimal scale factor was then chosen by fitting the simulated polarization ratios to the corresponding measured asymmetry ratios from the experimental data. This extraction procedure produced results for the ratio $g \equiv G_E^n/G_M^n$ that were acceptance-averaged and corrected for nuclear physics effects. The results for these three Q^2 points were then combined with the current

world data on G_E^n and fitted with a two-parameter Galster parameterization function. This “new” Galster fit was then fed back into the averaging procedure in place of the original Galster parameterization and a second iteration of the averaging procedure was performed. The changes between iterations were negligible and the two different methods for acceptance-averaging agreed to better than 2%.

One last result came out of the acceptance-averaging procedure dealing with the Q^2 value. This was alluded to earlier during the discussion on the G_M^n values. The acceptance-averaging procedure produced average Q^2 values, $\langle Q^2 \rangle$, of 0.45, 1.13, and 1.45 (GeV/c)².

5.6 Conclusion and Comments

A tabulation of the systematic errors discussed earlier are in Table 5.15. The total

Source	$\langle Q^2 \rangle$ (GeV/c) ²				
	0.45 ^a	1.13 ^a	1.13 ^b	1.45 ^a	1.45 ^b
Beam Polarization	1.4	0.8	0.4	1.7	0.3
Charge Exchange	<0.01	0.02	0.05	<0.01	0.2
Depolarization	<0.1	<0.1	0.2	0.1	0.6
Positioning/Traceback	0.2	0.3	0.3	0.4	0.4
Precession Angle	1.1	0.3	0.1	0.5	0.1
Radiative Corrections	0.7	0.1	0.1	0.05	0.05
Total	1.9	0.9	0.5	1.8	0.8

Table 5.15: Estimated systematic uncertainties in $\Delta g/g$ from all contributions except the time calibration which is assumed to be $\pm 2\%$.

(^a $\chi = \pm 40^\circ$ precession ^b $\chi = 0^\circ, \pm 90^\circ$ precession)

contribution to the uncertainty in $\Delta g/g$ is less than 2% for all data sets; the largest uncertainties occur for $\chi = \pm 40^\circ$ and $Q^2 = 0.45$ and 1.45 (GeV/c)². The final results of this analysis are reported in Table 5.16. Here the acceptance-averaged values of g are

listed with statistical and systematic errors. The first error is the statistical and the second error is the systematic. The values for $G_M^n/\mu_n G_D$ calculated by Jim Kelly[75] are also reported along with the extracted G_E^n values for each $\langle Q^2 \rangle$. The values for

$\langle Q^2 \rangle$ (GeV/c) ²	$g = G_E^n/G_M^n$	$G_M^n/\mu_n G_D$	G_E^n
0.447	$-0.0761 \pm 0.0083 \pm 0.0021$	1.003 ± 0.006	$0.0550 \pm 0.0060 \pm 0.0016$
1.132	$-0.131 \pm 0.010 \pm 0.003$	1.057 ± 0.017	$0.0394 \pm 0.0029 \pm 0.0012$
1.450	$-0.190 \pm 0.016 \pm 0.004$	1.044 ± 0.024	$0.0411 \pm 0.0035 \pm 0.0013$

Table 5.16: Results for $g = G_E^n/G_M^n$ and G_E^n and the values for $G_M^n/\mu_n G_D$ as obtained from Kelly's work[75]. Note that the first error report for g and G_E^n is the statistical error and the second error reported is the systematic error.

G_E^n from this analysis are plotted in Fig. 5.5 along with recent world data on polarized experiments. The analysis presented in this dissertation contributed to the results at $Q^2 = 1.13$ and 1.45 . Plotted on Fig. 5.5 are two curves. The dashed curve is the original Galster parameterization[28] that is a fit to G_E^n data below 0.71 (GeV/c)² available in 1971. The solid curve represents a new fit to the data represented on the figure using the Galster parameterization with two free parameters.

$$(5.27) \quad G_E^n = \frac{-a\mu_n\tau G_D}{1 + b\tau}$$

where $\tau = Q^2/4M_n^2$, $G_D = (1 + Q^2/\Lambda^2)^{-2}$, $\Lambda^2 = 0.71$ (GeV/c)², and a and b are the fitting parameters. The best fit to this data yields parameters of $a = 0.889 \pm 0.023$ and $b = 3.22 \pm 0.33$ [103].

The ratios $\mu_p G_E^p/G_M^p$ and $\mu_n G_E^n/G_M^n$ are compared with predictions of selected models in Fig. 5.6. Any model that attempts to reproduce the electromagnetic form factors of the nucleon should be able to reproduce, simultaneously, the form factors for both the proton and the neutron. The short dashed line in Fig. 5.6 is the

Figure 5.5: G_E^n vs. Q^2 for recent world data taken with polarized electron beams. The references for each data set are as follows: MIT-Bates $d(\vec{e}, e'\vec{n})$ [4], Jlab E93-026 $d(\vec{e}, e'n)$ [42], Mainz A1 $^3\vec{He}(\vec{e}, e'n)$ [93], Mainz A3 $d(\vec{e}, e'\vec{n})$ [94, 95], Mainz A $^3\vec{He}(\vec{e}, e'n)$ [96, 97], NIKHEF $\vec{d}(\vec{e}, e'n)$ [98], Schiavilla & Sick[41].

chiral soliton model[104] which reproduces the linear decrease observed in the proton form factor ratio but fails to reproduce the neutron data. The light-cone diquark model[105] represented by the long dashes agrees with both the proton and neutron data at low Q^2 but falls below the proton data and above the neutron data at higher Q^2 . The dotted line is the prediction from a chiral constituent quark model[106]. This model, like the previous one, achieves low Q^2 agreement but diverges for both the proton and neutron data as Q^2 increases. A light-front constituent quark within a cloud of pions[107] predicts the neutron data well(thick solid line) but falls below the proton data above $Q^2 \sim 2$ (GeV/c) 2 . Another light-front calculation using a one-gluon exchange and constituent quark form factors[108] (dash-dots line) fits the

neutron data well but deviates from the proton data above $3.5 \text{ (GeV}/c)^2$. The vector meson dominance coupled with perturbative QCD[109], represented here with a thin solid line, agree with the entire range of proton data but fall below the neutron data for $Q^2 > 1.2 \text{ (GeV}/c)^2$.

The conclusion here is there are presently no models that can successfully predict both the proton and neutron data at the same time. A successful confinement model should be able to predict all nucleon form factors. Although, some models have managed to qualitative fit the proton form factor data, the neutron electric form factor is very sensitive to small changes in the nucleon wave function. These differences tend to increase with increasing Q^2 . The results from this experiment, presented in this dissertation, provide a new challenge to present and future confinement models. Finally, the data at $Q^2 = 1.45 \text{ (GeV}/c)^2$ supports the idea that the G_E^n data doesn't follow the Galster parameterization at higher Q^2 and further investigations at even higher Q^2 are necessary.

Figure 5.6: $\mu_p G_E^p / G_M^p$ and $\mu_n G_E^n / G_M^n$ versus Q^2 . The MIT-Bates $p(\vec{e}, e'\vec{p})$ data comes from [99], the Mainz A1 $p(\vec{e}, e'\vec{p})$ data comes from [100], and the Jlab Hall A $p(\vec{e}, e'\vec{p})$ data comes from [101, 102]. The data points shown in the lower plot are the same as the ones in Fig. 5.5.

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APPENDIX A

Analysis of the Background Two-Step Process: ${}^2\text{H}(\vec{e}, e'\vec{p}) + \text{Pb}(\vec{p}, \vec{n})$

The analysis of background events from the charge-exchange events in the lead curtain was performed by all students on the experiment. The final tabulation and calculations were performed by Richard Madey, Brad Plaster, and William Tireman. Their work was reported in E93-038 internal report [117] and was adapted for use in this appendix.

A.1 Introduction

In E93-038, we made measurements with a liquid hydrogen target in order to assess the false asymmetry or the dilution of the asymmetry from the two-step process $[{}^2\text{H}(\vec{e}, e'\vec{p}) + \text{Pb}(\vec{p}, \vec{n})]$. The false asymmetry from this two-step process vanishes if the polarization-transfer coefficient $D_{SS'}^{Pb}$ for the $\text{Pb}(\vec{p}, \vec{n})$ conversion vanishes, which is approximately the case for 795 MeV protons incident on Pb; Prout et al.[110] reported $D_{SS'}^{Pb} = 0.014 \pm 0.013$ for 795 MeV protons. Even though $D_{SS'}^{Pb} = 0$, there remains a dilution of the asymmetry such that the true asymmetry is given by the measured asymmetry divided by the fraction of events that are real. At lower proton energies from 135 to 392 MeV, $D_{SS'}^{Pb} \simeq -0.24$ over a large region of excitation energies[111][113]. The polarization-transfer coefficient $D_{LS'}$ vanishes at 0° for all targets and all energies because parity invariance prohibits polarization transfer from the scattering plane to the normal direction so that $D_{LN'} \equiv 0$; and because of rotational invariance, $D_{LS'}(0^\circ) = D_{LN'}(0^\circ) = 0$.

In an early report prepared during the planning stage of E93-038, Madey, Eden, and Ahmidouch[114] estimated LH_2 run times to make the relative uncertainty in $g[\equiv G_E/G_M]$ for the two-step process smaller than the relative statistical uncertainty in g for the quasi-elastic scattering process from the neutron in deuterium. In a previous report, Madey, Tireman, and Plaster [116] assessed the false asymmetry and/or dilution from the first-pass analysis of the two-step process [$^2\text{H}(\vec{e}, e'\vec{p}) + \text{Pb}(\vec{p}, \vec{n})$].

A.2 Analysis

In E93-038, we extract the ratio $G_E/G_M \equiv g$ from measurements of two neutron scattering asymmetries. The basic equations are:

$$(A.1) \quad I_o D_{LS'} = -G_M G_E K_S \quad D_{LS'} \equiv \frac{P_{S'}}{P_L}$$

$$(A.2) \quad I_o D_{LL'} = G_M^2 K_L \quad D_{LL'} \equiv \frac{P_{L'}}{P_L}$$

$$(A.3) \quad I_o \equiv G_E^2 + K_O G_M^2$$

The kinematic functions K_S , K_L , and K_O depend on θ and Q^2 .

$$(A.4) \quad K_S \equiv 2[\tau(1 + \tau)]^{1/2} \tan(\theta_e/2)$$

$$(A.5) \quad K_L \equiv 2\tau \sqrt{(1 + \tau)[1 + \tau \sin^2(\theta_e/2)]} \frac{\sin(\theta_e/2)}{\cos^2(\theta_e/2)}$$

$$(A.6) \quad K_O \equiv \tau[1 + 2(1 + \tau) \tan^2(\theta_e/2)]$$

where

$$\tau = \frac{Q^2}{4M^2}$$

Dividing eq.(A.1) by eq.(A.2), we have

$$(A.7) \quad -K_R \frac{D_{LS'}}{D_{LL'}} = \frac{G_E}{G_M} \equiv g \quad K_R \equiv \frac{K_L}{K_S}$$

or

$$(A.8) \quad g = -K_R \frac{P_{S'}}{P_{L'}}.$$

We utilize the neutron spin-precession dipole magnet, Charybdis, in two different ways to measure $P_{S'}$ and $P_{L'}$. In the first case, with the magnet off, we measure the scattering asymmetry $\xi_{S'}$ from the sideways component of polarization $P_{S'}$. Then with the magnet current selected to precess the neutron polarization vector through 90° , we measure the scattering asymmetry $\xi_{L'}$ from the longitudinal component of polarization $P_{L'}$. The ratio $P_{S'}/P_{L'}$ is equal to $\xi_{S'}/\xi_{L'}$ provided the beam polarization does not change during the sequential measurements of $P_{S'}$ and $P_{L'}$. Thus,

$$(A.9) \quad g = -K_R \frac{\xi_{S'}}{\xi_{L'}} \equiv -K_R \eta_{S'L'} \quad \eta_{S'L'} \equiv \frac{\xi_{S'}}{\xi_{L'}}$$

In the second case, we select the Charybdis current to precess the neutron spin through $\pm\chi$ degrees. The top-bottom asymmetry measured in the neutron polarimeter is proportional to the projection of the neutron polarization vector on the axis that is perpendicular to the neutron momentum direction:

$$(A.10) \eta \equiv \frac{\xi_-}{\xi_+} = \frac{P_-^x}{P_+^x} = \frac{P_{S'} \cos(-\chi) + P_{L'} \sin(-\chi)}{P_{S'} \cos(\chi) + P_{L'} \sin(\chi)} = \frac{(P_{S'}/P_{L'}) \cos(\chi) - \sin(\chi)}{(P_{S'}/P_{L'}) \cos(\chi) + \sin(\chi)}.$$

Solving for $P_{S'}/P_{L'}$, we obtain

$$(A.11) \quad \frac{P_{S'}}{P_{L'}} = -\frac{(\eta + 1) \sin(\chi)}{(\eta - 1) \cos(\chi)} = -\frac{(\eta + 1)}{(\eta - 1)} \tan(\chi)$$

and substituting eq.(A.11) into eq.(A.8), we have

$$(A.12) \quad g = -K_R \frac{(1 + \eta_{\pm})}{(1 - \eta_{\pm})} \tan(\chi) \quad \eta_{\pm} \equiv \frac{\xi_{-}}{\xi_{+}}.$$

The measured asymmetry $\xi_{t'}^m$ from any transverse -polarization component is:

$$(A.13) \quad \xi_{t'}^m = f_R \xi_{t'}^R + f_B \xi_t^B = f_R \xi_{t'}^R \left[1 + \frac{f_B}{f_R} \frac{\xi_{t'}^B}{\xi_{t'}^R} \right]$$

Here ξ_t is a generic notation to denote $\xi_{S'}(\chi = 0^\circ)$, $\xi_{L'}(\chi = \pm 90^\circ)$, $\xi_{-}(-\chi)$, or $\xi_{+}(+\chi)$.

The symbol f_R represents the neutron fraction of real events from the $(e, e'n)$ process,

f_B represents the neutron fraction from the two-step background process [$^2\text{H}(\vec{e}, e'\vec{p}) + \text{Pb}(\vec{p}, \vec{n})$], and $\xi_{t'}^R(\xi_{t'}^B)$ denotes the asymmetry from the real (background) process.

Because of parity and rotational invariance, the polarization-transfer coefficient $D_{LS'}$ vanishes at 0° ; hence, only the transverse-polarization component contributes to the background asymmetry. The ratio $\xi_{t'}^B/\xi_{t'}^R$ is

$$(A.14) \quad \frac{\xi_{t'}^B}{\xi_{t'}^R} = \frac{P_{t'}^B}{P_{t'}^R} = \frac{P_{t'}^p}{P_L} \frac{D_{t't''}}{D_{L't'}^n} = \frac{D_{L't'}^p D_{tt'}^{Pb}}{D_{L't'}^n}$$

Here $D_{L't'}^p[D_{L't'}^n]$ is the transverse-polarization coefficient from the $(\vec{e}, e'\vec{p})[(\vec{e}, e'\vec{n})]$ reaction. We calculate the polarization-transfer coefficients $D_{L't'}^n$ and $D_{L't'}^p$ from the basic equation for an electron colliding elastically with a free nucleon.

The background event rate B from LH_2 represents a fraction f_B of the total event rate (B+R).

$$(A.15) \quad f_B = \frac{B}{B + R} = \frac{B/R}{B/R + 1}$$

where B/R is the ratio of the background rate B to the real counting rate R.

We rewrite eq.(A.13) to express $\xi_{t'}^R$ in terms of $\xi_{t'}^m$ and a correction factor $c_{t'}$ for all rotations of the neutron polarization vector:

$$(A.16) \quad \xi_{t'}^R = \frac{\xi_{t'}^m - f_B \xi_{t'}^B}{1 - f_B} = \xi_{t'}^m (1 + c_{t'})$$

where the correction factor is

$$(A.17) \quad c_{t'} = \frac{f_B}{1 - f_B} (1 - \xi_{t'}^B / \xi_{t'}^m)$$

with

$$(A.18) \quad \xi_{t'}^B / \xi_{t'}^m = \frac{D_{Lt'}^p D_{tt'}^{Pb}}{f_R D_{Lt'}^n + f_B D_{Lt'}^p D_{tt'}^{Pb}}$$

The ratio $\xi_{t'}^B / \xi_{t'}^m$ is positive because both $D_{Lt'}^p$ and $D_{tt'}^{Pb}$ are negative and $D_{Lt'}^n$ is positive. Even when $D_{tt'}^{Pb} = 0$, the correction factor $c_{t'} [= f_B / (1 - f_B)]$ is needed to account for the dilution of the measured asymmetry from the unpolarized Pb(p,n) reaction; in this case,

$$(A.19) \quad \xi_{t'}^R = \frac{\xi_{t'}^m}{f_R} \quad (D_{tt'}^{Pb} = 0).$$

The relative error $\Delta c_{t'} / c_{t'}$ in the correction factor is:

$$(A.20) \quad \frac{\Delta c_{t'}}{c_{t'}} = \frac{(\xi_{t'}^B / \xi_{t'}^m)}{1 - \xi_{t'}^B / \xi_{t'}^m} \left[\left(\frac{\Delta \xi_{t'}^B}{\xi_{t'}^B} \right)^2 + \left(\frac{\Delta \xi_{t'}^m}{\xi_{t'}^m} \right)^2 \right]^{1/2}.$$

In the approximation that $(\Delta \xi_{t'}^m / \xi_{t'}^m)^2$ is negligible in comparison with $(\Delta \xi_{t'}^B / \xi_{t'}^B)^2$, eq.(A.20) simplifies:

$$(A.21) \quad \frac{\Delta c_{t'}}{c_{t'}} \simeq \frac{(\xi_{t'}^B / \xi_{t'}^m)}{1 - \xi_{t'}^B / \xi_{t'}^m} \frac{\Delta \xi_{t'}^B}{\xi_{t'}^B}$$

This approximation becomes valid as $\xi_{t'}^B$ becomes small or approaches zero.

The uncertainty in these corrected asymmetries $\xi_{t'}^R$ is calculated for all rotations of the neutron polarization vector:

$$(A.22) \quad \Delta \xi_{t'}^R = \xi_{t'}^R \left[\left(\frac{\Delta \xi_{t'}^m}{\xi_{t'}^m} \right)^2 + \left(\frac{\Delta c_{t'}}{1 + c_{t'}} \right)^2 \right]^{1/2}.$$

Examination of eq. (A.22) reveals that the relative uncertainty in the transverse asymmetry $\xi_{t'}$ for the two-step process $[^2\text{H}(\vec{e}, e'\vec{p}) + \text{Pb}(\vec{p}, \vec{n})]$ is

$$(A.23) \quad \left(\frac{\Delta \xi_{t'}}{\xi_{t'}} \right)_{2\text{-step}} = \left(\frac{\Delta c_{t'}}{1 + c_{t'}} \right)$$

The corrected ratio $\eta_{t'_1 t'_2}^R$ is calculated from

$$(A.24) \quad \eta_{t'_1 t'_2}^R = \frac{\xi_{t'_1}^R}{\xi_{t'_2}^R}$$

and the uncertainty in this corrected ratio for all rotations is

$$(A.25) \quad \Delta\eta_{t'_1 t'_2}^R = \eta_{t'_1 t'_2}^R \left[\left(\frac{\Delta\xi_{t'_1}^R}{\xi_{t'_1}^R} \right)^2 + \left(\frac{\Delta\xi_{t'_2}^R}{\xi_{t'_2}^R} \right)^2 \right]^{1/2}$$

where $\xi_{t'_1}^R$ and $\xi_{t'_2}^R$ denote different rotations of the neutron polarization vector for the same method (viz, $\pm 40^\circ$ or $0^\circ, \pm 90^\circ$).

The corrected g value can be calculated from the corrected ratio η^R , which denotes that the values are corrected for the two-step process $[^2\text{H}(\vec{e}, e'\vec{p}) + \text{Pb}(\vec{p}, \vec{n})]$. For the $0^\circ, \pm 90^\circ$ method, we have

$$(A.26) \quad g^R = -K_R \eta_{S'L'}^R.$$

The relative uncertainty in this value is simply the relative uncertainty in the corrected ratio $\eta_{S'L'}^R$.

$$(A.27) \quad \left(\frac{\Delta g^R}{g^R} \right)_{0^\circ, \pm 90^\circ} = \frac{\Delta \eta_{S'L'}^R}{\eta_{S'L'}^R}$$

Applying the same idea to the $\pm\chi$ method, we find that the corrected g value is

$$(A.28) \quad g^R = -K_R \frac{(1 + \eta_\pm^R)}{(1 - \eta_\pm^R)} \tan(\chi).$$

The relative uncertainty is found from propagation of errors to be

$$(A.29) \quad \left(\frac{\Delta g^R}{g^R} \right)_{\pm\chi} = \frac{2}{((\eta_\pm^R)^2 - 1)} \Delta \eta_\pm^R.$$

Now we want to obtain expressions for $(\Delta g^R / g^R)_{2\text{-step}}$ for both the $0^\circ, \pm 90^\circ$ case and for the $\pm 40^\circ$ case. In the $0^\circ, \pm 90^\circ$ case, the “corrected” value of g , g^R , can be

written as

$$\begin{aligned}
 g^R &= -K_R \eta_{S'L'}^R \\
 &= -K_R \eta_{S'L'}^m \left(\frac{1 + c_{S'}}{1 + c_{L'}} \right) \\
 (A.30) \quad &= g^m C_{S'L'},
 \end{aligned}$$

where

$$(A.31) \quad C_{S'L'} \equiv \left(\frac{1 + c_{S'}}{1 + c_{L'}} \right).$$

As we now have $g^R = g^R(g^m, C_{S'L'})$, we apply the standard formula for propagation of errors,

$$(A.32) \quad (\Delta g^R)^2 = \left(\frac{\partial g^R}{\partial g^m} \right)^2 (\Delta g^m)^2 + \left(\frac{\partial g^R}{\partial C_{S'L'}} \right)^2 (\Delta C_{S'L'})^2,$$

to derive

$$\begin{aligned}
 \left(\frac{\Delta g^R}{g^R} \right)_{2\text{-step}}^2 &\equiv \left(\frac{\partial g^R}{\partial C_{S'L'}} \right)^2 \left(\frac{\Delta C_{S'L'}}{g^R} \right)^2 \\
 &= \left(\frac{\Delta C_{S'L'}}{C_{S'L'}} \right)^2 \\
 (A.33) \quad &= \left(\frac{\Delta c_{S'}}{1 + c_{S'}} \right)^2 + \left(\frac{\Delta c_{L'}}{1 + c_{L'}} \right)^2.
 \end{aligned}$$

For the $\pm 40^\circ$ case, the “corrected” value of g , g^R , can be written as

$$\begin{aligned}
 g^R &= K \tan \chi \left(\frac{\eta_{\pm}^R + 1}{\eta_{\pm}^R - 1} \right) \\
 (A.34) \quad &= K \tan \chi \left(\frac{\eta_{\pm}^m C_{\mp} + 1}{\eta_{\pm}^m C_{\mp} - 1} \right),
 \end{aligned}$$

where

$$(A.35) \quad C_{\mp} \equiv \left(\frac{1 + c_{-}}{1 + c_{+}} \right).$$

As we now have $g^R = g^R(g^m, C_{\mp})$, we can once again use the standard formula for propagation of errors,

$$(A.36) \quad (\Delta g^R)^2 = \left(\frac{\partial g^R}{\partial g^m} \right)^2 (\Delta g^m)^2 + \left(\frac{\partial g^R}{\partial C_{\mp}} \right)^2 (\Delta C_{\mp})^2,$$

to derive a formula for $\left(\frac{\Delta g^R}{g^R}\right)_{2\text{-step}}^2$. For $\pm\chi$ precession, the “corrected” value of g , g^R , can be written as

$$\begin{aligned} g^R &= K_R \tan\chi \left(\frac{\eta_{\pm}^R + 1}{\eta_{\pm}^R - 1} \right) \\ (A.37) \quad &= K_R \tan\chi \left(\frac{\eta_{\pm}^m C_{\mp} + 1}{\eta_{\pm}^m C_{\mp} - 1} \right), \end{aligned}$$

where

$$(A.38) \quad C_{\mp} \equiv \left(\frac{1 + c_-}{1 + c_+} \right).$$

We want to derive an expression for g^R of the form $g^R = g^R(g^m, C_{\mp})$. As

$$(A.39) \quad g^m = K_R \tan\chi \left(\frac{\eta_{\pm}^m + 1}{\eta_{\pm}^m - 1} \right),$$

it is easy to show that

$$(A.40) \quad g^R = K_R \tan\chi \left[\frac{C_{\mp}(g^m + K_R \tan\chi) + (g^m - K_R \tan\chi)}{C_{\mp}(g^m + K_R \tan\chi) - (g^m - K_R \tan\chi)} \right].$$

As we now have $g^R = g^R(g^m, C_{\mp})$, we can use the standard formula for propagation of errors,

$$(A.41) \quad (\Delta g^R)^2 = \left(\frac{\partial g^R}{\partial g^m} \right)^2 (\Delta g^m)^2 + \left(\frac{\partial g^R}{\partial C_{\mp}} \right)^2 (\Delta C_{\mp})^2,$$

to derive an expression for the relative *statistical* uncertainty in g^R ,

$$(A.42) \quad \left(\frac{\Delta g^R}{g^R} \right)_{\text{statistical}}^2 \equiv \left(\frac{\partial g^R}{\partial g^m} \right)^2 \left(\frac{\Delta g^m}{g^R} \right)^2$$

and an expression for the relative *systematic* uncertainty in g^R ,

$$(A.43) \quad \left(\frac{\Delta g^R}{g^R} \right)_{2\text{-step}}^2 \equiv \left(\frac{\partial g^R}{\partial C_{\mp}} \right)^2 \left(\frac{\Delta C_{\mp}}{g^R} \right)^2.$$

Evaluation of the partial derivatives is straight forward and enables us to write Eqs. (A.42) and (A.43) as

$$(A.44) \quad \left(\frac{\Delta g^R}{g^R} \right)_{\text{statistical}}^2 = \frac{16C_{\mp}^2 K_R^4 \tan^4 \chi}{[C_{\mp}(g^m + K_R \tan \chi) - (g^m - K_R \tan \chi)]^4} \left(\frac{\Delta g^m}{g^R} \right)^2,$$

and

$$(A.45) \quad \left(\frac{\Delta g^R}{g^R} \right)_{2\text{-step}}^2 = \frac{4K_R^2 \tan^2 \chi [K_R^2 \tan^2 \chi - (g^m)^2]^2}{[C_{\mp}(g^m + K_R \tan \chi) - (g^m - K_R \tan \chi)]^4} \left(\frac{\Delta C_{\mp}}{g^R} \right)^2.$$

Finally, using Eq. (A.40) and

$$(A.46) \quad \left(\frac{\Delta C_{\mp}}{C_{\mp}} \right)^2 = \left(\frac{\Delta c_-}{1 + c_-} \right)^2 + \left(\frac{\Delta c_+}{1 + c_+} \right)^2,$$

we arrive at the following expressions for the relative statistical and systematic uncertainties in g^R

$$(A.47) \quad \left(\frac{\Delta g^R}{g^R} \right)_{\text{statistical}}^2 = \frac{16C_{\mp}^2 K_R^2 \tan^2 \chi}{[C_{\mp}^2 (g^m + K_R \tan \chi)^2 - (g^m - K_R \tan \chi)^2]^2} (\Delta g^m)^2,$$

and

$$(A.48) \quad \left(\frac{\Delta g^R}{g^R} \right)_{2\text{-step}}^2 = \frac{4C_{\mp}^2 [K_R^2 \tan^2 \chi - (g^m)^2]^2}{[C_{\mp}^2 (g^m + K_R \tan \chi)^2 - (g^m - K_R \tan \chi)^2]^2} \left[\left(\frac{\Delta c_-}{1 + c_-} \right)^2 + \left(\frac{\Delta c_+}{1 + c_+} \right)^2 \right].$$

APPENDIX B

Evaluation of the Systematic Uncertainty in g from the Electron Scattering Angle θ_e

In order to obtain an expression for the systematic uncertainty in g in terms of the position and traceback uncertainties in the electron angle, $(\theta_e)_{pos}$ and $(\theta_e)_{tra}$ respectively, it is necessary to evaluate Eq. 5.15. The following derivation was first shown by Brad Plaster and Richard Madey in their report to the collaboration[77] and I have adopted it for use in this appendix.

The expression

$$(B.1) \quad (\Delta g)_{\theta_e}^2 \equiv \left(\frac{\partial g}{\partial \theta_e} \right)^2 (\Delta \theta_e)^2$$

must be evaluate to obtain the desired expression connecting the uncertainties in the positioning and traceback to the uncertainty in g . The partial derivative in Eq. B.1 has the following form:

$$(B.2) \quad \frac{\partial g}{\partial \theta_e} = \frac{\partial g}{\partial K_L} \frac{dK_L}{d\theta_e} + \frac{\partial g}{\partial K_S} \frac{dK_S}{d\theta_e}.$$

The four factors in Eq. B.2 can be defined as A, B, C, and D in the following way

$$(B.3) \quad A \equiv \frac{\partial g}{\partial K_L} = \frac{g}{K_L},$$

$$(B.4) \quad B \equiv \frac{\partial g}{\partial K_S} = -\frac{g}{K_S},$$

$$(B.5) \quad C \equiv \frac{dK_L}{d\theta_e},$$

$$(B.6) \quad D \equiv \frac{dK_S}{d\theta_e}.$$

The expressions for the kinematic functions are

$$(B.7) \quad K_S = 2\sqrt{\tau(1+\tau)} \tan(\theta/2),$$

$$(B.8) \quad K_L = 2\tau\sqrt{(1+\tau)(1+\tau\sin^2(\theta/2))} \sec(\theta_e/2) \tan(\theta_e/2)$$

and

$$(B.9) \quad \tau = \frac{Q^2}{4M^2},$$

$$(B.10) \quad Q^2 = 4EE' \sin^2(\theta_e/2),$$

$$(B.11) \quad E' = \frac{E}{1 + 2\left(\frac{E}{M}\right) \sin^2(\theta_e/2)}.$$

Equations C and D require the evaluation of the K_L and K_S with respect to θ_e . It follows that Eq. C and D can be evaluated as

$$(B.12) \quad C = \frac{\partial K_L}{\partial \tau} \frac{\partial \tau}{\partial \theta_e} + \frac{\partial K_L}{\partial \theta_e} \equiv EF + G,$$

$$(B.13) \quad D = \frac{\partial K_S}{\partial \tau} \frac{\partial \tau}{\partial \theta_e} + \frac{\partial K_S}{\partial \theta_e} \equiv HF + I.$$

The partial derivatives of E, F, G, H, and I can be calculated directly.

$$(B.14) \quad E = \frac{\sec(\theta_e/2) \tan(\theta_e/2)}{\{(1+\tau)[1+\tau\sin^2(\theta_e/2)]\}^{1/2}} [2 + 3\tau + (3+4\tau)\tau\sin^2(\theta_e/2)]$$

$$(B.15) \quad F = \frac{\left(\frac{E}{M}\right)^2 \sin(\theta_e/2) \cos(\theta_e/2)}{[1 + 2\left(\frac{E}{M}\right) \sin^2(\theta_e/2)]^2}$$

$$(B.16) \quad G = \frac{\tau(1+\tau)^{1/2}}{[1+\tau\sin^2(\theta_e/2)]^{1/2}} \times \left\{ \tau\sin^2(\theta_e/2) \sec(\theta_e/2) + \sec^3(\theta_e/2)[1+\tau\sin^2(\theta_e/2)][1+\sin^2(\theta_e/2)] \right\}$$

$$(B.17) \quad H = \frac{(1+2\tau) \tan(\theta_e/2)}{[\tau(1+\tau)]^{1/2}}$$

$$(B.18) \quad I = [\tau(1+\tau)]^{1/2} \sec^2(\theta_e/2)$$

All of that completely specifies $(\Delta g)_{\theta_e}^2$ in terms of τ and θ_e through these definitions of A, B, C, D, E, F, G, H, and I. Notice that from Eqs. B.2, B.3, and B.4 it is possible to recognize that $(\Delta g/g)_{\theta_e}^2$ is completely independent of g .