Lattice QCD and nucleon resonances

R.G. Edwards^a, H.R. Fiebig^b, G. Fleming^a, D.G. Richards^a (LHP Collaboration)

^aJefferson Laboratory, MS 12H2, 12000 Jefferson Avenue, Newport News, VA 23606, USA

^bPhysics Department, FIU, Miami, Florida 33199, USA

Lattice calculations provide an ab initio means for the study of QCD. Recent progress at understanding the spectrum and structure of nucleons from lattice QCD studies is reviewed.

Measurements of the masses of the lightest particles for the lowest spin values are described and related to predictions of the quark model. Measurements of the mass of the first radial excitation of the nucleon, the so-called Roper resonance, obtained using Bayesian statistical analyses, are detailed. The need to perform calculations at realistically light values of the pion mass is emphasised, and the exciting progress at attaining such masses is outlined.

The talk concludes with future prospects, emphasising the importance of constructing a basis of interpolating operators that is sensitive to three-quark states, to multi-quark states, and to excited glue.

1. INTRODUCTION

The investigation of the spectrum is the classic tool for gleaning information about the dynamics of a system. In this talk, we describe recent progress at lattice studies of the spectrum of excited baryon resonances. The aims of these studies are manifold: the testing of, and comparison with, models of hadronic structure, such as quark models, large- N_c , and boson-exchange models, the qualification of the status of the missing resonances of the quark models, and finally the interpretation of resonances as three-quark, multi-quark or molecular states, an issue brought into sharp focus by the observation of a candidate pentaquark state.

We begin by briefly outlining the theoretical and computational issues in the determination of the nucleon spectrum. Recent lattice results are then reviewed, in particular emphasising the importance of continuing the studies to physical values of the light-quark masses, and to larger volumes. The talk concludes with a discussion of prospects for future calculations.

2. RECENT RESULTS

The calculation of the light hadron spectrum, i.e. the lightest states of given quantum numbers, has historically been one of the benchmark calculations of lattice QCD. In order

to confront experiment, such calculations require a careful study of the systematic uncertainties, in particular finite-volume and discretisation errors, and the need to extrapolate to the physical values of the light quark masses. In the quenched approximation, the measured spectrum agrees with experiment to around 10% in most channels [1], and, in the meson sector, this discrepancy is removed in full QCD.

Figure 1. The left-hand plot shows of the lightest positive- and negative-parity nucleon resonances using two independent interpolating operators [5], while the right-hand plot shows also the spin-3/2 and spin-1/2 masses of both parities [6], obtained using the FLIC fermion action.

The observation of excited resonances in lattice QCD is invariably more demanding than that of the ground states since, for hadrons composed of light quarks, the signal-to-noise ratio for excited-state correlators increases severely as the excitation energy increases. None-the-less, there has recently been a flurry of activity aimed at computing the excited nucleon spectrum, and in particular the masses of the lightest spin-1/2 and spin-3/2 states of both parities [2–8]. These calculations employ a variety of fermion discretisation, each has quarks with masses around that of the strange quark, uses local interpolating operators, and each finds a spectrum broadly in line with quark-model expectations $m_N < m_{N^{1/2-}} < m_{N^*}$, as illustrated in Figure 1. In particular, none of these calculations reveal evidence of two of the more puzzling observations in the nucleon spectrum, the anomalously light Roper resonance, and a light $\Lambda(1405)^-$.

The preceding calculations have several limitations: they were all in the quenched approximation to QCD, they employed unphysically large values of the quark masses, were performed on relatively small volumes, and measured only local, “S-wave” interpolating operators constructed from three valence quarks that might be expected to be insensitive to molecular or multi-quark states. The elimination of the quenched approximation, and calculations at smaller values of the quark masses, are both computationally highly demanding, and it is therefore imperative to extract the greatest physical information possible from a calculation. The use of Bayesian statistics, with appropriately chosen priors, affords the prospect of so doing. A Bayesian fit to the standard nucleon correlator

Figure 2. The masses of the nucleon, its parity partner, and the first radial excitation of the nucleon obtained from a calculation using overlap fermions at a lattice spacing $a = 0.2$ fm [11].

in a full QCD calculation, once again with quark masses around those of the strange quark, reveals the same quark-model ordering of states as those observed in the quenched approximation to QCD, with no evidence for a light Roper resonance [9].

The discovery of lattice fermion actions possessing an exact analogue of chiral symmetry has promised calculations at the physical light-quark masses. A particular realisation of the action is through overlap fermions, which admit the use of shift-mass inverters allowing the simultaneous calculation of propagators at a range of quark masses. A Bayesian fit to the standard nucleon interpolating operator obtained in the quenched approximation to QCD using overlap fermions, with pion masses as low as 180 MeV, is reported in ref. [10], and reveals a dramatic qualitative change in the spectrum for pion masses below around 300 MeV. A concern is the contribution to the correlator from the non-unitary $\eta'N$ ghost states; by comparing calculations at two different volumes, the $\eta'N$ and Roper resonance are delineated[11]. The masses of the nucleon, its parity partner and the first radial excitation of the nucleon are shown in Figure 2, showing the dramatic reversal of the ordering of the states at light pion masses, and indicating that the experimentally observed $N(1440)$ Roper is indeed a simple three-quark state.

A particular choice of Bayesian prior, the Shannon-Jaynes entropy, yields the Maximum Entropy Method (MEM), which aims to extract the spectral function of the correlator. Here we apply the method to a calculation employing domain-wall fermions (DWF) for the valence quarks, computed on a set of 200 $N_f = 2 + 1$ improved staggered dynamical configurations generated by the MILC collaboration [12]; to ensure a sufficiently small residual mass, the configuration are smeared using a non-perturbative HYP blocking scheme. The spectral function obtained from the smeared-smeared correlators is shown in

Figure 3. The left-hand plot is the spectral density $\rho(\omega)$ vs. the energy ωa , where a is the lattice spacing, at the lightest valence quark mass, clearly showing two peaks. The right-hand plot shows the masses of the ground state (circle) and first excited state (triangle) of the nucleon obtained from the spectral function at three values of the valence pion mass. Also shown as the diamonds (offset for clarity) are the nucleon masses obtained from conventional maximum-likelihood fits to the data. The lattice spacing $a = 0.124$ fm is obtained from the 1P-1S splitting in the Upsilon spectrum[13]. Finally, the experimental nucleon and Roper masses are shown as the bursts.

Figure 3, together with the ground-state and first-excited-state masses at each of our three values of the valence quark mass; correlations between the data on different time slices are neglected. The smearing employed was designed to give the greatest overlap onto the ground-state proton, and is thus not optimal for our task; the smeared-point correlators show more dramatic evidence of an excited-state peak, with a central value around one standard deviation higher, but in this case we cannot guarantee that the spectral function is positive definite, a prerequisite for the application of the method. The excited-state mass does not exhibit the dramatic behaviour with m_π of the overlap-fermion calculation, but our lightest pion mass is only around 320 MeV; however, our determination of the excited-state mass is somewhat lower than that reported in ref. [11] at a common pion mass. A more complete, and perhaps satisfactory, analysis using the variational method and a matrix of correlators is in preparation[14].

A further study of the Roper resonance for the Wilson fermion action using the MEM was performed in [15]. Though this was performed once again for quark masses in the region of the strange-quark mass, there were indications of substantial finite-volume effects, emphasising the need to use lattice extents approaching 3 fm even at these quark masses.

3. FUTURE

The calculation of the nucleon resonance spectrum in lattice QCD is in its infancy, but already two issues are clear: the need to attain light pion mass, and to explore sufficiently large volumes. A more complete calculation of the spectrum will require the

use of variational techniques, for which it is crucial to construct a set of operators that provide a reasonably faithful description of the lowest-lying states.

The Lattice Hadron Physics Collaboration has embarked on a program to construct and measure such operators, using as a basis a set of gauge-invariant operators that explore the radial and orbital structure of the states[14,16]. For states at rest, the necessary operators must transform irreducibly under the lattice cubic group, which possesses only three double-valued representations of each parity. Thus the full angular-momentum structure of the higher- J states will only be revealed in the continuum limit. The need to include multi-quark operators in any complete description has been brought into sharp focus by the recent experimental observations of a “pentaquark” $S = 1$ state, and our group theory construction is readily adaptable to such operators.

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