Conformal kernel for NLO BFKL equation in $\mathcal{N}=4$ SYM

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The high-energy behavior of perturbative amplitudes is given by the BFKL pomeron \cite{1}. In the leading order, the BFKL equation is conformally invariant under Möbius SL(2,C) group of transformations of the transverse plane. In the next-to-leading order (NLO) the BFKL kernel in QCD is not invariant because of the running coupling, but the kernel in $\mathcal{N} = 4$ SYM is expected to be invariant. The eigenvalues of this conformal kernel are known from the calculation of forward NLO BFKL in the momentum space \cite{2}. In the conformal theory it is possible to recover the amplitude of the non-forward scattering of two reggeized gluons from the forward scattering amplitude. Using the NLO kernel for evolution of color dipoles in QCD \cite{3} we guess the Möbius invariant kernel for $\mathcal{N}=4$ SYM and check that it reproduces the known eigenvalues \cite{2}.

At high energies the typical forward scattering amplitude has the form

$$ A(s, t) = \frac{s}{q^2} \int d^2q \frac{d^2q'}{q'^2} F_A(q) F_B(q') \int_a^{s+i\infty} \frac{d\omega}{2\pi} \left( \frac{s}{qq'} \right)^\omega G_\omega(q, q') $$

where $G_\omega(q, q')$ is the partial wave of the forward reggeized gluon scattering amplitude satisfying the BFKL equation

$$ \omega G_\omega(q, q') = \delta^2(q - q') + \int d^2p K(p, q) G_\omega(p, q') $$

and $F_A(q), F_B(q')$ are the impact factors. In $\mathcal{N}=4$ SYM the kernel $K(p, q)$ \cite{2} is known up to the next-to-leading order

$$ \int d^2p K(p, q) f(p) = \frac{\alpha_s N_c}{\pi^2} \int d^2p \left\{ \frac{1}{(q - p)^2} \left[ 1 - \frac{\alpha_s N_c \pi}{12} f(p) - \frac{q^2}{2 p^2} f(q) \right] + \frac{\alpha_s N_c}{4\pi} \left[ \Phi(p, q) - \ln \frac{q^2}{p^2} \right] f(p) \right\} + \frac{3 \alpha_s^2 N_c^2}{2\pi^2} \zeta(3) f(p) \quad (3) $$

where $\zeta$ is the Riemann zeta-function and

$$ \Phi(p, q) = \frac{(q^2 - p^2)}{(q - p)^2 (q + p)^2} \left[ \ln \frac{q^2}{p^2} \ln \frac{q^2 p^2 (q - p)^4}{(q + p)^4} \right. $$

$$ + 2 \ln \frac{q^2}{p^2} \left[ 2 \ln \frac{q^2}{p^2} - 1 - \frac{(q^2 - p^2)^2}{(q - p)^2 (q + p)^2} \right] \times \left[ \int_0^1 - \int_1^{\infty} \frac{du}{(qu - p)^2} \ln \frac{u^2 q^2}{p^2} \right] \quad (4) $$

Here $\ln$ is the dilogarithm.

The eigenvalues of the kernel (3) are \cite{2}

$$ \int d^2 p (\frac{p^2}{q^2})^{\frac{1}{2} + i \nu} e^{i \nu \phi} K(q, p) = \omega(n, \nu), \quad (5) $$

$$ \omega(n, \nu) = \frac{\alpha_s}{\pi} N_c \left[ \chi(n, 1 + 2i\nu) + \frac{\alpha_s N_c \pi}{4\pi} \delta(n, 1 + 2i\nu) \right], $$

$$ \delta(n, \gamma) = 6 \zeta(3) - \frac{\pi^2}{3} \chi(n, \gamma) - \chi^*(n, \gamma) - 2\Phi(n, \gamma) $$

and $\chi(n, \gamma)$ is $2\psi(1) - \psi(\gamma + \frac{n}{2}) - \psi(1 - \gamma + \frac{n}{2})$ and

$$ \Phi(n, \gamma) = \int_0^1 \frac{dt}{1 + t} \left[ \psi(t) - \psi(1) \right] \psi \left( \frac{n}{2} + \frac{1}{2} \right) $$

$$ - \ln \ln \left[ \frac{1}{k + n} \right] + \sum_{k=1}^{\infty} \frac{(-t)^k}{(k + n)^2} \left[ \ln 1 - (-1)^k \right] \quad (6) $$

The Regge limit of the amplitude $A(x, y; x', y')$ in the coordinate space can be achieved as

$$ x = \lambda x_1 p_1 + x_\perp, \quad y = \lambda y_1 p_2 + y_\perp, $$

$$ x' = \rho x'_1 p_1 + x'_\perp, \quad y' = \rho y'_1 p_2 + y'_\perp $$

with $\lambda, \rho \to \infty$ and $x_\perp > 0 > y_\perp, x'_\perp > 0 > y'_\perp$.

Hereafter we use the notations $x_\perp = p_1^a x_\mu, x_\perp = p_2^a x_\mu$ where $p_1$ and $p_2$ are light-like vectors normalized by $2(p_1, p_2) = s$.

These “Sudakov variables” are related to usual light-cone coordinates $x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^3)$ by $x_\perp = x^+ \sqrt{s/2}, \quad x_\perp = - x^- \sqrt{s/2}$ so $x = \frac{1}{2} x_1 p_1 + \frac{1}{2} x_2 p_2 + x_\perp$.

We use the (1,-1,1,-1) metric so $x^2 = \frac{1}{8} x_\perp x_\perp - x_\perp^2$.}

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In the Regge limit (7) the full conformal group reduces to Möbius subgroup SL(2,C) leaving the transverse plane \((0, 0, z_\perp)\) invariant. In a conformal theory the four-point amplitude \(A(x, y; x', y')\) depends on two conformal ratios which can be chosen as

\[
R = \frac{(x - x')(y - y')^2}{(x - x')^2(y - y')^2},
\]

\[
r = \frac{R}{1 - (x - y')^2(y - x')^2(x - x')^2(y - y')^2 + \frac{1}{R}^2}
\]  

(8)

The conformal ratio \(R\) scales as \(\lambda^2 \rho^2\) while \(r\) does not depend on \(\lambda\) or \(\rho\). Following Ref. [4] it is convenient to introduce two SL(2,C)-invariant vectors

\[
\kappa = \frac{\sqrt{s}}{2x_p}(p_1 - \frac{x^2}{s}p_2 + x_\perp) - \frac{\sqrt{s}}{2y_p}(p_1 - \frac{y^2}{s}p_2 + y_\perp)
\]

\[
\kappa' = \frac{\sqrt{s}}{2x'_p}(p_1 - \frac{x'^2}{s}p_2 + x'_\perp) - \frac{\sqrt{s}}{2y'_p}(p_1 - \frac{y'^2}{s}p_2 + y'_\perp)
\]

(9)

such that

\[
\kappa^2 \kappa'^2 = \frac{1}{R} \quad \text{and} \quad 4(\kappa \cdot \kappa')^2 = \frac{r}{R}
\]  

(10)

(here \(x^2 = -x'^2\), \(x'^2 = -x^2\) and similarly for \(y\)). In the coordinate space the analog of Eq. (1) has the form:

\[
A(x, y; x', y') = \int d^2z_1d^2z_2d^2z_1'd^2z_2' I_A(x, y; z_1, z_2) 
\]

\[
\times \int \frac{d\omega}{2\pi} R^\omega G_\omega(z_1, z_2; z'_1, z'_2) I_B(x', y'; z'_1, z'_2)
\]

(11)

where the partial wave of the reggeized gluon scattering amplitude satisfies the equation

\[
\omega G_\omega(z_1, z_2; z'_1, z'_2) = \ln^2 \left(\frac{z_1 - z'_1}{z_2 - z'_2}\right)^2
\]

\[
\frac{(z_2 - z'_1)(z_2 - z'_2)(z_1 - z'_1)(z_1 - z'_2)}{(z_2 - z'_2)(z_1 - z'_1)}
\]

\[
+ \int d^4t_1d^4t_2 K(z_1, z_2; t_1, t_2)G_\omega(t_1, t_2; z'_1, z'_2)
\]

(12)

Here the first term in the r.h.s. is the leading-order contribution coming from two-gluon exchange.

The meaning of the Eq. (11) is that the amplitude is factorized into the product of three terms \(I_A\), \(I_B\), and \(G_\omega\) corresponding to rapidities \(\eta_A\), \(\eta_B\), and \(\eta_\omega\) respectively. With conformally invariant factorization of the amplitude into such product the impact factors and \(G_\omega\) should be separately conformally invariant leading to invariant kernel \(K(z_1, z_2; t_1, t_2)\). The eigenfunctions of a conformal kernel are \([5]\)

\[
E_{\nu, n}(z_10, z_20) = \left[\frac{\tilde{z}_{12}}{\tilde{z}_{1020}}\right]^{\frac{1}{2}+iv} + \frac{n}{2} \left[\frac{\tilde{z}_{12}}{\tilde{z}_{1020}}\right]^{\frac{1}{2}+iv} - \frac{n}{2}
\]

(13)

where \(\tilde{z} = z_2 + iz_\perp, \tilde{z} = z_2 - iz_\perp\) and \(z_{10} \equiv z_1 - z_0\) etc. Denoting the eigenvalues of the kernel \(K\) by \(\omega(n, \nu)\)

\[
\int d^2t_1d^2t_2 K(z_1, z_2; t_1, t_2)E_{\nu, n}(t_1 - z_0, t_2 - z_0)
\]

\[
= \omega(n, \nu)E_{\nu, n}(z_{10}, z_{20})
\]

(14)

and substituting the formal solution of the Eq. (12) into Eq. (11) we obtain

\[
A(x, y; x', y') = \sum_{n=-\infty}^{\infty} \int \frac{d\nu}{\pi^2} \frac{-2(\nu^2 + \frac{\kappa^2}{4})R_{2\omega}(n, \nu)}{[\nu^2 + (n-1)^2][\nu^2 + (n+1)^2]}
\]

\[
\times \int d^2z_0d^2z_1d^2z_2 I_A(x, y; z_1, z_2)E_{\nu, n}(z_{10}, z_{20})
\]

\[
\times \int d^2z'_1d^2z'_2 I_B(x', y'; z'_1, z'_2)E_{\nu, n}(z'_1 - z_0, z'_2 - z_0)
\]

As demonstrated in Ref. [4] the impact factors depend on one conformal (Möbius invariant) ratio

\[
I_A(x, y; z_1, z_2) = \frac{1}{z_{12}^4} I_A \left(\frac{\kappa^2(z_1 \cdot z_2)}{2(\kappa \cdot z_1)(\kappa \cdot z_2)}\right),
\]

\[
I_B(x', y'; z'_1, z'_2) = \frac{1}{z_{12}^4} I_B \left(\frac{\kappa'^2(z'_1 \cdot z'_2)}{2(\kappa' \cdot z'_1)(\kappa' \cdot z'_2)}\right)
\]

where \(z_1 \equiv p_1 + \frac{x_2^2}{s}p_2 + z_\perp\) and similarly for other \(z_i\). This enables us to carry out the integrations over \(z_i\) and \(z'_i\). The formulas are especially simple when we consider the correlator of four scalar currents such as \(\text{Tr}(Z^2)\) \((Z = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2))\) so that only the term with \(n = 0\) contributes. From conformal (Möbius) invariance we get [4]

\[
\int d^2z_1d^2z_2 \frac{I_A(z_1 \cdot z_2)}{z_{12}^8} = \frac{1}{8\pi} \frac{\Gamma(1 - 2\nu)}{\Gamma(2 - 2\nu)} \frac{\kappa^2}{(\kappa \cdot z_1)(\kappa \cdot z_2)} I_A(\nu)
\]

(15)

\[
\text{where} \quad z_0 \equiv p_1 + \frac{x_2^2}{s}p_2 + z_\perp\text{ and therefore (cf. Ref. [4])}
\]

\[
(x - y)^4(x' - y')^4 \frac{1}{2} \omega(\nu, 0)
\]

(16)

\[
\Omega(\nu, \nu) = \frac{\nu^2}{\pi^3} \int d^2z \frac{\kappa^2}{(\kappa \cdot z)^2} \frac{1}{2 + iv} \frac{\kappa'^2}{(\kappa' \cdot z)^2} \frac{1}{2 - iv}
\]

(17)

(Since the integral (17) does not scale with \(\lambda, \rho\) it can depend only on \(\frac{\kappa^2}{\kappa \cdot z} = \frac{\kappa'}{\kappa' \cdot z}\). Note that all the dependence on large energy \((\approx \lambda, \rho)\) is contained in \(R_{2\omega}(\nu, 0)\). It is worth noting that in the leading order in perturbation theory \(I(\nu) = \frac{2\pi^2 a_2}{\cos \pi \nu \sqrt{1 - \frac{\nu^2}{\pi^2}}}
\)

To restore the NLO BFKL kernel in the coordinate representation (11) from the eigenvalues (5) in the momentum representation we must prove that Eq. (16) agrees with Eq. (1) with the same set of \(\omega(0, \nu)\). (Strictly speaking, we need to demonstrate this property for arbitrary \(n\) but here we will do it only for \(n = 0\)).

In order to perform Fourier transformation of the correlator (16) we need to relax the limit (7) by allowing
small $x_\ast \sim y_\ast \sim 1/\lambda$ and $x'_\ast \sim y'_\ast \sim 1/\rho$. The conditions
(10) for vectors (9) are now satisfied up to $1/\lambda$ and $1/\rho$
corrections. The correlator (16) takes the form

$$
(x - y)^4 (x' - y')^4 (O(x) O(y) O(x') O(y'))
$$

(18)

$$
= - \frac{1}{2\pi^2} \int \! dv \nu \tanh\nu \left[ \frac{x_\ast y_\ast x'_\ast y'_\ast}{s^2(x - y)^2(x' - y')^2} \right] \omega(\nu)
$$

$$
\times \int \! d^2 z_0 \left[ \frac{(x - z)^2}{x} \left( \frac{(y - z)^2}{y} \right) - \frac{4}{s} (x - y) \right] \frac{1}{z_0^4} I_A(\nu)
$$

$$
\times \left[ \left( \frac{(x - z)^2}{x} \left( \frac{(y - z)^2}{y} \right) - \frac{4}{s} (x - y) \right) \right] \frac{1}{z_0^4} I_B(\nu)
$$

where $\omega(\nu) \equiv \omega(0, \nu)$. The forward scattering amplitude can be defined as (cf. Ref. [6])

$$
A(p_A, p_B) = -i \int \! d^4 x d^4 y e^{-i p_A x - i p_B y}
$$

$$
\langle O(x_\ast, x_\ast, x_\ast, x_\ast) O(y_\ast, y_\ast, y_\ast) O(x'_\ast, y'_\ast) O(y'_\ast, y'_\ast) \rangle
$$

(19)

where $p_A = p_1 + \frac{s}{x} p_2$ and $p_B = p_2 + \frac{s}{x} p_1$. Substituting Eq. (18) in Eq. (19) and performing the
coordinates we obtain

$$
A(s, 0) = -i \pi^2 \frac{s}{16 \lambda^2 p_A p_B} \int \! dv \nu I_A(\nu) I_B(\nu)
$$

$$
\times \left[ \frac{s^2}{16 p_A p_B} \right]^{1/2} \Gamma^2 \left( \frac{1}{2} + i \nu \right) \frac{1}{\Gamma(1 - 2i \nu)} \Gamma(1 + 2i \nu)
$$

$$
\times \Gamma^2 \left( \frac{1}{2} + \frac{1}{2} \nu \right) \nu \Gamma^2 \left( \frac{1}{2} + \frac{\nu}{2} + i \nu \right) \Gamma(1 + \nu)
$$

(20)

This should be compared with Eq. (1) which takes the form

$$
A(s, 0) = i s \int \! d^2 k \frac{d^2 k'}{k^2 k'^2} F_A(k) F_B(k')
$$

$$
\times \int \! dv \nu \left( k^2 \right)^{-1/2 + \frac{\nu}{2}} \left( k'^2 \right)^{-1/2 - \frac{\nu}{2}} \left( \frac{s}{|k||k'|} \right)^{\omega(\nu)}
$$

$$
= i s \int \! dv \nu F_A(\nu) F_B(\nu) \left( \frac{s}{4 p_A p_B} \right)^{\omega(\nu)}
$$

(21)

It is clear that Eq. (20) and Eq. (21) coincide after the
redefinition of the impact factor

$$
F_A(\nu) = \frac{\pi^{5/2} I_A(\nu)}{4 p_A p_B} \Gamma^2 \left( \frac{1}{2} + \nu + \frac{\nu}{2} \right) \frac{1}{\Gamma(1 - \nu + \omega(\nu))} \Gamma(1 + \nu)
$$

Now we are in a position to restore $K_{NLO}(z_1, z_2; t_1, t_2)$ from the
eigenvalues (5). At the leading-order level $K$ is given by the BFKL
kernel in the dipole form (linear part of the BK equation [7, 8])

$$
K_{LO}(z_1, z_2; z_3, z_4) = \frac{\alpha_s N_c}{2\pi^2} \left( \frac{z_1^2}{z_2^2} \frac{d^2}{d^2(\Delta)} \right) + \frac{z_1^2 d^2}{z_2^2 (\Delta)}
$$

$$
- \frac{\delta^2}{z_1^2} \delta^2(z_24) \int \! d^2 z \left( \frac{z_1^2}{z_2} \right)^{1/2 + \nu + \frac{\nu}{2} - e^{-\nu t}} \left[ - \frac{1}{(z_1 - z_2)^2 + \nu^2} \right]
$$

Using the eigenvalues $\omega(n, \nu)$ and the requirement of conformal
invariance it is possible to restore the conformal kernel for the
NLO BFKL kernel in QC [3].

The equation (12) with the kernel (22) is obviously
conformally invariant. Let us prove that its eigenvalues are given by Eq. (5). The integral

$$
\int \! dz_3 dz_4 K_{NLO}(z_1, z_2; z_3, z_4) E_{n, \nu}(n_3, n_4)
$$

(23)

$$
= [c(n, \nu) + \frac{\alpha_s N_c}{2\pi^2 (6\zeta(3) - \frac{\pi^2}{3} \chi(n, \nu))}] E_{n, \nu}(n_1, n_2)
$$

can be reduced to

$$
\frac{\alpha_s N_c}{2\pi^2} \int \! dz_3 dz_4 \frac{z_2^2}{z_1^2 z_3^2 z_4^2} \left( \ln \frac{z_2^2 z_3^2 z_4^2}{z_1^2 z_2^2 + z_3^2 z_4^2} + 1 \right) \ln \frac{z_2^2 z_3^2}{z_1^2}
$$

$$
\left( \ln \frac{z_3^2 z_4^2}{z_1^2 z_2^2} \right) = \frac{c(n, \nu)}{2\pi^2}
$$

by setting $z_0 = 0$ and making the inversion $x_1 \rightarrow x_1/z^2$. Taking now $z_2 = 0$ we obtain

$$
\frac{\alpha_s N_c}{2\pi^2} \int \! dz' \frac{z_1^2}{z_2^2} \frac{z_2^2}{z_1^2} \left( \ln \frac{z_1^2}{z_2^2} \right) \frac{1}{(z_1 - z_2)^2 + \nu^2} \frac{1}{(z_1 + z_2)^2 + \nu^2}
$$

$$
\times \left( \ln \frac{z_1 - z_2}{z_1 + z_2} \right) = \frac{1}{(z_1 - z_2)^2 + \nu^2}
$$

(24)

Using now the integral

$$
\int \! dz' \frac{(z_1 - z_2)^2 + \nu^2}{(z_1 - z_2)^2 + \nu^2} = \frac{1}{(z_1 - z_2)^2 + \nu^2}
$$

(25)

and the integral $J_{13}$ from Ref. [9]

$$
\int \! dz' \left[ 1 + \frac{z_1^2 z_2^2}{(z_1 - z_2)^2 + \nu^2} \right] \frac{z_1^2 z_2^2}{(z_1 - z_2)^2 + \nu^2} \frac{1}{(z_1 - z_2)^2 + \nu^2} = \Phi(z_1, z)
$$

(26)

(see Eq. (4) for the definition of $\Phi$) we obtain

$$
\frac{\alpha_s N_c}{4\pi^2} \int \! dz \left( \frac{z_1^2 z_2^2}{z_1^2 + \nu^2} \right) e^{-i \omega} \left[ \frac{1}{(z_1 - z_2)^2 + \nu^2} \right]
$$

(27)
where $\phi$ is the angle between $\vec{z}$ and $\vec{z}_1$. The final step is to use integrals
\[
\int \frac{d^2 z}{2\pi} \frac{1}{(z_1 - z)^2} (z^2/z_1^2) \gamma e^{-i n \phi} \ln^2 \frac{z^2}{z_1^2} = \chi''(n, \gamma)
\tag{28}
\]
and
\[
\int \frac{d^2 z}{2\pi} \left( \frac{z^2}{z_1^2} \right)^{c-1} e^{-i n \phi} \Phi(z_1, z) = -\Phi(n, \gamma) - \Phi(n, 1 - \gamma)
\tag{29}
\]
Comparing to Eq. (3) we see that \(c(n, \nu) = \frac{N_c^2}{4\pi^2} \left[ -\chi''(n, \nu) - 2\Phi(n, \frac{1}{2} + i\nu) - 2\Phi(\frac{1}{2} - i\nu) \right]\) which corresponds to \(\omega_{\text{NLO}}\) from Eq. (5).

In conclusion, let us comment on the results in the literature that NLO BFKL in the coordinate space is not conformally invariant [10]. As we mentioned above, the conformal result for the NLO BFKL kernel (22) corresponds to the factorization in rapidity consistent with Möbius invariance. In other words, this kernel should describe the evolution of the color dipole with the conformally invariant rapidity cutoff. At present, there is no obvious way to impose such a cutoff although we believe that it can be done by constructing a “composite operator” for a color dipole, order by order in the perturbation theory. We also think that the Fourier transform of Eq. (18) in the non-forward case should give the precise cutoff for the longitudinal integrations in the momentum space and cure the discrepancy with the results of Ref. [10].

One can also restore the NLO QCD kernel with the same rapidity cutoff implicitly defined above to satisfy the requirement of the conformal invariance of the \(N = 4\) kernel (22). Using the results of [3, 11] one obtains
\[
K_{\text{NLO}}^{\text{QCD}}(z_1, z_2; z_3, z_4) = K_{\text{NLO}}(z_1, z_2; z_3, z_4)
\tag{30}
\]
\[
+ \frac{\alpha_s}{4\pi} \left( b \ln \frac{z_{12}^2 \mu^2}{9 N_c} - \frac{10}{9} n_f \right) K_{\text{LO}}(z_1, z_2; z_3, z_4)
\]
\[
+ \frac{\alpha_s^2 N_c}{8\pi^3} \left[ \delta^2(z_{13})(\frac{1}{z_{14}^2} - \frac{1}{z_{24}^2}) \ln \frac{z_{14}^2}{z_{24}^2} + \delta^2(z_{24})(\frac{1}{z_{14}^2} - \frac{1}{z_{23}^2}) \ln \frac{z_{14}^2}{z_{23}^2} \right]
\]
\[
+ \frac{\alpha_s^2 N_c^2}{8\pi^4 z_{13}^2} \left\{ -3 \frac{z_{12}^2 z_{14}^2}{z_{23}^2 z_{24}^2} \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} + (1 + \frac{n_f}{N_c}) \left[ \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} - \frac{2}{3} \frac{z_{14}^2 z_{24}^2}{z_{23}^2 z_{13}^2} \ln \frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2} \right] \right\}
\]
where \(b = \frac{11}{3} N_c - \frac{2}{3} n_f \) and \(\mu\) is the normalization point in the MS scheme. This kernel has the QCD eigenvalues \(\omega(n, \nu)\) from Ref. [12]. Note that Eq. (30) is different from the NLO BK kernel for the evolution of color dipoles in Ref. [3] since the “rigid cutoff” \(\alpha > \sigma\) adopted in that paper is not conformally invariant.

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