A High Precision Measurement of the Proton Elastic Form Factor Ratio at Low Q^2

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On Behalf of E08-007 Collaboration

- Physics background
- Experiment
- Analysis
- Preliminary results
- Impacts & Future outlook

Proton Form Factors

Electron elastic scattering from proton:

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott} \frac{1}{1 + \tau} \left[G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right]$$

$$(\tau = \frac{Q^2}{4M^2}, \varepsilon = [1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2}]^{-1})$$

Sachs Form Factors:

Electric: $G_E(Q^2)$

Magnetic: $G_M(Q^2)$

$$Q^2=0$$

$$G_{Ep}(0) = 1 \quad G_{Mp}(0) = \mu_p$$

$$G_{En}(0) = 0 \quad G_{Mn}(0) = \mu_n$$

Dipole form

$$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2} \right)^{-2}$$

$$\mu_p \frac{G_E}{G_M} = 1$$

$$J_{hadronic}^\mu = e \bar{N}(p') [\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2)] N(p)$$

$$G_E \equiv F_1 - \tau F_2$$

$$G_M \equiv F_1 + F_2$$

Lowest order perturbation theory in QED
(single photon exchange)

Rosenbluth Separation

- Unpolarized beam on unpolarized target, cross section measurement.

$$\sigma_R \equiv \frac{d\sigma}{d\Omega} \frac{\varepsilon(1+\tau)}{\sigma_{Mott}} = \tau G_M^2(Q^2) + \varepsilon G_E^2(Q^2)$$

Method: Vary ε at fixed Q^2 , fit linearly

Slope $\longrightarrow G_E^2$
 Intercept $\longrightarrow G_M^2$

Difficulties:

- σ is not sensitive to G_E at large Q^2 and to G_M at small Q^2
- Limited by accuracy of cross section measurement at different settings.
- Radiative correction, two-photon exchange.

Recoil Polarization

- Direct measurement of form factor ratios by measuring the ratio of the transferred polarization P_t and P_l .

$$I_0 P_t = -2\sqrt{\tau(1+\tau)} G_E G_M \tan \frac{\theta_e}{2}$$

$$I_0 P_l = \frac{E_e + E_{e'}}{M} \sqrt{\tau(1+\tau)} G_M^2 \tan^2 \frac{\theta_e}{2}$$

$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{(E_e + E_{e'})}{2M} \tan \frac{\theta_e}{2}$$

Advantages:

- Only one measurement is needed for each Q^2 .
- Much better precision than a cross section measurement.
- Two-photon exchange effect small.

Proton Form Factor at Low Q^2

• FFs are measureable physical manifestation of the nucleon constituents and dynamics, but needs to be related to the spatial densities.

• Small $Q^2 \rightarrow$ larger length scale, $0.04 \text{ GeV}^2 \sim 1\text{fm}$

- pQCD not applicable at small Q^2 region, many competing theories:
 - VMD, Lattice, RCQM, Di-Quark, pChQM, CBM/LFCBM...
- 2003 - Fit by Friedrich & Walcher Eur. Phys. J. A17, 607 (2003):
 - Smooth dipole form + "bump & dip"
 - All four FFs exhibit similar structure at small momentum transfer.
 - Proposed interpretation: effect of pion cloud.

$$\tilde{\rho}_{ch}(k) = G_E(Q^2)(1 + \tau)^{\lambda_E}$$
$$\mu\tilde{\rho}_m(k) = G_M(Q^2)(1 + \tau)^{\lambda_M}$$
$$k^2 = \frac{Q^2}{1 + \tau}$$

$$\langle r_{E,M}^2 \rangle = \frac{-6}{G_{E,M}(0)} \left[\frac{d}{dQ^2} G_{E,M}(Q^2) \right]_{Q^2=0} - \frac{3\lambda}{2m^2}$$

High Precision Data at Low Q^2

- Zemach radius and hydrogen hyperfine splitting:

$$E_{hfs} = (1 + \Delta_{QED} + \Delta_{hvp}^p + \Delta_{\mu vp}^p + \Delta_{weak}^p + \Delta_S) E_F^p$$

$$\Delta_S = \Delta_Z + \Delta_R^p + \Delta_{pol} \quad \Delta_Z = -2\alpha Z \frac{m_e m_p}{m_e + m_p} r_Z$$

$$r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{\mu_N}{\mu_p} G_E(Q^2) G_M(Q^2) - 1 \right]$$

- **BLAST** result consistent with 1.
- Substantial deviation is observed in **LEDEX** (Ron. et al.).
- Inconsistent with F&W fit.
- Led to this new dedicated experiment **E08-007**.

- Strange form factor through parity violation:

$$A^{PV} + G_{E,M}^{p,nv} + G_A^{pZ} \text{ (calculated)} \rightarrow G_{E,M}^s$$

- **G. Ron et al** indicated HAPPEX-I shifted by $\sim 0.5\sigma$ towards 0 due to smaller G_E .
- Similar change in form factor in HAPPEX-III kinematics would lead to $\sim 1\sigma$ shift

E08-007

- A **high precision** (<1%) survey of the proton FF ratio by recoil polarization method.
- Data taking from May 15th ~ June 9th 2008.
- 8 Q^2 data points: **0.3 ~ 0.7** (GeV/c)².
- 3 different spectrometer settings for each kinematics for FPP and spin transport systematic check.
- **Successfully completed in Jefferson Lab Hall A.**

Experiment Setup

$E'_e: \sim 1\text{ GeV}$

BigBite

- Δp : 200-900MeV
- $\Delta\Omega$: 96msr
- Scint. + PS + SH

$E_e: 1.192\text{GeV}$
Polarization:
 $\sim 83\%$

e^-

p

LH₂ target

LHRs

- $\Delta p/p_0: \pm 4.5\%$,
- $\Delta\Omega: 6.7\text{msr}$
- Dipole bending angle 45deg
- VDC+FPP
- $P_p: 0.55 \sim 0.93\text{ GeV}/c$

BigBite Shower

- PS + Scint.+ SH
- Detect scattered electrons.
- Only elastic-peak blocks were in the trigger.
- Background minimized with tight elastic cut.

Elastic Events Selection

- HRS acceptance cut:
 - out of plane: ± 60 mr
 - in plane: ± 30 mr
 - momentum: ± 0.04 (dp/p_0)
 - reaction vertex cut
- FPP cuts:
 - scattering angle θ_{fpp} $5^\circ \sim 30^\circ$
 - reaction vertex (carbon door)
 - conetest cut
- Other cuts:
 - Coin. Timing cut
 - Coin. event type (trigger)
 - single track event
 - proton angle vs. momentum (graphical cut)

Focal Plane Polarimeter (FPP)

- Left-right asymmetry gives the vertical component while the up-down asymmetry gives the horizontal component.

- Consists of 4 straw chambers.
- Carbon analyzer: 23, 16, 8, 4, 2 cm thick.
- Need well determined scattering azimuthal angle ϕ_{fpp} .
- Alignment checked with straight through.

Focal Plane Asymmetry

- Detection probability at focal plane with azimuthally angle ϕ_{fpp}

$$f^{\pm} = \frac{1}{2\pi} \xi [1 \pm A_y(\theta_{fpp})(P_x^{fpp} \sin(\phi_{fpp}) - P_y^{fpp} \cos(\phi_{fpp}))]$$

- Helicity difference:

$$f^{diff} = f^{+} - f^{-} \approx \frac{1}{\pi} [A_y(P_x^{fpp} \sin(\phi_{fpp}) - P_y^{fpp} \cos(\phi_{fpp}))] = C \cos(\phi + \delta)$$

$$C = \frac{1}{\pi} A_y \sqrt{(P_x^{fpp})^2 + (P_y^{fpp})^2}$$

$$\tan \delta = \frac{P_y^{fpp}}{P_x^{fpp}}$$

- By simple dipole approximation:

$$R = \mu_p \frac{G_E}{G_M} \approx \sin \chi \frac{P_x^{fpp}}{P_y^{fpp}} \times K$$

(K: kinematic factor)

Maximum likelihood Method

- Dipole approximation too simple: dipole fringe field, quadrupoles, misalignment...
- Full spin precession by COSY:
 - differential algebra-based.
 - defines the geometry and related setup of the transport system.
 - quadrupole alignment study in 2001.

Focal plane

$$\begin{pmatrix} P_x^{fpp} \\ P_y^{fpp} \\ P_z^{fpp} \end{pmatrix} = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix} \begin{pmatrix} P_x^{tg} \\ P_y^{tg} \\ P_z^{tg} \end{pmatrix}$$

Scattering frame

$$S_{ij} = \sum_{k,l,m,n,p} C_{ij}^{klmp} x^k \theta^l y^m \phi^n \delta^p$$

- Extract target polarization components by maximizing the product of all the individual probabilities:

$$L(P_x^{tg}, P_z^{tg}) = \prod_{i=1}^N \left\{ \frac{1}{2\pi} [1 + (c_1 - \varepsilon_i A_y (S_{xx} h P_x^{tg} + S_{xz} h P_z^{tg}) + A_y S_{xy} P_y^{tg}) \cos \phi_i] \right. \\ \left. + (s_1 + \varepsilon_i A_y (S_{yx} h P_x^{tg} + S_{yz} h P_z^{tg}) + A_y S_{yy} P_y^{tg}) \sin \phi_i \right\}$$

$$\frac{\partial \ln L}{\partial P_x^{tg}} = \frac{\partial \ln L}{\partial P_y^{tg}} = \frac{\partial \ln L}{\partial P_z^{tg}} = 0 \quad \longrightarrow \quad P_x^{tg}, P_y^{tg}, P_z^{tg} \quad \longrightarrow \quad \mu_p \frac{G_E}{G_M}$$

Spin Transport in HRS

Systematic Budget

- Spin transport: OPTICS and COSY---major uncertainty
- FPP resolution---moderate
 - angular ~ 1 mr
- False asymmetry correction---small
 - statistical fluctuation
- VDC resolution---negligible
 - angular ~ 1 mr
- Spectrometer angle and beam energy---negligible
- Charge asymmetry---negligible
- Aluminum contamination---negligible

Preliminary results

- Smooth slow falling off. A few percent below typical expectations.

- No obvious indication of "Structure".

- Agreement with independent analysis of **Paolone** at 0.8 GeV^2 .

- No obvious trend to rise quickly to unity at the lowest Q^2 point.

- Disagreement with

- **GEp-I** : reanalysis at 0.5 GeV^2 , 0.8 GeV^2 are underway.
- **LEDEX** : preliminary reanalysis lowers into agreement.
- **BLAST**: origin of difference to be investigated.

Impacts and Future Outlook I

- Global fits of individual form factors by **John Arrington**.
- Old fit (black) : include all previous data (AMT).
- New fit (red) : add new data and remove lowest point of GEp-I, and highest point of LEDEX.
- Preliminary fits suggest lower G_E ($\sim 2\%$).

Impacts and Future Outlook II

• Proton Zemach radius:

$$\Delta_Z = -2\alpha Z \frac{m_e m_p}{m_e + m_p} r_Z$$
$$r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{\mu_N}{\mu_p} G_E(Q^2) G_M(Q^2) - 1 \right]$$

• FFs at Low Q^2 ($<1 \text{ GeV}^2$) accounts for $>70\%$ of r_Z . It would shift in the right direction with smaller G_E .

Form factor	Δ_Z	year
Dipole	-39.29	-
FW	-40.22	2003
Kelly	-40.99	2004
AS	-41.85	2007
AMT	-41.43	2007
New fit	-41.21	2009

Quantity	value (ppm)	uncertainty (ppm)
$(E_{\text{hfs}}(e^-p)/E_F^p) - 1$	1 103.48	0.01
Δ_{QED}	1 136.19	0.00
$\Delta_{\mu\text{vp}}^p + \Delta_{\text{hvp}}^p + \Delta_{\text{weak}}^p$	0.14	
Δ_Z (using [31])	-41.43	0.44
Δ_R^p (using [31])	5.85	0.07
Δ_{pol} (this work, using [31])	1.88	0.64
Total	1102.63	0.78
Deficit	0.85	0.78

Carlson, Nazaryan, and Griffioen, arXiv:0805.2603v1

Impacts and Future Outlook III

- Second half of the experiment (DSA) is scheduled in early 2012

$$A_{\text{phys}} = \frac{v_z \cos \theta' G_M^2 + v_x \sin \theta' \cos \phi' G_E G_M}{(\epsilon G_{Ep}^2 + \tau G_{Mp}^2) / [\epsilon(1 + \tau)]}$$

- Opportunity to see the FFR behavior at even lower Q^2 region.
- Direct comparison with **BLAST**.
- Challenges: Solid polarized proton target & effect of target field to septum magnets.
- Stay tuned ...

E08-007 Collaboration

- ◆ Argonne National lab
- ◆ Jefferson Lab
- ◆ Rutgers University
- ◆ St. Mary's University
- ◆ Tel Aviv University
- ◆ UVa
- ◆ CEN Saclay
- ◆ Christopher Newport University
- ◆ College of William & Mary
- ◆ Duke University
- ◆ Florida International University
- ◆ Institut de Physique Nuclaire d'Orsay
- ◆ Kent State University
- ◆ MIT
- ◆ Norfolk State University
- ◆ Nuclear Research Center Negev
- ◆ Old Dominion University
- ◆ Pacific Northwest National Lab
- ◆ Randolph-Macon College
- ◆ Seoul National University
- ◆ Temple University
- ◆ Universite Blaise Pascal
- ◆ University of Glasgow
- ◆ University of Maryland
- ◆ University of New Hampshire
- ◆ University of Regina
- ◆ University of South Carolina

Thank you!

Back up slides

Analyzing Power Fit

- Fit A_y and fed in to PALM.
- A_y as a function of p_p and θ_{fpp} .

$$A_y = \frac{a \times r}{1 + b \times r^2 + c \times r^4 + d \times r^6}$$

$$r = p \sin \theta_{fpp}$$

$$a(b, c, d) = a_0 + a_1(p - p_0) + a_2(p - p_0)^2 + a_3(p - p_0)^3 + a_4(p - p_0)^4$$

- Global fit over all the kinematics θ_{fpp} $5^\circ \sim 30^\circ$, best fit: $\chi^2/\text{ndf} = 234/215 = 1.09$

COSY Spin Precession Matrix

- Different SP matrix were generated by changing the default settings in COSY:

- dipole radius, drift distances, quadrupoles alignment
- central bending angle: 0.4 deg
- use COSY transport map to reconstruct target variables

- Uncertainties on target variables (OPTICS):

- dp: 0.001
- y_tg: 0.001 m
- ph_tg: 0.5 mr
- th_tg: 1 mr

False Asymmetry Analysis

- FA are a function of many variables, binning on the most sensitive ones:

$$S1 = \frac{\sum_i \sin \phi_i}{\sum_i \sin^2 \phi_i} \longrightarrow S1 = \frac{\sum_i \sin \phi_i \times A_y(\theta)}{\sum_i \sin^2 \phi_i \times A_y(\theta)}$$

- Add more weight to the events at moderate scattering angle (larger A_y).
- How FA affect ratio extraction?

$$L(P_x^{tg}, P_y^{tg}, P_z^{tg}) = \prod_{i=1}^N \frac{1}{2\pi} (1 + \lambda_{0,i} + \lambda_{x,i} hP_x^{tg} + \lambda_{y,i} P_y^{tg} + \lambda_{z,i} hP_z^{tg})$$

$$\lambda_{0,i} = s1 \sin \phi_i + c1 \cos \phi_i + s2 \sin 2\phi_i + c2 \cos 2\phi_i$$

- With infinite statistics $\int \lambda_{x,i} \lambda_{0,i} (1 + \lambda_{0,i} + \lambda_{x,i} hP_x^{tg} + \lambda_{y,i} P_y^{tg} + \lambda_{z,i} hP_z^{tg}) d\phi \xrightarrow{i \rightarrow +\infty} 0$

$$\begin{pmatrix} \sum_i \lambda_{x,i} (1 - \lambda_{0,i}) \\ \sum_i \lambda_{y,i} (1 - \lambda_{0,i}) \\ \sum_i \lambda_{z,i} (1 - \lambda_{0,i}) \end{pmatrix} = \begin{pmatrix} \sum_i \lambda_{x,i} \lambda_{x,i} & \sum_i \lambda_{x,i} \lambda_{y,i} & \sum_i \lambda_{x,i} \lambda_{z,i} \\ \sum_i \lambda_{y,i} \lambda_{x,i} & \sum_i \lambda_{y,i} \lambda_{y,i} & \sum_i \lambda_{y,i} \lambda_{z,i} \\ \sum_i \lambda_{z,i} \lambda_{x,i} & \sum_i \lambda_{z,i} \lambda_{y,i} & \sum_i \lambda_{z,i} \lambda_{z,i} \end{pmatrix} \begin{pmatrix} hP_x^{tg} \\ P_y^{tg} \\ hP_z^{tg} \end{pmatrix}$$