Nucleon Form Factor Measurements and Interpretation

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The data base for the form factors of the nucleon obtained from elastic ep scattering is discussed, as well as some the recent developments in their calculation.

I will first introduce the subject of measurements of the two form factors of the nucleon, G_E and G_M , or alternatively F_1 and F_2 . A short description of the two methods used to obtain them, cross section and double polarization measurements follows, illustrated by displays of the complete data base for proton and neutron. The now well known “discrepancy”, namely the systematic and significant difference between results from cross section-, and from polarization experiments, will be discussed next, together with radiative corrections. Following will be a succinct description of recent progress in calculating the nucleon form factors, and current understanding of the structure of the nucleon, as revealed by elastic scattering experiments. I will conclude with an outline of forthcoming experiments dedicated to the characterization of the nucleon form factors. Recent reviews of the subject can be found in [1–4].

Electromagnetic form factors are observables within the context of the Born approximation. They contain structure information on the many-body system of quarks and gluon which is the nucleon. However, the form factors extracted from experiment are relativistic invariants only to the extent that the probe is the single virtual photon exchanged between electron and nucleon. Higher order terms, regarded as radiative corrections, destroy this invariance. The new data have revealed that traditional radiative corrections were insufficiently accurate and incomplete.

In Born approximation the transition matrix is the product of the electron current, $j_\mu = \langle e' | \gamma_\mu | e \rangle$ and of the nucleon current $J_\mu = \langle p' | \Gamma_\mu | p \rangle$, where Γ_μ , describes the structure of the nucleon, and is of the form $\Gamma_\mu = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2(Q^2)$. F_1 and F_2 are the Dirac (helicity con-

Fig. 1. Complete data base for G_{Ep}/G_D , from Rosenbluth experiments.

Fig. 2. Same as Fig. 1, for $G_{Mp}/\mu_p G_D$.

serving) and Pauli (helicity non-conserving) form factors; they contain the information on the spatial extent of the charge and magnetic distributions inside the nucleon. Of equally common usage are the electric- and magnetic Sachs form factors:

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2) \text{ and } G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

The Sachs form factor appear in the Rosenbluth cross by their square:

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} (G_E^2 + \frac{\tau}{\epsilon} G_M^2),$$

but as $G_E \times G_M$ in double polarization observables; for example, the polarization of the recoiling nucleon emerging from the elastic scattering of longitudinally polarized electrons, has the two in-the-reaction plane components:

$$hP_e P_t = -hP_e 2\sqrt{\tau(1+\tau)} G_E G_M \tan \frac{\theta_e}{2} / I_0 \text{ and}$$

$$hP_e P_\ell = hP_e \frac{(E_e + E_{e'})}{M} \sqrt{\tau(1+\tau)} G_M^2 \tan^2 \frac{\theta_e}{2} / I_0,$$

where h and P_e are the beam helicity and polarization, respectively, and $I_o = G_E^2 + \frac{\epsilon}{\tau} G_M^2$. The ratio of G_E and G_M is then directly obtained from

Fig. 3. $\mu_p G_{Ep}/G_{Mp}$ from recoil polarization experiments.

Fig. 4. $G_{Mp}/\mu_p G_D$ after reanalyze in Ref. [9].

the ratio of the two polarization components:

$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{(E_e + E_{e'})}{2M} \tan\left(\frac{\theta_e}{2}\right).$$

The results from all Rosenbluth measurements published so far, divided by the dipole form factor $G_D = (1 + Q^2/0.71)^{-2}$, are shown in Figs. 1 and 2. The unacceptable scatter of data seen in Fig. 1 existed in 1987 when the first proposal to use recoil polarization as a way to obtain G_{Ep}/G_{Mp} with much greater sensitivity and smaller systematics, was presented [5]. In stark contrast is the internal consistency of the ratio data obtained in the first JLab recoil polarization experiment seen in Fig 3 [6–8]. Assuming for the moment that in fact Rosenbluth data above $Q^2 = 1\text{--}2 \text{ GeV}^2$ should be ignored, the G_{Mp} data become even more internally consistent, as shown in the analysis of Ref. [9] shown in Fig. 4.

So what causes the different results, from cross section and from polarization? Part of the answer is that radiative corrections at large Q^2 are both large and strongly ϵ dependent. This is best demonstrated in Fig. 5, where the Rosenbluth data of Andivahis *et al* [10] are shown as the reduced cross

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section:

$$\sigma_R = \frac{(1 + \tau)}{\tau\epsilon} \left(\frac{d\sigma}{d\Omega} \right)_{ep} / \left(\frac{d\sigma}{d\Omega} \right)_{Mott} = \frac{\epsilon}{\tau} G_{Ep}^2 + G_{Mp}^2,$$

versus ϵ , before radiative correction at the bottom, and after radiative correction at the top. The striking feature seen in Fig. 5 is the change of

Fig. 5. Reduced cross sections without and with radiative corrections [10].

Fig. 6. Several estimates of radiative corrections: [10,14,15,17].

slope resulting from the radiative correction at $Q^2 = 5 \text{ GeV}^2$; the slope is G_{Ep}^2/τ . It is evident that the accuracy of the radiative correction has to be of order 1% at $\epsilon = 1$ for this or larger Q^2 . In Fig. 6 I show the radiative correction applied in the Andivahis paper [10], which is based on the Mo and Tsai [11] approach, as modified by Walker *et al* [12] (solid red line). Also shown is the result of a calculation based on the improved Maximon and Tjon approach [13], as obtained from a code developed by Vanderhaeghen [14] (green dot-dashed line). The difference is about 3% at $\epsilon = 1$, i.e. half of the discrepancy between the Andivahis and the JLab polarization data illustrated with the dashed cyan line. Also shown is the result of a new calculation by Bystritskiy *et al* [15]; even though it does not appear that the same energy cuts have been applied in this calculation as in [10], the point is made that radiative correction calculations are not stable at the required level of a few %.

The JLab results have led to a reexamination of the two-photon contribution, which is included in standard radiative corrections only in the limit of

one of the photon energies being small. Recent calculations of the contribution of two-photon exchange with both photons approximately sharing the momentum transfer, first by Guichon *et al* [16], then Afanasev *et al* [17], and Blunden, Melnitchouk and Tjon [18], have shown that it probably contributes a correction to the standard radiative correction of several % at $\epsilon=1$. As an example the calculation of Afanasev *et al* [17] is shown as the dotted red line in Fig. 6. The final answer to the question of the origin of the discrepancy will have to come from experiments. There are several directly- and indirectly related experiments designed to provide answer to the question. For example:

- 1) ϵ -dependence of the G_E/G_M ratio at constant Q^2 , obtained from recoil

Fig. 7. Data base for G_{E_n} .

Fig. 8. Data base for G_{M_n} .

polarization: E04-019 in Hall C, Fall 2007 [19].

- 2) cross section difference in e^+ and e^- proton scattering; in Hall B [20].
- 3) non-linearity of Rosenbluth plot; E05-017, data taking completed July 2007.

All three determine the real part of the 2-photon amplitude.

- 4) Also, the imaginary part of the two-photon amplitude can be obtained from measurements of the induced, out-of-plane polarization component; such information is likely to be a by-product of E04-019, and single spin target asymmetry on the neutron: E05-015 [23] with a $^3\text{He} \uparrow$ -target.

- 5) Single spin beam asymmetry measurement at Bates [24] and Mainz

Fig. 9. F_2/F_1 and perturbative QCD.

Fig. 10. Logarithmic scaling according to Belitzky et al.

[25], likewise related to the imaginary part of the two-photon amplitude but with asymmetries at the level of parts per million.

The corresponding data for the neutron have improved too, but do not compare with the quality of the proton data; all recent experiments measuring G_{En} have used the double polarization technique; the current data base is shown in Fig. 7. New data up to 3.4 GeV^2 from Hall C are currently being analyzed [26]. The complete data base for G_{Mn} is shown in figure 8; included in this Figure are older results probably affected by systematics (black symbols), as well as the preliminary Hall B data of Brooks et al. [22]. As shown in Fig. 9, the long standing pQCD prediction from Brodsky and Ferrar [27] is not followed by the proton form factors F_1 and F_2 ; the ratio F_2/F_1 exhibits instead a $1/Q$ behavior. In Fig. 10 it is shown that scaling may occur at rather low Q^2 for the ratio $Q^2 F_1 F_2 / \ln^2(Q^2/\Lambda^2)$; Belitsky et al [28] have argued that this behavior would be a signature of the contribution of quark orbital angular momentum.

The earliest attempt at understanding proton form factor data were based on the Vector Dominance Model, with the work of Iachello [29], the Hoehler group [30] and Gari and Krumpelmann [31]. More recent work from the Bonn and COSY group includes [32,33]; independently, Lomon has systematically refined the Gari and Krumpelman approach [34]; the results of these analyses are in Fig. 11. In Fig. 12 a few of the typical relativistic constituent quark model (CQM), calculations are shown. The model was

Fig. 11. VMD models of the form factor ratio; data from Refs. [7,8].

Fig. 12. CQM models of the form factor ratio; data from Refs. [7,8].

Fig. 13. Isovector form factor in lattice QCD—citeedwards

Fig. 14. Error bars of planned measurements of \$G_{Ep}/G_{Mp}\$.

originally proposed by Isgur and Karl [35], and the 3 forms of dynamics possible in the relativistic version of the model, had been already discussed by Dirac [36]: instant, point-like and light-front. An early example of the point-like form is Chung and Coester [37]. Front-form calculations from Miller and collaborators [38]. More recent example of point-like approximation from Cardarelli [39]. Also shown in this figure are the covariant

spectator predictions of Gross and Agbakpe [40], and Gross, Ramalho and Peña [41]; these are multi-parameter fits to the data.

The nucleon form factor can also be calculated from first principle, solving the QCD Lagrangian on the lattice. Predictions are still limited by computer power to pion mass m_π larger than natural. The isovector form factor $F_1^V = F_1^p - F_1^n$ can be obtained without the need to include disconnected $q\bar{q}$ -loops. As seen in Fig. 13, the latest calculations of F_1^V are in fair agreement with experiment [42].

The measurement of nucleon form factors to the highest possible Q^2 is one of the prime tenets of the JLab 12 GeV upgrade. Several experiments will still be done with the 6 GeV beam; in particular E-04-108, which will measure G_{Ep}/G_{Mp} to close to 9 GeV². A letter of intent has been submitted [44], and a proposal approved [45], to pursue these recoil polarization experiments to the region of Q^2 13 to 15 GeV² with an 11 GeV beam; the anticipated statistical error bars of all future G_{Ep}/G_{Mp} measurements are shown in Fig. 14.

To conclude, the experimental characterization of the form factors of the nucleon, and their phenomenological understanding, have changed drastically since 1998, year of the first recoil polarization at JLab. The rapid decrease of G_{Ep} should not have been a complete surprise, as it had been predicted in at least 3 papers [29,38,43], but ignored by experimentalists 10 years ago. A full understanding of the implications of the JLab findings may not be in yet. Where does the pQCD behavior kick in? are the nucleons spherical in their ground state? Are we "really" seeing the effect of quark orbital angular momentum? If the shape of the form factors is not diffractive, what does $GEp=0$ or $GEpj0$ mean? Also to come is a full understanding of two-photon effects, and revision of standard radiative correction calculation codes. Several of the forthcoming experiments will address these questions.

References

1. H.Y. Gao, Int. J. Mod. Phys. E **12**, 1 (2003).
2. C.E. Hyde-Wright and K. de Jager, Ann. Rev. Nucl. Part. Sci **54**, 217 (2004)
3. J. Arrington, C. D. Roberts and J. M. Zanotti, J. Phys. G **34**, S23 (2007)
4. C.F. Perdrisat, V. Punjabi and M. Vanderhaeghen, Journal of Progress in Particle and Nuclear Physics, (2007): <http://sciencedirect.com/science/journal/01466410>, Articles in press.
5. "Electric Form Factor of the Proton bt Recoil Polarization", C.F. Perdrisat and V. Punjabi, E-93-027 (1987).

6. M.K. Jones, *et al*, Phys. Rev. Lett. **84**, 1398 (2000).
7. V. Punjabi *et al*, Phys. Rev.C **71**, 055202 (2005, errat. ibid C **71**, 069902 (2005).
8. O. Gayou *et al*, Phys. Rev. Lett. **88**, 092301 (2001); also O. Gayou, William and Mary PhD thesis unpublished (2002). <http://www.jlab.org/perdrisa/publications/gep99007->.
9. E.J. Brash, A. Kozlov, S. Li and G.M. Huber, Phys. Rev. C **65**, 051001 (2002).
10. L. Andivahis *et al*, Phys. Rev. D **50**, 5491 (1994).
11. L.W. Mo and Y.S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).
12. Walker R.C. *et al*, Phys. Rev. D **49**, 5671 (1994).
13. L.C. Maximon and J.A. Tjon, Phys. Rev. C **62**, 054320 (2000).
14. M. Vanderhaeghen *et al*, Phys. Rev. C **6**, 025501 (2000).
15. Yu. M. Bystritskiy, E.A. Kuraev and E. Tomasi-Gustafsson, Phys. Rev. C **75**, 015207 (2007).
16. P.A.M. Guichon, and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003).
17. A.V. Afanasev, *et al*, Phys. Rev. D **71**, 013008 (2005).
18. P.G. Blunder, W. Melnitchouk and T.A. Tjon, Phys. Rev.Lett. **91**, 142304 (2003), and Phys. Rev. C **72**, 034612 (2005).
19. R. Gilman, L. Pentchev, C.F. Perdrisat and R. Suleiman, Hall C experiment E04-019 (2004).
20. A. Afanasev *et al*, E07-005 (alias E04-116).
21. "A measurement of Two-Photon Exchange in Unpolarized Elastic *ep* Scattering, J. Arrington E-05-017 (2005).
22. W.K. Brooks and J.D. Lachniet, Nucl. Phys. A **755**, 261 (2005).
23. T. Averett *et al*. E-05-015 (2005).
24. S.P. Wells *et al*, Phys. Rev. C **63**, 064001 (2001).
25. F.E. Maas *et al*, Phys. Rev. Lett **94**, 082001 (2005).
26. G.Cates, K. McCormick, B. Reitz and B. Wojtsekhowski, JLab experiment E-02-013 (2002).
27. S. Brodsky and G.R. Farrar, Phys. Rev. D **11**, 1309 (1975).
28. A.V. Belitsky, X. Ji and F. Yuan, Phys. Rev. Lett. **91**, 092003 (2003).
29. F. Iachello, A.D. Jackson and A. Landé, Phys. Lett. B **43**, 191 (1973).
30. G. Hoehler, E. Pietarinen, I. Sabba-Stefanescu, F. Borkowski, G.G. Simon, V.H. Walter and R.W. Wendling, Nucl. Phys. B [bf 114, 505 (1976).
31. M.F. Gari and W. Kruempelmann, Z. Phys. A **322**, 689 (1985) and Phys. Lett. V **274**, 159 (1992).
32. P. Mergell, U.G. Meissner and D. Drechsel, Nucl. Phys. A **596**, 367 (1996).
33. M.A. Belushkin, H.W. Hammer and U.G. Meissner, Phys. Lett. B **633**, 507 (2006).
34. E.L. Lomon, Phys. Rev. C **64**, 035204 (2001), Phys. Rev. C **66**, 045501 (2002) and arXiv:nucl-th/0609020 (2006)..
35. N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978); **19**, 2653 (1979); **20** 1191 (1979); C. Hayne and N. Isgur, *ibid* **25**, 1944 (1982).
36. P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
37. Chung P. L. and F. Coester, Phys. Rev. D **44**, 229 (1991).

38. M.R. Frank, B.K. Jennings, and G.A. Miller, Phys. Rev. C **54**, 920 (1996);
G.A. Miller and M.R. Frank, Phys. Rev. C **65**, 065205 (2002).
39. Cardarelli F. and S. Simula, Phys. Rev. C **62**, 65201 (2000).
40. F. Gross and P. Agbakpe, Phys. Rev. C **73**, 015203 (2006).
41. F. Gross, G. Ramalho and M.T. Peña, arXiv:nucl-th/0606029 (2006).
42. R.G. Edwards *et al*, (LHPC collaboration), arXiv:hep-lat/0610007 (2006).
43. G. Holzwarth, Z. Phys. A **356**, 339 (1996).
44. “ G_{Ep}/G_{Mp} with an 11 GeV beam”, C.F. Perdrisat, V.Punjabi, M.K. Jones
and E.J. Brash. JLab LOI12-06-103 (2006).
45. “Large Acceptance Proton Form Factor Ratio Measurements at 13 and 15
GeV² Using Recoil Polarization”, L. Pentchev, C.F. Perdrisat, E. Cisbani,
V.Punjabi and B. Wojtsekhowski, E12-07-109.