Electron Scattering from the Deuteron Using the Gross Equation

J.W. Van Orden^{1,3}, N. Devine³, F. Gross^{2,3}

¹ Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA

² Department of Physics, College of William and Mary, Williamsburg, Virginia 23185, USA

³ The Continuous Electron Beam Accelerator Facility, 12000 Jefferson Ave., Newport News, Virginia 23606, USA

Abstract. The elastic electromagnetic form factors for the deuteron are calculated in the context of a one-boson-exchange model using the Gross or Spectator equation [1]. The formalism is manifestly covariant and gauge invariant. Results are shown for the impulse approximation and for $\rho\pi\gamma$ exchange currents. The impulse approximation results are quite close to the available data which suggests that only a relatively small exchange current contribution is required. It is shown that by using a soft form factor for the exchange current, the model provides a very good representation of the data.

1 Introduction

With CEBAF now coming on line, it will be routine to probe nuclear systems with electron scattering where the energy and momentum transfers will be well in excess of the nucleon mass. Under such circumstances, the usual nonrelativistic description of the nucleus is no longer reliable. It is, therefore, necessary to develop relativistically covariant models of the nuclear system. In addition, it is important to maintain gauge invariance in such calculations if they are to be reliable. This is a difficult problem and at present this program can be carried out completely in the deuteron. Here, we will present a calculation of the electromagnetic form factors of the deuteron in the context of the Gross equation [1]. This calculation is manifestly Lorentz covariant and has been constructed to be gauge invariant. We will show that by carefully constraining the interaction model to fit the nucleon-nucleon phase shifts and by using only minimal exchange currents we are capable of obtaining an excellent description of the available data.

Figure 1. Feynman diagrams representing the homogeneous integral equation for the Gross wave function.

2 The Model

The Gross equation [1] can best be understood as an example of a quasipotential equation [2, 3]. The common characteristic of all quasipotential equations is the replacement of the free intermediate-state two-nucleon propagator in the Bethe-Salpeter equation by a new propagator that includes a delta-function constraining the relative energy of the intermediate states. This reduces the four-dimensional Bethe-Salpeter equation to a three-dimensional integral equation. In the case of the Gross equation the relative energy constraint is obtained by keeping only the positive energy pole of one of the nucleons in performing energy loop integrals. This is illustrated by Fig. 1 which shows the Feynman diagrams representing the homogeneous Gross wave equation. The ovals represent deuteron vertex functions, the single lines nucleon propagators and the rectangular box the interaction kernel. The crosses on nucleon propagators represent particles placed on their positive energy mass-shells. In this case we have chosen to always place the left-hand nucleon (particle 1) on shell. Note that in order to obtain a consistent integral equation, the external propagator for particle 1 is placed on shell. Note also that in defining the wave function, the external nucleon lines represent nucleon propagators rather than simply spinors. The on-shell nucleon propagator simply becomes a positive energy projection operator. As will be shown below, calculation of current matrix elements requires the deuteron vertex function with both external nucleons off mass shell. This is represented by the Feynman diagrams in Fig. 2. The dots on the vertex function and the kernel represent attachment points for nucleon propagators. Note that on the right hand side of this equation the deuteron vertex has one nucleon on mass shell and this vertex along with the on- and off-shell propagators is identical to the Gross wave function as defined by Fig. 1. Therefore, the off-shell deuteron vertex function can be obtained by quadrature using the

Figure 2. Feynman diagrams representing the deuteron vertex function.

Gross wave function and the kernel with three external momenta off mass shell.

This vertex function for the deuteron can be written as [4]

$$(\Gamma(p, P) \cdot \xi_{\lambda_d}(P) \mathcal{C})_{ab} \quad (1)$$

where P is the four-momentum of the deuteron, p is the relative momentum of the external nucleons, $\xi_{\lambda_d}(P)$ is the polarization four-vector for the deuteron, $\mathcal{C}$ is the Dirac charge conjugation matrix, the subscripts a and b are indices in the Dirac spinor space, and Γ^μ can be determined by basic symmetry arguments to have the general form:

$$\begin{aligned} \Gamma^\mu(p, P) = & f_1(p^2, p \cdot P) \gamma^\mu + g_1(p^2, p \cdot P) \frac{p^\mu}{m} \\ & + \frac{\not{p}_2 - m}{m} \left(f_2(p^2, p \cdot P) \gamma^\mu + g_2(p^2, p \cdot P) \frac{p^\mu}{m} \right) \\ & + \left(f_2(p^2, -p \cdot P) \gamma^\mu + g_2(p^2, -p \cdot P) \frac{p^\mu}{m} \right) \frac{\not{p}_1 + m}{m} \\ & + \frac{\not{p}_2 - m}{m} \left(f_4(p^2, p \cdot P) \gamma^\mu + g_4(p^2, p \cdot P) \frac{p^\mu}{m} \right) \frac{\not{p}_1 + m}{m} \quad (2) \end{aligned}$$

where $p_1 = \frac{P}{2} + p$ and $p_2 = \frac{P}{2} - p$. A generalization of the Pauli symmetry requires that

$$\Gamma^\mu(p, P) = -C \Gamma^{\mu T}(-p, P) C^{-1}. \quad (3)$$

Note that for given values of p and P , the vertex function depends upon eight scalar functions, two pairs of which are related by inversion of p . In the case of the constrained spectator vertex function where particle 1 is on its positive energy mass shell, only four of these functions contribute. Therefore, the general off-shell vertex function can be represented by eight radial functions while the constrained constrained Gross wave functions have only four radial wave functions.

Since the on-shell constraint used in construction the Gross wave function and the fully-off-shell vertex function is not symmetric for the intermediate state nucleons, satisfying the generalized Pauli symmetry given by Eq. (3) requires some care. It has been shown in ref. [5] that the Pauli symmetry can be satisfied by requiring that the kernel of the integral equation be antisymmetrized. This has the unwanted effect of introducing unphysical singularities into the wave and scattering integral equations. But it has been shown that the quantitative effects of this defect are small and that the solution of the integral equations can be achieved at the expense of some technical complexity.

The Gross wave functions are normalized in the deuteron rest frame, where $P = (M_d, 0)$, according to [5]

$$1 = \int \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} \bar{\psi}(p, P) \gamma^0 \psi(p, P) - \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} \bar{\psi} \frac{\partial}{\partial M_d} V(p', p; P) \psi(p, P) \quad (4)$$

where M_d is the deuteron mass, $\psi(p, P)$ is the Gross wave function and $V(p', p; P)$ is the interaction kernel. The derivative term accounts for possible energy dependence of the kernel. Although we have written this normalization expression as it appears in the deuteron rest frame where it is of a relatively simple form, the normalization can be written in a covariant form. Indeed, as a check of some elements of the numerical calculation of the deuteron form factors, we have verified that the covariant normalization condition is satisfied in a variety of Lorentz frames.

Reference [5] describes in considerable detail the application of the spectator equation to nucleon-nucleon scattering and the deuteron bound state. Four models are presented for the NN interaction. Each model has been fitted to the NN phase shift data of Arndt and Roper, SP89 [6] with the aid of the error matrix obtained in the phase shift fit. These fits are constrained such that the deuteron bound state mass is correct. The resulting phase shift calculations are then compared to the data base and typically obtain a χ^2 per datum of approximately 2 for energies from 0 MeV to 225 MeV of laboratory kinetic energy. This is quite good for a one-boson-exchange model.

An unusual feature of these calculations is that they allow for off-shell meson-nucleon couplings not usually present in such models. For example, the basic form of the πNN coupling is chosen to be [4, 5]:

$$g_\pi \left[\lambda_\pi \gamma^5 + (1 - \lambda_\pi) \frac{(\not{p} - \not{p}')}{2m} \gamma^5 \right] \quad (5)$$

where λ_π is a parameter which extrapolates between pseudoscalar and pseudovector coupling. Note that this coupling is independent of λ_π when the nucleons are on mass shell, so models with differing values of λ_π differ in their off-shell content. All meson-nucleon interactions also include factorable form factors.

The meson-nucleon-nucleon form factors depend upon the invariant masses of the three virtual particles connected to the interaction vertex. That is, if p and p' are the initial and final nucleon four-momenta, and $\ell = p - p'$ is the meson four-momentum, the general form of the form factor is $F(p'^2, p^2, \ell^2)$. For simplicity, we assume that the form factor can be written in a factorable form [5, 7]

$$F(p'^2, p^2, \ell^2) = h(p'^2)h(p^2)f(\ell^2) \quad (6)$$

where the meson form factor is taken to be

$$f(\ell^2) = \frac{(\Lambda_\mu^2 - \mu^2)^2 + \Lambda_\mu^4}{(\Lambda_\mu^2 - \ell^2)^2 + \Lambda_\mu^4} \quad (7)$$

and the nucleon form factor is

$$h(p^2) = \frac{2(\Lambda_n^2 - m^2)^2}{(\Lambda_n^2 - p^2)^2 + (\Lambda_n^2 - m^2)^2} \quad (8)$$

where Λ_μ and Λ_n are meson and nucleon form factor masses. The presence of the nucleon form factor is an unusual feature of the calculations presented here. This form factor allows the four-momenta of the nucleons to be controlled such that the contributions from highly-virtual nucleons can be limited.

The interaction model used in the calculations shown here is a variation on model IIB of ref. [5]. The parameters of the model have been adjusted to fit the Nijmegen energy dependent np phase shifts [11]. The results are compared to the data base in SAID [6] to give a χ^2 per datum of 1.89 for energies of 1 to 250 MeV and of 2.53 for 1 to 350 MeV. This model uses a one-boson-exchange kernel containing six mesons: π , η , σ , σ_1 , ω and ρ . The σ_1 meson is a scalar-isovector companion to the σ with a mass comparable to the σ mass. The pion mixing parameter was fixed at $\lambda_\pi = 0$ for pure pseudovector coupling. A total of thirteen parameters were adjusted in the fitting procedure.

The deuteron wave functions for this model are shown in Fig. 3. There are four wave functions, the usual S and D waves that appear in the nonrelativistic treatment of the deuteron and singlet and triplet P waves of relativistic origin. The contributions to the normalization of the wave function from these components are: 92.979% for the S wave, 5.015% for the D wave, 0.049% for the triplet P wave and 0.009% for the singlet P wave. Note that this gives a total of only about 98%. The remaining 2% is associated with the derivative term in (4). This is almost entirely the result of energy dependence introduced into the kernel by the nucleon form factor (8). Note that the signs of the singlet and triplet P waves are opposite for this model.

3 Current Matrix Elements and the Gross Equation

There are two problems associated with the calculation of the electromagnetic current matrix elements using these wave functions. The first of these is common to all Bethe-Salpeter based calculations of the deuteron electromagnetic

Figure 3. Gross deuteron wave functions for Model IIB. The S wave is denoted by u , the D wave by w and the singlet and triplet P waves by v_t and v_s .

properties. This is associated with the use of form factors at the strong and electromagnetic vertices in such models. If the couplings were truly pointlike, the construction of the appropriate one- and two-body electromagnetic current operators would straightforward. The single-nucleon current operator could be taken directly from the effective lagrangian of the model. The two-nucleon current keeping two-body contributions could be calculated from all two-nucleon irreducible diagrams constructed by attaching a virtual photon to all nucleon or meson lines with charges or magnetic moments in the diagrams representing the irreducible Bethe-Salpeter current. These diagrams can be easily shown to satisfy the necessary one- and two-body Ward-Takahashi identities [8, 7].

The complication comes from the introduction of form factors at the strong and electromagnetic vertices to account for the finite sizes of the nucleons and mesons. Simply multiplying the various strong and electromagnetic vertices with form factors will violate the Ward-Takahashi identities. This problem is discussed in detail in ref. [7] for models containing factorable form factors such as is defined by (6). Using the arguments of this reference it is possible to construct a minimal description of the single-nucleon current operator given by [3]:

$$J^{(i)\mu}(p', p) = F_1(Q^2)f_0(p'^2, p^2)\gamma^\mu + \frac{F_2(Q^2)}{2m}h_0(p'^2, p^2)i\sigma^{\mu\nu}q_\nu \\ + F_3(Q^2)g_0(p'^2, p^2)\frac{\not{p}' - m}{2m}\gamma^\mu\frac{\not{p} - m}{2m} \quad (9)$$

where

$$f_0(p'^2, p^2) \equiv \frac{h(p'^2)}{h(p'^2)}\frac{m^2 - p'^2}{p^2 - p'^2} + \frac{h(p'^2)}{h(p^2)}\frac{m^2 - p^2}{p'^2 - p^2}, \quad (10)$$

$$g_0(p'^2, p^2) \equiv \left(\frac{h(p'^2)}{h(p'^2)} - \frac{h(p'^2)}{h(p^2)} \right) \frac{4m^2}{p'^2 - p^2} \quad (11)$$

and $h_0(p'^2, p^2)$ is an arbitrary function subject only to the constraint that $h_0(m^2, m^2) = 1$. In the calculations presented here, this function is chosen to be $h_0(p'^2, p^2) = f_0(p'^2, p^2)$, for simplicity.

For elastic scattering from the deuteron, only isoscalar two-body exchange currents can contribute. The only possible isoscalar contributions for the one-boson-exchange model used here are of the type $\rho\pi\gamma$, $\omega\eta\gamma$, $\omega\sigma\gamma$, etc. These currents have couplings that are individually gauge invariant and therefore require no complicated modification of the off-shell behavior of the vertex functions and form factors in order to maintain gauge invariance.

The second problem is associated with the use of a quasipotential formalism. The quasipotential prescriptions impose constraints on the phase space available to nucleons propagating in intermediate states. In general this constraint must be applied consistently to the calculation of the current matrix elements which will lead to an effective current which differs from the Bethe-Salpeter current operator. The procedure for obtaining the effective current operator for a general quasipotential reduction has not yet been sufficiently studied. However, the correct form of the matrix element for the spectator or

Figure 4. Feynman diagrams representing the Gross current matrix element.

Gross equation has been described in ref. [7], and a description of the matrix element for the Blankenbecler-Sugar equation is described in ref. [9]. In the case of the Gross equation, the correct expression for the current matrix element can be obtained by keeping only the positive energy nucleon poles for particle 1 in the evaluation of the energy loop integrals of the Bethe-Salpeter current matrix element. For the elastic matrix elements this leads to the Feynman diagrams displayed in Fig. 4. Diagram 4a, where the virtual photon is absorbed on particle 2, has particle 1 constrained on shell for both the initial and final state vertex functions and can be written in the form of a matrix element of the single-nucleon current operator between two Gross wave functions. However, if the virtual photon is absorbed on particle 1, the positive energy pole can be picked up for the propagator before the absorption of the virtual photon or the one after. This leads to diagrams 4b and 4c. In these diagrams only the initial or final vertex function is on shell and the other must be off shell. These two diagrams do not have the simple form associated with the nonrelativistic impulse approximation as does diagram 4a. The equation for the off-shell vertex function represented by Fig. 2 can be used to write the matrix elements for these diagrams in terms of the constrained Gross wave function. The resulting diagrams may be viewed as interaction current contributions which are necessary to accommodate the on-shell constraint. It should be noted that only by calculating diagrams 4a, 4b and 4c can the proper normalization of the charge be recovered from the charge form factor in the limit $Q^2 \rightarrow 0$. Diagrams involving two-body Bethe-Salpeter currents will have two internal energy loops which can be constrained independently to give diagram 4d. Note that the meson exchange currents must be symmetrized. Also, it should be noted that the Bethe-Salpeter and Gross two-body meson exchange currents will only be the same at the one-boson-exchange level.

Prior to ref. [7], it was assumed that the proper form of the Gross current matrix element was described by diagram 4a along with a symmetric diagram where the photon attaches to particle 1 and particle 2 is placed on mass shell

[10]. Because of the symmetry of the matrix element, the contribution of the second diagram is equivalent to diagram 4a. Thus this approximation is equivalent to simply calculating $2 \times$ diagram 4a. Since the form of this approximation looks like a matrix element of a single-nucleon current between spectator wave functions, it is referred to as the relativistic impulse approximation (RIA). Since the combination of diagrams 4a, 4b and 4c are related to the Bethe-Salpeter relativistic impulse approximation but represent a complete gauge-invariant description of the Gross one-body current matrix elements we will refer to it here as the complete impulse approximation (CIA).

In calculating the diagrams of Fig. 4 it is possible to proceed in two basically different ways. The scalar coefficient functions for half-off-shell and fully-off-shell vertex functions as represented in Eq. (2) can be determined and the matrix element can be evaluated as a trace in the Dirac matrix space. Alternatively the various elements of the Feynman diagrams can be projected onto positive and negative energy plane wave Dirac spinors and the wave and vertex functions expanded in a partial wave basis in the deuteron rest frames. It is then necessary to boost the wave and vertex functions to the Breit frame where the calculations are performed. The various elements of the CIA have been calculated independently in both ways and are in agreement. The exchange current contributions have only been calculated using the spinor expansion due to their greater complexity.

4 Results

Figure 5 shows the structure functions $A(Q^2)$ and $B(Q^2)$ as calculated with our model for variations on the impulse approximation. The relativistic impulse approximation of Hummel and Tjon [12] is shown for reference and is labelled "Tjon, RIA". This calculation uses the Höhler single nucleon electromagnetic form factors [13] while our calculations use the dipole parameterization of Galster [14]. Three versions of the impulse approximation are calculated using our model. The calculation labelled "RIA" uses the old definition of the RIA with a current matrix element calculated as twice the result of diagram 4a and using only an on-shell form of the single nucleon current operator obtained by setting $f_0 = h_0 = 1$ and $g_0 = 0$ in (9). The curve labelled "RIA, off shell" is the same as the first but with the full form of the off-shell single-nucleon current operator, and the curve labelled "CIA" is the complete impulse approximation corresponding to diagrams 4a, 4b and 4c with the completely off-shell single-nucleon current operator. For $A(Q^2)$, the RIA and the RIA of Hummel and Tjon are in reasonable agreement except at large Q^2 . Use of the off-shell current operator increases the size of $A(Q^2)$ to more closely describe the data above $Q^2 = 1 \text{ GeV}^2$. The CIA decreases the size of $A(Q^2)$ from 1 to 5 GeV^2 moving away from the data but remaining above the RIA.

For $B(Q^2)$, the minimum of the RIA is at larger Q^2 than is the calculation of Hummel and Tjon. This appears to be the result of dynamical differences in the interaction models used. In particular the position of the minimum of

Figure 5. A and B structure functions calculated in the impulse approximation.

$B(Q^2)$ is particularly sensitive to the sign of the singlet P wave v_s as can be seen by simply changing the sign of this wave function component in the calculation of the RIA as is shown in the curve labelled "RIA, $-v_s$ ". The effect of this change is to produce a large downward shift in the position of the minimum. This causes little change in $A(Q^2)$ or T_{20} , however. The use of the off-shell current operator in the RIA moves the minimum to even larger values of Q^2 . The CIA is very close to the off-shell RIA and both are in remarkably close agreement to the data. These curves show that only small contributions from the exchange currents are required to bring the CIA into good agreement with the data.

As mentioned above, the isoscalar exchange currents are all independently gauge invariant and are thus not needed to obtain a gauge invariant calculation of the form factors and structure constant. However, the $\rho\pi\gamma$ exchange current is related to the AAV anomaly [15] and the coupling and size of the contribution to the form factors at $Q^2 = 0$ is reasonably well constrained. The form factor for the $\rho\pi\gamma$ vertex is not known however and is a source of uncertainty in the calculations. The coupling constant for the $\omega\eta\gamma$ can be extracted from existing data but with less accuracy than in the previous case. Again the form factor is not known. Since a σ meson of 500 MeV does not exist except as a possible 2π -exchange enhancement, the existence and role of the $\omega\sigma\gamma$ exchange current is unclear. If we assume that $\omega \rightarrow \gamma + 2\pi$ is related to our $\omega \rightarrow \gamma\sigma$, then coupling constant for the $\omega\sigma\gamma$ can be extracted from existing data. Again the

Figure 6. A and B structure functions with exchange currents.

form factor is not known. The couplings and form factors for the other possible exchange currents can be predicted by quark models, but are not otherwise constrained. The contributions of the exchange currents to the elastic form factors of the deuteron have been calculated by Hummel and Tjon [12] in an approximate fashion using the Blankenbecler-Sugar equation. The form factors for all contributions were taken to be given by the vector dominance model (VMD). It was found that $\rho\pi\gamma$ and $\omega\sigma\gamma$ exchange currents were needed to obtain any agreement with the data and that contributions of the $\omega\eta\gamma$ exchange currents were small. These calculations for $A(Q^2)$ and $B(Q^2)$ are shown in Fig. 6 labelled as “Tjon, RIA+ $\rho\pi\gamma$ + $\omega\sigma\gamma$ ”. The CIA calculation is also shown for reference to the previous figures. Calculations of the contributions of the $\rho\pi\gamma$ exchange current are shown for our model as calculated with the VMD form factors (labelled “CIA+ $\rho\pi\gamma$ (VMD)”), and quark model form factors as calculated by Gross and Ito [16] (labelled “CIA+ $\rho\pi\gamma$ (Gross-Ito)”) and by Mitchell and Tandy [17] (labelled “CIA+ $\rho\pi\gamma$ (Mitchell-Tandy)”). Both of the quark model form factors are softer than the VMD. The $\rho\pi\gamma$ exchange currents tend to increase the size of $A(Q^2)$ and to move the minimum of $B(Q^2)$ to lower Q^2 . In both cases the VMD form factors produce much too large an effect while the softer quark model form factors give smaller effects. Indeed the calculation with the Mitchell-Tandy form factor is remarkably close to the data. Contributions from $\omega\sigma\gamma$ and $\omega\eta\gamma$ have also been calculated in our model but the results are still under examination.

Figure 7. The tensor polarization T_{20} .

Figure 7 shows the quantity

$$T_{20}(Q^2) = \tilde{t}_{20}(Q^2) = -\frac{\sqrt{2}Q^2}{3M_d^2} \frac{G_C G_Q + (Q^2/12M_d^2)G_Q^2}{G_C^2 + (Q^4/18M_d^4)G_Q^2} \quad (12)$$

for the various calculations. This quantity is relatively insensitive to variations in the impulse approximation and all such calculations are consistent with the data. A somewhat greater sensitivity is seen for the exchange current contributions. The quality of the data is not yet sufficient to distinguish among the various models, however.

Figure 8 shows the charge and quadrupole form factors $G_C(Q^2)$ and $G_Q(Q^2)$, comparing the impulse approximation and exchange current calculations of this work and Tjon, et al. to the data. The position of the zero in $G_C(Q^2)$ is quite sensitive to the underlying dynamics of the impulse approximation and to the exchange currents. The complete exchange current calculation of Hummel and Tjon seems to give a zero within the range required by the data, while our exchange current calculation with the Mitchell-Tandy form factor seems give the zero at too small a value of Q^2 . Our calculation seems to be a little too large at smaller Q^2 . As should be expected from the T_{20} calculations, the quadrupole form factor $G_Q(Q^2)$ is less sensitive to the dynamics. Our calculations seem to be slightly too large at larger Q^2 while those of Hummel and Tjon are slightly too small.

Figure 8. The charge and quadrupole form factors G_C and G_Q .

5 Summary

We have constructed a complete, relativistically covariant and gauge invariant model of elastic electron scattering from the deuteron using the Gross equation. The calculation includes the complete impulse approximation and $\rho\pi\gamma$ exchange currents. We find that the structure function $B(Q^2)$ is extremely sensitive to the presence of small P -wave components of the deuteron wave function of relativistic origin. By using a soft $\rho\pi\gamma$ electromagnetic form factor we have been able to obtain an excellent description of the data.

This, however, should be viewed as a preliminary result since we have not yet studied the sensitivity of the model to the choice of single-nucleon electromagnetic form factors and to the arbitrary decision to place $h_0 = f_0$ in the representation of the off-shell single-nucleon current operator. There are also some additional problems with the interaction model IIB. In particular, the deuteron quadrupole moment and the asymptotic D/S ratio are too small for this model while the magnetic moment is too large. We are presently looking for more accurate models. The Gross equation is also being applied to the calculation of the triton binding energy [18] and we expect that this will result in some additional constraints on acceptable interaction models. It is possible that our best interaction models may produce different results for the deuteron form factors.

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