g_A and Octet Axial Charges¹

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Abstract. We review recent progress on lattice calculations of nucleon axial coupling constants, as well as couplings of other octet members. With a combined SU(3) fit to all octet baryons, we find a better determination of $g_A = 1.18(4)_{\text{stat}}(6)_{\text{syst}}$. Our predictions for $g_{\Sigma\Sigma} = 0.450(21)_{\text{stat}}(27)_{\text{syst}}$ and $g_{\Xi\Xi} = -0.277(15)_{\text{stat}}(19)_{\text{syst}}$ are better determined than previous theoretical estimations. Finally, we describe a preliminary first full-QCD calculation of semileptonic decay quantity $(g_1(0)/f_1(0))_{\Sigma \rightarrow n} = -0.348(37)$.

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Introduction. The baryon axial couplings play important roles in hadronic physics and help us understand the structure of hadrons. However, these are long-distance physical quantities, in the regime where perturbative QCD no longer converges. Lattice QCD allows us to calculate these quantities from first principles. By discretizing space and time in the path integral, we can evaluate it numerically with the aid of the the world's large and ever-increasing computer resources.

Hadronic structure has been a major focus of the lattice QCD community. In recent years, lattice calculations have made remarkable progress in computing hadron properties from first principles. The nucleon axial coupling g_A has been computed[1, 2, 3, 4, 5] by various groups with improved precision and lighter quarks. There have also been studies of the structure of other octet baryons that are experimentally not so well known[2, 6]. We hope to gain insight into the axial charges $g_{\Sigma\Sigma}$ and $g_{\Xi\Xi}$ to compensate for shortcomings in measurements and provide better theoretical estimations.

The various systematic errors that enter a lattice calculation can be controlled, giving us the ability to compute the predictions of QCD precisely. Since the real world is effectively continuous and infinitely large, we will have to take the limits $a \rightarrow 0$ and $V \rightarrow \infty$ to eliminate the artifacts introduced in a discretized finite box. With the most state-of-the-art supercomputer, we are close but yet to simulate at the physical pion mass. Using calculations at multiple heavier pion masses, which are affordable for available computer resources, we can apply chiral perturbation theory to extrapolate quantities of interest to the physical limit. A recent work by the BMW collaboration[7] calculating multiple lattice spacings, volumes and pion masses as light as 180 MeV provided an excellent demonstration of how ground-state hadron masses with fully understood and controlled systematics are consistent with experiment. Calculations with multiple pion

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FIGURE 1. (left) The Axial charge versus pseudoscalar mass squared from various lattice groups[3, 1, 8, 5, 4]. (right) Comparison of nucleon axial charges with experimental measurements. The solid lines indicate statistical error while the dashed lines are systematic error (0f flavor calculations should add an extra 10–30% systematic for “quenching” effect). An SU(3) constrained fit gives Lin et al.[2] smaller statistical error in extrapolation than LHPC, which uses similar calculations.

masses also help determine the low-energy constants of chiral effective theory.

To calculate the octet baryon structure, one usually uses a simple octet baryon interpolating field $\chi_B = \sum_{\vec{x}, a, b, c} e^{i\vec{p} \cdot \vec{x}} \epsilon^{abc} [(q_a^1)^T C \gamma_5 q_b^2] q_c^3$. The baryon three-point (structure matrix element) Green functions are obtained from

$$\Gamma^{(3)}(t_{\text{src}}, t, t_{\text{snk}}) = \langle \chi_B(t_{\text{snk}}, \vec{p}_{\text{snk}}) \mathcal{A}(t, \vec{q}) \chi_B^\dagger(t_{\text{src}}, \vec{p}_{\text{src}}) \rangle, \quad (1)$$

where $\mathcal{A}$ is the axial-vector current $\bar{\psi} \gamma_\mu \gamma_5 \psi$. In this proceeding, we calculate isovector quantities, in which disconnected diagrams cancel due to isospin. Such diagrams in which the inserted quark current is not contracted with the valence quarks in the baryon interpolating fields are very difficult to estimate in lattice QCD.

Nucleon Axial Charge g_A . The axial charge of the nucleon is a fundamental hadronic observable, and it is well measured from neutron β -decay experiments. Therefore, it is a natural candidate for demonstrating how well lattice QCD can be extrapolated to the physical-pion point. The isovector axial charge g_A is defined as the isovector axial-vector form factor at zero four-momentum transfer, $g_A = G_A^+(0)$, where G_A^+ is the axial-vector form factor given by the nucleon matrix element of the isovector axial-vector current.

Many groups around the world have calculated axial charges using various gauge ensembles and fermion actions. Results from various collaborations are shown on left-hand side of Figure 1. We see that RBC’s 2f[3] and 0f[8] results are consistent with QCDSF’s 2f clover[4]. The 2+1-flavor (DWF sea and valence quarks) RBC/UKQCD data[1] (blue filled circles) have box sizes 2.7 and 1.8 fm and pion masses 330–670 MeV. The results are consistent with the LHPC data[5] (DWF valence with staggered sea) with box sizes of 2.5 and 2.7 fm and 350–870 MeV pions, except for the lightest point. Further investigations of the source-sink separation should be carried out to resolve this discrepancy. More 2+1-flavor results will become available during this summer’s lattice conference. A summary of g_A after the chiral extrapolation to the physical pion point is shown on the right-hand-side of Figure 1.

Hyperon Axial Charges. The hyperon axial couplings are important parameters entering the low-energy effective field theory description of the octet baryons. At the leading

FIGURE 2. Left panel: (Top) The SU(3) symmetry breaking measure $\delta_{\text{SU}(3)}$. The circles are the measured values at each pion mass, the square is the extrapolated value at the physical point, and the shaded region is the quadratic extrapolation and its error band. (Bottom) $\delta_{\text{SU}(3)}/x^2$ plot. Symbols as above, but the band is a constant fit. Right panel: Lattice data (circles) for g_A , $g_{\Sigma\Sigma}$ and $g_{\Xi\Xi}$ and chiral extrapolation (lines and bands). The square is the extrapolated value at the physical point.

order of SU(3) heavy-baryon chiral perturbation theory, these coupling constants are linear combinations of the universal coupling constants D and F , which enter the chiral expansion of every baryonic quantity, including masses and scattering lengths. These coupling constants are needed in the effective field theory description of both the non-leptonic decays of hyperons, and the hyperon-nucleon and hyperon-hyperon scattering phase shifts[9]. Hyperon-nucleon and hyperon-hyperon interactions are essential in understanding the physics of neutron stars, where hyperon and kaon production may soften the equation of state of dense hadronic matter.

We have calculated the axial coupling constants for Σ and Ξ strange baryons using lattice QCD for the first time[2]. We have done the calculation using 2+1-flavor staggered dynamical configurations with pion masses as light as 350 MeV. We first study the SU(3) symmetry breaking of the axial couplings through the quantity $\delta_{\text{SU}(3)}$,

$$\delta_{\text{SU}(3)} = g_A - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi} = \sum_n c_n x^n, \quad (2)$$

where x is $(M_K^2 - M_\pi^2)/(4\pi f_\pi^2)$. The left-hand side of Figure 2 shows $\delta_{\text{SU}(3)}$ as a function of x . Note that the value increases monotonically as we go to lighter pion masses. Our lattice data suggest that a $\delta_{\text{SU}(3)} \sim x^2$ dependence is strongly preferred, as the plot of $\delta_{\text{SU}(3)}/x^2$ versus x in the left-hand side of Figure 2 also demonstrates. A quadratic extrapolation to the physical point gives 0.227(38), telling us that SU(3) breaking is roughly 20% at the physical point, where $x = 0.332$ using the PDG values[10]. We compare the result of heavy-baryon SU(3) chiral perturbation theory[11] for $\delta_{\text{SU}(3)}$ and find that the coefficient of the term linear in x in Eq. 2 does not vanish, implying that an accidental cancellation of the low-energy constants is responsible for this behavior.

The right-hand side of Figure 2 shows our lattice data as a function of $(M_\pi/f_\pi)^2$ with the corresponding chiral extrapolation; the band shows the jackknife uncer-

tainty. We conclude that $g_A = 1.18(4)_{\text{stat}}(6)_{\text{syst}}$, $g_{\Sigma\Sigma} = 0.450(21)_{\text{stat}}(27)_{\text{syst}}$ and $g_{\Xi\Xi} = -0.277(15)_{\text{stat}}(19)_{\text{syst}}$. In addition, the SU(3) axial coupling constants are estimated to be $D = 0.715(6)_{\text{stat}}(29)_{\text{syst}}$ and $F = 0.453(5)_{\text{stat}}(19)_{\text{syst}}$. The axial charge couplings of Σ and Ξ baryons are predicted with significantly smaller errors than estimated in the past. By using the hyperon axial charges (which have smaller statistical errors) for simultaneous chiral extrapolation, the chiral extrapolation is better constrained, and thus we obtain a better determined nucleon axial charge than LHPC, who had similar statistics and numerical setup. Using combined fits across the octet in this way seems to be a promising direction for reducing statistical error.

Semileptonic $\Delta S \neq 0$ Axial Charges. The hyperons differ from the nucleons by their strangeness, and although hyperon decays via the weak interaction have been known for more than half a century, interest in their study has not decayed with time. They provide an ideal systems in which to study SU(3) flavor symmetry breaking and offer unique opportunities to understand baryon structure and decay mechanisms. So far, there are only two “quenched” (where the fermion masses in the sea sector are infinitely heavy) lattice calculations of hyperon beta decay, and they are in different channels, $\Sigma \rightarrow n$ and $\Xi^0 \rightarrow \Sigma^+$. Guadagnoli et al.[12] calculated the matrix element $\Sigma \rightarrow n$ with all of the pion masses larger than 700 MeV. Sasaki et al.[13] used lighter pion masses in the range 530–650 MeV and DWF to look at the Ξ^0 decay channel. In this work, we report a preliminary analysis[14] of the $\Sigma \rightarrow n$ channel and remove the quenched approximation, which often causes notoriously large systematic error. Our fermion sea sector contains degenerate up and down quarks plus the strange. We use pion masses as light as 350 MeV, which ameliorates some of the uncertainty in the extrapolation to the physical pion mass.

Experimentally, only a combination of axial form factors $|g_1(0)/f_1(0) - 0.133g_2(0)/f_1(0)|$ is determined, and $g_2(0)/f_1(0)$ often assumed to be zero when extracting g_1/f_1 at zero momentum transfer. We adopt naive linear mass combinations ($M_K^2 + M_\pi^2$) and ($M_K^2 - M_\pi^2$) and momentum dependence to extrapolate and find $g_1(0)/f_1(0) = -0.348(37)$. The SU(3)-vanishing weak-electric form factor ratio $g_2(0)/f_1(0)$ is 0.29(20), which is 1.5 standard deviations from zero.

REFERENCES

1. T. Yamazaki, et al., *Phys. Rev. Lett.* **100**, 171602 (2008), 0801.4016.
2. H.-W. Lin, and K. Orginos, *Phys. Rev.* **D79**, 034507 (2009), 0712.1214.
3. H.-W. Lin, T. Blum, S. Ohta, S. Sasaki, and T. Yamazaki, *Phys. Rev.* **D78**, 014505 (2008), 0802.0863.
4. A. A. Khan, et al., *Phys. Rev.* **D74**, 094508 (2006), hep-lat/0603028.
5. R. G. Edwards, et al., *Phys. Rev. Lett.* **96**, 052001 (2006), hep-lat/0510062.
6. H.-W. Lin, and K. Orginos, *Phys. Rev.* **D79**, 074507 (2009), 0812.4456.
7. S. Durr, et al., *Science* **322**, 1224–1227 (2008), 0906.3599.
8. S. Sasaki, K. Orginos, S. Ohta, and T. Blum, *Phys. Rev.* **D68**, 054509 (2003), hep-lat/0306007.
9. S. R. Beane, et al., *Nucl. Phys.* **A747**, 55–74 (2005), nucl-th/0311027.
10. W. M. Yao, et al., *J. Phys.* **G33**, 1–1232 (2006).
11. W. Detmold, and C. J. D. Lin, *Phys. Rev.* **D71**, 054510 (2005), hep-lat/0501007.
12. D. Guadagnoli, V. Lubicz, M. Papinutto, and S. Simula, *Nucl. Phys.* **B761**, 63–91 (2007), hep-ph/0606181.
13. S. Sasaki, and T. Yamazaki (2008), 0811.1406.
14. H.-W. Lin, *Nucl. Phys. Proc. Suppl.* **187**, 200–207 (2009), 0812.0411.