Lattice Results on Nucleon Properties

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Lattice QCD

- ◆ QCD observables are calculated from the path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int [dA][d\bar{\psi}][d\psi] O(\bar{\psi}, \psi, A) e^{i \int d^4x \mathcal{L}^{\text{QCD}}(\bar{\psi}, \psi, A)}$$

- ◆ Strong-coupling regions: expansions no longer converge
- ◆ Lattice QCD is a discrete version of continuum QCD theory

- ◆ Numerical integration to calculate the path integral
- ◆ Take $a \rightarrow 0$ and $V \rightarrow \infty$ in the continuum limit

Lattice Actions

◆ Symanzik Improvement

- ◆ Order-by-order in a improvement of the action and operators
- ◆ Systematic error due to discretization under control

◆ Gauge Actions

- ◆ Most gauge actions used today are $O(a^2)$ improved
- ◆ Small discretization effects ($\sim O(\Lambda_{\text{QCD}}^3 a^3)$) due to gauge choices

◆ Fermion Actions

- ◆ Most fermion actions are only $O(a)$ improved ($O(\Lambda_{\text{QCD}}^2 a^2)$)
- ◆ Differences are benign once all systematics are included
- ◆ Different choices of fermion action are confined by limits of computational and human power and by personal interest
- ◆ Commonly used actions:
 - domain-wall fermions, overlap fermions,
 - Wilson/clover fermions, twisted-Wilson fermions

Operators

- ◆ Conventional choices, if known quantum number
- ◆ For example, simple hadrons:

$$\pi^+(x) = \bar{u}^a(X) \gamma_5 d^a(x)$$

$$\rho_\mu^+(x) = \bar{u}^a(X) \gamma_\mu d^a(x)$$

$$p_\delta(x) = (u^a(X) C \gamma_\mu d^b(x)) u_\delta^c \epsilon_{abc}$$

$$\Delta_{\mu,\delta}^{++}(x) = (u^a(X) C \gamma_\mu u^b(x)) u_\delta^c \epsilon_{abc}$$

- ◆ Since these are the lightest particles in their respective channels, their masses are easy to determine using lattice QCD.
- ◆ Many physical quantities are calculated using these simple operators: masses, form factors, axial coupling constants, scattering lengths...

Spectroscopy

- ◆ Calculate the two-point Green function

$$\begin{aligned}\langle O \rangle &= \frac{1}{Z} \int [dU][d\psi][d\bar{\psi}] e^{-S_F(U, \psi, \bar{\psi}) - S_G(U)} O(U, \psi, \bar{\psi}) \\ &= \frac{1}{Z} \int [dU] \det M e^{-S_G(U)} O(U)\end{aligned}$$

- ◆ Spin projection

$$\sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J(X_{\text{snk}}) J(X_{\text{src}}) \rangle_{\alpha, \beta}$$

- ◆ Momentum projection

Two-point correlator

$$\Gamma_{AB}^{(2),T}(t; \vec{p}) = \sum_n \frac{E_n + M_n}{2E_n} Z_{n,A} Z_{n,B} \boxed{e^{-E_n(\vec{P})t}} \text{Exp decay}$$

At large enough t , the ground-state signal dominates.

Nucleon Mass

◆ Updates from the Lattice 2008 conference

statistical errorbars only (see [C. Jasen, LAT2008] for a full report)

Agreement among
many actions

Universality!

Lattice in the News (in *Science*)!

◆ Post-dictions of well known quantities

Example: BMW Collaboration, LAT2008

- ◆ Proves all the systematics are under control
- ◆ Next look at experimentally less-known quantities: excited-baryon spectrum, hybrid mesons, etc.

Roper Resonance on the Lattice

◆ Mostly done in “quenched” approx. N , P_{11} , S_{11} spectrum

Group	N_f	S_f	a_t^{-1} (GeV)	M_π (GeV)	L (fm)	Method	Extrapolation
Basak et al. [12]	0	Wilson	6.05	0.49	2.35	VM	N/A
Burch et al. [11]	0	CIDO	1.68, 1.35	0.35–1.1	2.4	VM	$a + bm_\pi^2$
Sasaki et al. [9]	0	Wilson	2.1	0.61–1.22	1.5, 3.0	MEM	$\sqrt{a + bm_\pi^2}$
Guadagnoli et al. [7]	0	Clover [13]	2.55	0.51–1.08	1.85	SBBM	$a + bm_\pi^2 + cm_\pi^4$
Leinweber et al. [8]	0	FLIC	1.6	0.50–0.91	2.0	VM	N/A
Mathur et al. [6]	0	Overlap [14]	1.0	0.18–0.87	2.4, 3.2	CCF	$a + bm_\pi + cm_\pi^2$

Roper Resonance on the Lattice

- ◆ Mostly done in “quenched” approx. N , P_{11} , S_{11} spectrum

◆ Questions

- ◆ Differences in analysis approach? (how does it affect the extracted masses?)
- ◆ Contamination by 2nd and higher excited states?
- ◆ How is the chiral extrapolation is performed?
- ◆ Other systematics? Finite-volume effect, sea N_f , lattice spacing, etc.

Finite-Volume Dependence

- ◆ Original summary plot in [Lasscock et. al. Phys.Rev.D76:054510 \(2007\)](#)

Finite-Volume Dependence

- ◆ Size matters! More so for excited states.

N_f Dependence

- ◆ $N_f = 0$, 2+1 isotropic clover action calculation ($L \sim 2.56, 1.93$ fm)

[K.F. Liu et. al., LAT2008] & in this conference

- ◆ Stay tuned for more 2+1-flavor calculations

Challenges

More on nucleon resonance?

◆ Operators:

- ◆ Allow us to pick out particular quantum numbers
- ◆ How do we create operators that
maximize overlap with the low-lying states of interest and
minimize mixing with other operators due to lattice artifacts?

◆ Euclidean space:

- ◆ Correlators with time-dependent form

dominated by ground state; challenge for excited states

$$\sum_n Z_{n,B} e^{-E_n(\vec{P})t}$$

Challenges

More on nucleon resonance?

◆ Operators:

- ◆ Allow us to pick out particular quantum numbers
- ◆ Good basis of operators

◆ Euclidean space:

- ◆ Correlators with time-dependent form $\sum_n Z_{n,E} e^{-E_n(\vec{P})t}$
dominated by ground state; challenge for excited states

Solution 2: increase resolution

Hadron Spectrum Collaboration (HSC)

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Dynamical Anisotropic Lattices

- ◆ $N_f = 2+1$ **anisotropic clover action** available ensemble list

R. Edwards et al., Phys. Rev. D78, 014505 (2008)

HWL et al., Phys. Rev. D78, 114508 (2008)

L_x	L_t	m_l	m_s	L (fm)	m_π L	m_π (MeV)
12	96	-0.0540	-0.0540	1.44	11.6	~ 1600
12	96	-0.0699	-0.0540	1.44	8.3	
12	96	-0.0794	-0.0540	1.44	5.9	
12	96	-0.0826	-0.0540	1.44	9.6	
16	96	-0.0826	-0.0540	1.92	6.3	~ 660
12	96	-0.0618	-0.0618	1.44	9.7	~ 1340
16	128	-0.0743	-0.0743	1.92	8.1	~ 850
16	128	-0.0808	-0.0743	1.92	5.6	
16	128	-0.0830	-0.0743	1.92	4.5	
16	128	-0.0840	-0.0743	1.92	3.8	
24	128	-0.0840	-0.0743	2.88	5.7	~ 360

- ◆ $24^3 \times 128$, $M_\pi \sim 220$ MeV in progress; LLNL on $32^3 \times 128$

Dynamical Anisotropic Lattices

- ◆ Preliminary octet masses extrapolation using modified NLO HB χ PT

- ◆ Dimensionless quantities to avoid lattice-spacing ambiguities

- ◆ Mass ratios with $\{l, s\}_{\Xi}$ coordinates

$$l_{\Xi} = \frac{9m_{\pi}^2}{4m_{\Xi}^2},$$

$$s_{\Xi} = \frac{9(2m_K^2 - m_{\pi}^2)}{4m_{\Xi}^2}$$

HWL et al., Phys. Rev. D79, 034502 (2009)

HWL et al., in preparation

Operator Design

- ◆ All baryon spin states wanted: $J = 1/2, 3/2, 5/2, \dots$
- ◆ Rotation symmetry is reduced due to discretization

rotation $SO(3) \Rightarrow$ octahedral O_h group

	I	P	6 C_4	8 C_6	8 C_3	6 C_8	6 C'_8	12 C_2
A_1	1	1	1	1	1	1	1	1
A_2	1	1	1	1	1	-1	-1	-1
E	2	2	2	-1	-1	0	0	0
G_1	2	-2	0	1	-1	$\sqrt{2}$	$-\sqrt{2}$	0
G_2	2	-2	0	1	-1	$-\sqrt{2}$	$\sqrt{2}$	0
T_1	3	3	-1	0	0	1	1	-1
T_2	3	3	-1	0	0	-1	-1	1
H	4	-4	0	-1	1	0	0	0

Baryons

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$
$\frac{9}{2}$	$G_1 \oplus 2 H$
$\frac{11}{2}$	$G_1 \oplus G_2 \oplus 2 H$
$\frac{13}{2}$	$G_1 \oplus 2 G_2 \oplus 2 H$
$\frac{15}{2}$	$G_1 \oplus G_2 \oplus 3 H$
$\frac{17}{2}$	$2 G_1 \oplus G_2 \oplus 3 H$
$\frac{19}{2}$	$2 G_1 \oplus 2 G_2 \oplus 3 H$
$\frac{21}{2}$	$G_1 \oplus 2 G_2 \oplus 4 H$
$\frac{23}{2}$	$2 G_1 \oplus 2 G_2 \oplus 4 H$

Operator Design

◆ Baryon field $\Phi_{\alpha\beta\gamma,ijk}^{ABC}(x) = \epsilon_{abc}[\tilde{D}_i^{(3)}\tilde{\psi}]_{Aa\alpha}(x)[\tilde{D}_j^{(3)}\tilde{\psi}]_{Bb\beta}(x)[\tilde{D}_k^{(3)}\tilde{\psi}]_{Cc\gamma}(x)$

- ◆ Classify states according to symmetry properties
- ◆ Projection onto irreducible representations of finite groups
- ◆ Number of operators:

N^+ Operator type	G_{1g}	H_g	G_{2g}
Single-Site	3	1	0
Singly-Displaced	24	32	8
Doubly-Displaced-I	24	32	8
Doubly-Displaced-L	64	128	64
Triply-Displaced-T	64	128	64
Total	179	321	144

S. Basak et al., Phys. Rev. D72, 094506 (2005)

Variational Method

- ◆ Calculate a correlator matrix from Green functions

$$C_{\Lambda}^{m,n}(t) = \sum_{\vec{x}} \sum_{\lambda} \langle 0 | B_{\lambda}^{\Lambda,m}(\vec{x}, t) \bar{B}_{\lambda}^{\Lambda,n}(0) | 0 \rangle$$

- ◆ Construct the matrix

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)^{\dagger} \mathcal{O}_j(0) | 0 \rangle$$

- ◆ Solve for the generalized eigensystem of

$$C(t)\psi = \lambda(t, t_0)C(t_0)\psi$$

with eigenvalues

$$\lambda_n(t, t_0) = e^{-(t-t_0)E_n} (1 + \mathcal{O}(e^{-|\delta E|(t-t_0)}))$$

C. Michael, Nucl. Phys. B 259, 58 (1985)

M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

- ◆ At large t , the signal of the desired state dominates.

$N_f = 2+1$ Study: Hyperons

- ◆ From isotropic $N_f = 2+1$
- ◆ Mixed action:
 - DWF on staggered sea
- ◆ $M_\pi \sim 300\text{--}750$ MeV
 $L \sim 2.5$ fm
- ◆ Naïve chiral extrapolation
HWL et al., in preparation

- ◆ From the pattern:
spin identification

- ◆ $\Xi(1690)$: $-1/2$
- ◆ $\Omega(2250)$: $-3/2$
- ◆ $\Omega(2380)$: $-1/2$
- ◆ $\Omega(2470)$: $1/2$

- ◆ Highly excited states can be extracted from
future dynamical anisotropic lattices

$N_f = 2$ Study: Nucleon

- ◆ $N_f = 2$, anisotropic Wilson action, *HSC, Phys.Rev.D79:034505 (2009)*
 $V = 24^3 \times 64$, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $M_\pi \sim 572$ MeV

$N_f = 2$ Study: Nucleon

- ◆ $N_f = 2$, anisotropic Wilson action, *HSC, Phys.Rev.D79:034505 (2009)*
 $V = 24^3 \times 64$, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $M_\pi \sim 400$ MeV

Jefferson Lab
Exploring the Nature of Matter

- Exp (GeV)

$N_f = 2+1$ Study: Nucleon

- ◆ $N_f = 2+1$, anisotropic clover action,
 $V = 20^3, 16^3 \times 128$, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3.5$, $M_\pi \sim 366, 580$ MeV

P_{11} Form Factors

In collaboration with

Saul Cohen, Robert Edwards, and David Richards (JLab)

7n cluster @ JLab

Three-Point Green Functions

- ◆ Three-point function with connected piece only

$$C_{3\text{pt}}^{\Gamma, \mathcal{O}}(\vec{p}, t, \tau) = \sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_{\beta}(\vec{p}, t) \mathcal{O}(\tau) \bar{J}_{\alpha}(\vec{p}, 0) \rangle$$

$\mathcal{O}$: $V_{\mu} = \bar{q}\gamma_{\mu}q$, $A_{\mu} = \bar{q}\gamma_{\mu}\gamma_5q$, or others

$$J = \epsilon^{abc} [q_1^{aT}(x) C \gamma_5 q_2^b(x)] q_1^c(x)$$

- ◆ Two topologies:

- ◆ Isovector quantities O^{u-d} : disconnected diagram cancelled

Form Factors

- ◆ The form factors are buried in the amplitudes

$$\begin{aligned} \Gamma_{\mu,AB}^{(3),T}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) \\ = a^3 \sum_n \sum_{n'} \frac{1}{Z_j} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{4E'_n(\vec{p}_f) E_n(\vec{p}_i)} e^{-(t_f-t)E'_n(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)} \\ \times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f, s')_{\beta} \langle N_{n'}(\vec{p}_f, s') | j_{\mu}(0) | N_n(\vec{p}_i, s) \rangle \bar{u}_n(\vec{p}_i, s)_{\alpha} \end{aligned}$$

- ◆ Nucleon form factor ($n = n' = 0$)

$$\langle N | V_{\mu} | N \rangle(q) = \bar{u}_N(p') \left[\gamma_{\mu} F_1(q^2) + \sigma_{\mu\nu} q_{\nu} \frac{F_2(q^2)}{2m} \right] u_N(p) e^{-iq \cdot x}$$

- ◆ Nucleon-Roper form factor ($n = 0, n' = 1$ or $n = 1, n' = 0$)

$$\langle N_2 | V_{\mu} | N_1 \rangle_{\mu}(q) = \bar{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_{\mu} - \frac{q_{\mu}}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_{\nu} \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq \cdot x}$$

- ◆ Need best possible input from two-point correlators

Nucleon Form Factors

◆ $N_f = 0$ anisotropic lattices, $M_\pi \sim 480, 720, 1080$ MeV

Nucleon Form Factors

◆ $N_f = 2+1$ anisotropic lattices, $M_\pi \sim 580, 875$ MeV

Nucleon-Roper Form Factors

- Completed exploratory study on quenched lattices

Proton- P_{11}

720-MeV Pion

Neutron- P_{11}

- Possible decaying state (circled above) [HWL et. al, Phys.Rev.D78:114508 \(2008\)](#)
- 200 configurations give us reasonable signal
- Lower pion mass will shift the time-like region to space-like region

Nucleon-Roper Form Factors

- ◆ $N_f = 2+1$ anisotropic lattices, $M_\pi \sim 580, 875$ MeV

Very Preliminary

- ◆ Need higher precision!

Precision Baryon Calculation

- ◆ Recent demonstration by NPLQCD [S. Beane et. al., 0903.2990\[hep-lat\]](#)
on 366 MeV 2+1f HSC anisotropic clover lattices
- ◆ < 0.2% accuracy

M_π	=	390.3(0.7)(0.3)(2.5) MeV,	M_K	=	546.0(0.6)(0.2)(3.6) MeV,
M_N	=	1163.9(1.8)(0.6)(7.6) MeV,	M_Λ	=	1252.4(1.6)(0.3)(8.2) MeV,
M_Σ	=	1283.7(1.6)(1.0)(8.4) MeV,	M_Ξ	=	1356.1(1.4)(0.2)(8.8) MeV,
$E_{N(1/2^-)}$	=	1610(06)(11)(11) MeV,	$E_{\Lambda(1/2^-)}$	=	1679(05)(02)(11) MeV,
$E_{\Sigma(1/2^-)}$	=	1727(06)(06)(11) MeV,	$E_{\Xi(1/2^-)}$	=	1825(6)(5)(12) MeV,

- ◆ Brute force! Used 292,500 sets of measurements on 1194 gauge configurations and a full year of a collaboration's resources for one pion mass, lattice spacing and volume.
- ◆ Can we find a smarter way of doing the calculation?

A New Technique

◆ Quark Distillation

M. Peardon et. al. (HSC), in preparation

A new technique to improve the contraction time and signal

◆ First peek at baryon signals:

- ◆ Noise-to-signal scales like N

NPLQCD,0903.2990[hep-lat]

Little Groups

- ◆ Continuum symmetries
SO(3) rotational symmetry
- ◆ Any non-zero momentum breaks the symmetry
Little group for any $\vec{p} \neq 0$: SO(2)
- ◆ Each spin irrep breaks up into its singlet $I_{\hat{p}}$ projections
Spin-1 ($J, J_{\hat{p}}$) = $\{(1,-1), (1,0), (1,+1)\} \rightarrow \{-1\}, \{0\}, \{+1\}$

SO(3)

SO(2)

Little Groups

- ◆ Lattice symmetry
Octahedral (cubic) group
- ◆ Non-zero momenta breaks the symmetry differently
Different little groups for different momentum directions
- ◆ Spin-1 $\rightarrow T_1 \rightarrow A_1 \oplus E_2$ or $A_1 \oplus B_1 \oplus B_2$ or $3A$

Summary and Outlook

- ◆ Lattice QCD calculations can resolve the mysteries of the baryon resonances
 - ◆ Ordering of the Roper and S_{11} could be resolved in 2+1-flavor calculations call for more independent calculations at large volume
 - ◆ Group-irrep operators provide powerful probes to extract highly excited resonances for nucleons $N_f=2$ and 2+1 (preliminary)
 - ◆ Quantum numbers can be indentified by their mass ordering, e.g. hyperons
 - ◆ We calculate large Q^2 momentum N - N form factors and N - N^* form factors (can be further improved with more statistics)
 - ◆ New technique will greatly improve the precision of the calculations
- ◆ In the near future...
 - ◆ Pion mass $\ll 300$ MeV with multiple volumes and lattice spacing
 - ◆ Multi-particle operators are needed to distinguish the resonances
 - ◆ Identify the spin of resonances directly
 - ◆ Chiral perturbation theory for extrapolations or to check its correctness

Backup Slides

Nucleon-Roper Form Factors

- ◆ Converting experimental data ($\gamma^* N_1 \rightarrow N_2$)

$$A_{1/2}(Q^2)/\kappa_A = G_M(Q^2) = F_1^*(Q^2) + F_2^*(Q^2)$$

$$S_{1/2}(Q^2)/\kappa_S = G_E(Q^2) = F_1^*(Q^2) - F_2^*(Q^2) Q^2 / (M_{N_1} + M_{N_2})^2$$

with

$$k_A(Q^2) \equiv \sqrt{2\pi\alpha} \frac{Q^2 + (M_{N_1} - M_{N_2})^2}{M_{N_1} (M_{N_1}^2 - M_{N_2}^2)}$$

$$k_S(Q^2) \equiv k_A(Q^2) \frac{M_{N_1} + M_{N_2}}{2\sqrt{2}Q^2 M_{N_2}} \sqrt{Q^2 + (M_{N_1} - M_{N_2})^2} \\ \times \sqrt{Q^2 + (M_{N_1} + M_{N_2})^2}$$

- ◆ Use CLAS analysis and PDG results to solve for $F_{1,2}^*(Q^2)$

I. Aznauryan et al., arXiv:0804.0447[nucl-ex], arXiv:0711.1120[nucl-th];

K. Park et al., Phys. Rev. C77, 015208 (2008);

V. I. Mokeev et. al, AIP Conf. Proc. 842, 339 (2006).

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