A Measurement of Inclusive
Quasi-elastic Electron Cross Sections
at $X > 1$ and High Q^2

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Abstract

Experiment E89-008 measured inclusive electron scattering cross sections from different nuclei in Hall C at Jefferson Laboratory. Cross sections on the low energy loss side of the quasi-elastic peak ($x_{\text{Bj}} > 1$) are extracted for carbon, aluminum, iron and gold. The data cover four-momentum transfers squared of 0.97 to 5.73 GeV²/c². The measured cross sections are compared to cross sections calculated using a microscopic spectral function.

The cross section results are also analyzed in terms of the two scaling functions $F(y)$ and $f(\psi')$. For both the data is found to be independent of the momentum transfer (scaling of the first kind). For $f(\psi')$ the data is in addition independent of the mass number A (scaling of the second kind) and thus exhibits superscaling properties.

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1 Introduction

Electron scattering is a powerful method to investigate the structure of a nuclear system. Since the electromagnetic part of the interaction is well described by QED, electron scattering is a well understood probe to study nuclear physics. The electromagnetic interaction of electrons with the target system is weak compared to the strong interaction. The interaction therefore can be well described by the exchange of a single virtual photon which transfers its momentum and energy to the probed target.

This experiment was designed to study the nuclear structure by measuring inclusive cross sections from several nuclei over a wide kinematical region. In inclusive reactions, where only the scattered electron is detected, the cross section for a fixed electron scattering angle θ depends on two variables: for example one can choose the momentum transfer q and the energy transfer ν which are carried by the virtual photon. The energy transfer is defined as $\nu \equiv \varepsilon - \varepsilon'$, where ε and ε' are the energies of the incoming and scattered electron.

Inclusive cross sections measured in multi-GeV electron scattering experiments are expected to provide stringent tests for the nuclear many body theory (NMBT) [1,2]. This theory has already been successful in explaining a number of nuclear properties [3]. However, some problems, such as the inclusive response at small energy loss, remained to be fully resolved. The region of small energy loss is of particular interest for studying the effects of final state interactions (FSI) and with it the possible influence of color transparency (CT) [1]. CT leads to a reduction of the FSI. Since the cross sections drop rapidly in this region, only a few experiments were previously performed in this kinematics [4]. The high luminosity of the JLab ¹ electron accelerating facility and the large acceptance of the Hall C spectrometer allowed a cross section measurement at least two orders of magnitudes smaller than previously accessible (see Table 1).

Table 1: *The performance of the NE3 experiment at SLAC [4] compared to E89-008 at JLab.*

SLAC (NE3)		JLab (E89 – 008)
$\sim 8 \mu\text{A}$	Beam Current	$\sim 80 \mu\text{A}$
0.7 msr	Solid Angle (HMS)	6.83 msr
$\pm 3 \%$	$\Delta p/p$ (HMS)	$\pm 10 \%$

¹The Jefferson Laboratory (JLab) is located in Virginia on the east coast of the United States.

The application of scaling is a powerful tool to study the reaction mechanism and the structure of a nuclear system. Scaling in scattering experiments is observed where a weakly interacting probe scatters from constituents bound in a composite system and a constituent is ejected quasi-freely from the system. Scaling has been found in a variety of different scattering processes. The best known is probably the deep inelastic scattering of GeV leptons from quarks within the nucleon [5]. There scaling was found at $x_{Bj} < 1$, where the Bjorken scaling variable is defined as

$$x_{Bj} \equiv \frac{Q^2}{2m_N\nu}. \quad (1)$$

Here m_N is the mass of a nucleon and $Q^2 \equiv q^2 - \nu^2$ is the four momentum transfer squared. It was found that the inclusive structure functions only depend on $x_{Bj}(Q^2, \nu)$ and not on Q^2 and ν independently. This functional independence is called scaling and is a signature of scattering off constituent particles in the target. In this case the constituents are quarks inside the nucleon.

In the present experiment the focus is on protons and neutrons in nuclei as the elementary constituents. Scaling can be observed when electrons scatter quasi-elastically off nucleons and the nucleons are ejected in a quasi-free manner, i.e. the nucleons leave the nucleus without strong effects from final state interactions. Dividing the inclusive cross section by an elementary electron-nucleon cross sections leads to functions which only depend on a single variable $y = y(q, \nu)$ or $\psi' = \psi'(q, \nu)$ [6–8] (see equations 22 and 34). These scaling functions then contain interesting information about the dynamical properties of the nuclear ground state. The fact that scaling occurs provides useful information about the reaction mechanism itself.

The scale of the structure studied by electron scattering depends on the energy and the momentum of the virtual photon. For scattering off a nucleus at $x_{Bj} = A$ (where A is the mass number), the energy transfer is very low and the electrons scatter coherently off the nucleus. For higher energy transfers incoherent scattering dominates, and the nucleons inside the nucleus can be resolved. Quasi-elastic scattering becomes the dominant process at $x_{Bj} = 1$. For even higher energy transfers, $x_{Bj} < 1$, the quarks inside the nucleons can be seen in the kinematical region known as deep inelastic scattering (DIS). The low energy loss side of the quasi-elastic peak, $x_{Bj} > 1$, is the region of interest for this experiment. In this region the data is expected to scale in terms of the variables y or ψ' .

Figure 1 summarizes the kinematical settings that were analyzed. The data were taken at a beam energy of 4.032 GeV. The scattered electrons were detected by the High Momentum Spectrometer (HMS) of Hall C at JLab.

Measurements over a range of momentum were taken for six spectrometer angles (gray dashed lines in Figure 1). Neighboring momentum settings always have a small overlap. The largest values of x_{Bj} are reached with the kinematics at 23° and 30° .

Figure 1: The kinematics of E89-008 are shown as a function of x_{Bj} and Q^2 . The points correspond to the kinematics for the central angle and the central momentum of the spectrometer.

2 Experiment

E89-008 took data in the late summer of 1996 in Hall C at JLab. JLab has three separate halls that can run experiments with different energies (with some restrictions) and with different intensities simultaneously. By the time E89-008 started, Hall C was the only hall commissioned to run experiments.

For E89-008 JLab delivered a continuous wave (CW) unpolarized electron beam at a nominal energy of 4.045 GeV with a maximum intensity of $82 \mu\text{A}$. In the hall, the electrons scattered off hydrogen, deuterium, carbon, aluminum, iron and gold targets. Hydrogen was only used for calibrations and normalizations. The scattered electrons were detected by either the High Momentum Spectrometer (HMS) or the Short Orbit Spectrometer (SOS). Data was taken over a wide range of scattering angles and energies.

2.1 Accelerator

Figure 2: *Overview of the electron accelerating facility at JLab.*

In the injector, the electrons leave the hot cathode of a thermionic gun. This gun produces a 5 mA DC-beam with an energy of 100 keV. Afterwards two circular apertures some distance apart define the emittance of the beam and intercept roughly 50 % of its intensity. Two chopping cavities and chopping apertures then put the RF-structure to the beam and select different intensities for each hall. Both cavities are driven by two 499 MHz transverse deflecting modes with a 90 degree phase shift. The first cavity deflects the

beam onto a cone producing a circle over the three chopping apertures (see figure 3). The size of these apertures determines the beam intensity and the width of the beam bursts each hall receives (~ 1.67 ps).

Figure 3: *A downstream view of the chopping apertures. The apertures for the three halls are located along a circle spaced 120 degrees apart. A special device called THE CENTRAL PLUG (located to the left above the chopping apertures) can be inserted. It collimates everything away that is not supposed to propagate beyond this point.*

After the three collimated beams have been recombined by the second cavity of the chopper the electrons are accelerated by 18 Niobium cavities (or 2.25 cryo modules) to an energy of 45 MeV. The cavities are driven by clystrons that operate at a radio frequency (RF) of $3 \cdot 499 \text{ MHz} = 1.497 \text{ GHz}$.

Figure 4: *A cavity made with five cells as it is used at JLab. A cryo module contains eight cavities.*

In the main accelerator the electrons gain their energy in two superconducting linear accelerators (linacs). Each linac has 20 cryo modules with a total of 160 cavities (see Figure 4) and accelerates the electrons by 400 MeV (45 MeV/400 MeV corresponds to 2.25/20 cryo modules since they use the same RF)². The two linacs are connected by two arcs that consist of five

²45 MeV and 400 MeV was the status of 1996. Nowadays the linacs can be tuned to reach higher energies.

(east) and four (west) lines. In this manner it is possible for the electrons to pass through each linac five times. E89-008 always ran with five-pass beam.

2.2 Hall C Arc

After acceleration the electrons are extracted in the beam-switchyard (BSY) and start their way through the 42m long Hall C arc. The arc bends the electrons by 34.4° and is used to measure the beam energy. In and after the arc the electrons pass several devices along the beam line. These devices are needed to raster the beam, to monitor the beam position and its profile and to measure the beam current (see Figure 5).

Figure 5: The Hall C arc deflects the electrons into Hall C.

2.2.1 Beam Position

During E89-008 the two beam profile monitors (BPMs) *H00A* and *H00B*, which are located closest to the target (see Figure 5), were used to monitor the beam position, its shape and its angle at the target. The spatial resolution of the BPMs is ~ 1 mm for absolute and ~ 0.2 mm for relative measurements [9]. Each BPM consists of a cavity with four antennae mounted at $\pm 45^\circ$

relative to the horizontal and vertical plane (see Figure 6). The antennae pick up the signals of the beam frequency. The intensities of these signals are proportional to the distance to the beam. The beam position therefore can be calculated by dividing the difference over the sum of the signals of the antennae lying opposite of each other. Calculating a ratio means that the measured position is in first order independent of the beam current.

Figure 6: A BPM with the four antennae mounted at $\pm 45^\circ$. This angle was chosen to prevent damage by synchrotron radiation.

From the information of the two BPMs the beam position and angle at the target were calculated on a run by run basis. In the analysis the relative position and angle offset to Run 9035 (a run in the beginning of the experiment, where midplane symmetry for both spectrometers was checked), lead to corrections as described in chapter 3.1.1.

2.2.2 Beam Current

To measure absolute cross sections the integrated charge of each run has to be known precisely. The beam current in Hall C is measured by using beam cavity monitors (BCMs) and a parametric current transformer (Unser monitor). For this experiment BCM2 was used as the main monitor (see Figure 5).

BCMs are cylindrical resonant cavities mounted on the beam line. The passing electron beam excites different resonance modes of wire loop antennae. The size, shape and material of the cavities determine the possible modes that can be picked up [10]. These parameters are chosen to match the frequency at which the accelerator is operated. The amplitudes of all the modes are proportional to the beam current, and some special modes (such as mode TM_{010} [10] that was used) are relatively insensitive to the beam position.

The BCMs show a small temperature dependence of $\sim 0.25\%/^\circ C$ within a range of $\sim 2^\circ C$ of the temperature the cavity was tuned for. Outside this range the temperature dependence is stronger. During E89-008 the temperature of the BCM2 cavity was well inside this range and was stable to $\pm 0.2^\circ C$. Thus, the error is negligible. However, the temperature in the electronics

room of the counting house can also lead to an uncertainty. That temperature was stable to $\pm 0.5^\circ\text{C}$ which results in a error of the 0.2% level [11].

The Unser monitor was used to calibrate the gains of the BCMs [12–15]. The Unser monitor has an extremely stable gain but can have varying zero-offsets. Before E89-008 started, the gain of the Unser monitor was calibrated by feeding a precisely known current through a wire running through the Unser cavity [16]. Calibration runs were taken throughout the experiment to check the gains of the BCMs. Since the gains were stable within 0.5%, a single set of gains and offsets was used for the BCMs and an absolute error of 1% was assigned to the charge measurement [11].

2.2.3 Beam Rastering

Since the electron beam has a very small width of $\sigma \simeq 140\ \mu\text{m}$ it needs to be rastered. Otherwise it would locally melt the target. The fast raster is used for this purpose. It is mounted 21 m in front of the target (see Figure 5) and rasters the beam with 24.2 kHz horizontally and 17.0 kHz vertically. The two frequencies produce a moving Lissajous figure which covers an area of $\sim 4\text{ mm}^2$ on the target. The sinusoidal rastering of the beam causes the electron density in the corners of the rastered surface to be higher than in the center (see Figure 7).

Figure 7: *The rastered beam measured with BPM H00B is shown for run 9555.*

2.3 Targets

During E89-008 a cryo target and a solid target ladder were mounted in the scattering chamber.

The cryo target ladder contained three loops (of which two were in operation) and two dummy targets. Each loop provides two targets of $\sim 4\text{ cm}$ and $\sim 15\text{ cm}$ length.

- Loop1 was operated with liquid hydrogen at a temperature of 19 K and an absolute pressure of 1.65 bar. The density of the liquid hydrogen was $0.07230(36) \text{ g/cm}^3$ [17]. The purity of the hydrogen gas was 99.8 %. Since the contamination was mainly nitrogen and oxygen, which freeze on the surface of the heat exchanger, no correction for impurities was applied.
- Loop3 was operated with liquid deuterium at a temperature of 22 K and an absolute pressure of 1.38 bar. The density of the deuterium was $0.1670(8) \text{ g/cm}^3$ [17]. The purity of the deuterium was 99.6 %.
- The dummy targets were made of two thin aluminum plates mounted 4.01 cm and 14.9 cm apart, at roughly the same location as the end-caps of the cryo target-cells. The dummy target is ~ 10 times thicker than the cryo target end-caps (see Table 2) to acquire enough statistics within a reasonable amount of time.

The temperature of the cryo targets was stabilized using a heat exchanger for cooling, a low power heater and a high power heater. The high power heater compensated the heat load of the beam on the target. Typical heat loads for a $50 \mu\text{A}$ beam on the deuterium targets were 80 Watt for the short and 300 Watt for the long target. The loops were operated in *constant heat load* mode, which means that the high power heater stayed at a constant heat load while the low power heater corrected for fluctuations.

The uncertainty of the target density without beam is 0.5 % [17]. With beam the special raster motion of the fast raster leads to an uncertainty due to local boiling. As it can be seen in Figure 7 the beam rastered over 4 mm^2 spends much more time at the edges and in the corners. This leads to an uncertainty of $0.1\%/10 \mu\text{A}$ [11, 18].

Since the cell windows are convex, the beam position can affect the target length. A 2 mm radial offset causes a target length change of less than 0.5 %. During E89-008 the beam offset for the cryo target runs analyzed was never more than 1 mm. The corresponding uncertainty can be neglected.

In order to calculate the energy loss of the electrons and to correct for electrons scattered off the aluminum target cells the thickness of the target cell end-caps and side-walls needs to be known. All cell side-walls measure $0.013(5) \text{ cm}$ [17]. The other quantities needed are listed in Table 2.

The solid target ladder, cooled with water, was able to carry five targets. Two carbon targets of a purity of 99.5 %, two iron targets of a purity of 99.9 % and one gold target of a purity of 99.99 % were mounted. All the target lengths with their uncertainties are summarized in Table 3.

Table 2: The end-cap thicknesses and the offsets to the target center in beam direction are listed for the cryo target cells and the dummy targets [17].

Target		Entrance [cm]	Exit [cm]	z_{offset} [cm]
LH_2	4cm	0.0071(3)	0.0137(5)	-0.06(3)
	15cm	0.0071(3)	0.0119(5)	0.09(3)
LD_2	4cm	0.0071(3)	0.0114(5)	0.16(3)
	15cm	0.0071(3)	0.0135(5)	0.15(3)
Dummy	4cm	0.09596	0.09667	0.31(6)
	15cm	0.09601	0.09675	0.19(6)

Table 3: The target thicknesses (t_{Tgt}) and the errors assigned to it ($\delta t_{Tgt}/t_{Tgt}$), the radiation lengths (RL) and the target length (l_{Tgt}) are given for all targets used for the cross section analysis. The thicknesses of the solid targets were measured by $t_{Tgt} = \text{weight}/\text{area}$. The error for the cryo targets cover a beam current range of $10\mu\text{A} - 50\mu\text{A}$.

Target	$t_{Tgt} [g/cm^2]$	$\delta t_{Tgt}/t_{Tgt} [\%]$	RL [%]	$l_{Tgt} [cm]$
LH_2	0.3152(17)	0.51 - 0.71	0.51	4.36
	1.1091(62)	0.51 - 0.71	1.81	15.34
LD_2	0.6964(37)	0.55 - 0.73	0.57	4.17
	2.5250(138)	0.55 - 0.73	2.06	15.12
^{12}C	0.8915(12)	0.52	2.09	0.50
	2.5096(103)	0.64	5.88	1.42
^{27}Al	0.2572(6) + 0.2643(6)	0.35	1.09 + 1.12	0.095 + 0.096
^{56}Fe	0.2129(3)	0.14	1.54	0.03
	0.8034(11)	0.14	5.80	0.10
^{197}Au	0.3768(6)	0.16	5.83	0.02

2.4 High Momentum Spectrometer (HMS)

After scattering in the target the electrons are detected by the two spectrometers sitting in Hall C (see Figure 8). The High Momentum Spectrometer (HMS) sits on the right side of the beam pipe looking downstream. The Short Orbit Spectrometer (SOS) is located on the left side. The two spectrometers have similar detection system. As the analysis presented here covers only the data taken with the HMS, a detailed description of the HMS only is given.

On their way to the HMS, the scattered electrons loose energy (in addition to the energy they loose in the target after scattering) when passing through

Figure 8: *Hall C side view with the HMS (SOS) in forward (backward) angle position. The beam is entering the hall from the left. Due to the different sizes of the spectrometers, the beam pipe does not go through the center of the hall.*

two vacuum windows and an air gap. The total energy loss after scattering needs to be added to the energy detected by the HMS to get the correct energy of the scattered electron at the interaction. The thicknesses and the radiation lengths of the materials other than target are listed in Table 4.

Table 4: *On their way to the HMS the electrons pass a 16 mil aluminum window, a 15 cm air gap and a 17 mil kevlar / 5 mil mylar window.*

Material	thickness [g/cm ²]	RL [%]
chamber exit window	0.1097	0.46
15 cm air gap	0.0182	0.05
HMS vacuum window	0.0496	0.10

2.4.1 Coordinate System

The coordinate system used here follows the *COSY* conventions [19]. There x always points to the dispersive direction, z points to the direction of the central ray and y is chosen appropriately to get a right handed coordinate system ($x \times y = z$). Since the HMS is bending the electrons upwards, positive x is pointing down, positive y is pointing left (towards smaller angles !!!) and positive z goes along with the central electron ray.

COSY does not use angles in contrast to the *TRANSPORT* conventions [20]. So, $yp \equiv y' \equiv \frac{dy}{dz}$ is a slope and not an angle.

2.4.2 Collimators

The electrons pass the HMS slit system at a distance of $z = 127.7$ cm from the center of the scattering chamber. Two different collimators were used during E89-008 (see Figure 9). Due to a wrong alignment of the slit-box the collimators have a vertical offset of $x_{off} = 0.496$ cm and a horizontal offset of $y_{off} = -0.004$ cm.

Figure 9: *The front shape of the two collimators used during the experiment. Positive x points down and positive y points to the left (see section 2.4.1).*

The large collimator is octagonal (covering $7/8$ of the rectangular area including the corners). It is made of tungsten ($\rho = 17$ g/cm³) and is 6.35 cm thick. It accepts electrons within a solid angle of 6.83 msr for a point target.

The sieve slit is used to check the optics of the spectrometer. It is also made of tungsten and is 3.175 cm thick. Electrons can only pass through a grid of holes. Except for the center hole the diameter of the holes is 0.508 cm. The diameter of the central hole is only 0.254 cm. It can be used to measure the angular resolution. The holes are separated horizontally by 1.524 cm and vertically by 2.540 cm. Two holes are missing to give a clear identification of left-right and top-bottom.

2.4.3 Performance

After the electrons pass the collimator they are focused and bent by the HMS magnets. The HMS is a QQQD spectrometer (three quadrupoles and one dipole magnet). It was operated in a point-to-point tune.

Quadrupole fields exhibit the property of focusing in one plane and of defocusing in the plane perpendicular to it. Since the effect of focusing gets

stronger with increasing distance from the central ray, one can focus in both planes with at least two quadrupole magnets which have their fields rotated by 90° relative to each other. For the HMS, Q1 and Q3 were set to focus in the dispersive plane while Q2 was focusing in the nondispersive plane.

The dipole bends the electrons up by 25° on a radius of 12.06 m. The magnetic field of the dipole, which needs to be precisely known, is regulated by an NMR probe located in a region where the field is uniform. The performance of the HMS is shown in Table 5.

HMS - Performance	
p_{max}	7.5 GeV/c
$\Delta p/p$	$\pm 10\%$
$\delta p/p$	0.1 %
θ_{min}	12.5°
$\delta\theta$ (in plane)	± 1 mrad
$\delta\phi$ (out of plane)	± 1 mrad
$d\Omega$ (no collimator)	8.1 msr

Table 5: *The HMS performance in the point-to-point tune used for E89-008.*

2.4.4 HMS Hut

After the magnets the electrons reach the HMS hut where they are detected. The detecting system is equipped with two drift chambers, two pairs of hodoscopes, a gas Čerenkov detector and a lead glass shower counter. The focal plane (FP) to which the electrons are focused lies half way in between the two drift chambers (see Figure 10).

- The drift chambers (DCs) are used to extract tracking information. As a pair they provide two positions (y_{fp} , x_{fp}) and two slopes (yp_{fp} , xp_{fp}) at the focal plane. Each chamber contains six layers of sensing wires that cover an active area of 52 cm horizontally and 113 cm vertically.
- Since the hodoscopes can provide fast signals, they are used to form part of the trigger (see section 2.4.5). Each pair of hodoscopes is made of two scintillator arrays with 16 scintillators in the horizontal and 10 scintillators in the vertical direction covering 75.5 cm horizontally and 120.5 cm vertically. The bars overlap each other by 5 mm and have photomultipliers (PMs) on each side.
- The threshold gas Čerenkov detector is used to reject pions. The threshold to produce Čerenkov light is given by the speed of light in the gas inside the Čerenkov tank. The gas is chosen in a way that electrons are

Figure 10: A side view of the HMS detector stack inside the shielding hut.

above this threshold, and thus produce light, and that pions are below the threshold, and are thus rejected.

- The shower counter is also used to reject pions. It is made out of four layers containing 13 lead glass bars each. The entire lead glass stack is slightly tilted in order to have no holes particles could pass through. In lead glass electrons deposit their full energy in a shower. They produce photons through bremsstrahlung. These photons produce e^-e^+ pairs which produce photons again. The Čerenkov light, which is produced by the charged particles of this shower, is collected by PMs on one side of the lead glass bar. Pions can be rejected with a cut on the energy of the shower counter since they deposit only a fraction of their energy.

More information about the design, calibration and performance of all these parts can be found in [11, 14, 21–23].

2.4.5 e^- Trigger

The electronics and in particular the electron trigger is mainly the work of John Arrington. It is in detail described in his thesis [11]. Therefore, just the basic ideas of the electron trigger including pion rejection are summarized here.

Due to the high pion to electron ratio for kinematics with low spectrometer momentum settings, it is necessary to incorporate particle identification (ID) through the Čerenkov detector and the shower counter in the hardware trigger. The real electron trigger (ELREAL) can be formed by two separate coincidences (ELLO, ELHI). The two coincidences differ in the way they include particle ID (see Figure 11). Together they are almost 100 % efficient even if one of the detectors is not working properly.

Figure 11: *The heart of the HMS electron trigger.*

(ELLO) requires a Čerenkov signal and one of the following conditions:

- At least one plane of each hodoscope pair S1, S2 is firing (STOF), and the sum of the first layer of the shower counter is above a lower threshold (PRLO). A weak hodoscope condition comes together with a low threshold first layer shower counter condition.
- Three out of the four hodoscope planes S1X, S1Y, S2X, S2Y are firing (SCIN). Since this includes the (STOF) condition it does not require anything from the shower counter to satisfy the two out of three condition for (ELLO).

(ELHI) is a triple coincidence not including a Čerenkov signal. It is based on particle identification through the shower counter and requires:

- Three out four hodoscope planes S1X, S1Y, S2X, S2Y must fire (SCIN).
- The sum of the first layer of the shower counter is above a higher threshold (PRHI).
- The sum of all the layers of the shower counter is above a certain threshold (SHLO).

3 Analysis

The primary goal of the analysis is to extract the cross sections as a function of the energy loss for a fixed spectrometer angle. This chapter discusses the way the cross sections were extracted from the raw data.

3.1 Event Reconstruction

In order to extract the physics quantities for each event several quantities from the detectors of the spectrometer need to be determined. The standard Hall C analysis code was used for that purpose. It contains the calibrations of the different detector elements and provides the following quantities for each event:

- A Čerenkov value in units of [#photo-electrons].
- The sum of the first layer and the total sum of the energy deposited in the shower counter.
- Two positions and two slopes at the focal plane ($y_{fp}, x_{fp}, yp_{fp}, xp_{fp}$) obtained by fitting a track to the position information from the drift chambers, the hodoscopes and the shower counter. In case of multiple tracks the one with the smallest χ^2 is chosen as the best track.

From the four quantities at the focal plane, one can determine the trajectory of the electron at the interaction point in the target. With a set of matrix elements (MEs) the four focal plane quantities are transferred from the focal plane to the target. This reconstruction is done by computing COSY sums as it is shown for yp_{tar} :

$$yp_{tar} = \sum_{i,j,k,l=0}^{i+j+k+l \leq 5} \text{coeff}_{i,j,k,l} \cdot x_{fp}^i \cdot xp_{fp}^j \cdot y_{fp}^k \cdot yp_{fp}^l \quad (2)$$

$yp_{tar}, xp_{tar}, y_{tar}, \Delta p/p$ are the four reconstructed quantities. $yp_{tar} \equiv \frac{dy}{dz}$ is the slope of the electron in the horizontal plane relative to the central ray. As positive y points left for the HMS, positive yp_{tar} goes towards smaller angles. $xp_{tar} \equiv \frac{dx}{dz}$ is the slope in the vertical plane with positive x pointing down. y_{tar} is the position at the target along the horizontal axis that passes through the center of the scattering chamber and is perpendicular to the central ray of the HMS. With the knowledge of the spectrometer angle (θ_{HMS}), one can calculate the target position along the beam line by $z_{tar} = y_{tar} / \sin(\theta_{HMS})$. $\Delta p/p$ is the deviation of the electron momentum to the momentum of the

central ray of the spectrometer. It is usually given in units of % of the central momentum.

There are different sets of MEs with different properties to do the reconstruction. Since E89-008 was going over large momentum ranges, the set of matrix elements with the largest and best $\Delta p/p$ reconstruction was chosen for the analysis. Figure 12 shows a comparison of three different set of MEs.

Figure 12: Three kinematics of raw iron cross sections at 30° with a large momentum overlap are compared to a fit (top left plot). The other three plots compare the cross sections analyzed with different sets of matrix elements (σ_{Exp}) with the fit (σ_{Fit}).

A fit was made to the data of three kinematics which had a large momentum overlap (top left plot of Figure 12). Looking at the ratios of the experimental data points to the fit as a function of $\Delta p/p$ shows the momentum range for which the set of MEs can be used. The choice fell on a set of

5th order COSY MEs (top right plot of Figure 12). 5th order means that the sum of the exponents in equation 2 is at maximum 5. The typical momentum range used in the analysis covers -9.5% to $+10.5\%$ in $\Delta p/p$.

3.1.1 Beam Angle and Beam Position Correction

The beam angle and the beam position at the target are monitored for each run using the BPMs (see section 2.2.1). Since the beam was not perfectly stable over the full run-period, each kinematical setting was corrected for the in-plane beam angle and the vertical beam position at the target relative to run 9035 (see section 2.2.1).

The in-plane beam angle at the target goes straight into yp_{tar} as an offset. The vertical beam position at the target has an effect on the momentum since the HMS was run in a point-to-point tune. If the beam is low, the reconstructed momentum is lower than the actual value. In other words: A positive x_{tar} offset causes a negative $\Delta p/p$ offset. This dependence was taken into account by a first order correction based on the COSY MEs. A 1 mm beam offset in x_{tar} gives a -0.07902% correction for $\Delta p/p$.

An example of such a correction is shown in Figure 16. The data point indicated by the violet star is not corrected for the beam being 2.3 mm too high. Applying the correction leads to the black open circle above the violet star.

3.1.2 Sieve Slit Data

Figure 13 shows data taken with the sieve-slit. The data is reconstructed to the front edge of the sieve-slit using the 5th order COSY MEs (see section 3.1). Figure 13 shows the data at the slit (bottom right plot) and its projections onto the horizontal and vertical axes. The two missing holes and the smaller hole in the center give a clear definition of the orientation of the slit. If the reconstruction did not satisfy the needs of the experiment, this data could have been used to determine part of the MEs [24]. Here it is simply used to check the optics of the HMS with its reconstruction.

The nominal distance between the holes is 1.524 cm horizontally and 2.54 cm vertically. Fitting Gaussians to the peaks on the projected axes and calculating an average hole-to-hole distance gives 1.531(7) cm for the horizontal and 2.502(5) cm for the vertical distance. The horizontal reconstruction is in fair agreement with the nominal grid. The vertical reconstruction is a little bit too narrow, but the error is small enough to have a negligible effect on the scattering angle.

The sieve-slit data can also be used for an absolute calibration of the

Figure 13: *Run9405: Carbon data taken at $\theta_{HMS} = 23^\circ$ and $p_{HMS} = 2.02$ GeV/c with the sieve-slit inserted. The large vertical offset of the slit-box (see section 2.4.2) can be seen in the bottom figures.*

vertical reconstruction if the sieve-slit position relative to the beam line is well known. This procedure leads to an absolute offset for $x_{p_{tar}}$ of 6 mrad. The offset is rather large but comes from the fact that the MEs were created before the 0.496 cm offset of the slit-box was discovered.

It is not possible to get an absolute offset for $y_{p_{tar}}$ from the sieve slit data. There are two components contributing to the offset of $y_{p_{tar}}$: an intrinsic offset, as we have it for $x_{p_{tar}}$, and a global angular offset for the spectrometer. Thus a global $y_{p_{tar}}$ offset can only be found by looking at data which have a physical correlation on $y_{p_{tar}}$, such as elastic scattering off hydrogen.

3.2 Elastic Data

Elastic data taken on a hydrogen target provide an extremely useful tool to do checks, calibrations and comparisons to models since the invariant mass

and the cross sections are known. For this reason elastic hydrogen data was taken for all kinematics using the 4 cm hydrogen target. The background coming from the electrons scattering off the cell entrance and exit windows was measured with aluminum targets sitting at the corresponding positions and was subtracted. Due to drift chamber problems the hydrogen data for 37° ($Q^2 = 3.511 \text{ GeV}^2/c^2$) could not be used.

3.2.1 yp_{tar} Distributions

Figure 14: yp_{tar} distributions for the 15° , 23° , 45° and 55° data. The simulation is shown as a red line lying on top of the data points. For all kinematics the elastic peak is in the center of the $\Delta p/p$ acceptance.

In order to find a global yp_{tar} offset the yp_{tar} distributions of elastic data

for all spectrometer angles are compared to a simulation³. The simulation starts at the target with an event distribution that is weighted with the elastic cross section. The events are transported through the spectrometer up to the focal plane including multiple scattering. If the tracks collide with magnets or vacuum pipes on their way up to the focal plane the events are thrown away. The events are then reconstructed back to the target. The same cuts are applied to the simulation and the data. Figure 14 shows this comparison for four angles.

The best match for all the angles is achieved by applying *no* yp_{tar} offset. Only the 23° data show a small mismatch at the peak. This may result from the fact that instead of the 4 cm, the 15 cm dummy target was used to do the target-cell background subtraction.

3.2.2 Invariant Mass

The elastic hydrogen data, that was taken, covers almost all of the kinematics settings used for the inelastic data. For elastic scattering off hydrogen, the

Figure 15: *The Feynman diagram for electrons scattering elastically off hydrogen.*

invariant mass, which is the mass of the final hadronic state, must be the proton mass (m_p). The square of the invariant mass equals the square of the two 4-vectors coming to the nucleon vertex (see Figure 15):

$$\begin{aligned} W^2 &= [(q, \nu) + (0, m_p)]^2 \\ W^2 &= m_p^2 + 2 \cdot m_p \cdot (\epsilon - \epsilon') - 4 \cdot \epsilon \cdot \epsilon' \cdot \sin^2(\theta/2) \end{aligned} \quad (3)$$

The invariant mass squared depends on three variables: the initial and final electron energy, ϵ and ϵ' , and the electron scattering angle θ .

Figure 16 shows the difference of the invariant mass to the proton mass for all elastic hydrogen data taken during E89-008. The left plot shows the results of the analysis done with the nominal beam energy. The data points show a slope over the momentum range covered. The systematical effect of

³The standard Hall C Monte Carlo SIMC was used to do this comparison.

each of the three variables mentioned above is also shown. The blue dashed-dotted line shows the effect of a 0.1 % global systematical offset in $\Delta p/p$. The red dashed line, which shows a slope, displays what happens when a 0.1 % global offset is applied to the beam energy. The green dotted line shows the change of this difference when a 1 mrad global offset is applied to the scattering angle.

It is not possible to determine all three offsets with this data. But since the angular offset is fixed by comparing the yp_{tar} distributions to the Monte Carlo (see section 3.2.1), the other two offsets can be determined by a fit. The fit is shown as a black solid line in the left plot of Figure 16 and leads to a beam energy of $\varepsilon = 4.032(4)$ GeV and a $\Delta p/p$ -offset of -0.22% . The results of the analysis with these offsets is shown in the right plot of Figure 16. A comparison of this fitted beam energy with beam energy measurements performed with the Hall C arc is given in appendix C.

Even though the different runs were not taken at the same time, an effect coming from a constantly increasing beam energy during the runtime can be excluded. The run-numbers shown on the plot indicate that the data were not taken in a chronological order.

Figure 16: *The results of the analysis with $\varepsilon = 4.045$ GeV (left) and with $\varepsilon = 4.032$ GeV (right). The open circles come from data where the elastic peak was not exactly in the center of the $\Delta p/p$ -acceptance. The purple star in the left plot is not corrected for beam position and angle as described in section 3.1.1. It corresponds to the open circle above after the correction is applied. The gray dashed lines in the right plot indicate the slope corresponding to the systematical uncertainty assigned to the beam energy.*

The uncertainty of the beam energy measurements based on various measurements over the last years is 0.1 % [25]. An additional uncertainty of

0.05 % comes from the error of the fit shown in the left plot of Figure 16 and is added in quadrature. This leads to a total uncertainty of the beam energy of 0.11 %. This error is indicated by the gray dashed lines in the right plot of Figure 16.

3.2.3 Elastic Hydrogen Cross Sections

In order to look for absolute cross section normalization the elastic cross sections are compared to a Monte Carlo (SIMC). The events of the Monte Carlo are weighted by an electron-proton cross section calculated using the proton form factor parameterization of Mergell *et al.* [26]. The number of events generated by the Monte Carlo are normalized to the target thickness and the collected charge of the measured elastic data. The same cuts were applied for the data and the Monte Carlo.

For the 4 cm hydrogen target the θ -acceptance of events that make their way through the HMS is roughly $\pm 2^\circ$. Since θ and ε' are correlated, only the central part of the $\Delta p/p$ -acceptance is illuminated by elastically scattered events. In order to get rid of events scattering off the cell-windows, the data taken with the dummy target was subtracted.

Figure 17: Ratios of measured to simulated elastic cross sections. Only the statistical errors are shown.

The real data and the Monte Carlo data were binned in θ -bins. The left plot in Figure 17 shows a cross section comparison for each theta bin over the full θ -range for elastic data taken at $\theta_{HMS} = 30^\circ$. In the right plot all θ -bins for one kinematics are averaged. Ratios of real to Monte Carlo data are then shown as a function of the spectrometer angle. To get a more complete picture, the ratios calculated with different form factor parameterizations are shown on the right plot. The gray zone shows a comparison to a local fit of

the world data [27], and indicates to which degree the elastic cross sections are known from experiments independent of the global parameterizations such as used by Mergell and other authors.

Including a systematic error of $\sim 3\%$ (see section 3.4) the elastic cross sections are consistent with the Monte Carlo and with the fit to the world data. No correction to the inelastic cross sections was applied.

3.2.4 Elastic Scattering from the Deuteron at 15°

The elastic hydrogen data cover almost the full momentum range, except for the highest momentum setting at 15° . For a central momentum of $3.76 \text{ GeV}/c$ no hydrogen data was taken because the HMS minimum angle, determined by the beam line, is larger than the 10.55° needed for this kinematics. But the cross section for elastic scattering off deuterons is still large enough to be used for a calibration. Since the inelastic cross sections are extremely steep for this kinematics (see Figures 35 - 38), it is important to have a calibration.

Figure 18: *The elastic deuteron peak for $\theta_{HMS} = 15^\circ$ and $p_{HMS} = 3.76 \text{ GeV}/c$ before (left) and after (right) dummy subtracting. The solid red line shows the fit to the data.*

For $\varepsilon = 4.032 \text{ GeV}$ and $\theta = 15.0^\circ$ the elastic deuteron peak is expected to be at $\nu = 0.275 \text{ GeV}$. Due to the kinematical broadening of the θ -acceptance, a correction needs to be applied to the energy transfer ν to be able to see the elastic peak on a one dimensional plot. The correction is based on the $\theta - \varepsilon'$ correlation for elastic scattering and corrects ν of all the events for the

difference of the measured scattering angle to the central spectrometer angle:

$$\begin{aligned} \nu_{corr} &= \varepsilon - \varepsilon' - (\varepsilon'_{HMS} - \varepsilon'_{calc}) \\ \varepsilon'_{calc} &= \frac{2 m_d \varepsilon}{4 \varepsilon \sin^2(\theta/2) + 2 m_d}, \end{aligned} \quad (4)$$

where m_d is the deuteron mass. A Gaussian is fitted on top of a linear background as it is shown in Figure 18. It demonstrates that a small but noticeable correction of -4 MeV has to be applied for this kinematics. This offset is already included in the plots of Figure 18.

A rough cross section estimation can be made to verify that the small peak in Figure 18 is the elastic $e - D$ peak. The cross section determined by a fit to the world data is $0.076(4)$ nb/sr [27]. The integral of the Gaussian fitted together with a linear background is 0.045 nb/sr. Taking radiative corrections of $\sim 20\%$ into account and considering that the linear background overestimates the background subtraction, this result is in fair agreement with the fit to the word data.

3.3 Inelastic Data

Once the calibration with the well known elastic data is done, the inclusive differential cross sections of inelastic electron-nucleon scattering can be extracted. A differential cross section describes the probability of an incoming electron with the energy ε to interact with the target and to be scattered at an angle θ with an energy loss ν into a solid angle Ω_e . The inclusive cross section is then a function of the initial and final electron energy and the electron scattering angle. It can be written as

$$\frac{d^3\sigma}{d\Omega d\nu}(\varepsilon, \theta, \nu) = \frac{e}{C} \cdot \underbrace{\frac{1}{N_A \cdot \frac{1}{m} \cdot d_{tgt}}}_{\mathcal{L}} \cdot \frac{1}{\Omega_e \Delta\nu} \cdot \frac{\#electrons}{\mathcal{E}_{ff}}, \quad (5)$$

where e is the elementary charge, C is the collected charge, N_A is the Avogadro number, m is the mass of the target nucleus, d_{tgt} is the target thickness, $\Delta\nu$ is the ν bin width and $\mathcal{E}_{ff}$ is the total efficiency. The things that go into the luminosity ($\mathcal{L}$) are already discussed.

In the following chapters the remaining quantities going into equation 5 and the cuts applied to the data will be looked at.

3.3.1 Electron Identification

In order to select the electron events and to reject other particles (mostly π^-) both a Čerenkov and a shower counter cut are used. This is a stronger

electron selection as it is done by the hardware trigger (see section 2.4.5). There, either one of the two detectors can give a trigger, which guarantees a high efficiency even if one of the detectors is not working properly.

Figure 19: *Gold data: The left plot shows a low π/e -ratio (15° , $3.21 \text{ GeV}/c$). The right plots has the worst π/e -ratio of all kinematics (55° , $1.11 \text{ GeV}/c$).*

Figure 19 shows that depending on the kinematics it is necessary to use both a Čerenkov and a shower counter cut, to get a clean electron sample. The shower counter energy as shown in Figure 19 is divided by the energy selected by the tracking algorithm. Since the electrons deposit the full energy in the detector, they show up at 1.0 in the shower counter spectrum. The electrons show a Čerenkov signal of ~ 10 photoelectrons as an average.

Most of the pions don't produce a Čerenkov signal. They deposit roughly $\sim 0.3 \text{ GeV}$ in the shower counter with a tail on the high energy side that goes up to the full energy (see Figure 19). Some pions show a Čerenkov signal greater than zero when they knock out electrons from material in front of the Čerenkov detector.

In the electron trigger the threshold on the Čerenkov signal was set to ~ 1.5 photoelectrons. For the offline analysis the threshold for this cut was set slightly higher to 2 photoelectrons. The efficiency of this cut is calculated by applying a hard shower counter cut ($E_{Cal}/\varepsilon' > 1.0$) on runs with a π/e -ratio of less than 0.2. By comparing counts with and without the Čerenkov cut an efficiency of 99.86 % was found for which the data is corrected. Due to pedestal shifts an uncertainty of 0.5 % is assigned.

A fiducial cut was applied to the shower counter. The cut requires the electrons to hit the shower counter more than 5 cm away from the edge. This fiducial cut makes sure that the full shower of the electron is inside the shower counter. The resolution of the shower counter is $6.5 \text{ \%}/\sqrt{\varepsilon'}$. The threshold for the shower counter cut was set to 0.6 which leads to an electron efficiency of

$> 99.99\%$. For some kinematics at large angles and low momenta where the π/e -ratio is very high (> 100) an uncertainty of 0.5% for pion contamination has to be allowed for.

3.3.2 Background Rejection

There were three sources of background electrons which needed to be eliminated: (i) a few electrons which were outside of the acceptance managed to get back into the acceptance by multiple scattering. These events were eliminated by applying loose cuts on the reconstructed quantities. Table 6 lists the cuts used.

Table 6: *The cuts used on the reconstructed quantities.*

$$\begin{aligned} \text{ABS}(y_{tar}/\sin(\theta_{HMS})) &< 10 \text{ cm} + l_{tgt}/2 \\ \text{ABS}(yp_{tar}) &< 0.07 \\ \text{ABS}(xp_{tar}) &< 0.10 \end{aligned}$$

(ii) Some electrons did scrape along the dipole magnet and made it to the focal plane. (iii) Background electrons from the hall did scatter back into the acceptance via the vacuum pipe at the exit of the HMS-dipole.

Figure 20: *Carbon data taken at $\theta_{HMS} = 23^\circ$, $p_{HMS} = 3.76 \text{ GeV}/c$ is reconstructed to the exit of the dipole magnet ($z_{fp} = -576 \text{ cm}$). The form of the vacuum pipe at the dipole exit is drawn as a black line in the left plot.*

These events of (ii) and (iii) can only be seen when the cross section gets extremely small. The worst case is shown in the left plot of Figure 20 where

no software cuts are applied. Electrons scattering back into the acceptance can be nicely separated at the exit of the dipole magnet 576 cm in front of the focal plane. Since all these electrons have too low momenta, the shower counter cut takes care of almost all of these events. The right plot of Figure 20 shows what is left after applying the usual software cuts. It is not even necessary to apply a fiducial cut at the dipole exit. Nevertheless, the cut was applied.

3.3.3 Efficiencies and Dead-times

The quantities that go into the variable $\mathcal{E}_{ff}$ of equation 5 are efficiencies for the Čerenkov cut (see section 3.3.1), for the electron trigger and for the tracking, the pre-scaling and the electronic and the computer dead-time. Everything but the Čerenkov efficiency is calculated on a run by run basis from the standard Hall C analyzing code. They are just briefly explained here. A more detailed discussion can be found in [11].

The trigger efficiency was usually high ($> 99\%$). To determine it, an efficiency for each hodoscope plane is calculated by dividing the number of hits for each plane by the number of times the plane should have fired. For this calculation a reduced fiducial volume of only the center part of each hodoscope paddle is used. With these efficiencies the efficiency for 3 out of 4 hodoscope planes, which formed the trigger, was calculated.

The tracking efficiency varied between 93 % and 97 %. It is determined in a similar way as the trigger efficiency. A particle identification cut and a fiducial cut covering the central and low momentum area, where the electron cross section is the largest, is used for the calculation. The tracking efficiency is then the number of tracks found divided by the number of *good* events. A good event has a 3/4 trigger, passes a time of flight cut and has at least one drift chamber with less than 15 hits. From the tracking efficiency an uncertainty of $\pm 0.5\%$ is assigned to the cross section.

The electronic dead-time arising from the 50 ns gate-width of the signals coming from the hodoscopes was never more than 0.2 %.

The computer dead-time leads for some runs to a significant correction. It is simply calculated as the ratio of the triggers formed (pre-triggers) to the triggers accepted by the data acquisition. Running in *buffered mode* the conversion time for one event is roughly 100 μ s. So, for a pre-trigger rate of 2 kHz a dead-time of roughly 17 % has to be taken into account.

3.3.4 Solid Angle and Spectrometer Acceptance

Due to the shape of the collimator, the magnets, the vacuum pipe and the detectors, the spectrometer is not uniformly illuminated over its acceptance.

Figure 21: *The one dimensional HMS acceptances are shown for the thick carbon target ($l_{tgt} = 1.42$ cm) at 30° . The asymmetric shape of the θ -acceptance comes from the fact that the θ -bins are curved over the surface of the collimator with its octagonal shape (see Figure 9).*

This means that the number of electrons need to be weighted by a spectrometer specific acceptance function. Generally the acceptance is a function of the three target coordinates x , y , z and the three spectrometer coordinates $x_{p_{tar}}$, $y_{p_{tar}}$ and $\Delta p/p$. Since the inclusive cross section neither depends on the target coordinates nor on the out of plane angle, only a two dimensional acceptance function $A(\theta, \Delta p/p)$, or equivalently $A(\theta, \nu)$, needs to be generated.

The data is therefore binned in (θ, ν) -bins. The acceptance function is created with a Monte Carlo. The Monte Carlo throws events uniformly distributed into a phase space that covers more than the full solid angle of the collimator and more than the $\Delta p/p$ -acceptance used for the cross section extraction. A solid angle of ± 40 mrad (in plane) times ± 90 mrad (out of plane) and a $\Delta p/p$ -acceptance of $\pm 14\%$ was typically covered. The generated events are transported up to the focal plane and reconstructed as already mentioned in section 3.2.1. They undergo the same cuts and the same binning as the data. After the reconstruction each Monte Carlo bin contains an acceptance weighted solid angle times ν bin width information. The last factor, going into equation 5, which is needed to determine the cross

section can then be calculated as

$$\Omega_e \Delta\nu = PhaseSpace \cdot \frac{\#counts(\theta, \nu)}{GenEvents} , \quad (6)$$

where $GenEvents$ is the number of events generated by the Monte Carlo and $PhaseSpace$ is the phase space in which the events were generated. $\#counts(\theta, \nu)$ is the number of Monte Carlo events in each bin after the reconstruction.

Figure 22: *Carbon cross sections taken at 15° . The ratios are described in the text.*

After a cross section for each (θ, ν) -bin has been determined, all the θ -bins for a given energy transfer ν are combined and averaged which leads to a cross section $\sigma(\theta_{HMS}, \nu)$ for a fixed spectrometer angle θ_{HMS} . This *averaging* over the θ -acceptance leads to a *bin centering* correction (see section 3.3.6) since the cross section is usually neither flat nor linear over the averaged θ -region.

The ν bin width chosen for the analysis is 20 MeV. A θ -acceptance of $\pm 1.4^\circ$ was usually divided up into 16 θ -bins. Figure 22 shows the ratios of the averaged cross sections analyzed with more than one θ -bin to the cross sections analyzed with only a single θ -bin. One single bin means that all electron events are weighted equally. Figure 22 shows that it is necessary to use at least 8 θ -bins over a θ -acceptance of $\pm 1.4^\circ$.

3.3.5 Raw Cross Sections

The cross sections can now be extracted from the raw data according to equation 5. For a given spectrometer angle (θ_{HMS}), the cross section is calculated for each ν -bin. The different kinematics were chosen in a way

that there was always an overlap in ν . As an example, Figure 23 shows all the kinematics taken with the thin iron target for one spectrometer angle. The plots show that in the overlap regions the cross sections for a given energy transfer ν , measured with a different central momentum of the spectrometer, are in fair agreement over a wide range of energy transfers. The dotted lines in the bottom plot mark a $\pm 5\%$ deviation of the ratios from 1.

Figure 23: *Uncorrected iron cross section taken with the thin iron target (1.5% RL) are shown for $\theta_{HMS} = 30^\circ$. The bottom plot shows the cross section ratios in the overlap regions. The open circles were taken with the thick iron target.*

These cross sections now need to be corrected for the (θ, ν) bin size (see section 3.3.6) and after that for radiative effects (see section 3.3.7). The radiative effects can already be seen in Figure 23. The black open circles are raw cross sections extracted from the data taken with the thick iron target (5.80%RL). A 10 – 20 % difference can be seen between the cross sections of two iron targets of different thicknesses. The corrections are applied to the data for each target separately after combining all the kinematics for one angle.

3.3.6 Bin Centering Correction

Since the raw cross section as shown in section 3.3.5 are extracted from (θ, ν) -bins of a finite size they represent an integral over the bin size. But the aim is to have an exact cross section at the bin center. As long as the cross section is a linear function of the variables, the cross section at the bin center is equal to the integrated cross section. But any higher order variation leads to a correction for which the raw cross sections need to be divided by

$$BinCorr = \frac{\int \int d\theta d\nu \cdot \sigma(\theta, \nu)}{\Delta\theta \Delta\nu \cdot \sigma(\theta_C, \nu_C)}, \quad (7)$$

where θ_C and ν_C are the angle and the energy transfer in the bin center.

While the cross section varies only little over the ν bin size ($\Delta\nu = 20$ MeV) the variation over the θ bin size ($\Delta\theta = \pm 1.4^\circ$) can be large which can lead to a large correction.

Figure 24: *Two fits to $F(y)$ are shown for Gold at 15° . For $y < -0.1$ GeV/c a $F(y)$ model-function [4] is fitted locally to the data. Otherwise a quadratic function is fitted. The right plot indicates how far out $F(y)$ has to be extrapolated for the lowest data point. The blue dashed line in the right plot shows a linear fit which was used to estimate the systematic uncertainty.*

The calculation of the two dimensional integral in equation 7 takes advantage of the fact that the data is scaling in the variable y (see section 4.3.1 and 5.2). When scaling, $F(y)$ depends only on y and not on θ and ν , and it can be used to evaluate the two dimensional integral of equation 7. The integral is calculated numerically. y and $\tilde{\sigma}_{eN}$ are calculated for each point according to its kinematics. $F(y)$ is then determined from a local fit to the data (see Figure 24), and the corresponding cross section can be calculated using equation 26.

Except for some kinematics taken at 15° and 23° , a θ -acceptance of $\pm 1.4^\circ$ was used in the analysis. At very forward angles $F(\theta, y)$ does not scale perfectly in y (see chapter 5.2.1) and changes significantly in θ . Using the full θ -acceptance leads to a systematic error in the bin centering correction for these kinematics. Figure 25 points out this problem for 15° . The blue cruces

Figure 25: The cross sections of 15° carbon data are shown for several cuts of the theta acceptance. The ratios show the systematic effect coming from the not perfect y -scaling.

are cross sections analyzed with the full θ -acceptance. They are roughly 7 % below the cross sections analyzed with a tight cut in θ . Instead of using the full θ -acceptance, a limited acceptance of $\Delta\theta = \pm 0.70^\circ$ (for the 15° data) and $\Delta\theta = \pm 1.05^\circ$ (for the 23° data) was used for large energy transfers where this problem occurred.

In order to estimate the systematic uncertainty of the fitting procedure, the integral is again calculated by using a linear function for the local fits to $F(y)$ (see Figure 24). This gives a fair estimate of the systematic uncertainty. Figure 26 shows the bin centering correction calculated with the two different fit functions for two kinematics taken with the gold target.

From the ratios of the cross sections calculated with the two different fit functions (see the bottom plots of Figure 26) an uncertainty of 2.5 % is assigned to the bin centering correction. At the lower edge of ν the systematic uncertainty can get as high as 20 % since $F(y)$ needs to be extrapolated in this region.

Figure 26: The bin centering corrections calculated for Gold at 15° (left) and 45° (right) by fitting a $F(y)$ model function (black line) and a linear function (blue dashed line) locally to the data. The bottom plots show the ratios of the cross sections calculated with the two different fit functions.

3.3.7 Radiative Correction

Another sometimes large correction, which is applied after the bin centering corrections, accounts for radiative effects due to internal and external Bremsstrahlung. The Feynman diagrams for the main contributions are shown in Figure 27.

Figure 27: The left and the center diagram show the dominant Feynman graphs for the radiative corrections. Other contributions as the one shown in the right graph where the radiated photon is connected with two vertices are of higher order and are less important.

Internal Bremsstrahlung means that the electrons radiate real or virtual photons in processes involved with the nucleus scattered off. It is independent of the target thickness. External Bremsstrahlung means, that electrons radiate in the Coulomb field of any nucleus other than the one involved in the scattering process. It therefore depends on the target thickness and on other materials the electrons pass through (see Tables 2, 3, 4).

In order to correct for these effects, a code developed and used in experiments at the Stanford linear accelerator (SLAC) is used [4]. The code uses the formulae of Stein *et al.* [28] which are based on a discussion in [29] followed by the work of Mo and Tsai [30, 31].

Figure 28: Cross section ratios of the thin to the thick iron target are shown for $\theta_{HMS} = 15^\circ$ (left) and $\theta_{HMS} = 30^\circ$ (right) before and after the radiative corrections.

The radiative corrections are calculated with an iterative procedure. It starts with a model cross section and computes the integrals needed for the radiative folding. The radiated (folded) model is then compared to the data. The ratio of the data to the radiated model gives a correction to the initial model which is then used for the next iteration. This *loop* is repeated until the radiated model matches the data. After the $(i)^{th}$ iteration the data is multiplied with

$$RadCorr = \frac{\sigma_{(i)}(Model)}{\sigma_{(i)}(radiatedModel)}. \quad (8)$$

The uncertainty for this correction arising from external Bremsstrahlung can be estimated by analyzing targets of different thicknesses. Figure 28 shows the corrections for iron calculated for two kinematics. The left plot was taken at kinematics where the cross section is very steep and where the statistics get poor. The right plot corresponds to kinematics where the cross section is flat and high (see Figure 37). After the correction (blue stars in Figure 28) the weighted means of all data points are calculated for each kinematics. The results are 0.975(6) for the 15° -data and 1.016(5) for the 30° -data.

In addition, an uncertainty coming from the sensitivity to the choice of a model, used for the iterative procedure, can be estimated. The left plot in Figure 29 shows three initial models used for this estimation for carbon at $\theta_{HMS} = 15^\circ$. The carbon data on this plot is already radiatively corrected. Otherwise it can not be compared with the initial models. The model dependence is always below 1 % (see bottom right plot of Figure 29).

Figure 29: Carbon data taken at $\theta_{HMS} = 15^\circ$ with the three different models are shown on the left. The black line represents the model that was finally used [4]. The radiative corrections and the ratios of the cross sections corrected with the different models are shown on the right.

Adding both contributions quadratically a systematic uncertainty of 2.2 % is assigned to the radiative corrections.

3.4 Systematic Uncertainties

A summary of all the systematic uncertainties is given in Table 7. The table is split up in two parts. The first part gives the systematic uncertainties that apply to the cross section and for which a total uncertainty is calculated.

The second part lists the kinematical uncertainties that are correlated with each other. Their effects are not added to the total systematic uncertainty because the cross sections are given as a function of an energy difference. A shift in ε would produce a large change for steep cross sections plotted as a function of ε' . But such a shift also requires a shift in ε' to get the correct invariant mass from the hydrogen data (see section 3.2.2).

Table 7: *List of the systematic uncertainties.*

Origin	Uncertainty	Section
Charge measurement	1 %	2.2.2
Target thickness	0.14 – 0.73 %	2.3
Particle ID	0.5 – 0.7 %	3.3.1
Efficiencies	0.5 %	3.3.3
Bin centering	2.5 %	3.3.6
Radiative effects	2.2 %	3.3.7
Total uncertainty	3.5 – 3.6 %	
Beam energy	0.11 %	3.2.2
HMS momentum	0.1 %	2.4.3
Scattering angle	1 mrad	2.4.3

4 Theory

This chapter describes the theoretical background needed for the interpretation of the measured cross sections and of the scaling functions. Section 4.1 gives a kinematical overview of quasi-elastic scattering in plane wave impulse approximation (PWIA). Section 4.2 explains the theoretical calculation of the inclusive cross section in local density approximation (LDA) to which the measured cross section will be compared. Section 4.3 shows how far one can go with the interpretation of the cross section in the framework of PWIA. The y -scaling formalism and the concept of superscaling are laid out in the sections 4.3.1 and 4.3.2.

4.1 Quasi-elastic Scattering in PWIA

It is a good approximation to treat the quasi-elastic scattering process for high momentum transfers in the framework of PWIA [6]. PWIA makes three basic assumptions: (i) The scattered electron transfers momentum and energy by only one virtual photon. (ii) This virtual photon couples to only one constituent of the nucleus. (iii) The final state interactions (FSI) of the knocked out nucleon with the $(A - 1)$ recoil or with other nucleons can be neglected.

Figure 30: *The Feynman diagram for quasi-elastic scattering in PWIA. The energy and momentum transfer take place through ONE virtual photon to ONE nucleon of the nucleus.*

The Feynman diagram for this approximation is shown in Figure 30. The

left side represents the electron part with an incoming and outgoing electron of energies ε and ε' . A virtual photon transfers the momentum q and the energy $\nu = \varepsilon - \varepsilon'$ to the nucleus. The four momentum transfer is defined as $Q^2 \equiv q^2 - \nu^2$. The virtual photon couples to a bound (off-shell) nucleon which carries an initial momentum $\vec{k}$ and an energy E . After the scattering process the knocked out nucleon carries a momentum $\vec{k} + \vec{q}$ while the remaining, in general excited, $(A - 1)$ system is left with a momentum $-\vec{k}$.

It is convenient to introduce an energy E^* characterizing how excited the recoiling $(A - 1)$ system is:

$$E^* \equiv \sqrt{M_{A-1}^{*2} + k^2} - \sqrt{M_{A-1}^2 + k^2} \geq 0, \quad (9)$$

where M_{A-1}^* is the mass of the residual nucleus in an excited state and M_{A-1} is the mass of the $(A - 1)$ system in the ground state.

The energy of the struck nucleon is then given from energy conservation by

$$E = M_A - \sqrt{M_{A-1}^2 + k^2} - E^* \quad (10)$$

and in general yields $E^2 \neq k^2 + m_N^2$, since the struck nucleon is off-shell. Here M_A is the mass of the nucleus. The ground-state masses are related by the nuclear separation energy:

$$E_S = M_{A-1} + m_N - M_A, \quad (11)$$

where m_N is the nucleon mass. Energy conservation on the hadron side is given through

$$\nu = \sqrt{m_N^2 + (\vec{k} + \vec{q})^2} - M_A + \sqrt{M_{A-1}^2 + k^2} + E^*. \quad (12)$$

4.2 Cross Section Calculation in LDA

Inclusive e-N cross sections provide an interesting case to test assumptions made within the framework of nuclear many-body theory (NMBT). Here a forward calculation of the inclusive e-N cross section following the approach of [1] and [2] is summarized. The calculation will be compared to the measured cross sections in section 5.1.

There are two main problems for this calculation in applying NMBT to scattering of multi-GeV electrons. First, relativistic kinematics have to be used since the struck nucleons have momenta of the order of the nucleon mass and are therefore relativistic. Second, FSI lead to a folding of the cross section.

The cross section for the inclusive reaction, $e + A \rightarrow e' + \text{anything}$, is given by [32]

$$\frac{d^3\sigma}{d\Omega d\varepsilon'} = \frac{\alpha^2}{Q^4} \frac{\varepsilon'}{\varepsilon} \mathcal{L}^{\mu\nu} \mathcal{W}_{\mu\nu}^A(Q) \quad (13)$$

where $\alpha = 1/137$ is the fine structure constant and $\mathcal{L}^{\mu\nu}$ and $\mathcal{W}_{\mu\nu}^A$ are the lepton and the nuclear tensors, respectively. The lepton tensor is well described in QED. In PWIA the nuclear tensor can be described by the integral over the spectral function $S(\vec{k}, E)$ times a relativistic off-shell single e-N cross section. The calculation of the off-shell e-N cross section follows the approach of de Forest [33], generalized to describe not only elastic but also inelastic electron scattering off an off-shell nucleon [1]. The spectral function, which is the probability to find a nucleon with momentum $\vec{k}$ and energy E inside the nucleus, is split into a correlated and a single-particle part [2]:

$$S(\vec{k}, E) = S_{s.p.}(\vec{k}, E) + S_{corr}(\vec{k}, E) \quad (14)$$

$$S_{corr}(\vec{k}, E) = \sum n_i \cdot S_{i,NM}(\vec{k}, E) \quad (15)$$

The single particle part, $S_{s.p.}(\vec{k}, E)$, is obtained by a mean-field calculation using a Wood-Saxon potential. Since the correlated part, $S_{corr}(\vec{k}, E)$, is mostly sensitive to the short range part of the N-N interaction, it is expected to be unaffected by finite-size effects and can be calculated in local density approximation (LDA). In LDA spectral functions computed at different densities of nuclear matter ($S_{i,NM}$) are combined and weighted according to the probability distribution n_i of nucleons at densities i in the nucleus. Here nuclear matter spectral functions for densities of $\rho = 0.25, 0.50, 0.75, 1.00$ times nuclear matter density were used.

In order to estimate the effects of FSI it is convenient to treat the FSI in perturbation theory and to split the Hamiltonian occurring in the nuclear tensor $\mathcal{W}_{\mu\nu}^A$ (see [1]):

$$H = H_0 + H_I, \quad (16)$$

where H_0 is the unperturbed Hamiltonian and where H_I contains only the FSI of the struck nucleon with the other $A - 1$ nucleons. PWIA is obtained by neglecting H_I . As originally proposed by [34] the FSI can be described by representing H_I with an optical potential $V(Q, \nu, t) - iW(Q, \nu, t)$ which leads to a folding of the cross section and the nuclear tensor can be written as:

$$\mathcal{W}_{\mu\nu}^A(Q, \nu) = \int_0^\infty d\nu' F(\nu - \nu') \mathcal{W}_{\mu\nu, PWIA}^A(Q, \nu' - V(Q)), \quad (17)$$

where the folding function F is given by

$$F(\nu - \nu') = \frac{1}{\pi} \Re \left(\int_0^\infty dt e^{i(\nu - \nu')t} e^{-W(q,t)t} \right). \quad (18)$$

Figure 31: The correlated parts of the nuclear matter spectral functions are shown for $0.5 \cdot \rho_{NM}$ (left) and $1.0 \cdot \rho_{NM}$ (right).

It is assumed that $V(Q)$ is independent of ν and t and that $W(Q, t)$ is independent of t .

The main effects of the FSI are contained in the imaginary part $W(q, t)$ of the optical potential. Multiple-scattering Glauber theory [35] provides a framework for obtaining appropriate expressions for the time dependent $W(q, t)$, where the nucleon is at $r = 0$ and $t = 0$ when it is struck. Assuming that the particles surrounding the struck nucleon are uniformly distributed leads to the time independent Lorentzian folding function F_L shown as the red dashed-dotted line in Figure 32. In reality the particles surrounding the struck nucleon are not uniformly distributed at a density ρ [36]. The density is rather given by $\rho \cdot g(r)$, where $g(r)$ is the pair distribution function [37] as shown in Figure 33. F_G , the folding function computed in correlated Glauber approximation is shown as the black dashed line in Figure 32.

Two important parameters show up in the optical potential $V - iW$: $\sigma_{NN}(q)$ and $\beta(q)$. $\sigma_{NN}(q)$ is the free nucleon-nucleon cross section, and $\beta(q)$ is the slope of $\sigma_{NN}(q = 0)$ which is equal to the range of the potential associated with it. These two parameters are obtained by interpolating existing data [38,39] of N-N scattering.

In matter there are several corrections to $\sigma_{NN}(q)$ of which the ones due to the quark structure of the nucleons are most important at high energies. Since the folding function is sensitive to interactions of the struck nucleon within $1/q$ of where it was struck, it is possible that the N-N interaction differs significantly from the free N-N interaction. This phenomenon is known as *color transparency* [40–43]. Color transparency (CT) is predicted by quantum chromo dynamics (QCD) and is based on the assumption that interactions with high momentum transfers select *small* nucleons [43]. *Small* means

Figure 32: *The folding functions calculated for iron at 23°.*

Figure 33: *The NN pair distribution as a function of the N-N distance.*

that the colored quarks of the nucleon are in a compact point-like configuration. If a photon with momentum q is absorbed by one of the quarks of the nucleon, then $2 \cdot q/3$ must be transferred by gluons to the other two quarks. The uncertainty principle demands that these gluons can only travel over a distance of $c \cdot 3/q$ where c is the velocity of light. Photons with large momentum transfers, therefore, select *small* nucleons. Due to the small interquark distance the color is better shielded and the interaction of the nucleon with the medium is suppressed. The medium is more *transparent* for the nucleon which leads to a reduced FSI. F_{GCT} is the folding function obtained in correlated Glauber approximation including the effects of color transparency and is shown as the blue solid line in Figure 32.

4.3 Quasi-elastic Cross Sections and Scaling

In PWIA the inclusive quasi-elastic cross section can be written as an integral over all final states of the exclusive $(e, e'N)$ cross sections for both proton and neutron knockout:

$$\frac{d^3\sigma}{d\Omega d\nu} = \sum_{i=1}^A \int d\vec{k} \cdot \int dE \cdot S_i(\vec{k}, E) \cdot \sigma_{ei} \cdot \delta(Arg), \quad (19)$$

where $Arg = \nu + M_A - \sqrt{m_N^2 + (\vec{k} + \vec{q})^2} - \sqrt{M_{A-1}^{*2} + k^2}$ incorporates energy conservation for quasi-elastic scattering (equation 12). Here σ_{ei} is the relativistic electron-nucleon cross section [33] and $S_i(\vec{k}, E)$ is the spectral function.

The energy-conserving δ -function in equation 19 contains the argument $\nu - \mathcal{N}$, with

$$\mathcal{N} = \sqrt{m_N^2 + (\vec{k} + \vec{q})^2} - M_A + \sqrt{M_{A-1}^2 + k^2} + E^*, \quad (20)$$

where $(\vec{k} + \vec{q})^2 = k^2 + q^2 + 2kq \cos\theta_{kq}$ and θ_{kq} is the angle between $\vec{k}$ and $\vec{q}$. For unpolarized beam and target, the spectral function can be assumed to be spherically symmetric. This allows to use spherical coordinates $d\vec{k} = k^2 dk \cdot d\phi \cdot d(\cos(\theta_{kq}))$. Integrating over the azimuthal angle ϕ and over $\cos\theta_{kq}$ using the δ -function (see appendix E of [44]) leads to:

$$\frac{d^3\sigma}{d\Omega d\nu} = 2\pi \cdot \sum_{i=1}^A \int_{-y}^Y k dk \cdot \int_0^{E_M^*} dE^* \cdot S_i(k, E) \cdot \underbrace{\bar{\sigma}_{ei} \cdot k \cdot \left| \frac{\partial \mathcal{N}}{\partial(\cos\theta_{kq})} \right|^{-1}}_{\equiv K}, \quad (21)$$

where $\bar{\sigma}_{ei} \equiv \int d\phi \sigma_{ei} / 2\pi$ is the azimuthal-angle-averaged, single-nucleon cross section. The factor $|\partial \mathcal{N} / \partial \cos\theta_{kq}|^{-1}$ comes from changing variables in the δ -function. All factors in the integrand have to be evaluated at $\nu = \mathcal{N}$ (energy conservation for quasi-elastic scattering). In PWIA the kinematical factor K defined in equation 21 is $K = [m_N^2 + (\vec{k} + \vec{q})^2]^{1/2} / q$.

The integration limits of equation 21 require some discussion. Integrating over the δ -function yields a nonzero answer as long as $-1 \leq \cos\theta_{kq} \leq +1$. For values of ν less than ν_0 , where the quasi-elastic peak occurs, the situation shown in Figure 34 results. The curves show $E^*(k)$ for $\cos\theta_{kq} = -1$. The curves for $\cos\theta_{kq} = +1$ fall in the region where $E^* < 0$, so this case is never probed.

Figure 34: The (k, E^*) integration region calculated for several Q^2 at a fixed value of $y = -0.2$ GeV/c.

Figure 34 shows that the (k, E^*) -integration is restricted to the region bound by the two curves $E^* = 0$ and $E^*(k)$ for $\cos\theta_{kq} = -1$. The intersection

of the two curves define the limits of the k -integration:

$$y = [(M_A + \nu)\sqrt{\Lambda_2 - M_{A-1}^2 W^2} - q\Lambda]/W^2 \quad (22)$$

$$Y = [(M_A + \nu)\sqrt{\Lambda_2 - M_{A-1}^2 W^2} + q\Lambda]/W^2, \quad (23)$$

where $W \equiv \sqrt{(M_A + \nu)^2 - q^2}$ and $\Lambda \equiv (M_{A-1}^2 - m_N^2 + W^2)/2$.

The E^* integration runs between zero, where the recoiling $(A-1)$ system is in its ground state, and a maximum given by

$$E_M^* = \sqrt{m_N^2 + (q + y)^2} + \sqrt{M_{A-1}^2 + y^2} - \sqrt{m_N^2 + (q - k)^2} - \sqrt{M_{A-1}^2 + k^2}. \quad (24)$$

4.3.1 y-Scaling

For a given scattering angle the inclusive response function depends only on two independent variables. If a response function can be found which depends only on one variable, which itself is a function of the two other variables, then the response scales in this new variable and this is called scaling. The property of scaling results essentially from the kinematics of the scattering process where a constituent is ejected in a quasi-free manner from the target.

The seminal idea of y -scaling was already mentioned in the seventies by West [45] and first applied to data by Sick *et al.* [46]. A thorough discussion of the formalism is given in [6, 47].

For y -scaling the focus is on protons and neutrons as the elementary constituents of nuclei. Scaling is a consequence of the kinematics when scattering quasi-elastically off a nucleon inside the nucleus. Let the momentum transfer q and the energy transfer ν be the two variables the inclusive response depends on. It has been observed experimentally [4] that for $q > 0.5 \text{ GeV}/c$ the response function on the low energy loss side of the quasi-elastic peak, obtained from dividing the inclusive cross section by an elementary e-N cross section, is only a function of a single variable $y = y(q, \nu)$.

The y -scaling behavior can be derived by making a few additional assumptions to the framework of PWIA:

Assuming that the spectral function is isospin independent, equation 21 can be written as:

$$\frac{d^3\sigma}{d\Omega d\nu} = 2\pi \cdot \int_{-y}^Y k dk \cdot \int_0^{E_M^*} dE^* \cdot S(k, E^*) \cdot \tilde{\sigma}_{eN} \quad (25)$$

where $\tilde{\sigma}_{eN} = K \sum_{i=1}^A \tilde{\sigma}_{ei}$ and K is the kinematical factor defined in equation 21. The rapid falloff of the spectral function as k and E^* increase and the specific nature of the off-shell single-nucleon cross section (a good prescription known as *cc1* is given by de Forest [33] and can be found in appendix A) point to a dominance of the integrand around $k = -y$ and $E^* = 0$. It is therefore a good approximation [6] to calculate $\tilde{\sigma}_{eN}$ at $k = -y$ and $E^* = 0$. The quasi-elastic cross section can then be written as

$$\frac{d^3\sigma}{d\Omega d\nu} = F(q, y) \cdot \tilde{\sigma}_{eN}(k = -y, E^* = 0), \quad (26)$$

where

$$F(q, y) \equiv 2\pi \cdot \int_{-y}^Y k dk \cdot \tilde{n}(q, y), \quad (27)$$

$$\tilde{n}(q, y) \equiv \int_0^{E_M^*} dE^* \cdot S(k, E^*). \quad (28)$$

As shown in Figure 34 the upper integration limit Y of the k -integral grows rapidly with q . Since realistic spectral functions are steeply peaked in momentum (see Figure 31), Y can be replaced by ∞ for q being sufficiently large. If q is sufficiently large, the upper limit of the E^* -integral can be extended to

$$E_M^*(q \rightarrow \infty) = y + k + \sqrt{M_{A-1}^2 + y^2} - \sqrt{M_{A-1}^2 + k^2}, \quad (29)$$

yielding the q -independent result $\tilde{n}(\infty, y) \equiv \tilde{n}(y)$ (equation 28) and hence the q -independent scaling function $F(\infty, y) \equiv F(y)$ (equation 27). In the literature this functional independence of the momentum transfer is called *scaling of the first kind* [7,8].

With the assumption that $\tilde{\sigma}_{eN}$ is calculated at $k = -y$ and $E^* = 0$ and with the weaker assumption of isospin independence, y -scaling is derived from the initial assumptions of PWIA. The response function F , which is a function only of the scaling variable y represents a momentum integral of $\tilde{n}$, which is itself an energy integral at a fixed momentum over a particular region of the spectral function.

If it were possible to allow the upper limit of the E^* -integral to go to ∞ , then

$$n(k) = \int_0^\infty dE^* S(k, E) \quad (30)$$

would represent the momentum distribution of the A -body in the ground state. $F(y)$ would then become the probability to find a nucleon with the momentum component y parallel to the momentum transfer $\vec{q}$.

4.3.2 Superscaling

The idea of superscaling was introduced in [48] and elaborated in [49]. It has been further discussed and studied with a large body of inclusive electron scattering data in [7, 8].

As for y -scaling the focus of superscaling is on protons and neutrons in nuclei. The basic idea of the superscaling formalism is to find *universal* momentum distributions and spectral functions which are independent of the nuclear species, whereas the y -scaling formalism provides only information on momentum distributions and spectral functions of individual nuclei. The idea of superscaling was strongly motivated by the relativistic Fermi gas (RFG) model [48, 49]. The model predicts that when using a dimensionless scaling variable ψ and a slightly modified dimensionless scaling function f , scaling of the first kind (independence of q as it is found for y -scaling) and in addition scaling of the second kind (independence of the Fermi momentum k_F , i.e. independence of the nuclear species) can be expected. That is, the model predicts superscaling (scaling of the first and of the second kind).

Within the RFG model one finds a variable y_{RFG} , which for the RFG model is the analog of the usual y -scaling variable [49]:

$$y_{RFG} = m_N (\lambda \sqrt{1 + 1/\tau} - \kappa). \quad (31)$$

Here κ , λ and τ are the dimensionless versions of q , ν and Q^2 and can be found in the appendix A. The RFG model suggests to use the dimensionless variable ψ using the Fermi momentum as the scale:

$$\psi = \frac{y_{RFG}}{k_F} \left[1 + \mathcal{O}[\eta_F^2] \right]. \quad (32)$$

Here k_F stands for the Fermi momentum and $\eta_F \equiv k_F/m_N$ is its dimensionless counterpart. The full expression for the scaling variable ψ is given by

$$\psi = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{(1 + \lambda)\tau + \kappa\sqrt{\tau(1 + \tau)}}}, \quad (33)$$

where $\xi_F = \sqrt{1 + \eta_F^2} - 1$. Clearly the scaling variable ψ has a similar behavior as the usual y -scaling variable. The variable ψ , constructed from the RFG model in the $A \rightarrow \infty$ limit, contains no binding effects and reflects the missing energy dependence of the relativistic Fermi gas spectral function (see [50]). As in conventional y -scaling, where the nucleon binding can partly be accounted for by putting the initial nucleon off-shell by a constant average removal energy, binding effects can partially be accounted for when using ψ

by introducing an empirical shift in the energy transfer: $\nu' = \nu - E_{shift}$. The dimensionless variables λ and τ are modified to incorporate the shifted energy transfer, $\lambda' \equiv \nu'/2m_N$ and $\tau' \equiv \kappa^2 - \lambda'^2$, which is finally included in

$$\psi' \equiv \psi(\lambda \rightarrow \lambda', \tau \rightarrow \tau'). \quad (34)$$

Both, the variable ψ' constructed in the RFG model as well as $\Upsilon \equiv y/k_F$, the analog value of the PWIA, show the superscaling behavior and yield similar results [8]. But one has to keep in mind that the two different models make somewhat different assumptions about the spectral function.

In analogy to equation 26 the dimensionless scaling function as it is suggested in the RFG model is defined as:

$$f(\psi') \equiv k_F \cdot F(\psi') \equiv k_F \cdot \frac{d^3\sigma/d\Omega d\nu}{\tilde{\sigma}_{eN}^{RFG}} \quad (35)$$

Here $\tilde{\sigma}_{eN}^{RFG}$ represents the on-shell e-N cross section. Its expression can be found in appendix B. The function F is made dimensionless when multiplied by the Fermi momentum k_F . For the new scaling functions $f(\psi)$ or $f(\psi')$ the RFG model predicts scaling of the first and second kind. Thus, f exhibits superscaling properties.

5 Results

In the following chapter the results of the analysis of carbon, aluminum, iron and gold data are shown. The aluminum cross sections were extracted from the dummy targets which were otherwise only used for background subtractions of the liquid hydrogen and deuterium targets. An analysis was possible up to $\theta_{HMS} = 30^\circ$. The cross section were extracted following the same procedure as for the other targets, but the Monte Carlo used to create the acceptance function (see section 3.3.6) was modified to simulate the dummy target. For higher angles it was no longer possible to extract aluminum cross sections because the dummy target was moving to the very edge of the acceptance for larger spectrometer angles. There the acceptance function is not fully understood.

5.1 Cross Sections

Figures 35 - 38 show the extracted inclusive cross sections for the various targets as a function of the energy transfer. Only the statistical errors are shown. The systematic uncertainties are discussed in section 3.4. No Coulomb corrections are applied to the data. Tables containing the numerical values of the cross sections can be found in appendix D.

The cross sections are compared to the cross sections calculated in LDA as described in section 4.2. The FSI are included using the correlated Glauber approximation to calculate the folding function. The reduction of FSI due to the effects of color transparency is also taken into account. Table 8 lists the values for $\sigma_{NN}(q)$, the free N-N cross section, and for $\beta(q)$, the range of the associated potential, used to calculate the folding functions.

Table 8: *List of $\sigma_{NN}(q)$ and $\beta(q)$ used to calculate the folding functions. The values are interpolated from tables of Dobrovolsky et al. [38] and Silverman et al. [39].*

ε [GeV]	θ [°]	Q^2 [GeV ² /c ²]	q [GeV/c]	σ_{NN} [mb]	β [fm]
4.032	15	0.966	1.110	33.9	0.233
4.032	23	1.927	1.726	43.9	0.353
4.032	30	2.764	2.221	44.7	0.368
4.032	37	3.510	2.647	44.3	0.375
4.032	45	4.216	3.044	43.6	0.381
4.032	55	4.895	3.420	43.0	0.386

Figure 35: *Carbon cross sections* Figure 36: *Aluminum cross sections*

Figure 37: *Iron cross sections* Figure 38: *Gold cross sections*

The parameterization of Mergell *et al.* [26] was employed for the free nucleon form factors used to calculate the de Forest cross section.

The cross sections calculated in LDA are in good agreement with the measured cross sections. For carbon and aluminum there is a discrepancy in the low energy loss tail. In this region the calculated cross section is very sensitive to the tail of the folding function at large ν (see Figure 32). This behavior is not yet under full control for carbon and aluminum.

5.1.1 FSI and Color Transparency

With the JLab electron facility it was possible to measure inclusive cross sections at high q^2 and at very low values of energy transfers ν (large x_{Bj}). In this regime final state interactions (FSI) and with it the possible existence of color transparency (CT) of the knocked out nucleon and the remaining (A-1) nucleons are expected to become significant. This region thus provides a good case to test different models of how FSI have to be included.

Figure 39 shows again the iron cross sections measured at 23° . For this kinematics the cross sections were extracted for the largest values of x_{Bj} ($x_{Bj} = 5.4$ for the lowest data point). The quasi-elastic peak sits at $\nu = 1.030$ GeV.

Figure 39: Various calculations are compared to the measured iron cross sections at $\theta_{HMS} = 23^\circ$.

The gray dotted lines in Figure 39 indicate the quasi-elastic and the inelastic cross section contribution calculated in PWIA without taking FSI

into account (no folding). PWIA reproduces the cross sections near and above the quasi-elastic peak. At lower energy loss it by far underestimates the cross sections, which means that FSI have to be included.

The red dashed-dotted line presents the theoretical prediction including FSI, but without taking the short-range correlations of the nucleons into account. A uniform nucleon distribution at density ρ around the struck nucleon is assumed. As a consequence the cross section is folded with a Lorentzian folding function F_L (see Figure 32). The black dashed line results when a nucleonic density $\rho \cdot g(r)$ rather than ρ is used, where $g(r)$ represents the NN pair-distribution function (see Figure 33). This takes N-N correlations into account. The cross section is folded with F_G (see Figure 32), a folding function calculated in correlated Glauber approximation. For both models the cross sections are overestimated in the very low energy loss wing of the quasi-elastic peak.

Only the blue solid curve is able to reproduce the data at the smallest values of ν . Here the effects of color transparency are included which leads to reduced FSI. The corresponding folding function, F_{CGT} (see Figure 32), is calculated in correlated Glauber approximation. Color transparency reduces the FSI significantly for the extreme kinematics shown here. As a matter of fact, the effect of color transparency found in inclusive experiments is much larger than for $(e, e'p)$ experiments of the same momentum transfer. This can be explained by the fact that the distance over which (e, e') is sensitive to the interaction of the recoiling system, $1/q$, is smaller than the distance within which the small 3-quark state evolves back to a normal nucleon. Inclusive experiments therefore observe the full effect of color transparency on FSI. In exclusive experiments, where the knocked out nucleon is detected, the distance of sensitivity to interactions with the $(A - 1)$ system is of the order of the nuclear radius. As this quantity is much larger than the distance during which the recoil nucleon remains in its *small state*, the FSI of the recoiling nucleon in $(e, e'p)$ basically is the one of the normal nucleon. Thus the effect of color transparency on FSI is much smaller.

5.2 y-Scaling

Figures 40 - 43 represent the results of the y -scaling analysis. Again, only the statistical errors are shown.

Table 9 lists the empirical separation energies [8, 27] used to calculate the mass of the recoiling $(A - 1)$ system according to equation 11.

For not too large $|y|$ the momentum transfers are large enough that the nucleon is ejected from the nucleus in a quasi-free manner. The basic assumptions made within the framework of PWIA seem to be fine in the region of

Figure 40: y -scaling for carbon Figure 41: y -scaling for aluminum

Figure 42: y -scaling for iron Figure 43: y -scaling for gold

Table 9: *The empirical separation energies used for the analysis are listed.*

Target	E_s [MeV]
Carbon	10
Aluminum	12
Iron	15
Gold	25

negative y -values where the data exhibits a nice scaling behavior. For this region $F(y)$ represents the probability of finding a nucleon with the momentum component y parallel to the momentum transfer q for the given nucleus.

If only the quasi-elastic process would contribute the scaling function would be symmetric around $y = 0$. The dominance of the deep inelastic scattering for $y > 0$ is responsible for the fact that the data do not scale for positive y . If one would replace $\tilde{\sigma}_{eN}$ of equation 27 with the corresponding e-quark cross section for the deep inelastic part and use the corresponding scaling variable, one would also find a scaling behavior in this region.

5.2.1 Q^2 -Dependence

As discussed in section 4.3.1, $F(Q^2, y)$ is expected to converge to a certain limit for Q^2 going to infinity. In order to learn how good the approach of scaling is, the scaling function $F(Q^2, y)$ has to be plotted as a function of Q^2 for fixed values of y (see Figure 44). The data points shown in Figure 44 are interpolated from $F(y)$ by applying a local fit. The same $F(y)$ model function of [4] as used for the bin centering correction (section 3.3.6) was fitted. The error was calculated from the mean of the neighboring two data points, weighted with the difference in y to the fixed y -value.

The left plots of Figure 44 show the y -scaling function at a fixed value of $y = -0.2 \text{ GeV}/c$. The scaling function seems to converge for Q^2 getting larger. But for the largest values of Q^2 the scaling function increases. This means that for this value of y there is a significant contribution from deep inelastic scattering. For deep inelastic scattering, the assumptions made to extract $F(y)$ are not correct. $F(y)$ is determined by dividing the measured cross sections by the $cc1$ cross section of de Forest (see equation 26). The de Forest cross section is a function of the nucleon form factors which get smaller with increasing Q^2 . Since deep inelastic scattering corresponds to scattering off a point-like quark which has a form factor $\equiv 1$, a contribution of deep inelastic scattering leads to an increase of the y -scaling function for large momentum transfers.

Figure 44: The approach to y -scaling for carbon, aluminum, iron and gold is shown for two fixed values of y . Only the statistical errors are shown. The data from an earlier inclusive experiment, NE3 at SLAC [4], are also shown.

The right plots of Figure 44 show the y -scaling function at a fixed value of $y = -0.4$ GeV/ c . Here the scaling function seems to reach a plateau for large Q^2 .

5.3 Superscaling

The y-scaling function allows to study the momentum distributions of the individual nuclei as demonstrated in section 5.2. The basic goal of the superscaling formalism is to find *universal* momentum distributions which can only be gained from data which exhibits the superscaling property. The Fermi momenta, k_F , and the shifts of the energy transfer, E_{shift} , that were used in the analysis to calculate ψ' and $f(\psi')$ (see section 4.3.2) are listed in Table 10.

Table 10: *Empirically chosen Fermi momenta and shift energies used in the analysis for the various targets [8].*

Target	k_F [MeV/c]	E_{shift} [MeV]
Carbon	220	15
Aluminum	230	15
Iron	235	20
Gold	245	25

Figure 45 shows the scaling function discussed in section 4.3.2 for the full data set that was analyzed. The data shows the superscaling behavior for values of ψ' somewhat below 0. For positive values of ψ' scaling of the first kind is badly violated due to cross section contributions other than quasi-elastic scattering.

Figure 45: *The scaling function $f(\psi')$ is shown for the full set of data which was analyzed.*

Figure 46: *Scaling of the second kind is shown for two fixed kinematics at $Q^2 = 0.967 \text{ GeV}^2/c^2$ (left) and $Q^2 = 1.927 \text{ GeV}^2/c^2$ (right).*

In order to investigate the scaling of the second kind, $f(\psi')$ is shown for two fixed kinematics (see Figure 46). It can be observed that the data scales with high quality for negative ψ' , which in PWIA correspond to large initial momenta of the nucleon.

For positive ψ' the increase of the Fermi momentum k_F with the mass number A is responsible for breaking the scaling of the second kind (see the left plot of Figure 46). Larger widths of k_F cause larger overlaps of the quasi-elastic peak with the Δ peak, which corresponds to a larger overlap with non quasi-elastic scattering processes that violate scaling. Going to higher momentum transfers these contributions seem to be less dominant and scaling of the second kind can be observed also for positive ψ' (see right plot of Figure 46).

6 Summary and Conclusions

Due to the high performance of accelerator and spectrometer, inclusive electron cross sections were measured two orders of magnitude smaller than previously accessible. Cross sections for carbon, aluminum, iron and gold were extracted. They were taken with a beam energy of $\varepsilon = 4.032$ GeV at angles of 15° , 23° , 30° , 37° , 45° and 55° covering a four momentum square region of $0.9 \text{ GeV}^2/c^2 < Q^2 < 4.9 \text{ GeV}^2/c^2$. The largest values of x_{Bj} were reached with the data taken at 23° ($x_{Bj} = 5.4$).

The cross sections can best be described within the framework of the correlated Glauber theory which includes an appropriate treatment of the FSI of the recoil nucleon. At the largest values of x_{Bj} the cross sections can only be described when the effects of color transparency on the FSI are included.

Under the assumption of plane wave impulse approximation (PWIA) the $F(y)$ scaling functions were extracted from the cross sections for values of y as low as $-0.8 \text{ GeV}/c$. Scaling of the momentum transfer (scaling of the first kind) was observed for $y < 0$ which can be used to determine momentum distributions of nucleons in nuclei. While at $y = -0.2 \text{ GeV}/c$ the contributions from other processes than quasi-elastic scattering are not negligible and impair scaling, $F(y, Q^2)$ converges at $y = -0.4 \text{ GeV}/c$ and below with increasing Q^2 .

The dimensionless $f(\psi')$ scaling functions were extracted from the cross sections within the framework of the relativistic Fermi gas (RFG), where the Fermi momentum k_F sets the scale. For $\psi' < 0$ the data exhibit the superscaling property (independence of the momentum transfer and of the nuclear species) which means that they can be used to determine a general or *universal* momentum distribution of nucleons in matter. For $\psi' > 0$ scaling of the first and of the second kind are violated for the lowest Q^2 data. Surprisingly the data for $Q^2 \geq 1.9 \text{ GeV}^2/c^2$ also scale with the mass number A for positive ψ' ; this is not yet understood.

A cc1 Off-Shell e-N Cross Section

The formulae of de Forest known as *cc1* [33] are used to describe a single e-N cross section for a moving off-shell nucleon. The formulae are given as they are discussed in the appendix of [8]. In this form the off-shell character of *cc1* is pointed out in a clear way.

In PWIA the energy transfer for an off-shell nucleon that is struck by a virtual photon is given by $\nu = E_N - E$. According to Figure 30 the initial and final nucleon energies E and E_N can be written as

$$E = M_A^0 - \sqrt{(M_{A-1}^0)^2 + k^2} - E^* \quad E_N = \sqrt{m_N^2 + (\vec{k} + \vec{q})^2}, \quad (36)$$

where E^* characterizes how excited the remaining $(A-1)$ system is after the nucleon knockout.

Bringing the nucleon on its mass shell leads to an energy transfer $\bar{\nu} \equiv E_N - \bar{E}$ with the on-shell nucleon energy $\bar{E}$ as

$$E \neq \bar{E} = \sqrt{(m_N^2 + k^2)}. \quad (37)$$

Integrating over the azimuthal angles and summing over all nucleons assuming that the spectral function does not differ for protons and neutrons, the cross section can be written as

$$\begin{aligned} \tilde{\sigma}_{eN} &= \sigma_{Mott} \left[v_L \tilde{w}_L + v_T \tilde{w}_T \right] \\ \sigma_{Mott} &= \left[\frac{\alpha \cdot \cos(\theta/2)}{2 \cdot \varepsilon \cdot \sin^2(\theta/2)} \right]^2, \end{aligned} \quad (38)$$

where the two kinematical factors v_L and v_T are

$$v_L = \frac{Q^4}{q^4} \quad v_T = \frac{Q^2}{2q^2} + \tan^2(\theta/2). \quad (39)$$

The longitudinal and the transverse contribution may be written as

$$\begin{aligned} \tilde{w}_L &= \frac{1}{2\kappa\sqrt{1+\eta^2}} \left(\frac{\kappa^2}{\bar{\tau}} \right) \left[\tilde{G}_E^2 + \delta^2 (\tilde{W}_2 + \Delta\tilde{W}_1) \right. \\ &\quad \left. + (1 + \bar{\tau})\Delta\tilde{W}_1 + (1 + \tau)\Delta\tilde{W}_2 \right] \\ \tilde{w}_T &= \frac{1}{2\kappa\sqrt{1+\eta^2}} \left[2\bar{\tau}\tilde{G}_M^2 + \delta^2 (\tilde{W}_2 + \Delta\tilde{W}_1) \right] \end{aligned} \quad (40)$$

where

$$\delta^2 = \frac{\bar{\tau}}{\kappa^2} \left(\frac{E_N + \bar{E}}{2m_N} \right)^2 - (1 + \bar{\tau}). \quad (41)$$

Here the dimensionless variables $\eta \equiv k/m_N$, $\kappa \equiv q/2m_N$, $\lambda \equiv \nu/2m_N$ and $\tau \equiv \kappa^2 - \lambda^2$ are employed. The corresponding *on-shell* variables for λ and τ are $\bar{\lambda} \equiv \bar{\nu}/2m_N$ and $\bar{\tau} \equiv \kappa^2 - \bar{\lambda}^2$.

Finally the familiar Sachs form factors for the neutron and the proton enter the equations the following way:

$$\begin{aligned}\tilde{G}_E^2(\tau) &= ZG_{Ep}^2 + NG_{En}^2 \\ \tilde{G}_M^2(\tau) &= ZG_{Mp}^2 + NG_{Mn}^2 \\ \Delta\tilde{G}(\tau) &= ZG_{Ep}G_{Mp} + NG_{En}G_{Mn}\end{aligned}\tag{42}$$

Some parameterizations of fits to these form factors [26, 51–54] are compared in Figure 47.

Figure 47: The electric and magnetic form factors of the proton, G_{ep} and G_{mp} , and of the neutron, G_{en} and G_{mn} , divided by the empirical dipole-form of the form factors are plotted for a Q^2 -range that covers the data range of E89-008.

$$\begin{aligned}
\widetilde{W}_1(\tau) &= \tau \widetilde{G}_M^2 \\
\widetilde{W}_2(\tau) &= \frac{1}{1+\tau} [\widetilde{G}_E^2 + \tau \widetilde{G}_M^2] \\
\Delta \widetilde{W}_1(\tau, \bar{\tau}) &= \frac{\bar{\tau} - \tau}{(1+\tau)^2} [\widetilde{G}_E^2 + \widetilde{G}_M^2 - 2\Delta \widetilde{G}] \\
\Delta \widetilde{W}_2(\tau, \bar{\tau}) &= \frac{\bar{\tau} - \tau}{(1+\tau)^2} [\widetilde{G}_E^2 - \widetilde{G}_M^2] .
\end{aligned} \tag{43}$$

Figure 1 consists of two side-by-side plots showing the ratio of the nucleon dipole form factor to the nucleon elastic form factor, $\sigma_{eN}/\sigma_{eN}(\text{Dipol-FF})$, as a function of the virtuality v in GeV^2 . The left plot covers the range $v \in [0.5, 2.0]$ and the right plot covers $v \in [2.0, 3.0]$. Both plots show four theoretical curves: Bosted (1995) in red dashed, Mergell (1990) in black solid, Gari (1985) in red dash-dotted, and Hoehler 8.2 (1976) in blue dotted. The curves generally show a slight increase or decrease with v , with the Hoehler 8.2 (1976) curve being the highest and the Bosted (1995) curve being the lowest.

Figure 48: Ratios of $\tilde{\sigma}_{eN}$ calculated with different parameterizations are shown for Iron. The plots cover the data range taken at 30° (left) and at 55° (right).

B On-Shell e-N Cross Section

In the RFG model the on-shell e-N cross section summed over all protons and neutrons can be written as

$$\tilde{\sigma}_{eN}^{RFG} = \sigma_{Mott} [v_L G_L + v_T G_T] \quad (44)$$

with σ_{Mott} and the two kinematical factors v_L and v_T as defined in equation 39.

The on-shell longitudinal and transverse e-N responses G_L and G_T [49, 55, 56] are given by the expressions:

$$\begin{aligned} G_L(\kappa, \lambda) &= \frac{(\kappa^2/\tau)[\tilde{G}_E^2 + \tilde{W}_2\Delta]}{2\kappa[1 + \xi_F(1 + \psi^2)/2]} \\ G_T(\kappa, \lambda) &= \frac{2\tau\tilde{G}_M^2 + \tilde{W}_2\Delta}{2\kappa[1 + \xi_F(1 + \psi^2)/2]} \end{aligned} \quad (45)$$

with $\xi_F = \sqrt{1 + \eta_F^2} - 1$ and $\eta_F = k_F/m_N$ being the dimensionless Fermi kinetic energy and momentum, and

$$\Delta = \xi_F(1 - \psi^2) \left[\frac{\sqrt{\tau(1 + \tau)}}{\kappa} + \frac{1}{3}\xi_F(1 - \psi^2)\frac{\tau}{\kappa^2} \right]. \quad (46)$$

C Beam Energy Measurements

Two beam energy measurements were performed in 1996 for the setup of a 4.045 GeV beam. One was done on the 31st of March before E89-008 took data. The other one was performed on the 27th of October [57] roughly two months after E89-008 had finished taking data. For both measurements the Hall C arc was used as a spectrometer. Only the dipole magnets of the arc were turned on and the position and direction of the beam were measured at the beginning, the center and the end of the arc. Knowing the field integral ($\int Bdl$) and the absolute position of the beam, the energy can be determined. The measured energies were 4.045(4) GeV (March) and 4.055(4) GeV (October).

Later on it was discovered that the cycling procedure for the arc dipoles differed from the one used when the magnets were field-mapped. This led to a correction as shown in Figure 49. For a beam energy of 4.05 GeV the correction is 0.19(4) %. This results in corrected beam energies of $\varepsilon = 4.037(4)$ GeV (March) and $\varepsilon = 4.047(4)$ GeV (October).

Figure 49: *The correction applied to the beam energy measured with the Hall C arc is shown as a function of the beam energy. The correction has an uncertainty of 20 %.*

The beam energy deduced in this analysis is $\varepsilon = 4.032(4)$ GeV (see section 3.2.2). This value is 0.12 % lower than the energy measured in March and 0.37 % lower than what was measured in October.

D Cross Section Tables

The tables below contain the final radiatively unfolded cross sections with their statistical errors. The energy transfers ν are given in units of GeV and the cross sections are listed in units of nb/sr/GeV.

Table 11: *Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 15^\circ$*

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
0.040				.13(10)·10 ¹
0.060			.49(25)	.67(23)·10 ¹
0.080	.194(70)		.473(80)·10 ¹	.140(34)·10 ²
0.100	.105(17)·10 ¹	.179(60)·10 ¹	.83(11)·10 ¹	.206(35)·10 ²
0.120	.236(26)·10 ¹	.66(10)·10 ¹	.162(15)·10 ²	.694(69)·10 ²
0.140	.411(34)·10 ¹	.134(14)·10 ²	.302(21)·10 ²	.1020(86)·10 ³
0.160	.920(51)·10 ¹	.250(20)·10 ²	.475(26)·10 ²	.192(11)·10 ³
0.180	.1485(67)·10 ²	.474(29)·10 ²	.903(37)·10 ²	.362(17)·10 ³
0.200	.2485(92)·10 ²	.743(37)·10 ²	.1443(48)·10 ³	.548(21)·10 ³
0.220	.393(11)·10 ²	.1078(44)·10 ³	.2061(58)·10 ³	.774(24)·10 ³
0.240	.587(15)·10 ²	.1798(60)·10 ³	.3030(72)·10 ³	.1154(31)·10 ⁴
0.260	.933(20)·10 ²	.2811(81)·10 ³	.4743(95)·10 ³	.1736(40)·10 ⁴
0.280	.1466(28)·10 ³	.4033(98)·10 ³	.719(13)·10 ³	.2602(54)·10 ⁴
0.300	.2135(37)·10 ³	.592(13)·10 ³	.1046(16)·10 ⁴	.3762(70)·10 ⁴
0.320	.3048(49)·10 ³	.855(17)·10 ³	.1533(22)·10 ⁴	.5586(95)·10 ⁴
0.340	.4143(61)·10 ³	.1199(22)·10 ⁴	.2020(26)·10 ⁴	.690(11)·10 ⁴
0.360	.5790(84)·10 ³	.1622(28)·10 ⁴	.2713(33)·10 ⁴	.964(14)·10 ⁴
0.380	.784(11)·10 ³	.2000(31)·10 ⁴	.3497(39)·10 ⁴	.1203(17)·10 ⁵
0.400	.1031(14)·10 ⁴	.2384(36)·10 ⁴	.4408(47)·10 ⁴	.1409(21)·10 ⁵
0.420	.1264(16)·10 ⁴	.2758(45)·10 ⁴	.5397(65)·10 ⁴	.1783(25)·10 ⁵
0.440	.1437(17)·10 ⁴	.3329(52)·10 ⁴	.6094(70)·10 ⁴	.1986(27)·10 ⁵
0.460	.1701(22)·10 ⁴	.3688(57)·10 ⁴	.7025(78)·10 ⁴	.2209(29)·10 ⁵
0.480	.1883(20)·10 ⁴	.4029(61)·10 ⁴	.7703(83)·10 ⁴	.2451(31)·10 ⁵
0.500	.2015(21)·10 ⁴	.4319(49)·10 ⁴	.8170(87)·10 ⁴	.2610(33)·10 ⁵
0.520	.2101(21)·10 ⁴	.4571(51)·10 ⁴	.8571(84)·10 ⁴	.2646(32)·10 ⁵
0.540	.2192(22)·10 ⁴	.4770(53)·10 ⁴	.8753(84)·10 ⁴	.2851(34)·10 ⁵
0.560	.2265(22)·10 ⁴	.4836(53)·10 ⁴	.9327(89)·10 ⁴	.3030(32)·10 ⁵
0.580	.2271(22)·10 ⁴	.4835(53)·10 ⁴	.9492(89)·10 ⁴	.3092(32)·10 ⁵
0.600	.2303(22)·10 ⁴	.4857(52)·10 ⁴	.9673(90)·10 ⁴	.3313(73)·10 ⁵
0.620	.2271(36)·10 ⁴	.4944(53)·10 ⁴	.1035(12)·10 ⁵	.3452(75)·10 ⁵
0.640	.2241(36)·10 ⁴	.4987(53)·10 ⁴	.1056(29)·10 ⁵	.3554(76)·10 ⁵
0.660	.2238(35)·10 ⁴	.4960(52)·10 ⁴	.1045(28)·10 ⁵	.3536(75)·10 ⁵
0.680	.2290(36)·10 ⁴	.4850(50)·10 ⁴	.1025(28)·10 ⁵	.3547(74)·10 ⁵
0.700	.2230(35)·10 ⁴	.4879(51)·10 ⁴	.1041(28)·10 ⁵	.3514(73)·10 ⁵
0.720	.2222(34)·10 ⁴	.4800(50)·10 ⁴	.1070(28)·10 ⁵	.3580(73)·10 ⁵
0.740	.2208(34)·10 ⁴	.4788(50)·10 ⁴	.1043(27)·10 ⁵	.3470(70)·10 ⁵
0.760	.2247(35)·10 ⁴	.4873(50)·10 ⁴	.1045(28)·10 ⁵	.3546(72)·10 ⁵

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 15^\circ$ (continued)

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
0.780	.2255(34)·10 ⁴	.4918(51)·10 ⁴	.1043(27)·10 ⁵	.3488(70)·10 ⁵
0.800	.2319(36)·10 ⁴	.5226(54)·10 ⁴	.1075(28)·10 ⁵	.3782(74)·10 ⁵
0.820	.2240(34)·10 ⁴	.5244(80)·10 ⁴	.1095(28)·10 ⁵	.3702(72)·10 ⁵
0.840	.2379(36)·10 ⁴	.5304(82)·10 ⁴	.1073(27)·10 ⁵	.3741(73)·10 ⁵
0.860	.2400(36)·10 ⁴	.5329(82)·10 ⁴	.1085(28)·10 ⁵	.3689(72)·10 ⁵
0.880	.2336(35)·10 ⁴	.5471(84)·10 ⁴	.1048(27)·10 ⁵	.3702(71)·10 ⁵
0.900	.2328(35)·10 ⁴	.5400(83)·10 ⁴	.1082(27)·10 ⁵	.3740(72)·10 ⁵
0.920	.2359(35)·10 ⁴	.5540(84)·10 ⁴	.1107(28)·10 ⁵	.3791(72)·10 ⁵
0.940	.2476(37)·10 ⁴	.5614(85)·10 ⁴	.1136(28)·10 ⁵	.3963(74)·10 ⁵
0.960	.2386(35)·10 ⁴	.5439(82)·10 ⁴	.1093(27)·10 ⁵	.3833(72)·10 ⁵
0.980	.2420(36)·10 ⁴	.5472(82)·10 ⁴	.1145(28)·10 ⁵	.3927(73)·10 ⁵
1.000	.2462(36)·10 ⁴	.5419(81)·10 ⁴	.1191(29)·10 ⁵	.3855(72)·10 ⁵
1.020	.2322(34)·10 ⁴	.5290(79)·10 ⁴	.1111(28)·10 ⁵	.3841(71)·10 ⁵
1.040	.2393(35)·10 ⁴	.5231(78)·10 ⁴	.1130(28)·10 ⁵	.3848(71)·10 ⁵
1.060	.2448(36)·10 ⁴	.5362(79)·10 ⁴	.1140(28)·10 ⁵	.3870(71)·10 ⁵
1.080	.2555(37)·10 ⁴	.5503(82)·10 ⁴	.1153(28)·10 ⁵	.4063(74)·10 ⁵
1.100	.2587(38)·10 ⁴		.1223(29)·10 ⁵	.4062(74)·10 ⁵

Table 12: Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 23^\circ$

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
0.260	.48(28)·10 ⁻³		.18(13)·10 ⁻²	
0.280	.74(29)·10 ⁻³		.49(22)·10 ⁻²	
0.300	.120(35)·10 ⁻²		.176(41)·10 ⁻¹	
0.320	.150(38)·10 ⁻²	.57(33)·10 ⁻²	.156(34)·10 ⁻¹	
0.340	.330(56)·10 ⁻²	.82(42)·10 ⁻²	.381(54)·10 ⁻¹	
0.360	.614(79)·10 ⁻²	.280(75)·10 ⁻¹	.476(59)·10 ⁻¹	.286(72)
0.380	.137(12)·10 ⁻¹	.416(92)·10 ⁻¹	.767(72)·10 ⁻¹	.366(81)
0.400	.189(14)·10 ⁻¹	.72(13)·10 ⁻¹	.1200(93)	.503(94)
0.420	.311(18)·10 ⁻¹	.89(14)·10 ⁻¹	.239(14)	.66(11)
0.440	.486(22)·10 ⁻¹	.175(19)	.272(14)	.128(15)·10 ¹
0.460	.777(29)·10 ⁻¹	.224(22)	.441(18)	.144(16)·10 ¹
0.480	.1094(34)	.375(30)	.646(23)	.245(21)·10 ¹
0.500	.1570(41)	.563(37)	.903(27)	.298(23)·10 ¹
0.520	.2406(53)	.649(38)	.1232(32)·10 ¹	.473(30)·10 ¹
0.540	.3382(63)	.1010(47)·10 ¹	.1667(37)·10 ¹	.685(34)·10 ¹
0.560	.4383(74)	.1247(53)·10 ¹	.2220(44)·10 ¹	.784(35)·10 ¹
0.580	.5927(89)	.1862(67)·10 ¹	.3207(55)·10 ¹	.1048(41)·10 ²
0.600	.772(10)	.2287(75)·10 ¹	.4002(62)·10 ¹	.1411(48)·10 ²
0.620	.1087(20)·10 ¹	.2971(89)·10 ¹	.5440(79)·10 ¹	.1833(55)·10 ²
0.640	.1410(24)·10 ¹	.399(10)·10 ¹	.662(15)·10 ¹	.2257(62)·10 ²
0.660	.1707(27)·10 ¹	.475(11)·10 ¹	.879(18)·10 ¹	.2938(70)·10 ²
0.680	.2274(32)·10 ¹	.664(15)·10 ¹	.1091(20)·10 ²	.3668(80)·10 ²

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 23^\circ$ (continued)

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
0.700	.2967(39)·10 ¹	.866(18)·10 ¹	.1431(24)·10 ²	.4755(92)·10 ²
0.720	.3986(50)·10 ¹	.1166(22)·10 ²	.1852(28)·10 ²	.651(11)·10 ²
0.740	.5257(61)·10 ¹	.1474(25)·10 ²	.2427(34)·10 ²	.858(14)·10 ²
0.760	.7410(83)·10 ¹	.2003(32)·10 ²	.3374(45)·10 ²	.1128(17)·10 ³
0.780	.924(21)·10 ¹	.2366(66)·10 ²	.4335(57)·10 ²	.1466(44)·10 ³
0.800	.1235(25)·10 ²	.2850(74)·10 ²	.542(13)·10 ²	.1754(49)·10 ³
0.820	.1528(28)·10 ²	.3640(84)·10 ²	.669(14)·10 ²	.2134(54)·10 ³
0.840	.1879(32)·10 ²	.4537(96)·10 ²	.825(17)·10 ²	.2653(62)·10 ³
0.860	.2370(37)·10 ²	.533(11)·10 ²	.1021(19)·10 ³	.3153(67)·10 ³
0.880	.2735(41)·10 ²	.665(12)·10 ²	.1229(21)·10 ³	.3626(73)·10 ³
0.900	.3324(47)·10 ²	.789(13)·10 ²	.1415(23)·10 ³	.4221(80)·10 ³
0.920	.3986(53)·10 ²	.852(14)·10 ²	.1685(26)·10 ³	.5077(89)·10 ³
0.940	.4555(59)·10 ²	.976(15)·10 ²	.1888(28)·10 ³	.5707(95)·10 ³
0.960	.5012(62)·10 ²	.1123(16)·10 ³	.2025(28)·10 ³	.637(10)·10 ³
0.980	.5553(66)·10 ²	.1226(17)·10 ³	.2263(31)·10 ³	.684(11)·10 ³
1.000	.6196(74)·10 ²	.1334(17)·10 ³	.2497(27)·10 ³	.8080(96)·10 ³
1.020	.6507(75)·10 ²	.1461(17)·10 ³	.2691(28)·10 ³	.8558(98)·10 ³
1.040	.7043(79)·10 ²	.1529(18)·10 ³	.2889(29)·10 ³	.927(10)·10 ³
1.060	.7332(81)·10 ²	.1639(19)·10 ³	.3065(30)·10 ³	.989(11)·10 ³
1.080	.7923(87)·10 ²	.1740(19)·10 ³	.3329(32)·10 ³	.1053(11)·10 ⁴
1.100	.8491(92)·10 ²	.1853(20)·10 ³	.3586(34)·10 ³	.1138(12)·10 ⁴
1.120		.2112(32)·10 ³	.3882(59)·10 ³	.1255(20)·10 ⁴
1.140		.2121(32)·10 ³	.4072(60)·10 ³	.1347(21)·10 ⁴
1.160		.2238(33)·10 ³	.4319(62)·10 ³	.1402(21)·10 ⁴
1.180		.2345(34)·10 ³	.4502(63)·10 ³	.1473(22)·10 ⁴
1.200		.2322(34)·10 ³	.4602(64)·10 ³	.1522(22)·10 ⁴
1.220		.2539(36)·10 ³	.4746(65)·10 ³	.1619(23)·10 ⁴
1.240		.2573(35)·10 ³	.4971(67)·10 ³	.1642(23)·10 ⁴
1.260		.2783(38)·10 ³	.5087(67)·10 ³	.1683(23)·10 ⁴
1.280		.2925(39)·10 ³	.5455(71)·10 ³	.1779(24)·10 ⁴
1.300		.3019(61)·10 ³	.5573(71)·10 ³	.1845(24)·10 ⁴
1.320		.3266(66)·10 ³	.5873(74)·10 ³	.1898(24)·10 ⁴
1.340		.3330(66)·10 ³	.6174(76)·10 ³	.2016(25)·10 ⁴
1.360		.3508(68)·10 ³	.6304(78)·10 ³	.2167(27)·10 ⁴
1.380		.3664(70)·10 ³	.6625(80)·10 ³	.2201(27)·10 ⁴
1.400		.3615(69)·10 ³	.6813(82)·10 ³	.2309(28)·10 ⁴
1.420		.3901(73)·10 ³	.7295(86)·10 ³	.2412(29)·10 ⁴
1.440		.3947(73)·10 ³	.7545(88)·10 ³	.2450(29)·10 ⁴
1.460		.4152(75)·10 ³	.7760(89)·10 ³	.2558(30)·10 ⁴
1.480		.4201(76)·10 ³	.8116(92)·10 ³	.2656(31)·10 ⁴
1.500		.4428(79)·10 ³	.8432(95)·10 ³	.2775(32)·10 ⁴

Table 13: *Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 30^\circ$*

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
0.640	.122(88) $\cdot 10^{-3}$		.109(59) $\cdot 10^{-2}$	
0.660	.121(87) $\cdot 10^{-3}$		.265(95) $\cdot 10^{-2}$	
0.680	.39(16) $\cdot 10^{-3}$		.224(96) $\cdot 10^{-2}$	
0.700	.36(17) $\cdot 10^{-3}$		.37(12) $\cdot 10^{-2}$	
0.720	.92(25) $\cdot 10^{-3}$		.44(13) $\cdot 10^{-2}$	
0.740	.153(31) $\cdot 10^{-2}$		.124(20) $\cdot 10^{-1}$	
0.760	.208(28) $\cdot 10^{-2}$		.151(23) $\cdot 10^{-1}$	.53(39) $\cdot 10^{-1}$
0.780	.397(38) $\cdot 10^{-2}$		.177(24) $\cdot 10^{-1}$	.142(59)
0.800	.452(41) $\cdot 10^{-2}$	.113(65) $\cdot 10^{-1}$	.223(28) $\cdot 10^{-1}$	.71(42) $\cdot 10^{-1}$
0.820	.626(49) $\cdot 10^{-2}$	.107(62) $\cdot 10^{-1}$	.350(35) $\cdot 10^{-1}$	.113(51)
0.840	.990(63) $\cdot 10^{-2}$	.241(99) $\cdot 10^{-1}$	.513(43) $\cdot 10^{-1}$	.83(49) $\cdot 10^{-1}$
0.860	.1167(69) $\cdot 10^{-1}$	.225(92) $\cdot 10^{-1}$	.624(47) $\cdot 10^{-1}$	.195(71)
0.880	.1572(77) $\cdot 10^{-1}$	.44(14) $\cdot 10^{-1}$	.729(47) $\cdot 10^{-1}$	.367(93)
0.900	.2157(93) $\cdot 10^{-1}$	.49(14) $\cdot 10^{-1}$	.1103(62)	.269(79)
0.920	.290(11) $\cdot 10^{-1}$	.85(21) $\cdot 10^{-1}$	.1428(70)	.48(11)
0.940	.359(12) $\cdot 10^{-1}$	.118(23)	.1744(78)	.55(12)
0.960	.470(14) $\cdot 10^{-1}$	.126(24)	.2355(91)	.70(13)
0.980	.581(15) $\cdot 10^{-1}$	.160(31)	.2792(99)	.926(95)
1.000	.726(17) $\cdot 10^{-1}$	.259(36)	.388(12)	.120(11) $\cdot 10^1$
1.020	.952(19) $\cdot 10^{-1}$	.224(35)	.4893(97)	.168(13) $\cdot 10^1$
1.040	.1236(22)	.332(44)	.620(11)	.208(14) $\cdot 10^1$
1.060	.1560(25)	.457(47)	.759(12)	.282(17) $\cdot 10^1$
1.080	.2152(43)	.454(37)	.1016(14) $\cdot 10^1$	.335(18) $\cdot 10^1$
1.100	.2636(48)	.734(49)	.1317(17) $\cdot 10^1$	.421(20) $\cdot 10^1$
1.120	.3479(57)	.824(51)	.1725(29) $\cdot 10^1$	.485(22) $\cdot 10^1$
1.140	.4401(66)	.1249(70) $\cdot 10^1$	.2285(34) $\cdot 10^1$	.648(25) $\cdot 10^1$
1.160	.5636(77)	.1541(76) $\cdot 10^1$	.2881(40) $\cdot 10^1$	.898(29) $\cdot 10^1$
1.180	.7401(94)	.1869(84) $\cdot 10^1$	.3676(47) $\cdot 10^1$	.1090(33) $\cdot 10^2$
1.200	.947(11)	.248(10) $\cdot 10^1$	.4565(55) $\cdot 10^1$	.1346(37) $\cdot 10^2$
1.220	.1179(13) $\cdot 10^1$	.322(11) $\cdot 10^1$	.5727(65) $\cdot 10^1$	.1771(43) $\cdot 10^2$
1.240	.1568(32) $\cdot 10^1$	.3734(94) $\cdot 10^1$	.6995(74) $\cdot 10^1$	.2097(46) $\cdot 10^2$
1.260	.1892(35) $\cdot 10^1$	.447(10) $\cdot 10^1$	.8490(85) $\cdot 10^1$	.2585(52) $\cdot 10^2$
1.280	.2383(41) $\cdot 10^1$	.546(11) $\cdot 10^1$	.1046(10) $\cdot 10^2$	.3137(59) $\cdot 10^2$
1.300	.2776(45) $\cdot 10^1$	.666(15) $\cdot 10^1$	.1233(11) $\cdot 10^2$	.3913(87) $\cdot 10^2$
1.320	.3332(50) $\cdot 10^1$	.787(17) $\cdot 10^1$	.1448(13) $\cdot 10^2$	.4615(95) $\cdot 10^2$
1.340	.3945(56) $\cdot 10^1$	.932(18) $\cdot 10^1$	.1706(14) $\cdot 10^2$	.522(10) $\cdot 10^2$
1.360	.4623(62) $\cdot 10^1$	.1085(20) $\cdot 10^2$	.2007(16) $\cdot 10^2$	.610(11) $\cdot 10^2$
1.380	.5252(67) $\cdot 10^1$	.1264(21) $\cdot 10^2$	.2277(18) $\cdot 10^2$	.672(12) $\cdot 10^2$
1.400	.5772(71) $\cdot 10^1$	.1368(21) $\cdot 10^2$	.2532(19) $\cdot 10^2$	.750(12) $\cdot 10^2$
1.420	.6651(79) $\cdot 10^1$	.1481(22) $\cdot 10^2$	.2815(21) $\cdot 10^2$	.829(13) $\cdot 10^2$
1.440	.7457(86) $\cdot 10^1$	.1618(23) $\cdot 10^2$	.3139(23) $\cdot 10^2$	.910(12) $\cdot 10^2$
1.460	.8354(79) $\cdot 10^1$	.1864(25) $\cdot 10^2$	.3462(24) $\cdot 10^2$	.1054(13) $\cdot 10^3$
1.480	.9066(83) $\cdot 10^1$	.2051(25) $\cdot 10^2$	.3852(26) $\cdot 10^2$	.1165(13) $\cdot 10^3$

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 30^\circ$ (continued)

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
1.500	.9886(87)·10 ¹	.2263(26)·10 ²	.4172(27)·10 ²	.1267(14)·10 ³
1.520	.1123(10)·10 ²	.2420(27)·10 ²	.452(10)·10 ²	.1453(15)·10 ³
1.540	.1235(18)·10 ²	.2666(31)·10 ²	.5276(77)·10 ²	.1651(28)·10 ³
1.560	.1290(18)·10 ²	.2830(33)·10 ²	.5678(80)·10 ²	.1848(30)·10 ³
1.580	.1453(20)·10 ²	.3099(35)·10 ²	.6120(83)·10 ²	.2057(32)·10 ³
1.600	.1537(21)·10 ²	.3290(36)·10 ²	.6804(88)·10 ²	.2089(32)·10 ³
1.620	.1616(21)·10 ²	.3550(37)·10 ²	.6896(74)·10 ²	.2256(33)·10 ³
1.640	.1695(22)·10 ²	.3752(38)·10 ²	.7537(78)·10 ²	.2304(33)·10 ³
1.660	.1824(23)·10 ²	.4037(41)·10 ²	.7939(79)·10 ²	.2486(34)·10 ³
1.680	.1999(24)·10 ²	.4279(42)·10 ²	.8668(84)·10 ²	.2662(36)·10 ³
1.700	.2068(24)·10 ²	.4616(74)·10 ²	.9113(85)·10 ²	.2886(38)·10 ³
1.720	.2133(25)·10 ²	.4984(77)·10 ²	.9781(88)·10 ²	.3039(39)·10 ³
1.740	.2351(27)·10 ²	.5500(84)·10 ²	.1065(9)·10 ³	.3321(41)·10 ³
1.760	.2519(28)·10 ²	.5731(86)·10 ²	.1148(10)·10 ³	.3577(35)·10 ³
1.780	.2615(22)·10 ²	.6034(66)·10 ²	.1221(9)·10 ³	.3865(36)·10 ³
1.800	.2787(23)·10 ²	.6449(77)·10 ²	.1321(9)·10 ³	.4088(37)·10 ³
1.820	.3018(24)·10 ²	.6909(81)·10 ²	.1412(10)·10 ³	.4376(39)·10 ³
1.840	.3163(25)·10 ²	.7247(82)·10 ²	.1488(10)·10 ³	.4665(40)·10 ³
1.860	.3428(26)·10 ²	.7957(87)·10 ²	.1580(10)·10 ³	.4896(41)·10 ³
1.880	.3770(28)·10 ²	.8363(88)·10 ²	.1700(11)·10 ³	.5378(44)·10 ³
1.900	.3908(43)·10 ²	.888(12)·10 ²	.1803(16)·10 ³	.5823(73)·10 ³
1.920	.4068(44)·10 ²	.927(12)·10 ²	.1899(16)·10 ³	.6061(74)·10 ³
1.940	.4233(45)·10 ²	.937(12)·10 ²	.1982(17)·10 ³	.6493(78)·10 ³
1.960	.4474(46)·10 ²	.987(13)·10 ²	.2057(17)·10 ³	.6741(79)·10 ³
1.980	.4647(47)·10 ²	.1042(13)·10 ³	.2173(18)·10 ³	.7127(82)·10 ³
2.000	.4794(48)·10 ²	.1122(14)·10 ³	.2298(18)·10 ³	.7414(83)·10 ³
2.020	.5071(50)·10 ²	.1176(14)·10 ³	.2365(19)·10 ³	.7565(84)·10 ³
2.040	.5330(52)·10 ²	.1218(15)·10 ³	.2534(27)·10 ³	.7949(87)·10 ³
2.060	.5456(52)·10 ²	.1256(15)·10 ³	.2586(27)·10 ³	.8166(88)·10 ³
2.080	.5572(53)·10 ²	.1310(15)·10 ³	.2701(28)·10 ³	.8611(91)·10 ³
2.100	.5899(56)·10 ²	.1324(15)·10 ³	.2881(29)·10 ³	.8996(94)·10 ³
2.120	.6083(57)·10 ²	.1364(16)·10 ³	.2953(30)·10 ³	.9135(95)·10 ³
2.140	.6342(58)·10 ²	.1452(16)·10 ³	.3124(26)·10 ³	.9698(98)·10 ³
2.160		.1587(18)·10 ³	.3386(29)·10 ³	
2.180			.3497(55)·10 ³	
2.200			.3535(55)·10 ³	
2.220			.3595(55)·10 ³	
2.240			.3798(56)·10 ³	
2.260			.3831(56)·10 ³	
2.280			.3941(56)·10 ³	
2.300			.3972(57)·10 ³	
2.320			.4113(58)·10 ³	
2.340			.4169(58)·10 ³	

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 30^\circ$ (continued)

ν	<i>Carbon</i>	<i>Aluminum</i>	<i>Iron</i>	<i>Gold</i>
2.360			.4437(60)·10 ³	
2.380			.4390(59)·10 ³	
2.400			.4552(41)·10 ³	
2.420			.4738(42)·10 ³	
2.440			.4828(42)·10 ³	
2.460			.4993(43)·10 ³	
2.480			.5120(57)·10 ³	
2.500			.5242(57)·10 ³	
2.520			.5202(56)·10 ³	
2.540			.5201(56)·10 ³	
2.560			.5138(55)·10 ³	
2.580			.5386(56)·10 ³	
2.600			.5484(57)·10 ³	
2.620			.5510(43)·10 ³	
2.640			.5794(44)·10 ³	
2.660			.5939(44)·10 ³	
2.680			.6164(67)·10 ³	
2.700			.6091(66)·10 ³	
2.720			.6141(65)·10 ³	
2.740			.6249(66)·10 ³	
2.760			.6091(64)·10 ³	
2.780			.6365(66)·10 ³	
2.800			.6327(65)·10 ³	
2.820			.6504(44)·10 ³	
2.840			.6806(45)·10 ³	
2.860			.7005(46)·10 ³	
2.880			.7018(58)·10 ³	
2.900			.7084(58)·10 ³	
2.920			.7183(58)·10 ³	
2.940			.7036(57)·10 ³	
2.960			.6986(57)·10 ³	
2.980			.7055(57)·10 ³	
3.000			.7218(58)·10 ³	
3.020			.7285(60)·10 ³	

Table 14: *Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 37^\circ$*

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
1.100		.17(10)·10 ⁻²	
1.120		.26(14)·10 ⁻²	
1.140		.55(19)·10 ⁻²	
1.160		.38(15)·10 ⁻²	
1.180		.31(13)·10 ⁻²	

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 37^\circ$ (continued)

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
1.200		.39(16)·10 ⁻²	
1.220		.98(24)·10 ⁻²	
1.240		.90(23)·10 ⁻²	
1.260		.108(24)·10 ⁻¹	
1.280		.174(32)·10 ⁻¹	
1.300		.243(38)·10 ⁻¹	
1.320		.328(43)·10 ⁻¹	
1.340		.427(51)·10 ⁻¹	
1.360		.553(59)·10 ⁻¹	
1.380		.790(69)·10 ⁻¹	
1.400		.879(71)·10 ⁻¹	
1.420	.209(10)·10 ⁻¹	.1094(57)	.270(28)
1.440	.281(12)·10 ⁻¹	.1543(68)	.474(37)
1.460	.413(14)·10 ⁻¹	.1836(72)	.601(42)
1.480	.514(16)·10 ⁻¹	.2255(80)	.794(48)
1.500	.642(18)·10 ⁻¹	.2898(91)	.846(49)
1.520	.810(20)·10 ⁻¹	.373(11)	.1112(56)·10 ¹
1.540	.1044(23)	.536(18)	.1400(62)·10 ¹
1.560	.1313(26)	.646(20)	.1786(71)·10 ¹
1.580	.1702(30)	.852(23)	.2405(84)·10 ¹
1.600	.2087(35)	.1049(25)·10 ¹	.3011(94)·10 ¹
1.620	.2673(41)	.1267(28)·10 ¹	.363(10)·10 ¹
1.640	.3332(46)	.1560(31)·10 ¹	.446(11)·10 ¹
1.660	.4083(53)	.1868(35)·10 ¹	.553(13)·10 ¹
1.680	.5195(63)	.2312(40)·10 ¹	.658(14)·10 ¹
1.700	.6049(69)	.2815(44)·10 ¹	.757(15)·10 ¹
1.720	.7223(78)	.3334(48)·10 ¹	.932(17)·10 ¹
1.740	.8738(90)	.3969(55)·10 ¹	.1115(19)·10 ²
1.760	.1014(10)·10 ¹	.4600(60)·10 ¹	.1256(19)·10 ²
1.780	.1187(11)·10 ¹	.5268(64)·10 ¹	.1441(21)·10 ²
1.800	.1363(13)·10 ¹	.5900(69)·10 ¹	.1681(20)·10 ²
1.820	.1555(14)·10 ¹	.6878(78)·10 ¹	.1955(22)·10 ²
1.840	.1745(15)·10 ¹	.7633(73)·10 ¹	.2203(23)·10 ²
1.860	.1949(16)·10 ¹	.8653(80)·10 ¹	.2495(25)·10 ²
1.880	.2291(17)·10 ¹	.932(18)·10 ¹	.2900(27)·10 ²
1.900	.2605(38)·10 ¹	.1086(19)·10 ²	.3561(34)·10 ²
1.920	.2964(41)·10 ¹	.1228(21)·10 ²	.3985(66)·10 ²
1.940	.3284(43)·10 ¹	.1365(22)·10 ²	.4324(68)·10 ²
1.960	.3528(45)·10 ¹	.1484(23)·10 ²	.4535(69)·10 ²
1.980	.3904(48)·10 ¹	.1588(23)·10 ²	.4915(71)·10 ²
2.000	.4186(49)·10 ¹	.1719(24)·10 ²	.5327(73)·10 ²
2.020	.4637(52)·10 ¹	.1956(26)·10 ²	.5833(78)·10 ²
2.040	.4805(53)·10 ¹	.2076(27)·10 ²	.6411(81)·10 ²
2.060	.5418(58)·10 ¹	.2266(28)·10 ²	.7117(87)·10 ²
2.080	.6005(61)·10 ¹	.2504(30)·10 ²	.8350(98)·10 ²

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 37^\circ$ (continued)

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
2.100	.6399(64)·10 ¹	.2716(31)·10 ²	.894(10)·10 ²
2.120	.7044(69)·10 ¹	.2952(30)·10 ²	.949(10)·10 ²
2.140	.7914(61)·10 ¹	.3376(34)·10 ²	.1075(9)·10 ³
2.160	.8538(63)·10 ¹	.3600(35)·10 ²	.1160(9)·10 ³
2.180	.9458(68)·10 ¹	.3969(37)·10 ²	.1260(10)·10 ³
2.200	.1044(12)·10 ²	.4412(93)·10 ²	.1411(17)·10 ³
2.220	.1158(12)·10 ²	.4900(97)·10 ²	.1594(18)·10 ³
2.240	.1221(13)·10 ²	.541(10)·10 ²	.1678(19)·10 ³
2.260	.1321(13)·10 ²	.568(10)·10 ²	.1759(19)·10 ³
2.280	.1384(14)·10 ²	.583(10)·10 ²	.1833(19)·10 ³
2.300	.1451(14)·10 ²	.623(11)·10 ²	.1958(20)·10 ³
2.320	.1527(14)·10 ²	.655(11)·10 ²	.2104(21)·10 ³
2.340	.1617(15)·10 ²	.691(11)·10 ²	.2240(22)·10 ³
2.360	.1746(16)·10 ²	.745(12)·10 ²	.2411(23)·10 ³
2.380	.1833(16)·10 ²	.753(12)·10 ²	.2549(24)·10 ³
2.400	.1972(17)·10 ²	.808(12)·10 ²	.2736(25)·10 ³
2.420	.2037(18)·10 ²	.880(13)·10 ²	.2913(26)·10 ³
2.440	.2141(18)·10 ²	.902(13)·10 ²	.2940(26)·10 ³
2.460		.1000(14)·10 ³	

Table 15: *Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 45^\circ$*

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
1.560		.62(36)·10 ⁻³	
1.580		.81(41)·10 ⁻³	
1.600		.107(48)·10 ⁻²	
1.620		.181(65)·10 ⁻²	
1.640		.288(71)·10 ⁻²	
1.660		.405(85)·10 ⁻²	
1.680		.481(93)·10 ⁻²	
1.700		.563(98)·10 ⁻²	
1.720	.136(81)·10 ⁻²	.72(11)·10 ⁻²	
1.740	.218(99)·10 ⁻²	.113(14)·10 ⁻¹	
1.760	.32(12)·10 ⁻²	.144(17)·10 ⁻¹	
1.780	.303(56)·10 ⁻²	.171(17)·10 ⁻¹	.36(12)·10 ⁻¹
1.800	.466(55)·10 ⁻²	.184(17)·10 ⁻¹	.66(13)·10 ⁻¹
1.820	.598(62)·10 ⁻²	.276(16)·10 ⁻¹	.73(14)·10 ⁻¹
1.840	.624(63)·10 ⁻²	.392(19)·10 ⁻¹	.157(21)
1.860	.1131(85)·10 ⁻¹	.415(19)·10 ⁻¹	.173(21)
1.880	.1308(92)·10 ⁻¹	.618(24)·10 ⁻¹	.214(23)
1.900	.180(11)·10 ⁻¹	.777(27)·10 ⁻¹	.260(25)
1.920	.238(12)·10 ⁻¹	.1026(31)	.346(30)

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 45^\circ$ (continued)

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
1.940	.357(16)·10 ⁻¹	.1337(37)	.405(32)
1.960	.356(16)·10 ⁻¹	.1740(60)	.518(36)
1.980	.493(19)·10 ⁻¹	.2269(69)	.705(42)
2.000	.616(22)·10 ⁻¹	.2816(76)	.877(46)
2.020	.786(24)·10 ⁻¹	.3520(87)	.1066(51)·10 ¹
2.040	.1005(27)	.4251(94)	.1180(53)·10 ¹
2.060	.1280(32)	.541(11)	.1673(64)·10 ¹
2.080	.1540(35)	.678(12)	.2017(71)·10 ¹
2.100	.1825(38)	.827(14)	.2421(76)·10 ¹
2.120	.2208(37)	.1005(15)·10 ¹	.3042(73)·10 ¹
2.140	.2840(44)	.1192(17)·10 ¹	.3686(81)·10 ¹
2.160	.3452(49)	.1414(18)·10 ¹	.4173(84)·10 ¹
2.180	.431(12)	.1656(20)·10 ¹	.5031(92)·10 ¹
2.200	.506(13)	.1977(69)·10 ¹	.550(17)·10 ¹
2.220	.586(13)	.2169(72)·10 ¹	.679(19)·10 ¹
2.240	.653(14)	.2743(81)·10 ¹	.811(21)·10 ¹
2.260	.733(15)	.3241(88)·10 ¹	.930(22)·10 ¹
2.280	.846(16)	.3431(90)·10 ¹	.1081(24)·10 ²
2.300	.990(18)	.3999(98)·10 ¹	.1198(25)·10 ²
2.320	.1112(19)·10 ¹	.450(10)·10 ¹	.1348(27)·10 ²
2.340	.1246(20)·10 ¹	.517(11)·10 ¹	.1596(30)·10 ²
2.360	.1377(21)·10 ¹	.608(12)·10 ¹	.1804(32)·10 ²
2.380	.1577(22)·10 ¹	.656(12)·10 ¹	.2012(33)·10 ²
2.400	.1703(23)·10 ¹	.767(14)·10 ¹	.2268(36)·10 ²
2.420	.2021(26)·10 ¹	.839(14)·10 ¹	.2515(38)·10 ²
2.440	.2189(27)·10 ¹	.927(15)·10 ¹	.2856(40)·10 ²
2.460	.2622(32)·10 ¹	.1093(17)·10 ²	.3343(46)·10 ²

Table 16: *Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 55^\circ$*

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
2.100		.79(57)·10 ⁻³	
2.120		.147(52)·10 ⁻²	
2.140	.49(25)·10 ⁻³	.245(69)·10 ⁻²	
2.160	.54(25)·10 ⁻³	.315(75)·10 ⁻²	
2.180	.68(28)·10 ⁻³	.61(11)·10 ⁻²	
2.200	.194(49)·10 ⁻²	.67(11)·10 ⁻²	
2.220	.279(59)·10 ⁻²	.111(14)·10 ⁻¹	
2.240	.220(53)·10 ⁻²	.116(14)·10 ⁻¹	
2.260	.398(70)·10 ⁻²	.205(19)·10 ⁻¹	
2.280	.462(74)·10 ⁻²	.255(22)·10 ⁻¹	
2.300	.634(86)·10 ⁻²	.313(24)·10 ⁻¹	
2.320	.802(96)·10 ⁻²	.391(26)·10 ⁻¹	

Cross sections for $\varepsilon = 4.032$ GeV and $\theta = 55^\circ$ (continued)

ν	<i>Carbon</i>	<i>Iron</i>	<i>Gold</i>
2.340	.100(11)·10 ⁻¹	.503(30)·10 ⁻¹	
2.360	.131(13)·10 ⁻¹	.695(35)·10 ⁻¹	
2.380	.174(14)·10 ⁻¹	.791(37)·10 ⁻¹	
2.400	.238(15)·10 ⁻¹	.1046(43)	.261(29)
2.420	.318(18)·10 ⁻¹	.1420(39)	.448(38)
2.440	.422(20)·10 ⁻¹	.1693(43)	.528(41)
2.460	.523(24)·10 ⁻¹	.2273(52)	.755(49)
2.480	.702(58)·10 ⁻¹	.2971(90)	.833(51)
2.500	.813(62)·10 ⁻¹	.3573(98)	.1192(62)·10 ¹
2.520	.950(67)·10 ⁻¹	.461(11)	.1361(65)·10 ¹
2.540	.1131(73)	.556(12)	.1619(69)·10 ¹
2.560	.1462(81)	.624(13)	.1841(72)·10 ¹
2.580	.1737(88)	.767(14)	.2296(81)·10 ¹
2.600	.2119(96)	.909(16)	.2754(89)·10 ¹
2.620	.2623(68)	.1099(15)·10 ¹	.3177(95)·10 ¹
2.640	.3229(76)	.1313(17)·10 ¹	.4205(91)·10 ¹
2.660	.3991(84)	.1584(19)·10 ¹	.526(10)·10 ¹
2.680	.504(12)	.1967(23)·10 ¹	.611(18)·10 ¹
2.700	.605(13)	.2372(48)·10 ¹	.694(19)·10 ¹
2.720	.657(14)	.2653(51)·10 ¹	.835(21)·10 ¹
2.740	.776(15)	.3088(55)·10 ¹	.942(22)·10 ¹
2.760	.883(16)	.3635(59)·10 ¹	.1081(24)·10 ²
2.780	.991(17)	.4244(64)·10 ¹	.1230(25)·10 ²
2.800	.1146(18)·10 ¹	.4662(67)·10 ¹	.1422(27)·10 ²
2.820	.1300(19)·10 ¹	.5659(71)·10 ¹	.1717(26)·10 ²
2.840	.1509(18)·10 ¹	.6573(68)·10 ¹	.2009(28)·10 ²
2.860	.1775(20)·10 ¹	.7655(77)·10 ¹	.2495(33)·10 ²
2.880	.2024(41)·10 ¹	.875(14)·10 ¹	.2902(66)·10 ²
2.900	.2281(43)·10 ¹	.1034(15)·10 ²	.3260(70)·10 ²
2.920	.2578(46)·10 ¹	.1097(15)·10 ²	.3505(71)·10 ²
2.940	.2834(48)·10 ¹	.1230(16)·10 ²	.4017(76)·10 ²
2.960	.3248(51)·10 ¹	.1448(18)·10 ²	.4551(80)·10 ²
2.980	.3580(54)·10 ¹	.1569(18)·10 ²	.4939(84)·10 ²
3.000	.3957(57)·10 ¹	.1761(19)·10 ²	.5640(90)·10 ²
3.020	.4271(61)·10 ¹	.1941(21)·10 ²	.6355(99)·10 ²
3.040	.4823(90)·10 ¹		

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