Two Topics in the Physics of Light Nuclei

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Abstract. I review how tensor forces affect the ground-state structure of nuclei, and how isospin-symmetry-breaking corrections at the nucleon and nuclear level contaminate the asymmetry measured in parity-violating electron scattering from nuclei, complicating the extraction of the strange-quark form factors from these measurements.

1 Introduction

The last few years have witnessed substantive progress made in our understanding of the structure and dynamics of light nuclei (mass number $A \leq 12$). Realistic two and three-nucleon potentials and associated electroweak currents have been constructed that reproduce well the energy spectra and a variety of electromagnetic and weak transitions in these light systems, both at the low energies (keV regime) of relevance in nuclear astrophysics as well as at the high energies (GeV regime) probing the short range structure of nuclei. Parallel developments in chiral perturbation theory and power-counting rules have provided a QCD underpinning for these interactions and currents, while treatments of nuclear dynamics beyond the non-relativistic limit have also been carried out in the two- and three-nucleon systems. Lastly, the continuing evolution and refinement of calculational methods, such as quantum Monte Carlo and hyperspherical harmonics techniques, accompanied by the surge in available computer power, have led to significant progress in computing nuclear properties with the realistic potentials and currents mentioned above.

Some of this progress is reviewed in my talk at this conference [1]. In the present proceedings, however, I will only discuss two issues having to do with i) the role that tensor forces play in shaping the ground-state structure of nuclei [2], and ii) limitations in the use of nuclei as laboratories for the study of hadronic properties, specifically in the extraction, from parity-violating electron scattering off ^4He , of the electric form factor G_E^s , associated with the strange-quark content of the nucleon [3].

2 Tensor forces and the ground-state structure of nuclei

The interaction that binds protons and neutrons together in the atomic nucleus is characterized by a strong repulsion at short ($\lesssim 0.5$ fm) distances, and a strong coupling between spatial and spin degrees of freedom in pairs of nucleons at intermediate to large ($\gtrsim 1$ fm) separations, see the left panel of Fig. 1. This “tensor” character of the interaction binds the deuteron and couples S- and D-waves into a ground state with a large non-spherical component. This is in marked contrast to systems such as the hydrogen atom where the dominant $1/r$ Coulomb potential leads to an essentially spherical ground state.

These features of the nucleon-nucleon (NN) potential are reflected in the two-nucleon density distributions [4]. Of particular relevance are those in the $TS=01$ channel, illustrated in the right panel of Fig. 1. Because of the tensor force, these densities are strongly anisotropic. Furthermore, the densities $\rho_{0,1}^{M_S}(\mathbf{r}; A)$ in a nucleus with mass number A are proportional to those in the deuteron for inter-nucleon separations up to $\simeq 2$ fm. That the neutron-proton relative wave function in a nucleus is similar, at small separations, to that in the deuteron had been conjectured by Levinger and Bethe [5] over fifty years ago; indeed, Fig. 1 provides a microscopic justification for that conjecture, known as the “quasi-deuteron model.”

The two-nucleon density distributions in states with pair spin-isospin $TS=11$, 00 , and 10 are drastically different from those with $TS=01$ [4]. They are shown for ${}^4\text{He}$, ${}^6\text{Li}$, and ${}^{16}\text{O}$ in Fig. 2, where all curves have been normalized to have the same peak height as for ${}^{16}\text{O}$. The $TS=11$ interaction has a tensor component of opposite sign with respect to that of the $TS=01$ interaction. As a consequence, the $M_S=\pm 1$ ($=0$) density distributions (left panel of Fig. 2) are largest when the two nucleons are in the xy -plane (along the z -axis), namely the situation is the reverse of that illustrated in Fig. 1 for $TS=01$. However, the $TS=11$ densities are strongly A -dependent, in particular their anisotropy decreases as the number

Figure 1. Left panel: The expectation value of the static part of the NN interaction in the $TS=01$ channel for two different spin-spatial configurations: the NN pair in state $M_S=0$ located either along the z -axis or in the xy -plane. Right panel: Corresponding two-nucleon density distributions in a variety of nuclei.

of nucleons increases.

This strong A -dependence is also a feature of the $TS=00$ two-nucleon densities, as seen in the central panel of Fig. 2. In contrast, the $TS=10$ density distributions, shown in the right panel of Fig. 2, do display, for separation distances less than 2 fm, very similar shapes. This is not surprising, since the $TS=10$ interaction is attractive enough to almost support a bound state.

Tensor correlations also impact strongly the momentum distributions of NN pairs in the ground state of a nucleus and, in particular, lead to large differences in the np versus pp distributions at moderate values of the relative momentum in the pair [2]. The np and pp momentum distributions in ${}^3\text{He}$, ${}^4\text{He}$, ${}^6\text{Li}$, and ${}^8\text{Be}$ nuclei are shown in Fig. 3 as functions of the relative momentum q at fixed total pair momentum $Q=0$, corresponding to nucleons moving back to back. They are calculated with the Variational Monte Carlo method, and the associated statistical errors are displayed only for the pp pairs; they are negligibly small for the np pairs. The striking features seen in all cases are: i) the momentum distribution of np pairs is much larger than that of pp pairs for relative momenta in the range $1.5\text{--}3.0\text{ fm}^{-1}$, and ii) for the helium and lithium isotopes the node in the pp momentum distribution is absent in the np one, which instead exhibits a change of slope at a characteristic value of $q \simeq 1.5\text{ fm}^{-1}$. The nodal structure is much less prominent in ${}^8\text{Be}$. At small values of q the ratios of np to pp momentum distributions are closer to those of np to pp pair numbers, which in ${}^3\text{He}$, ${}^4\text{He}$, ${}^6\text{Li}$, and ${}^8\text{Be}$ are respectively 2, 4, 3, and $8/3$.

This strong isospin dependence of NN momentum distributions should lead to large differences in the cross sections for back-to-back pp - and np -pair knock-out. There are rather clear indications from a recent analysis of a BNL experiment [6], which measured cross sections for (p, pp) and (p, ppn) processes on ${}^{12}\text{C}$ in kinematics close to two nucleons being ejected back to back, that this is indeed the case. From this analysis an upper limit to the ratio of pp to np probabilities (denoted respectively as P_{pp} and P_{np}) with (approximately) vanishing total momentum and relative momenta in the range $(275\text{--}550)\text{ MeV}/c$ has been inferred: $P_{pp}/P_{np} \lesssim 0.04^{+0.09}_{-0.04}$. Hopefully, a more precise value for this ratio will become available in the near future, when the analysis of ${}^{12}\text{C}(e, e'np)$ and ${}^{12}\text{C}(e, e'pp)$ data, taken at Jefferson Lab, is completed. Experimental confirmation of the

Figure 2. The $\rho_{11}^M(r, \theta; A)$ (left panel), $\rho_{00}^0(r, \theta; A)$ (central panel), and $\rho_{10}^0(r, \theta; A)$ (right panel) density distributions for various nuclei.

smallness of this ratio would provide a direct verification of the crucial role that the tensor force plays in shaping the short-range structure of nuclei.

3 Isospin symmetry breaking corrections and G_E^s

One of the challenges of modern hadronic physics is to determine, at a quantitative level, the role that quark-antiquark pairs, and in particular $s\bar{s}$ pairs, play in the structure of the nucleon. Parity-violating (PV) electron scattering from nucleons and nuclei offers the opportunity to investigate this issue experimentally. The PV asymmetry (A_{PV}) arises from interference between the amplitudes due to exchange of photons and Z -bosons, which couple respectively to the electromagnetic and weak neutral currents. These currents involve different combinations of quark flavors, and therefore measurements of A_{PV} , in combination with electromagnetic form factor data for the nucleon, allow one to isolate, in principle, the electric and magnetic form factors G_E^s and G_M^s , associated with the strange-quark content of the nucleon.

Recently, the HAPPEX Collaboration [7] at Jefferson Lab reported on measurements of the PV asymmetry in elastic electron scattering from ^4He at four-momentum transfers of 0.091 (GeV/c)^2 and 0.077 (GeV/c)^2 . Because of the $J^\pi=0^+$ spin-parity assignments of this nucleus, transitions induced by magnetic and axial-vector currents are forbidden, and therefore these measurements can lead to a direct determination of the strangeness electric form factor G_E^s [8], provided that isospin symmetry breaking (ISB) effects in both the nucleon and ^4He , and relativistic and meson-exchange (collectively denoted with MEC) contributions to the nuclear electromagnetic and weak vector charge operators, are negligible. A realistic calculation of these latter contributions [8] found that they are in fact tiny at low momentum transfers. In the following I discuss a recent quantitative e:

Figure 3. Left panel: The np (lines) and pp (symbols) momentum distributions in various nuclei as functions of the relative momentum q at vanishing total pair momentum Q .

The PV asymmetry measured in (e, e') elastic scattering from ${}^4\text{He}$ can be written as [3]:

$$A_{PV} = \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \left[4\sin^2\theta_W - 2\frac{F^{(1)}(q)}{F^{(0)}(q)} - \frac{2G_E^I - G_E^s}{(G_E^p + G_E^n)/2} \right], \quad (1)$$

where G_μ is the Fermi constant as determined from muon decays, and here $\sin^2\theta_W$ is taken to incorporate radiative corrections. The terms G_E^I and $F^{(1)}(q)$ are the contributions to A_{PV} , associated with the violation of isospin symmetry at the nucleon and nuclear level, respectively. The most accurate measurement of the PV asymmetry, recently reported in Ref. [7] at $Q^2=0.077 \text{ (GeV/c)}^2$, gives $A_{PV} = [+6.40 \pm 0.23 \text{ (stat)} \pm 0.12 \text{ (syst)}]$ ppm, from which one obtains [3]

$$\Gamma \equiv -2\frac{F^{(1)}(q)}{F^{(0)}(q)} - \frac{2G_E^I - G_E^s}{(G_E^p + G_E^n)/2} = 0.010 \pm 0.038 \quad (2)$$

at $Q^2 = 0.077 \text{ (GeV/c)}^2$. This result is consistent with zero.

Kubis and Lewis [9] have recently estimated $G_E^I(Q^2)$, combining a leading-order calculation in chiral perturbation theory with estimates for low-energy constants using resonance saturation. Their prediction along with the associated uncertainty—mostly arising from uncertainties in the values of the vector meson coupling constants—for this form factor are shown in Fig. 4.

Figure 4. The isospin-violating nucleon form factor $G_E^I(Q^2)$. The band comprises a range of values for various vector-meson coupling constants, as well as an estimate of higher-order chiral corrections. For details, see Ref. [9].

A calculation of the ratio $F^{(1)}(q)/F^{(0)}(q)$ has been recently carried out [3] with quantum Monte Carlo methods and ${}^4\text{He}$ wave functions derived from a variety of Hamiltonian models with the hyperspherical harmonic expansion method (see Ref. [10] and references therein). These Hamiltonian models include explicit ISB induced by both the strong and electromagnetic interactions.

The form factors $F^{(0)}(q)$ and $F^{(1)}(q)$ are displayed in Fig. 5. The dashed (solid) curves represent the results of calculations including the one-body (one-body plus MEC) EM charge operators. In the inset of Fig. 5, we show the model dependence of the ratio $|F^{(1)}(q)/F^{(0)}(q)|$ (here, all calculations include MEC). Therefore, at $Q^2=0.077 \text{ (GeV/c)}^2$, both contributions $F^{(1)}/F^{(0)}$ and G_E^I are found of the same order of magnitude as the central value of Γ in Eq. (2).

Figure 5. The $F^{(0)}(q)$ and $F^{(1)}(q)$ form factors for the AV18/UIX Hamiltonian model. The $F^{(0)}(q)$ is compared with the experimental ${}^4\text{He}$ charge form factor [11]. The ratio $|F^{(1)}(q)/F^{(0)}(q)|$ (all calculations include MEC) is shown in the inset for a variety of Hamiltonian models.

Using in this equation the calculated values for $F^{(1)}/F^{(0)}$ (≈ -0.00157) and the chiral result for G_E^I (≈ -0.0017), one would obtain $G_E^s[Q^2 = 0.077 \text{ (GeV/c)}^2] = -0.001 \pm 0.016$ thus suggesting that the value of Γ is almost entirely due to isospin admixtures. Of course, the experimental error on Γ is still too large to allow us to draw a more definite conclusion. A recent estimate of G_E^s using lattice QCD input obtains [12] $G_E^s[0.1 \text{ (GeV/c)}^2] = +0.001 \pm 0.004 \pm 0.003$. An increase of one order of magnitude in the experimental accuracy would be necessary in order to be sensitive to G_E^s at low values of Q^2 . Finally, contributions to the asymmetry, originating from PV components in the nuclear potentials induced by hadronic weak interactions, are expected to be much smaller than the ISB corrections studied here [13].

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