The Results of the Comparative Research of Proposed Absolute Energy Measurements for CEBAF

I. P. Karabekov
(Yerevan Physics Institute)
(Yerevan, Alikhanian Brothers St. 2, Republic of Armenia)

Abstract

The need to measure the incident beam energy close to the target absolutely with precision no worse than one part in 10^4 is an important part of the CEBAF Hall A physics program. Since the appearance of the Conceptual Design Report of 13 April 1990, where the requirement was established, the problem has been considered at both CEBAF and the Yerevan Physics Institute.

The following methods were considered for solving the problem:

- Backscattering of a plane electromagnetic wave by the relativistic electron beam. Calculation shows that the intensity of the backscattered radiation in a bandwidth of 10^{-4} near the maximum frequency is about 1 photon per second at 4 GeV and 0.3 mA. (To be published.)
- Three- and four-magnet chicanes [1,2] with appropriate detector systems. Such a system was used at SLAC for absolute measurement of the SLC beam energy with precision 3×10^{-4} . Calculations show that similar accuracy can be achieved for CEBAF in both proposed systems. The tolerances for each type are presented in this paper.
- Angular distribution of synchrotron radiation^[3,4,5]. Calculations have shown that precision of about 2.5×10^{-5} is achievable for CEBAF. The tolerances are also presented in this paper [3,4,5].

The high precision energy measurement for CEBAF is discussed.

1.0 Introduction

The precision of beam mean energy determination using magnetic spectrometers depends on the accuracy by which field intensity, B , and bending radius, ρ , are measured. But as well as direct measurement of ρ is impossible for bending angle $\Theta_B < 2\pi$ this parameter will be determined using another measurable values as deflection of beam trajectory, H , bending angle, Θ , length of trajectory, δs .

All these parameters may be measured by limited accuracy and the main problem is to design the scheme of the beam interaction with magnetic field where the error of the measurements are minimal.

The main formula for energy determination in case of the extended magnetic field and adopted for case of bending angle measurement may be write as:

$$E = \left(\frac{\rho}{\delta s} \right) \int_{s_1}^{s_2} B ds. \quad (1)$$

The another form of the main expression adopted for deflection measurement is following

$$E = \left(\frac{300}{\delta s} \right) \int_{s_1}^{s_2} B ds \frac{H}{1 - \cos \Theta_B}. \quad (2)$$

Formulas (1) and (2) described the main functions of three and four magnet chicane accordingly and they are the basis for the calculations of the main principal and technical tolerances.

**The Tables of the Main Parameters of the Chicanes
Proposed for CEBAF**

Table 1. Main parameters of 3-magnet chicane

Parameter	Symbol	Size
1. Bending angle	Θ	10^{-1} rad
2. Magnet length	l_m	1.0 meter
3. SR path length	l_{SR}	7.0 meters
4. Vertical bend angle	Θ_v	$2 \cdot 10^{-3}$
5. Number of spectrometer magnets	N_B	1
6. Number of kicker magnets	N_k	4
7. Number of position detectors	n_{det}	2
8. Total length of chicane	L_T	8 meters

Table 2. Main parameters of 4-magnet chicane

Parameter	Symbol	Size
1. Bending angle	Θ_B	16×4
2. Dispersion	D_B	1.73 meters
3. Bending radius	ρ_B	11.1 meters
4. Number of magnets	n_B	4
5. Length of magnets	l_m	3.0 meters
6. Length of long straight section	l_{SL}	3.0 meters
7. Length of short straight section	l_{SS}	2.0 meters
8. Beam position monitor - resolution - number	BPM	10^{-4} rad 4
9. Beam entrance angle resolution	BEA	10^{-5} rad
10. Total length of chicane		20 meters

Schemes of these chicane presented in Figure 1.

The Proposed Schemes of Spectrometrical Chicanes

Figure 1

The Tables of the Errors Limiting of the Resolution

Table 3. Systematic error for 3-magnet chicane

Source of error	formula	size of error	contribution to energy error
Magnetic measurement	$\int B \cdot ds$ + monitoring	10^{-4}	$1. \times 10^{-4}$
Detector position	$\frac{\Delta z}{L_s} \sqrt{n_d}$	$0.86 \cdot 10^{-5}$	0.6×10^{-4}
Kicker rotation	$\Delta \phi \cdot \sin \theta_k \sqrt{n_k}$ $\Delta \phi = 2 \times 10^{-3}$	$0.8 \cdot 10^{-5}$	$0.8. \times 10^{-4}$
Survey error	$\frac{ \Delta z }{L_s}$ $ \Delta z = 30 \mu m$	10^{-5}	$1. \times 10^{-4}$

$$\text{Total } \Delta E = 1.73 E_0 \times 10^{-4}$$

Table 4. Systematic error for 4-magnet chicane

Source of error	formula	size of error	contribution to energy error
Magnetic measurement	$\frac{1}{\sqrt{n_m}} \int B \cdot ds$ + monitoring	0.5×10^{-7}	0.5×10^{-4}
BPM Measurement	$\sqrt{n_{BPM}} \Delta z $	1.73×10^{-4}	$1. \times 10^{-4}$
Beam entrance angle	$ \delta z = \delta \phi \rho \sin 32^\circ$ + $ \delta \phi 1.5 L$	$1.6 \cdot 10^{-4}$	$2.06 \cdot 10^{-4}$
Survey error	$ \Delta z \cdot \sqrt{n_{BPM}}$	5.2×10^{-5}	0.3×10^{-4}

$$\text{Total } \Delta E = 2.29 E_0 \times 10^{-4}$$

2.0 The Principal Scheme of the Beam Mean Energy Absolute Measurement Using Synchrotron Radiation

The principle of the method described in [3] is to measure the variation in the angular distribution of the synchrotron radiation (SR) in the vertical plane. This distribution, which depends upon $\gamma = E_0/mc^2$, is shown in Figure 2. If one can measure and compare the SR beam intensity at two angles, $\psi = 0$ and $\psi = \psi_1$ (see Figure 3), the energy can be determined absolutely, since the relation between the photon fluxes at the two angles can be calculated accurately for the given γ . Consider two horizontal slits of equal width, d , but different lengths ℓ_1 , ℓ_2 , so that the numbers of photons per unit time through the slits are equal:

$$\Delta\theta_1 N_K(\psi = 0) = \Delta\theta_2 N_K(\psi = \psi_1), \quad (1)$$

where N_K is the flux density of the radiation in the bandwidth $\Delta\lambda/\lambda = K$. The aperture angles are $\Delta\theta_1 = \ell_1/L$, $\Delta\theta_2 = \ell_2/L$, where L is the distance between the slits and the point on the trajectory where the SR beam originates.

The mean energy E_0 of the beam is determined absolutely when the two photon fluxes are equal, which is measured as the zero difference.

3.0 Synchrotron Radiation Vertical Distribution

Figure 2. SR photon distribution along the axis vertical to the orbit plane, ψ is the vertical radiation angle. γ is the Lorentz factor of the particle.

Figure 3. Separation of Photon Fluxes at the Angles $\psi = 0$ and $\psi = \psi_0$

The Tolerances and Resolution of the Synchrotron Radiation Analysis Method

The working point is $\gamma\psi = 0.801084$, where the second derivative of the function of photon vertical distribution is equal to zero.

Table 5. Main parameters for AEM-4GeV

The source of error	Symbol of Formula	Tolerance	Maximum Error $\Delta E/E_0$
angle between orbit plane and the slit system symmetry plane	$\Theta\psi$	$0.945 \cdot 10^{-5} \text{ rad}$	10^{-5}
quantum fluctuations	ΔN_k	$2.140 \cdot 10^5 \text{ ps}^{-1}$	$0.5 \cdot 10^{-5}$
the arc in bending	$\Delta\theta$	0.148 rad	10^{-5}
wavelength	$\Delta\lambda/\lambda$	$2.700 \cdot 10^{-3}$	10^{-5}
accuracy of measurement of field intensity	$\Delta B/B$	$0.600 \cdot 10^{-4}$	10^{-5}
accuracy of measurement of the monochromator angles	$\delta\theta$	$2.720 \cdot 10^{-5}$	10^{-5}
emittance parameters	$\delta\psi = y' + (y/L)$	$1.250 \cdot 10^{-5} \text{ rad}$ at $L = 5 \cdot 10^3 \text{ mm}$	10^{-5}

The total r.m.s. error $\Delta E = 2.5 \cdot 10^{-5} E$

4.0 An Optimal Variant of CEBAF High Precision Energy Measurement Complex

Above presented results of the investigations and technical possibilities at CEBAF permits to make following principal solution about beam mean energy measurement with the precision of one unit of 10^{-4} and higher. It is evident that this goal may be achieved only by proposed method of synchrotron radiation vertical distribution measurement [3,4] with the tolerances presented in Table 5. In the other hand for the creation of SR, high precision magnetic field is required. With this purpose special short magnet will be inserted in extracted beam transportation system as close as possible to target area (see Figure 4). The best place for this magnet is the place of penultimate dipole which for this case will be removed.

Figure 4

For the compensation of the distortion of electron beam path caused by inserted magnet the optical system like three magnet chicane is required. Such a kind of optics may be performed using neighboring ordinary magnets by giving additional bend angle (see Figure 5). This additional angle, $\delta\Theta$, may be received by increasing the current in magnet conductors. The tolerable increasing of the current is about 20% which creates accordingly $\delta\Theta = 0.2 \text{ rad}$.

Figure 5

THE PROPOSED SCHEMES OF SPECTROMETRICAL CHICANES

The Results of the Comparative Research of Proposed Absolute Energy Measurements for CEBAF

I. P. Karabekov
(Yerevan Physics Institute)
(Yerevan, Alikhanian Brothers St. 2, Republic of Armenia)

Abstract

The need to measure the incident beam energy close to the target absolutely with precision no worse than one part in 10^4 is an important part of the CEBAF Hall A physics program. Since the appearance of the Conceptual Design Report of 13 April 1990, where the requirement was established, the problem has been considered at both CEBAF and the Yerevan Physics Institute.

The following methods were considered for solving the problem:

- Backscattering of a plane electromagnetic wave by the relativistic electron beam. Calculation shows that the intensity of the backscattered radiation in a bandwidth of 10^{-4} near the maximum frequency is about 1 photon per second at 4 GeV and 0.3 mA. (To be published.)
- Three- and four-magnet chicanes [1,2] with appropriate detector systems. Such a system was used at SLAC for absolute measurement of the SLC beam energy with precision 3×10^{-4} . Calculations show that similar accuracy can be achieved for CEBAF in both proposed systems. The tolerances for each type are presented in this paper.
- Angular distribution of synchrotron radiation^[3,4,5]. Calculations have shown that precision of about 2.5×10^{-5} is achievable for CEBAF. The tolerances are also presented in this paper [3,4,5].

The high precision energy measurement for CEBAF is discussed.

1.0 Introduction

The precision of beam mean energy determination using magnetic spectrometers depends on the accuracy by which field intensity, B , and bending radius, ρ , are measured. But as well as direct measurement of ρ is impossible for bending angle $\Theta_B < 2\pi$ this parameter will be determined using another measurable values as deflection of beam trajectory, H , bending angle, Θ , length of trajectory, δs .

All these parameters may be measured by limited accuracy and the main problem is to design the scheme of the beam interaction with magnetic field where the error of the measurements are minimal.

The main formula for energy determination in case of the extended magnetic field and adopted for case of bending angle measurement may be write as:

$$E = \left(\frac{\rho}{\delta s} \right) \int_{s_1}^{s_2} B ds. \quad (1)$$

The another form of the main expression adopted for deflection measurement is following

$$E = \left(\frac{300}{\delta s} \right) \int_{s_1}^{s_2} B ds \frac{H}{1 - \cos \Theta_B}. \quad (2)$$

Formulas (1) and (2) described the main functions of three and four magnet chicane accordingly and they are the basis for the calculations of the main principal and technical tolerances.

**The Tables of the Main Parameters of the Chicanes
Proposed for CEBAF**

Table 1. Main parameters of 3-magnet chicane

Parameter	Symbol	Size
1. Bending angle	Θ	10^{-1} rad
2. Magnet length	l_m	1.0 meter
3. SR path length	l_{SR}	7.0 meters
4. Vertical bend angle	Θ_v	$2 \cdot 10^{-8}$
5. Number of spectrometer magnets	N_B	1
6. Number of kicker magnets	N_k	4
7. Number of position detectors	n_{det}	2
8. Total length of chicane	L_T	8 meters

Table 2. Main parameters of 4-magnet chicane

Parameter	Symbol	Size
1. Bending angle	Θ_B	16×4
2. Dispersion	D_B	1.73 meters
3. Bending radius	ρ_B	11.1 meters
4. Number of magnets	n_B	4
5. Length of magnets	l_m	3.0 meters
6. Length of long straight section	l_{SL}	3.0 meters
7. Length of short straight section	l_{SS}	2.0 meters
8. Beam position monitor - resolution - number	BPM	10^{-4} rad 4
9. Beam entrance angle resolution	BEA	10^{-5} rad
10. Total length of chicane		20 meters

Schemes of these chicane presented in Figure 1.

The Proposed Schemes of Spectrometrical Chicanes

Figure 1

The Tables of the Errors Limiting of the Resolution

Table 3. Systematic error for 3-magnet chicane

Source of error	formula	size of error	contribution to energy error
Magnetic measurement	$\int B \cdot ds$ + monitoring	10^{-4}	$1. \times 10^{-4}$
Detector position	$\frac{\langle \Delta x \rangle}{L_s} \sqrt{n_d}$	$0.86 \cdot 10^{-5}$	0.6×10^{-4}
Kicker rotation	$\Delta \phi \cdot \sin \theta_k \sqrt{n_k}$ $\Delta \phi = 2 \times 10^{-3}$	$0.8 \cdot 10^{-5}$	$0.8. \times 10^{-4}$
Survey error	$\frac{ \Delta x }{L_s}$ $ \Delta x = 30 \mu m$	10^{-5}	$1. \times 10^{-4}$

$$\text{Total } \Delta E = 1.73 E_0 \times 10^{-4}$$

Table 4. Systematic error for 4-magnet chicane

Source of error	formula	size of error	contribution to energy error
Magnetic measurement	$\frac{1}{\sqrt{n_m}} \int B \cdot ds$ + monitoring	0.5×10^{-7}	0.5×10^{-4}
BPM Measurement	$\sqrt{n_{BPM}} \Delta x $	1.73×10^{-4}	$1. \times 10^{-4}$
Beam entrance angle	$ \delta x = \delta \phi \rho \sin 32^\circ$ + $ \delta \phi l_{SL}$	$1.6 \cdot 10^{-4}$	$2.06 \cdot 10^{-4}$
Survey error	$ \Delta x \cdot \sqrt{n_{BPM}}$	5.2×10^{-5}	0.3×10^{-4}

$$\text{Total } \Delta E = 2.29 E_0 \times 10^{-4}$$

2.0 The Principal Scheme of the Beam Mean Energy Absolute Measurement Using Synchrotron Radiation

The principle of the method described in [3] is to measure the variation in the angular distribution of the synchrotron radiation (SR) in the vertical plane. This distribution, which depends upon $\gamma = E_0/mc^2$, is shown in Figure 2. If one can measure and compare the SR beam intensity at two angles, $\psi = 0$ and $\psi = \psi_1$ (see Figure 3), the energy can be determined absolutely, since the relation between the photon fluxes at the two angles can be calculated accurately for the given γ . Consider two horizontal slits of equal width, d , but different lengths l_1 , l_2 , so that the numbers of photons per unit time through the slits are equal:

$$\Delta\theta_1 N_K(\psi = 0) = \Delta\theta_2 N_K(\psi = \psi_1), \quad (1)$$

where N_K is the flux density of the radiation in the bandwidth $\Delta\lambda/\lambda = K$. The aperture angles are $\Delta\theta_1 = l_1/L$, $\Delta\theta_2 = l_2/L$, where L is the distance between the slits and the point on the trajectory where the SR beam originates.

The mean energy E_0 of the beam is determined absolutely when the two photon fluxes are equal, which is measured as the zero difference.

3.0 Synchrotron Radiation Vertical Distribution

Figure 2. SR photon distribution along the axis vertical to the orbit plane, ψ is the vertical radiation angle. γ is the Lorentz factor of the particle.

Figure 3. Separation of Photon Fluxes at the Angles $\psi = 0$ and $\psi = \psi_0$

The Tolerances and Resolution of the Synchrotron Radiation Analysis Method

The working point is $\gamma\psi = 0.801084$, where the second derivative of the function of photon vertical distribution is equal to zero.

Table 5. Main parameters for AEM-4GeV

The source of error	Symbol of Formula	Tolerance	Maximum Error $\Delta E/E_0$
angle between orbit plane and the slit system symmetry plane	$\Theta\psi$	$0.945 \cdot 10^{-5} \text{ rad}$	10^{-5}
quantum fluctuations	ΔN_k	$2.140 \cdot 10^5 \text{ ps}^{-1}$	$0.5 \cdot 10^{-5}$
the arc in bending	$\Delta\theta$	0.148 rad	10^{-5}
wavelength	$\Delta\lambda/\lambda$	$2.700 \cdot 10^{-3}$	10^{-5}
accuracy of measurement of field intensity	$\Delta B/B$	$0.600 \cdot 10^{-4}$	10^{-5}
accuracy of measurement of the monochromator angles	$\delta\theta$	$2.720 \cdot 10^{-5}$	10^{-5}
emittance parameters	$\delta\psi = y' + (y/L)$	$1.250 \cdot 10^{-5} \text{ rad}$ at $L = 5 \cdot 10^3 \text{ mm}$	10^{-5}

The total r.m.s. error $\Delta E = 2.5 \cdot 10^{-5} E$

4.0 An Optimal Variant of CEBAF High Precision Energy Measurement Complex

Above presented results of the investigations and technical possibilities at CEBAF permits to make following principal solution about beam mean energy measurement with the precision of one unit of 10^{-4} and higher. It is evident that this goal may be achieved only by proposed method of synchrotron radiation vertical distribution measurement [3,4] with the tolerances presented in Table 5. In the other hand for the creation of SR, high precision magnetic field is required. With this purpose special short magnet will be inserted in extracted beam transportation system as close as possible to target area (see Figure 4). The best place for this magnet is the place of penultimate dipole which for this case will be removed.

Figure 4

For the compensation of the distortion of electron beam path caused by inserted magnet the optical system like three magnet chicane is required. Such a kind of optics may be performed using neighboring ordinary magnets by giving additional bend angle (see Figure 5). This additional angle, $\delta\theta$, may be received by increasing the current in magnet conductors. The tolerable increasing of the current is about 20% which creates accordingly $\delta\theta = 0.2 \text{ rad}$.

Figure 5

The high precision magnets length is about 0.5 cm and new formed straight sections having more than 2m each are sufficient to install four 0.2m long kicker magnets. If SR swaths length will be taken about 7m long and the distance between them is measured by the accuracy $3 \times 10^{-5} m$, the beam absolute energy determination preliminary will be performed by accuracy of three units of 10^{-4} and definitively by the accuracy up to $2.5 \cdot 10^{-5}$ using SR analysis method.

The main parameters of inserted optics and the main source of the error which are limiting the resolution for magnetic analysis are presented in Tables 6 and 7.

Table 6. Main parameters of the proposed chicane

Parameter	Symbol	Size
1. Bending angle	Θ	4×10^{-2}
2. Magnet length	L_m	0.52 m
3. SR path length	L_{SR}	7.0 meters
4. Vertical bend angle	Θ_v	$2 \cdot 10^{-3} rad$
5. Number of spectrometer magnets	N_B	1
6. Number of kicker magnets	N_k	4
7. Number of position detectors	n_{det}	2
8. Total length of chicane	L_T	no additional space needed

Table 7. Systematic errors for the proposed chicane

Source of Error	Size of Error	Contribution to energy error
1. Magnetic measurement	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
2. Detector position resolution	$5.76 \times 10^{-6} rad$	1.44×10^{-4}
3. Kicker rotation	$8 \times 10^{-6} rad$	2.0×10^{-4}
4. Survey error	$0.5 \times 10^{-5} rad$	1.1×10^{-4}

$$\text{Total } \Delta E = 2.25 \times 10^{-4} E_0$$

5.0 References

- [1] J. Kent, M. King *et al.*, "PRECISION MEASUREMENT OF THE SLC BEAM ENERGY" SLAC- PUB 4922, LBL-26977, March 1989.
- [2] B. Bevins and P. Kloeppel. "PROPOSED HIGH ACCURACY BEAM ENERGY MEASUREMENT AT CEBAF" (presented to this workshop).
- [3] I. P. Karabekov. Nucl. Instrum. Methods, A286, 1990, p.37.
- [4] I. P. Karabekov, R. Rossmannith. "MEASUREMENTS OF THE ABSOLUTE VALUE OF THE BEAM ENERGY AND ITS DEVIATION AT CEBAF" CEBAF TN 90-0224 April 23, 1990.
- [5] I. P. Karabekov. "SPECIFICATION FOR CONSTRUCTION OF ABSOLUTE ENERGY MONITOR FOR CEBAF" CEBAF TN 91-045 July 8, 1991.