STRANGE HAPPENINGS:
A GLOBAL ANALYSIS OF THE STRANGE
VECTOR AND AXIAL FORM FACTORS
OF THE NUCLEON AND THEIR UNCERTAINTIES

BY

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DEDICATION

For my loving wife, Tiffany. Thank you for all your support and encouragement. I could not have finished this without you.

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ABSTRACT

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We studied the strange contributions to the elastic vector and axial form factors of the nucleon using all available elastic electroweak scattering data. Specifically, we combine elastic νp and $\bar{\nu} p$ scattering cross-section data from the Brookhaven E734 experiment with elastic ep and quasi-elastic ed and $e\ ^4\text{He}$ scattering parity-violating asymmetry data from the SAMPLE, HAPPEX, PVA4 and G^0 experiments. We not only determined these form factors at individual values of momentum-transfer (Q^2), as other groups have done recently, but also fit the Q^2 -dependence of these form factors using simple functional forms. I present an overview of the G^0 backward-angle experiment as well as the results of these fits using existing data, along with some expectations of how we can improve our knowledge of these form factors if the MicroBooNE collaboration completes their experiment.

CONTENTS

LIST OF TABLES	ix
LIST OF FIGURES	x
1 INTRODUCTION	1
2 THEORY	2
2.1 Currents and Form Factors	2
2.2 Parity-Violating Electron-Nucleon Scattering	6
2.3 Neutrino-Proton & Anti-Neutrino-Proton Scattering	11
3 EXPERIMENTAL APPARATUS	14
3.1 Beam	14
3.2 Target	15
3.3 Magnet	17
3.4 Detectors	18
3.5 Electronics	24
3.6 Data Acquisition	25
3.7 Analyzer and Database	26
3.8 Simulation Software	26
4 G^0 -BACKWARD: ANALYSIS	27
4.1 Event-by-Event Corrections	28
4.1.1 Pass 1: Raw Yields and Asymmetries	29
4.1.2 Pass 2: Scaler Counting Correction	29
4.1.3 Pass 3: Electronics Corrections	30
4.1.4 Pass 4: Linear Regression Correction	34
4.2 Run-Averaged Asymmetry Corrections	41
4.2.1 Background Corrections	41
4.2.2 Electromagnetic Radiative Corrections	42
4.2.3 Beam Polarization Correction	44
4.2.4 Q^2 Determination	48
4.3 Physics Asymmetries and Extracted Form Factors	48
5 FIT	51
5.1 Experiments	51
5.1.1 Brookhaven E734	52
5.1.2 G^0 -Forward	52
5.1.3 G^0 -Backward	52
5.1.4 SAMPLE	53
5.1.5 HAPPEX	53
5.1.6 PVA4	54
5.2 Model-Independent Extractions	54

5.3	Global Analysis	56
5.4	Upcoming Data	58
6	CONCLUSION	66
	REFERENCES	67

LIST OF TABLES

1	Weak isospin and electroweak charges of the elementary fermions.	3
2	PDG radiative parameters.	9
3	“One-quark” radiative corrections.	10
4	“One-quark” and “many-quark” axial radiative corrections.	10
5	G^0 backward beam parameter values.	15
6	G^0 backward angle running periods.	27
7	Blinding factors.	29
8	Deadtime contributions	35
9	Pass 3 and 4 asymmetries and asymmetry differences.	40
10	Linear regression corrections.	40
11	Primary background contributions to electron yield.	43
12	Hydrogen electromagnetic radiative corrections.	45
13	Deuterium electromagnetic radiative corrections.	46
14	Transverse corrections.	48
15	Q^2 sensitivities.	48
16	Beam energy, center of target energy and Q^2 by running period. .	49
17	G^0 backward asymmetries.	49
18	G^0 extracted form factors.	50
19	BNL E734 differential cross section data.	52
20	SAMPLE asymmetry data.	53
21	HAPPEX asymmetry data.	53
22	PVA4 asymmetry data.	54
23	Preliminary fit results.	57
24	Fit results as a function of fixed $\Lambda_{E,M}$ values.	57
25	Simulated MicroBooNE data.	63
26	Fit results without and with simulated MicroBooNE data.	64

LIST OF FIGURES

1	Three types of electroweak current and gauge boson couplings. . .	2
2	Electroweak interactions in elastic electron-nucleon scattering. . .	7
3	“One-quark” radiative correction diagrams.	8
4	G^0 cryogenic target.	16
5	Detailed view of G^0 cryogenic target manifold.	17
6	SMS octants and single-octant side view.	18
7	G^0 forward single-octant schematic.	19
8	G^0 (forward) detector azimuthal symmetry.	19
9	G^0 forward apparatus in hall C.	20
10	G^0 backward apparatus in hall C.	21
11	G^0 backward angle Čerenkov electron detector system.	21
12	G^0 backward single-octant schematic.	22
13	G^0 backward CED-FPD detector yield coincidence matrices. . . .	23
14	G^0 backward CED-FPD coincidence matrix locus definitions. . . .	23
15	Basic timing diagram of coincidence board signals.	24
16	G^0 data acquisition.	25
17	G^0 backward analysis strategy.	28
18	Cuts on yield and effects on asymmetry.	31
19	Locus definitions and electronics rate corrections.	32
20	Induced charge asymmetry vs. detector asymmetry, 362 MeV, LD2. .	36
21	Field scans for individual cells on inelastic edge of elastic locus. .	42
22	Amplitudes for tree-level and virtual internal radiative corrections. .	43
23	Amplitudes for real internal radiative corrections.	44
24	Fit of the G^0 forward angle results.	51
25	Model-independent extractions of strange form factors.	55
26	Model-independent extractions with strange form factor fits. . . .	59
27	Normalized deviations of fit from data with Gaussian fit.	60
28	G^0 forward asymmetries and fit.	60
29	E734 νp and $\bar{\nu} p$ cross-sections and fit.	61
30	Model-independent extractions with fits and 70% CL curves. . . .	62
31	Extractions and 70% CL fit using simulated MicroBooNE data. . .	65

The strangeness will wear off and I think we will discover the deeper meanings...

-Jackson Pollack

1 INTRODUCTION

Physicists currently believe the proton and neutron, or nucleons, are composed of three up and down valence quarks—quarks that are always present—gluons to hold the quarks together and the vacuum. Rather than empty nothingness, however, the vacuum is actually a bubbling sea of virtual quark-antiquark pairs constantly fluctuating into and out of existence. (Allowed by the uncertainty principle, virtual particle-antiparticle pairs may spring from nothingness as long as they annihilate each other within a short amount of time inversely proportional to the energy—including mass—they possess.) Because smaller masses equate to lower energy, virtual pairs of the lightest quarks are most likely to spring into existence and will last longest when they do. So the most prevalent of these virtual quarks are up and down—the same flavors as the valence quarks. But these virtual up and down quarks are not easily distinguishable from the valence quarks. Certainly, one can easily imagine a scenario in which a virtual quark-antiquark pair springs forth very near a valence quark of the same flavor, hurtles toward annihilation at the point where the valence quark lies and the virtual antiquark annihilates the valence quark, casting the formerly-valence into nothingness and bequeathing all of its presence and existence onto the formerly-virtual quark. We know no laws or properties to stop such a scenario from happening. Perhaps existence is like celebrity, for which quarks jealously struggle, or a burden that quarks try to shed and can slip into anonymity—who can say the motivations of quarks?

Strange quarks are more massive than up and down quarks but are significantly less massive than charm, bottom or top quarks. And because nucleons contain no strange valence quarks, the strange quark provides the best window on the contributions of the virtual quark sea to a particle's properties. For years physicists assumed that vacuum contributions were zero—a nucleon contains no net strange quarks, for example, so how could strange quarks contribute to the nucleon's properties? As physicists have been able to conduct more precise experiments, however, we have now reached a point where we might be able to measure such tiny contributions.

2 THEORY

According to the Standard Model of particle physics, elementary fermions interact by exchanging gauge bosons—if the fermions interact electromagnetically they exchange a photon (γ); if they interact via the weak nuclear force they exchange either a Z^0 , W^+ or W^- boson; if they interact via the strong nuclear force they exchange a gluon. (If the particles interact gravitationally we theorize that they exchange a graviton, but gravity is not part of the Standard Model.)

2.1 Currents and Form Factors

When we calculate the Lagrangian of an interaction we use fermion currents coupled to gauge bosons. Figure 1 shows Feynman diagrams representing the three types of electroweak interactions: electromagnetic, neutral weak and charged weak currents.

Figure 1: Three types of electroweak current and gauge boson couplings.

We can write the electromagnetic (EM) current as

$$J_\mu^{EM} = Q\bar{u}\gamma_\mu u \quad (1)$$

where Q is the EM coupling constant (electric charge) of the fermion in units of proton charge, $\bar{u}$ (u) is the fermion creation (annihilation) operator and γ_μ ($\mu = 0,1,2,3$) are the Dirac matrices, which take care of the vector transformation properties of EM interactions.

Weak nuclear interactions do not conserve parity so they contain vector and axial vector components. We can write the Z^0 exchange current, or neutral weak current (NC), as

$$J_\mu^{NC} = \bar{u}\gamma_\mu(c_V + c_A\gamma_5)u \quad (2)$$

where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ and c_V and c_A are the weak vector and axial coupling constants (weak vector and axial “charges”) of the fermion. Electroweak theory gives

these as

$$\begin{aligned} c_V &= 2T^3 - 4\sin^2\theta_W Q \\ c_A &= -2T^3 \end{aligned} \quad (3)$$

where $T^3 = \pm\frac{1}{2}$ is the z-component of the fermion's weak isospin and θ_W is the weak mixing angle. Table 1 shows the weak isospin and electroweak charges of the elementary fermions.

Table 1: Weak isospin and electroweak charges of the elementary fermions.

Fermion	T^3	Q	c_V	c_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0	1	-1
e, μ, τ	$-\frac{1}{2}$	-1	$-1 + 4\sin^2\theta_W$	1
u, c, t	$\frac{1}{2}$	$\frac{2}{3}$	$1 - \frac{8}{3}\sin^2\theta_W$	-1
d, s, b	$-\frac{1}{2}$	$-\frac{1}{3}$	$-1 + \frac{4}{3}\sin^2\theta_W$	1

Similarly, we can write the $W^\pm$ exchange current, or charged weak current (CC), as

$$J_\mu^{CC} = \bar{u}\gamma_\mu(1 - \gamma_5)v \quad (4)$$

where this time u and v are different fermion states because the $W^\pm$ boson carries one unit of charge; the beginning and ending fermion states must be different, with charges differing by one unit.

Because hadrons are extended particles with structure, we use form factors to contain the spatial and temporal information in expressions for the current. Two properties distinguish one form factor from another: the transition between beginning and ending hadronic states (elastic, inelastic, $N - \Delta$, etc.) and the tensor properties of the interaction (scalar, vector, axial-vector, etc.). We multiply the form factors by the appropriate coupling constants, or charges, relevant to the type of interaction (electromagnetic, weak, etc.). We can write the hadronic electromagnetic (photon exchange) current as

$$J_\mu^{EM,N} = \bar{u}(p') \left[\gamma_\mu F_1^{\gamma,N}(q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2M} F_2^{\gamma,N}(q^2) \right] u(p) \quad (5)$$

where u is the hadronic state (also denoted by N), with initial momentum p' and final momentum p , $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, M is the hadron mass, $q^2 = (p' - p)^2$ is the four-momentum transfer and $F_1^{\gamma,N}(q^2)$ and $F_2^{\gamma,N}(q^2)$ are the electromagnetic Dirac and Pauli (elastic vector) form factors of the specific hadron, N .

We can write the weak neutral current as

$$J_\mu^{NC,N} = \bar{u}(p') \left[\begin{array}{c} \gamma_\mu F_1^{Z,N}(q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2M} F_2^{Z,N}(q^2) + \\ \gamma_\mu \gamma_5 G_A^{Z,N}(q^2) + \frac{q_\mu}{M} \gamma_5 G_P^{Z,N}(q^2) \end{array} \right] u(p) \quad (6)$$

where $F_1^{Z,N}(q^2)$ and $F_2^{Z,N}(q^2)$ are now the neutral-current Dirac and Pauli (elastic vector) form factors, $G_A^{Z,N}(q^2)$ is the axial vector form factor and $G_P^{Z,N}(q^2)$ is the pseudo-scalar form factor—all of the specific hadron, N . (Because the pseudo-scalar form factor, G_P , does not contribute to parity-violating elastic eN scattering asymmetries, this dissertation will no longer include it.)

As mentioned above, form factors contain the spatial and temporal information about interactions. Elastic form factors, therefore, contain the spatial and temporal information about a specific particle. The Q^2 -dependence of elastic form factors is related to the distance scale of examination of the particle by the deBroglie wavelength

$$Q \approx \frac{h}{\lambda}. \quad (7)$$

As $Q^2 \rightarrow 0, \lambda \rightarrow \infty$ and we see the whole particle, so at $Q^2 = 0$, form factors are properties of the particle. For point-like particles, form factors are constants; because form factors of extended particles contain spatial information, as $Q^2 \rightarrow \infty, \lambda \rightarrow 0$ —the distance scale shrinks to zero—so form factors must also go to zero

Instead of the Dirac and Pauli form factors, nuclear physicists customarily use the Sachs electric and magnetic form factors[23], defined as

$$\begin{aligned} G_E^N &= F_1^N - \tau F_2^N \\ G_M^N &= F_1^N + F_2^N \end{aligned} \quad (8)$$

where $\tau = \frac{Q^2}{4M^2}$. (We redefine both the EM and NC Dirac and Pauli form factors in terms of EM and NC Sachs electric and magnetic form factors.) These Sachs form factors are convenient because, at $Q^2 = 0$, the photon exchange form factors are simply the electric charges and magnetic moments of the nucleons:

$$\begin{aligned} G_E^{\gamma,p} &= 1, & G_M^{\gamma,p} &= \mu_p \\ G_E^{\gamma,n} &= 0, & G_M^{\gamma,n} &= \mu_n. \end{aligned} \quad (9)$$

Additionally, in the Breit frame, where the exchanged photon is purely spacelike, physicists often interpret $G_E^{\gamma,N}$ and $G_M^{\gamma,N}$ as the Fourier transforms of the charge and magnetization densities of the nucleon[49].

Nucleons are extended particles with complex structures. However, because the quarks they contain are point-like, we may re-write the nucleon form factors

in terms of the individual quark contributions

$$G_{E,M}^\gamma = \sum_j Q^j G_{E,M}^j \quad (10)$$

$$G_{E,M}^Z = \sum_j c_V^j G_{E,M}^j \quad (11)$$

and

$$G_A^Z = \sum_j c_A^j G_A^j \quad (12)$$

where j differentiates between the six different quark flavors, $G_{E,M}^j$ are the Sachs vector form factors and G_A^j are the axial form factors for the individual quarks. Note that each of these individual quark form factors includes contributions from both the quark and its anti-quark counterpart. Because of their large masses the heavier quarks contribute negligibly to the nucleon structure[29]. In terms of u , d and s quarks, then, we can expand equations 10 and 11 for the proton explicitly to

$$G_{E,M}^{\gamma,p} = \frac{2}{3} G_{E,M}^u - \frac{1}{3} G_{E,M}^d - \frac{1}{3} G_{E,M}^s \quad (13)$$

and

$$G_{E,M}^{Z,p} = \left(1 - \frac{8}{3} \sin^2 \theta_W\right) G_{E,M}^u + \left(-1 + \frac{4}{3} \sin^2 \theta_W\right) G_{E,M}^d + \left(-1 + \frac{4}{3} \sin^2 \theta_W\right) G_{E,M}^s \quad (14)$$

where all of the individual quark form factors contain an implicit “ p ” superscript because they indicate the quark form factors for quarks inside a proton.

As with the nucleon form factors, these individual quark electric and magnetic form factors, multiplied by their electric charges, equal the Fourier transforms of the electric charge and magnetization densities of the individual quark flavors inside the proton. For nucleons, the u and d quark form factors include contributions from both valence and virtual quarks (from the virtual quark sea), while the s form factors include contributions only from virtual $s\bar{s}$ sea quarks. This is why strange quark form factors are so important—they show us contributions to particles’ properties from the virtual sea.

We can also expand equation 12 for the proton in terms of u , d and s quarks to

$$G_A^{Z,p} = -G_A^u + G_A^d + G_A^s \quad (15)$$

where, again, the individual quark form factors contain an implicit “ p ” superscript.

(In parallel with the Sachs vector form factors, the individual quark axial form factors, at $Q^2 = 0$, equal the contributions of the individual quark flavors to the proton's spin.) This flavor decomposition for the axial form factor is more difficult than for the vector form factors because it is difficult to separate the u and d contributions. The combination $-G_A^u + G_A^d$, however, is accessible in low energy charged-current scattering, so we call it G_A^{CC} and commonly re-write equation 15 as

$$G_A^{Z,p} = -G_A^{CC} + G_A^s. \quad (16)$$

Equations 13 and 14 break down the proton vector form factors into their constituent quark flavors. For the neutron we use charge symmetry, which states that exchanging the u ($\bar{u}$) and d ($\bar{d}$) quarks changes a proton to a neutron, and vice versa. Protons and neutrons vary from this symmetry at order 1% or less[38]. Because the u and d masses are so similar, switching them, and thus switching their distributions, should make little difference in the overall mass distributions between protons and neutrons. Because we expect the valence quarks to attract strange quarks and anti-quarks through kaon resonances, exchanging u and d quarks also exchanges s and $\bar{s}$, both of which contribute to the strange form factor. Therefore we also assume that the strange form factors are the same between nucleons. Switching u and d quarks in equation 13 gives

$$G_{E,M}^{\gamma,n} = \frac{2}{3}G_{E,M}^d - \frac{1}{3}(G_{E,M}^u + G_{E,M}^s) \quad (17)$$

for the neutron vector electromagnetic form factors. The three equations 13, 14 and 17 contain three unknowns (G_E^u , G_E^d and G_E^s) for the G_E^N form factors and three unknowns (G_M^u , G_M^d and G_M^s) for the G_M^N form factors so if we can measure the $G_{E,M}^N$ form factors we can solve these systems of equations.

We can also use equations 13 and 17 to express the up and down quark form factors in terms of the strange quark form factors and the nucleon form factors.

$$\begin{aligned} G_{E,M}^u &= 2G_{E,M}^{\gamma,p} + G_{E,M}^{\gamma,n} + G_{E,M}^s \\ G_{E,M}^d &= G_{E,M}^{\gamma,p} + 2G_{E,M}^{\gamma,n} + G_{E,M}^s \end{aligned} \quad (18)$$

The electromagnetic nucleon form factors have been well-measured at low Q^2 so this allows us to focus on the strange form factors.

2.2 Parity-Violating Electron-Nucleon Scattering

The electromagnetic interaction dominates elastic $e - N$ scattering, but the neutral weak interaction also contributes to a small degree. Figure 2 shows Feynman diagrams of these two interactions. We can write the amplitude of the scattering using the standard Feynman rules: sandwich the gauge boson propagator

Figure 2: Electroweak interactions in elastic electron-nucleon scattering.

between the two scattered particles' currents and multiply by the coupling constant of the interaction. So the electromagnetic scattering amplitude is

$$M_\gamma = J_\mu^{EM,e} \left(-\frac{e^2}{Q^2} \right) J_\mu^{EM,N} \quad (19)$$

where e is the electromagnetic coupling constant—the proton's charge—while the neutral weak scattering amplitude is

$$\begin{aligned} M_Z &= J_\mu^{NC,e} \frac{g^2 \sqrt{2}/8}{Q^2 + M_Z^2} J_\mu^{NC,N} \\ &\approx J_\mu^{NC,e} G_F J_\mu^{NC,N} \end{aligned} \quad (20)$$

for $Q^2 \ll M_Z^2$, where $g^2 = \frac{8}{\sqrt{2}} G_F M_{W,Z}^2$ is the dimensionless electroweak coupling constant and $G_F = 1.166 \times 10^{-5}$ [5] is the Fermi coupling constant. The elastic scattering cross-section is proportional to the square of the total elastic scattering amplitude

$$\begin{aligned} \frac{d\sigma}{d\Omega} &\propto |M|^2 = |M_\gamma + M_Z|^2 \\ &= |M_\gamma|^2 + 2\text{Re}(M_\gamma^* M_Z) + |M_Z|^2 \end{aligned} \quad (21)$$

If we scatter longitudinally-polarized electrons from unpolarized nucleons we can measure the asymmetry of the scattering cross sections. Because the neutral current contains an axial term, cross terms in M_Z change signs between helicity states, and remain after subtracting σ_+ and σ_- . We express the asymmetry as

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \approx \frac{M_\gamma M_Z}{|M_\gamma|^2} \quad (22)$$

where, in the last step we can neglect M_Z in the denominator because $\frac{M_Z}{M_\gamma} \sim 10^{-5}$ [41]. In terms of the proton's electromagnetic, neutral weak and axial form factors the asymmetry becomes

$$A^p = \left[\frac{-G_F Q^2}{4\sqrt{2}\pi\alpha} \right] \frac{\epsilon G_E^{\gamma,p} G_E^{Z,p} + \tau G_M^{\gamma,p} G_M^{Z,p} - (1 - 4\sin^2\theta_W) \epsilon' G_M^{\gamma,p} G_A^{Z,p}}{\epsilon (G_E^{\gamma,p})^2 + \tau (G_M^{\gamma,p})^2} \quad (23)$$

where $\tau = \frac{Q^2}{4M_p^2}$, $\epsilon = [1 + 2(1 + \tau) \tan^2(\theta_e/2)]^{-1}$ and $\epsilon' = \sqrt{\tau(1 + \tau)(1 - \epsilon^2)}$ are kinematic factors.

This expression contains only the tree-level interactions shown in figure 2. A complete expression of the asymmetry must also include higher-order electroweak corrections[11]. “One-quark” scattering corrections, in which only a single quark is involved[41, 60], typically dominate the corrections. Figure 3 shows two examples of these higher-order effects.

Figure 3: “One-quark” radiative correction diagrams.

We make corrections to the tree level values by

Tree Level	$\Rightarrow$	Rad. Corr. Included
$2c_V^u + c_V^d = 1 - 4\sin^2\theta_W$	$\Rightarrow$	$(1 + R_V^p)(1 - 4\sin^2\theta_W)$,
$2c_V^d + c_V^u = -1$	$\Rightarrow$	$-(1 + R_V^n)$,
$c_V^u + c_V^d + c_V^s = -1$	$\Rightarrow$	$-(1 + R_V^{(0)})$,
$\frac{1}{2}(c_A^u - c_A^d) = -1$	$\Rightarrow$	$-(1 + R_A^{T=1})$,
$\sqrt{3}(c_A^u + c_A^d) = 0$	$\Rightarrow$	$\sqrt{3}R_A^{T=0}$,
$c_A^u + c_A^d + c_A^s = 1$	$\Rightarrow$	$1 + R_A^{(0)}$

(24)

where the R parameters are common in parity-violation literature and can be expressed in terms of the more general radiative correction parameters defined by the Particle Data Group (PDG). The PDG introduces a set of constants, $C_{ij}, i = (1, 2), j = (u, d)$, to describe the parity-violating coupling of the electron

current to the quark current[5]

$$\begin{aligned}
C_{1u} &= \rho_1 \left(-\frac{1}{2} + \frac{4}{3} \kappa_1 \hat{s}_Z^2 \right) + \lambda_{1u} , \\
C_{1d} &= \rho_1 \left(\frac{1}{2} - \frac{2}{3} \kappa_1 \hat{s}_Z^2 \right) + \lambda_{1d} , \\
C_{2u} &= \rho_2 \left(-\frac{1}{2} + 2\kappa_2 \hat{s}_Z^2 \right) + \lambda_{2u} , \\
C_{2d} &= \rho_2 \left(\frac{1}{2} - 2\kappa_2 \hat{s}_Z^2 \right) + \lambda_{2d} ,
\end{aligned} \tag{25}$$

where $\hat{s}_Z^2 = \sin^2 \theta_W = 0.23120(15)$ for the renormalization scale at M_Z using the modified minimal subtraction($\overline{\text{MS}}$) renormalization scheme[4], and the ρ , κ and λ parameters are defined in the PDG book[5]. The C_{ij} parameters are related to the $c_{V,A}^{u,d,s}$ by

$$\begin{aligned}
c_V^{u,c,t} &= -2C_{1u} , & c_V^{d,s,b} &= -2C_{1d} , \\
c_A^{u,c,t} &= \frac{2C_{2u}}{1 - 4 \sin^2 \theta_W} , & c_A^{d,s,b} &= \frac{2C_{2d}}{1 - 4 \sin^2 \theta_W} .
\end{aligned} \tag{26}$$

Table 2 shows the values of the PDG radiative parameters. Table 3 shows the resulting “one-quark” radiative corrections. The complete radiative corrections, however, include “many-quark” contributions and heavy quark contributions as well:

$$R = R_{\text{one-quark}} + R_{\text{many-quark}} + R_{\text{heavy}} . \tag{27}$$

Table 2: PDG radiative parameters.

Parameter	SM Value
$\hat{s}_Z^2$	0.230
ρ_1	0.99
ρ_2	1.0004
κ_1	1.003
κ_2	1.03
λ_{1u}	-1.80×10^{-5}
λ_{1d}	3.6×10^{-5}
λ_{2u}	-0.012
λ_{2d}	0.0026

Table 3: “One-quark” radiative corrections.

R Parameter	Value
R_V^p	−0.054568
R_V^n	−0.011679
$R_V^{(0)}$	−0.011789
$R_A^{T=1}$	−0.173
$R_A^{T=0}$	−0.253
$R_A^{(0)}$	−0.550

The heavy quark contributions are small[29] so we neglect them. Table 4 shows “one-quark,” “many-quark” [60] and total axial radiative corrections.

Table 4: “One-quark” and “many-quark” axial radiative corrections.

	$R_A^{T=1}$	$R_A^{T=0}$	$R_A^{(0)}$
one-quark	−0.173	−0.253	−0.550
many-quark	$−0.087 \pm 0.35$	0.015 ± 0.20	N/A
total	$−0.26 \pm 0.35$	$−0.24 \pm 0.20$	$−0.55 \pm 0.55$

The axial part in particular becomes even more complicated with radiative corrections. We can express the axial form factor in terms of a triplet part (G_A^{CC}), an octet part ($G_A^8 = \frac{1}{2\sqrt{3}} (G_A^u + G_A^d - 2G_A^s)$), which becomes important when including radiative corrections, and a singlet part (which includes G_A^s). With the complicated radiative corrections we generally rename the axial form factor G_A^e , the “effective axial form factor.” The full expression for the effective axial form factor is[41]

$$G_A^e = - (1 + R_A^{T=1}) G_A^{CC} + \sqrt{3} R_A^{T=0} G_A^8 + (1 + R_A^{(0)}) G_A^s \quad (28)$$

Expanding equation 23 in terms of the strange quark form factors, and including radiative corrections[41], the full asymmetry expression, as a linear combina-

tion of G_E^s , G_M^s and G_A^s , is[43]¹

$$A^p = A_0^p + A_E^p G_E^s + A_M^p G_M^s + A_A^p G_A^s \quad (29)$$

where

$$\begin{aligned} A_0^p &= -K^p \left\{ \begin{array}{l} \varepsilon G_E^p [(1 - 4 \sin^2 \theta_W) (1 + R_V^p) G_E^p - (1 + R_V^n) G_E^n] + \\ \tau G_M^p [(1 - 4 \sin^2 \theta_W) (1 + R_V^p) G_M^p - (1 + R_V^n) G_M^n] - \\ \varepsilon' G_M^p (1 - 4 \sin^2 \theta_W) [- (1 + R_A^{T=1}) G_A^{CC} + \sqrt{3} R_A^{T=0} G_A^8] \end{array} \right\} \\ A_E^p &= K^p \{ \varepsilon G_E^p (1 + R_V^0) \} \\ A_M^p &= K^p \{ \tau G_M^p (1 + R_V^0) \} \\ A_A^p &= K^p \{ \varepsilon' G_M^p (1 - 4 \sin^2 \theta_W) (1 + R_A^0) \} \\ K^p &= \frac{G_F Q^2}{4\pi \sqrt{2} \alpha \varepsilon (G_E^p)^2 + \tau (G_M^p)^2}. \end{aligned}$$

To access the neutron's form factors we cannot have a neutron target from which to scatter electrons. Instead we scatter electrons from deuterium. To calculate the A_d , the deuterium scattering asymmetry, the static approximation gives

$$A_d = \frac{A_p \sigma_p + A_n \sigma_n}{\sigma_p + \sigma_n} \quad (30)$$

where A_p and A_n are the parity-violating ep and en scattering asymmetries, and σ_p and σ_n are the elastic ep and en scattering cross-sections. But this expression also needs nuclear corrections. So that we can write the deuterium expression in a form parallel to equation 29, Schiavilla calculated asymmetry coefficients for the expression

$$A^d = a_0^d + a_1 G_E^s + a_2 G_M^s + a_3 G_A^{e(T=1)} + a_4 G_A^s \quad (31)$$

using a nuclear model of deuterium, including two-body effects[52]. He calculated coefficients for the specific kinematics of each SAMPLE and G^0 deuterium measurement, using nuclear response functions, Argonne V18 potentials and nucleon form factors. For SAMPLE he used Dipole/Galster form factor parameterizations while, for G^0 , he used Kelly parameterizations [30].

2.3 Neutrino-Proton & Anti-Neutrino-Proton Scattering

Equation 23 shows that the electron's weak charge, $(1 - 4 \sin^2 \theta_W) \approx 0.075$, suppresses the axial term in the elastic electron-nucleon scattering asymmetry. (The kinematic factor ϵ' also suppresses it for forward-angle scattering.) Therefore,

¹Because the G^0 analysis extracts G_A^e instead of G_A^s , the G^0 collaboration prefers to split our A_0^p term into a_0 and $a_3 G_A^{e(T=1)}$ and merge the resulting a_0 term with our $A_A^p G_A^s$ term.

parity-violating electron-nucleon scattering alone cannot determine the strange axial form factor to any precision. The axial form factor dominates neutrino-proton and anti-neutrino-proton scattering cross-sections, however, and because these are pure weak processes, radiative corrections are negligible at low Q^2 [37]. Combining electron-nucleon scattering data with neutrino-proton scattering data allows us to determine all three strange form factors.

The νp and $\bar{\nu} p$ elastic scattering cross-sections measure quadratic combinations of the weak form factors. The full expression is[3]

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{2\pi} \frac{Q^2}{E_\nu^2} (A \pm BW + CW^2) \quad (32)$$

using the $+$ sign for ν and the $-$ sign for $\bar{\nu}$ scattering, and where

$$\begin{aligned} W &= 4 \left(\frac{E_\nu}{M_p} - \tau \right) \\ \tau &= \frac{Q^2}{4M_p^2} \\ A &= \frac{1}{4} \left[(G_A^Z)^2 (1 + \tau) - \left((F_1^Z)^2 - \tau (F_2^Z)^2 \right) (1 - \tau) + 4\tau F_1^Z F_2^Z \right] \\ B &= -\frac{1}{4} G_A^Z (F_1^Z + F_2^Z) \\ C &= \frac{1}{64\tau} \left[(G_A^Z)^2 + (F_1^Z)^2 + \tau (F_2^Z)^2 \right]. \end{aligned}$$

The νp and $\bar{\nu} p$ charged-current (inelastic) scattering cross-sections also measure quadratic combinations of the weak form factors. The full expression is[32]

$$\frac{d\sigma}{dQ^2}_{CC} = \frac{G_F^2}{2\pi} \frac{\cos^2 \theta_C}{E_\nu^2} (A \pm BW + CW^2) \quad (33)$$

using the + sign for ν and the - sign for $\bar{\nu}$ scattering, and where

$$\begin{aligned}
W &= 4 \left(\frac{E_\nu}{\bar{M}_d} - \tau_{CC} - \frac{M_\mu^2}{4\bar{M}_d^2} \right) \\
\tau_{CC} &= \frac{Q^2}{4\bar{M}_d^2} \\
A &= \frac{1}{4} (M_\mu^2 + Q^2) \left[\frac{(G_A^{CC})^2 (1 + \tau_{CC}) - ((F_1^v)^2 - \tau_{CC} (F_2^v)^2) (1 - \tau_{CC}) + 4\tau_{CC} F_1^v F_2^v}{(F_1^v)^2 - \tau_{CC} (F_2^v)^2} \right] \\
B &= \frac{Q^2}{4} G_A^{CC} (F_1^v + F_2^v) \\
C &= \frac{Q^2}{64\tau_{CC}} \left[(G_A^{CC})^2 + (F_1^v)^2 + \tau_{CC} (F_2^v)^2 \right],
\end{aligned}$$

M_μ is the muon mass, $\bar{M}_d = (M_p + M_n)/2$ is the average nucleon mass in a deuteron, θ_C is the Cabibbo angle for d and s quark mixing, and $F_{1,2}^v = F_{1,2}^p - F_{1,2}^n$.

Neutrino physicists use the Dirac and Pauli form factors instead of converting to Sachs form factors.

3 EXPERIMENTAL APPARATUS

Like other parity-violating electron-nucleon scattering experiments, G^0 scattered longitudinally-polarized electrons from non-polarized nucleon targets. The forward-angle experiment detected forward-scattered protons from a liquid hydrogen (LH2) target, while the backward-angle experiment detected backward-scattered electrons from a liquid hydrogen and a liquid deuterium (LD2) target.

3.1 Beam

To measure asymmetries on the order of 10 ppm between beam polarization states requires a well-controlled and highly-polarized beam—the Continuous Electron Beam Accelerator Facility (CEBAF) beam at the Thomas Jefferson National Accelerator Facility suited the collaboration’s needs. To generate the polarized beam, a 5 Watt fiber laser dislodges electrons from a GaAs cathode electron source via the photoelectric effect [53]. Before striking the cathode, the laser first passes through a static polarizer to become linearly-polarized, then through a Pockels cell to become circularly-polarized². A Pockels cell is made from a birefringent crystal whose indices of refraction change in an electric field. By applying the correct voltage, the Pockels cell becomes a quarter-wave plate, which shifts linearly polarized light into circularly polarized light. The circular polarization of the laser light gives the liberated electrons from the cathode longitudinal polarization. By changing the sign of the applied Pockels cell voltage, we flip the helicity of the polarized light and the resulting polarized electrons.

For the G^0 experiment the accelerator operators flipped the helicity every 1/30 second so that the asymmetry measurements were insensitive to the 60 Hz noise common to all North American electrical systems; we call the 1/30 second unit a “macropulse” (MPS). The G^0 beam has a quartet polarization structure, so that the MPSs of electrons come in groups of four, with their respective polarizations either $+- -+$ or $-+ +-$, selected pseudo-randomly. Additionally, the laser beamline also includes an insertable half wave plate (IHWP) that we can insert into or remove from the beamline [53]. This allowed us to flip all electron helicities manually on a macroscopic timescale; while running we changed the IHWP state every day or two. We used the IHWP to eliminate systematic helicity-correlated asymmetries from electronics or beam transport.

The G^0 collaboration chose beam specifications to limit helicity-correlated false beam parameter asymmetries to 5% or less of the statistical uncertainty of the measurements. Table 5 shows our target beam specifications and what we actu-

²For circularly-polarized light, we say it has “positive helicity” if its polarization—defined by the right-hand rule—is in the direction of propagation, and “negative helicity” if its polarization is in the opposite direction.

ally achieved. The JLab accelerator group exceeded our specifications in every category.

Table 5: G^0 backward beam parameter values.

Parameter	Achieved	Specification
Δx (nm)	$-19. \pm 3.$	40
Δy (nm)	$-17. \pm 2.$	40
$\Delta \theta_x$ (nrad)	-0.8 ± 0.2	4
$\Delta \theta_y$ (nrad)	0.0 ± 0.1	4
ΔE (eV)	2.5 ± 0.5	34
A_Q (ppm)	0.09 ± 0.08	2

3.2 Target

To keep systematic uncertainties less than 5% of the measured asymmetries, the G^0 target system had to accommodate large power deposits into the target liquid while avoiding density fluctuations. The target had to be reliable over many months' continuous running and could not interfere with the magnetic field. The G^0 target system was a closed recirculating cryogenic system constructed of 6061 aluminum and Type 304 stainless steel and cooled by a helium heat exchanger [21].

Figure 4 shows a schematic of the G^0 target. The target loop contains a total volume of about 6.5 L, circulated by a cryogenic pump immersed in the fluid that operates at 30 Hz, moving about 0.2 L of target liquid per pump revolution. A heat exchanger on the opposite side of the target loop cools the target liquid with gaseous He, at 15 K and 12 atm, from the JLab end station refrigerator at a flow rate of about 6 g/s. The exchanger cooled at a rate of about 50 W per g/s of coolant flow.

Figure 5 shows a more detailed schematic of the target manifold, which houses two cylindrical aluminum cells coaxial with the beam. The beam travels through the secondary helium cell before entering the primary target liquid cell. The helium cell was filled with He gas, at the same temperature and pressure as the target liquid, with an exit window—the target liquid cell entrance window—the same shape as the target liquid cell exit window. The helium cell defined the length of the target liquid cell as 20 cm and helped to provide azimuthal symmetry for

Figure 4: G^0 cryogenic target.

particles scattered from the target liquid within the range of the magnet's angular acceptance. The target liquid entered the target cell through a conical aluminum tube (light blue in Figure 5) with holes in it to increase turbulence and, thereby, heat transfer.

The helium cell entrance window was 0.228 mm thick; the helium cell exit window/target liquid cell entrance window was 0.178 mm thick; the target liquid cell exit window was 0.0762 mm thick. To distribute beam heating in the target windows and liquid, two small magnets steer the beam with a frequency of 25 kHz over a target raster area 2 mm x 2 mm.

During running we kept the LH2 at 19 K (2 degrees below the boiling point) and 1.7 atm; we kept the LD2 at 22 K (3 degrees below the boiling point) and 1.7 atm. Keeping the target loop connected to a ballast tank allowed us to maintain the loop pressure. The target system kept temperature fluctuations after changes in the beam current under 0.2 K.

For testing purposes we also had two dummy aluminum targets available to put into the beamline to approximate the entrance and exit windows of the target cell. The "frame" target was a section of the arm holding the G^0 target system that could be inserted into the beamline instead of the cryogenic target, located 1.7 cm upstream from the entrance window position, with a thickness of 0.307 cm. The "flyswatter" target could be inserted into the beamline with or without the cryogenic target in place, located 1.0 cm downstream of the exit window position, with a thickness of 0.076 cm. We could also place a tungsten target or a carbon target into the beamline for other tests.

Figure 5: Detailed view of G^0 cryogenic target manifold.

3.3 Magnet

The G^0 superconducting magnet system (SMS) contains eight azimuthally-symmetric toroidal superconducting coils [47]. Figure 6 shows the azimuthal symmetry of the detectors and magnet, as well as single-octant schematic. In the left picture, the detector octants are numbered while the magnet coils are lettered. The beam travels into the page and green arrows indicate the direction of the magnetic field. The right picture shows a single-octant schematic showing how the magnetic field pushes the scattered particles to the detectors—this time the magnetic field comes out of the page. Each octant, defined by collimators near the target, accepted 20° in azimuthal angle.

Liquid He cooled the magnet's superconducting NbTi coils to 4.5 K. The coils functioned reliably with current up to 5000 A to generate a maximum field integral of 1.6 T·m [40]. For the backward-angle experiment we operated the SMS at currents of 3500 A during the 687 MeV running and 2650 A during the 362 MeV running. Two 1000 Hz turbo-modular vacuum pumps maintained an SMS vacuum at 10^{-6} Torr.

Figure 6: SMS octants and single-octant side view.

3.4 Detectors

The G^0 experiment consisted of two parts. Figure 7 shows a schematic for a single octant of the G^0 forward experiment. G^0 forward scattered a 3 GeV electron beam from a liquid hydrogen (LH2) target and measured forward-scattered protons, whose paths were bent by a magnetic field to 16 pairs of scintillator focal plane detectors (FPDs). (The collaboration used pairs of FPDs so they could require a coincidence, in order to eliminate random FPD signals.) Each FPD was attached to a PMT on each end. Each FPD accepted protons at a different angle, from 52° to 76° , and a different Q^2 value, from 0.12 GeV^2 to 1.0 GeV^2 . The G^0 forward analysis distinguished protons from pions in the detectors by using time-of-flight data.

Figure 8 shows the azimuthal symmetry of the G^0 detector. From the target, the scattered protons curve out to the detectors, arranged in a “Ferris wheel” of eight octants. North American collaborators designed the detectors and electronics in the odd-numbered octants and French collaborators designed the detectors and electronics in the even-numbered octants.

Figure 9 shows the G^0 forward-angle apparatus in Hall C of Jefferson Lab. For the backward angle experiment we turned the Ferris wheel and magnet around and added another array of nine scintillator detectors, called “cryostat exit detectors” (CEDs), and a Čerenkov detector to each octant—housed in the “mini-Ferris wheel.” Figure 10 shows the G^0 backward-angle apparatus in Hall C, with the entire apparatus turned around from the forward-angle, and the mini-Ferris wheel sandwiched between the Ferris wheel and the magnet. The inset picture shows the

Figure 7: G^0 forward single-octant schematic.

Figure 8: G^0 (forward) detector azimuthal symmetry.

detector structures pulled away from the magnet, so that the Čerenkov detectors (white) are visible.

Figure 9: G^0 forward apparatus in hall C.

Figure 11 shows the structure of the Čerenkov detectors. The Aerogel, with an index of refraction of 1.03, distinguishes between electrons and pions—electrons in the gel generate Čerenkov light but pions do not. Each Čerenkov detector is attached to four PMTs—“n-CER#” in the diagram represents the tube number scheme use by the DAQ.

In the G^0 backward-angle experiment we measured the electrons recoiling from either a liquid hydrogen or liquid deuterium (LD2) target. The forward-angle collaboration optimized the detector layout for the forward-angle kinematics, but, because of the backward-angle kinematics, the Q^2 of electrons detected by the FPDs does not vary significantly. So, for a given beam energy, we group all detected electrons into a single Q^2 bin, at a scattering angle of 108° . To make measurements at two different Q^2 values we used two different beam energies—a 362 MeV beam gives a scattered Q^2 of 0.221 GeV^2 and a 687 MeV beam gives a scattered Q^2 of 0.628 GeV^2 . To eliminate random scintillator signals the backward-angle collaboration used FPD signals in coincidence with CED signals, so we needed only one detector of the FPD pairs—we used the front detector. Each of the CEDs and FPDs is attached to a PMT at each end. To eliminate random Čerenkov signals

Figure 10: G^0 backward apparatus in hall C.

Figure 11: G^0 backward angle Čerenkov electron detector system.

we required a coincidence between at least two of the four PMTs attached to the

Čerenkov detector³.

Figure 12 shows how particles with different trajectories will pass through different combinations of CEDs and FPDs. We form coincidence matrices, shown in Figure 13, where the horizontal axis is FPD number and the vertical is CED number. These are our yield matrices for each of the four target-energy combinations.

Figure 12: G^0 backward single-octant schematic.

Electrons recoiling from elastic scattering have different kinematics from electrons recoiling from inelastic scattering, so elastic signals will show up in a different part of the coincidence matrices than inelastic signals. Above the elastic and inelastic loci is the background locus, which is mostly pions, and we call the area below “superelastic,” which contains no physics. Figure 14 shows the coincidence matrix locus definitions for the two different beam energies. The high-energy hydrogen matrix in Figure 13 shows a clear separation between the elastic locus and the inelastic locus.

Originally, we used Photonis XP4572/B PMTs with borosilicate windows for the Čerenkov detectors. Unfortunately, when we switched our target liquid from hydrogen to deuterium, the rates recorded by the PMTs increased to a level at

³During one period we required a coincidence between at least three PMTs for a Čerenkov signal, to study the efficiency.

Figure 13: G^0 backward CED-FPD detector yield coincidence matrices.

Figure 14: G^0 backward CED-FPD coincidence matrix locus definitions.

which we had to use a greatly reduced beam current to avoid swamping the PMTs. By isolating a PMT from the Čerenkov detector we verified that more than 80% of the PMT count rate came from the PMT itself. Testing revealed that the greatly increased rates came from neutron capture within the borosilicate windows and that using PMTs with quartz and lime-glass windows would reduce the signals from neutron capture by up to 95% [17]. During December, 2006, we replaced the borosilicate window PMTs with Photonis XP4578/B PMTs with quartz windows.

3.5 Electronics

The G^0 backward-angle experiment required coincidences between FPD signals and CED signals. The electronics generated the trigger signal by connecting all FPD signals with logical OR operators and, separately, all CED signals with ORs, within a given octant, then connecting these combined signals together within an 8 ns window with a logical AND operator. Figure 15 shows a schematic of this process with a basic timing scenario. If a trigger, CED and FPD signal coincided then the electronics sent a 15 ns signal to the scalars. If this signal coincided with the 20 ns Čerenkov signal then the coincidence signal went to the electron electronics; otherwise, the coincidence signal went to the pion electronics.

Figure 15: Basic timing diagram of coincidence board signals.

A logic board identified which scintillator detectors fired during a detected event. Constant fraction discriminators (CFDs) distinguished signals from PMTs, then meantimers (MT) used the signals from each pair of PMTs attached to a given CED or FPD to calculate the mean times each pair of PMTs received its event signals. (In this way the timing was independent of where in the detector the particle passed.) [36] Events where a single CED fired with a single FPD dominated, but the electronics also tracked multihits (MHs). We call events during which

two CEDs fired with one FPD or two FPDs fired with one CED “multihit12,” and events where two CEDs fired with two FPDs “multihit22.” The coincidence rate was usually of order 100 kHz. Programmable logic chips controlled the coincidence logic. The North American electronics coded the coincidence information into single eight-bit words, with which we had a problem, discussed in section 4.1.2.

3.6 Data Acquisition

During running, Jefferson Lab’s CEBAF On-line Data Acquisition (CODA) [19] handled and stored the raw output from the G^0 electronics. Figure 16 shows a schematic for the G^0 data acquisition [36]. “Read-Out controllers” (ROCs) read out information from detectors, including rates from all detector PMTs, MT rates from all detector PMT pairs, Čerenkov PMT rates and CED-FPD coincidence rates, both with and without Čerenkov coincidence. The ROCs buffered the data and transmitted it over the network. Because data for any particular event was spread over multiple data streams, the “event builder” merged concurrent data from different ROCs and formatted the outgoing data.

Figure 16: G^0 data acquisition.

RunControl, the main graphical user interface (GUI) allowed shift workers to start and stop runs. This GUI also allowed us to switch many hardware configurations quickly and monitor experimental and diagnostic data in real time. We generally limited production runs during the data-taking to one hour to keep the data files small enough to handle manageably.

3.7 Analyzer and Database

The G^0 analyzer is an analysis package written in ROOT and C++ [48]. For the backward-angle experiment we adapted the forward-angle analyzer [15]. Raw G^0 data files contain the scaler and timing information from the electronics for every MPS. When it analyzed a run, the G^0 analyzer could generate data files that contained ntuples and histograms for MPS-by-MPS offline analysis. It could also output run-averaged values to the G^0 MySQL database [42].

3.8 Simulation Software

G0GEANT is a Fortran-based simulation tool for modeling elementary particles passing through matter [46]. It includes a definition of the physical layout of the G^0 experimental apparatus, including the target, magnet and detectors [54] (measured both pre- and post-run), and is based on the GEANT3 package created at CERN [26].

We used Monte Carlo simulations to compare with many quantities we measured during G^0 . For the most part, our systematic corrections and uncertainties come from measured quantities. But extensive comparisons enabled us to fine-tune the simulation and verify that we understood the experiment well. We used simulations often to validate our methods for correcting the measured asymmetries, and occasionally to estimate a few values (such as π^0 background decay rates) that we could not measure directly.

G0GEANT allowed us to select initial parameters such as beam energy, magnet current, target, target temperature, minimum and maximum electron scattering angles (in both θ and ϕ) and the total number of Monte Carlo events [22]. For each event, G0GEANT generates an electron at a random point along the central axis of the target cylinder, with random scattering angles within the specified ranges and a random scattering energy. G0GEANT then propagates the electron in the scattered direction, through the target, magnetic field and detectors. G0GEANT records information (such as time-of-flight, detector number and particle position, angle and energy) for any particle that reaches a detector or detectors. GEANT stores all the event variables so that users can easily eliminate non-physical events by making cuts on parameters. For each each type of process GEANT calculates a cross-section and uses it to weight each event's probability to occur.

4 G^0 -BACKWARD: ANALYSIS

For analysis purposes we divided the G^0 backward-angle data into different sets based on target, beam energy and Čerenkov phototube multiplicity. (During most of the running, if at least two Čerenkov PMTs fired, we counted that as a Čerenkov signal; during the first part of the 687 MeV deuterium running we counted a Čerenkov signal only if at least three of the PMTs fired.) Table 6 shows the different running periods. For the final results, after the corrections we averaged together the results from the three 687 MeV LD2 periods; for the 687 MeV LH2 result we didn't end up using the LH2 687a data because it was very noisy and required large electronics corrections, thus adding a great amount of systematic uncertainty⁴.

During the running shift leaders marked the purpose and data quality of the runs. Before analysis we divided the runs into sets, each of which a student was responsible for checking the data quality. If certain runs showed strange yield, asymmetry, beam parameter or linear regression slope values we checked the log-book to see what was happening during that time. In this way we eliminated runs that did not contain data that was clearly valid. For example, after we had discovered the North American scaler counting problem, but before we had discovered the cause, we conducted tests using different configurations of the electronics in octant 1. Therefore, a significant portion of the runs during the 362 MeV LD2 running contain junk data in octant 1, so we do not use the octant 1 data for those runs.

Table 6: G^0 backward angle running periods.

Target	Energy	Dates	Beam Current	Runs	Name
H	687 MeV	Mar/Apr 2006	55 μ A	100	LH2687a
H	362 MeV	Jul/Aug 2006	20 μ A	475	LH2362
H	687 MeV	Sep/Oct 2006	60 μ A	548	LH2687b
D	687 MeV	Nov/Dec 2006	20 μ A	352	LD2687aM3
D	687 MeV	Dec 2006	20 μ A	180	LD2687aM2
D	362 MeV	Jan/Feb 2007	35 μ A	648	LD2362
D	687 MeV	Mar 2007	20 μ A	332	LD2687bM2

Figure 17 shows the G^0 backward-angle analysis strategy. First the analyzer forms blinded asymmetries for each run from the raw yields. Then the analyzer

⁴The collaboration decided that having cleaner data with low systematic error was better than having more statistics but also high systematic error.

makes scaler counting corrections, rate corrections and helicity-correlated beam corrections to the yields. We do the four analyzer steps for each run and get a run-averaged blinded asymmetry for each running period. Then we correct the run-averaged asymmetries for inelastic electrons, backgrounds and pion contamination. We unblind the asymmetries and make radiative corrections and beam polarization corrections to get the physics asymmetry, which we combine with the forward-angle results to extract the three strange form factors.

Figure 17: G^0 backward analysis strategy.

4.1 Event-by-Event Corrections

The G^0 analyzer used four passes to fully analyze a run: it takes the raw yields and forms blinded asymmetries, then makes a scaler counting correction, electronics corrections and linear regression correction. To analyze a given run the G^0 analyzer accesses at the database to check whether the selected run has been analyzed for the previous pass. If so, it retrieves the necessary values from the database from the previous pass. (If not, the user must analyze the run for the previous pass first.) The analyzer then forms histograms from the raw data and writes the means and widths to the database as values and uncertainties. Users

can later access the yields and asymmetries, and their respective uncertainties, in the database.

4.1.1 Pass 1: Raw Yields and Asymmetries

As pointed out in section 3.1, the G^0 beam has a quartet polarization structure, so that MPSs come in groups of four, polarized either $+- -+$ or $-+ +-$. In pass 1 the analyzer forms an asymmetry for each quartet using the raw yields

$$A = \frac{Y_1^+ - Y_1^- + Y_2^+ - Y_2^-}{Y_1^+ + Y_1^- + Y_2^+ + Y_2^-} \quad (34)$$

where Y_1^+ is yield of the first positively-polarized macropulse, Y_1^- is yield of the first negatively-polarized macropulse, Y_2^+ is yield of the second positively-polarized macropulse and Y_2^- is yield of the second negatively-polarized macropulse. As the analyzer works through all the data in the raw data file it tracks the running average of the asymmetry. When the analyzer has analyzed all the data for a run it multiplies the asymmetry by a blinding factor and stores the blinded asymmetry⁵ in the database. Each of the four target/energy running periods has a different blinding factor, randomly generated, between 0.75 and 1.25—Table 7 lists them.

Table 7: Blinding factors.

Data Set	Blinding Factor
687 MeV H	1.23932
687 MeV D	1.12257
362 MeV D	1.09657
362 MeV H	1.01295

4.1.2 Pass 2: Scaler Counting Correction

The G^0 electronics sorts CED/FPD coincidences into separate scaler channels with an FPGA-based system in the North American octants—the odd-numbered octants. During the experimental running the G^0 analyzer measured the statistical quality of the asymmetry distribution for each run by the ratio to counting

⁵We use these blinding factors to shield the actual values of the asymmetries in order to avoid biased analysis.

statistics (RCS)

$$RCS = \frac{\sigma_A}{\sigma_{\text{counting}}} = \frac{\sigma_A}{\sqrt{N}} \quad (35)$$

for the N events in a run. For most of the backward-angle running this ratio stayed near 1, but early during the low energy deuterium period, the RCS values for the North American octants were about 1.2, indicating a strong non-statistical component to the asymmetry distributions.

Because of an error in the programming of the North American FPGA chips, short pulses, say about 3 nanoseconds, could be sent to the scaler in less than 7 nanoseconds—the time-resolution for a 140 megahertz scaler [45]. These short pulse pairs sometimes caused the scaler to add or drop a random bit in the data. We found that the effect on the asymmetry was just a small fraction of the statistical uncertainty, which we tested by making different cuts on the data and comparing with data from the French octants. And this was confirmed by the unchanged asymmetry after we reprogrammed the FPGA chips. Fortunately, we confirmed that the problem was completely blind to helicity, and only about 1% of the events have these miscounts.

Figure 18 shows yield and asymmetry histograms from one coincidence cell, for 15 runs chained together. We made different cuts on the yield (top row) and checked the effects on the asymmetry (bottom row). These plots show that a 5-sigma cut removes the shoulders from the asymmetry histograms. The cut removes only about 1% of the data for the low-energy deuterium running period, and less for the other running periods [18]. In pass 2 the analyzer makes this cut.

4.1.3 Pass 3: Electronics Corrections

In pass 3 the analyzer makes the electronics corrections. In general, we must account for two electronics effects—deadtime and randoms. The sizes of both corrections depend on the beam current.

Deadtime is simply the time during which particles may scintillate in the detectors but cannot be detected by the electronics because the electronics are busy processing previously-detected particles. The fraction of time that a given electronics channel is “dead” to incoming signals is

$$DT = 1 - e^{-\tau R} \approx \tau R \quad (36)$$

where τ is the deadtime and R is the rate at which the electronics receives the signals⁶. Signals in the CEDs and FPDs are verified by the discriminators (CFD) and meantimers (MT) as well as the Čerenkov trigger, which leads to the full

⁶This is where the beam current dependence comes in.

Figure 18: Cuts on yield and effects on asymmetry.

deadtime expression [57]

$$Y_{i,j}^{\text{meas}} = Y_{i,j}^{\text{corr},DT} (1 - DT_{CFD_i}) (1 - DT_{MT_i}) (1 - DT_{CFD_j}) (1 - DT_{MT_j}) \cdot (1 - DT_{\text{trig}}) (1 - MH22_{i,j}) (1 - MH12_{i,j}) \quad (37)$$

where i and j distinguish between CEDs and FPDs, respectively, $Y_{i,j}^{\text{meas}}$ is the measured yield of coincidence cell i, j , $Y_{i,j}^{\text{corr}}$ is the corrected yield, DT_{CFD} and DT_{MT} are the deadtime fractions for the discriminators and meantimers, $MH12_{i,j}$ is the “multihit12”—the fraction of events where two of the scintillators for a given (CED or FPD) detector fire in coincidence with one of the scintillators for the other (CED or FPD) detector for the i, j coincidence cell—and $MH22_{i,j}$ is the “multihit22”—the fraction of events where two of the scintillators for a given (CED or FPD) detector fire in coincidence with two of the scintillators for the other (CED or FPD) detector for the i, j coincidence cell.

Random coincidences come from coincidences between detector signals that do not come from detecting the same particle. As with the deadtime correction, we must account for random coincidences between CED and FPD signals, as well as random signals from the Čerenkov trigger in coincidence with CED/FPD signals. We corrected for the second type of random signals by correcting the yield quartet-by-quartet, while we corrected for the first by estimating the correction

and applying it to the final asymmetries.

We measured the real CED/FPD signals in coincidence with random Čerenkov signals rates directly by using the pion electronics in “randoms” mode, or we computed the rates in the analysis code from measured pion and Čerenkov rates in “pions” mode. The randoms corrections in these cases are

$$Y_{i,j}^{\text{corr,rdm}} = \frac{Y_{i,j}^{\text{corr,DT,e}} - Y_{i,j}^{\text{corr,DT,r}} \frac{RC_2}{RC_1}}{1 - RC_2 - DT_C} \quad (38)$$

and

$$Y_{i,j}^{\text{corr,rdm}} = \frac{\left(Y_{i,j}^{\text{corr,DT,e}} + Y_{i,j}^{\text{corr,DT,\pi}} \right) RC_2 - Y_{i,j}^{\text{corr,DT,e}}}{RC_2 + DT_C - 1}, \quad (39)$$

respectively, in “randoms” and “pions” modes, where $Y_{i,j}^{\text{corr,DT,e,\pi,r}}$ are the dead-time corrected electron, pion and random yields, and $RC_{1,2}$ are the Čerenkov randoms signals uncorrelated and correlated with the trigger.

The deadtime and randoms corrections have opposite effects. The deadtime correction adds rate to account for the time that the electronics are busy and cannot detect particles; the randoms correction subtracts rate to account for the signals that were counted as wrongly counted as electrons. Figure 19 shows how different corrections dominate in different CED/FPD coincidence cell. The center plot shows the locus definitions for the 362 MeV running periods while the outer plots show the rate correction for two different coincidence cells. The left figure shows the uncorrected and corrected yields for CED 2 FPD 3, an inelastic cell. Scattering with a 362 MeV beam does not produce many elastic electrons, so the randoms correction dominates the electronics correction in this low-rate cell and the correction is negative. The right figure shows the rate correction for CED 9 FPD 14, a high-rate elastic cell. The deadtime correction dominates in this cell and the correction is positive.

Figure 19: Locus definitions and electronics rate corrections.

The plots of both cells show the rate dependence of the electronics effects. The horizontal axes are the beam currents while the vertical axes are the resulting

yields. Before the electronics corrections lines determined by the measured yields, as functions of the beam current, have non-zero slopes. After the corrections lines determined by the corrected slopes have little slope.

The analyzer applies these deadtime and randoms corrections cell-by-cell and mps-by-mps, then re-calculates a rate-corrected asymmetry for each quartet, again writing the quartet-averaged asymmetries to the database. For the uncertainty we can break the measured asymmetry into [58]

$$A_{\text{meas}} = A_{\text{phys}} + A_{DT} + A_{\text{contam}} \quad (40)$$

where A_{DT} includes three parts

$$A_{DT} = A_{\text{singles}} + A_{\text{trig}} + A_{MH12_{i,j}}. \quad (41)$$

The asymmetry from singles effects together with random coincidences is

$$\begin{aligned} A_{\text{singles}} = & -f_{\text{rdm}} A_{DT_{\text{corr}}} \\ & -(1 - f_{\text{rdm}}) \left(\frac{DT_{MT_i}}{1 - DT_{CFD_i} - DT_{MT_i}} A(DT_{MT_i}) \right. \\ & + \frac{DT_{CFD_i}}{1 - DT_{CFD_i} - DT_{MT_i}} A(DT_{CFD_i}) \\ & + \frac{DT_{MT_j}}{1 - DT_{CFD_j} - DT_{MT_j}} A(DT_{CFD_j}) \\ & + \frac{DT_{CFD_j}}{1 - DT_{CFD_j} - DT_{MT_j}} A(DT_{CFD_j}) \\ & \left. + (f_{\text{rdm}}) A_{\text{rdm}} \right) \end{aligned} \quad (42)$$

where f_{rdm} is the random coincidence fraction, $A_{DT_{\text{corr}}}$ is the complete deadtime corrected coincidence asymmetry, $A(DT_{CFD_{i,j}, MT_{i,j}})$ are the CED and FPD discriminator and meantimer deadtime asymmetries, and A_{rdm} is the asymmetry from CED/FPD randoms. The asymmetry from the trigger electronics is

$$A_{\text{trig}} = -\frac{DT_{\text{trig}}}{1 - DT_{\text{trig}} - MH22_{i,j}} A(DT_{\text{trig}}) - \frac{MH22_{i,j}}{1 - DT_{\text{trig}} - MH22_{i,j}} A(MH22_{i,j}) \quad (43)$$

where $A(DT_{\text{trig}})$ is the trigger deadtime asymmetry and $A(MH22_{i,j})$ is the mul-

tihit22 asymmetry. The multihit12 asymmetry is

$$\begin{aligned}
A_{MH12_{i,j}} &= A(1 - MH12_{i,j}) \\
&= - \frac{MH12_{i,j}}{1 - MH12_{i,j}} \left[\frac{Y_{MT_i}}{Y_{MT_i} + Y_{MT_j}} A_{MT_i} + \frac{Y_{MT_j}}{Y_{MT_i} + Y_{MT_j}} A_{MT_j} + A_Q \right]
\end{aligned} \tag{44}$$

where $Y_{MT_{i,j}}$ are the CED and FPD meantimer yields, $A_{MT_{i,j}}$ are the CED and FPD meantimer asymmetries and A_Q is the charge asymmetry. Finally, the contamination asymmetry is

$$\begin{aligned}
A_{\text{contam}} &= f_{\text{contam},\pi} (A_{\text{phys}}^\pi - A_{\text{phys}}^e) - (1 - f_{\text{contam},\pi}) \frac{DT_C}{1 - DT_C} A(DT_C) + \\
&\quad f_{\text{contam},\pi} A(RC_2)
\end{aligned} \tag{45}$$

where $f_{\text{contam},\pi}$ is the pion contamination fraction, $A(DT_C)$ is the Čerenkov dead-time asymmetry and $A(RC_2)$ is the asymmetry of the Čerenkov randoms correlated with the trigger. Table 8 shows the all of the pass 3 corrections and associated uncertainties.

To test the electronics corrections we took a few series of runs in which we purposely induced large charge asymmetries and plotted the charge asymmetries versus the detected asymmetries, before and after the electronics corrections [57]. Figure 20 shows these plots, for French octants on the left and North American octants on the right. The red points are the measured asymmetries with no electronics corrections, the green with only deadtime corrections and the blue with both deadtime and randoms corrections. Ideally, the slope of the line determined by the fully corrected points would be zero because the electronics corrections would account for any charge asymmetry. Realistically, however, there will still be a residual slope because of helicity-correlated effects, corrected in pass 4.

4.1.4 Pass 4: Linear Regression Correction

In pass 4 the analyzer corrects for helicity-correlations in the beam. Variations in the beam affect measured yields. Particularly, if the beam varies as it changes helicity, these changes can induce false asymmetries as part of the asymmetries measured by the experiment. The G^0 experiment tracks six different beam parameters—charge (or current), x -position, y -position, angle in x , angle in y and energy ($q, x, y, \theta_x, \theta_y, E$)—as it tracks the detector yields. On an mps-by-mps basis, the analyzer calculates the averages of the six beam parameters; on a

Table 8: Deadtime contributions: a) LH2362, b) LH2687a, c) LH2687b, d) LD2362, e) LD2687a, f) LD2687b; i/o represents in and out IHWP state.

	A_{meas}	A_{Single}	A_{Trig}	$A_{MH_{12}}$	A_{contam}^{sys}	A_{syst}^{tot}
a.i.	8.3591 ± 0.8117	-0.0144 ± 0.0016	-0.1745 ± 0.0021	-0.1107 ± 0.0034	-0.0027 ± 0.0376	-0.3040 ± 0.0379
a.o.	-11.0175 ± 0.8380	0.0037 ± 0.0017	0.1900 ± 0.0022	0.0851 ± 0.0036	-0.0052 ± 0.0390	0.2582 ± 0.0392
b.i.	44.0498 ± 2.9288	-0.0058 ± 0.0021	-0.1154 ± 0.0017	-0.0688 ± 0.0053	-0.9959 ± 0.1640	-1.1897 ± 0.1641
b.o.	-47.4462 ± 2.8033	0.0076 ± 0.0020	0.1281 ± 0.0016	0.0764 ± 0.0051	0.8812 ± 0.1573	1.0845 ± 0.1574
c.i.	37.1536 ± 7.5210	-0.0004 ± 0.0057	-0.1093 ± 0.0044	-0.1384 ± 0.0191	-0.4530 ± 0.6697	-0.7366 ± 0.6700
c.o.	-43.9244 ± 6.4785	0.0063 ± 0.0049	0.1016 ± 0.0038	0.0877 ± 0.0165	0.5687 ± 0.5854	0.7839 ± 0.5857
d.i.	13.7828 ± 0.8532	-0.0143 ± 0.0020	-0.2584 ± 0.0028	-0.1475 ± 0.0051	-0.3590 ± 0.1981	-0.7619 ± 0.1982
d.o.	-15.8797 ± 0.8737	0.0092 ± 0.0020	0.2497 ± 0.0028	0.1375 ± 0.0052	0.3958 ± 0.2145	0.7843 ± 0.2146
e.i.	38.3347 ± 4.3703	-0.0094 ± 0.0025	-0.0891 ± 0.0031	-0.0558 ± 0.0065	-4.4857 ± 1.8645	-4.6409 ± 1.8645
e.o.	-43.2201 ± 5.2456	0.0031 ± 0.0031	0.0626 ± 0.0038	0.0377 ± 0.0080	8.0601 ± 2.1743	8.1540 ± 2.1743
f.i.	39.8650 ± 4.6953	-0.0053 ± 0.0025	-0.1007 ± 0.0030	-0.0558 ± 0.0065	-2.7965 ± 1.7740	-2.9619 ± 1.7740
f.o.	-45.8315 ± 4.2360	-0.0004 ± 0.0023	0.0784 ± 0.0029	0.0406 ± 0.0062	7.2656 ± 1.5978	7.3943 ± 1.5978

Figure 20: Induced charge asymmetry vs. detector asymmetry, 362 MeV, LD2.

quartet-by-quartet basis the analyzer also calculates the beam charge asymmetry

$$A_q = \frac{q_1^+ - q_1^- + q_2^+ - q_2^-}{q_1^+ + q_1^- + q_2^+ + q_2^-} \quad (46)$$

and the beam parameter differences for the other five parameters

$$dP_k = \frac{P_{k1}^+ + P_{k2}^+}{2} - \frac{P_{k1}^- + P_{k2}^-}{2} \quad (47)$$

where k differentiates between the non-charge beam parameters $(x, y, \theta_x, \theta_y, E)$. Using the beam charge asymmetry and the average charge we can also form the charge difference

$$\begin{aligned} dq = 2\langle q \rangle A_q &= 2 \frac{(q_1^+ + q_1^- + q_2^+ + q_2^-)}{4} \frac{q_1^+ - q_1^- + q_2^+ - q_2^-}{q_1^+ + q_1^- + q_2^+ + q_2^-} \\ &= \frac{q_1^+ + q_2^+}{2} - \frac{q_1^- + q_2^-}{2}. \end{aligned} \quad (48)$$

We write the change in detector yield due to variations in the beam parameters

$$\delta Y = \sum_i \frac{\partial Y}{\partial P_i} \delta P_i \quad (49)$$

where i differentiates between all six beam parameters, $\frac{\partial Y}{\partial P_i}$ are the detector sensitivities, or linear regression slopes, for the different beam parameters and

$\delta Y \equiv Y - \langle Y \rangle$ and $\delta P_i \equiv P_i - \langle P_i \rangle$ are the variations of the yield and beam parameters from their means. If we include this change in yield in the asymmetry equation

$$\begin{aligned}
A &= \frac{(Y_1^+ + \delta Y_1^+) - (Y_1^- + \delta Y_1^-) + (Y_2^+ + \delta Y_2^+) - (Y_2^- + \delta Y_2^-)}{(Y_1^+ + \delta Y_1^+) + (Y_1^- + \delta Y_1^-) + (Y_2^+ + \delta Y_2^+) + (Y_2^- + \delta Y_2^-)} \\
&= \frac{\left\{ \left(Y_1^+ + \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ - \langle P_i \rangle) \right) - \left(Y_1^- + \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^- - \langle P_i \rangle) \right) \right.}{\left\{ \left(Y_1^+ + \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ - \langle P_i \rangle) \right) + \left(Y_1^- + \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^- - \langle P_i \rangle) \right) \right.} \\
&\quad \left. + \left(Y_2^+ + \sum_i \frac{\partial Y}{\partial P_i} (P_{i2}^+ - \langle P_i \rangle) \right) - \left(Y_2^- + \sum_i \frac{\partial Y}{\partial P_i} (P_{i2}^- - \langle P_i \rangle) \right) \right\}}{\left. + \left(Y_2^+ + \sum_i \frac{\partial Y}{\partial P_i} (P_{i2}^+ - \langle P_i \rangle) \right) + \left(Y_2^- + \sum_i \frac{\partial Y}{\partial P_i} (P_{i2}^- - \langle P_i \rangle) \right) \right\}} \\
&= \frac{Y_1^+ - Y_1^- + Y_2^+ - Y_2^- + \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ - P_{i1}^- + P_{i2}^+ - P_{i2}^-)}{Y_1^+ + Y_1^- + Y_2^+ + Y_2^- + \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ + P_{i1}^- + P_{i2}^+ + P_{i2}^- - 4\langle P_i \rangle)} \\
&\approx \frac{Y_1^+ - Y_1^- + Y_2^+ - Y_2^-}{Y_1^+ + Y_1^- + Y_2^+ + Y_2^-} + \frac{\sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ - P_{i1}^- + P_{i2}^+ - P_{i2}^-)}{Y_1^+ + Y_1^- + Y_2^+ + Y_2^-} \tag{50}
\end{aligned}$$

where in the final step we have taken advantage of the fact that $Y_1^+ + Y_1^- + Y_2^+ + Y_2^- \gg \sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ + P_{i1}^- + P_{i2}^+ + P_{i2}^- - 4\langle P_i \rangle)$, because $P_{i1}^+ + P_{i1}^- + P_{i2}^+ + P_{i2}^- - 4\langle P_i \rangle \approx 0$. Subtracting out the physics asymmetry leaves the helicity-correlated false asymmetry

$$\begin{aligned}
A_{\text{false}} &= \frac{\sum_i \frac{\partial Y}{\partial P_i} (P_{i1}^+ - P_{i1}^- + P_{i2}^+ - P_{i2}^-)}{Y_1^+ + Y_1^- + Y_2^+ + Y_2^-} \\
&= \frac{\sum_i \frac{\partial Y}{\partial P_i} 2dP_i}{4\langle Y \rangle} \\
&= \frac{1}{2\langle Y \rangle} \sum_i \frac{\partial Y}{\partial P_i} dP_i \tag{51}
\end{aligned}$$

where $\langle Y \rangle$ is the average yield per mps.

To calculate the linear regression slopes, $\frac{\partial Y}{\partial P_i}$, the analyzer uses a multi-dimensional least squares method that takes into account correlations between the beam parameters. We start by rewriting equation 49, the change in detector yield due to variations in the beam parameters,

$$\delta Y = \sum_i s_i \delta P_i \quad (52)$$

where we have rewritten the slopes $\frac{\partial Y}{\partial P_i} = s_i$ to simplify the notation. If we take many measurements of the yield and beam parameters we can define

$$\chi^2 = \sum_m \frac{\left(\delta Y - \sum_i s_i \delta P_i \right)_m^2}{\sigma_m^2} \quad (53)$$

where σ_m is the uncertainty of the m th measurement of $\delta Y - \sum_i s_i \delta P_i$. The best values of s_i are those that minimize χ^2 [14] so, taking the derivatives of the χ^2 function with respect to the slopes and setting them equal to zero gives the six equations

$$-2 \sum_m (\delta P_j)_m \frac{\left(\delta Y - \sum_i s_i \delta P_i \right)_m}{\sigma_m^2} = 0. \quad (54)$$

By assuming that all the statistical uncertainties of the measurements, σ_m , are equal we can simplify these equations to

$$\langle \delta P_j \delta Y \rangle = \sum_i \langle \delta P_j \delta P_i \rangle s_i \quad (55)$$

where $\langle \rangle$ indicate the average over all the measurements and i and j each differentiates between the beam parameters. If we write these equations in matrix form

$$\begin{bmatrix} \langle \delta q \delta Y \rangle \\ \langle \delta x \delta Y \rangle \\ \vdots \\ \langle \delta E \delta Y \rangle \end{bmatrix} = \begin{bmatrix} \langle \delta q \delta q \rangle & \langle \delta q \delta x \rangle & \dots & \langle \delta q \delta E \rangle \\ \langle \delta x \delta q \rangle & \langle \delta x \delta x \rangle & \dots & \langle \delta x \delta E \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle \delta E \delta q \rangle & \langle \delta E \delta x \rangle & \dots & \langle \delta E \delta E \rangle \end{bmatrix} \begin{bmatrix} s_q \\ s_x \\ \vdots \\ s_E \end{bmatrix} \quad (56)$$

we can easily see that, to obtain the linear regression slopes, we need to invert the beam parameter correlation matrix and multiply it by the yield-beam parameter

correlation vector.

We use the relation

$$\langle \delta a \delta b \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle \quad (57)$$

to calculate the correlation elements. While analyzing a run the analyzer keeps track of the average beam parameters, yields and their products to use in these calculations.

To find the statistical uncertainty of the slopes we can use the curvature matrix of χ^2 [14]

$$\alpha_{ij} \equiv \frac{1}{2} \frac{\partial^2 \chi^2}{\partial s_i \partial s_j} = \sum_m \frac{(\delta P_i)_m (\delta P_j)_m}{\sigma_m^2} = \frac{N \langle \delta P_i \delta P_j \rangle}{\sigma^2} \quad (58)$$

where m again differentiates between individual measurements, N is the total number of measurements and we have continued to assume that the measurement uncertainties, σ_m , are equal. The error matrix is simply the inverse of the curvature matrix

$$\begin{aligned} \epsilon &\equiv \alpha^{-1} \\ \epsilon_{ij} &= \frac{\sigma^2}{N} \langle \delta P \delta P \rangle_{ij}^{-1} \end{aligned} \quad (59)$$

For σ^2 we use the standard deviation of the $\delta Y - \sum_i s_i \delta P_i$ measurements

$$\begin{aligned} \sigma^2 &= \frac{1}{N-6} \sum_m \left(\delta Y - \sum_i s_i \delta P_i \right)_m^2 \\ &\approx \frac{1}{N} \sum_m \left(\delta Y - \sum_i s_i \delta P_i \right)_m^2 \\ &= \langle \delta Y^2 \rangle - 2 \sum_i s_i \langle \delta P_i \delta Y \rangle + \sum_i \sum_j s_i s_j \langle \delta P_i \delta P_j \rangle \end{aligned} \quad (60)$$

where $N \gg 6$. The diagonal elements of the resulting error matrix are the squares of the statistical uncertainties of the linear regression slopes.

Table 9 shows, for all six running periods, the pass 3 and pass 4 physics asymmetries and uncertainties, the differences in the pass 3 and pass 4 asymmetries—the linear regression correction—and estimates of the uncertainty of this difference from subtracting the pass 3 asymmetry uncertainty in quadrature from the pass 4 asymmetry uncertainty

$$\Delta A_{\text{lin.reg.corr}} = \sqrt{(\Delta A_4)^2 - (\Delta A_3)^2} \quad (61)$$

where ΔA_3 and ΔA_4 are the pass 3 and 4 asymmetry uncertainties.

Table 9: Pass 3 and 4 asymmetries and asymmetry differences.

Running Period	Pass 3 Asym (ppm)	Pass 4 Asym (ppm)	Difference
362MeV LH2	-10.0738 $\pm$ 0.6281	-10.0971 $\pm$ 0.6281	0.0233 $\pm$ 0.0061
687MeV LH2, a	-42.602 $\pm$ 5.43109	-42.315 $\pm$ 5.43121	-0.287 $\pm$ 0.036
687MeV LH2, b	-46.939 $\pm$ 2.15757	-47.133 $\pm$ 2.15584	0.194 $\pm$ 0.086
362MeV LD2	-15.38 $\pm$ 0.736224	-15.39 $\pm$ 0.736599	0.010 $\pm$ 0.024
687MeV LD2, a	-50.467 $\pm$ 3.94575	-50.376 $\pm$ 3.94606	-0.091 $\pm$ 0.049
687MeV LD2, b	-48.565 $\pm$ 4.46647	-48.590 $\pm$ 4.46656	0.025 $\pm$ 0.028

We tried several different methods to duplicate the corrections by using values from the database [51]. But the analyzer makes the pass 4 corrections mps-by-mps (using the slope, yield and beam parameter values calculated in pass 3) while the database stores only run-averaged values, so we were unable to duplicate the corrections using database values. Therefore, we could not accurately determine the uncertainties associated with the linear regression corrections—plugging database values into equation 51 gives false asymmetries that are 1-2 orders of magnitude smaller than the asymmetry differences in Table 9. So we estimate the linear regression correction uncertainty as 30% of the asymmetry difference. Table 10 summarizes the total linear regression correction and uncertainty for each running period.

Table 10: Linear regression corrections.

Running Period	Correction (ppm)
362MeV LH2	0.0233 $\pm$ 0.0070
687MeV LH2, a	-0.2873 $\pm$ 0.086
687MeV LH2, b	0.1941 $\pm$ 0.058
362MeV LD2	0.0098 $\pm$ 0.0029
687MeV LD2, a	-0.0906 $\pm$ 0.027
687MeV LD2, b	0.0252 $\pm$ 0.0076

4.2 Run-Averaged Asymmetry Corrections

We then corrected the asymmetries for background, pion contamination and polarization. The background came primarily from the aluminum target windows and accounted for about 12% of the yield in all four running periods, generating an asymmetry that was about equal to the deuterium asymmetry. The pion contamination during the 687 MeV deuterium running accounted for about 5% of the yield, with no asymmetry. Finally, we corrected for the polarization, which was about 86%.

4.2.1 Background Corrections

In general, the CED-FPD coincidence asymmetries measured in the elastic locus are sums of elastic asymmetries and background asymmetries, each weighted by the fraction each contributes to the yield:

$$A_{\text{measured}} = f_{\text{elastic}} A_{\text{elastic}} + \sum_i f_{\text{background}_i} A_{\text{background}_i} \quad (62)$$

where

$$f_{\text{background}_i} = \frac{Y_{\text{background}_i}}{Y_{\text{total}}} \quad (63)$$

and

$$f_{\text{elastic}} + \sum_i f_{\text{background}_i} = 1. \quad (64)$$

Comparisons between data and simulation show significant background contributions from aluminum scattering, inelastic electron electron scattering, π^- contamination and π^0 decays (from the collimators).

To be sure we understood the geometrical setup and momentum acceptance of our detectors we took series of runs with each target/energy combination over a range of magnetic field settings of the superconducting magnet and made simulations over a range of magnetic field settings. Increasing the magnetic field shifts particles with higher momentum closer to the target—toward lower numbered CEDs and FPDs—toward the inelastic locus. Figure 21 shows plots of these magnetic field scans for individual CED-FPD coincidence cells on the inelastic edge of the elastic locus for each of the four target-energy combinations [39]. The horizontal axis shows the magnetic field setting of the magnet while the vertical axis shows the resulting yields. The vertical black dashed lines indicate the nominal magnetic field setting for that target and energy. We simulated the (quasi)elastic yields as well as the yields for each of the background processes. The total simulated yield, in red, agrees well with the measured yield, in green, indicating that we understand the setup and acceptance well and that our simulations are trust-

worthy. We applied the background corrections to each individual coincidence cell in each octant.

Figure 21: Field scans for individual cells on inelastic edge of elastic locus.

The primary background to the elastic electron asymmetries comes from the aluminum target windows. We took a number of runs with hydrogen gas ($<1\%$ liquid density) in the target to study the background from the target windows. We also took runs with the frame target or the flyswatter to simulate the entrance and exit windows. Finally, we also took runs on the carbon target and the tungsten target. We compared the results from all of these different target runs to simulations to verify the simulations for extracting the contributions from the aluminum [20].

The inelastic and π^0 decay fractions turned out to be small, so in our calculations we combined them into f_{other} . The π^- fraction turned out to be zero in all but the 687 MeV LD2 running periods. Table 11 shows the yield fractions for each of the background contributions, for each of the target-energy combinations [39].

4.2.2 Electromagnetic Radiative Corrections

Because scattered electrons radiate in the presence of nuclei and other electrons, which affects measured rates, asymmetries and momentum-transfer, we must account for radiative corrections when looking at eN scattering. The scattered electron must emit an infinite number of photons, the sum of whose energies

Table 11: Primary background contributions to electron yield.

Target	Energy	f_{al}	f_{pi^-}	f_{other}	f
H	362	0.129 ± 0.064	0. ± 0.001	0.003 ± 0.003	0.132 ± 0.064
H	687	0.110 ± 0.055	0. ± 0.001	0.023 ± 0.023	0.133 ± 0.060
D	362	0.099 ± 0.050	0. ± 0.002	0.005 ± 0.005	0.104 ± 0.050
D	687	0.061 ± 0.031	0.04 ± 0.015	0.029 ± 0.029	0.13 ± 0.045

must be finite [16]. Because the probability of emitting the i th photon follows a q/k_i dependence, for a single photon to contain nearly all of the radiated energy is much more likely than for many photons to share the energy [56]. Therefore, in the G^0 analysis we followed the generally-accepted convention of assuming a one-photon exchange between electronic and hadronic currents for electromagnetic radiative corrections [22].

We separate into two major categories the energy losses of the scattered electron. The scattered electron loses energy “internally” when the nucleus from which it scatters prompts the electron to radiate; the scattered electron loses energy “externally” when other nuclei prompt it to radiate. The effects of internal processes are proportional to the number of target scatterers, or target thickness. External processes require two scatterers, so these effects are proportional to the square of the target thickness. Figure 22 shows the tree-level diagram along with different internal virtual radiative processes. Figure 23 shows different internal real radiative processes. Both groups of radiative effects alter the initial and final energy distributions of scattered electrons by broadening the elastic or quasi-elastic peak and forming a radiative tail. All of these processes reduce the magnitude of the measured asymmetry because the asymmetry is a function of Q^2 .

Figure 22: Amplitudes for tree-level and virtual internal radiative corrections.

Figure 23: Amplitudes for real internal radiative corrections.

To determine the correction we need to calculate both the tree-level asymmetry of single-boson exchange, A_{tree} , and the asymmetry that includes the radiative effects, A_{RC} . The radiative correction, RC is simply the ratio

$$RC = \frac{A_{\text{tree}}}{A_{RC}}. \quad (65)$$

We calculated these asymmetries using the G0GEANT simulation code, using beam energy, spectrometer current, target type, target temperature, minimum and maximum scattering angles and total number of Monte Carlo events, following the prescription by Tsai [56].

We write the measured asymmetry as

$$A = a_0 + a_1 G_E^s + a_2 G_M^s + a_3 G_A^{e(T=1)} + a_4 G_A^s \quad (66)$$

so that we can calculate a correction for each term and apply it to each coefficient. We studied the effect of calculating and applying corrections for all four terms and for only the a_0 term. The difference in the corrected asymmetry was less than 0.05% so we decided to apply the correction only for a_0 . Tables 12 and 13 show the tree-level asymmetries, those with radiative effects and radiative corrections for each term in Equation 66 for all of the different running periods [22].

4.2.3 Beam Polarization Correction

We measured the beam polarization using the Hall C Møller and Mott polarimeters. Each time we measured the polarization we halted G^0 running, removed the target from the beamline and spent a few hours taking polarization runs. We measured the asymmetry with the Møller polarimeter to determine the polarization using [44]

$$A_{\text{Møller}} = |P_z^B| |P_z^T| A_{ZZ}(\theta) \quad (67)$$

Table 12: Hydrogen electromagnetic radiative corrections.

361.90 MeV LH2			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-10.434	-10.064	1.037 ± 0.002
a1	10.200	10.110	1.009 ± 0.001
a2	8.781	8.388	1.047 ± 0.003
a3	2.684	2.587	1.037 ± 0.002
a4	1.203	1.156	1.037 ± 0.002
684.86 MeV LH2			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-37.951	-36.612	1.037 ± 0.002
a1	21.760	21.542	1.010 ± 0.001
a2	62.523	59.198	1.056 ± 0.003
a3	12.042	11.508	1.046 ± 0.003
a4	5.400	5.159	1.046 ± 0.003
685.57 MeV LH2			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-38.026	-36.697	1.036 ± 0.002
a1	21.783	21.557	1.010 ± 0.001
a2	62.735	59.420	1.056 ± 0.003
a3	12.074	11.544	1.046 ± 0.003
a4	5.412	5.174	1.046 ± 0.003

Table 13: Deuterium electromagnetic radiative corrections.

363.05 MeV LD2			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-15.492	-15.019	1.032 ± 0.004
a1	7.158	7.089	1.010 ± 0.001
a2	1.966	1.883	1.044 ± 0.005
a3	2.882	2.801	1.029 ± 0.003
a4	0.251	0.243	1.033 ± 0.004
689.42 MeV LD2			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-52.445	-50.797	1.032 ± 0.004
a1	13.755	13.609	1.011 ± 0.001
a2	13.666	12.994	1.052 ± 0.006
a3	11.273	10.851	1.039 ± 0.004
a4	0.983	0.947	1.038 ± 0.004
689.61 MeV LD2			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-53.537	-52.487	1.035 ± 0.004
a1	13.752	13.623	1.010 ± 0.001
a2	13.684	12.969	1.055 ± 0.006
a3	11.284	10.834	1.042 ± 0.005
a4	0.905	0.945	1.041 ± 0.005
689.61 MeV LD2, Multiplicity 3			
	A_{tree} (ppm)	A_{RC} (ppm)	R_c
a0	-52.347	-50.586	1.035 ± 0.004
a1	13.798	13.662	1.010 ± 0.001
a2	13.627	12.912	1.055 ± 0.006
a3	11.246	10.796	1.042 ± 0.005
a4	0.980	0.942	1.041 ± 0.004

where $A_{ZZ}(\theta) = -\frac{7}{9}$ is the analyzing power, $P_z^T = 8.036 \pm 0.015\%$ is the known polarimeter target polarization and P_z^B is the beam polarization that we are trying to determine.

Normally, the Hall C Møller polarimeter requires a minimum beam energy of 800 MeV to run. Moving the first quadrupole closer to the superconducting solenoid, however, allowed us to use the Møller during the 687 MeV running. We had only partial success with only one Møller measurement during the 362 MeV running. However, this partial measurement is consistent with our final polarization determination [25].

We used the Mott polarimeter to verify the Møller results during the 687 MeV running, to within 2%. When analyzed separately the 687 MeV and 362 MeV Mott measurements agree. Therefore, we used the same polarization value for the 362 MeV running that we determined during the 687 MeV running using the Møller, with an added term in quadrature in the systematic uncertainty from variation in the Mott measurements.

During the 687 MeV running we could not completely reproduce the tune of the polarimeter—for the same beam position and the same quad settings we did not always reproduce the same beam tune. We attributed this to the Møller quadrupole running with a very low current, so that we did not always reproduce the same magnetic field. This unproducibility was a primary source of systematic uncertainty.

We made eighteen separate polarization measurements, each the average result of a series of polarization runs. We determined the final polarization values

$$P_{687} = 85.78 \pm 0.07(stat) \pm 1.38(syst)\% \quad (68)$$

and

$$P_{362} = 85.78 \pm 0.07(stat) \pm 1.95(syst)\% \quad (69)$$

by fitting the polarization measurements to a constant [25].

The polarization correction is

$$A_{\text{corr}} = \frac{A_{\text{meas}}}{P} + K_T \quad (70)$$

where P is the polarization and K_T comes from the transverse component of the beam polarization. Using data from runs with a transversely-polarized beam, we determined that this transverse component is of order 0.01 ppm for all four target-energy combinations. Because asymmetry variations from octant-to-octant could vary more than this transverse correction, we did not actually make this correction, but rather included it as a systematic uncertainty term in the final asymmetry results. Table 14 shows the correction for each target-energy combination.

Table 14: Transverse corrections.

	362 LH2	362 LD2	687 LH2	687 LD2
K_S (ppm)	0.022 ± 0.003	0.036 ± 0.002	0.008 ± 0.007	0.012 ± 0.013

4.2.4 Q^2 Determination

Because the asymmetries are functions of form factors and the form factors are functions of Q^2 , the accuracy of an asymmetry measurement depends closely on the accuracy of Q^2 . To determine Q^2 we compared simulated yields with the yields we measured in our data, to make sure we understood the detector acceptances and the radiative losses in the experiment. We used Hall C surveys to verify the positions of the target, collimators and detectors relative to each other [54]. We also used simulation to study sensitivities to detector position (in z and radially), target position, beam energy and magnetic field of the toroidal magnet [35]. Table 15 shows that all of these sensitivities are much smaller than the uncertainties in our measurements, except for beam energy, which is still slightly smaller. The simulation beam energies are 687 MeV and 362 MeV and Q^2 values are 0.6301 GeV² and 0.2222 GeV², respectively.

Table 15: Q^2 sensitivities.

x	362 LH2	362 LD2	687 LH2	687 LD2	Δx
z_{det}	19	18	23	10	1.0cm
r_{det}	5	3	8	5	0.5cm
z_{targ}	5	4	10	6	0.5cm
E_{beam}	2	2	4	5	1.0MeV
B_{magnet}	6	23	7	16	1.3%

Table 16 shows the beam energy, center of target energy and Q^2 for each running period. The Q^2 uncertainty does not include the systematic uncertainty from energy variations [35].

4.3 Physics Asymmetries and Extracted Form Factors

Once we had a fully corrected asymmetry for each CED-FPD coincidence cell in each octant we averaged over all cells in the locus, weighted by the asymmetry

Table 16: Beam energy, center of target energy and Q^2 by running period.

Running Period	Sys. Err (%)	Beam Energy (MeV)	Energy (MeV)	$Q^2(\text{GeV}^2)$ (GeV^2)
LH2 687a	0.21	685.57 ± 0.92	682.45 ± 0.92	0.6275 ± 0.003
LH2 362	0.27	361.90 ± 0.50	358.75 ± 0.51	0.2217 ± 0.001
LH2 687b	0.27	684.86 ± 0.92	681.74 ± 0.92	0.6264 ± 0.003
LD2 687a (M2)	0.19	689.61 ± 0.93	686.07 ± 0.93	0.6300 ± 0.003
LD2 687a (M3)	0.19	689.61 ± 0.93	686.05 ± 0.93	0.6287 ± 0.003
LD2 362	0.22	363.05 ± 0.66	359.51 ± 0.66	0.2193 ± 0.001
LD2 687b	0.19	689.42 ± 0.93	686.87 ± 0.93	0.6299 ± 0.003

uncertainties, for each octant separately (excluding octant 1 CED 5 cells and octant 5 CED 8 cells because of sometimes-faulty PMTs [50]). We then averaged over all octants, again weighted by the asymmetry uncertainties. Table 17 shows the final G^0 backward-angle physics asymmetries for the four different target- Q^2 combinations. The uncertainties shown are statistical, point-to-point and global.

Table 17: G^0 backward asymmetries.

Target	Q^2 (GeV^2)	θ_e ($^\circ$)	A_{PV} (10^{-6})
H ₂	0.222	108	$-11.25 \pm 0.86 \pm 0.27 \pm 0.43$
D ₂	0.220	108	$-16.93 \pm 0.81 \pm 0.41 \pm 0.21$
H ₂	0.626	108	$-45.9 \pm 2.4 \pm 0.8 \pm 1.0$
D ₂	0.629	108	$-55.5 \pm 3.3 \pm 2.0 \pm 0.7$

As this table shows, the Q^2 values for the different 687 MeV and 362 MeV running periods differ slightly. To extract the strange form factors at specific Q^2 values we adjusted our asymmetries to common Q^2 values of 0.221 GeV^2 and 0.628 GeV^2 using kinematic sensitivities $\frac{\delta A}{\delta Q^2} = -123.0, -113.8 \frac{\text{ppm}}{\text{GeV}^2}$ (from calculating the derivatives) for the high and low energy energy hydrogen asymmetries, respectively, and $-138.6, -111.2 \frac{\text{ppm}}{\text{GeV}^2}$ for the high and low energy energy deuterium asymmetries, respectively. We then use the resulting backward-angle asymme-

tries

$$\begin{aligned}
A_{phys}^H(0.221 \text{ GeV}^2) &= -11.10 \pm 0.86 \pm 0.27 \pm 0.43 \text{ ppm} \\
A_{phys}^D(0.221 \text{ GeV}^2) &= -17.07 \pm 0.81 \pm 0.42 \pm 0.21 \text{ ppm} \\
A_{phys}^H(0.628 \text{ GeV}^2) &= -46.05 \pm 2.41 \pm 0.83 \pm 1.03 \text{ ppm} \\
A_{phys}^D(0.628 \text{ GeV}^2) &= -55.38 \pm 3.31 \pm 1.97 \pm 0.71 \text{ ppm}
\end{aligned} \tag{71}$$

for extracting the form factors.

We can calculate A_{NVS} from equation 23 by setting $G_E^s = G_M^s = 0$. Subtracting A_{NVS} from A_{phys} leaves only a linear combination G_E^s and G_M^s —the form in which the G^0 forward collaboration expressed their results. We tried different methods for fitting the G^0 forward results to interpolate results at $Q^2=0.221 \text{ GeV}^2$ and 0.628 GeV^2 . Figure 24 shows the forward angle results together with linear and quadratic fits (blue and purple lines, respectively) to the highest 11 and 13 Q^2 points (solid and dashed lines, respectively). We used the values

$$\begin{aligned}
A_{phys}^f(0.221 \text{ GeV}^2) &= -4.77 \pm 0.36 \pm 0.21 \pm 0.23 \text{ ppm} \\
A_{phys}^f(0.628 \text{ GeV}^2) &= -20.71 \pm 0.78 \pm 0.90 \pm 1.31 \text{ ppm}
\end{aligned} \tag{72}$$

from the linear fit of the 11 highest Q^2 points (solid blue line) to extract the values of the form factors at $Q^2=0.221 \text{ GeV}^2$ and 0.628 GeV^2 . Table 18 shows these extracted values, together with the statistical, point-to-point systematic and global systematic uncertainties.

Table 18: G^0 extracted form factors.

	$Q^2=0.221 \text{ GeV}^2$	$Q^2=0.628 \text{ GeV}^2$
G_E^s	$-0.0142 \pm 0.0356 \pm 0.0182 \pm 0.0176$	$0.1102 \pm 0.0488 \pm 0.0296 \pm 0.0237$
G_M^s	$0.0834 \pm 0.1834 \pm 0.0855 \pm 0.0781$	$-0.1235 \pm 0.1095 \pm 0.0614 \pm 0.0317$
G_A^E	$-0.5012 \pm 0.3171 \pm 0.1930 \pm 0.0878$	$-0.1973 \pm 0.4254 \pm 0.2568 \pm 0.0949$

Figure 24: Fit of the G^0 forward angle results.

5 FIT

This analysis uses 33 parity-violating electron scattering asymmetry data points from forward-angle electron-proton and electron- ^4He and backward-angle electron-proton and electron-deuteron scattering provided by the HAPPEX [1, 7], SAMPLE [13], PVA4 [34, 12] and G^0 [10, 6] experiments, in combination with 14 νp and $\bar{\nu} p$ elastic scattering cross-section data points from Brookhaven E734 [3]. These experiments have provided data over the momentum-transfer range $0.03 < Q^2 < 1.05 \text{ GeV}^2$.

5.1 Experiments

Because these experiments are all elastic or quasi-elastic, the beam energy, scattering angle and momentum-transfer are related for any given measurement. The momentum-transfer and beam energy appear in the neutrino-scattering cross-section formula, so Table 19 lists these; the scattering angle and momentum-transfer appear in the electron-scattering asymmetry formula, so the other data tables below list these.

5.1.1 Brookhaven E734

Brookhaven’s E734 experiment measured cross-sections for elastic νp and $\bar{\nu} p$ scattering in a hydrocarbon target/detector system over a momentum-transfer range of $0.45 < Q^2 < 1.05 \text{ GeV}^2$ using a 1.25 GeV neutrino beam [3]. The total uncertainties in table 19 include statistical, Q^2 -dependent systematic, and Q^2 -independent systematic contributions, all added in quadrature. Table 19 also lists the correlation coefficient ρ for the ν and $\bar{\nu}$ data at each value of Q^2 . There are also correlations between the data points at different values of Q^2 , not shown here; a ROOT script with a full error matrix calculation is available.

Table 19: BNL E734 differential cross section data.

Q^2 (GeV ²)	$d\sigma/dQ^2(\nu p)$ (10 ⁻¹² (fm/GeV) ²)	$d\sigma/dQ^2(\bar{\nu} p)$ (10 ⁻¹² (fm/GeV) ²)	Correlation Coefficient
0.45	0.165 ± 0.033	0.0756 ± 0.0164	0.13
0.55	0.109 ± 0.017	0.0426 ± 0.0062	0.26
0.65	0.0803 ± 0.0120	0.0283 ± 0.0037	0.29
0.75	0.0657 ± 0.0098	0.0184 ± 0.0027	0.26
0.85	0.0447 ± 0.0090	0.0129 ± 0.0023	0.16
0.95	0.0294 ± 0.0073	0.0108 ± 0.0022	0.12
1.05	0.0205 ± 0.0063	0.0101 ± 0.0027	0.07

5.1.2 G^0 -Forward

The G^0 forward-angle experiment in Hall C at Jefferson Lab measured parity-violating asymmetries in forward-angle ep elastic scattering. G^0 -Forward scattered a 3.031 GeV electron beam from a liquid hydrogen target. Arrays of scintillator detectors separated the detected protons into Q^2 bins for a momentum-transfer range of $0.122 < Q^2 < 0.997 \text{ GeV}^2$ [10]. The G^0 -Forward results webpage⁷ contains the asymmetries and uncertainties, as well as other information. A ROOT script with a full error matrix calculation is available.

5.1.3 G^0 -Backward

Table 17 in section 4.3 shows the final G^0 backward-angle physics asymmetries for the four different target- Q^2 combinations. The uncertainties shown are statistical, point-to-point and global.

⁷<http://g0web.jlab.org/DocDB/0005/000549/002/index.html>

5.1.4 SAMPLE

The SAMPLE experiment at the MIT-Bates laboratory measured parity-violating asymmetries in backward-angle ep elastic and ed quasi-elastic scattering at low Q^2 . The experiment consisted of three measurements: SAMPLE-I scattered a 200 MeV electron beam from a hydrogen target; SAMPLE-II scattered a 200 MeV beam from a deuterium target; and SAMPLE-III scattered a 125 MeV beam from a deuterium target [13]. Table 20 shows the results. Uncertainties are statistical and systematic, respectively. Betsy Beise provided the mean scattering angles.

Table 20: SAMPLE asymmetry data.

Target	Q^2 (GeV ²)	θ_e ($^\circ$)	A_{PV} (10^{-6})	Reference
H ₂	0.1	144.4	$-5.61 \pm 0.67 \pm 0.88$	[55]
D ₂	0.091	144.0	$-7.77 \pm 0.73 \pm 0.62$	[27]
D ₂	0.038	143.2	$-3.51 \pm 0.57 \pm 0.58$	[28]

5.1.5 HAPPEX

The HAPPEX experiment in Hall A at Jefferson Lab measured parity-violating asymmetries in forward-angle ep elastic and $e^4\text{He}$ quasi-elastic scattering at various momentum transfers. HAPPEX used an approximately 3 GeV electron beam in each case [1, 7]. Table 21 shows the results.

Table 21: HAPPEX asymmetry data.

Target	Q^2 (GeV ²)	θ_e ($^\circ$)	A_{PV} (10^{-6})	Reference
H ₂	0.477	12.3	$-15.05 \pm 0.98 \pm 0.56$	[7]
H ₂	0.099	6.0	$-1.14 \pm 0.24 \pm 0.06$	[8]
⁴ He	0.091	5.7	$6.72 \pm 0.84 \pm 0.21$	[9]
H ₂	0.109	6.0	$-1.58 \pm 0.12 \pm 0.04$	[1]
⁴ He	0.077	5.8	$6.40 \pm 0.23 \pm 0.12$	[1]

5.1.6 PVA4

The PVA4 experiment at the MAMI facility in Mainz measured parity-violating asymmetries in forward- and backward-angle ep elastic scattering at different Q^2 values. The experiment consisted of three measurements: forward scattering of an 854.3 MeV electron beam from a liquid hydrogen target, forward scattering using a 570.4 MeV beam and backward scattering using a 315.1 MeV beam [12]. Table 22 shows the results. Liu, et al, provided the forward-scattering angles [31]; S. Baunack provided the backward-scattering Q^2 [12], from which we calculated the backward-scattering angle.

Table 22: PVA4 asymmetry data.

Target	Q^2 (GeV ²)	θ_e ($^\circ$)	A_{PV} (10^{-6})	Reference
H ₂	0.230	35.45	$-5.44 \pm 0.54 \pm 0.26$	[33]
H ₂	0.108	35.52	$-1.36 \pm 0.29 \pm 0.13$	[34]
H ₂	0.2241	144.8	$-17.23 \pm 0.82 \pm 0.89$	[12]

5.2 Model-Independent Extractions

A number of groups have analyzed different combinations of these electroweak datasets to extract the strange form factors at specific Q^2 points. These extractions use only experimental data and the standard model expressions for cross sections and asymmetries—they use no parameterizations or models. Model-dependent global analyses (like the one presented in this dissertation) must agree with these model-independent extractions.

Figure 25 shows plots of these model-independent extractions. To derive the green square points Liu, McKeown and Ramsey-Musolf combined all available data at $Q^2=0.1$ GeV²—from HAPPEX, G^0 -Forward, SAMPLE and PVA4 [31]. To derive the open black circle points Pate, Papavassiliou and McKee combined the HAPPEX and E734 data; to derive the closed black circle points they combined G^0 -Forward and E734 data [43]. The PVA4 and G^0 collaborations combined their own forward- and backward-angle data to derive the red squares [12] and blue triangles [6], respectively.

Figure 25: Model-independent extractions of strange form factors.

5.3 Global Analysis

We used functional forms with seven parameters (ρ_s , μ_s , ΔS , Λ_E , Λ_M , Λ_A , S_A),

$$G_E^s = \frac{\rho_s \tau}{\left(1 + \frac{Q^2}{\Lambda_E^2}\right)^2}, \quad G_M^s = \frac{\mu_s}{\left(1 + \frac{Q^2}{\Lambda_M^2}\right)^2}, \quad \text{and} \quad G_A^s = \frac{\Delta S + S_A Q^2}{\left(1 + \frac{Q^2}{\Lambda_A^2}\right)^2}$$

to fit the form factors; ρ_s is the strangeness radius, $\tau = Q^2/4M_p$, μ_s is the strange quark contribution to the magnetic moment and ΔS is the strange quark contribution to the nucleon spin. These forms are the simplest functions of only even powers of Q^2 that match the boundary conditions⁸. The Q^2 -dependence of elastic form factors is related to the distance scale of examination of the particle by the deBroglie wavelength. As $Q^2 \rightarrow 0$, $\lambda \rightarrow \infty$ and we see the whole particle. We know that the net contribution of strange quarks and anti-quarks to a nucleon's charge is zero, so as $Q^2 \rightarrow 0$, $G_E^s \rightarrow 0$ —the τ in the denominator of G_E^s allows this. At the opposite boundary, because form factors contain spatial information, as $Q^2 \rightarrow \infty$, $\lambda \rightarrow 0$ —the distance scale shrinks to zero—all of the form factors must also go to zero—the $\left(1 + \frac{Q^2}{\Lambda_E^2}\right)^2$ form for the denominator allows this. At first we tried using only ΔS in the numerator of the axial fit function, but our fit did not agree with the model-independent determinations and also resulted in large negative values for ΔS . Adding the S_A term allowed the fit to agree with the model-independent determinations and to avoid large negative ΔS .

For our fit code we used the built-in TMinuit class in ROOT which minimizes the input function. We chose to minimize the χ^2 function, defined as the sum of the terms

$$\chi_{\text{non-corr},i}^2 = \left[\frac{(A_{\text{data},i} - A_{\text{fit},i})}{dA_{\text{data},i}} \right]^2 \quad (73)$$

for each independent experimental measurement, i , and the terms

$$\chi_{\text{corr},j}^2 = (A_m W_{m,n} A_n)_j \quad (74)$$

for each set, j , of correlated experimental measurements, where

$$A_m = (A_{\text{data},m} - A_{\text{fit},m}) \quad (75)$$

and $W_{m,n}$ is the weight matrix—the inverse of the correlation matrix—for the set of correlated measurements. E734, G^0 -Forward and G^0 -Backward contained correlated measurements, and so each contributed a single complicated term to

⁸The form factors can be functions of only even powers of Q^2 [59].

the χ^2 function, while HAPPEX, SAMPLE and PVA4 each contributed 5, 3 and 3 simple terms, respectively.

Our best 7-parameter fit gives large Λ_E and Λ_M values (> 1000) with large uncertainties; large Λ_E and Λ_M means $G_E^s \simeq \rho_s \tau$ and $G_M^s \simeq \mu_s$. Since the fit determines Λ_E and Λ_M so poorly we fixed these parameters at 1000 and re-fit the data with only five free parameters. Table 23 shows the resulting values for the five-parameter fit.

Table 23: Preliminary fit results.

Parameter	Value
ρ_s	0.13 ± 0.21
μ_s	0.035 ± 0.053
ΔS	-0.27 ± 0.41
Λ_S	1.3 ± 1.9
S_A	0.32 ± 0.48

Table 24 shows the effect on the fit of fixing Λ_E and Λ_M at a variety of values. Based on the trend in the χ^2 values, we cannot make any strong statement concerning Λ_E and Λ_M . We hope that the upcoming HAPPEX and PVA4 data at $Q^2=0.62 \text{ GeV}^2$ will help determine Λ_E and Λ_M , or at least set useful lower limits.

Table 24: Fit results as a function of fixed $\Lambda_{E,M}$ values.

$\Lambda_{E,M}$	χ^2/dof	ρ_s	μ_s	ΔS
0.1	54.5/42	$3. \pm 86.$	$26. \pm 28.$	-0.3 ± 0.5
1.0	51.2/42	0.43 ± 0.39	0.02 ± 0.11	-0.22 ± 0.36
10.	50.4/42	0.13 ± 0.22	0.034 ± 0.055	-0.26 ± 0.41
100.	50.4/42	0.13 ± 0.21	0.035 ± 0.053	-0.27 ± 0.41
1000.	50.4/42	0.13 ± 0.21	0.035 ± 0.053	-0.27 ± 0.41

In our 5-parameter fit (Table 23) the uncertainty of μ_s is small compared to other recent determinations. This is likely an artifact of fixing $\Lambda_M=1000$, so that we fit G_M^s to a constant and the resulting value and uncertainty arise from averaging the data over the full Q^2 range. Since the existing data determine Λ_M poorly, we can say only that if G_M^s really is flat at low Q^2 then our result for μ_s will be valid. Our fit also shows that ρ_s , the slope of G_E^s , is consistent with

zero, in agreement with the model-independent determinations of G_E^s which are consistent with zero at each Q^2 . Finally, the lack of neutrino scattering data at low Q^2 determines the overall shape of G_A^s poorly.

Figure 26 shows plots of the model-independent extraction points and the G_E^s , G_M^s and G_A^s functions (in black) using our 5-parameter fit. Our fits agree well with the model-independent determinations of G_E^s , G_M^s and G_A^s —within one σ of most of the model-independent points.

Figure 27 shows a histogram of the normalized deviations of our fit from the data, and the fit of a Gaussian to the histogram. The mean of the Gaussian is consistent with 0 and the σ is consistent with 1, so the data is distributed normally with respect to our fit. The histogram also contains no outliers.

Figure 28 shows a plot of the G^0 -Forward asymmetry data points (in red) with the asymmetries predicted by our fit (in black). Because the asymmetries are negative we have taken the negatives of the asymmetries to plot them on a logarithmic vertical scale. Figures 27 and 28 both show that the fit agrees well with the G^0 -Forward data.

Figure 29 shows a plot of the E734 cross-section data points together with the cross-sections predicted by our fit (lines). Figures 27 and 29 both show that the fit agrees well with the E734 data.

Figure 30 again shows plots of the model-independent extractions and our G_E^s , G_M^s and G_A^s fits, this time with 70% confidence-level (CL) curves for the fit. Table 23 shows that all of the fit parameters are consistent with zero, and so are all poorly determined to some extent. But the sizes of the G_E^s and G_M^s error bands in Figure 30 compare reasonably with the sizes of the error bars on the model-independent vector form factor extraction points, while the error band is huge for G_A^s —much bigger than the error bars on the model-independent axial extraction points. This very wide error band, particularly at $Q^2 < 0.45 \text{ GeV}^2$, comes from a lack of neutrino data. The lack of low- Q^2 neutrino-scattering data means that we cannot determine the Q^2 -dependence of G_A^s .

5.4 Upcoming Data

Several experiments will provide new data soon. HAPPEX and PVA4 are measuring forward-angle electron-proton asymmetries at a $Q^2 = 0.62 \text{ GeV}^2$. Sci-BooNE released preliminary results⁹ for neutral current elastic neutrino-proton cross sections and the final results should cover a momentum-transfer range of $0.2 < Q^2 < 1$.

Perhaps most exciting, the MicroBooNE collaboration¹⁰ has received prelimi-

⁹H. Takei's slides at the Sixth International Workshop on Neutrino-Nucleus Interactions in the Few-GeV Region, 2009

¹⁰<http://www-microboone.fnal.gov/>

Figure 26: Model-independent extractions with strange form factor fits.

Figure 27: Normalized deviations of fit from data with Gaussian fit.

Figure 28: G^0 forward asymmetries and fit.

Figure 29: E734 νp and $\bar{\nu} p$ cross-sections and fit.

Figure 30: Model-independent extractions with fits and 70% CL curves.

nary approval to begin building a new liquid argon time projection chamber (TPC) to use with the Booster and NUMI beams at Fermilab in order to study the low energy excess observed by MiniBooNE [2]. MicroBooNE will also measure low energy neutrino cross sections using the yield ratio

$$R_{NC/CC} = \frac{N(\nu p \rightarrow \nu p)}{N(\nu n \rightarrow \mu^- p)} \quad (76)$$

where $N(\nu p \rightarrow \nu p)$ is the yield of the elastic, or neutral-current, $\nu p \rightarrow \nu p$ reaction, given by equation 32, and $N(\nu n \rightarrow \mu^- p)$ is the yield of the inelastic charged-current $\nu n \rightarrow \mu^- p$ reaction, given by equation 33.

This ratio is a more attractive observable than the neutral-current cross-section because many experimental uncertainties, such as efficiencies and flux uncertainties, cancel; some theoretical uncertainties, like nuclear corrections, may also cancel. The MicroBooNE experiment may begin as early as 2012. Collaborators plan to run for 2-3 years, with a goal of 6×10^{20} protons-on-target. Currently the collaboration plans to measure down to a momentum-transfer of $Q^2 < 0.1$.

To estimate how the MicroBooNE results might help our fit MicroBooNE collaborators simulated 2×10^{20} protons-on-target—about one year of data—made some reasonable cuts and estimated the statistical uncertainty of the yield ratio [24]. This uncertainty includes no estimates of systematic errors, but also does not include the full statistics—it is only a rough indicator of the *uncertainties* we might expect. We then used our fit parameters to generate *values* for this simulated yield ratio data. Table 25 shows the simulated MicroBooNE data. We then used this data as additional input data for our fit program to look at how it might affect the sizes of our error bands. (Note: with this exercise we are trying to get an idea only of how the MicroBooNE data might affect the fit parameter *uncertainties*; we make no claim about how it might affect the fit parameter *values*.)

Table 25: Simulated MicroBooNE data.

Q^2	$R_{NC/CC}$
0.08-0.2	0.206 ± 0.005
0.2 -0.4	0.181 ± 0.005
0.4 -0.6	0.156 ± 0.007
0.6 -0.8	0.136 ± 0.009
0.8 -1.0	0.118 ± 0.012
1.0 -1.2	0.101 ± 0.015

Table 26 shows the resulting fit parameters without and with the simulated MicroBooNE data. The middle column shows the fit results using the existing data while the last column shows the fit results after also including the simulated MicroBooNE data. Comparing the uncertainties in the middle and last columns shows that the simulated data reduces the sizes of the uncertainties of the vector form factor fit parameters, ρ_s and μ_s , only slightly but that it reduces the sizes of the uncertainties of the axial fit parameters, ΔS , Λ_A and S_A , up to an order of magnitude.

Table 26: Fit results without and with simulated MicroBooNE data.

Parameter	Fit to Existing Data	Including MicroBooNE
ρ_s	0.13 ± 0.21	0.13 ± 0.19
μ_s	0.035 ± 0.053	0.034 ± 0.046
ΔS	-0.27 ± 0.41	-0.265 ± 0.038
Λ_S	1.3 ± 1.9	1.34 ± 0.38
S_A	0.32 ± 0.48	0.322 ± 0.071

Figure 31 again shows the model-independent extractions, now with new G_E^s , G_M^s and G_A^s fits that include the simulated MicroBooNE data, and 70% confidence-level curves. Comparing Figure 31 with Figure 30, the simulated data hardly affects the GEs and GMs error bands, but it greatly narrows the GAs error band; the width of the error band using the simulated MicroBooNE data is comparable to the sizes of the model-independent extraction error bars.

Figure 31: Extractions and 70% CL fit using simulated MicroBooNE data.

6 CONCLUSION

Current parity-violating electron-nucleon and neutrino-proton, anti-neutrino-proton scattering data constrains the sizes of the strange vector form factors, but isn't yet extensive enough to reveal any detailed Q^2 -dependence. The data also hints at the Q^2 -dependence of the strange axial form factor but leaves ΔS unconstrained, mostly because of lack of data at low Q^2 values. Fortunately, some new data is coming that may help constrain the fits. HAPPEX and PVA4 will both soon measure forward-angle electron-proton asymmetries at $Q^2=0.62 \text{ GeV}^2$. Also, SciBooNE will soon release final results for neutral current neutrino-proton cross sections over $0.2 < Q^2 < 1.0 \text{ GeV}^2$. Finally, Fermilab has preliminarily approved the MicroBooNE collaboration's Large liquid Argon Time Projection Chamber (TPC) to measure the yield ratio

$$R_{NC/CC} = \frac{N(\nu p \rightarrow \nu p)}{N(\nu n \rightarrow \mu^- p)} \quad (77)$$

down to $Q^2 < 0.1 \text{ GeV}^2$, possibly beginning as early as 2012. The MicroBooNE results particularly could greatly reduce the G_A^s error band.

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