Study of the N to Delta Transition via $p(\vec{e}, e'\vec{p})\pi^0$ Reaction

by

Zhengwei Chai

B.Sc., University of Science and Technology of China (1993)

M.Sc., Institute of High Energy Physics, China (1996)

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Physics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2003

© Massachusetts Institute of Technology 2003. All rights reserved.

Author
Department of Physics
July 31, 2003

Certified by
William Bertozzi
Professor of Physics
Thesis Supervisor

Certified by
Shalev Gilad
Principal Research Scientist
Thesis Supervisor

Accepted by
Professor Thomas J. Greytak
Associate Department Head for Education

Study of the N to Delta Transition via $p(\vec{e}, e'\vec{p})\pi^0$ Reaction

by

Zhengwei Chai

B.Sc., University of Science and Technology of China (1993)

M.Sc., Institute of High Energy Physics, China (1996)

Submitted to the Department of Physics
on July 31, 2003, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy in Physics

Abstract

A precise measurement of the $p(\vec{e}, e'\vec{p})\pi^0$ reaction at the $\Delta(1232)$ resonance with $Q^2=0.9, 1.1(GeV/c)^2$ was conducted in experimental Hall A at the Jefferson Lab. The angular distributions of the coincidence cross section and recoil polarization in the pion electro-production process were used for the extraction of both the unpolarized and polarized response functions. By including the recoil polarization observables, a large number of response functions are obtained for a model-independent multipole analysis to extract both the dominant and small amplitudes in the $p(e, e'p)\pi^0$ reaction, which are important for understanding nucleon structure and the $N\rightarrow\Delta$ transition mechanism.

This thesis describes the experimental setup and the details of the absolute cross section analysis. A data-defined R-Function method was developed for the precise analysis of the small acceptance of the Hall A spectrometer. A simple and efficient analysis procedure for absolute cross section extraction with extensive use of simulation was designed and used. The extracted cross section results were parameterized for the extraction of polarized response functions from recoil polarizations. The cross section results and preliminary polarization and multipole results are presented. The $N\rightarrow\Delta$ quadrupole transition ratios R_{EM} and R_{SM} extracted from the differential cross sections are compared with those from the preliminary multipole analysis results.

Thesis Supervisor: William Bertozzi
Title: Professor of Physics

Thesis Supervisor: Shalev Gilad
Title: Principal Research Scientist

Contents

1	Introduction	11
1.1	Nucleon and Δ	11
1.2	$N \rightarrow \Delta$ Quadrupole Transitions	14
1.2.1	Quadrupole Measurements	14
1.2.2	Sources of Quadrupole Transitions	17
1.3	Polarization Observables in $p(\vec{e}, e'\vec{p})\pi^0$ Reaction	23
1.3.1	Response Functions	23
1.3.2	Multipole Amplitudes	27
1.4	Pion Electroproduction Models	31
1.4.1	SAID	32
1.4.2	MAID and DMT	34
1.4.3	S-L Model	36
2	The Experiment Setup	38
2.1	Experiment Overview	38
2.2	Accelerator and Polarized Source	39
2.3	Hall A Instrumentation	42
2.3.1	Beam Line Instruments	42
2.3.2	Cryogenic Target	52
2.3.3	High Resolution Spectrometer	55
2.3.4	Detector Packages	56

2.3.5	Trigger Electronics	66
2.3.6	Data Acquisition	68
2.3.7	Analysis Software	70
2.3.8	HRS Optics Optimization	71
2.4	Experiment Kinematics	75
3	Data Analysis	78
3.1	Analysis Overview	78
3.2	Acceptance Analysis with R-Function	80
3.2.1	General R-Function Definition	80
3.2.2	HRS Acceptance Analysis with R-Function	82
3.3	Monte Carlo Simulation	92
3.3.1	MCEEP	92
3.3.2	EPIPROD	94
3.3.3	Radiative Effects	94
3.3.4	Experiment Specification	97
3.3.5	Acceptance Model with R-Function	99
3.3.6	Resolution Smearing and Kinematic Adjustment	100
3.4	Experiment Data Selection	103
3.4.1	Event Selection	103
3.4.2	Background Subtraction	106
3.5	Experiment Data Corrections	109
3.5.1	Computer Deadtime	109
3.5.2	Electronic Deadtime	109
3.5.3	Tracking Efficiency	111
3.5.4	Trigger Efficiency	113
3.5.5	Proton Absorption	114
3.5.6	Luminosity Correction	115
3.6	$p(e, e'p)\pi^0$ Cross Section	117

3.6.1	Kinematic Binning in $(W, Q^2, \theta_\pi, \phi_\pi)$ Space	117
3.6.2	Unpolarized Cross Section	117
3.7	Systematic Uncertainty	120
3.7.1	System Error from Various Factors	120
3.7.2	Additional System Error	121
3.8	Unpolarized Response Functions	124
3.8.1	Response Function Fit on ϕ_π Distribution	124
3.8.2	Legendre Fit on (θ_π, ϕ_π) Distribution	125
3.8.3	R'_{LT} Analysis	127
3.9	Polarization and Polarized Response Function Analysis	129
3.10	Multipole Analysis	130
4	Results and Discussion	131
4.1	Cross Sections	131
4.2	Unpolarized Responses and Legendre Coefficients	137
4.3	Preliminary Polarizations and Polarized Responses	143
4.4	Preliminary Multipoles	151
4.5	Summary	157

List of Figures

1-1	$M1$ single spin-flip transition of nucleon to Δ	13
1-2	Q^2 dependence of the R_{EM} and R_{SM} from previous experimental measurements. Only statistical errors are shown.	16
1-3	Four vector current $J^\mu = (\rho, \vec{J})$: (a) one-body current $J_{[1]}^\mu$, (b),(c) and (d) two-body gluon and pion exchange currents $J_{[2]}^\mu$	17
1-4	$N \rightarrow \Delta$ quadrupole transition via the one-body operator $\hat{Q}_{[1]}$	20
1-5	Double spin flip in $N \rightarrow \Delta$ quadrupole transition via the two-body quadrupole operator $\hat{Q}_{[2]}$. The two-body exchange currents can be either gluons or pions.	21
1-6	Kinematics for $p(\vec{e}, e'p)\pi^0$ Reaction	24
1-7	Diagrams contribution to pion electro-production: (A) direct, (B) crossed nucleon pole, (C) pion pole, (D) contact term, (E) s channel isobar excitation, (F) u channel isobar excitation, (G) photon pole, vector meson exchange, (H)re-scattering	31
1-8	SAID $E2/M1$ and $S2/M1$ ratios vs Q^2 (solid curves)	33
1-9	$E2/M1$ and $S2/M1$ ratios vs Q^2 (solid curves:MAID,dashed curves:DMT)	35
1-10	Imaginary parts of the multipole amplitudes in S-L model for $\gamma^*N \rightarrow \pi N$ at $\Delta(1232)$ region. Solid curves are full calculations, and the dotted curves are calculations without non-resonant interaction.	37
2-1	The Continuous Electron Beam Accelerator Facility at JLAB	39

2-2	The layers of the strained GaAs cathode (Left) and the energy levels of conduction and valance bands (Right)	40
2-3	Hall A Configuration	42
2-4	Hall A Arc Energy Measurement	43
2-5	Hall A eP Energy Measurement	44
2-6	Hall A Møller Polarimeter	46
2-7	Hall A Compton Polarimeter	47
2-8	Hall A Beam Current Monitor	49
2-9	Side view of the Hall A cryotarget	53
2-10	Side view of the Hall A High Resolution Spectrometer	56
2-11	Hadron Arm Detector Package	57
2-12	Electron Arm Detector Package	57
2-13	VDC assembly layout (not to scale)	59
2-14	Scintillator Counter Layout	61
2-15	Hall A Focal Plane Polarimeter False Asymmetry Simulation	64
2-16	Trigger Electronics Setup	67
2-17	E91-011 optics check (E-Arm z_{react} in meter)	74
2-18	E91-011 optics check (E-Arm x_{siev} vs y_{siev} in meter)	74
2-19	E91-011 kinematic acceptance in (W, Q^2) plane	77
2-20	E91-011 kinematic acceptance in (θ, ϕ) plane	77
3-1	R-Function Definition in xy plane	80
3-2	Areas selected by R Cuts in xy plane	82
3-3	Data-defined boundary examples for R cut in spectrometer acceptance analysis	85
3-4	Normalized kinematic variables distribution comparison between the data(solid lines) and simulation(dotted lines) for kinematics thcm0	88
3-5	Normalized target variables distribution comparison between the data(solid lines) and simulation(dotted lines) for kinematics thcm0	89

3-6	Yield ratio of the simulation and data versus different R Cuts for kinematics them0	90
3-7	Normalized missing mass distribution comparisons between the data(solid lines) and the simulation(dotted lines). The missing mass unit is MeV/c ² 102	
3-8	Polygon cuts on the gmean-hbeta plane to suppress the π^+ background for kinematics setting them135	104
3-9	The timing spectra with various cuts for kinematic setting them155 .	108
3-10	The missing mass spectra with various cuts for kinematic setting them155	108
3-11	The relative luminosity versus beam current	116
4-1	2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1220MeV$ and $Q^2 = 0.9GeV^2$. The $x = \cos \theta_\pi$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.	133
4-2	2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1300MeV$ and $Q^2 = 0.9GeV^2$. The $x = \cos \theta_\pi$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.	134
4-3	2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1220MeV$ and $Q^2 = 0.9GeV^2$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.	135
4-4	2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1300MeV$ and $Q^2 = 0.9GeV^2$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.	136
4-5	Response functions for $p(e, e'p)\pi^0$ at $W = 1260MeV$ and $Q^2 = 0.9GeV^2$. Dashed: MAID; Dotted: SAID.	138
4-6	Legendre coefficients from the angular distribution fit to differential cross section for $p(e, e'p)\pi^0$ at $Q^2 = 0.9GeV^2$. Solid: DMT; Dashed:MAID	140
4-7	Legendre coefficients from the angular distribution fit to differential cross section for $p(e, e'p)\pi^0$ at $Q^2 = 1.1GeV^2$. Solid: DMT; Dashed:MAID	141
4-8	Proton polarization P_n for $W=1200-1240MeV$ and $Q^2=0.8-1.0GeV^2$.	144

4-9	Proton polarization P_t for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$	145
4-10	Proton polarization P_l for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$	146
4-11	Proton polarization P'_n for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$	147
4-12	Proton polarization P'_t for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$	148
4-13	Proton polarization P'_l for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$	149
4-14	Polarized responses for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$	150
4-15	Preliminary 1+ multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 0.9\text{GeV}^2$. Solid: MAID; Dashed:DMT	152
4-16	Preliminary non-resonant multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 0.9\text{GeV}^2$. Solid: MAID; Dashed:DMT	153
4-17	Preliminary 1+ multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 1.1\text{GeV}^2$. Solid: MAID; Dashed:DMT	154
4-18	Preliminary non-resonant multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 1.1\text{GeV}^2$. Solid: MAID; Dashed:DMT	155
4-19	Response functions for $p(e, e'p)\pi^0$ at $W = 1180\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.	170
4-20	Response functions for $p(e, e'p)\pi^0$ at $W = 1180\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.	170
4-21	Response functions for $p(e, e'p)\pi^0$ at $W = 1220\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.	171
4-22	Response functions for $p(e, e'p)\pi^0$ at $W = 1220\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.	171
4-23	Response functions for $p(e, e'p)\pi^0$ at $W = 1260\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.	172
4-24	Response functions for $p(e, e'p)\pi^0$ at $W = 1260\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.	172
4-25	Response functions for $p(e, e'p)\pi^0$ at $W = 1300\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.	173
4-26	Response functions for $p(e, e'p)\pi^0$ at $W = 1300\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.	173
4-27	Response functions for $p(e, e'p)\pi^0$ at $W = 1340\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.	174
4-28	Response functions for $p(e, e'p)\pi^0$ at $W = 1340\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.	174

List of Tables

1.1	Truncated Legendre expansion of the differential cross section	27
1.2	Truncated Legendre expansion of the helicity-dependent polarization	28
1.3	Truncated Legendre expansion of the helicity-independent polarization	28
2.1	JLab Beam Parameters	41
2.2	E91-011 Beam Energy Measurements	45
2.3	General characteristics of Hall A High Resolution Spectrometer . . .	58
2.4	E91-011 Kinematic Settings	76
4.1	Legendre Fit Coefficients	142
4.2	1+ multipoles and $E2/M1$ and $C2/M1$ ratios	151

Chapter 1

Introduction

This thesis presents the cross section analysis and results of the $N \rightarrow \Delta$ transition experiment[1] which was performed in Hall A at Jefferson Lab in 2000. The polarization analysis from the same experiment can be found in another thesis[2]. The preliminary polarization results are presented here as they were used in the response function and multipole analysis.

The first chapter discusses the $N \rightarrow \Delta$ quadrupole transition measurements and the $p(\vec{e}, e'\vec{p})\pi^0$ reaction formalism, which are the physics motivations of the present experiment. The experimental setup is described in chapter 2. The detail of the cross section analysis is given in chapter 3. The last chapter presents and discusses the experimental results.

1.1 Nucleon and Δ

In the quark model[3], the ordinary baryons consist of three quarks and the quarks have two internal degree of freedoms: flavor and color. The baryon state function can be described as:

$$|qqq\rangle_A = |color\rangle_A |space, spin, flavor\rangle_S \quad (1.1)$$

where the subscripts A and S indicate antisymmetry and symmetry under the exchange of any two quarks respectively. The three lightest quarks, u , d and s , are considered with strongly broken $SU(3)$ flavor symmetry. With the assumption of the spin-independent quark-quark interaction, the flavor symmetry $SU(3)$ may be coupled with the spin symmetry $SU(2)$, resulting the $SU(6)$ multiplets of flavor and spin states. Thus for ordinary baryons with only u , d and s quarks, the flavor and spin are combined to form 6 basic states: $u \uparrow$, $u \downarrow$, $d \uparrow$, $d \downarrow$, $s \uparrow$ and $s \downarrow$, here $\uparrow$ and $\downarrow$ are the two components of the $1/2$ spin. $SU(6)$ multiplets can be formed from three of these basic states:

$$6 \otimes 6 \otimes 6 = 20_A \oplus 70_M \oplus 70_M \oplus 56_S \quad (1.2)$$

where the subscript M denotes mixed permutation symmetry. These $SU(6)$ multiplets can be decomposed according to their spin and flavor contents:

$$20 = {}^2 8 \oplus {}^4 1 \quad (1.3)$$

$$70 = {}^2 10 \oplus {}^4 8 \oplus {}^2 8 \oplus {}^2 1 \quad (1.4)$$

$$56 = {}^4 10 \oplus {}^2 8 \quad (1.5)$$

where the superscript $2S + 1$ denotes the spin S of the 3 quark states in the $SU(3)$ flavor multiplets.

Both the nucleon(isospin $I=1/2$) and $\Delta(1232)(I=3/2)$ belong to the 56-plet in which the orbital angular momenta of the 3 quarks are zero. The nucleon is in the $J^P = 1/2^+$ octet and Δ is in the $J^P = 3/2^+$ decuplet. The 20-plet and 70-plet are 3 quarks states with nonzero orbital angular momenta which are classified as $SU(6) \otimes O(3)$ super-multiplets, with the assumption of rotational invariance in the three-dimensional space.

In this simple $SU(6)$ quark model, both the nucleon and Δ have a spherically symmetric spatial distribution since their three quarks are in S -wave states ($L = 0$).

The electromagnetic excitation from the nucleon to the Δ is described as a pure magnetic dipole transition($M1$) which is a result of spin-flip of a single quark, as shown in Fig. 1-1.

Figure 1-1: $M1$ single spin-flip transition of nucleon to Δ

1.2 $N \rightarrow \Delta$ Quadrupole Transitions

1.2.1 Quadrupole Measurements

There has been intense experimental and theoretical interest in the deviation of the nucleon structure from spherical symmetry ever since such a possibility was suggested by Glashow[4]. The non-S characteristics of the nucleon state was first regarded as a signal of the q-q color hyperfine interaction in nucleon[5]. The experimental measurement of the nucleon deformation is of fundamental importance, because it can provide knowledge on the internal dynamics of nucleon and q-q interaction involving non-valence quark degree of freedom. The structure and dynamics of the baryons ought to be understood ultimately in terms of the strong interactions derived from the Quantum chromodynamics (QCD). The deformation of the nucleon can be determined from the quadrupole moment measurement. However, the quadrupole deformation of the nucleon can not be directly measured because the nucleon has spin $1/2$. It can only be inferred with a physics model using the quadrupole amplitude for a transition from the nucleon to a state with spin greater than or equal to $3/2$, such as $\Delta(1232)$ which has spin $3/2$.

The $N \rightarrow \Delta$ transition can be studied in either pion photo- or electro-production on nucleon process: $\gamma N \rightarrow \Delta \rightarrow \pi N$ where γ can be either real or virtual photons. The allowed multipole transitions are $M1$, $E2$ and $C2$ according to the spin-parity selection rules. The $M1$ is the magnetic dipole transition, $E2$ the electric quadrupole transition and $C2$ the coulomb quadrupole transition. The $M1$ and $E2$ are excited by transverse photons, and the $C2$ is excited by longitudinal photon. The multipole transitions are also denoted by the index $(l\pm)$ where the l is the orbital angular momentum of the pion with respect to the recoil nucleon and the total angular momentum of the πN system is $J = l \pm 1/2$. Thus the allowed electromagnetic multipoles $M1$, $E2$ and $C2$ are corresponding to the pion-production transverse multipoles M_{1+} , E_{1+} and longitudinal multipole L_{1+} respectively. The longitudinal multipoles are related

to the scalar multipoles by $\omega S_{l\pm} = |\vec{q}| L_{l\pm}$ where ω is the photon energy and $\vec{q}$ is the photon momentum.

Because of the strong interaction in the hadronic πN system, the multipoles have complex values. If the nucleon and the Δ are not spherical, the quadrupole multipoles E_{1+} and S_{1+} will not be zero. Pion cloud contributions in the $N \rightarrow \Delta$ transition can also lead non-zero quadrupole amplitudes. The quadrupole strengths in the $N \rightarrow \Delta$ transition are usually extracted as the ratios of quadrupoles and magnetic dipole:

$$R_{EM} = \frac{E_{1+}}{M_{1+}} = \frac{\text{Re}(E_{1+}^* M_{1+})}{|M_{1+}|^2} \quad (1.6)$$

$$R_{SM} = \frac{S_{1+}}{M_{1+}} = \frac{\text{Re}(S_{1+}^* M_{1+})}{|M_{1+}|^2} \quad (1.7)$$

Previous experiments results from DESY[6], NINA[7], CEA[8], ELSA[9], MAMI[10], LEGS[11], JLAB[12], BATES[13] confirmed that the M_{1+} is dominant in the $N \rightarrow \Delta$ transition. However, these experiments also found significantly nonzero quadrupole strengths, as shown in Fig. 1-2. Although the early measurements for R_{EM} are scattered, recent data are concentrated at $\sim -2\%$ with little Q^2 dependence. The recent R_{SM} data show significant Q^2 dependence. Both R_{EM} and R_{SM} are not zero, indicating there is quadrupole deformation of the nucleon structure. However, the experimental quadrupole strength is still far from the perturbative QCD predication $E_{1+} = M_{1+}$ as $Q^2 \rightarrow \infty$ resulting from the quark helicity conservation. In pQCD, for the electromagnetic $N \rightarrow \Delta$ transition, the helicity amplitude G_- ($\propto E_{1+} - M_{1+}$) is asymptotically zero relative to the helicity-conserving amplitude G_+ ($\propto E_{1+} + M_{1+}$) because of different Q^2 dependence ($G_- \propto 1/Q^2 G_+$), which leads to $E_{1+} = M_{1+}$ for sufficiently high Q^2 [14].

Figure 1-2: Q^2 dependence of the R_{EM} and R_{SM} from previous experimental measurements. Only statistical errors are shown.

1.2.2 Sources of Quadrupole Transitions

There are several different possible sources contributing to the deformation of the baryons. First, the tensor forces between quarks can produce the D waves in the nucleon and Δ state functions. Such configuration admixtures lead to the deformation of the valence quark distribution from spherical symmetry. The tensor force is generated from the one-gluon exchange interactions between quarks[5]. Thus quadrupole transition can be induced by exciting an S state quark in the nucleon to a D state in the Δ , or a D state quark in the nucleon to an S state in the Δ via the one-body current in Fig. 1-3(a). Second, in the physical baryons, there are also $q\bar{q}$ pairs and gluons. They can also contribute to the observed quadrupole transitions through two-body currents, as in Fig. 1-3(b-d)[15, 16]. Third, in the dynamical models[17, 18] for the pion photo- or electro-production reactions, the pion cloud surrounding the nucleon and Δ is found to be the dominant source of the quadrupole transitions. It should be noted that the second and the third sources are intrinsically related. Both are the non-valence degree of freedom. The difference is the former is in terms of a microscopic quark model, and the latter is considered in terms of a phenomenological dynamic reaction model.

Figure 1-3: Four vector current $J^\mu = (\rho, \vec{J})$: (a) one-body current $J_{[1]}^\mu$, (b),(c) and (d) two-body gluon and pion exchange currents $J_{[2]}^\mu$.

One-body transition: D state admixture

The quark hyperfine interaction producing D waves in the nucleon and Δ is of the form[5]:

$$H_{hyp}^{ij} = \frac{2\alpha_s}{3m_i m_j} \left[\frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} (3\vec{S}_i \cdot \vec{r}_{ij} \vec{S}_j \cdot \vec{r}_{ij} - \vec{S}_i \cdot \vec{S}_j) \right] \quad (1.8)$$

where $\alpha_s = \frac{g^2}{4\pi}$ is the strong coupling constant, m_i and m_j are quark masses, $\vec{S}_i$ and $\vec{S}_j$ are quark spins, and r_{ij} is the distance of the two interacting quarks. Such an interaction is the color analog of the magnetic dipole-dipole interaction of electromagnetism. The above hyperfine interaction breaks the $SU(6)$ symmetry and introduces non S-wave impurities in the baryonic ground states. The second part of the interaction, tensor term, induces D waves in the nucleon and Δ .

The one-body quadrupole operator, which can be obtained from the multipole expansion of the one-body charge operator, is proportional to the spherical harmonic $Y^2(\vec{r}_i)$ with rank two::

$$\hat{Q}_{[1]} = \sqrt{\frac{16\pi}{5}} \sum_{i=0}^3 e_i r_i^2 Y_0^2(\vec{r}_i) = \sum_i e_i (3z_i^2 - r_i^2) \quad (1.9)$$

where the sum is over the three constituent quarks in the baryon. This one-body charge operator needs a D state component in the nucleon or Δ to make a nonzero contribution. Considering only S and D waves, the nucleon and Δ state functions can be written as:

$$|N\rangle = a_S |(S = \frac{1}{2}, L = 0) J = \frac{1}{2}\rangle + a_D |(S = \frac{3}{2}, L = 2) J = \frac{1}{2}\rangle \quad (1.10)$$

$$|\Delta\rangle = b_S |(S = \frac{3}{2}, L = 0) J = \frac{3}{2}\rangle + b_D |(S = \frac{1}{2}, L = 2) J = \frac{3}{2}\rangle \quad (1.11)$$

The D -states in the above equations have mixed permutation symmetry with respect to the two quark exchange. The positive D -state amplitude $b_D = 0.07$ in the Δ

indicates a prolate deformation of its valence quark distribution, while the negative D -state amplitude $a_D = -0.04$ in nucleon corresponds to an oblate deformation[5].

For the $N \rightarrow \Delta$ transition as shown in Fig. 1-4, applying the one-body quadrupole operator $\hat{Q}_{[1]}$ between the nucleon and Δ state functions $|N\rangle$ and $|\Delta\rangle$ gives the following quadrupole transition moment:

$$Q_{p \rightarrow \Delta} = -b^2 \frac{4}{\sqrt{30}} (a_S b_D - a_D b_S) \quad (1.12)$$

which corresponds to exciting a D -state quark in nucleon to an S -state in Δ and an S -state quark in nucleon to a D -state in Δ . b is the harmonic oscillator parameter describing the spatial extension of the baryon wave function, also referred as the quark core radius.

Using the quark core radius $b \simeq 0.5 fm$ which is related to the average single-quark energy $\omega = 1/(m_q b^2) \simeq 500 MeV$, one gets only 20% of the experimental quadrupole strength $Q_{p \rightarrow \Delta}^{exp} \simeq -0.11 fm^2$ by considering only the one-body current as shown in Fig. 1-3[19]

The $E2/M1$ ratio from the spatial single quark currents calculation is -0.1% [20]. This is much less than the experimental $E2/M1 \simeq -2.5\%$. Thus the single quark transition is not the dominant source of the $N \rightarrow \Delta$ quadrupole transition.

Two-body transition: Deformed pion cloud

A two-body operator can also generate a quadrupole moment even when the valence quarks are pure S -states[15]. This two-body quadrupole operator is obtained from the multipole expansion of the two-body gluon and pion exchange currents, involving the interaction between the photon and a $q\bar{q}$ pair, as shown in Fig. 1-3 on page 17:

$$\hat{Q}_{[2]} = B \sum_{i \neq j=1}^3 e_i (3\sigma_{iz}\sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j) \quad (1.13)$$

Figure 1-4: $N \rightarrow \Delta$ quadrupole transition via the one-body operator $\hat{Q}_{[1]}$

where constant B has dimension fm^2 containing the orbital and color matrix elements. This two-body operator acts in quark spin and isospin space. Because it is a spin-tensor operator with rank 2, it can simultaneously flip the spin of two quarks without changing the orbital angular momentum of the three quark state. Such double spin flip yields the quadrupole $N \rightarrow \Delta$ transition shown in Fig. 1-5.

Although the operator $\hat{Q}_{[2]}$ acts on the valence quark spin space, it reflects that the physical nucleon contains $q\bar{q}$ pairs which have a deformed distribution. Because both the Q_Δ and the neutron charge radius r_n^2 are dominated by exchange currents between valence quarks, they can be related as the following[16]:

$$\sqrt{2}Q_{p \rightarrow \Delta^+} = Q_{\Delta^+} = 4B = r_n^2 \quad (1.14)$$

The quadrupole moment calculated from the experimental neutron charge radius agrees well with the transition quadrupole moments extracted from the $E2/M1$ and

Figure 1-5: Double spin flip in $N \rightarrow \Delta$ quadrupole transition via the two-body quadrupole operator $\hat{Q}_{[2]}$. The two-body exchange currents can be either gluons or pions.

$C2/M1$ [19]. Since the two-body $q\bar{q}$ pair currents provide the major contribution to the quadrupole moment, the nucleon deformation can be regarded as mainly residing in the $q\bar{q}$ cloud.

Pion Cloud

As mentioned before, in the dynamical model for the $N \rightarrow \Delta$ reaction, the non-resonant interaction involving pion re-scattering in the final state is found to be the dominant source of the quadrupole strength in the $N \rightarrow \Delta$ transition. This is consistent with the two-body interaction assumption in which the photon interacts with a $q\bar{q}$ pair including the pion in the nucleon. In the pion cloud model of the nucleon and Δ , the bare nucleon and Δ have symmetric shapes; however, the pion cloud surrounding the bare nucleon and Δ has a deformed shape thus inducing a quadrupole $N \rightarrow \Delta$ transition. In the dynamical model DMT[18], the bare Δ is almost spherical and the electric $E2$ and Coulomb $C2$ quadrupole excitations of the physical Δ are saturated by pion cloud contribution. In the meson-exchange model S-L[17], the non-resonant contribution is dominant, but varies with Q^2 with a maximum near the photon point ($Q^2 = 0$). The pion cloud contribution is also found to play an important role in

the electro-production of the Δ resonance in a chiral chromodielectric model[21], in which the pion cloud is described as a coherent state and the physical nucleon and Δ are obtained using the spin and isospin projection.

1.3 Polarization Observables in $p(\vec{e}, e'\vec{p})\pi^0$ Reaction

The R_{EM} can be extracted from both the pion photo- and electro-production reactions. R_{SM} can only be extraction from the pion electro-production reaction, since the coulomb quadrupole transition $C2$ (S_{1+}) can only be excited by longitudinally polarized virtual photons. To study the Q^2 evolution of the quadrupole amplitudes in $N \rightarrow \Delta$ transition, the virtual photon in the electro-production reaction is needed since the real photon corresponds to $Q^2 = 0$. The main goal of the pion electro-production experiments is to measure the $M1$, $E2$ and $C2$ resonance amplitudes. The difficulty in extraction of the small $E2$ and $C2$ is that there is always the contributions from the non-resonant and the higher mass resonance. The existing measurements of $E2$ and $C2$ are mainly from differential cross section measurements, which only measured a few observables with large uncertainties, thus resulting an unsatisfactory knowledge of the quadrupole amplitudes. Because there are not enough observables in the cross section measurements to determine the background contribution, the extraction of the resonant $M1$, $E2$ and $C2$ amplitudes relies on the reaction model which could lead to large systematic model errors[13].

The recoil polarization measurement for reaction $p(\vec{e}, e'\vec{p})\pi^0$ can provide more than 10 polarization observables in addition to the ones obtained in differential cross section measurement. These additional polarization observables provide certain sensitivity to the background contribution in the $N \rightarrow \Delta$ transition. A better extraction of the $N \rightarrow \Delta$ transition amplitudes is made possible through a multipole analysis of the angular distributions of the differential cross section and recoil polarizations.

1.3.1 Response Functions

The kinematics for pion electro-production in the $p(\vec{e}, e'\vec{p})\pi^0$ reaction in one photon exchange approximation is shown in Fig. 1-6, in which a polarized electron is incident on an unpolarized hydrogen target and the scattering electron and recoil proton are

Figure 1-6: Kinematics for $p(\vec{e}, e'\vec{p})\pi^0$ Reaction

detected and the proton's polarization is also measured. $\vec{k}_i$ and $\vec{k}_f$ are the electron's initial and final momentum respectively. $\vec{P}_p$ is the recoil proton's momentum and $\vec{P}_\pi$ is the neutral pion's momentum. $\vec{q}$ is the 3-momentum transfer. ϕ is the proton's azimuthal angle, which is the angle between the electron scattering plane and reaction plane containing the pion and the proton. The pion's azimuthal angle ϕ_π is equal to $180^\circ + \phi_p$. The recoil polarization is measured in the reaction frame defined by the following basis vectors:

$$\hat{n} = \frac{\vec{q} \times \vec{P}_p}{|\vec{q} \times \vec{P}_p|} \quad \hat{l} = \frac{\vec{P}_p}{|\vec{P}_p|} \quad \hat{t} = \hat{n} \times \hat{l} \quad (1.15)$$

The differential cross section for $p(\vec{e}, e'\vec{p})\pi^0$ reaction can be expressed as[22]:

$$\frac{d^5\sigma}{dk_f d\Omega_e d\Omega_\pi^*} = \Gamma_v \sigma_v [1 + \mathbf{P} \cdot \mathbf{S} + h P_e (A + \mathbf{P}' \cdot \mathbf{S})] \quad (1.16)$$

where Γ_v is the virtual photon flux:

$$\Gamma_v = \frac{\alpha}{2\pi^2} \frac{k_f}{k_i} \frac{k_\gamma}{Q^2} \frac{1}{1 - \epsilon} \quad (1.17)$$

for initial (final) electron momenta $k_i(k_f)$. Here Q^2 is virtual photon's 4-momentum transfer squared, $k_\gamma = \frac{W^2 - m_p^2}{2m_p}$ with W the invariant mass of the pion proton system, $\epsilon = (1 + 2\frac{|\vec{q}|^2}{Q^2} \tan^2 \frac{\theta_e}{2})^{-1}$ is the virtual photon's transverse polarization. σ_v is the virtual photon cross section, $\mathbf{P}$ is the induced or helicity independent recoil proton polarization, $\mathbf{P}'$ is the transferred or helicity dependent recoil proton polarization, A is the electron beam analyzing power, and $h = \pm 1$ is the beam helicity, P_e is the longitudinal electron polarization.

The cross section and polarization can be decomposed into expressions of the kinematic factors ν_α , which depend on electron kinematics, and response functions $R_\alpha(Q^2, W, \theta)$, and the azimuthal dependence:

$$\sigma_v = \nu_L R_L + \nu_T R_T + \nu_{LT} R_{LT} \cos \phi_\pi + \nu_{TT} R_{TT} \cos 2\phi_\pi \quad (1.18)$$

$$A\sigma_v = \nu'_{LT} R'_{LT} \sin \phi_\pi \quad (1.19)$$

$$P_n \sigma_v = \nu_L R_L^n + \nu_T R_T^n + \nu_{LT} R_{LT}^n \cos \phi_\pi + \nu_{TT} R_{TT}^n \cos 2\phi_\pi \quad (1.20)$$

$$P_m \sigma_v = \nu_{LT} R_{LT}^m \sin \phi_\pi + \nu_{TT} R_{TT}^m \sin 2\phi_\pi \quad (m = l, t) \quad (1.21)$$

$$P'_n \sigma_v = \nu'_{LT} R'^n_{LT} \sin \phi_\pi \quad (1.22)$$

$$P'_m \sigma_v = \nu'_{LT} R'^m_{LT} \cos \phi_\pi + \nu'_{TT} R'^m_{TT} \quad (m = l, t) \quad (1.23)$$

where the subscripts L, T, LT and TT indicate the longitudinal, transverse, longitudinal-transverse interference and transverse-transverse interference components of the hadronic tensor respectively, and primes indicate the dependence on the electron beam helicity.

The kinematic factors ν_α are defined as:

$$\nu_0 = k/k_\gamma \quad (1.24)$$

$$\nu_L = \nu_0 \epsilon_S \quad (1.25)$$

$$\nu_T = \nu_0 \quad (1.26)$$

$$\nu_{TT} = \nu_0 \epsilon \quad (1.27)$$

$$\nu_{LT} = \nu_0 \sqrt{2\epsilon_S(1+\epsilon)} \quad (1.28)$$

$$\nu'_{LT} = \nu_0 \sqrt{2\epsilon_S(1-\epsilon)} \quad (1.29)$$

$$\nu'_{TT} = \nu_0 \sqrt{1-\epsilon^2} \quad (1.30)$$

with $\epsilon_S = Q^2/q^2\epsilon$ and k is the proton momentum in the center of mass system.

These 18 response functions can be grouped into the following types according to their characteristics and the methods for the extraction of individual response functions:

handle	Real type	Imaginary type
Left/Right	$R_{LT}, R_{LT}^{l,t}, R_{TT}^{l,t}$	R_{LT}^n
Out of Plane	$R_{TT}, R_{LT}^n, R_{TT}^n$	$R'_{LT}, R_{LT}^{l,t}, R_{TT}^{l,t}$
Rosenbluth	R_L, R_T	R_L^n, R_T^n

- Left/Right: in-plane measurements ($\phi = 0^\circ$ and 180°) are needed to separate these response functions associated with $\cos \phi_\pi$ terms.
- Out of Plane: out-of-plane measurements ($\phi \neq 0^\circ$ or 180°) are required to extract these response functions associated with $\cos 2\phi_\pi$ or $\sin \phi_\pi$ terms.
- Rosenbluth: different beam energies are needed to vary the kinematic factors for the separation of L and T type response functions.

These response function functions can provide various insights into the nucleon structure and the $N \rightarrow \Delta$ transition mechanism.

1.3.2 Multipole Amplitudes

The responses in the $p(\vec{e}, e'\vec{p})\pi^0$ reaction are functions of the four-momentum transfer Q^2 , invariant mass of pion nucleon W and pion polar angle θ_π in the hadronic center of mass system. The response can be described by a Legendre expansion as the following:

$$R_\eta = (1 - x^2)^{\alpha_\eta/2} \sum_{n=0}^{\infty} A_{\eta n} P_n(x) \quad (1.31)$$

where α_η is a constant describing the dominant angular dependence of the response function R_η , and $x = \cos(\theta_\pi)$. The Legendre coefficients $A_{\eta n}$ are function of W and Q^2 . These Legendre coefficients are bilinear combinations of the multipole amplitudes which can be extracted from the angular distributions of the experimental cross sections and recoil polarizations.

Considering the S and P waves contribution only and assuming M_{1+} dominance, the relation of the experimental observables to the multipoles can be described as in the following tables (Table 1.1, 1.3, 1.2) [23].

R_η	α_η	n	$A_{\eta n}$
R_L	0		
R_T	0	0	$2 M_{1+} ^2$
		1	$2\text{Re}(M_{1+}E_{0+}^*)$
		2	$- M_{1+} ^2 + 6\text{Re}(M_{1+}E_{1+}^*) - 2\text{Re}(M_{1+}M_{1-}^*)$
R_{TT}	2	0	$-\frac{3}{2} M_{1+} ^2 - 3\text{Re}(M_{1+}E_{1+}^*) - 3\text{Re}(M_{1+}M_{1-}^*)$
R_{LT}	1	0	$\text{Re}(M_{1+}S_{0+}^*)$
		1	$6\text{Re}(M_{1+}S_{1+}^*)$
R'_{LT}	1	0	$-Im(M_{1+}S_{0+}^*)$
		1	$-6Im(M_{1+}S_{1+}^*)$

Table 1.1: Truncated Legendre expansion of the differential cross section

The dominant amplitude for the reaction $p(\vec{e}, e'\vec{p})\pi^0$ in the Δ resonance region is M_{1+} . The only other two resonant amplitudes are the quadrupoles E_{1+} and S_{1+} which are very small compared to M_{1+} . The precise determination of these resonant

R_η	α_η	n	$A_{\eta n}$
R_{LT}^n	0	0	$Re(M_{1+}S_{1-}^*)$
		1	$Re(M_{1+}S_{0+}^*)$
		2	$4Re(M_{1+}S_{1+}^*)$
R_{LT}^l	1	0	$-2(Re(M_{1+}S_{1-}^*) + Re(M_{1+}S_{1+}^*))$
		1	$-3Re(M_{1+}S_{0+}^*)$
		2	$-12Re(M_{1+}S_{1+}^*)$
R_{LT}^t	0	0	$-Re(M_{1+}S_{0+}^*)$
		1	$Re(M_{1+}S_{1-}^*) - \frac{16}{5}Re(M_{1+}S_{1+}^*)$
		2	$2Re(M_{1+}S_{0+}^*)$
		3	$\frac{36}{5}Re(M_{1+}S_{1+}^*)$
R_{TT}^l	0	1	$- M_{1+} ^2 + \frac{6}{5}Re(M_{1+}E_{1+}^*) + 2Re(M_{1+}M_{1-}^*)$
		2	$2Re(M_{1+}E_{0+}^*)$
		3	$-\frac{36}{5}Re(M_{1+}E_{1+}^*)$
R_{TT}^t	1	0	$-2 M_{1+} ^2 + Re(M_{1+}M_{1-}^*)$
		1	$-3Re(M_{1+}E_{0+}^*)$
		2	$-12Re(M_{1+}E_{1+}^*)$

Table 1.2: Truncated Legendre expansion of the helicity-dependent polarization

R_η	α_η	n	$A_{\eta n}$
R_L^n	1		
R_T^n	1	0	$Im(M_{1+}E_{0+}^*)$
		1	$-3Im(M_{1+}M_{1-}^*)$
R_{LT}^n	0	0	$Im(M_{1+}S_{1-}^*)$
		1	$Im(M_{1+}S_{0+}^*)$
		2	$4Im(M_{1+}S_{1+}^*)$
R_{LT}^l	1	0	$2(Im(M_{1+}S_{1-}^*) + Im(M_{1+}S_{1+}^*))$
		1	$3Im(M_{1+}S_{0+}^*)$
		2	$12Im(M_{1+}S_{1+}^*)$
R_{LT}^t	0	0	$Im(M_{1+}S_{0+}^*)$
		1	$-Im(M_{1+}S_{1-}^*) + \frac{16}{5}Im(M_{1+}S_{1+}^*)$
		2	$2Im(M_{1+}S_{0+}^*)$
		3	$-\frac{36}{5}Im(M_{1+}S_{1+}^*)$
R_{TT}^n	1	0	$-3Im(M_{1+}E_{0+}^*)$
		1	$-12Im(M_{1+}E_{1+}^*) + 3Im(M_{1+}M_{1-}^*)$
R_{TT}^l	2	0	$-3Im(M_{1+}E_{0+}^*)$
		1	$-18Im(M_{1+}E_{1+}^*)$
R_{TT}^t	1	0	$-3Im(M_{1+}M_{1-}^*)$
		1	$3Im(M_{1+}E_{0+}^*)$
		2	$12Im(M_{1+}E_{1+}^*)$

Table 1.3: Truncated Legendre expansion of the helicity-independent polarization

quadrupoles is necessary in understanding the nucleon structure and the $N \rightarrow \Delta$ transition mechanism. As mentioned before, in the $p(\vec{e}, e'\vec{p})\pi^0$ process, there are also non-resonant and higher mass resonance contributions as background to yields in the Δ region. These background amplitudes must also be constrained by experimental measurements or model calculations for a reliable and precise extraction of the quadrupoles S_{1+} and E_{1+} in addition to the M_{1+} amplitude.

Because S_{1+} , E_{1+} and background amplitudes are very small with respect to M_{1+} in the Δ resonance region, the best way to extract them is through the simultaneous measurement of those observables which depend linearly on the interferences between the dominant amplitude (M_{1+}) and small amplitudes (resonant and non-resonant).

As can be seen in Table 1.1, the differential cross section measurement provides observables (R_L , R_T , R_{LT} and R_{TT}) which can access the real parts of the interferences. The beam polarization and helicity-dependent recoil proton polarization measurements provide observables which are the imaginary parts of the interferences, such as R'_{LT} in Table 1.1 and the responses (R''_{LT} , R^l_{LT} , R^t_{LT} , R''_{TT} and R^l_{TT}) shown in Table 1.2. And the helicity independent recoil polarization observables (R^n_L , R^n_T , R^n_{LT} , R^l_{LT} , R^t_{LT} , R^n_{TT} , R^l_{TT} , R^t_{TT}) provide access to the real parts of the interferences, as shown in Table 1.3. Thus the recoil polarization together with the differential cross section enable one access both the real and imaginary parts of the interference terms.

A scattering process involving n particles with spin s_i is described by $N = \prod_{i=1}^n (2s_i + 1)$ scattering amplitudes[24]. All the observables are bilinear products of these amplitudes. For parity-conserving process, the number of independent complex amplitudes is $N/2$, resulting $N - 1$ real parameters by excluding an overall arbitrary phase. In the one photon exchange approximation, the reaction $p(\vec{e}, e'\vec{p})\pi^0$ is described by the electromagnetic process $\gamma_v p \rightarrow p\pi^0$. Since the virtual photon spin is 1, pion spin is 0 and proton spin is 1/2, $N = (2 \times 1 + 1)(2 \times 1/2 + 1)(2 \times 1/2 + 1)(2 \times 0 + 1) = 12$, resulting 6 independent complex scattering amplitudes with 11 real parameters which have to be determined from the experimental measurements.

Using the azimuthal distribution to separate R_{LT} and R_{TT} and varying beam energy to separate R_L and R_T , the measurement of the differential cross sections alone can provide 4 observables. To facilitate a model-independent determination of 11 real parameters, at least 7 more observables are needed. The angular distribution of recoil polarization can provide 13 polarization observables and the beam polarization measurement can provide 1 observable. The more response functions extracted from the experimental measurement, the better constraints one has on the scattering amplitudes in the multipole analysis. The polarization measurement provides more observables for the multipole analysis. However, its usefulness is limited due to relatively small angular coverage. On the other hand, the cross section measurement can usually provide quite significant angular coverage. Since a sufficient number of well-determined response functions are needed for the model-independent multipole analysis, it is necessary to combine both the cross sections for greater angular coverage and recoil polarizations for more observables for a better extraction of the relevant multipole amplitudes.

1.4 Pion Electroproduction Models

Figure 1-7: Diagrams contribution to pion electro-production: (A) direct, (B) crossed nucleon pole, (C) pion pole, (D) contact term, (E) s channel isobar excitation, (F) u channel isobar excitation, (G) photon pole, vector meson exchange, (H) re-scattering

If there are not enough experimental observables for the model-independent analysis, a reaction model or the truncated multipole expansion is needed for the extraction the transition multipole amplitudes to provide knowledge of the structure and dynamics of the nucleon, as was done in previous $N \rightarrow \Delta$ transition experiments . Even with enough observables for a model-independent multipole analysis, the reaction model is still useful in the analysis by giving a unified description on different measurements and clarifying the different contributions in the pion electro-production process. In this experimental analysis, some phenomenological pion electro-production models are

used for comparison with data and understanding the physics of the results. These models are described in the following sections.

In the pion electro-production process, the amplitudes have contributions from both the resonance and non-resonance pieces. In Fig. 1-7, (A), (B), (C) and (D) are non-resonant contributions, they are called Born background terms. (G) is the vector meson exchange (ρ and ω) with photon pole. (H) is the pion re-scattering contribution, which is also called a final state interaction. In the $\Delta(1232)$ region, the contribution from isobar excitation to higher mass resonances ((E) and (F)) is also treated as background.

1.4.1 SAID

The SAID is a phase shift analysis on the world's data. The SAID pion electro-production[25] fit is an extension of its analysis of pion photo-production and elastic pion-nucleon scattering in the Δ region[26]. The SAID database contains about 40K data points from electro-production of pions off a proton target. It is concentrated in the region of Q^2 values below 1 GeV^2 and forward angles(θ_π). The Born terms, containing pion and vector-meson exchanges, are included as the background contributions. Several phenomenological structure functions are used in the parameterized multipole expressions.

The extracted $E2/M1$ and $S2/M1$ ratios in SAID fit are shown in Fig. 1-8. The filled circles are values extracted from the Q^2 dependent fit and the filled triangles are from single Q^2 fit on the world data. The solid lines are the best fits on the Q^2 -dependent solutions. Because there is little data in the region of $Q^2 > 1.0 \text{ GeV}^2$, the single Q^2 points in this region have correspondingly large uncertainties. The dash-dotted and dashed curves are from MAID[27] and S-L[17] calculations respectively. Other data points are experimental results. The $S2/M1$ ratio shows a significant Q^2 dependence as seen in Fig.1-2 on page. 16. SAID fit for $E2/M1$ ratio approaches and crosses over to positive values when $Q^2 < 5 \text{ GeV}^2$. MAID fit also shows the same

Figure 1-8: SAID $E2/M1$ and $S2/M1$ ratios vs Q^2 (solid curves)

cross-over trend. However, more precise data are needed to reduce the large data uncertainty to determine whether such trend could be regarded as a signature of the pQCD limit of 100% for E2/M1 ratio.

1.4.2 MAID and DMT

MAID is a unitary isobar model for single pion photo and electro-production off protons and neutrons[27]. In this model, the non-resonant background contains Born terms and vector meson exchange contributions, the nucleon resonance excitations is described by Breit-Wigner functions. The t matrix in α channel $t_{\gamma\pi}^\alpha$ consists of the background part $t_{\gamma\pi}^{B,\alpha}$ and resonance part $t_{\gamma\pi}^{R,\alpha}$, $t_{\gamma\pi}^\alpha = t_{\gamma\pi}^{B,\alpha} + t_{\gamma\pi}^{R,\alpha}$.

These born term contributions are dominant part of the background in this model. The other background contribution includes vector meson exchange contributions.

The background contributions are unitarized using the K-matrix method:

$$t_{\gamma\pi}^{B,\alpha}(MAID) = e^{i\delta_\alpha} \cos \delta_\alpha v_{\gamma\pi}^{B,\alpha}(W, Q^2) \quad (1.32)$$

and the resonance excitations are unitarized using a unitary phase ϕ_R :

$$t_{\gamma\pi}^{R,\alpha}(W, Q^2) = \bar{\mathcal{A}}_\alpha^R(Q^2) \frac{f_{\gamma R}(W) \Gamma_R M_R f_{\pi R}(W)}{M_R^2 - W^2 - i M_R \Gamma_R} e^{i\phi_R} \quad (1.33)$$

where δ_α are the elastic pion-nucleon scattering phases for a specific channel $\alpha = \{l, j, t\}$ below the inelastic two-pion production threshold, $v_{\gamma\pi}^{B,\alpha}$ is the background transition potential, $\bar{\mathcal{A}}_\alpha^R(Q^2)$ is the bare form factor which includes a dipole form factor $(1/(1 + Q^2/0.71)^2)$, and $f_{\pi R}$ is the Breit-Wigner factor describing the decay of a resonance R with total width $\Gamma_R(W)$ and physical mass M_R . The phase ϕ_R is used to adjust the total multipole to equal to the corresponding πN phase shift δ_α .

Unitarity is satisfied by the appropriate choice of the phases. It was found that the unitarization is important for the extraction of $M_{1+}^{3/2}$, $E_{1+}^{3/2}$ and $S_{1+}^{3/2}$ resonance multipoles especially at low Q^2 .

Figure 1-9: $E2/M1$ and $S2/M1$ ratios vs Q^2 (solid curves:MAID,dashed curves:DMT)

With adjustable phenomenological parameters for relevant multipoles, the MAID agrees well with existing pion photo- and electro-production data on the nucleon. This experiment is also using the MAID as a baseline model to fit the experimental data for the extraction of multipole amplitudes.

In the dynamical model DMT[18], the resonance terms are the same as in MAID, but the background terms are different:

$$t_{\gamma\pi}^{B,\alpha}(DMT) = e^{i\delta_\alpha} \cos \delta_\alpha [v_{\gamma\pi}^{B,\alpha}(W, Q^2) + P \int_0^\infty dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q, q') v_{\gamma\pi}^{B,\alpha}(q')}{W - E_{\pi N}(q')}] \quad (1.34)$$

where $R_{\pi N}^{(\alpha)}$ is the full πN scattering reaction matrix. The pion loop effects are generated dynamically as a principal value integral over the reaction matrix. In this model, the pion re-scattering shown as the principle value integral is the dominance source of the quadrupole strength in the $N \rightarrow \Delta$ transition.

8 nucleon resonances are included in the MAID and DMT: $P_{33}(1232)$, $S_{31}(1620)$

and $D_{33}(1700)$ for Deltas, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{11}(1650)$ and $F_{15}(1650)$ for N^* s.

The Q^2 evolution of the $E2/M1$ and $S2/M1$ ratios of the $N \rightarrow \Delta(1232)$ with the MAID and DMT fit is shown in Fig. 1-9. As in the recent SAID fit, the MAID results for R_{EM} exhibits a clear tendency to cross zero and change sign as Q^2 increases. Data at higher Q^2 is needed to see whether such a trend continues to slowly approach the pQCD region.

1.4.3 S-L Model

The meson exchange model by Sato and Lee(S-L model[17]) is another dynamical model which addresses the pion cloud effect in pion electro-production process. Using a unitary transformation to a model Lagrangian with $N, \Delta, \pi, \rho, \omega$ and γ fields, an effective Hamiltonian, consisting of bare $\Delta \leftrightarrow \pi N, \gamma N$ vertex and meson-exchange $\pi N \leftrightarrow \pi N, \gamma N$ transition operators, is derived. In this model, the pion cloud effect is considered in the non-resonant interaction dressed $\gamma N \rightarrow N$ vertex. The effects of the pion cloud as a function of Q^2 from this model calculation are shown in Fig. 1-10. The solid lines are the full calculations including the pion cloud effects. The dotted lines are calculations without non-resonant interactions. The E2 and C2 amplitudes are dominated by the pion cloud effects, especially at the low Q^2 region.

It should be noted that none of the existing pion electro-production models is completely satisfactory because they are all built on insufficient experimental data, most of which involve cross section measurements with large uncertainties. With the advancement of experimental measurement, more polarization data and higher precision cross section data are becoming available, and correspondingly the theoretical models need to be improved for a better knowledge of nuclear structure and the $N \rightarrow \Delta$ transition mechanism. This experiment provides invaluable experimental inputs to theoretical model development by measuring both the cross sections and recoil polarizations in the pion electro-production process in the Δ resonance region.

Figure 1-10: Imaginary parts of the multipole amplitudes in S-L model for $\gamma^*N \rightarrow \pi N$ at $\Delta(1232)$ region. Solid curves are full calculations, and the dotted curves are calculations without non-resonant interaction.

Chapter 2

The Experiment Setup

2.1 Experiment Overview

To study the spin observables in the $N \rightarrow \Delta$ transition, the experiment E91-011[1] was performed in Hall A at Jefferson Laboratory between May 19 and August 1, 2000. A polarized electron beam from the accelerator with the energy 4.5 GeV and longitudinal polarization 70% was incident on the 15 cm liquid hydrogen target in Hall A. Two high resolution spectrometers (HRSe and HRSh) were used to detect the scattering electrons and recoil protons respectively. The focal plane polarimeter (FPP) in the hadron arm was also used to measure the recoil proton's polarization. The first part of this chapter presents the CEBAF (Continuous Electron Beam Accelerator Facility) accelerator at Jefferson Lab. The second part describes the Hall A experimental setup and instrumentation. The experimental kinematic settings and acceptance coverages are summarized in the last part of this chapter.

Figure 2-1: The Continuous Electron Beam Accelerator Facility at JLAB

2.2 Accelerator and Polarized Source

The Thomas Jefferson National Accelerator Facility (Jefferson Lab) is located in Newport News, Virginia. It is the world's premier medium energy electron scattering laboratory whose primary mission is to conduct basic research of the atom's nucleus at the quark level. There are three complementary experiment halls utilizing the electron beams to investigate the structure of nuclei and the fundamental interactions.

The CEBAF[28] accelerator (see Fig. 2-1) was designed to deliver continuous wave electron beams up to 5.7 GeV to three halls simultaneously. A strained GaAs cathode in the polarized gun is illuminated by a 1497 MHz gain-switched diode laser. This electron source can provide maximum beam current $200 \mu\text{A}$ with the longitudinal polarization about 70%. At the injector, the electron polarization is measured with a 5 MeV Mott polarimeter and the polarization vector is oriented with a Wien filter. The polarized electrons are then accelerated to 45 MeV and injected into the north linac. There are two superconducting linacs, each accelerating the electron by an

Figure 2-2: The layers of the strained GaAs cathode (Left) and the energy levels of conduction and valance bands (Right)

energy 400 MeV per pass. The energy gain per pass can be tuned to exceed 500 MeV if necessary. The north and south linacs are joined by two 180° arcs, forming a racetrack type recirculating beam line. At the beam switch-yard, one of three interleaved 499 MHz bunch trains can be extracted and delivered to any one of the experimental halls using RF separation and septa.

The polarized electron source is an improved version of GaAs cathode used at SLAC[29]. It was manufactured by growing layers of GaAs on the GaAs substrate (see Fig. 2-2). The shorter lattice spacing of GaAs_{0.72}P_{0.28} (5.5968 Å) causes the natural spacing of GaAs (5.6533 Å) to shrink slightly and creates a strain in the GaAs crystal. The strain splits the energy levels of the P_{3/2} electrons in the valance band of GaAs. When the cathode is illuminated by a circularly polarized laser of proper frequency, electrons in the P_{3/2} and m=3/2 state can be excited to S_{1/2} and m=1/2 state of the conduction band. A thin film of CsF is deposited onto the GaAs layer to lower the work function of the electron in the conduction band, thus allowing the excited electrons to escape from the surface. Since the strain maintains a wide energy gap in the P_{3/2} sub-states, the electrons in the m=1/2 state cannot be excited

Energy	Energy Spread	Current	Emittance	Instantaneous Size
0.8 - 6.0 GeV	2.5×10^{-5}	1-190 μA	2×10^{-9} m·r	35 μm

Table 2.1: JLab Beam Parameters

to the conduction band by the laser. Thus theoretically, the electrons escaping from the cathode surface are 100% polarized.

The present JLab beam parameters are shown in Table 2.1.

During this experiment period, an averaged 70% longitudinally polarized CW beam was delivered into Hall A for production data taking, with the helicity flipping at 30 Hz and the current varying from 20 μA to 100 μA .

Figure 2-3: Hall A Configuration

2.3 Hall A Instrumentation

The following Hall A experimental instruments[30] were used in E91-011 experiment: beam line instruments for measuring various electron beam parameters, cryogenic target in the scattering chamber at the Hall A center and two high resolution spectrometers. The schematic layout of Hall A is shown in Fig. 2-3.

2.3.1 Beam Line Instruments

Arc and eP Beam Energy Measurements

The absolute beam energy is measured with two independent methods: the Arc energy method and the eP method.

The Arc-Energy method, developed by the French collaboration of Pascal Bernin and the Saclay group, measures the energy as a function of the bend angle of the arc and the field integral of eight-dipole magnets located in the forty-meter arc between the accelerator and the Hall A end station (see Fig. 2-4). The nominal bend angle of the beam line in the arc section is 34.3° . When beam is tuned in dispersive mode

Figure 2-4: Hall A Arc Energy Measurement

for the energy measurement, the beam momentum (p in GeV/c) is a function of the field integral of eight dipoles ($\int \vec{B} \cdot d\vec{l}$ in Tm) and the net bend angle through the arc section (θ in radian):

$$p = c \frac{\int \vec{B} \cdot d\vec{l}}{\theta} \quad (2.1)$$

where $c = 0.299792 \text{ GeV rad T}^{-1} \text{ m}^{-1}$. The actual bend angle of the arc is measured by four wire scanners, two at the upstream entrance of the arc and two at the downstream exit of the arc. The magnetic field integral in the eight bending dipoles in the arc is measured by a ninth reference magnet located in a separate building, which is identical to and powered in series with the other magnets.

The eP method, developed by the Clermont Ferrand group, determines the energy based on the emission angles of particles in an electron-proton elastic reaction (see Fig. 2-5). The method requires the beam to be directed to a stand alone device in the beam line 17 m upstream of the target. The eP device utilizes a target consisting of a rotating 10-30 μm thick tape of CH₂. The scattering angle of the electron θ_e

Figure 2-5: Hall A eP Energy Measurement

and the recoil angle of the proton θ_p in the $p(e, e'p)$ elastic reaction are related to the beam energy E by:

$$E = M_p \frac{\cos \theta_e + \sin \theta_e / \tan \theta_p - 1}{1 - \cos \theta_p} + O(m_e^2/E^2) \quad (2.2)$$

where M_p and m_e are the masses of the proton and electron respectively. Silicon strip detectors are used to detect the scattering electrons and recoil protons in the reaction plane. Seven electron detectors are placed at each arm, detecting scattering electrons in the range of 9° to 41° . A proton detector is placed at exactly 60° in each arm for the detection of the recoil protons.

The beam energy measurements of the arc and eP methods agree with each other within their respective uncertainties ($\leq 3 \times 10^{-4}$). The energy measurement results from the two methods during E91-011 experiment are shown in Table 2.2[31].

In the following data analysis and simulation, the measured beam energy 4530.6 MeV was used for kinematic calculation.

Date	Pass	Arc Energy (MeV)	eP (MeV)	Nominal Energy (MeV)
05/23/2000	4	4530.6±0.9	—	4537.98
06/20/2000	4	4531.0±1.0	4530.0±1.0	4537.99

Table 2.2: E91-011 Beam Energy Measurements

Møller and Compton Polarimeters

In addition to the measurement of the recoil protons' polarization with FPP, the measurement of the beam polarization is also needed for the extraction of helicity dependent response functions. Two polarimeters, Møller and Compton polarimeters, equipped in the Hall A beam line, were used to measure the polarization of the electron beam.

The Møller scattering of the polarized electrons off polarized atomic electrons in magnetized foil ($e^- + e^- \rightarrow e^- + e^-$) is used for the beam polarization measurement with the Møller polarimeter. The reaction cross section is related to the beam polarization P_{beam} and target polarization P_{target} by:

$$\sigma \propto [1 + \sum_{i=X,Y,Z} (A_{ii} \cdot P_{target,i} \cdot P_{beam,i})] \quad (2.3)$$

where $i = X, Y$ or Z is the projection direction of the polarizations. The analyzing power A is decided by the scattering angle in the center of mass frame θ_{CM} . Assuming beam is along the Z -axis and the scattering plane is ZX plane:

$$A_{ZZ} = -\frac{\sin^2 \theta_{CM} \cdot (7 + \cos^2 \theta_{CM})}{(3 + \cos^2 \theta_{CM})^2}, A_{XX} = -A_{YY} = -\frac{\sin^4 \theta_{CM}}{(3 + \cos^2 \theta_{CM})^2} \quad (2.4)$$

At $\theta_{CM} = 90^\circ$ the analyzing power has its maximum $A_{ZZ,max} = 7/9$. A beam with polarization transverse to the scattering plane leads to lower analyzing power $A_{XX,max} = A_{ZZ,max}/7$. The main purpose of the polarimeter is to measure the longitudinal component of the beam polarization.

The Møller polarimeter consists of a polarized target, a magnetic channel consist-

Figure 2-7: Hall A Compton Polarimeter

The Compton scattering process is used to measure the beam polarization concurrently with the experiments running in the hall. The beam polarization is extracted from the counting rate asymmetry for opposite helicities in the scattering of a circularly polarized photon beam off the polarized electron beam. The Compton polarimeter (see Fig 2-7) is installed at the entrance of the hall (see Fig. 2-3 on page 42 for Hall A layout). It consists of a magnetic chicane, a photon source, an electromagnetic calorimeter and an electron detector. The electron beam is deflected vertically by four dipoles of the chicane and crosses the photon beam at the Compton interaction point (CIP). After Compton scattering, the backscattered photons are detected in the calorimeter and the electrons are detected in the silicon strip electron detector. The photon source is a primary 230 mW CW Nd:YAG laser beam ($\lambda = 1064$ nm)

which is amplified by a 85 cm long resonant Fabry-Pérot cavity. The amplification factor of 7300 has been measured, corresponding to a photon beam power of 1680W inside of the cavity. The circular polarization of laser is greater than 99% for both right and left helicity states. The Compton luminosity is maximized by selecting a small crossing angle between the electron and photon beams. The background contribution is measured without the photon beam. For JLab 4.5 GeV and 40 μ A electron beam, the asymmetry measured in Compton scattering is about 5.5% with a relative statistical accuracy of 1% for half an hour data taking.

The total relative systematic error of Compton measurement is about 1.7%[30].

Beam Current Monitor

The Beam Current Monitor (BCM) (see Fig. 2-8) is designed for stable, low noise, non-intercepting beam current measurements. It consists of an Unser monitor, two RF cavities, the electronics and a data acquisition system. The cavities and the Unser monitor are enclosed in a box to improve magnetic shielding and temperature stabilization. The box is located 25 m upstream of the target. A cavity monitor and a Faraday Cup at the Injector section of the accelerator are used to provide an absolute reference during calibration runs.

The Unser monitor is a Parametric Current Transformer for absolute reference of the beam current. The monitor is calibrated by passing a known current through a wire inside the beam pipe and has a nominal output of 4 mV/ μ A. The reference current is generated by a high precision current source. As the Unser monitor's output signal drifts significantly on a time scale of several minutes, it cannot be used to continuously monitor the beam current. However, this drift is measured during the calibration runs (by taking a zero current reading) and removed in calibrating the two RF cavities. The more stable cavities are then used to determine the beam current and charge for each run.

The two resonant RF cavity monitors on either side of the Unser Monitor are

Figure 2-8: Hall A Beam Current Monitor

stainless steel cylindrical high Q (~ 3000) waveguides, which are tuned to the frequency of the beam (1.497 GHz) resulting in voltage levels at their outputs which are proportional to the beam current. Each of the RF output signals from the two cavities are split into two parts. One part of the signal is converted to 10 kHz signals by a down-converter and fed into an RMS-to-DC converter board with a 50 kHz bandpass filter to eliminate noise, split to two sets of outputs, which are recorded in the data stream. These two outputs lead to the sampled and integrated data in the data stream.

For the sampled data, one of the amplifier outputs is sent to a high precision digital AC voltmeter. Each second this device provides a digital output which represents the RMS average of the input signal during that second. The resulting number is proportional to the beam charge accumulated during the corresponding second. Signals from both cavities' multi-meters, as well as from the multi-meter connected to the Unser, are transported through GPIB ports to a computer where they are recorded every 1 to 2 seconds via the data-logging process. They are also recorded in the data stream of the data acquisition system at quasi-regular intervals, typically every two to five seconds.

For the integrated data, the other amplifier output is sent to an RMS-to-DC converter which produces an analog DC voltage level. This level drives a Voltage-To-Frequency (VTOF) converter whose output frequency is proportional to the input DC voltage level. These signals are then fed to Fastbus scalars and are finally injected into the data stream along with other scalar information. These scalars simply accumulate during the run, resulting in a number which is proportional to the time integrated voltage level and therefore more accurately represents the true integral of the current and hence the total beam charge. The regular RMS to DC output is linear for currents from about $5\ \mu\text{A}$ to well above $200\ \mu\text{A}$. A set of amplifiers with differing gains ($\times 3$ and $\times 10$) allow the non-linear region to be extended to lower currents at the expense of saturation at the very high currents. Hence there are 3 signals coming from each

BCM. All 6 signals from the 2 BCMs are fed to scaler inputs of each spectrometer, providing a redundancy of 12 scaler outputs for determining the beam charge during a run.

The BCMs usually are calibrated every 2 to 3 months. A series of measurements were performed to calibrate the Hall A beam current monitors at the beginning of this experiment on May 21, 2000. Detailed calibration analysis can be found in reference[32]. For the E91-011 experiment, the beam charge measured from the upstream BCM was used since the upstream BCM was found to be more stable than the downstream BCM during the experiment period. The relative systematic error of the beam charge measurement was estimated to be about 0.5%.

Beam Position Monitor

Two Beam Position Monitors (BPMs), located at distances 7.524 m and 1.286 m upstream of the target position, are used to determine the position and the direction of the beam on the target point in Hall A. The standard difference-over-sum technique is then used to determine the relative position of the beam to within $100\text{ }\mu\text{m}$ for currents above $1\text{ }\mu\text{A}$. The absolute position of the BPMs is calibrated with respect to wire scanners (super-harps) which are located adjacent to each of the BPMs (at 7.353 m and at 1.122 m upstream of the target). These super-harps are regularly surveyed to determine the absolute position in Hall A coordinate system.

The position information from the BPMs can be recorded in two ways: 1. The averaged position over 0.3 seconds is logged into the EPICS database (1 Hz updating frequency) and injected into the data stream every 3-4 seconds, unsynchronized but with a time stamp. 2. Event-by-event information from the BPMs are recorded in the data stream from each of the 8 BPM antennas (2×4) from which the position of the beam can be reconstructed.

2.3.2 Cryogenic Target

Scattering Chamber

The scattering vacuum chamber consists of several rings, supported on a 607 mm diameter central pivot post. The stainless steel base ring has one vacuum pump-out port and other ports for viewing and electrical feed-throughs. The middle ring is made out of aluminum and located at beam height with 152 mm vertical cutouts on each side of the beam over the full angular range ($12.5^\circ \leq \theta \leq 165^\circ$). The cutouts are covered with a pair of flanges with 0.38 mm aluminum foils. It also has entrance and exit beam ports. The upper ring is used to house the cryotarget. The scattering chamber has two flat covers, one for the middle ring and one for the top hat. The chamber vacuum is maintained at 10^{-6} Torr level to serve as an heat insulator as well as reducing multiple scattering.

Cryogenic Target

The cryogenic target (15 cm liquid hydrogen) was used for the experiment production data taking. It is mounted inside the scattering chamber along with subsystems for cooling, gas handling, temperature and pressure monitoring, target control and target motion, and an attached dummy and solid target ladder(see Fig. 2-9).

The targets are arranged in a vertical stack, which can be moved from one position to another under remote control to select different target. Besides the 5 target positions for cryotargets (10 cm helium, 15 cm and 4 cm LH₂, and 15 cm and 4 cm LD₂), there are three dummy targets (which were used for optics data taking in this experiment) and five solid targets including the carbon foil (also used for optics data taking). The dummy targets contain two thin pieces of aluminum, separated by a distance of 10 cm, 15 cm or 4 cm, which can be used for subtraction of the window contributions. The solid target ladder has five target positioning usually occupied by a BeO target an empty target, a ¹²C target and two special-purpose targets.

Figure 2-9: Side view of the Hall A cryotarget

There are three independent target loops in the basic cryogenic target: a liquid hydrogen (LH₂) loop, a liquid deuterium (LD₂) loop and a gaseous helium loop. The LH₂ target is operated at a temperature of 19 K and a pressure of about 0.17 MPa (approximately 3 K sub-cooling, which means the the temperature of the liquid is below its boiling temperature at this specified pressure), having a density of 0.0723 g/cm³. The liquid loop has two horizontal cylindrical target cells, 15 cm and 4 cm long, with their axes along the beam direction. The 15 cm LH₂ target cell was used for this experiment. The cells are made of aluminum with a diameter of 63.5 mm. The upstream window is 71 μ m thick and the downstream window about 102 μ m. The side wall is 178 μ m thick. The upstream window has a thick ring holder with an inner diameter of 19mm, which is large enough to let the beam pass through.

The target coolant (helium) is supplied by the End Station Refrigerator (ESR) at either 4.5 K or 15 K to provide the cooling for the liquid hydrogen target. The liquid target uses the 15 K coolant and returns it at a little over 20 K. The total 15 K cooling power can be over 1 kW. Up to 130 μ A of beam current has been delivered on th 15 cm LH₂ target. The beam heating alone is about 700 W. Adding the heating from the target circulating fans and a small amount of heater power to keep the target stable in temperature, the heat load can reach 1 kW. The coolant supply is controlled with Joule-Thompson (JT) valves, which can be adjusted either locally or remotely.

The target pressures are monitored with pressure transducers at several locations on the gas panel. The precision on the pressure measurement is better than 0.34 MPa, which contributes less than 0.1% to the density uncertainty. The target temperature is monitored at several locations in the loop. The primary temperature sensors are LakeShore Cernox, a special type of semiconductor. These are precision temperature sensors with high radiation resistance. The precision of the temperature measurement is about 0.05 K, which contributes less than 0.1% uncertainty to the density. Two other types of temperature sensors, vapor-pressure transducers and resistors, are also used as checks. Diode temperature sensors are used in the coolant lines. The resistor

and diode sensors are read out with an Oxford Temperature Monitor Model ITC 501, while the Cernoxs are read out with an Oxford Temperature Control Model ITC 502. Low-power heaters, up to 60 W, are controlled by the ITC 502 to keep the temperature stable. High-power heaters, up to 1kW, are used to keep the target temperature stable when the beam is off. The temperature control is automated by a computer PID (proportion, integral and derivative) feedback loop.

All target positions are surveyed before and after experiments. The motion during vacuum pump-down and cool-down is monitored and corrected in the position determination. The targets are usually centered to about 1mm with a typical position precision of 0.5 mm.

The intrinsic beam spot size is very small ($\sim 100 \mu\text{m}$) and can cause local damage to the target cell at high current. The beam is rastered on target with an amplitude of a few mm to prevent overheating of the liquid cryo target. With rastered beam the density fluctuation due to beam heating is only a few percent at a beam current of 100 μA . During this experiment, it was found that the target relative density fluctuation is less than 2% for the current range of 20-60 μA .

2.3.3 High Resolution Spectrometer

The Hall A spectrometers were designed to measure the particle momentum, position and angle with high resolution. Each of the two spectrometers consists of three $\cos 2\theta$ quadrupoles (Q1, Q2 and Q3) and one dipole (D). These superconducting magnets are arranged in the QQDQ configuration (see Fig. 2-10). The bending angle is 45° in vertical plane. The momentum range is from 0.3 to 4 GeV/c. The relative momentum acceptance is about 9% with the resolution of 10^{-4} .

Each spectrometer is point-to-point in the dispersive direction. Q1 is convergent in the dispersive (vertical) plane. Q2 and Q3 provide transverse focusing. The dipole's entrance and exit are both inclined at 30° with respect to the central axis. The magnetic field of the dipole increases with the radial distance, which provides a natural

Figure 2-10: Side view of the Hall A High Resolution Spectrometer

focusing in the dispersive direction.

The general characteristics of Hall A High Resolution Spectrometers are shown in Table 2.3. The design detail of the High Resolution Spectrometers can be found in reference[30].

2.3.4 Detector Packages

The detector package of each spectrometer has the following components: two scintillators for trigger and time-of-flight measurements, a pair of Vertical Drift Chambers (VDC) for charged particle tracking information, Čerenkov detectors (aerogel and gas) and lead-glass shower counters for particle identification. The Hadron spectrometer (Right Arm) has a focal plane polarimeter (FPP) to measure the polarization of detected protons.

For the E91-011 experiment, a pair of VDCs and two scintillators in the Electron Arm (Left Arm, see Fig. 2-12) were used for event trigger and tracking. The Hadron Arm (Right Arm, see Fig. 2-11) detectors included 3 scintillators, a pair of VDCs and

Figure 2-11: Hadron Arm Detector Package

Figure 2-12: Electron Arm Detector Package

Momentum Range	0.3-4.0 GeV/c
Configuration	QQDQ
Bend Angle	45°
Optical Length	23.4 m
Momentum acceptance	$\pm 4.5\%$
Dispersion (D)	12.4 cm/%
Radial Linear Magnification (M)	2.5
D/M	5
Momentum Resolution (FWHM)	10^{-4}
Horizontal Angular Acceptance	± 28 mr
Vertical Angular Acceptance	± 60 mr
Solid Angle	~ 6.7 msr
Horizontal Angular Resolution (FWHM)	0.6 mr
Vertical Angular Resolution (FWHM)	2.0 mr
Transverse Length Acceptance	± 5 cm
Transverse Position Resolution (FWHM)	1.5 mm
Spectrometer Angle Determination Accuracy	0.1 mr

Table 2.3: General characteristics of Hall A High Resolution Spectrometer

FPP in the first period of data taking. In the second period of data taking, one of the scintillators (S0) in Hadron Arm was removed and the aerogel Čerenkov detector was installed to serve as the pion rejector.

Figure 2-13: VDC assembly layout (not to scale)

Vertical Drift Chamber

The Vertical Drift Chambers[33] provide a precise measurement of the position and angle of incidence of charged particles at the spectrometer focal planes. The tracking information from the VDC measurement is combined with the knowledge of the spectrometer optics to reconstruct the position, angle and momentum of the particles in the target.

Each spectrometer has a pair of identical VDCs. The focal plane of the spectrometer is in between the top and bottom VDC planes. The top VDC is placed about 35 cm above the bottom VDC and shifted about 35 cm with respect to the bottom one in the dispersive direction. Each VDC is composed of two wire planes in a standard UV configuration - the wires of each plane are oriented at 90° to one another, and each plane is oriented at 45° with respect to the nominal particle trajectories (see Fig. 2-13). There are a total of 368 sense wires in each plane.

The VDC chambers are operated at -4.0 kV with a gas mixture of argon (Ar,

62%) and ethane (C_2H_6 , 38%), with a flow rate 5 liter/hour. The charged particles produce ionization in the chamber gas when passing through the VDCs. The ions and electrons generated in the ionization process drift along the electric field lines between the grounded wires and the cathode planes, which are held at about -4 kV potential. In the region close to the wires, the drifting electrons accelerate because the electric field intensifies, resulting further ionizations to an avalanche which induces an electrical pulse signal on the signal wire. The pulses are then amplified, discriminated and used to start multi-hit TDCs, the TDCs are stopped by the event trigger.

Each VDC wire is positioned with the precision of about $30\text{ }\mu\text{m}$. The position of VDCs is known to $100\text{ }\mu\text{m}$ accuracy. At the focal plane, the position resolution ($\sigma_{x,y}$) is about $100\text{ }\mu\text{m}$, and the angular resolution ($\sigma_{\theta,\phi}$) is about 0.5 mrad. The VDC wire efficiency is usually greater than 99%. The wire efficiency can drop to below 95% when there is gas problem, thus it is necessary to monitor the VDC efficiency frequently to assure the data quality.

Scintillator

There are two planes of trigger scintillators S1 and S2 in each spectrometer. S1 is located 1.5 meters away from the center of the first VDC plane. The S2 is 2 meters away from S1 (see Fig. 2-11 and Fig. 2-12 for S1 and S2 in each spectrometer)

S1 and S2 each consist of six paddles(see Fig. 2-14). The active area of each S1 paddle is about 29.5 cm by 35.5 cm. The active area of each S2 paddle is about 37.0 cm by 54.0 cm. The S1 paddles are installed at a small angle to the average plane and overlap by 10 mm. The S2 paddles have only 5 mm overlap. These paddles are made of 5 mm thick plastic scintillator. A 2-inch photo multiplier tube at each side is used to collect the photons and generate signals for both the Analog-Digital Converter (ADC) and the Time-Digital Converter (TDC).

The scintillators are mainly used to generate triggers for the data acquisition system. The time resolution of each plane is about 0.30 nsec (σ). Besides the use for

Figure 2-14: Scintillator Counter Layout

trigger purpose, the scintillators can also be used for particle identification. Time-of-flight (TOF) between the S1 and S2 planes can be used to measure the particle's speed β in terms of light speed c in vacuum. In this experiment, the scintillator ADC was also used to distinguish the protons from charged pions because of the response difference of the protons and pions. In general the ADC amplitude of the protons is greater than that of the pions. The background charged pions can be effectively separated from the protons using both the scintillator ADC and the β from the TOF measurement. The detail of using the scintillator ADC for particle identification is described in next chapter of data analysis.

Aerogel Čerenkov counter

During the second period of this experimental data taking, an aerogel Čerenkov counter was removed from the Electron Arm and installed in Hadron Arm. The Čerenkov signals were used in the Hadron Arm trigger to reject the charged pion

events since most of the aerogel hits were caused by the charged pions. The aerogel Čerenkov's material is hygroscopic aerogel material[34] with a refraction index $n = 1.03$ at 9cm thickness. 26 PMT's are used to collect the Čerenkov light caused by charged particles passing through the detector with speed greater than the light speed (c/n) in this material. Using the cosmic rays, all the 26 PMT's high voltages were carefully adjusted to align the single photon electron peaks. The sum of the signals from all these PMTs were used to distinguish pions and protons.

Since the Čerenkov light is emitted at an angle of $\cos^{-1}(1/n\beta)$ with respect to the charged particle momentum, the threshold speed of a charged particle for emitting the light is c/n . Thus the threshold momentum for the protons is 3.92 GeV/c, charged pions 0.58 GeV/c and electrons 0.002 GeV/c. The threshold momentum can be calculated from $m_0c/\sqrt{1-\beta^2}$ where the m_0 is the rest mass of the charged particle and β is the particle's speed in terms of the vacuum light speed. Since the hadron arm momentum was set in the range from 0.7 to 1.4 GeV/c, the momentum setting exceeded the pion threshold but still lower than the proton threshold. However, because the knock-out electrons caused by protons can also activate the Čerenkov light, some Čerenkov signals were also observed for the proton events. When the Čerenkov signals were used in the trigger in Hadron Arm for the pion rejection, some protons which generating knock-out electrons were also rejected. Because there is not enough information to determine the fraction of protons whose knock-out electrons emitted the Čerenkov light, the data taken with the pion rejector in the trigger was not used for the absolute cross section measurement.

Focal Plane Polarimeter

The focal plane polarimeter (FPP) in the hadron arm(see Fig. 2-11 on page 57) was used to measure the polarization of recoil protons at the Hadron Arm focal plane in this experiment.

It consists of front and rear straw chambers and a carbon analyzer. The front

straw chamber 1 and 2 are separated by about 114 cm, and are located before and after the gas Čerenkov detector (the gas Čerenkov was not in its place during the E91-011 experiment period). The carbon analyzer is located right after the scintillator 2. The carbon analyzer consists of 5 carbon blocks which were used to adjusted the total scattering carbon thickness. The block thicknesses, from front to rear, are 22.9cm, 15.2cm, 7.6cm, 3.8cm and 1.9cm. The rear chamber 3 and 4, are separated by 38 cm and are immediately behind of the carbon analyzer. The straw chamber planes are designated as X, U, and V planes. The central ray defines the z axis. X wires measure position along the dispersive direction. The UV coordinate system is created by a 45 degree rotation in the transverse plane of the XY coordinate system, with +U between the +X and +Y axes, and +V between the +Y and -X axes. There are 3 U planes and 3 V planes in each of the chamber 1, 2 and 4. The rear chamber 3 has 2 U planes, 2 V planes and 2 X planes. The straw number in each plane ranges from 160 to 276.

The front chamber 1 and 2 are used to determine the proton trajectories before they are scattered in the carbon analyzer. The rear chamber 3 and 4 are used to determine the proton trajectories after they are scattered in the carbon analyzer. When the protons have transverse components of the polarization, the scattering angular distribution has an azimuthal asymmetry due to the proton's spin-orbit interaction with the analyzing nucleus. The proton's transverse polarization can then be extracted from a Fourier analysis of the angular asymmetry. A spin precession matrix modeled by the COSY[35] using the spectrometer magnetic field maps of different elements is used to reconstruct the proton polarization at the target from the measured focal plane polarization.

Using the VDC track as reference, the spatial alignment offsets of each straw chamber plane are determined by the straight-through events taken without the carbon analyzer in. The eP elastic data was used to determine the FPP's instrumental asymmetry (false asymmetry) since the protons from such reaction do not have the

transverse polarization. The false asymmetry has to be properly corrected when analyzing the recoil polarization of the production data.

The FPP straw inefficiency was suggested to be an important source of the false asymmetry besides the possible contribution from imperfect chamber alignment. A GEANT[36] Monte Carlo software written by Ed Brash[37] was modified to include the effect of the measured straw efficiencies for the study of the false asymmetry.

Figure 2-15: Hall A Focal Plane Polarimeter False Asymmetry Simulation

The straw efficiency is defined as the fraction of good tracks in all the tracks measured by FPP. For a specific straw, if the straw was not fired and a fitted track

passes through that straw, that track was defined as a bad-hit for this straw. A good hit for the straw requires both the fitted track passes through the straw and the straw is fired by that track. The GEANT simulation software has an event generator to generate the transport[38] variables of the protons tracks at the spectrometer's focal plane. The detector elements in simulation include Scintillators, FPP front chambers, Carbon Analyzer and FPP rear chambers. The position, dimension and material specifications of each element are corresponding to that of the hadron spectrometer. The FPP tracking routines of the program ESPACE[39] were used for the tracking of the simulated protons before and after the carbon analyzer. The false asymmetry was determine by fitting the normalized azimuthal distribution of the scattering protons with the following formula: $1 + P_2 \cos \phi + P_3 \sin \phi$ where ϕ is the azimuthal angle, P_2 and P_3 are the false asymmetries.

Fig. 2-15 shows the comparison between the simulated FPP false asymmetry and the false asymmetry measured from the eP elastic data with two typical Hadron Arm momentum settings. Although the simulation did not reproduce the measured false asymmetry exactly, it simulated the P_3 ($\sin \phi$ term) relative momentum (δ) dependence qualitatively. A comparison simulation with ideal straw efficiency (100%) showed no significant false asymmetry. This simulation proved the straw inefficiency is one of the important FPP false asymmetry sources. Another false asymmetry source is the FPP chamber misalignment. In fact, in this experiment, the false asymmetry was reduced significantly using a new FPP alignment method. Since the proton in the eP elastic scattering with unpolarized electron beam is not polarized, the FPP false asymmetry was measured using such data and parameterized so that it could be taken into account in the recoil proton polarization analysis. Although there are a lot of charged pions in the polarization data, they don't contribute to the asymmetry of the secondary scattering in FPP since the pion has spin zero. The detail of the polarization analysis of this experimental data can be found in R. Roche's thesis[2].

2.3.5 Trigger Electronics

The two scintillators make the primary trigger in each spectrometer arm, and a coincidence is formed between the spectrometer arms. A simplified trigger setup for this experiment is shown in Fig. 2-16. During the second period of the experiment data taking (which was mainly for the recoil polarization measurement), the Aerogel Čerenkov signal was also included in the H-Arm trigger to reject the π^+ background events since most of the signals in the Aerogel Čerenkov was generated by the passing π^+ .

The trigger system was built from commercial CAMAC and NIM discriminators, delay units, logic units and memory lookup units (MLU). First the signals from both PMTs in each paddle are required to form a coincidence for that paddle. The logical OR of the signals from these 6 paddles is then formed for that scintillator plane. The main trigger S-Ray is formed from the logical AND of two scintillator plans with the requirement that the fired paddle combinations in the two planes are in an allowed set. A memory lookup (MLU) is used to decide whether the combination is valid. The allowed combinations are for S-Ray tracks that are at an approximately 45 degree angle with respect to the hall floor, with the difference of fired paddle number in the two planes equal to 0 or 1. The timing of this trigger is determined by a strobe on the MLU which in most events comes from the right-side PMTs of the second scintillator plane. The coincidence trigger S5 is formed in an overlap AND circuit with a 110 nsec window between the singles trigger S1 (from the Electron Arm) and S3 (from Hadron Arm). During this experiment the coincidence timing window was adjusted to 70 nsec to suppress the accidentals. A looser trigger S2 is formed if the S-Ray requirement could not be satisfied or two of the 3 detectors fired: two scintillators and Čerenkov for electron arm. For hadron arm, the looser trigger S4 is formed if the S-Ray requirement could not be satisfied or only one of the two scintillators was fired. The S2 and S4 trigger are mainly used for the measurement of the trigger inefficiencies. All these trigger types were sent to a scaler unit for counting and

Figure 2-16: Trigger Electronics Setup

to a Trigger Supervisor (TS) unit which has both the pre-scale function and MLU function. The prescale factor is an integer greater than 0. If the prescale for a specific trigger type is N , then only 1 out of N trigger of that type is recorded in the data stream. During the production data taking, all the triggers except S5 were pre-scaled to reduce the computer deadtime of the DAQ system. The event trigger types in the CODA[40] data stream are called T1, T2, T3, T4 and T5 which are corresponding to the triggers S1, S2, S3, S4 and S5 respectively.

For the $(e, e'p)$ coincidence measurement, the scattered electrons are all relativistic with the speed of light c while the recoil protons' speed is less than the light speed. The trigger setup has to make appropriate change of the delay on the Hadron Arm for each momentum setting (proton momenta above 360 MeV/c can be accommodated by the hardware at present).

Since this experiment was run with very high event rates (in the order of 1MHz) in the scintillator planes, the electronic deadtime correction became significant for the precise cross section measurement. A partial scheme for the measurement of the electronic dead time was installed during this experiment. A well-defined recognizable electronic pulse was added in the analog circuitry to the scintillator pulses before the discriminator. The electronic deadtime correction was determined from the fraction of pulses which could not go through the trigger supervisor at which point the DAQ was triggered.

2.3.6 Data Acquisition

The Hall A data acquisition (DAQ) system uses CODA (CEBAF On-line Data Acquisition)[40] developed by the Jefferson Lab Data Acquisition Group.

There are two fast-bus crates in the hadron spectrometer and one fast-bus crate in the electron spectrometer. The fast-bus modules for the detectors are:

- LeCroy model 1877 TDCs: operating in common-stop with 0.5 nsec resolution for drift chambers (VDCs) and straw chambers (FPP).

- LeCroy model 1875 TDCs: operating in common-start with 0.1 nsec resolution for scintillators and trigger diagnostics.
- LeCroy model 1881M ADCs: for signals from scintillators, Čerenkov, and lead-glass detectors.

The VME systems provide the scalers and event-driven readout of the beam position and raster current. The custom-made module trigger supervisors (TS) are used to synchronize the readout crates and handle the dead-time logic of the system, and to prescale various trigger inputs.

The main custom components of the CODA are:

- a readout controller (ROC) which runs on the front-end crates to facilitate the communication between CODA and detectors.
- an event builder (EB) which collects all the ROC data fragments, and incorporates all the necessary CODA header information needed to describe and label an event and the data fragments to build the event.
- an event recorder (ER) to write the data (built by event builder) to disk.
- an event transfer (ET) system which allows distributed access to the data stream from user processes and inserts additional data into the data stream every a few seconds from the control system, scalers or some text information.
- a graphical user interface (Run-Control) to set experimental configuration, control runs, and monitor CODA components.

There are several types of data in the data stream. The EPICS[41] status events from the slow control system are included into each data run files to record the beam line, target and spectrometer status. The scalers are read from the trigger supervisor (TS) by the DAQ every a few seconds and incorporated into the data stream. Most of the events in the data stream are physics events which contain information from two

spectrometers. For this experiment, the average size of a typical coincidence event is 1.4 Kilobytes. In total, about 5 Terabytes data was collected during the experiment.

2.3.7 Analysis Software

In both the online and offline Hall A data analysis, the raw data taken by the DAQ are usually analyzed by the event analyzer ESPACE (Event Scanning Program for hall A Collaboration Experiments)[39] which decodes and analyzes the data and writes out data summary and HBOOK[42] files containing ntuples and histograms. The HBOOK files can be analyzed by PAW[43] for the diagnosis of the detector status or extraction of physics results. In this experiment data analysis, the HBOOK files created by ESPACE were also converted to ROOT[44] files for further cross section and other physical analysis with the C++ based ROOT.

The ESPACE is the Hall A general purpose analysis software written in FORTRAN 77. It was designed to analyze the data from the two Hall A High Resolution Spectrometers (HRSe and HRSh) whose detectors consist of 2 or 3 scintillator planes, vertical drift chambers, gas and Aerogel Čerenkov, shower counters and focal plane polarimeter. Since most of the Hall A experiments have been using it as the event analyzer for the past several years, it is relatively well debugged although it still has a rigid structure and is difficult to maintain. The histogram filling, graphics and macro processing functions are implemented using standard subroutines from CERNLIB.

ESPACE has the following main features:

- It has a CODA event decoder, tracking and reconstruction routines, the COOLHANDS[45] histogramming package, and KUIP[46] user interface for analysis program control.
- It can also be used for the calibration of the detectors' gain and timing constants ("optimizations").
- Conditional tests (cuts) can be defined and applied for the selection of event

data to output interested variables from raw detector signals (e.g. TDCs, ADCs and helicities) to basic physical quantities such as the coordinates of the reaction vertex and kinematic variables (e.g. four momentum transfer Q^2 and invariant mass W in the reaction $(e, e'X)$).

2.3.8 HRS Optics Optimization

The JLab Hall A HRS's have the focusing properties which are point-to-point in the dispersive direction. Using the optics tensor elements, the coordinates $(y_{tg}, \delta, \theta_{tg}, \phi_{tg})$ of the interaction vertex can be reconstructed from the coordinates $(x_{fp}, y_{fp}, \theta_{fp}, \phi_{fp})$ measured at the focal plane. A detailed description of the various Hall A coordinate systems can be found in the ESPACE manual[39]. The quality of such transformation is of vital importance to the success of the spectrometer's small acceptance analysis.

The optics data taken with 4, 10 and 15 cm dummy cells and carbon were used for the optimization of the y_{tg} for both the E-Arm and H-Arm. The sieve slit data taken with the liquid hydrogen target were used for the optimization of the E-Arm θ_{tg} and ϕ_{tg} . The Hall A event analyzer ESPACE was modified to write out the VDC coordinates reconstructed from VDC measurements in the detector coordinate system as N-tuples on an event-by-event basis. Such N-tuple events were uniformly selected in the 4-dimensional acceptance region defined by the distribution of the events' target coordinate $(y_{tg}, \theta_{tg}, \theta_{tg}, \phi_{tg})$ to avoid potential bias in the optics optimization. A C++ optics optimization software written by Nilanga Liyanage[47] was used for this experiment optics optimization with the modification to include the nonzero beam position effect.

The following is a summary of the optics optimization process.

First the coordinates of each event in the detector coordinate system at focal plane were reconstructed by the VDC tracking subroutine of ESPACE. These detector coordinates $(x, y, \theta, \phi)_{det}$ were corrected with the detector offsets from the central ray of the spectrometer and transformed to the focal plane coordinates $(x, y, \theta, \phi)_{fp}$. The

focal plane coordinates were then transformed to target coordinates $(\delta, y, \theta, \phi)_{tg}$ using the optics tensor elements.

A fifth order expansion ($j + k + l = 5$) of the focal plane coordinates for the target variables was used as the following:

$$\begin{aligned} y_{tg} &= \sum_{jkl} Y_{jkl} \theta_{fp}^j y_{fp}^k \phi_{fp}^l & \theta_{tg} &= \sum_{jkl} T_{jkl} \theta_{fp}^j y_{fp}^k \phi_{fp}^l \\ \phi_{tg} &= \sum_{jkl} P_{jkl} \theta_{fp}^j y_{fp}^k \phi_{fp}^l & \delta &= \sum_{jkl} D_{jkl} \theta_{fp}^j y_{fp}^k \phi_{fp}^l \end{aligned}$$

where the tensors Y_{jkl} , T_{jkl} , P_{jkl} and D_{jkl} are polynomials in x_{fp} :

$$\begin{aligned} Y_{jkl} &= \sum_i Y_{jkl,i} x_{fp}^i & T_{jkl} &= \sum_i T_{jkl,i} x_{fp}^i \\ P_{jkl} &= \sum_i P_{jkl,i} x_{fp}^i & D_{jkl} &= \sum_i D_{jkl,i} x_{fp}^i \end{aligned}$$

The goal of the optics optimization is to search for the optimal tensor elements $Y_{jkl,i}$, $T_{jkl,i}$, $P_{jkl,i}$ and $D_{jkl,i}$ using the known $(\delta, y, \theta, \phi)$ from the surveys and the VDC measured $(x, y, \theta, \phi)_{fp}$. Three variables were calculated from the target variables:

$$z_{react} = \frac{-(y_{tg} - z_{off} \sin \theta_0)}{\cos \theta_0 \phi_{tg} + \sin \theta_0} + x_{beam} \frac{1 - \phi_{tg} \tan \theta_0}{\phi_{tg} + \tan \theta_0} \quad (2.5)$$

$$x_{siev} = (\theta_{tg} + 0.6 x_{tg}) Z_0 + x_{tg} \quad (2.6)$$

$$y_{siev} = \phi_{tg} Z_0 + y_{tg} \quad (2.7)$$

where θ_0 is the spectrometer's angle in the Hall A coordinate system, which is the angle between the electron beam direction and the spectrometer's central line, Z_0 is the distance from the hall center, defined by the intersection of the electron beam and the vertical symmetry axis of the target system, to the central sieve slit hole at the entrance of the spectrometer, z_{off} the distance from the hall center to the crossing point of the spectral central ray and the beam line, and x_{tg} is the vertical coordinate which is calculated from θ_{tg} , z_{react} and the vertical mis-pointing of the spectrometer

from its ideal position. The z_{react} is reaction vertex position along the beam line, and the x_{siev} and y_{siev} are the vertical and horizontal positions respectively at the sieve plane. The x_{beam} is the beam's horizontal position.

The optics tensor elements are obtained by minimizing the following χ^2 functions:

$$\Delta(z_{react}) = \sum_i (z_{react,i} - z_{react}^0)^2 \quad (2.8)$$

$$\Delta(siev) = \sum_i [(x_{siev,i} - x_{siev}^0)^2 + (y_{siev,i} - y_{siev}^0)^2] \quad (2.9)$$

$$\Delta = \sum_i [(y_{tg,i} - y_{tg}^0)^2 + (\phi_{tg,i} - \phi_{tg}^0)^2 + (\theta_{tg,i} - \theta_{tg}^0)^2] \quad (2.10)$$

where z_{react}^0 , x_{siev}^0 , y_{siev}^0 , y_{tg}^0 , ϕ_{tg}^0 and θ_{tg}^0 are calculated “nominal” values from the survey results.

The Fig. 2-17 shows the reconstructed z_{react} distribution compared with the nominal positions (solid vertical lines). The Fig. 2-18 shows the reconstructed x_{siev} vs y_{siev} compared with the nominal sieve slit hole positions (crosses). The two large sieve holes, which are used to determine the sieve orientation, are clearly shown. As shown both Fig. 2-17, 2-18, the optics reconstruction at the acceptance edge region is not as good as in the center acceptance region. For the precise acceptance evaluation in the absolute cross section analysis, the acceptance edge events were excluded using the R-Function cut which is discussed in detail in next chapter.

Figure 2-17: E91-011 optics check (E-Arm z_{react} in meter)

Figure 2-18: E91-011 optics check (E-Arm x_{siev} vs y_{siev} in meter)

2.4 Experiment Kinematics

Experiment Kinematic Settings

This experiment measures both the cross section and the recoil polarization of the $N \rightarrow \Delta$ transition in the $p(\vec{e}, e'\vec{p})\pi^0$ reaction. It was designed to extract 14 out of the total 18 response functions using the out-of-plane angle ϕ_π dependence of the experimental cross section $\sigma(W, Q^2, \theta_\pi, \phi_\pi)$ and recoil polarization $P(W, Q^2, \theta_\pi, \phi_\pi)$. The L and T type response functions are not separated because the data was taken at only one beam energy point. The physics goal of this experiment is to extract various multipoles with partial wave analysis using the polar angle θ_π dependence of the response functions $R(W, Q^2, \theta_\pi)$. Here W is the invariant mass of the hadronic system and Q^2 is the space-like four-momentum transfer. The measurement of recoil polarization observables has the advantage of access to interferences between the small amplitudes and dominant amplitude M_{1+} . Those small amplitudes include resonant amplitudes E_{1+} and S_{1+} , non-resonant background amplitudes E_{0+} and S_{0+} , etc.

The electron beam was set to nominal energy $E_{beam}=4537\text{MeV}$ with electron polarization around 70% and current varying from 20 to 100 μA . The Electron Arm was used to detect the scattering electrons with fixed spectrometer angle $\theta_E=14.05^\circ$ and central momentum $p_E=3667.6\text{MeV}/c$ to set the nominal $Q^2=1.0\text{GeV}^2/c^2$ and $W=1232\text{MeV}/c^2$ in $p(\vec{e}, e'\vec{p})\pi^0$ reaction. The Hadron Arm was used to detect the recoil protons and measure the recoil polarizations at the focal plane. Twelve complementary kinematic settings were selected to maximize the acceptance coverage in the (θ_p, ϕ_p) plane at the nominal $Q^2=1.0\text{GeV}^2/c^2$ and $W=1232\text{GeV}/c^2$. Table 2.4 shows the momentum and angle settings of the Hadron Arm for these 12 kinematics.

Experimental Acceptance Coverage

The kinematic acceptance coverage in (W, Q^2) plane is shown in Fig. 2-19. The limited coverage in (W, Q^2) plane allows the binning of the experiment data for the

Kinematics	polar angle θ_{pq} ($^{\circ}$)	azimuthal angle ϕ_{pq} ($^{\circ}$)	H-Arm angle θ_H ($^{\circ}$)	H-Arm momentum P_H (GeV/c)
thcm0	0	-	42.3	1.378
thcm25	25	0	38.1	1.350
thcm-25	25	180	46.3	1.350
thcm50	50	0	34.3	1.270
thcm-50	50	180	50.2	1.270
thcm90	90	0	29.8	1.066
thcm-90	90	180	54.8	1.066
thcm135	135	0	30.8	0.819
thcm-135	135	180	53.6	0.819
thcm155	155	0	34.7	0.742
thcm-155	155	180	49.7	0.742
thcm180	180	-	42.3	0.703

Table 2.4: E91-011 Kinematic Settings

study of W and Q^2 dependence of the multipoles. The kinematic acceptance coverage in (θ, ϕ) plane is shown in Fig. 2-20. The event concentration in the $\phi = 0^{\circ}$ and $\phi = 180$ region is necessary for the left/right type response functions (R_{LT}, R_{LT}^n , etc.) separation. The nearly full out-of-plane angle coverage in the low θ_p region and limited coverage in other regions are utilized for the separation of out-of-plane type response functions, such as R_{TT} , R'_{LT} , etc.

Figure 2-19: E91-011 kinematic acceptance in (W, Q^2) plane

Figure 2-20: E91-011 kinematic acceptance in (θ, ϕ) plane

Chapter 3

Data Analysis

3.1 Analysis Overview

The data analysis of the E91-011 experiment consists of two main parts. The first part is the absolute cross section analysis, which is discussed in detail in the following sections of this chapter. The second part is the recoil proton's polarization analysis, the detail of which can be found in R. Roche's thesis[2]. Both the cross section and polarization results are needed for the extraction of the multipole amplitudes with partial wave analysis.

There are several important problems in the E91-011 experiment that need to be carefully handled in the cross section analysis:

- High accidental event rate. For some kinematic settings, the signal to noise ratio is less than 10% according to the coincidence timing spectrum.
- Significant DAQ electronic dead time. Because the singles rate was of the order of a MHz at the focal plane, the experimental data has to be corrected for the electronic dead time in the cross section analysis.
- High fraction of multiple track events in both spectrometers due to high singles rate.

- Precise analysis of the small spectrometer acceptance for the absolute cross section measurement.

In this cross section analysis, Monte Carlo simulation was extensively and effectively used to simplify the extraction of radiation-corrected and acceptance-unfolded differential cross section in the center of mass frame for available $(W, Q^2, \theta_\pi, \phi_\pi)$ points. The simulation, data selection and efficiency correction are performed for each of the 12 kinematics settings. Because there are overlapped kinematic regions among different settings, both the simulated data and corrected experimental data of different settings are combined for kinematic binning in the $(W, Q^2, \theta_\pi, \phi_\pi)$ space for cross section analysis.

First, the detail of the R-Function acceptance analysis method is described. The Monte Carlo simulation for the cross section analysis, the experimental data selection and efficiency correction are presented following the discussion of R-Function method. Then cross section extraction, unpolarized response function separation and experimental cross section parametrization are described. The recoil polarization and polarized response function analysis with maximum likelihood method, and multipole amplitude extraction with partial wave analysis are summarized at the end of this chapter.

3.2 Acceptance Analysis with R-Function

The R-Function Method has been used to solve boundary value problems with various boundary conditions on geometrical domains[48, 49]. In this data analysis, the basic idea of the R-Function was adopted in designing a data-boundary-defined R-Function for effective and precise acceptance analysis in the absolute cross section measurement.

3.2.1 General R-Function Definition

Figure 3-1: R-Function Definition in xy plane

In this analysis, the R-Function is defined as the minimum distance from a test point to the boundary of a polygon in a plane. The R-Function is < 0 if the test point is outside of the polygon and > 0 if inside of the polygon.

Suppose a polygon is defined by n boundary points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ in xy plane as shown in Fig. 3-1.

The polygon can be formed in arbitrary shape by connecting n boundary points with the requirement that there is no intersection between any two of these boundary

lines. As shown in Fig. 3-1, for the inside point (x, y) , the minimum distance from it to the boundary lines is the length of the dotted line perpendicular to the boundary line connecting two points (x_2, y_2) and (x_3, y_3) . This distance is defined as the R-Function at (x, y) ($R(x, y)$) with respect to the polygon boundary and $R(x, y) > 0$ since the point (x, y) is an inside point. The R-Function at (x', y') $R(x', y') < 0$ since the point (x', y') is outside of the polygon.

Suppose $d_{xy,ij}$ is the distance from a point (x, y) to the boundary line connecting two points (x_i, y_i) and (x_j, y_j) (the length of the line is denoted as l_{ij}). Three distances are calculated to determine $d_{xy,ij}$: the perpendicular distance d_{ij} from point (x, y) to the line connecting points (x_i, y_i) and (x_j, y_j) , the distance d_i from point (x, y) to point (x_i, y_i) and the distance d_j from point (x, y) to point (x_j, y_j) . If $l_{ij}^2 + d_i^2 - d_j^2 > 0$ and $l_{ij}^2 + d_j^2 - d_i^2 > 0$, $d_{xy,ij} = d_{ij}$. If $l_{ij}^2 + d_i^2 - d_j^2 < 0$ or $l_{ij}^2 + d_j^2 - d_i^2 < 0$, $d_{xy,ij} = \min(d_i, d_j)$.

The minimum of the n distances from the point (x, y) to n boundary lines is defined as the absolute value of the R-Function at point (x, y) with respect to the polygon boundary defined by n points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$.

The following algorithm is designed to determine whether a test point (x, y) is inside or outside of the polygon.

Suppose the polygon boundary points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ are arranged counterclockwise in xy plane. Define the angle of the point (x, y) with respect to the boundary line connecting points (x_i, y_i) and (x_j, y_j) as the angle needed to counterclockwise rotate the line connecting points (x, y) and (x_i, y_i) to be collinear with the line connecting points (x, y) and (x_j, y_j) . If such rotation angle is $> \pi$, it is subtracted by 2π . The sum of n angles of the point (x, y) with respect to the n boundary lines is equal to 2π if the point (x, y) is inside of the polygon and 0 if outside of the polygon. Then a minus sign is assigned to the R-Function for an outside point and a plus sign for an inside point.

Thus for any point (x, y) in a xy plane where a polygon is defined by n boundary points, a R-Function $R(x, y)$ can be constructed with its absolute value represent-

ing the minimum distance from point (x, y) to the polygon boundary and the sign indicating whether it is inside or outside of the polygon.

Because the R-Function is the minimum distance from a point to a reference polygon boundary, the description of the inside or outside area of the polygon proportionally transformed from the reference polygon in a xy plane can be simplified using the R-Function cut (R cut). As shown in the descriptive Fig. 3-2 in which the middle polygon is the reference polygon, all the points in the central shaded area are the points satisfying $R(x, y) > 0.05$ (here the length unit is arbitrary). The shaded band area are all the points satisfying $-0.1 < R(x, y) < 0$.

Figure 3-2: Areas selected by R Cuts in xy plane

3.2.2 HRS Acceptance Analysis with R-Function

Using the VDC measurement, the focal plane coordinates $(x, y, \theta, \phi)_{fp}$ of each event are obtained from the ESPACE tracking analysis. The target coordinates $(\delta, y, \theta, \phi)_{tg}$ of that event are reconstructed from the focal plane coordinates using

a transformation with the optics tensor elements. The quality of the optics elements at the HRS acceptance edge is not as good as that in the central acceptance area. However a well-defined HRS acceptance boundary is critical for accurate evaluation of the acceptance integration in phase space (which is also the effective acceptance) in the absolute cross section measurement. Since we must apply acceptance cuts on these target variables or the kinematic variables constructed from the target variables, the HRS optics has to be optimized as well as possible. In practice, the HRS optics database can never be optimized to be perfect, thus it is important to devise an appropriate acceptance analysis method to reduce the effect of imperfect optics in the acceptance edge region on the precision of the absolute acceptance evaluation required by the experiment measurement. The events at the acceptance edge may be excluded to suppress the effect of the imperfect optics. The R-Function designed in the previous section is an excellent choice for such event selection since it provides a convenient single cut variable to control the area of a polygon in a plane. When the reference polygon boundary is defined from the experimental data distribution, applying different R cuts can remove a different number of events at the acceptance edge.

For 4 target coordinates $(\delta, y, \theta, \phi)_{tg}$ from one spectrometer measurement, there are 6 different combinations: (δ, y_{tg}) , (δ, θ_{tg}) , (δ, ϕ_{tg}) , (y_{tg}, θ_{tg}) , (y_{tg}, ϕ_{tg}) and (θ_{tg}, ϕ_{tg}) .

The R-Function is defined in each of these 6 planes with the polygon boundary points defined from the distribution of the events' target variables. The minimum of the 6 R-Functions for one arm is defined as that spectrometer's R-Functions. For coincidence measurement, the minimum of the H-Arm and E-Arm R-Functions is defined as the R-Function of the coincidence event. Thus a single R-Function cut is constructed to control the acceptance cuts in 12 different planes in the 8-dimensional acceptance space expanded by 8 independent target variables.

Since the target coordinates have different units, all the coordinates have to be appropriately scaled before the calculation of the R-Functions. Two different scalings

are applied:

- Coordinates scaling:

$$x' = \frac{x}{x_{max} - x_{min}} \quad (3.1)$$

$$y' = \frac{y}{y_{max} - y_{min}} \quad (3.2)$$

where x_{max} , x_{min} , y_{max} and y_{min} are the respective maxima and minima of the x and y coordinates of all the boundary points in xy plane.

- R-Function scaling:

$$R' = \frac{R}{\sqrt{A}} \quad (3.3)$$

where A is the inside area of the polygon defined by all the boundary points in a plane.

For experiment E91-011, the coincidence data was used to define the acceptance polygon boundary. The R cut defined by the coincidence data has the advantage of suppressing the accidental background using the kinematic correlation between the tracks in the two spectrometers. The phase space enclosed by the coincidence-data-defined-boundaries is also slightly closer in volume to actual phase space than the phase space enclosed by the singles-data-defined-boundaries. Since the background level and the effective acceptance varied for different kinematic settings, each of the 12 kinematic setting's data was used to define its own set of 12 polygon boundaries. Two typical data-defined polygon boundaries in the (θ_{tg}, ϕ_{tg}) and (δ, y_{tg}) planes are shown in Fig. 3-3. In the above plots, the lines connecting the boundary points (asterisks) are used as the boundary lines for the initial acceptance cut.

For each coincidence event selected from the raw data, the R-Function is calculated with respect to the 12 polygon boundaries defined by data of that kinematic setting. Monte Carlo simulation with MCEEP[50] and EPIPROD[51] was conducted for each

Figure 3-3: Data-defined boundary examples for R cut in spectrometer acceptance analysis

of the 12 kinematics settings. For the $p(e, e'p)\pi^0$ reaction in this experiment, MAID[52] was used as the theory model in the simulation. The accidental background in the data was also subtracted using the coincidence timing spectra. The boundaries defined by data were also used in MCEEP to calculate the R-Function for each simulated $p(e, e'p)\pi^0$ coincidence event. The same R cut was applied to both the data and simulated events to select the common effective phase acceptance volume for the experimental data and simulated data. The detail of event selection and simulation is described in the following sections.

Various normalized yield distribution comparisons between the data and the simulation are shown in Fig. 3-4 and Fig. 3-5. In Fig. 3-4, ϕ_p and θ_{pq} are the proton's azimuthal and polar angles in the center of mass frame respectively, $Miss_m$ is the missing mass, the neutral pion mass in $p(e, e'p)\pi^0$ reaction. In Fig. 3-5, the top 4 plots are the electron arm's y_{tg} , θ_{tg} , ϕ_{tg} and δ distributions, and the bottom 4 plots are the hadron arm's target coordinates distributions. These comparisons are for the kinematic setting *thcm0* (for definition of the kinematic settings, see Table 2.4 on page 76).

As shown in the bottom-left plot of the Fig. 3-4, the normalized coincidence R-Functions (R_{eh}) of the data and simulation have the same distribution when $R_{eh} > 0.05$. In Fig. 3-6, the yield ratio of the data and the simulation versus R cut becomes flat when $R = 0.05$. These two plots demonstrate that the cut $R > 0.05$ can select the same effective acceptance volume for the data and simulation. Both plots can be used to select the appropriate R-Function acceptance cut. Although much more analysis work is required to generate the second plot than that for the first plot, one can also estimate the relative acceptance error using the yield ratio variation in the flat region. As shown in Fig. 3-4 and Fig. 3-5, the simulated target and kinematic variables' distributions agree very well with that of the experimental data. Because the th_{tg} , which is θ_{tg} , the track's out-of-horizontal plane angle in spectrometer coordinate frame, is usually the worst reconstructed target variable, the

small deviations in the th_{tg} distributions are more likely caused by imperfect optics other than small difference between the experimental cross section and the theoretical predictions at this kinematic setting.

In general, the good agreement between these normalized data and simulated distributions demonstrates the high precision of the acceptance analysis results. Such precise acceptance analysis has not been achieved using other known Hall-A acceptance analysis methods, such as default spectrometer aperture tests with Lero's forward function map[53] and data defined aperture tests with the same forward function map. However, the data-defined R-Function cut should not be misused as the physical aperture cuts of the spectrometer because of the way R-Function is defined. All the data-defined boundaries in 6 planes are the 2-dimensional projections of the event distribution in the 4-dimensional space. The volume of the object in the 4-dimensional space enclosed by the 6 boundaries is slightly greater than that of the event distribution from which these boundaries are defined. This can be easily understood if one takes a polygon in a 2-dimensional plane as an example. One can project the polygon distribution to two orthogonal axes to get two 1-dimensional cuts. The two 1-dimensional cuts define a rectangle with an area greater than the original polygon in the plane. However, one can tighten the two 1-dimensional cuts to shrink its size until it is completely enclosed in the original polygon. Similar reasoning can be applied to the R-Function acceptance cut. In the acceptance analysis, the most important thing is to have the same effective acceptance in the data and simulation after using some acceptance cut, since the effective acceptance is evaluated in the simulation. Although the acceptance volume of the event distribution defined by the initial 2-dimensional boundaries of the data is greater than the actual volume of the data distribution, which can be seen from both the bottom left plot in Fig. 3-4 and the yield ratio curve in Fig. 3-6, one can always increase the R cut to select a common region in which the data and the simulated event distributions have the same effective acceptance volume.

Figure 3-4: Normalized kinematic variables distribution comparison between the data(solid lines) and simulation(dotted lines) for kinematics thcm0

Figure 3-5: Normalized target variables distribution comparison between the data(solid lines) and simulation(dotted lines) for kinematics thcm0

Figure 3-6: Yield ratio of the simulation and data versus different R Cuts for kinematics thcm0

Because the data-defined initial boundaries in those 2-dimensional planes cannot completely determine the spectrometer's physical apertures, one should try to define the initial data boundary in such way that the resulted volume of the simulated events is as close to the effective volume of the data events as possible. Several ways can be used to improve the precision of the acceptance approximation of the initial data-defined boundaries. One way is to project the data's event distribution to more planes. The more boundary planes are used, the smaller R cut can be used to get the common acceptance region. This simply involves more manual work. The other way is to use the kinematic correlation between the two arms for coincidence measurement to define the initial boundaries. The second way was used in the acceptance analysis of this work. The coincidence data boundaries not only can define an acceptance region in simulation closer to that in the data, but also can help suppress the accidental backgrounds with the coincidence event's kinematic correlations. As one can see from those comparison plots, the acceptance analysis with the data-defined R-Function cut

also has the disadvantage that certain number of the data events have to be excluded to select a common acceptance region. For those experiments with critical statistic requirement, using more data-defined boundary planes to define the R cut can be an effective solution for keeping more data events in the cross section analysis.

3.3 Monte Carlo Simulation

The Monte Carlo simulation was extensively used in the cross section analysis. Its main purpose is to evaluate the effective acceptance in the phase space for each $(W, Q^2, \theta_\pi, \phi_\pi)$ bin. The second use of the simulation is to apply the radiative correction. Using the MAID model[52] in the simulation for the calculation of the reference cross section, the radiation-corrected experimental cross section in the center of mass hadronic system corresponding to the point-acceptance can be conveniently extracted from the yields ratio of the data and simulation for a specific $(W, Q^2, \theta_\pi, \phi_\pi)$ point. The effective use of the Monte Carlo simulation simplified the acceptance unfolding and radiative correction in the absolute cross section analysis.

3.3.1 MCEEP

The simulation software MCEEP (**M**onte **C**arlo for **(e, e'p)**)[50] was designed to fold theoretical models over the appropriate experimental acceptance. It is different from the traditional importance sampling technique by using a uniform random sampling method to populate the experimental acceptance.

In MCEEP, an event is defined as any combination of variables which completely specifies the reaction in the laboratory. For each event, weights corresponding to the relevant observables such as cross sections are calculated. Such weights can be accumulated and inspected from the output histograms. For the simulation of our coincidence experiment, MAID cross section for $p(e, e'p)\pi^0$ was attached to each uniformly sampled event and summed to get the simulation yields in a certain experimental acceptance region for comparison with the measured yields. The reaction vertex can be randomly selected from a general 3-dimensional beam-target interaction region so that the effects of extended targets and rastered beams are effectively evaluated.

For the $p(e, e'p)\pi^0$ reaction which is a bound state case, MCEEP performs a 5-dimensional integral, in which the proton momentum is calculated from the bound

state missing mass (π^0 mass) and the values of the other five randomly selected kinematic variables (the electron momentum, in-plane and out-plane angles and the proton in-plane and out-plane angles). MCEEP simulation includes radiative effects and realistic multiple scattering.

The simulation of the spectrometer systems is achieved by using a series of user-defined spectrometer elements which act on the transport[38] vector of each event. The spectrometer acceptance is modeled with the data-defined aperture boundaries. Resolution effects are simulated by smearing the transport vector with a Gaussian resolution function.

The Ntuple output (for both kinematics variables and transport variables weighted by theoretical cross section) of the MCEEP simulation is converted by an external program (mceep_util) to a PAW-compatible HBOOK file. The HBOOK file is then converted to a ROOT file for the cross section analysis.

The MCEEP yield for $p(e, e'p)\pi^0$ is determined from:

$$Y_i = L\Delta t \int_{\Delta v_i} \frac{d^5\sigma^{theory}}{dk_e d\Omega_e d\Omega_p} dk_e d\Omega_e d\Omega_p \quad (3.4)$$

where L is the average luminosity and Δt is the counting time. The Δv_i is multi-dimensional phase space consistent with the experimental acceptance volume. The k_e is the final electron momentum, $d\Omega_e$ and $d\Omega_p$ are the electron and proton solid angles respectively. The $\frac{d^5\sigma^{theory}}{dk_e d\Omega_e d\Omega_p}$ is the theoretical 5-fold differential cross section in the laboratory coordinate system. The subscript i indicates the integral extends only over the volume of a given kinematic bin ($W, Q^2, \theta_\pi, \phi_\pi$).

The average luminosity is obtained from:

$$L = \frac{1}{\Delta t} \int \frac{I\rho(I)dN_A}{eA} dt \quad (3.5)$$

where the I is the beam current, $\rho(I)$ is target density, d the target thickness, A the atomic weight, e the electron charge and N_A the Avogadro constant.

The simulated yield including radiative effects is:

$$Y_i = L\Delta t \int_{\Delta v_i} \frac{d^5\sigma^{theory}}{dk_e d\Omega_e d\Omega_p} \otimes f_R dk_e d\Omega_e d\Omega_p \quad (3.6)$$

where $\frac{d^5\sigma}{dk_e d\Omega_e d\Omega_p} \otimes f_R$ is the convolution of the theoretical cross section with the radiative tail f_R .

3.3.2 EIPROD

EIPROD[51] is a program which calculates various quantities for the electro-production of pseudo-scalar mesons from nucleons in which the polarization of the recoil nucleon is detected. In this analysis, EIPROD reads the multipole amplitudes using the sophisticated dynamical model MAID[52] (a unitary isobar model for pion photo- and electro-production on the nucleon) and then calculates the unpolarized 2-fold differential cross sections in the center of hadronic mass system corresponding to a specific kinematic point $(W, Q^2, \theta_\pi, \phi_\pi)$ with a specific electron beam energy. This 2-fold differential cross section was used as the point acceptance reference cross section to extract the experimental cross section in the center of mass of the hadronic system. EIPROD was also used as the event generator in the MCEEP simulation by calculating the laboratory cross section from MAID multipole amplitudes for each uniformly sampled event.

3.3.3 Radiative Effects

For the $(e, e'p)$ reaction, the charged particles can experience other interactions in addition to the primary reaction. These interactions involve the radiation of real and virtual photons, energy loss and momentum change. They can significantly change the cross sections and must be corrected to extract the radiation-corrected cross sections for the comparison with theoretical calculations.

These effects can be classified into two types:

- Internal effects

The interaction occurs on the same atom as for the primary reaction. The electron radiates real and virtual photons in the presence of the Coulomb field of the target nucleus of the primary $(e, e'p)$ reaction. This is also called internal bremsstrahlung radiation. The soft photon(real photons with the energy below the photon cutoff energy) and virtual photon contribute the “Schwinger” correction to the cross section. The hard photon (real photon with energy above the cutoff energy) emission leads to the radiative tail.

- Schwinger Correction

In MCEEP, the Schwinger correction is given as[54]:

$$C_{Schwinn} = e^{-\delta_r}(1 - \delta_v) \quad (3.7)$$

where δ_r is the soft photon contribution and δ_v is the virtual photon contribution. For the inelastic scattering of the pion electro-production process, they are given as:

$$\delta_r = \frac{\alpha}{\pi} \left(\ln \frac{Q^2}{m_e^2} - 1 \right) \ln \left[\frac{k}{\xi^2} \frac{ee'}{\Delta^2} \right] \quad (3.8)$$

$$\delta_v = \frac{\alpha}{\pi} \left[\frac{28}{9} - \frac{13}{6} \ln \frac{Q^2}{m_e^2} + \frac{1}{2} \ln^2 \frac{e}{e'} + \frac{\pi^2}{6} - L_2(\cos^2 \frac{\theta_e}{2}) \right] \quad (3.9)$$

where

$$\xi = 1 + \frac{e}{M}(1 - \cos \theta_e) \quad (3.10)$$

$$k = 1 + \frac{\omega}{M}(1 - \cos \theta_e) \quad (3.11)$$

$L_2(x)$ is the Spence function defined by

$$L_2(x) = - \int_0^x \frac{\ln(1-y)}{y} dy \quad (3.12)$$

and Δ is the photon cutoff energy. The Schwinger correction is applied to the cross section as an overall multiplicative factor where the kinematics are evaluated without radiation.

– Radiative Tail

The radiation of a hard photon can affect the kinematics of the scattering event thus changing the yield distribution with respect to the missing mass. The peaking approximation is used, where the photon is assumed to be radiated either along the incident or scattered electron directions. Since the $p(e, e'p)\pi^0$ is a bound state case, the prescription of Borie and Drechsel[55] is used for the radiative tail in peaking approximation for MCEEP simulation. It gives the 6-fold cross section including photon emission in terms of the 5-fold differential radiation free cross sections:

$$\frac{d^6\sigma}{dk_\gamma dk_e d\Omega_e d\Omega_p} = \frac{\alpha}{\pi k_\gamma} \left[\frac{(e' + k_\gamma)^2 + e'^2}{(e' + k_\gamma)^2} \frac{d^5\sigma(e, e' + k_\gamma)}{dk_e d\Omega_e d\Omega_p} \ln \frac{2e'}{m_e} \right. \\ \left. \frac{e^2 + (e - k_\gamma)^2}{e^2} \frac{d^5\sigma(e - k_\gamma, e')}{dk_e d\Omega_e d\Omega_p} \ln \frac{2e}{m_e} \right] \quad (3.13)$$

where the first term corresponds to radiation after the primary scattering and the second corresponds to the radiation before the primary scattering.

– Multi-Photon Correction

The full multi-photon contribution to the radiative tail in MCEEP is given by:

$$f_{mp} = (1 - \delta_v) \frac{(1 - e^{-\delta_r})}{\delta_r} \quad (3.14)$$

where δ_r and δ_v are the same as defined in the Schwinger correction. This factor is applied to the tail cross section for each event. The kinematics used to calculate δ_r and δ_v are the vertex kinematics.

• External effects

The mean hadron and electron energy losses by collisions in the target are calculated from the Bethe-Bloch formula[56]. One of the three distributions (Landau, Vavilov and Gaussian) is chosen to calculate energy loss fluctuation according to the ratio between the mean energy loss and the maximum energy transfer possible in a single collision. Multiple scattering due to Coulomb scattering from nuclei is well represented by the Moliere theory[57]. For small deflection angle the Coulomb scattering is approximated by a Gaussian distribution. The external Bremsstrahlung, in which the charged particles lose energy by interaction with the Coulomb field of the nuclei of the traversed medium, is also accounted for in MCEEP simulation.

3.3.4 Experiment Specification

When running the MCEEP simulation, the bound state reaction with radiation and multi-photon process was selected for $p(e, e'p)\pi^0$ simulation. The EPIPROD event generator was used in the simulation.

The experimental condition for each kinematic setting was specified in the MCEEP input file. The following is a summary of these specifications:

- Masses

The proton's mass was set as the ejectile mass and the neutral pion mass was set as the missing mass.

- Kinematics

Each of the 12 kinematic settings needs a separate input file to specify the different experimental conditions. The electron beam energy (4.5306 GeV) was used for all the 12 kinematics settings. The E-Arm central momentum and angle were used as the momentum and the in-plane angle of the scattered electron. The H-Arm central momentum and angle were used as the momentum and the in-plane angle of the recoil protons. The recoil proton's momentum and angles

are different for different kinematic setting.

- Acceptance

The MCEEP input file also specifies the nominal spectrometer acceptance in which the event is uniformly sampled. The momentum and solid angle acceptance of the electron and hadron arm are set to some values slightly greater than their nominal values to allow appropriate acceptance cut in the later analysis.

- Rate parameters

The Luminosity in each kinematics is specified as $\bar{I}d\rho$ (in $\mu\text{A}\cdot\text{g}/\text{cm}^2$). The $\bar{I}$ is the average beam current for all the analyzed runs of one kinematic setting. The d is liquid hydrogen target thickness. The ρ is the Hall A liquid hydrogen target density operating on the regular condition. The total beam time for the analyzed runs was also specified to properly calculate the simulated yield. These parameters vary for different kinematics.

- Target parameters

The Hall A liquid hydrogen target parameters were used and the MCEEP ELOSS_MOD=2 was used to account for the energy loss and multiple scattering in the target but remove the mean energy loss. This allows comparison with data corrected for the mean energy loss.

- Beam parameters

For each choice of kinematics, the center position of a typical run data was used as the beam offset in the input file. The rectangular raster size from the data was also used to specify the corresponding beam spot size in simulation. Thus the reaction region in simulation is a 3-d region approximately the same as the actual beam reaction vertex distribution.

The experimental conditions specified in the simulation are set the same as the actual data taking conditions, so that the ratio between the simulated yield and

the efficiency corrected data yield can be conveniently used for the extraction of the experiment cross section with known theoretical cross section.

3.3.5 Acceptance Model with R-Function

The standard MCEEP was modified to implement the data-defined R-Function acceptance model which is described in previous sections. Besides the nominal spectrometer acceptance specified in the simulation input file, a more realistic spectrometer is necessary for the acceptance analysis of the absolute cross section. In the MCEEP simulation, the data-defined initial boundaries are read when the simulation starts. For each event generated in the nominal spectrometer acceptance region, the R-Function is calculated. Only those events with $R > 0$ are kept for further cross section analysis.

Since the same boundaries are used to calculate the R-Function event by event in both the simulation and the replayed experimental data, the R-Function provides a convenient acceptance cut to select a common acceptance region in the data and the simulation.

As is discussed in previous sections, the R-Function proved to be an effective and precise acceptance analysis method. Before the R-Function cut was used, several other acceptance analysis methods were tried and none of them could provide the high precision acceptance analysis result which is necessary in our absolute cross section measurement. This is because the precise analysis of the spectrometer acceptance needs a well-defined boundary of the particle distributions. The HRS acceptance includes not only geometric acceptance, but also the momentum acceptance. The small geometric acceptance is correlated to the momentum acceptance because the HRS is a magnetic spectrometer in which the particles have to be deflected by the magnetic field to reach the focal plane to be detected. The physical aperture of the spectrometer depends on the spectrometer setting. Other acceptance analysis methods cannot avoid the effects of imperfect optics at the acceptance edge region and do not have

a sufficiently accurate function map to simulate the particle transportation in the spectrometer for high precision absolute cross section measurement. The R-Function method developed in this analysis avoids the use of an inaccurate function map by defining the acceptance region with polygon boundaries on the projected planes of the target coordinates distribution. It also reduces the effects of imperfect optics by excluding the events in the acceptance edge region. Although the R-Function method has the disadvantage of reducing statistics, it still provides much higher precision acceptance analysis results.

3.3.6 Resolution Smearing and Kinematic Adjustment

In principle, the transport coordinates can be smeared with an appropriate Gaussian to simulate the experimental resolution effect in the missing mass distribution. Because the Hadron Arm settings were different for different kinematics, different Gaussian smearing is needed to properly reconstruct the different experimental missing mass width in simulation. After many tests, it was found a simple Gaussian smearing on the VDC track's position coordinates can be used for all the 12 different kinematics and approximately reconstructs the varying missing mass resolution effects.

Besides the resolution smearing in the simulation, the kinematic correction was also made to align the missing mass peak position in the data and the simulation. Since for kinematics thcm155, the missing mass position was more sensitive to the small change in the electron scattering angle than other kinematic settings, an appropriate angle offset was chosen from several tests to align the missing mass peak of the data and simulation. The same offset also correctly aligned the missing mass peaks of the data and simulation for other kinematic settings.

With the resolution smearing in simulation, kinematic correction and R cut in both the data and simulation for each of the 12 kinematic settings, all the target variables and six kinematic variables are compared between the normalized simulation and the

background subtracted data using the timing spectra.

The good agreement in these comparison plots shows the high precision acceptance analysis which is achieved with the effective use of the MCEEP simulation. A typical comparison can be found in Fig. 3-4 on page 88 and Fig. 3-5 on page 89. The missing mass of the reaction $p(e, e'p)\pi^0$ distribution comparisons for all the 12 kinematic settings are shown in Fig. 3-7. In Fig. 3-7, for the data yields, the accidental backgrounds were subtracted using the coincidence timing spectra. Although the resolution smearing for some kinematics is not perfect, the kinematic adjustment works for all the 12 kinematics (for definition of kinematic settings, see Table 2.4 on page 76). The radiative tails in the data are also reproduced in the simulation. These missing plots demonstrate that the radiative effects in simulation are well accounted and there are very little accidental contributions in the background-subtracted data yield distributions.

Figure 3-7: Normalized missing mass distribution comparisons between the data(solid lines) and the simulation(dotted lines). The missing mass unit is MeV/c^2

3.4 Experiment Data Selection

3.4.1 Event Selection

The event selection cuts were carefully designed to suppress the accidental background level. These cuts inevitably also excluded some pion electro-production events which are from the reaction of interest. Thus the selection of the event cuts must allow the accurate evaluation of the efficiency for the interested coincidence events from $p(e, e'p)\pi^0$ reaction.

The event cuts consists of selections at two stages in the analysis process:

- Event selection in ESPACE replay

Because of the high singles rate in each Arm, there is a relatively large number of multiple track(the number of tracks>1) events in both Arm. For such events, the ESPACE reconstructed kinematic variables W , Q^2 , θ_π and ϕ_π may not be from a real coincidence event of the $p(e, e'p)\pi^0$ reaction. There is also the possibility that the ESPACE reconstructs more than one track when there is only one event because of the imperfect tracking algorithm of ESPACE.

Only the single track events (reconstructed by ESPACE) were selected for the cross section analysis. In addition to the one track cut, the number of the hits in each of the 4 VDC planes is required to be in the range of 3-8 to improve the resolution of the missing mass distribution for the convenience of comparison with simulation. A cut on the coincidence time was also used to reduce the file size of selected events.

- Event selection in cross section analysis

The polygon cut on the $gmean - hbeta$ distribution was also used to suppress the π^+ background in the selected data. The $hbeta$ is the proton's speed β in terms of light speed c , as calculated by the time-of-flight between the scintillator S1 and S2 in hadron arm. The calculation of the $gmean$ is as the following: first,

Figure 3-8: Polygon cuts on the gmean-hbeta plane to suppress the π^+ background for kinematics setting them135

if the both the 2 ADCs from the left and right sides of one scintillator paddle are greater than 100 (the unit is ADC channel), the geometric mean of the 2 ADCs, $\sqrt{ADC_l \times ADC_r}$, is calculated, then the maximum of the geometric means from the 6 paddles in one scintillator plane is defined as the gmean of that plane for the event. In cross section analysis, the gmean of the H-Arm S1 plane was used for charged pion suppression. The ADC is proportional to the mean energy loss per unit path length DE/DX of the charged particle passing through the scintillator. For the same momentum, the average DE/DX of the π^+ is less than that of the protons. From the time-of-flight between the two scintillator planes S1 and S2, the particle speed beta (in terms of the light speed c) can also be calculated and named *hbeta*. For the same momentum, the *hbeta* of π^+ is greater than that of the proton. Due to the insufficient resolution, the proton and the charged pion cannot be well separated by using a single beta or gmean cut. However, the protons are well separated from the pions in the plane of *gmean* – *hbeta*. Thus the polygon cut on the *gmean* – *hbeta* plane was used to exclude the charged pions. To avoid excluding the protons, a relatively loose polygon cut was used. For each kinematics setting, various polygon cuts were tried and the resulted background subtracted yields were calculated. The appropriate polygon cut was selected around which the change of the polygon cut does not change the yield of the protons significantly. The typical *gmean* – *hbeta* cut is shown in Fig. 3-8, where the background π^+ level is the highest among all the 12 kinematic settings. In Fig. 3-8, the events enclosed in those polygons are dominated by the charged pions. These pion events show a flat distribution in the coincidence timing spectra because the hadron arm was set for the measurement of protons. Such flat distributions can be utilized for effective suppression of charged pion background without using too tight PID cut.

It is worth pointing out that the kinematic cuts on $(W, Q^2, \theta_\pi, \phi_\pi)$ in the cross section

binning have very strong kinematic constraint which greatly help to select the true coincident events and suppress the accidental background events. In fact, in the Δ resonance region, the *gmean* vs *hbeta* polygon cut can be removed without significant change in the evaluation of the data yield.

One additional cut used in the event selection is the missing mass cut ($> 75 \text{ MeV}/c^2$) to suppress the elastic radiative tail background contributions in kinematic setting thcm-50 and thcm-90. As shown in Fig. 3-7 on page 102, this cut effectively removed the elastic events and didnot cause significant loss of pion production events.

3.4.2 Background Subtraction

After the acceptance cut (*R* Cut), PID cut (*gmean* – *hbeta* polygon cut), and kinematic cut on $(W, Q^2, \theta_\pi, \phi_\pi)$ are applied, there are still some accidental events in the selected event sample from the experimental data. These accidental background yield can be evaluated from the coincidence timing spectra since the accidental events have a flat distribution in the timing spectra.

As shown in Fig. 3-9 and Fig. 3-10, the acceptance *R* cut and kinematic cuts on *W* and Q^2 can exclude a large amount of the accidental events. The Fig. 3-9 is a typical plot showing the cut effects on the timing spectra (for kinematic setting thcm155). In the cross section analysis, the timing cut $208.7 < \text{tc_cor} < 214.7$ was used to calculate the total yield (including the background and real coincidence events) under the coincidence timing peak. The yield of the background contribution under this peak was evaluated from the yield in the side regions $200.7 < \text{tc_cor} < 208.7$ and $214.7 < \text{tc_cor} < 224.7$. Here the *tc_cor* is the ESPACE corrected coincidence time between the two arms. As shown in Fig. 3-10, these cuts and the flat background subtraction worked very well in selecting the real coincidence events from the $p(e, e'p)\pi^0$ reaction. After applying various cuts and the background subtraction, the normalized missing mass distribution of the experimental data agrees very well with the MCEEP simulation in which the MAID and MCEEP standard radiation treat-

ment were used. The flat background subtraction was applied for the data sample in every $(W, Q^2, \theta_\pi, \phi_\pi)$ bin with the R-Function acceptance cut since the background contribution is kinematics dependent.

A typical normalized missing mass distribution comparison between the background subtracted experimental data(blue solid line) and the simulated data(red solid line) can be found in Fig. 3-7. As shown in this figure, not only is the missing mass peak of the simulation aligned with that of the data, but also the radiative tail of the missing mass distribution of the simulation is well reproduced.

Figure 3-9: The timing spectra with various cuts for kinematic setting them155

Figure 3-10: The missing mass spectra with various cuts for kinematic setting them155

3.5 Experiment Data Corrections

The number of the events selected from the previous cuts and flat background subtraction is less than the number of the actual reaction events within the acceptance cut. To measure the absolute cross section, the data yield has to be corrected by various correction factors. The following sections discuss the deadtime, tracking and trigger efficiency, proton absorption and luminosity corrections in the cross section measurement.

3.5.1 Computer Deadtime

Because the data acquisition system needs about $700\ \mu\text{s}$ to process one event and the coincidence event rate was a few hundred Hz, some of the coincidence events cannot be recorded to the data file. Such deadtime is called computer deadtime and the correction factor for coincidence event number can be calculated from:

$$\epsilon_{cdt} = \frac{S_5}{T_5} \quad (3.15)$$

where S_5 is the total number of coincidence event triggers, and T_5 is the number of the events recorded in the data file. The number of the coincidence event trigger is counted by a scaler. The number of the events recorded in the data file can be either counted from an ESPACE reply of the data or analyzed from run-by-run epics files.

3.5.2 Electronic Deadtime

Electronic deadtime is due to the deadtime of various electronic components and the pulse width of the trigger signals. Such deadtime can be around 100 ns. In this experiment, because the event rates at the scintillators planes were in the order of 1 MHz, the electronic deadtime of the order 5 – 10% was expected. It was the first time an electronic deadtime measurement was made in Hall A experiment because it

can not be neglected as before. The detail of the electronic deadtime analysis can be found in Mark Jones' report[58]. A summary of it is given in the following.

The electronic deadtime was measured with the pulser events in which a pulser signal was sent to the linear-or with the S1 paddle 1 and S2 paddle 1 in each arm and to a TDC channel to tag the pulser event. The pulser was also sent to a scaler. The electronic deadtime (EDT) can be measured from the computer deadtime (CDT) and the combination of computer and electronic deadtime (ECDT) as shown in the following:

$$\text{ECDT} = 1 - (1 - \text{EDT})(1 - \text{CDT}) \quad (3.16)$$

$$\text{EDT} = \frac{\text{ECDT} - \text{CDT}}{1 - \text{CDT}} \quad (3.17)$$

The ECDT is determined from:

$$\text{ECDT} = 1 - \frac{\Sigma N_{pulse}}{\Sigma S_{pulse}/PS} \quad (3.18)$$

where the N_{pulse} is the number of the pulser events in the data file and S_{pulse} is the number of the pulser events measured by the scaler.

The CTD is determined from:

$$\text{CDT} = 1 - \frac{\Sigma N_i}{\Sigma S'_i/PS_i} \quad (3.19)$$

where N_i is the number of trigger type i in the data file and S'_i is the corrected trigger type i recorded by the scaler.

Using the data of 26 runs between run 3296 and run 2954, the measured EDT was fitted as a function of the memory lookup strobe rate on each spectrometer. The strobe rate again was fit as a function of the sum of the rates in the right scintillator paddles in all planes in an arm since the strobe rate was not available from some runs

while the sum of the scintillator paddles rates was available for all the runs.

Thus a scheme of using the sum of the rates in the scintillator paddles to calculate the electronic deadtime was established. The systematic error of the EDT was about 1%.

Using the measured EDT (about 5-10%), the electronic deadtime correction factor is define as:

$$\epsilon_{edt} = \frac{1}{1 - \text{EDT}} \quad (3.20)$$

3.5.3 Tracking Efficiency

As mentioned before, only 1-track events the ESPACE reconstructed satisfying the multiple hit cut were selected to evaluate the yield for cross section measurement.

Due to the high singles rate, more than one event can be recorded in the same timing window of the DAQ. For E91-011 coincidence data, 1 – 12% of the H-Arm events have multiple tracks and 10 – 12% of the E-Arm events have multiple tracks, depending on the kinematic settings. The singles rates in the E-Arm varied slightly with different beam current (the E-Arm nominal setting was the same) while the singles rates in the H-Arm significantly changed for different kinematic settings. Although the ESPACE has relatively satisfactory tracking algorithm, only one track events were selected for cross section analysis to avoid the difficulty in handling the pairing problem of selecting the coincident tracks from two arms. It was assumed that the good event fraction in the multiple events is the same as that in the one track events to make the tracking correction.

In addition to the one track selection, the multiple hit cut was also used to improve the quality of the reconstructed one track events' kinematics. In fact, most of events passing the multiple hit cut have only one reconstructed track. In this analysis, both conditions were used to ensure the quality of the reconstructed kinematic variables since we need to make cuts on those variables for kinematic binning.

The tracking efficiency correction can be evaluated from the following:

$$\epsilon_{trk} = \epsilon_{eh,trk} \quad (3.21)$$

$$\epsilon_{eh,trk} = \frac{N_{total}}{N_{trk,mult}} \quad (3.22)$$

or from:

$$\epsilon_{trk} = \epsilon_{e,trk} \epsilon_{h,trk} \quad (3.23)$$

$$\epsilon_{e,trk} = \frac{N_{total}}{N_{e,trk,mult}} \quad (3.24)$$

$$\epsilon_{h,trk} = \frac{N_{total}}{N_{h,trk,mult}} \quad (3.25)$$

where the $1/\epsilon_{eh,trk}$ is the tracking efficiency evaluated from applying the 1-track and multiple cuts on both the E-Arm and H-Arm simultaneously, the $1/\epsilon_{e,trk}$ is the tracking efficiency in E-Arm events and $1/\epsilon_{h,trk}$ is the tracking efficiency in H-Arm events. The N_{total} is the number of the coincidence events used for the evaluation of the tracking efficiency, the $N_{e,trk,mult}$ is the number of events passing the E-Arm one track and multiple hit cuts and the $N_{h,trk,mult}$ is the number of the events passing the H-Arm one track and multiple hit cuts.

If the multiple track was caused by the high single rate, then $\epsilon_{eh,trk} = \epsilon_{e,trk} \epsilon_{h,trk}$. It in fact proved be correct for all the kinematic settings except for the kinematic setting thcm-50 and thcm-90 where the differences are a few percent. It was found the difference was resulted from the elastic scattering events in these two settings. Because of the strong kinematic correlation, the elastic scattering events were concentrated in some specific region at the focal plane. It is likely the scintillator efficiency had to a certain extent some dependence on position. Since in this experiment analysis, the most important thing is to get the global correction factors, we simply excluded those elastic scattering events in the calculation of tracking efficiency. The polygon cut on the H-Arm focal plane coordinates proved to be an effective cut in excluding

the unwanted elastic scattering events. After excluding the elastic scattering events with such polygon cut, the evaluated tracking efficiencies from the Eqn. (3.22) are the same as that from Eqn.(3.23).

3.5.4 Trigger Efficiency

Since the scintillators are used as the trigger counter in the experiment and the scintillators are not 100% efficient, the event loss due to scintillator inefficiency has also to be corrected. The trigger efficiency correction factor for the coincidence measurement is defined as the following:

$$\epsilon_{trig} = \frac{1}{\epsilon_{e,s1}\epsilon_{e,s2}\epsilon_{h,s1}\epsilon_{h,s2}} \quad (3.26)$$

where $\epsilon_{e,s1}$ is the E-Arm scintillator S1 efficiency, $\epsilon_{e,s2}$ is the E-Arm scintillator S2 efficiency, $\epsilon_{h,s1}$ is the H-Arm scintillator S1 efficiency and $\epsilon_{h,s2}$ is the H-Arm scintillator S2 efficiency.

The detail of the scintillator efficiency analysis can be found in[2]. The following is a summary of this analysis.

The E-Arm scintillator efficiencies are determined from:

$$\epsilon_{e,s1} = \frac{N(T1.\check{C}erenkov.1track)}{N(T1.\check{C}erenkov.1track) + N(T2.\check{C}erenkov.S2.1track)} \quad (3.27)$$

$$\epsilon_{e,s2} = \frac{N(T1.\check{C}erenkov.1track)}{N(T1.\check{C}erenkov.1track) + N(T2.\check{C}erenkov.S1.1track)} \quad (3.28)$$

where $N(T1.\check{C}erenkov.1track)$ is the number of one track T1 events with a Čerenkov hit, $N(T2.\check{C}erenkov.S2.1track)$ is the number of one track T2 events with an S2 and Čerenkov hit, $N(T2.\check{C}erenkov.S1.1track)$ is the number of one track T2 events with S1 and T2 hits. The denominator for the above equations is the total number of events which have signals at least in one of the scintillators and the Čerenkov. The numerator is the number of events which have signals in both scintillators and the

Čerenkov.

The H-Arm scintillator efficiencies are determined from:

$$\epsilon_{h,s1} = \frac{N(\text{T3.S0.1track})}{N(\text{T3.S0.1track}) + N(\text{T4.S0.S2.1track})} \quad (3.29)$$

$$\epsilon_{h,s2} = \frac{N(\text{T3.S0.1track})}{N(\text{T3.S0.1track}) + N(\text{T4.S0.S1.1track})} \quad (3.30)$$

where $N(\text{T3.S0.1track})$ is the number of one track T3 events with an S0 hit, $N(\text{T4.S0.S2.1track})$ is the one track T4 events with an S2 and an S0 hit, $N(\text{T4.S0.S1.1track})$ is the number of one track T4 events with an S1 and a T2 hits. The denominator for the above equations is the total number of events which at least have signals in one of the scintillators and the S0. And the numerator is the number of events which have signals in both scintillators and S0.

The trigger efficiency was evaluated to be 96.7% from the 4 scintillator efficiencies with an estimated relative systematic error 1%.

3.5.5 Proton Absorption

There is a certain probability of the proton being absorbed before it makes the trigger at the focal plane. The main absorption happened in the target and the proton absorption in the scintillator was already accounted from the previous trigger efficiency correction. The material between the target and the scintillators can be approximated by the protons with the density of that material, with the assumption of equal proton-proton and proton-neutron total cross section, which is approximately valid in our hadron arm settings. It was assumed that once a nuclear reaction happens the proton cannot make the trigger. The proton absorption correction factor is calculated from the following:

$$\epsilon_{pcorr} = 1 + \left[\frac{0.0723t_{lh2} + 2.7t_{wall}}{\sin \theta_p} + 2.7t_{exit} + 0.0013t_{air} + 1.4t_{spec} \right] N_0 \sigma(p_p) \quad (3.31)$$

where

t_{lh2} is the radius of the liquid hydrogen target (3.175cm),

t_{wall} is the thickness of the target cell side wall (0.0175cm Aluminum),

t_{exit} is the thickness of the scattering chamber exit window (0.0406cm Aluminum),

t_{air} is the thickness of the air from the target to the entrance of the spectrometer (58cm),

t_{spec} is the thickness of the spectrometer entrance window (0.0178cm kapton),

N_0 is the Avogadro constant (6.022e23/mol), σ_{pp} is the total cross section of the pp reaction for a proton with momentum p_p .

The total cross section of the pp reaction can be found at http://pdg.lbl.gov/~sbl/pp_total.dat. The calculated proton absorption factors range from 1.008 to 1.017 (mainly dependent on the varying proton momenta).

3.5.6 Luminosity Correction

The target density variation can be monitored through the relative luminosity. Since the E-Arm setting was the same for all the 12 kinematics, its single rate can be used as a luminosity monitor since the single rate is proportional to the beam current and the target density. The ratio of the E-Arm single scalar and the beam charge will be the same if the target density did not change even if the beam current was different. For each of the runs used in cross section analysis, the E-Arm single scalar S1 was corrected for the electronic deadtime and divided by the total beam charge of that run. The relative luminosity (L_{rel}) for each run is defined as:

$$L = \frac{S_1}{(1 - \text{EDT})Q} \quad (3.32)$$

$$L_{rel} = \frac{L}{\bar{L}} \quad (3.33)$$

Figure 3-11: The relative luminosity versus beam current

where S_1 is the number of T1 events recorded by the scaler, EDT is the electronic deadtime as calculated from the previous section and Q is the beam charge of the run. $\bar{L}$ is the averaged luminosity of all the runs in the cross section analysis. L_{rel} is a good quantity for monitoring the effective target density.

Fig. 3-11 shows the relative luminosity versus different beam currents. The error was calculated from the electronic deadtime error ($\sim 1\%$), which dominates the luminosity fluctuation. As can be seen from this figure, the target density was very stable except for a few runs. Since there is relative big error in the evaluation of the electronic deadtime for the low current runs, it was decided not to make the absolute target density correction. To get a better global luminosity consistency, a relative target density correction was made in which the target density was corrected with respect to the averaged luminosity of the runs used in cross section analysis.

3.6 $p(e, e'p)\pi^0$ Cross Section

3.6.1 Kinematic Binning in $(W, Q^2, \theta_\pi, \phi_\pi)$ Space

Since the spectrometer acceptance is limited, the out-of-plane coverage in the (θ_π, ϕ_π) is also limited. To extract the maximum information with the response function fit from the cross section measurement, a reasonable kinematic binning in $(W, Q^2, \theta_\pi, \phi_\pi)$ is needed.

From the analysis of the data event distribution in the $(W, Q^2, \theta_\pi, \phi_\pi)$ 4-dimensional space, the following binning was used for the cross section extraction:

- W : 5 bins on the invariant mass of the $\pi^0 p$ system between 1160 MeV/c² and 1360 MeV/c² with bin size 40 MeV/c².
- Q^2 : 2 bins on the 4-momentum transfer between 0.8 GeV²/c² and 1.2 GeV²/c² with bin size 0.2 GeV²/c².
- θ_π : 10 bins on the proton's polar angle in the $\pi^0 p$ center of mass system between $\cos \theta_\pi = -1.0$ and $\cos \theta_\pi = 1.0$ with the bin size 0.2.
- ϕ_π : 12 bins on the proton's azimuthal angle in the $\pi^0 p$ center of mass system between 0° and 360° with bin size 30°.

3.6.2 Unpolarized Cross Section

The raw yield of the data in each $(W, Q^2, \theta_\pi, \phi_\pi)$ bin was first subtracted by the yield of accidental background contribution using the coincidence timing spectra, and then corrected with the previously discussed correction factors. The simulation yield in each $(W, Q^2, \theta_\pi, \phi_\pi)$ bin was also evaluated. Since the yield is proportional to the laboratory cross section, the ratio of the corrected data yield and the simulation yield is equal to the ratio of the acceptance averaged experimental cross section and the

acceptance averaged theoretical cross section:

$$\frac{Y_{dat}(i)}{Y_{mc}(i)} = \frac{\bar{\sigma}_{dat}(i)}{\bar{\sigma}_{mc}(i)} \quad (3.34)$$

where $Y_{dat}(i)$ is the corrected data yield of the i -th $(W, Q^2, \theta_\pi, \phi_\pi)$ bin, $Y_{mc}(i)$ is the simulation yield of the i -th $(W, Q^2, \theta_\pi, \phi_\pi)$ bin, $\bar{\sigma}_{dat}(i)$ is the acceptance averaged experimental cross section, and $\bar{\sigma}_{mc}(i)$ is the acceptance averaged theoretical cross section.

Since it is not convenient to compare the acceptance averaged experimental cross section with the point acceptance theoretical cross section, the acceptance has to be unfolded to get the point acceptance cross section for the response function separation and multipole analysis. The above equation can be projected to get the experimental cross section corresponding to point acceptance:

$$\frac{\sigma_{dat}(i)}{\sigma_{mc}(i)} = \frac{\bar{\sigma}_{dat}(i)}{\bar{\sigma}_{mc}(i)} \quad (3.35)$$

where $\sigma_{dat}(i)$ is the point acceptance experimental cross section, and $\sigma_{mc}(i)$ is the point acceptance theoretical cross section. Such projection is valid only when the cross section in the kinematic $(W, Q^2, \theta_\phi, \phi_\pi)$ bin does not change dramatically which is true in the kinematic region accessible in this experiment.

Thus the following relation can be used for the extraction of point acceptance experimental cross section $\sigma_{dat}(i)$:

$$\frac{Y_{dat}(i)}{Y_{mc}(i)} = \frac{\sigma_{dat}(i)}{\sigma_{mc}(i)} \quad (3.36)$$

Furthermore, since the simulation yield is the summed weight of cross section folded with radiation, if $\sigma_{mc}(i)$ is the theoretical MAID cross section without radiation, then $\sigma_{dat}(i)$ will be the radiation corrected experimental cross section corresponding to the point acceptance.

The EPIP ROD was used to calculate the MAID cross section at each $(W, Q^2, \theta_\pi, \phi_\pi)$ point with the E91-011 beam energy 4.5306 GeV. The 5-fold differential cross section for the $p(e, e'p)\pi^0$ can be expressed as:

$$\frac{d^5\sigma}{dk_e d\Omega_e d\Omega_p^*} = \Gamma_v \sigma_v(W, Q^2, \theta_\pi, \phi_\pi, \epsilon) \quad (3.37)$$

where Γ_v is the virtual photon flux, σ_v is the center of mass cross section for the virtual photo-production $\gamma_v + N \rightarrow x + N$, ϵ is the transverse polarization of the virtual photon.

$$\Gamma_v = \frac{\alpha}{2\pi^2} \frac{k_f}{k_i} \frac{k_\gamma}{Q^2} \frac{1}{1 - \epsilon} \quad (3.38)$$

$$\epsilon = \frac{1}{1 + 2 \frac{|\vec{q}|^2}{Q^2} \tan^2 \frac{\theta_e}{2}} \quad (3.39)$$

where $k_\gamma = (W^2 - m_p^2)/2m_p$ is the laboratory energy for a real photon to excite the proton to the resonance state with invariant mass W , k_i and k_f are the electron's initial and final momentum respectively, θ_e is the electron scattering angle.

For the convenience of the response function and multipole analysis, the center of mass cross section was extracted for the middle point of each $(W, Q^2, \cos \theta_\pi, \phi_\pi)$ bin.

3.7 Systematic Uncertainty

3.7.1 System Error from Various Factors

The following systematic uncertainties are considered in evaluating the system error of the measured cross section:

- Acceptance uncertainty

As it is discussed in the previous sections, the acceptance analysis with the R-Function cut is quite precise. Since the R-Function is defined with kinematics dependent boundaries and the R cut is different for different kinematic setting, the acceptance analysis error is evaluated for each kinematics respectively. It was estimated using the yield ratio variation in the flat region as shown in Fig. 3-6 on page 90. The estimated acceptance error is from 0.6% to 2.4%. The relatively big acceptance analysis error 2.4% in kinematics setting *thcm135* is related to the high background level. For kinematic setting *thcm0*, the acceptance error is only 0.6% where there are very few background events.

- Particle identification uncertainty

The charged pion background was suppressed from the polygon cut on $gmean - hbeta$ distribution. The remaining accidental background contribution was subtracted using the timing spectrum. The system uncertainty related to this background exclusion was estimated as the following: Different $gmean - hbeta$ polygon cuts as show in Fig. 3-8 on page 104 were applied, then resulted background subtracted data yields were calculated. There is a flat region in the plot of data yield versus different polygon cuts(from the center loose cut to outside tight cut). The variation of the yields in the flat region was used as the estimation of the particle identification uncertainty in the cross section. Such uncertainty is from 0 – 1% depending on kinematic settings.

- Electronic deadtime uncertainty

The electronic deadtime uncertainty is about 1% which slightly varies for different kinematic setting and beam currents due to the change of the singles rates.

- Tracking efficiency error

It was assumed that the multiple tracks event was due to the high single rate. The variation in the E-Arm tracking efficiencies was used as a conservative estimate of the tracking efficiency correction uncertainty. This error is estimated to be 1.2% for each of the 12 kinematics.

- Trigger efficiency error

This was estimated from the scintillator efficiency error. A conservative estimate is 0.8%.

- Beam energy error

Since the beam energy (4.5306 GeV) is accurate to 0.5 MeV, the MAID was used to estimate the corresponding cross section change for such energy variation. The resulted cross section change is about 1%.

- Beam charge error

This was estimated to be about 0.5%.

The total system uncertainty from the above contributions is between 2.3% to 3.3% depending on the background level in the data.

3.7.2 Additional System Error

It was noticed there are a few data points with extracted cross sections off the fitted curve by more than a few estimated standard error. These data points were identified as the points located at the acceptance edge region. Because the yield distribution at the acceptance edge region drops rapidly to 0, a small acceptance error can result big deviation in the extracted experimental cross section using the yield ratio between

the data and the simulation. The background subtraction using the timing spectra may also introduce some addition error due to insufficient statistics. For these data points, the acceptance error is greater than other data points. A special procedure was designed to search for these abnormal data points at the acceptance edge region and adjust the systematic error for the cross sections.

The cross section was extracted for each $(W, Q^2, \theta_\pi, \phi_\pi)$ bin.

For a specific (θ_π, ϕ_π) bin, the data and MC yields in each (W, Q^2) are normalized in such way that both the sum of the data and that of MC for all (W, Q^2) points corresponding to this (θ_π, ϕ_π) are equal to 1. Then the ratio of the normalized data and MC yields in each $(W, Q^2, \theta_\pi, \phi_\pi)$ bin is calculated as R_1 . For other (θ_π, ϕ_π) bins, The R_1 is calculated in the same way.

Similarly, R_2 is defined for (W, Q^2) , R_3 for (W, θ_π) , R_4 for (W, ϕ_π) , R_5 for (Q^2, θ_π) and R_6 for (Q^2, ϕ_π) .

Thus 6 normalized ratios for each $(W, Q^2, \theta_\pi, \phi_\pi)$ are calculated.

Define R and R' as the following:

$$R = R_1 R_2 R_3 R_4 R_5 R_6 \quad (3.40)$$

$$R' = \frac{1}{R_1 R_2 R_3 R_4 R_5 R_6} \quad (3.41)$$

The following algorithm was designed to search for the acceptance edge bins: First consider the acceptance in (W, Q^2) plane. For a specific (W, Q^2) , there are 4 immediate nearby points $(W - \delta W, Q^2)$, $(W + \delta W, Q^2)$, $(W, Q^2 - \delta Q^2)$ and $(W, Q^2 + \delta Q^2)$, where δW and δQ^2 are the half sizes of the W bin and Q^2 bin respectively. Similarly, the acceptance in (W, θ_π) , (W, ϕ_π) , (Q^2, θ_π) , (Q^2, ϕ_π) and (θ_π, ϕ_π) planes is examined using the same procedure.

If $R < 0.1 - \textit{Statistic_Error}$ or $R' < 0.1 - \textit{Statistic_Error}$ and the data point is at the acceptance edge region, this data point is excluded. Here the *Statistic_Error* is the statistic fluctuation of the calculated value of R or R' . If $R < 0.7 - \textit{Statistic_Error}$

or $R' < 0.7 - \textit{Statistic_Error}$ and the data point is at the acceptance edge region, the data point is kept but assigned greater system error. The adjusted systematic error is calculated as the following:

$$\textit{New_System_Error} = \frac{(1 - R)^6 R_{yield}}{R} \quad (R < R') \quad (3.42)$$

$$\textit{New_System_Error} = \frac{(1 - R')^6 R_{yield}}{R'} \quad (R > R') \quad (3.43)$$

where R_{yield} is the ratio of the data and MC yields.

3.8 Unpolarized Response Functions

3.8.1 Response Function Fit on ϕ_π Distribution

The virtual photo-production cross section in the center of mass frame σ_v can be written as:

$$\sigma_v = \nu_0(\nu_L R_L + \nu_T R_T + \nu_{LT} R_{LT} \cos \phi_\pi + \nu_{TT} R_{TT} \cos 2\phi_\pi) \quad (3.44)$$

$$(3.45)$$

where ϕ_π is the π^0 's azimuthal angle in the center of mass frame, ν_α ($\alpha = L, T, LT$ or TT) is the kinematic factor which depends only on electron kinematics, $R_\alpha(W, Q^2, \theta_\pi)$ ($\alpha = L, T, LT$ or TT) is the response function carrying the hadron information, ν_0 is an overall kinematic factor. In this analysis, the following convention for the kinematic factor definition is used:

$$\nu_0 = \frac{k}{q_0} \quad (3.46)$$

$$\nu_L = \epsilon_S \quad (3.47)$$

$$\nu_T = 1 \quad (3.48)$$

$$\nu_{LT} = \sqrt{2\epsilon_S(1 + \epsilon)} \quad (3.49)$$

$$\nu_{TT} = \epsilon \quad (3.50)$$

where k is recoil momentum in the center of mass frame, q_0 is the momentum needed by a real photon to excite a state with invariant mass W , ϵ is the transverse polarization of the virtual photon as defined before, ϵ_S is the scalar polarization.

If W , Q^2 and E_{beam} are known, the following equations can be used to calculate the above kinematic factors (for $p(e, e'p)\pi^0$ reaction):

$$E_i = E_{beam} \quad (3.51)$$

$$E_f = E_i - \frac{W^2 + Q^2 - m_p^2}{2m_p} \quad (3.52)$$

$$\omega = E_i - E_f \quad (3.53)$$

$$\theta_e = 2 \sin^{-1} \sqrt{\frac{Q^2}{4E_i E_f}} \quad (3.54)$$

$$\epsilon = \frac{1}{1 + 2 \frac{Q^2 + \omega^2}{Q^2} \tan^2 \frac{\theta_e}{2}} \quad (3.55)$$

$$\epsilon_S = \frac{Q^2}{(Q^2 + \omega^2)} \frac{W^2}{m_p^2} \epsilon \quad (3.56)$$

$$E_\pi = \frac{W^2 - m_p^2 + m_\pi^2}{2W} \quad (3.57)$$

$$k = \sqrt{E_\pi^2 - m_\pi^2} \quad (3.58)$$

$$q_0 = \frac{W^2 - m_p^2}{2W} \quad (3.59)$$

$$\nu_0 = \frac{k}{q_0} \quad (3.60)$$

where m_p ($=938.3\text{MeV}/c^2$) and m_π ($=134.9\text{MeV}/c^2$) are the rest masses of the proton and neutral pion respectively.

Using the above equations, $\nu_L R_L + \nu_T R_T$, R_{LT} and R_{TT} are extracted for available (W, Q^2, θ_π) points by fitting the cross section's ϕ_π dependences.

R_L and R_T can not be separated without varying the kinematic factor ϵ_S .

3.8.2 Legendre Fit on (θ_π, ϕ_π) Distribution

To extract as much information as possible from the measured unpolarized cross section and provide the parametrized experimental cross section for the polarized response function separation, the simultaneous fit on the cross section's (θ_π, ϕ_π) dependence was also performed.

The response functions can be expressed in multipoles up to $L = 1$:

$$\nu_L R_L + \nu_T R_T = A_0 + A_1 P_1(\cos \theta_\pi) + A_2 P_2(\cos \theta_\pi) \quad (3.61)$$

$$R_{TT} = C_0 \sin^2 \theta_\pi \quad (3.62)$$

$$R_{LT} = (D_0 + D_1 P_1(\cos \theta_\pi)) \sin \theta_\pi \quad (3.63)$$

Using the multipole decompositions of the response functions[27] for S and P waves, the following Legendre coefficients can be derived:

$$\begin{aligned} A_0 = & \epsilon_S |S_{0+}|^2 + 4\epsilon_S |S_{1+}|^2 + \epsilon_S |S_{1-}|^2 - 4\epsilon_S \text{Re}(S_{1+}^* S_{1-}) \\ & + |E_{0+}|^2 + \frac{1}{2} |2M_{1+} + M_{1-}|^2 + \frac{1}{2} |3E_{1+} - M_{1+} + M_{1-}|^2 \\ & + \frac{1}{3} [12\epsilon_S (|S_{1+}|^2 + \text{Re}(S_{1+}^* S_{1-})) + |3E_{1+} + M_{1+} - M_{1-}|^2 \\ & - \frac{1}{2} |2M_{1+} + M_{1-}|^2 - \frac{1}{2} |3E_{1+} - M_{1+} + M_{1-}|^2] \end{aligned} \quad (3.64)$$

$$A_1 = 2\text{Re}(E_{0+}^* (3E_{1+} + M_{1+} - M_{1-})) + 2\epsilon_S \text{Re}(S_{0+}^* (4S_{1+} + S_{1-})) \quad (3.65)$$

$$\begin{aligned} A_2 = & \frac{2}{3} [12\epsilon_S (|S_{1+}|^2 + \text{Re}(S_{1+}^* S_{1-})) + |3E_{1+} + M_{1+} - M_{1-}|^2 \\ & - \frac{1}{2} |2M_{1+} + M_{1-}|^2 - \frac{1}{2} |3E_{1+} - M_{1+} + M_{1-}|^2] \end{aligned} \quad (3.66)$$

$$C_0 = 3[\frac{3}{2} |E_{1+}|^2 - \frac{1}{2} |M_{1+}|^2 - \text{Re}(E_{1+}^* (M_{1+} - M_{1-}) + M_{1+}^* M_{1-})] \quad (3.67)$$

$$D_0 = -\text{Re}(S_{0+}^* (3E_{1+} - M_{1+} + M_{1-}) - (2S_{1+}^* - S_{1-}^*) E_{0+}) \quad (3.68)$$

$$D_1 = -6\text{Re}(S_{1+}^* (E_{1+} - M_{1+} + M_{1-}) + S_{1-}^* E_{1+}) \quad (3.69)$$

Assuming M_{1+} dominance and partial wave expansion of the unpolarized response functions for S and P waves, one can have:

$$A_0 = 2|M_{1+}|^2 \quad (3.70)$$

$$A_1 = 2\text{Re}(M_{1+} E_{0+}^*) \quad (3.71)$$

$$A_2 = -|M_{1+}|^2 + 6\text{Re}(M_{1+} E_{1+}^*) - 2\text{Re}(M_{1+} M_{1-}^*) \quad (3.72)$$

$$C_0 = -\frac{3}{2} |M_{1+}|^2 - 3\text{Re}(M_{1+} E_{1+}^*) - 3\text{Re}(M_{1+} M_{1-}^*) \quad (3.73)$$

$$D_0 = \text{Re}(M_{1+} S_{0+}^*) \quad (3.74)$$

$$D_1 = 6\text{Re}(M_{1+} S_{1+}^*) \quad (3.75)$$

where P_1 and P_2 are the Legendre polynomials.

The R_{EM} and R_{SM} can be extracted from the above truncated multipole expansions:

$$R_{EM} = \frac{Re(E_{1+}M_{1+}^*)}{|M_{1+}|^2} = \frac{3A_2 - 2C_0}{12A_0} \quad (3.76)$$

$$R_{SM} = \frac{Re(S_{1+}M_{1+}^*)}{|M_{1+}|^2} = \frac{D_1}{3A_0} \quad (3.77)$$

3.8.3 R'_{LT} Analysis

When the electron beam is polarized and its polarization is measured, an additional helicity dependent response function R'_{LT} can be extracted from the measured cross section.

The Eqn. 3.44 on page 124 can be modified as:

$$\sigma_v = \nu_0(\nu_L R_L + \nu_T R_T + \nu_{LT} R_{LT} \cos \phi_\pi + \nu_{TT} R_{TT} \cos 2\phi_\pi + P_e \nu'_{LT} R'_{LT} \sin \phi_\pi) \quad (3.78)$$

where P_e is electron beam polarization including the helicity sign, $\nu'_{LT} = \sqrt{2\epsilon_S(1-\epsilon)}$ is helicity dependent kinematic factor and $R'_{LT}(W, Q^2, \theta_\pi)$ is the helicity dependent response function.

The R'_{LT} was extracted using the following procedure.

For each $(W, Q^2, \theta_\pi, \phi_\pi)$ bin, the helicity plus and minus data yields N_+ and N_- were determined using the same procedure in the cross section analysis, the averaged electron beam polarization P_e was also calculated, thus the electron beam asymmetry A'_{LT} was determined from:

$$A'_{LT} = \frac{1}{|P_e|} \frac{N_+ - N_-}{N_+ + N_-} \quad (3.79)$$

The R'_{LT} is proportional to A'_{LT} :

$$R'_{LT} = \frac{\sigma_0 A'_{LT}}{\nu'_{LT} \sin \phi_\pi} \quad (3.80)$$

where σ_0 is the unpolarized cross section determined from previous analysis. To reduce the statistic variation in the R'_{LT} , the parameterized experimental cross section was used to calculate the $\sigma_0(W, Q^2, \theta_\pi, \phi_\pi)$.

The R'_{LT} only depends on (W, Q^2, θ_π) . For each $(W, Q^2, \theta_\pi, \phi_\pi)$, one can evaluate the R'_{LT} according Eqn.(3.80) . If there are more than one data R'_{LT} for a (W, Q^2, θ_π) point, the weighted average of these values is defined as the R'_{LT} for that (W, Q^2, θ_π) point. The following weighting is equivalent to the χ^2 fit on the ϕ_π distribution to extract the R'_{LT} :

$$R'_{LT}(W, Q^2, \theta_\pi) = \frac{\sum_i \frac{R'_{LT}(i)}{\Delta R'_{LT}(i)^2}}{\sum_i \frac{1}{\Delta R'_{LT}(i)^2}} \quad (3.81)$$

where $R'_{LT}(i)$ and $\Delta R'_{LT}(i)$ are the i -th helicity dependent response function and its estimated error respectively.

3.9 Polarization and Polarized Response Function Analysis

The detail of the polarization and polarized response function analysis can be found in R. Roche's thesis[2]. The following is a summary of the polarization analysis.

The maximum likelihood method was used to extract the recoil polarization at target and the corresponding response functions. The likelihood function is expressed as:

$$L = \prod_{i=1}^N \frac{1}{2\pi} (1 + \epsilon_x \sin \phi_{fpp} + \epsilon_y \cos \phi_{fpp}) \quad (3.82)$$

where

$$\epsilon_\alpha = \xi_\alpha + A(\theta_{fpp}) \sum_{\beta} S_{\alpha\beta} (P_\beta + hP'_\beta) \quad (\alpha = x, y) \quad (3.83)$$

is the azimuthal asymmetry at focal plane for a recoil polarization $\vec{\Pi} = \vec{P} + h\vec{P}'$ with helicity-independent and helicity-dependent contributions, $\vec{P}$ and $\vec{P}'$, in the center of hadronic mass system at the target transformed to the focal plane coordinate system by the spin-precession matrix $S_{\alpha\beta}$. $A(\theta_{fpp})$ is the FPP analyzing power. ξ_α is the experimentally measured FPP false asymmetry.

To extract the polarized response functions, the polarizations are also expanded into response functions (R_η) using the parametrized experimental cross section (σ):

$$\sigma P_\beta = \sum_{\eta} \nu_{\beta\eta}(\epsilon, \phi_\pi) R_\eta \quad (3.84)$$

where $\nu_{\beta\eta}$ are the kinematic coefficients, depending on the transverse polarization of the virtual photon ϵ , and the charged pion's azimuthal angle ϕ_π .

3.10 Multipole Analysis

EPIPROD is used to extract the multipole amplitudes from partial wave analysis[51]. The multipole analysis is performed by studying the sensitivity of the data to variations of the reference multipole amplitudes (MAID). The experimental multipole amplitude $A_i(W, Q^2)$ is expressed as:

$$A_i(W, Q^2) = A_i^{(0)}(W, Q^2) + \delta A_i(W, Q^2) \quad (3.85)$$

where $A^{(0)}$ is obtained from MAID 2000, and δA is an adjustable parameter to be fitted to the data. The multipole fit is performed on all available cross section and polarized response function data in the (W, Q^2) bin simultaneously. Adjustments to the magnetic, electric and scalar amplitudes for a subset of the contributing partial waves are fit while the other amplitudes are maintained at their input (reference) values. In the preliminary multipole analysis, only S and P wave multipole amplitudes in MAID are adjusted: resonant multipoles M_{1+} , E_{1+} , S_{1+} and non-resonant multipoles E_{0+} , S_{0+} , M_{1-} , S_{1-} .

Chapter 4

Results and Discussion

4.1 Cross Sections

The 2-fold differential cross sections for pion electro-production on the proton are extracted for 944 $(W, Q^2, \cos\theta_\pi, \phi_\pi)$ points in the center of mass system. All the differential cross section results are included in Appendix A.

Some of the typical cross section plots are included here. Fig. 4-1 and 4-2 show azimuthal distributions of the cross sections corresponding to different $x = \cos\theta_\pi$. Fig. 4-3 and 4-4 show the polar distributions of the cross sections corresponding to different ϕ_π . In these plots, the SAID and MAID calculations are also shown together with the data Legendre fit results. In the Δ resonance peak region($W = 1220\text{MeV}/c^2$), both SAID and MAID agree with the data although some small difference can still be identified. However, in the Δ tail region, the calculations from SAID and MAID are significantly different. MAID agrees with the data better than SAID. This is due to the different background considerations in SAID and MAID. In the resonance peak region, the background contribution is not dominant, thus the SAID and MAID have similar predictions. In the Δ tail region($W = 1300\text{MeV}/c^2$), the background contributions are significant, the SAID and MAID have different cross sections. From the comparisons with our cross sections, the MAID is more accurate

than SAID in considering the background contributions.

Most of the cross sections results have about 3% systematic errors, resulting less than 5% total error for many data points. Such high precision cross sections in this kinematic region will be very useful for improving pion electro-production models.

Figure 4-1: 2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1220\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$. The $x = \cos \theta_\pi$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.

Figure 4-2: 2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1300\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$. The $x = \cos\theta_\pi$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.

Figure 4-3: 2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1220\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.

Figure 4-4: 2-fold differential cross section for $p(e, e'p)\pi^0$ at $W = 1300\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$. Solid: Legendre fit on data; Dashed: MAID; Dotted: SAID.

4.2 Unpolarized Responses and Legendre Coefficients

Response Functions

The usual way to extract more information from the cross sections is to fit its ϕ_π distributions to separate different response functions. The response function fit requires enough coverage of ϕ_π distributions. However, as can be seen in Fig. 4-1, 4-2, 4-3 and 4-4, there is not enough out-of-plane coverage for many of the $(W, Q^2, \cos \theta_\pi)$ bins. Thus, except for the R'_{LT} , the response functions extracted from the differential cross sections are not used for further multipole analysis. Instead, the differential cross sections are directly used in the multipole fits to make the best use of available cross section measurement.

Although the E91011 experiment was not designed to measure the R'_{LT} , using the beam helicity and the measured cross sections, R'_{LT} is extracted for all the available $(W, Q^2, \cos \theta_\pi)$ bins. Some of the R'_{LT} are significantly nonzero, indicating the linear contribution of non-resonant backgrounds. Fig. 4-28 shows $\cos \theta_\pi$ distributions of the separated response functions for $W = 1260 MeV$ and $Q^2 = 0.9 GeV^2$. This is in the Δ tail region, one can clearly see MAID is better than SAID in terms of agreement with our measurement. All the extracted R'_{LT} are used in the multipole analysis.

The response functions extracted from the cross section measurement are included in Appendix B.

Figure 4-5: Response functions for $p(e, e'p)\pi^0$ at $W = 1260 \text{ MeV}$ and $Q^2 = 0.9 \text{ GeV}^2$. Dashed: MAID; Dotted: SAID.

Legendre Fit Results

Fig. 4-6 and 4-7 show the Legendre coefficients from fit on the angular distributions of cross sections. The Legendre coefficients are defined by Eqn. 3.61, 3.62 and 3.63 on page 125 and 126. The 2nd order Legendre fit on the cross sections are quite good, as can be seen from Fig. 4-1,4-2,4-3 and 4-4 on page 133 through 136. The MAID and DMT calculations use the S and P wave multipole expansions for the corresponding Legendre coefficients.

Table. 4.1 on page 142 shows the Legendre fit results. From the fitted Legendre coefficients, R_{EM} and R_{SM} are extracted. Since the expressions for R_{EM} and R_{SM} with the Legendre coefficients are only valid when the M_{1+} dominance assumption is good, the R_{EM} and R_{SM} extracted with this method are only meaningful in the Δ resonance region.

Figure 4-6: Legendre coefficients from the angular distribution fit to differential cross section for $p(e, e'p)\pi^0$ at $Q^2 = 0.9\text{GeV}^2$. Solid: DMT; Dashed:MAID

Figure 4-7: Legendre coefficients from the angular distribution fit to differential cross section for $p(e, e'p)\pi^0$ at $Q^2 = 1.1 \text{ GeV}^2$. Solid: DMT; Dashed: MAID

W (MeV)	1180	1180	1220	1220	1260	1260	1300	1300	1340
Q^2 (GeV ²)	0.9	1.1	0.9	1.1	0.9	1.1	0.9	1.1	0.9
E_{beam} (GeV)	4.531	4.531	4.531	4.531	4.531	4.531	4.531	4.531	4.531
χ^2	83	106	183	219	264	370	151	134	159
ndf	54	84	117	129	127	128	95	75	68
A_0 (μb)	7.61	5.80	10.71	7.83	5.89	4.27	3.11	2.39	2.31
ΔA_0 (μb)	0.41	0.21	0.09	0.05	0.05	0.03	0.06	0.14	0.08
A_1 (μb)	1.80	1.66	1.21	1.00	0.30	0.19	0.01	-0.02	0.50
ΔA_1 (μb)	0.25	0.36	0.04	0.04	0.03	0.02	0.08	0.23	0.09
A_2 (μb)	-2.80	-1.76	-5.13	-3.68	-3.41	-2.43	-2.07	-1.56	-1.29
ΔA_2 (μb)	0.23	0.18	0.12	0.06	0.06	0.04	0.04	0.11	0.04
D_0 (μb)	-0.88	-0.95	-0.27	-0.29	0.18	0.09	0.12	0.00	-0.16
ΔD_0 (μb)	0.36	0.06	0.02	0.02	0.01	0.01	0.04	0.08	0.06
D_1 (μb)	-1.81	-1.61	-2.04	-1.62	-0.62	-0.54	0.10	-0.03	0.04
ΔD_1 (μb)	0.21	0.09	0.04	0.04	0.02	0.02	0.05	0.10	0.06
C_0 (μb)	-2.66	-1.68	-6.29	-4.55	-4.07	-3.07	-2.07	-1.37	-0.99
ΔC_0 (μb)	0.33	0.09	0.18	0.09	0.09	0.06	0.05	0.07	0.06
v_0	0.828	0.828	0.876	0.876	0.906	0.906	0.926	0.926	0.940
ϵ	0.958	0.947	0.956	0.944	0.953	0.940	0.949	0.936	0.946
ϵ_S	0.931	0.896	0.941	0.910	0.947	0.920	0.948	0.926	0.946
R_{EM}	-3.4%	-2.8%	-2.2%	-2.1%	-2.9%	-2.3%	-5.6%	-6.8%	-6.8%
ΔR_{EM}	1.4%	0.9%	0.4%	0.3%	0.4%	0.3%	0.5%	1.5%	0.7%
R_{SM}	-7.9%	-9.3%	-6.3%	-6.9%	-3.5%	-4.2%	1.0%	-0.4%	0.6%
ΔR_{SM}	1.2%	0.8%	0.1%	0.2%	0.1%	0.2%	0.5%	1.4%	0.8%

Table 4.1: Legendre Fit Coefficients

4.3 Preliminary Polarizations and Polarized Responses

The results of preliminary polarization and polarized response functions can be found in reference[2]. For completeness, some results are included in this section.

In these polarization plots, the solid circles are experimental polarization values averaged in each ϕ_{pq} bin. Only statistic errors are shown. The open squares are the acceptance averaged MAID values. For helicity-dependent polarizations, there is little difference between the MAID and SAID calculations, and the preliminary data agrees well with the theoretical predictions. However, for helicity-independent polarizations, there is significant difference between the MAID and SAID calculations, and MAID is better in terms of the agreement with the preliminary data although neither of the models can accurately describe the data. For the polarized response functions, unlike the responses extracted from cross section measurement, the difference between MAID and SAID is quite significant, indicating the polarization observables have greater sensitivity to the small multipole contributions.

Figure 4-8: Proton polarization P_n for $W=1200-1240$ MeV and $Q^2=0.8-1.0$ GeV 2

Figure 4-9: Proton polarization P_t for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$

Figure 4-10: Proton polarization P_l for $W=1200-1240$ MeV and $Q^2=0.8-1.0$ GeV 2

Figure 4-11: Proton polarization P'_n for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$

Figure 4-12: Proton polarization P'_t for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$

Figure 4-13: Proton polarization P'_l for $W=1200-1240$ MeV and $Q^2=0.8-1.0$ GeV 2

Figure 4-14: Polarized responses for $W=1200-1240\text{MeV}$ and $Q^2=0.8-1.0\text{GeV}^2$

4.4 Preliminary Multipoles

To perform a good multipole analysis, both the cross section and polarization results are needed. Since the polarization results[2] are not finalized yet, the multipole analysis results are still preliminary. The multipole results from the preliminary analysis of J. J. Kelly[23] are compared with the MAID and DMT predictions.

Fig. 4-15 and 4-16 show the comparisons of the extracted preliminary multipoles and the theoretical predictions for $Q^2 = 0.9\text{GeV}^2$. Fig. 4-17 and 4-18 show the comparisons of the extracted preliminary multipoles and the theoretical predictions for $Q^2 = 1.1\text{GeV}^2$. These preliminary multipoles show little Q^2 dependence in the 1.0GeV^2 region. Both MAID and DMT correctly predict the trend of the resonance multipoles' W dependence although significant difference between the our measurements and theoretical predictions is evident. For non-resonant multipoles, neither MAID nor DMT is good in describing the experimental results.

Table. 4.2 shows the preliminary multipoles extracted at $(W, Q^2) = (1220\text{MeV}, 0.9\text{GeV}^2)$ and $(W, Q^2) = (1220\text{MeV}, 1.1\text{GeV}^2)$.

	$W = 1220\text{MeV}, Q^2 = 0.9\text{GeV}^2$			$W = 1220\text{MeV}, Q^2 = 1.1\text{GeV}^2$		
	DATA	MAID	DMT	DATA	MAID	DMT
$ReM_{1+} (10^{-2}fm)$	1.028 ± 0.054	0.475	0.465	1.020 ± 0.048	0.401	0.395
$ImM_{1+} (10^{-2}fm)$	1.927 ± 0.030	2.164	2.144	1.567 ± 0.030	1.829	1.798
$ReE_{1+} (10^{-2}fm)$	0.0010 ± 0.0057	0.036	0.036	0.0018 ± 0.0055	0.029	0.030
$ImE_{1+} (10^{-2}fm)$	-0.081 ± 0.004	-0.063	-0.050	-0.073 ± 0.004	-0.053	-0.038
$ReS_{1+} (10^{-2}fm)$	-0.050 ± 0.005	-0.010	-0.011	-0.053 ± 0.006	-0.0083	-0.011
$ImS_{1+} (10^{-2}fm)$	-0.135 ± 0.004	-0.158	-0.156	-0.125 ± 0.004	-0.133	-0.140
$\frac{Re(M_{1+}^* E_{1+})}{ M_{1+} ^2} (\%)$	-3.3 ± 0.2	-2.4	-1.9	-3.2 ± 0.3	-2.4	-1.7
$\frac{Re(M_{1+}^* S_{1+})}{ M_{1+} ^2} (\%)$	-6.6 ± 0.2	-7.1	-7.1	-7.2 ± 0.3	-7.1	-7.5

Table 4.2: $1+$ multipoles and $E2/M1$ and $C2/M1$ ratios

It is helpful to compare the quadrupole strength R_{EM} and R_{SM} extracted from the Legendre fit on cross sections and those calculated from model-independent multipole

Figure 4-15: Preliminary 1+ multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 0.9 \text{ GeV}^2$. Solid: MAID; Dashed: DMT

Figure 4-16: Preliminary non-resonant multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 0.9 \text{ GeV}^2$. Solid: MAID; Dashed: DMT

Figure 4-17: Preliminary 1+ multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 1.1 \text{ GeV}^2$. Solid: MAID; Dashed: DMT

Figure 4-18: Preliminary non-resonant multipoles for $p(e, e'p)\pi^0$ at $Q^2 = 1.1 \text{ GeV}^2$. Solid: MAID; Dashed: DMT

analysis results which utilize both the cross section and polarization observables.

From the Legendre fit on cross sections,

$$R_{EM} = (-2.2 \pm 0.4)\% \quad R_{SM} = (-6.3 \pm 0.1)\% \quad \text{for } Q^2 = 0.9\text{GeV}^2$$

$$R_{EM} = (-2.1 \pm 0.3)\% \quad R_{SM} = (-6.9 \pm 0.2)\% \quad \text{for } Q^2 = 1.1\text{GeV}^2$$

From the preliminary multipole fit on cross section and polarization,

$$R_{EM} = (-3.3 \pm 0.2)\% \quad R_{SM} = (-6.6 \pm 0.2)\% \quad \text{for } Q^2 = 0.9\text{GeV}^2$$

$$R_{EM} = (-3.2 \pm 0.3)\% \quad R_{SM} = (-7.2 \pm 0.3)\% \quad \text{for } Q^2 = 1.1\text{GeV}^2$$

The R_{SM} from the two analyses have small differences. However, the R_{EM} from the cross section analysis is smaller than that from multipole analysis. This may indicate there is significant model error in the extraction of the R_{EM} from the cross section measurements. These quadrupole strengths are clearly not zero around $Q^2 = 1.0\text{GeV}^2$. Our results have the same sign and similar magnitude as the existing measurements.

4.5 Summary

The absolute cross sections for the $p(e, e'p)\pi^0$ reaction at Δ resonance with $Q^2 = 0.9, 1.1(\text{GeV}/c)^2$ have been measured. Using the R-Function acceptance analysis method, high precision cross section results are obtained with the systematic error at about 3%, which is better than the existing measurement at the same kinematics. The quadruple strength in $N \rightarrow \Delta$ transition R_{EM} and R_{SM} extracted from the cross sections are consistent with existing measurements. Both the MAID and SAID calculations are close to the experimental cross sections, but neither of them are accurate, especially in the Δ tail region.

With the measurement of polarization observables, a direct multipole analysis to extract both the real and imaginary parts of the multipoles is made possible for the first time. Both the dominant amplitudes and small amplitudes can be extracted from the experimental data without using the truncated multipole expansions, resulting much less model error in the quadrupole measurement. Although the polarization analysis is not finalized, preliminary polarization and multipole results show significant sensitivity to the small amplitudes contributions. Thus we can expect a better experimental determination of those small amplitudes once the polarization analysis is finished. The angular distribution of the recoil polarization results and high precision cross sections are also invaluable experimental inputs for theoretical model development for the study of nucleon structure.

Appendix A E91-011 Differential Cross Sections

The following tables include the differential cross sections of reaction $p(e, e'p)\pi^0$ for different $(W, Q^2, \cos \theta_\pi, \phi_\pi)$ in the $\pi^0 p$ center of mass system. W is the invariant mass of the $\pi^0 p$ system, Q^2 is the 4-momentum transfer, ϵ is the transverse polarization of the virtual photon, $x = \cos \theta_\pi$, θ_π and ϕ_π are the pion polar and azimuthal angles in the center of mass system. σ is the 2-fold differential cross section, Sta is statistic error, Sys is systematic error, $\Delta\sigma$ is the total error($= \sqrt{\text{Sta}^2 + \text{Sys}^2}$).

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1180	0.9	0.958	-0.95	45	3.49	0.71	0.09	0.72
1180	0.9	0.958	-0.95	315	2.97	0.56	0.09	0.57
1180	0.9	0.958	-0.95	345	3.29	0.57	0.09	0.57
1180	0.9	0.958	-0.75	15	4.40	0.12	0.11	0.17
1180	0.9	0.958	-0.75	45	5.30	0.28	0.12	0.31
1180	0.9	0.958	-0.75	315	4.28	0.22	0.12	0.25
1180	0.9	0.958	-0.75	345	3.96	0.11	0.10	0.15
1180	0.9	0.958	-0.65	15	3.76	0.23	0.15	0.93
1180	0.9	0.958	-0.65	45	5.76	0.41	0.13	0.43
1180	0.9	0.958	-0.65	315	4.68	0.37	0.13	0.40
1180	0.9	0.958	-0.65	345	4.35	0.25	0.11	0.27
1180	0.9	0.958	-0.55	15	4.45	0.69	0.12	0.70
1180	0.9	0.958	-0.45	15	4.56	0.15	0.12	0.19
1180	0.9	0.958	-0.45	345	4.00	0.12	0.26	0.29
1180	0.9	0.958	-0.35	15	4.69	0.09	0.11	0.14
1180	0.9	0.958	-0.35	345	4.18	0.07	0.12	0.14
1180	0.9	0.958	-0.25	15	4.85	0.07	0.12	0.14
1180	0.9	0.958	-0.25	345	4.39	0.06	0.11	0.13
1180	0.9	0.958	-0.15	15	4.70	0.07	0.26	0.27
1180	0.9	0.958	-0.15	345	4.53	0.07	0.11	0.13
1180	0.9	0.958	-0.05	15	4.80	0.09	0.69	0.69
1180	0.9	0.958	-0.05	345	4.38	0.09	0.10	0.13
1180	0.9	0.958	0.05	15	4.57	0.18	0.28	0.33
1180	0.9	0.958	0.05	345	4.13	0.16	0.10	0.19
1180	0.9	0.958	0.45	15	4.49	0.89	1.80	2.70
1180	0.9	0.958	0.45	45	6.05	0.38	0.13	0.40
1180	0.9	0.958	0.45	315	5.33	0.19	0.13	0.23
1180	0.9	0.958	0.45	345	4.16	1.70	0.07	1.70
1180	0.9	0.958	0.55	15	3.49	0.10	0.07	0.12
1180	0.9	0.958	0.55	45	5.62	0.12	0.12	0.17
1180	0.9	0.958	0.55	315	4.87	0.09	0.36	0.37
1180	0.9	0.958	0.55	345	3.86	0.10	0.54	0.55
1180	0.9	0.958	0.65	15	3.46	0.04	0.12	0.13
1180	0.9	0.958	0.65	45	5.71	0.09	0.33	0.34
1180	0.9	0.958	0.65	315	4.65	0.08	0.65	0.65
1180	0.9	0.958	0.65	345	3.78	0.05	0.55	0.55
1180	0.9	0.958	0.75	15	3.74	0.05	0.12	0.13
1180	0.9	0.958	0.75	45	5.51	0.16	0.12	0.20
1180	0.9	0.958	0.75	75	8.97	0.44	0.52	2.10
1180	0.9	0.958	0.75	315	4.62	0.12	0.69	0.70
1180	0.9	0.958	0.75	345	3.88	0.05	0.32	0.32
1180	0.9	0.958	0.85	15	3.30	0.12	0.07	0.14
1180	0.9	0.958	0.85	45	4.85	0.06	0.11	0.12
1180	0.9	0.958	0.85	75	7.29	0.21	1.40	1.40
1180	0.9	0.958	0.85	165	6.33	2.60	0.23	2.60
1180	0.9	0.958	0.85	225	8.35	1.60	0.26	1.60
1180	0.9	0.958	0.85	255	6.38	0.89	2.30	3.30

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1260	0.9	0.953	-0.25	195	3.00	0.06	0.10	0.11
1260	0.9	0.953	-0.25	345	4.57	0.05	0.11	0.12
1260	0.9	0.953	-0.15	15	4.30	0.04	0.11	0.12
1260	0.9	0.953	-0.15	165	3.10	0.05	0.10	0.11
1260	0.9	0.953	-0.15	195	3.20	0.05	0.12	0.13
1260	0.9	0.953	-0.15	345	4.29	0.04	0.11	0.12
1260	0.9	0.953	-0.05	15	4.26	0.05	0.11	0.12
1260	0.9	0.953	-0.05	165	3.34	0.07	0.12	0.14
1260	0.9	0.953	-0.05	195	3.29	0.07	0.16	0.17
1260	0.9	0.953	-0.05	345	4.17	0.05	0.11	0.12
1260	0.9	0.953	0.05	15	4.31	0.08	0.10	0.13
1260	0.9	0.953	0.05	165	3.43	0.13	0.11	0.17
1260	0.9	0.953	0.05	195	3.60	0.14	0.11	0.18
1260	0.9	0.953	0.05	345	4.09	0.08	0.10	0.13
1260	0.9	0.953	0.15	15	3.32	0.50	0.54	0.73
1260	0.9	0.953	0.15	345	3.62	0.65	0.10	0.65
1260	0.9	0.953	0.45	15	3.93	0.34	1.80	1.90
1260	0.9	0.953	0.45	45	5.33	0.14	0.49	0.51
1260	0.9	0.953	0.45	165	3.60	0.31	0.12	0.33
1260	0.9	0.953	0.45	195	4.03	0.29	0.12	0.31
1260	0.9	0.953	0.45	315	6.60	0.13	0.19	0.23
1260	0.9	0.953	0.45	345	4.83	0.41	1.50	1.50
1260	0.9	0.953	0.55	15	3.14	0.06	0.33	0.33
1260	0.9	0.953	0.55	45	5.18	0.07	0.41	0.42
1260	0.9	0.953	0.55	165	3.74	0.16	0.17	0.23
1260	0.9	0.953	0.55	195	4.33	0.17	0.22	0.28
1260	0.9	0.953	0.55	315	5.48	0.08	0.12	0.14
1260	0.9	0.953	0.55	345	3.77	0.07	0.07	0.10
1260	0.9	0.953	0.65	15	2.64	0.04	0.06	0.07
1260	0.9	0.953	0.65	45	4.78	0.10	0.14	0.17
1260	0.9	0.953	0.65	75	7.62	1.30	0.15	1.30
1260	0.9	0.953	0.65	135	6.97	1.10	1.30	3.00
1260	0.9	0.953	0.65	165	3.61	0.14	0.12	0.18
1260	0.9	0.953	0.65	195	4.19	0.15	0.32	0.35
1260	0.9	0.953	0.65	285	7.23	1.60	0.15	1.60
1260	0.9	0.953	0.65	315	4.76	0.10	0.15	0.18
1260	0.9	0.953	0.65	345	3.00	0.05	0.06	0.07
1260	0.9	0.953	0.75	15	2.82	0.15	0.36	0.39
1260	0.9	0.953	0.75	45	3.70	0.06	0.21	0.22
1260	0.9	0.953	0.75	75	5.45	0.39	0.36	0.53
1260	0.9	0.953	0.75	135	4.74	0.18	0.14	0.23
1260	0.9	0.953	0.75	165	3.32	0.22	0.11	0.24
1260	0.9	0.953	0.75	195	4.12	0.22	0.66	0.69
1260	0.9	0.953	0.75	225	5.83	0.21	0.73	0.76
1260	0.9	0.953	0.75	315	4.63	0.07	0.51	0.51
1260	0.9	0.953	0.75	345	3.00	0.14	1.20	1.20
1260	0.9	0.953	0.85	15	2.56	0.06	0.46	0.47

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1180	0.9	0.958	0.85	345	4.10	0.12	1.30	1.30
1180	0.9	0.958	0.95	15	4.54	0.06	0.13	0.14
1180	0.9	0.958	0.95	45	4.85	0.07	0.10	0.12
1180	0.9	0.958	0.95	75	7.05	0.81	0.23	0.85
1180	0.9	0.958	0.95	105	5.89	0.44	0.20	0.48
1180	0.9	0.958	0.95	135	6.41	0.30	0.21	0.37
1180	0.9	0.958	0.95	165	7.30	0.46	0.22	0.51
1180	0.9	0.958	0.95	225	6.41	0.29	0.21	0.36
1180	0.9	0.958	0.95	255	5.94	0.43	1.00	1.10
1180	0.9	0.958	0.95	315	4.75	0.06	0.26	0.27
1180	0.9	0.958	0.95	345	4.65	0.06	0.39	0.39
1180	1.1	0.947	-0.95	15	2.31	0.06	0.06	0.09
1180	1.1	0.947	-0.95	45	2.51	0.07	0.06	0.09
1180	1.1	0.947	-0.95	105	2.22	0.05	0.05	0.07
1180	1.1	0.947	-0.95	135	1.94	0.05	0.05	0.07
1180	1.1	0.947	-0.95	165	1.95	0.06	0.04	0.07
1180	1.1	0.947	-0.95	195	1.86	0.06	0.05	0.07
1180	1.1	0.947	-0.95	225	2.13	0.05	0.05	0.07
1180	1.1	0.947	-0.95	255	2.41	0.05	0.07	0.08
1180	1.1	0.947	-0.95	285	2.46	0.07	0.12	0.14
1180	1.1	0.947	-0.95	315	2.45	0.06	0.07	0.09
1180	1.1	0.947	-0.95	345	2.44	0.06	0.06	0.08
1180	1.1	0.947	-0.85	15	2.77	0.07	0.07	0.10
1180	1.1	0.947	-0.85	45	2.97	0.06	0.08	0.10
1180	1.1	0.947	-0.85	75	3.33	0.09	0.08	0.12
1180	1.1	0.947	-0.85	105	3.05	0.07	0.07	0.10
1180	1.1	0.947	-0.85	135	2.69	0.10	0.07	0.12
1180	1.1	0.947	-0.85	165	2.01	0.06	0.06	0.08
1180	1.1	0.947	-0.85	195	2.08	0.06	0.08	0.10
1180	1.1	0.947	-0.85	225	2.55	0.09	0.06	0.11
1180	1.1	0.947	-0.85	255	3.09	0.07	0.07	0.10
1180	1.1	0.947	-0.85	285	3.18	0.08	0.22	0.23
1180	1.1	0.947	-0.85	315	2.95	0.06	0.08	0.10
1180	1.1	0.947	-0.85	345	2.84	0.07	0.07	0.10
1180	1.1	0.947	-0.75	15	3.02	0.06	0.08	0.10
1180	1.1	0.947	-0.75	45	3.58	0.08	0.09	0.12
1180	1.1	0.947	-0.75	75	4.48	0.13	1.10	1.10
1180	1.1	0.947	-0.75	105	4.54	0.12	0.69	0.70
1180	1.1	0.947	-0.75	135	2.92	0.08	0.11	0.13
1180	1.1	0.947	-0.75	165	2.32	0.06	0.16	0.17
1180	1.1	0.947	-0.75	195	2.49	0.06	0.07	0.09
1180	1.1	0.947	-0.75	225	3.05	0.09	0.10	0.13
1180	1.1	0.947	-0.75	255	3.56	0.11	0.09	0.14
1180	1.1	0.947	-0.75	285	3.62	0.11	0.10	0.15
1180	1.1	0.947	-0.75	315	3.45	0.07	0.10	0.12
1180	1.1	0.947	-0.75	345	3.03	0.06	0.15	0.16
1180	1.1	0.947	-0.65	15	3.09	0.05	0.08	0.09

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1260	0.9	0.953	0.85	105	4.85	0.75	0.15	0.76
1260	0.9	0.953	0.85	135	4.05	0.11	0.17	0.20
1260	0.9	0.953	0.85	165	3.24	0.12	0.09	0.15
1260	0.9	0.953	0.85	195	3.68	0.12	0.09	0.15
1260	0.9	0.953	0.85	225	4.64	0.13	0.36	0.38
1260	0.9	0.953	0.85	255	5.30	0.82	0.15	0.83
1260	0.9	0.953	0.85	285	5.98	1.10	0.14	1.10
1260	0.9	0.953	0.85	315	3.84	0.06	0.18	0.19
1260	0.9	0.953	0.85	345	2.82	0.06	0.08	0.10
1260	0.9	0.953	0.95	15	2.45	0.20	0.05	0.21
1260	0.9	0.953	0.95	45	2.71	0.53	0.07	0.54
1260	0.9	0.953	0.95	75	3.43	0.27	0.36	0.45
1260	0.9	0.953	0.95	105	3.31	0.33	0.26	0.42
1260	0.9	0.953	0.95	135	3.02	0.47	0.92	1.00
1260	0.9	0.953	0.95	165	3.06	0.20	0.08	0.21
1260	0.9	0.953	0.95	225	3.28	0.49	0.09	0.50
1260	0.9	0.953	0.95	255	4.01	0.35	0.10	0.36
1260	0.9	0.953	0.95	285	3.74	0.29	0.21	0.36
1260	0.9	0.953	0.95	315	3.34	0.56	0.75	0.94
1260	0.9	0.953	0.95	345	2.64	0.21	0.05	0.21
1260	1.1	0.940	-0.95	15	2.01	0.07	0.12	0.14
1260	1.1	0.940	-0.95	45	1.88	0.05	0.05	0.07
1260	1.1	0.940	-0.95	105	1.93	0.08	0.05	0.10
1260	1.1	0.940	-0.95	135	1.62	0.05	0.05	0.07
1260	1.1	0.940	-0.95	165	1.54	0.05	0.05	0.07
1260	1.1	0.940	-0.95	195	1.65	0.05	0.07	0.09
1260	1.1	0.940	-0.95	225	1.88	0.05	0.18	0.19
1260	1.1	0.940	-0.95	255	2.29	0.09	0.17	0.19
1260	1.1	0.940	-0.95	285	2.11	0.04	0.05	0.07
1260	1.1	0.940	-0.95	315	2.03	0.05	0.05	0.07
1260	1.1	0.940	-0.95	345	2.12	0.07	0.05	0.08
1260	1.1	0.940	-0.85	15	2.20	0.05	0.06	0.08
1260	1.1	0.940	-0.85	45	2.43	0.06	0.10	0.12
1260	1.1	0.940	-0.85	75	2.69	0.04	0.07	0.08
1260	1.1	0.940	-0.85	105	2.46	0.06	0.30	0.30
1260	1.1	0.940	-0.85	135	1.90	0.04	0.06	0.07
1260	1.1	0.940	-0.85	165	1.41	0.09	0.05	0.10
1260	1.1	0.940	-0.85	195	1.46	0.08	0.05	0.10
1260	1.1	0.940	-0.85	225	2.26	0.04	0.08	0.09
1260	1.1	0.940	-0.85	255	2.86	0.06	0.09	0.11
1260	1.1	0.940	-0.85	285	3.25	0.05	0.07	0.09
1260	1.1	0.940	-0.85	315	2.75	0.07	0.07	0.10
1260	1.1	0.940	-0.85	345	2.21	0.05	0.06	0.08
1260	1.1	0.940	-0.75	15	2.44	0.05	0.13	0.14
1260	1.1	0.940	-0.75	45	2.96	0.05	0.10	0.11
1260	1.1	0.940	-0.75	75	3.70	0.06	0.16	0.17
1260	1.1	0.940	-0.75	105	3.47	0.06	0.22	0.23

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1180	1.1	0.947	-0.65	105	6.04	0.30	2.30	2.60
1180	1.1	0.947	-0.65	135	3.93	0.10	0.50	0.51
1180	1.1	0.947	-0.65	165	3.12	0.13	0.10	0.16
1180	1.1	0.947	-0.65	195	3.22	0.12	0.08	0.15
1180	1.1	0.947	-0.65	225	3.65	0.11	0.10	0.15
1180	1.1	0.947	-0.65	255	5.19	0.32	0.28	1.30
1180	1.1	0.947	-0.65	285	4.23	0.18	0.41	0.45
1180	1.1	0.947	-0.65	315	3.51	0.08	0.10	0.12
1180	1.1	0.947	-0.65	345	3.11	0.05	0.08	0.09
1180	1.1	0.947	-0.55	15	3.27	0.06	0.08	0.10
1180	1.1	0.947	-0.55	45	4.82	0.10	0.15	0.18
1180	1.1	0.947	-0.55	75	6.79	0.48	0.57	1.90
1180	1.1	0.947	-0.55	135	5.72	0.21	3.10	3.10
1180	1.1	0.947	-0.55	165	3.35	0.23	0.09	0.25
1180	1.1	0.947	-0.55	195	3.15	0.20	0.09	0.22
1180	1.1	0.947	-0.55	225	4.03	0.20	0.25	0.32
1180	1.1	0.947	-0.55	255	5.04	1.00	0.14	1.00
1180	1.1	0.947	-0.55	285	5.94	1.20	0.87	2.80
1180	1.1	0.947	-0.55	315	3.90	0.09	0.11	0.14
1180	1.1	0.947	-0.55	345	3.23	0.06	0.08	0.10
1180	1.1	0.947	-0.45	15	3.50	0.07	0.12	0.14
1180	1.1	0.947	-0.45	45	5.05	0.15	0.20	0.25
1180	1.1	0.947	-0.45	105	8.19	1.80	0.16	1.80
1180	1.1	0.947	-0.45	135	5.50	1.50	0.13	1.60
1180	1.1	0.947	-0.45	165	4.13	0.14	0.11	0.76
1180	1.1	0.947	-0.45	195	3.73	0.13	0.11	0.16
1180	1.1	0.947	-0.45	225	5.47	1.70	0.13	1.70
1180	1.1	0.947	-0.45	315	3.86	0.11	0.15	0.19
1180	1.1	0.947	-0.45	345	3.24	0.07	0.08	0.10
1180	1.1	0.947	-0.35	15	3.63	0.10	0.21	0.23
1180	1.1	0.947	-0.35	45	5.71	0.23	0.28	1.20
1180	1.1	0.947	-0.35	135	6.91	0.60	3.00	3.60
1180	1.1	0.947	-0.35	165	4.41	0.13	0.12	0.18
1180	1.1	0.947	-0.35	195	4.15	0.13	0.12	0.17
1180	1.1	0.947	-0.35	225	5.28	0.59	0.15	0.61
1180	1.1	0.947	-0.35	315	3.94	0.15	0.12	0.19
1180	1.1	0.947	-0.35	345	3.29	0.09	0.08	0.12
1180	1.1	0.947	-0.25	15	4.19	0.24	3.30	3.40
1180	1.1	0.947	-0.25	135	6.92	0.62	0.16	0.64
1180	1.1	0.947	-0.25	165	5.07	0.24	0.13	0.27
1180	1.1	0.947	-0.25	195	4.66	0.23	0.13	0.26
1180	1.1	0.947	-0.25	315	4.26	0.39	0.11	0.41
1180	1.1	0.947	-0.25	345	4.13	0.23	2.10	2.30
1180	1.1	0.947	-0.15	15	3.49	0.31	0.08	0.32
1180	1.1	0.947	-0.15	345	3.09	0.27	0.08	0.29
1180	1.1	0.947	-0.05	15	3.48	0.17	0.09	0.77
1180	1.1	0.947	-0.05	345	3.34	0.16	0.07	0.74

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1260	1.1	0.940	-0.75	195	1.67	0.07	0.07	0.09
1260	1.1	0.940	-0.75	225	2.78	0.06	0.10	0.12
1260	1.1	0.940	-0.75	255	4.05	0.07	0.10	0.13
1260	1.1	0.940	-0.75	285	4.22	0.07	0.24	0.25
1260	1.1	0.940	-0.75	315	3.33	0.06	0.22	0.22
1260	1.1	0.940	-0.75	345	2.64	0.05	0.39	0.40
1260	1.1	0.940	-0.65	15	2.77	0.08	0.07	0.10
1260	1.1	0.940	-0.65	45	3.63	0.06	0.11	0.12
1260	1.1	0.940	-0.65	75	5.18	0.10	0.16	0.19
1260	1.1	0.940	-0.65	105	4.82	0.09	0.41	0.42
1260	1.1	0.940	-0.65	135	2.76	0.06	0.61	0.61
1260	1.1	0.940	-0.65	165	1.54	0.04	0.06	0.07
1260	1.1	0.940	-0.65	195	1.71	0.04	0.05	0.07
1260	1.1	0.940	-0.65	225	3.20	0.07	0.09	0.12
1260	1.1	0.940	-0.65	255	4.90	0.11	0.25	0.27
1260	1.1	0.940	-0.65	285	5.08	0.11	0.20	0.23
1260	1.1	0.940	-0.65	315	3.77	0.06	0.12	0.13
1260	1.1	0.940	-0.65	345	3.21	0.08	0.79	0.80
1260	1.1	0.940	-0.55	15	2.87	0.05	0.07	0.09
1260	1.1	0.940	-0.55	45	4.39	0.08	0.11	0.13
1260	1.1	0.940	-0.55	75	6.97	0.22	0.60	0.64
1260	1.1	0.940	-0.55	105	6.11	0.20	0.29	0.35
1260	1.1	0.940	-0.55	135	3.26	0.06	0.12	0.13
1260	1.1	0.940	-0.55	165	1.57	0.05	0.06	0.07
1260	1.1	0.940	-0.55	195	1.83	0.05	0.06	0.08
1260	1.1	0.940	-0.55	225	3.80	0.07	0.19	0.20
1260	1.1	0.940	-0.55	255	5.91	0.35	0.48	0.60
1260	1.1	0.940	-0.55	285	5.43	0.29	0.23	0.37
1260	1.1	0.940	-0.55	315	4.36	0.09	0.11	0.14
1260	1.1	0.940	-0.55	345	2.94	0.05	0.10	0.11
1260	1.1	0.940	-0.45	15	2.80	0.04	0.08	0.09
1260	1.1	0.940	-0.45	45	4.99	0.09	0.12	0.15
1260	1.1	0.940	-0.45	75	10.60	1.00	11.00	11.00
1260	1.1	0.940	-0.45	105	8.32	0.64	0.74	0.97
1260	1.1	0.940	-0.45	135	3.96	0.07	0.14	0.15
1260	1.1	0.940	-0.45	165	1.54	0.11	0.06	0.12
1260	1.1	0.940	-0.45	195	2.12	0.16	0.06	0.58
1260	1.1	0.940	-0.45	225	4.30	0.08	0.28	0.29
1260	1.1	0.940	-0.45	285	5.91	1.30	0.16	1.30
1260	1.1	0.940	-0.45	315	4.66	0.09	0.18	0.20
1260	1.1	0.940	-0.45	345	3.19	0.04	0.18	0.19
1260	1.1	0.940	-0.35	15	2.84	0.05	0.13	0.14
1260	1.1	0.940	-0.35	45	5.93	0.12	0.22	0.25
1260	1.1	0.940	-0.35	135	4.39	0.14	0.21	0.25
1260	1.1	0.940	-0.35	165	2.29	0.69	0.06	0.69
1260	1.1	0.940	-0.35	225	5.04	0.17	1.50	1.50
1260	1.1	0.940	-0.35	315	4.75	0.11	0.12	0.17

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1180	1.1	0.947	0.15	345	3.29	0.29	0.20	1.00
1180	1.1	0.947	0.75	15	2.68	0.85	0.05	0.86
1180	1.1	0.947	0.75	345	2.79	0.85	0.05	0.85
1220	0.9	0.956	-0.95	15	4.99	0.11	0.13	0.17
1220	0.9	0.956	-0.95	45	5.11	0.14	0.22	0.26
1220	0.9	0.956	-0.95	105	4.96	0.15	0.12	0.19
1220	0.9	0.956	-0.95	135	4.18	0.16	0.11	0.20
1220	0.9	0.956	-0.95	165	3.37	0.12	0.16	0.19
1220	0.9	0.956	-0.95	195	3.44	0.12	0.10	0.16
1220	0.9	0.956	-0.95	225	4.18	0.16	0.25	0.29
1220	0.9	0.956	-0.95	255	4.49	0.13	0.13	0.18
1220	0.9	0.956	-0.95	285	5.24	0.15	0.13	0.20
1220	0.9	0.956	-0.95	315	4.96	0.13	0.19	0.22
1220	0.9	0.956	-0.95	345	4.82	0.11	0.13	0.17
1220	0.9	0.956	-0.85	15	6.17	0.19	0.60	0.63
1220	0.9	0.956	-0.85	45	7.47	0.19	0.31	0.37
1220	0.9	0.956	-0.85	75	9.23	0.66	0.19	2.50
1220	0.9	0.956	-0.85	105	7.95	0.52	1.10	1.20
1220	0.9	0.956	-0.85	135	4.53	0.17	0.15	0.22
1220	0.9	0.956	-0.85	165	3.71	0.13	0.11	0.17
1220	0.9	0.956	-0.85	195	3.37	0.12	0.11	0.16
1220	0.9	0.956	-0.85	225	4.91	0.17	0.15	0.23
1220	0.9	0.956	-0.85	255	6.47	0.53	0.17	0.56
1220	0.9	0.956	-0.85	285	8.09	0.46	1.60	1.60
1220	0.9	0.956	-0.85	315	6.17	0.17	0.18	0.25
1220	0.9	0.956	-0.85	345	5.77	0.18	0.15	0.24
1220	0.9	0.956	-0.75	15	6.58	0.08	0.16	0.18
1220	0.9	0.956	-0.75	45	9.37	0.27	1.30	1.30
1220	0.9	0.956	-0.75	165	3.59	0.16	0.19	0.25
1220	0.9	0.956	-0.75	195	3.71	0.16	0.12	0.20
1220	0.9	0.956	-0.75	225	5.12	0.49	0.83	1.80
1220	0.9	0.956	-0.75	255	9.33	2.70	0.25	2.80
1220	0.9	0.956	-0.75	315	6.89	0.19	0.47	0.51
1220	0.9	0.956	-0.75	345	5.99	0.07	0.16	0.17
1220	0.9	0.956	-0.65	15	6.74	0.10	0.17	0.19
1220	0.9	0.956	-0.65	45	10.30	0.21	0.22	0.31
1220	0.9	0.956	-0.65	135	7.57	0.64	0.21	0.67
1220	0.9	0.956	-0.65	165	4.15	0.10	0.13	0.16
1220	0.9	0.956	-0.65	195	3.98	0.10	0.13	0.16
1220	0.9	0.956	-0.65	225	8.07	0.68	0.21	2.40
1220	0.9	0.956	-0.65	315	8.19	0.18	0.22	0.29
1220	0.9	0.956	-0.65	345	6.70	0.10	0.17	0.19
1220	0.9	0.956	-0.55	15	6.98	0.21	0.24	0.32
1220	0.9	0.956	-0.55	45	11.80	0.37	1.40	1.50
1220	0.9	0.956	-0.55	165	4.69	0.16	0.14	0.21
1220	0.9	0.956	-0.55	195	4.46	0.15	0.17	0.23
1220	0.9	0.956	-0.55	225	8.76	0.61	0.24	0.65

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1260	1.1	0.940	-0.25	45	7.33	0.26	2.60	2.60
1260	1.1	0.940	-0.25	135	5.18	0.29	0.13	0.32
1260	1.1	0.940	-0.25	315	4.59	0.20	0.22	0.30
1260	1.1	0.940	-0.25	345	3.13	0.09	0.08	0.12
1260	1.1	0.940	-0.15	165	1.99	0.29	0.17	0.77
1260	1.1	0.940	-0.15	195	1.95	0.24	0.56	0.61
1260	1.1	0.940	-0.05	15	3.20	0.10	0.08	0.13
1260	1.1	0.940	-0.05	165	2.23	0.08	0.08	0.12
1260	1.1	0.940	-0.05	195	2.60	0.08	0.16	0.18
1260	1.1	0.940	-0.05	345	2.88	0.09	0.08	0.12
1260	1.1	0.940	0.05	15	3.24	0.08	0.08	0.11
1260	1.1	0.940	0.05	165	2.44	0.07	0.08	0.11
1260	1.1	0.940	0.05	195	2.55	0.07	0.08	0.10
1260	1.1	0.940	0.05	345	2.98	0.07	0.08	0.10
1260	1.1	0.940	0.15	15	3.03	0.11	0.07	0.13
1260	1.1	0.940	0.15	165	2.33	0.07	0.23	0.24
1260	1.1	0.940	0.15	195	2.55	0.08	0.09	0.12
1260	1.1	0.940	0.15	345	2.83	0.10	0.07	0.13
1260	1.1	0.940	0.25	165	2.58	0.13	0.21	0.24
1260	1.1	0.940	0.25	195	2.45	0.14	0.13	0.19
1260	1.1	0.940	0.35	165	2.36	0.34	0.09	0.35
1260	1.1	0.940	0.55	15	2.12	0.16	0.08	0.17
1260	1.1	0.940	0.55	45	4.13	0.80	0.09	0.81
1260	1.1	0.940	0.55	315	4.60	0.90	0.73	2.20
1260	1.1	0.940	0.55	345	2.22	0.29	0.05	0.30
1260	1.1	0.940	0.65	15	1.90	0.05	0.05	0.07
1260	1.1	0.940	0.65	45	4.60	0.24	2.60	2.60
1260	1.1	0.940	0.65	165	2.60	0.15	0.09	0.17
1260	1.1	0.940	0.65	195	2.58	0.14	0.09	0.16
1260	1.1	0.940	0.65	315	3.30	0.25	0.07	0.26
1260	1.1	0.940	0.65	345	2.00	0.05	0.04	0.07
1260	1.1	0.940	0.75	15	1.76	0.07	0.10	0.12
1260	1.1	0.940	0.75	165	2.67	0.09	0.08	0.12
1260	1.1	0.940	0.75	195	2.70	0.09	0.08	0.12
1260	1.1	0.940	0.75	225	3.18	0.73	1.10	1.90
1260	1.1	0.940	0.75	315	2.82	0.20	0.30	0.36
1260	1.1	0.940	0.75	345	1.77	0.08	0.04	0.09
1260	1.1	0.940	0.85	15	1.70	0.05	0.04	0.06
1260	1.1	0.940	0.85	45	2.78	0.08	0.12	0.14
1260	1.1	0.940	0.85	135	3.75	0.11	0.30	0.32
1260	1.1	0.940	0.85	165	2.53	0.15	0.07	0.16
1260	1.1	0.940	0.85	195	2.63	0.15	0.07	0.17
1260	1.1	0.940	0.85	225	3.24	0.10	0.09	0.13
1260	1.1	0.940	0.85	315	2.37	0.08	0.05	0.09
1260	1.1	0.940	0.85	345	1.63	0.05	0.10	0.12
1260	1.1	0.940	0.95	15	1.67	0.10	0.03	0.11
1260	1.1	0.940	0.95	45	2.52	0.33	0.47	0.57

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1220	0.9	0.956	-0.45	15	6.80	0.16	0.19	0.25
1220	0.9	0.956	-0.45	165	6.08	1.90	0.15	1.90
1220	0.9	0.956	-0.45	345	6.89	0.16	0.19	0.24
1220	0.9	0.956	-0.35	15	7.30	0.09	0.20	0.22
1220	0.9	0.956	-0.35	345	7.24	0.08	0.18	0.20
1220	0.9	0.956	-0.25	15	7.17	0.07	0.18	0.19
1220	0.9	0.956	-0.25	165	5.94	0.44	0.20	0.49
1220	0.9	0.956	-0.25	195	6.17	0.43	0.20	0.47
1220	0.9	0.956	-0.25	345	7.16	0.07	0.18	0.19
1220	0.9	0.956	-0.15	15	7.01	0.07	0.18	0.19
1220	0.9	0.956	-0.15	165	7.08	0.21	0.22	0.31
1220	0.9	0.956	-0.15	195	6.61	0.20	0.25	0.32
1220	0.9	0.956	-0.15	345	7.03	0.06	0.18	0.19
1220	0.9	0.956	-0.05	15	7.12	0.08	0.23	0.24
1220	0.9	0.956	-0.05	165	7.60	0.18	0.23	0.30
1220	0.9	0.956	-0.05	195	6.98	0.17	0.23	0.29
1220	0.9	0.956	-0.05	345	6.89	0.08	0.17	0.19
1220	0.9	0.956	0.05	15	6.72	0.14	0.16	0.22
1220	0.9	0.956	0.05	165	8.26	0.28	0.25	0.37
1220	0.9	0.956	0.05	195	7.81	0.27	0.25	0.37
1220	0.9	0.956	0.05	345	6.82	0.14	0.16	0.22
1220	0.9	0.956	0.15	15	5.53	2.50	0.15	2.50
1220	0.9	0.956	0.45	15	6.58	0.73	0.20	0.76
1220	0.9	0.956	0.45	45	9.22	0.18	0.35	0.39
1220	0.9	0.956	0.45	165	8.62	0.23	0.34	0.41
1220	0.9	0.956	0.45	195	8.12	0.21	0.30	0.36
1220	0.9	0.956	0.45	315	9.48	0.17	0.23	0.29
1220	0.9	0.956	0.45	345	6.66	1.40	0.11	1.40
1220	0.9	0.956	0.55	15	4.73	0.08	0.10	0.13
1220	0.9	0.956	0.55	45	9.14	0.09	0.20	0.22
1220	0.9	0.956	0.55	165	8.66	0.13	0.28	0.31
1220	0.9	0.956	0.55	195	8.30	0.13	0.33	0.35
1220	0.9	0.956	0.55	315	8.42	0.09	0.38	0.39
1220	0.9	0.956	0.55	345	5.34	0.10	0.10	0.15
1220	0.9	0.956	0.65	15	4.69	0.05	0.10	0.11
1220	0.9	0.956	0.65	45	8.79	0.11	0.34	0.36
1220	0.9	0.956	0.65	75	13.20	1.40	0.28	1.40
1220	0.9	0.956	0.65	135	11.20	0.62	0.34	0.71
1220	0.9	0.956	0.65	165	9.24	0.15	0.33	0.37
1220	0.9	0.956	0.65	195	8.64	0.14	0.30	0.33
1220	0.9	0.956	0.65	225	10.80	0.45	0.34	0.56
1220	0.9	0.956	0.65	285	13.00	0.64	0.28	0.69
1220	0.9	0.956	0.65	315	7.58	0.10	0.45	0.46
1220	0.9	0.956	0.65	345	5.13	0.05	0.30	0.31
1220	0.9	0.956	0.75	15	4.52	0.08	0.09	0.12
1220	0.9	0.956	0.75	45	6.70	0.09	0.43	0.44
1220	0.9	0.956	0.75	75	11.50	0.23	1.80	1.80

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1260	1.1	0.940	0.95	135	2.63	0.12	0.06	0.13
1260	1.1	0.940	0.95	165	2.30	0.07	0.06	0.09
1260	1.1	0.940	0.95	225	2.73	0.13	0.20	0.24
1260	1.1	0.940	0.95	255	3.00	0.45	0.07	0.45
1260	1.1	0.940	0.95	285	2.73	0.22	0.15	0.26
1260	1.1	0.940	0.95	315	2.54	0.50	0.06	0.50
1260	1.1	0.940	0.95	345	1.52	0.10	0.03	0.10
1300	0.9	0.949	-0.95	15	1.16	0.02	0.03	0.04
1300	0.9	0.949	-0.95	45	1.21	0.02	0.03	0.04
1300	0.9	0.949	-0.95	105	1.29	0.03	0.05	0.05
1300	0.9	0.949	-0.95	135	1.26	0.02	0.04	0.04
1300	0.9	0.949	-0.95	165	1.15	0.03	0.06	0.06
1300	0.9	0.949	-0.95	195	1.17	0.03	0.04	0.05
1300	0.9	0.949	-0.95	225	1.29	0.02	0.05	0.05
1300	0.9	0.949	-0.95	255	1.45	0.03	0.04	0.05
1300	0.9	0.949	-0.95	285	1.36	0.03	0.04	0.05
1300	0.9	0.949	-0.95	315	1.23	0.02	0.03	0.04
1300	0.9	0.949	-0.95	345	1.17	0.02	0.04	0.04
1300	0.9	0.949	-0.85	15	1.32	0.03	0.04	0.05
1300	0.9	0.949	-0.85	45	1.64	0.03	0.07	0.08
1300	0.9	0.949	-0.85	75	2.03	0.03	0.21	0.21
1300	0.9	0.949	-0.85	105	1.95	0.03	0.13	0.13
1300	0.9	0.949	-0.85	135	1.65	0.04	0.05	0.07
1300	0.9	0.949	-0.85	165	1.34	0.03	0.04	0.05
1300	0.9	0.949	-0.85	195	1.41	0.03	0.04	0.05
1300	0.9	0.949	-0.85	225	1.82	0.05	0.08	0.09
1300	0.9	0.949	-0.85	255	2.17	0.03	0.06	0.07
1300	0.9	0.949	-0.85	285	2.06	0.04	0.05	0.06
1300	0.9	0.949	-0.85	315	1.78	0.04	0.05	0.06
1300	0.9	0.949	-0.85	345	1.35	0.03	0.04	0.05
1300	0.9	0.949	-0.75	15	1.55	0.03	0.08	0.08
1300	0.9	0.949	-0.75	45	2.15	0.04	0.06	0.08
1300	0.9	0.949	-0.75	75	3.00	0.07	0.07	0.10
1300	0.9	0.949	-0.75	105	2.87	0.06	0.10	0.11
1300	0.9	0.949	-0.75	135	1.91	0.04	0.07	0.08
1300	0.9	0.949	-0.75	165	1.31	0.04	0.06	0.07
1300	0.9	0.949	-0.75	195	1.43	0.04	0.04	0.06
1300	0.9	0.949	-0.75	225	2.26	0.04	0.06	0.07
1300	0.9	0.949	-0.75	255	3.02	0.07	0.08	0.11
1300	0.9	0.949	-0.75	285	2.90	0.08	0.09	0.12
1300	0.9	0.949	-0.75	315	2.23	0.04	0.06	0.08
1300	0.9	0.949	-0.75	345	1.60	0.03	0.07	0.07
1300	0.9	0.949	-0.65	15	1.81	0.03	0.05	0.06
1300	0.9	0.949	-0.65	45	2.75	0.06	0.08	0.10
1300	0.9	0.949	-0.65	75	4.83	0.26	3.40	3.40
1300	0.9	0.949	-0.65	105	4.54	0.28	1.00	1.50
1300	0.9	0.949	-0.65	135	2.34	0.04	0.17	0.17

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1220	0.9	0.956	0.75	195	8.27	0.09	0.26	0.27
1220	0.9	0.956	0.75	225	10.30	0.19	0.34	0.39
1220	0.9	0.956	0.75	285	9.14	0.22	0.24	0.32
1220	0.9	0.956	0.75	315	7.66	0.10	0.77	0.77
1220	0.9	0.956	0.75	345	4.81	0.08	0.10	0.13
1220	0.9	0.956	0.85	15	4.67	0.07	0.72	0.72
1220	0.9	0.956	0.85	45	6.06	0.05	0.19	0.19
1220	0.9	0.956	0.85	75	9.60	0.27	2.00	2.00
1220	0.9	0.956	0.85	105	10.00	0.48	0.31	0.57
1220	0.9	0.956	0.85	135	10.70	0.21	1.40	1.40
1220	0.9	0.956	0.85	165	8.01	0.07	0.23	0.24
1220	0.9	0.956	0.85	195	8.02	0.07	0.23	0.24
1220	0.9	0.956	0.85	225	9.50	0.21	0.67	0.70
1220	0.9	0.956	0.85	255	10.70	0.50	1.30	1.40
1220	0.9	0.956	0.85	285	8.48	0.33	0.90	0.96
1220	0.9	0.956	0.85	315	6.52	0.05	0.39	0.39
1220	0.9	0.956	0.85	345	4.81	0.07	0.09	0.12
1220	0.9	0.956	0.95	15	4.95	0.07	0.19	0.20
1220	0.9	0.956	0.95	45	5.57	0.13	0.12	0.17
1220	0.9	0.956	0.95	75	7.79	0.54	0.21	0.58
1220	0.9	0.956	0.95	105	7.73	0.20	0.23	0.30
1220	0.9	0.956	0.95	135	8.34	0.36	0.39	0.53
1220	0.9	0.956	0.95	165	7.97	0.27	0.21	0.34
1220	0.9	0.956	0.95	225	7.93	0.36	0.23	0.42
1220	0.9	0.956	0.95	255	7.96	0.21	0.23	0.31
1220	0.9	0.956	0.95	285	7.59	0.56	0.21	0.60
1220	0.9	0.956	0.95	315	5.62	0.14	0.12	0.18
1220	0.9	0.956	0.95	345	5.01	0.07	0.12	0.14
1220	1.1	0.944	-0.95	15	3.51	0.07	0.10	0.12
1220	1.1	0.944	-0.95	45	3.81	0.08	0.14	0.16
1220	1.1	0.944	-0.95	105	3.34	0.05	0.09	0.10
1220	1.1	0.944	-0.95	135	2.90	0.07	0.08	0.10
1220	1.1	0.944	-0.95	165	2.53	0.05	0.08	0.10
1220	1.1	0.944	-0.95	195	2.55	0.05	0.11	0.12
1220	1.1	0.944	-0.95	225	2.94	0.07	0.08	0.10
1220	1.1	0.944	-0.95	255	3.54	0.05	0.11	0.12
1220	1.1	0.944	-0.95	285	3.82	0.05	0.09	0.11
1220	1.1	0.944	-0.95	315	3.83	0.08	0.15	0.17
1220	1.1	0.944	-0.95	345	3.66	0.07	0.09	0.12
1220	1.1	0.944	-0.85	15	4.25	0.06	0.11	0.13
1220	1.1	0.944	-0.85	45	4.64	0.07	0.17	0.18
1220	1.1	0.944	-0.85	75	4.97	0.07	0.13	0.14
1220	1.1	0.944	-0.85	105	4.82	0.08	0.13	0.15
1220	1.1	0.944	-0.85	135	3.28	0.05	0.16	0.16
1220	1.1	0.944	-0.85	165	2.46	0.03	0.08	0.09
1220	1.1	0.944	-0.85	195	2.57	0.04	0.08	0.09
1220	1.1	0.944	-0.85	225	3.65	0.05	0.11	0.12

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1300	0.9	0.949	-0.65	225	2.71	0.05	0.08	0.10
1300	0.9	0.949	-0.65	255	3.79	0.23	0.10	0.25
1300	0.9	0.949	-0.65	285	3.93	0.26	0.09	0.28
1300	0.9	0.949	-0.65	315	2.73	0.07	0.08	0.10
1300	0.9	0.949	-0.65	345	1.81	0.03	0.05	0.06
1300	0.9	0.949	-0.55	15	1.91	0.03	0.06	0.06
1300	0.9	0.949	-0.55	45	3.27	0.09	0.08	0.12
1300	0.9	0.949	-0.55	135	2.86	0.08	0.08	0.12
1300	0.9	0.949	-0.55	225	3.32	0.11	0.14	0.17
1300	0.9	0.949	-0.55	315	2.83	0.08	0.13	0.15
1300	0.9	0.949	-0.55	345	2.00	0.03	0.06	0.06
1300	0.9	0.949	-0.45	15	2.08	0.04	0.07	0.08
1300	0.9	0.949	-0.45	45	3.83	0.11	0.09	0.14
1300	0.9	0.949	-0.45	135	4.04	0.29	2.50	2.50
1300	0.9	0.949	-0.45	225	3.24	0.42	0.09	0.43
1300	0.9	0.949	-0.45	315	3.48	0.10	0.09	0.14
1300	0.9	0.949	-0.45	345	2.14	0.04	0.06	0.08
1300	0.9	0.949	-0.35	15	2.57	0.10	0.16	0.19
1300	0.9	0.949	-0.35	45	4.65	0.27	1.20	1.20
1300	0.9	0.949	-0.35	165	2.06	0.30	0.13	0.79
1300	0.9	0.949	-0.35	195	2.15	0.28	0.11	0.78
1300	0.9	0.949	-0.35	315	3.16	0.26	0.21	0.33
1300	0.9	0.949	-0.35	345	2.51	0.10	0.08	0.13
1300	0.9	0.949	-0.25	15	2.48	0.04	0.07	0.08
1300	0.9	0.949	-0.25	165	1.46	0.23	0.29	0.65
1300	0.9	0.949	-0.25	195	1.94	0.22	0.12	0.25
1300	0.9	0.949	-0.25	345	2.52	0.04	0.08	0.09
1300	0.9	0.949	-0.15	15	2.42	0.03	0.07	0.08
1300	0.9	0.949	-0.15	165	1.77	0.29	0.06	0.29
1300	0.9	0.949	-0.15	195	2.21	0.36	0.52	0.63
1300	0.9	0.949	-0.15	345	2.55	0.03	0.07	0.08
1300	0.9	0.949	-0.05	15	2.27	0.03	0.07	0.07
1300	0.9	0.949	-0.05	165	1.78	0.85	0.06	0.85
1300	0.9	0.949	-0.05	345	2.39	0.03	0.07	0.07
1300	0.9	0.949	0.05	15	2.35	0.05	0.07	0.08
1300	0.9	0.949	0.05	345	2.37	0.05	0.07	0.08
1300	0.9	0.949	0.15	15	2.10	0.18	0.39	0.43
1300	0.9	0.949	0.15	345	2.21	0.19	0.06	0.20
1300	0.9	0.949	0.45	15	2.49	0.27	0.05	0.27
1300	0.9	0.949	0.45	45	2.99	0.18	0.79	0.81
1300	0.9	0.949	0.45	315	4.07	0.18	0.14	0.23
1300	0.9	0.949	0.45	345	3.30	0.31	0.86	1.30
1300	0.9	0.949	0.55	15	2.11	0.07	0.04	0.08
1300	0.9	0.949	0.55	45	2.59	0.09	0.22	0.54
1300	0.9	0.949	0.55	315	3.04	0.10	0.43	0.44
1300	0.9	0.949	0.55	345	2.69	0.09	0.68	0.84
1300	0.9	0.949	0.65	15	1.69	0.07	0.04	0.35

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1220	1.1	0.944	-0.85	315	4.76	0.07	0.13	0.14
1220	1.1	0.944	-0.85	345	4.29	0.06	0.19	0.20
1220	1.1	0.944	-0.75	15	4.46	0.08	0.12	0.14
1220	1.1	0.944	-0.75	45	5.64	0.07	0.24	0.25
1220	1.1	0.944	-0.75	75	6.74	0.10	0.16	0.19
1220	1.1	0.944	-0.75	105	6.20	0.11	0.57	0.58
1220	1.1	0.944	-0.75	135	4.21	0.05	0.15	0.16
1220	1.1	0.944	-0.75	165	2.69	0.06	0.09	0.11
1220	1.1	0.944	-0.75	195	2.82	0.06	0.10	0.12
1220	1.1	0.944	-0.75	225	4.31	0.05	0.13	0.14
1220	1.1	0.944	-0.75	255	6.11	0.11	0.18	0.21
1220	1.1	0.944	-0.75	285	6.70	0.11	0.32	0.33
1220	1.1	0.944	-0.75	315	5.65	0.07	0.15	0.16
1220	1.1	0.944	-0.75	345	4.58	0.09	0.12	0.15
1220	1.1	0.944	-0.65	15	4.89	0.07	0.13	0.15
1220	1.1	0.944	-0.65	45	6.52	0.09	0.17	0.19
1220	1.1	0.944	-0.65	75	8.67	0.16	0.92	0.93
1220	1.1	0.944	-0.65	105	7.85	0.17	0.69	0.71
1220	1.1	0.944	-0.65	135	5.21	0.09	0.15	0.17
1220	1.1	0.944	-0.65	165	2.93	0.05	0.10	0.11
1220	1.1	0.944	-0.65	195	3.00	0.05	0.10	0.11
1220	1.1	0.944	-0.65	225	5.42	0.09	0.15	0.18
1220	1.1	0.944	-0.65	255	7.60	0.19	0.20	0.28
1220	1.1	0.944	-0.65	285	7.46	0.16	0.26	0.31
1220	1.1	0.944	-0.65	315	6.42	0.10	0.19	0.21
1220	1.1	0.944	-0.65	345	4.85	0.07	0.12	0.14
1220	1.1	0.944	-0.55	15	5.03	0.06	0.12	0.14
1220	1.1	0.944	-0.55	45	7.37	0.11	0.17	0.20
1220	1.1	0.944	-0.55	75	12.40	0.36	5.50	5.50
1220	1.1	0.944	-0.55	105	10.10	0.28	0.24	0.37
1220	1.1	0.944	-0.55	135	5.89	0.11	0.51	0.52
1220	1.1	0.944	-0.55	165	3.18	0.04	0.11	0.12
1220	1.1	0.944	-0.55	195	3.31	0.04	0.11	0.11
1220	1.1	0.944	-0.55	225	5.97	0.12	0.18	0.22
1220	1.1	0.944	-0.55	255	9.31	0.37	0.52	0.64
1220	1.1	0.944	-0.55	285	8.23	0.40	0.22	0.46
1220	1.1	0.944	-0.55	315	6.72	0.10	0.20	0.22
1220	1.1	0.944	-0.55	345	5.13	0.06	0.15	0.16
1220	1.1	0.944	-0.45	15	4.94	0.07	0.12	0.14
1220	1.1	0.944	-0.45	45	8.92	0.13	0.35	0.38
1220	1.1	0.944	-0.45	105	14.40	0.79	8.30	8.90
1220	1.1	0.944	-0.45	135	7.02	0.12	0.35	0.36
1220	1.1	0.944	-0.45	165	3.68	0.05	0.11	0.12
1220	1.1	0.944	-0.45	195	3.75	0.05	0.11	0.12
1220	1.1	0.944	-0.45	225	6.85	0.12	0.19	0.23
1220	1.1	0.944	-0.45	255	12.00	1.60	8.60	8.70
1220	1.1	0.944	-0.45	285	8.10	1.80	1.50	4.10

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1300	0.9	0.949	0.65	345	1.89	0.08	0.04	0.08
1300	0.9	0.949	0.85	75	1.78	0.67	0.06	0.67
1300	0.9	0.949	0.85	285	2.19	0.58	0.06	0.58
1300	0.9	0.949	0.95	15	1.50	0.46	1.70	1.70
1300	0.9	0.949	0.95	45	1.72	0.28	0.04	0.28
1300	0.9	0.949	0.95	75	1.77	0.32	0.59	0.67
1300	0.9	0.949	0.95	105	3.34	2.30	0.04	2.30
1300	0.9	0.949	0.95	285	2.04	0.28	0.68	0.73
1300	0.9	0.949	0.95	315	1.63	0.44	0.04	0.44
1300	0.9	0.949	0.95	345	1.51	0.70	0.03	0.70
1300	1.1	0.936	-0.95	15	1.09	0.12	0.40	0.54
1300	1.1	0.936	-0.95	45	1.09	0.06	0.20	0.21
1300	1.1	0.936	-0.95	105	1.22	0.08	0.11	0.14
1300	1.1	0.936	-0.95	135	1.02	0.08	0.03	0.09
1300	1.1	0.936	-0.95	165	0.96	0.12	0.03	0.12
1300	1.1	0.936	-0.95	195	1.05	0.12	0.10	0.37
1300	1.1	0.936	-0.95	255	0.97	0.08	0.04	0.28
1300	1.1	0.936	-0.95	285	1.12	0.13	0.03	0.13
1300	1.1	0.936	-0.95	315	1.06	0.07	0.02	0.07
1300	1.1	0.936	-0.95	345	1.10	0.08	0.21	0.35
1300	1.1	0.936	-0.85	15	1.16	0.08	0.06	0.10
1300	1.1	0.936	-0.85	45	1.18	0.05	0.03	0.06
1300	1.1	0.936	-0.85	75	1.30	0.05	0.30	0.31
1300	1.1	0.936	-0.85	105	1.32	0.07	0.15	0.17
1300	1.1	0.936	-0.85	135	1.00	0.45	0.04	0.46
1300	1.1	0.936	-0.85	165	1.28	0.16	0.18	0.49
1300	1.1	0.936	-0.85	225	1.33	0.20	0.04	0.20
1300	1.1	0.936	-0.85	255	1.63	0.08	0.05	0.09
1300	1.1	0.936	-0.85	285	1.72	0.06	0.07	0.09
1300	1.1	0.936	-0.85	315	1.63	0.06	0.14	0.15
1300	1.1	0.936	-0.85	345	1.17	0.38	0.03	0.38
1300	1.1	0.936	-0.75	15	1.28	0.04	0.04	0.06
1300	1.1	0.936	-0.75	45	1.52	0.06	0.09	0.11
1300	1.1	0.936	-0.75	75	1.98	0.06	0.06	0.09
1300	1.1	0.936	-0.75	105	1.68	0.07	0.92	0.99
1300	1.1	0.936	-0.75	135	1.84	0.20	0.23	0.30
1300	1.1	0.936	-0.75	225	1.90	0.20	0.10	0.22
1300	1.1	0.936	-0.75	255	2.46	0.10	0.24	0.26
1300	1.1	0.936	-0.75	285	2.37	0.07	0.05	0.09
1300	1.1	0.936	-0.75	315	1.78	0.07	0.11	0.13
1300	1.1	0.936	-0.75	345	1.46	0.05	0.04	0.06
1300	1.1	0.936	-0.65	15	1.41	0.06	0.11	0.13
1300	1.1	0.936	-0.65	45	1.89	0.05	0.07	0.09
1300	1.1	0.936	-0.65	75	2.72	0.09	0.25	0.26
1300	1.1	0.936	-0.65	105	2.55	0.12	0.15	0.20
1300	1.1	0.936	-0.65	135	1.78	0.14	0.06	0.15
1300	1.1	0.936	-0.65	195	1.36	0.26	0.12	0.60

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1220	1.1	0.944	-0.35	15	4.94	0.09	0.12	0.15
1220	1.1	0.944	-0.35	45	9.72	0.22	0.72	0.75
1220	1.1	0.944	-0.35	135	7.94	0.15	0.21	0.26
1220	1.1	0.944	-0.35	165	4.15	0.08	0.13	0.15
1220	1.1	0.944	-0.35	195	4.30	0.09	0.12	0.15
1220	1.1	0.944	-0.35	225	7.18	0.15	0.21	0.26
1220	1.1	0.944	-0.35	315	7.78	0.18	0.32	0.37
1220	1.1	0.944	-0.35	345	4.96	0.08	0.13	0.15
1220	1.1	0.944	-0.25	15	5.27	0.14	0.12	0.19
1220	1.1	0.944	-0.25	45	11.60	0.40	5.30	5.30
1220	1.1	0.944	-0.25	135	10.00	0.25	1.10	1.10
1220	1.1	0.944	-0.25	165	4.67	0.22	0.20	0.30
1220	1.1	0.944	-0.25	195	4.42	0.23	0.13	0.26
1220	1.1	0.944	-0.25	315	7.32	0.33	0.33	0.47
1220	1.1	0.944	-0.25	345	5.45	0.15	0.12	0.19
1220	1.1	0.944	-0.15	15	5.08	0.34	0.13	0.36
1220	1.1	0.944	-0.15	165	5.86	0.68	0.65	2.10
1220	1.1	0.944	-0.15	345	4.91	0.35	0.23	0.42
1220	1.1	0.944	-0.05	15	5.21	0.12	0.12	0.17
1220	1.1	0.944	-0.05	165	5.36	0.17	0.17	0.24
1220	1.1	0.944	-0.05	195	4.99	0.17	0.17	0.93
1220	1.1	0.944	-0.05	345	4.91	0.12	0.12	0.17
1220	1.1	0.944	0.05	15	5.33	0.10	0.12	0.15
1220	1.1	0.944	0.05	165	6.10	0.11	0.22	0.24
1220	1.1	0.944	0.05	195	5.84	0.11	0.20	0.23
1220	1.1	0.944	0.05	345	4.96	0.10	0.12	0.15
1220	1.1	0.944	0.15	15	5.38	0.15	1.20	1.20
1220	1.1	0.944	0.15	165	6.49	0.11	0.25	0.27
1220	1.1	0.944	0.15	195	6.32	0.11	0.19	0.22
1220	1.1	0.944	0.15	345	4.29	0.13	0.11	0.17
1220	1.1	0.944	0.25	165	7.12	0.16	0.48	0.51
1220	1.1	0.944	0.25	195	6.65	0.16	1.40	1.40
1220	1.1	0.944	0.35	165	5.15	0.26	0.59	1.30
1220	1.1	0.944	0.35	195	5.57	0.31	0.20	0.37
1220	1.1	0.944	0.65	15	3.69	0.11	0.20	0.23
1220	1.1	0.944	0.65	165	6.85	0.27	0.20	0.34
1220	1.1	0.944	0.65	195	7.01	0.28	0.20	0.35
1220	1.1	0.944	0.65	315	5.15	1.10	0.40	2.40
1220	1.1	0.944	0.65	345	3.73	0.12	0.07	0.14
1220	1.1	0.944	0.75	15	3.46	0.12	0.07	0.13
1220	1.1	0.944	0.75	45	6.56	1.90	0.11	1.90
1220	1.1	0.944	0.75	165	6.92	0.25	0.19	0.32
1220	1.1	0.944	0.75	195	6.91	0.26	0.19	0.32
1220	1.1	0.944	0.75	225	9.76	4.00	0.22	4.00
1220	1.1	0.944	0.75	315	4.94	1.90	0.11	1.90
1220	1.1	0.944	0.75	345	3.18	0.11	0.07	0.13
1220	1.1	0.944	0.85	15	3.14	0.10	0.07	0.12

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1300	1.1	0.936	-0.65	285	2.87	0.13	0.06	0.15
1300	1.1	0.936	-0.65	315	1.97	0.06	0.06	0.08
1300	1.1	0.936	-0.65	345	1.75	0.08	0.04	0.09
1300	1.1	0.936	-0.55	15	1.63	0.07	0.10	0.12
1300	1.1	0.936	-0.55	45	2.30	0.07	0.10	0.12
1300	1.1	0.936	-0.55	75	4.14	0.54	0.09	0.55
1300	1.1	0.936	-0.55	135	1.77	0.11	0.92	0.93
1300	1.1	0.936	-0.55	225	2.55	0.15	0.11	0.19
1300	1.1	0.936	-0.55	285	2.66	0.73	0.08	0.73
1300	1.1	0.936	-0.55	315	2.21	0.10	0.07	0.12
1300	1.1	0.936	-0.55	345	1.74	0.07	0.11	0.13
1300	1.1	0.936	-0.45	15	1.64	0.04	0.05	0.06
1300	1.1	0.936	-0.45	45	3.18	0.11	0.22	0.25
1300	1.1	0.936	-0.45	135	2.18	0.18	0.07	0.64
1300	1.1	0.936	-0.45	225	2.35	0.29	0.07	0.30
1300	1.1	0.936	-0.45	315	2.30	0.11	0.09	0.14
1300	1.1	0.936	-0.45	345	1.79	0.04	0.05	0.06
1300	1.1	0.936	-0.35	15	1.63	0.04	0.05	0.06
1300	1.1	0.936	-0.35	45	3.23	0.10	0.07	0.12
1300	1.1	0.936	-0.35	135	2.24	0.74	0.07	0.75
1300	1.1	0.936	-0.35	315	2.67	0.10	0.07	0.12
1300	1.1	0.936	-0.35	345	1.86	0.05	0.05	0.07
1300	1.1	0.936	-0.25	15	1.69	0.12	0.29	0.32
1300	1.1	0.936	-0.25	45	4.08	0.20	1.30	1.30
1300	1.1	0.936	-0.25	315	2.41	0.23	0.49	0.54
1300	1.1	0.936	-0.25	345	1.72	0.17	0.05	0.54
1300	1.1	0.936	-0.15	165	1.33	0.63	0.05	0.63
1300	1.1	0.936	-0.15	195	1.52	0.62	0.05	0.63
1300	1.1	0.936	-0.05	15	1.83	0.13	0.05	0.14
1300	1.1	0.936	-0.05	165	1.68	0.49	0.50	1.00
1300	1.1	0.936	-0.05	345	1.70	0.13	0.06	0.47
1300	1.1	0.936	0.05	15	1.89	0.07	0.05	0.08
1300	1.1	0.936	0.05	195	1.81	0.59	0.05	0.59
1300	1.1	0.936	0.05	345	1.81	0.06	0.05	0.08
1300	1.1	0.936	0.15	15	2.01	0.10	0.05	0.11
1300	1.1	0.936	0.15	345	1.98	0.10	0.05	0.11
1300	1.1	0.936	0.55	15	1.21	0.31	0.22	0.65
1300	1.1	0.936	0.65	15	1.07	0.14	0.16	0.42
1300	1.1	0.936	0.65	315	1.69	0.64	0.04	0.64
1300	1.1	0.936	0.65	345	1.06	0.14	0.57	0.69
1300	1.1	0.936	0.95	45	1.72	0.90	0.03	0.90
1300	1.1	0.936	0.95	75	1.87	1.30	0.03	1.30
1340	0.9	0.945	-0.95	15	0.54	0.02	0.02	0.03
1340	0.9	0.945	-0.95	45	0.56	0.02	0.02	0.03
1340	0.9	0.945	-0.95	105	0.78	0.02	0.02	0.03
1340	0.9	0.945	-0.95	135	0.79	0.05	0.02	0.06
1340	0.9	0.945	-0.95	165	0.95	0.08	0.15	0.17

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1220	1.1	0.944	0.85	165	6.81	0.20	0.17	0.27
1220	1.1	0.944	0.85	195	6.23	0.20	0.17	0.27
1220	1.1	0.944	0.85	225	7.61	0.35	0.19	0.40
1220	1.1	0.944	0.85	315	4.62	0.18	0.10	0.20
1220	1.1	0.944	0.85	345	3.27	0.11	0.07	0.13
1220	1.1	0.944	0.95	15	3.57	0.13	0.07	0.15
1220	1.1	0.944	0.95	45	4.17	0.39	0.09	0.40
1220	1.1	0.944	0.95	75	5.06	0.64	0.49	1.90
1220	1.1	0.944	0.95	105	6.27	0.71	0.75	1.00
1220	1.1	0.944	0.95	135	7.28	0.78	1.20	1.40
1220	1.1	0.944	0.95	165	5.66	0.20	0.15	0.25
1220	1.1	0.944	0.95	225	6.55	0.75	0.15	0.77
1220	1.1	0.944	0.95	255	7.43	0.78	0.57	2.50
1220	1.1	0.944	0.95	285	6.06	0.72	0.15	0.74
1220	1.1	0.944	0.95	315	5.53	0.72	3.70	3.70
1220	1.1	0.944	0.95	345	3.56	0.13	0.07	0.15
1260	0.9	0.953	-0.95	15	2.48	0.05	0.07	0.08
1260	0.9	0.953	-0.95	45	2.61	0.05	0.08	0.09
1260	0.9	0.953	-0.95	105	2.67	0.06	0.07	0.09
1260	0.9	0.953	-0.95	135	2.20	0.04	0.07	0.08
1260	0.9	0.953	-0.95	165	1.88	0.03	0.06	0.07
1260	0.9	0.953	-0.95	195	1.91	0.03	0.06	0.07
1260	0.9	0.953	-0.95	225	2.25	0.04	0.08	0.09
1260	0.9	0.953	-0.95	255	2.62	0.06	0.07	0.09
1260	0.9	0.953	-0.95	285	2.71	0.04	0.07	0.08
1260	0.9	0.953	-0.95	315	2.69	0.05	0.07	0.08
1260	0.9	0.953	-0.95	345	2.54	0.05	0.07	0.08
1260	0.9	0.953	-0.85	15	2.96	0.05	0.08	0.10
1260	0.9	0.953	-0.85	45	3.75	0.07	0.36	0.37
1260	0.9	0.953	-0.85	75	4.33	0.08	0.16	0.18
1260	0.9	0.953	-0.85	105	4.06	0.08	0.53	0.54
1260	0.9	0.953	-0.85	135	2.87	0.04	0.13	0.14
1260	0.9	0.953	-0.85	165	1.96	0.05	0.06	0.08
1260	0.9	0.953	-0.85	195	2.02	0.05	0.09	0.10
1260	0.9	0.953	-0.85	225	2.83	0.04	0.09	0.10
1260	0.9	0.953	-0.85	255	3.98	0.08	0.12	0.14
1260	0.9	0.953	-0.85	285	4.32	0.08	0.10	0.13
1260	0.9	0.953	-0.85	315	3.42	0.06	0.10	0.12
1260	0.9	0.953	-0.85	345	2.93	0.05	0.12	0.13
1260	0.9	0.953	-0.75	15	3.52	0.05	0.09	0.10
1260	0.9	0.953	-0.75	45	4.79	0.10	0.75	0.76
1260	0.9	0.953	-0.75	105	5.78	0.16	0.31	0.35
1260	0.9	0.953	-0.75	135	3.69	0.07	1.00	1.00
1260	0.9	0.953	-0.75	165	1.98	0.03	0.07	0.07
1260	0.9	0.953	-0.75	195	2.03	0.03	0.07	0.07
1260	0.9	0.953	-0.75	225	3.49	0.07	0.12	0.14
1260	0.9	0.953	-0.75	255	5.66	0.16	0.15	0.22

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1340	0.9	0.945	-0.95	255	0.85	0.03	0.03	0.04
1340	0.9	0.945	-0.95	285	0.70	0.07	0.02	0.07
1340	0.9	0.945	-0.95	315	0.64	0.02	0.02	0.03
1340	0.9	0.945	-0.95	345	0.52	0.02	0.01	0.02
1340	0.9	0.945	-0.85	15	0.68	0.02	0.02	0.03
1340	0.9	0.945	-0.85	45	0.85	0.02	0.05	0.05
1340	0.9	0.945	-0.85	75	1.12	0.04	0.03	0.05
1340	0.9	0.945	-0.85	105	1.10	0.04	0.04	0.06
1340	0.9	0.945	-0.85	135	1.27	0.11	0.04	0.12
1340	0.9	0.945	-0.85	165	1.14	0.12	0.03	0.12
1340	0.9	0.945	-0.85	195	1.13	0.14	0.03	0.14
1340	0.9	0.945	-0.85	225	1.48	0.10	0.12	0.15
1340	0.9	0.945	-0.85	255	1.37	0.04	0.04	0.06
1340	0.9	0.945	-0.85	285	1.16	0.04	0.03	0.05
1340	0.9	0.945	-0.85	315	1.04	0.02	0.03	0.03
1340	0.9	0.945	-0.85	345	0.69	0.02	0.04	0.05
1340	0.9	0.945	-0.75	15	0.78	0.02	0.04	0.04
1340	0.9	0.945	-0.75	45	1.21	0.03	0.05	0.06
1340	0.9	0.945	-0.75	75	1.66	0.05	0.04	0.07
1340	0.9	0.945	-0.75	105	1.72	0.07	0.05	0.09
1340	0.9	0.945	-0.75	135	1.38	0.10	0.10	0.13
1340	0.9	0.945	-0.75	225	1.94	0.10	0.80	0.81
1340	0.9	0.945	-0.75	255	1.80	0.08	0.05	0.10
1340	0.9	0.945	-0.75	285	1.69	0.06	0.04	0.07
1340	0.9	0.945	-0.75	315	1.27	0.04	0.04	0.05
1340	0.9	0.945	-0.75	345	0.83	0.02	0.03	0.03
1340	0.9	0.945	-0.65	15	0.97	0.02	0.04	0.04
1340	0.9	0.945	-0.65	45	1.62	0.04	0.12	0.13
1340	0.9	0.945	-0.65	75	2.59	0.13	0.06	0.15
1340	0.9	0.945	-0.65	105	3.27	0.42	3.30	3.50
1340	0.9	0.945	-0.65	135	1.65	0.14	0.05	0.15
1340	0.9	0.945	-0.65	225	2.14	0.18	0.33	0.38
1340	0.9	0.945	-0.65	285	2.17	0.17	0.06	0.18
1340	0.9	0.945	-0.65	315	1.52	0.05	0.05	0.07
1340	0.9	0.945	-0.65	345	1.02	0.02	0.03	0.04
1340	0.9	0.945	-0.55	15	1.13	0.02	0.04	0.04
1340	0.9	0.945	-0.55	45	2.19	0.08	0.05	0.10
1340	0.9	0.945	-0.55	135	2.75	0.80	1.50	2.10
1340	0.9	0.945	-0.55	315	1.53	0.08	0.09	0.12
1340	0.9	0.945	-0.55	345	1.18	0.02	0.04	0.04
1340	0.9	0.945	-0.45	15	1.23	0.02	0.04	0.05
1340	0.9	0.945	-0.45	45	2.28	0.08	0.15	0.17
1340	0.9	0.945	-0.45	315	2.04	0.08	0.06	0.10
1340	0.9	0.945	-0.45	345	1.32	0.03	0.04	0.05
1340	0.9	0.945	-0.35	15	1.39	0.06	0.05	0.08
1340	0.9	0.945	-0.35	45	2.90	0.15	0.29	0.33
1340	0.9	0.945	-0.35	315	2.50	0.37	0.06	0.37

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1260	0.9	0.953	-0.75	345	3.27	0.04	0.09	0.10
1260	0.9	0.953	-0.65	15	3.78	0.04	0.10	0.11
1260	0.9	0.953	-0.65	45	5.72	0.12	0.44	0.46
1260	0.9	0.953	-0.65	135	4.28	0.07	0.50	0.50
1260	0.9	0.953	-0.65	165	2.18	0.03	0.07	0.08
1260	0.9	0.953	-0.65	195	2.23	0.04	0.08	0.09
1260	0.9	0.953	-0.65	225	4.15	0.07	0.12	0.14
1260	0.9	0.953	-0.65	255	6.61	0.66	0.18	0.69
1260	0.9	0.953	-0.65	285	5.12	1.10	0.67	2.50
1260	0.9	0.953	-0.65	315	4.90	0.11	0.25	0.28
1260	0.9	0.953	-0.65	345	3.65	0.04	0.10	0.10
1260	0.9	0.953	-0.55	15	3.90	0.06	0.10	0.12
1260	0.9	0.953	-0.55	45	6.78	0.14	0.15	0.20
1260	0.9	0.953	-0.55	135	5.31	0.10	0.34	0.36
1260	0.9	0.953	-0.55	165	2.43	0.09	0.07	0.11
1260	0.9	0.953	-0.55	195	2.48	0.09	0.07	0.12
1260	0.9	0.953	-0.55	225	4.83	0.10	0.14	0.17
1260	0.9	0.953	-0.55	315	5.60	0.13	0.15	0.19
1260	0.9	0.953	-0.55	345	3.91	0.06	0.10	0.12
1260	0.9	0.953	-0.45	15	4.35	0.14	0.31	0.34
1260	0.9	0.953	-0.45	45	8.09	0.34	3.10	3.50
1260	0.9	0.953	-0.45	165	2.15	0.62	0.64	1.30
1260	0.9	0.953	-0.45	225	5.31	0.33	0.15	0.36
1260	0.9	0.953	-0.45	315	6.19	0.28	0.18	0.33
1260	0.9	0.953	-0.45	345	4.04	0.14	0.25	0.29
1260	0.9	0.953	-0.35	15	4.43	0.08	0.11	0.14
1260	0.9	0.953	-0.35	165	2.88	0.15	0.09	0.17
1260	0.9	0.953	-0.35	195	3.07	0.13	0.09	0.16
1260	0.9	0.953	-0.35	345	4.46	0.08	0.11	0.14
1260	0.9	0.953	-0.25	15	4.45	0.05	0.11	0.12
1260	0.9	0.953	-0.25	165	3.00	0.07	0.10	0.12

W MeV	Q ² GeV ²	ε	X	ϕ	σ μb	Sta μb	Sys μb	$\Delta\sigma$ μb
1340	0.9	0.945	-0.25	345	1.72	0.06	0.05	0.07
1340	0.9	0.945	-0.15	15	1.67	0.03	0.05	0.05
1340	0.9	0.945	-0.15	345	1.73	0.03	0.05	0.06
1340	0.9	0.945	-0.05	15	1.64	0.03	0.05	0.05
1340	0.9	0.945	-0.05	345	1.68	0.03	0.05	0.05
1340	0.9	0.945	0.05	15	1.68	0.04	0.05	0.06
1340	0.9	0.945	0.05	345	1.61	0.05	0.05	0.06
1340	0.9	0.945	0.15	15	1.90	0.22	0.17	0.67
1340	0.9	0.945	0.85	45	2.68	0.73	0.03	0.73
1340	0.9	0.945	0.85	75	1.75	0.43	0.03	0.43
1340	0.9	0.945	0.85	285	3.34	0.58	0.03	0.58
1340	0.9	0.945	0.85	315	1.93	0.57	0.03	0.57
1340	0.9	0.945	0.95	15	1.71	0.35	0.02	0.35
1340	0.9	0.945	0.95	45	1.43	0.22	0.22	0.31
1340	0.9	0.945	0.95	75	1.38	0.67	0.02	0.67
1340	0.9	0.945	0.95	285	1.24	0.45	0.02	0.45
1340	0.9	0.945	0.95	315	1.55	0.22	0.43	0.48
1340	0.9	0.945	0.95	345	1.56	0.27	0.02	0.27
1340	1.1	0.932	-0.75	75	1.91	0.94	0.03	0.94
1340	1.1	0.932	-0.75	345	0.94	0.37	0.02	0.38
1340	1.1	0.932	-0.65	45	1.52	0.26	0.36	0.72
1340	1.1	0.932	-0.55	15	1.18	0.46	0.03	0.46
1340	1.1	0.932	-0.55	45	1.35	0.26	0.09	0.60
1340	1.1	0.932	-0.55	345	0.96	0.47	0.03	0.47
1340	1.1	0.932	-0.45	15	0.91	0.09	0.25	0.38
1340	1.1	0.932	-0.45	45	1.53	0.40	0.29	0.83
1340	1.1	0.932	-0.35	15	1.08	0.10	0.03	0.11
1340	1.1	0.932	-0.35	45	1.87	0.30	0.05	0.30
1340	1.1	0.932	-0.35	345	1.26	0.21	0.03	0.21
1340	1.1	0.932	-0.25	45	2.91	0.85	0.76	1.70
1340	1.1	0.932	0.15	15	1.01	0.40	0.03	0.41

Appendix B Response Functions Extracted from Cross Section Results

The follow response functions are extracted from cross section results for available (W, Q^2) points: $R_T + \epsilon_S R_L$, R_{LT} , R_{TT} and R'_{LT} . The polar distributions($\cos\theta_\pi$ dependencies) of these response functions are shown here. The dashed lines are MAID calculations, and the dotted lines are SAID calculations.

Figure 4-19: Response functions for $p(e, e'p)\pi^0$ at $W = 1180 MeV$ and $Q^2 = 0.9 GeV^2$.

Figure 4-20: Response functions for $p(e, e'p)\pi^0$ at $W = 1180 MeV$ and $Q^2 = 1.1 GeV^2$.

Figure 4-21: Response functions for $p(e, e'p)\pi^0$ at $W = 1220 \text{ MeV}$ and $Q^2 = 0.9 \text{ GeV}^2$.

Figure 4-22: Response functions for $p(e, e'p)\pi^0$ at $W = 1220 \text{ MeV}$ and $Q^2 = 1.1 \text{ GeV}^2$.

Figure 4-23: Response functions for $p(e, e'p)\pi^0$ at $W = 1260\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.

Figure 4-24: Response functions for $p(e, e'p)\pi^0$ at $W = 1260\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.

Figure 4-25: Response functions for $p(e, e'p)\pi^0$ at $W = 1300\text{MeV}$ and $Q^2 = 0.9\text{GeV}^2$.

Figure 4-26: Response functions for $p(e, e'p)\pi^0$ at $W = 1300\text{MeV}$ and $Q^2 = 1.1\text{GeV}^2$.

Figure 4-27: Response functions for $p(e, e'p)\pi^0$ at $W = 1340 \text{ MeV}$ and $Q^2 = 0.9 \text{ GeV}^2$.

Figure 4-28: Response functions for $p(e, e'p)\pi^0$ at $W = 1340 \text{ MeV}$ and $Q^2 = 1.1 \text{ GeV}^2$.

Bibliography

- [1] S. Frullani, J. Kelly and A. Sarty (Spokespersons), Investigation of the N to Delta Transition via Polarization Observables in Hall A, JLAB Experiment E91-011.
- [2] Rikki Roche, PhD thesis, Florida State University (2003).
- [3] K. Hagiwara *et al.*, Phys. Rev. **D 66**, 010001 (2002).
- [4] S. L. Glashow, Physica **A 96**, 27(1979).
- [5] N. Isgur, G. Karl and R. Koniuk, Phys. Rev. **D 25**, 2394(1982).
- [6] W. Albrecht *et al.*, Nucl. Phys. **B 25**, 1(1970); W. Albrecht *et al.*, Nucl. Phys. **B 27**, 615(1971); J. C. Alder *et al.*, Nucl. Phys. **B 46**, 573(1972).
- [7] R. Siddle *et al.*, Nucl. Phys. **B 35**, 93(1971).
- [8] C. Mistretta *et al.*, Phys. Rev. **184**, 1487(1969).
- [9] F. Kalleicher *et al.*, Z. Phys. **A 359**, 201(1997).
- [10] R. Beck *et al.*, Phys. Rev. Lett. **78**, 606(1997).
- [11] G. Blanpied *et al.*, Phys. Rev. Lett. **79**, 4337(1997).
- [12] V. V. Frolov *et al.*, Phys. Rev. Lett. **82**, 45(1999); K. Joo *et al.*, Phys. Rev. Lett. **88**, 122001-1(2002).
- [13] C. Mertz *et al.*, Phys. Rev. Lett. **86**, 2963(2001).

- [14] C. E. Carlson *et al.*, Phys. Rev. **D 34**, 2704(1986).
- [15] A. J. Buchmann and E. M. Henley, Phys. Rev. **C 63**, 015202(2001).
- [16] A. J. Buchmann, E. Hernandez and K. Yazaki, Phys. Lett. **B 269**, 35(1991); Nucl. Phys. **A 569**, 661(1994); A. J. Buchmann, E. Hernandez and Amand Faessler, Phys. Rev. **C 55**, 448(1997).
- [17] T. Sato and T. -S. Lee, Phys. Rev. **C 63**, 055201-1(2001) T. Sato and T. -S. Lee, Phys. Rev. **C 54**, 2660(1996).
- [18] S. S. Kamalov and S. N. Yang, Phys. Rev. Lett. **83**, 4494(1999); S. S. Kamalov and S. N. Yang, D. Drechsel, O. Hanstein and L. Tiator, Phys. Rev. **C 64**, 032201(2001); S. S. Kamalov and S. N. Yang, nucl-th/0005007.
- [19] G. Blanpied *et al.*, Phys. Rev. Lett. **79**, 4337(1997).
- [20] S. Capstick and G. Karl, Phys. Rev. **D 41**, 2767(1990).
- [21] M. Fiolhais, B. Golli and S. Sirca, Phys. Lett. **B 373**, 229(1996).
- [22] C. Giusti, F. D. Pacati, Nuc. Phys. **A 504**, 685 (1989).
- [23] James J. Kelly, private communication.
- [24] M. Simonius, Phys. Rev. Lett. **19**, 279(1967).
- [25] R. A. Arndt, I. I. Strakovsky and R. L. Workman, Pi N Newslett **16**, 150(2002), arXiv:nucl-th/0110001.
- [26] R. A. Arndt *et al.*, Phys. Rev. **C 42**, 1853(1990); R. A. Arndt *et al.*, Phys. Rev. **C 42**, 1864(1990); Z. Li *et al.*, Phys. Rev. **C 47**, 2759(1993); R. A. Arndt *et al.*, Phys. Rev. **C 53**, 430(1996).

- [27] L. Tiator, D. Drechsel, S. Kamalov and S. N. Yang, Pi N Newslett **16**,41(2002), arXiv:nucl-th/0111062; L. Tiator, D. Drechsel, O. Hanstein, S. S. Kamalov and S. N. Yang, Nucl. Phys. **A 689**,205(2001); D. Drechsel, O. Hanstein, S. S. Kamalov and L. Tiator, Nucl. Phys. **A 645**,145(1999); D. Drechsel and L. Tiator, J. Phys. G:Nucl. Part. Phys. **18**,449(1992).
- [28] C. W. Leemann and D. R. Douglas and G. A. Krafft, Annu. Rev. Nucl. Part. Sci. **51**,413 (2001).
- [29] R. Alley *et al.*, Nucl. Instr. Meth. **A 365**, 1 (1995).
- [30] B. D. Anderson *et al.*, Basic Instrumentation for Hall A at Jefferson Lab, Nucl. Instr. Meth., to be published.
- [31] Arun Saha, private communication.
- [32] Mark Jones, Report on BCM Calibration, E91-011 internal report.
- [33] K. G. Fissum *et al.*, Nucl. Instr. Meth. **A 449**, 505 (2000).
- [34] Airglass AB, BOX 150, 245 22 Staffanstorps, Sweden.
- [35] M. Berz, H. C. Hoffmann and H. Wollmick, Nucl. Instr. Mech. bf A 258,402 (1987).
- [36] GEANT - Detector Description and Simulation Tool, <http://wwwinfo.cern.ch/asd/geant/index.html>.
- [37] Ed Brash, private communication.
- [38] K. L. Brown, F. Rothacher, D. C. Carey and C. H. Iselin, TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, SLAC-91, Rev. 2, UC-28(1/A).
- [39] E.A.J.M. Offermann, ESPACE Users Guide (1997).

- [40] Jefferson Lab Data Acquisition Group, The CODA Data Acquisition System, <http://coda.jlab.org>.
- [41] Experimental Physics and Industrial Control System, <http://www.aps.anl.gov/epics>.
- [42] CERN Program Library Y250, http://wwwinfo.cern.ch/asdoc/hbook_html3/hboomain.html.
- [43] Physics Analysis Workstation, <http://wwwinfo.cern.ch/asd/paw>.
- [44] ROOT - An Objected-Oriented Data Analysis Framework, <http://root.cern.ch>.
- [45] J.B. Mandeville, PhD thesis, University of Illinois at Urbana-Champaign (1993).
- [46] KUIP, <http://wwwinfo.cern.ch/asdoc/kuip/kuipoverview.html>.
- [47] Nilanga Liyanage, private communication.
- [48] V. L. Rvachev, T. I. Sheiko, V. Shapiro and I. Tsukanov, Computational Mechanics **25**, 305 (2000).
- [49] V. L. Rvachev, T. I. Sheiko, V. Shapiro and I. Tsukanov, Computer-Aided Geometric Design **18**, 195 (2001).
- [50] Paul Ulmer, MCEEP, <http://www.physics.odu.edu/~ulmer/mceep/mceep.html>.
- [51] James J. Kelly, EPIPROD.
- [52] D. Drechsel, O. Hanstein, S.S. Kamalov and L. Tiator, Nucl. Phys. **A645**, 145 (1999) or <http://www.kph.uni-mainz.de/MAID/maid2000/maid2000.html>.
- [53] John Lerose, HRS Spectrometer Transfer Functions, <http://hallaweb.jlab.org/news/minutes/tranferfuncs.html>.
- [54] S. Penner, Nuclear Structure Physics, Proceedings of the 18th Scottish Univ. Summer School in Physics, 284(1977).

- [55] E. Borie and D. Drechsel, Nucl. Phys. **A 167**, 369 (1971).
- [56] William R. Leo, Techniques for Nuclear and Particle Physics Experiments, Springer Verlag, 1994.
- [57] G. R. Lynch, O. I. Dahl, Nucl. Inst. Meth. in Phys. Rev. **B 58**, 6(1991).
- [58] Mark Jones, Report on Electronic Deadtime, E91-011 internal report.

Acknowledgements

Many people provided important help for the accomplishment of this thesis, and they deserve my great appreciation.

First I would like to thank my research advisor Prof. William Bertozzi for his continuing guidance, attention, encouragement and support throughout my graduate study at MIT. Many enlightening discussions with him not only enhanced my physics insight, but also helped me understand how to become a successful physicist.

I would like to thank Dr. Shalev Gilad for helpful discussions and friendly suggestions throughout my time with the Nuclear Interaction Group. Being involved in all the details of my work, he frequently provided helpful advices for many difficult problems in my thesis work.

I would like to thank my thesis committee members: Dr. William Donnelly and Prof. Aron Bernstein for their valuable comments and corrections to this thesis.

I would like to thank Prof. James Kelly for his leading role as one of the spokespersons in this experiment. His high standard requirement for the experimental work greatly improved the quality of my analysis results. I would also like to thank the other two spokespersons of this experiment: Prof. Adam Sarty and Prof. Salvatore Frullani for their tireless efforts in the planning and performing of this experiment.

I would like to thank the post-docs with our group who kindly helped me in my research projects. They are: Drs. Kevin Fissum, Jianguo Zhao, David Rowntree, Zilu Zhou, Riad Sulein, Simon Sirca and Douglas Higinbotham.

I would like to thank Drs. Juncai Gao and Nilanga Liyange, who were graduate students with our group, for their invaluable help to my work in Hall A at Jefferson Lab. I was lucky to work with Nilanga Liyange on the optimization of the Hall A spectrometer optics. The optics optimization work deepened my understanding of the spectrometer acceptance and inspired the design of the new acceptance analysis method.

I would like to thank Dr. Mark Jones for his strategic guidance and kind help in

our analysis work with his rich experimental experience. I would also like to thank Dr. Xiaodong Jiang for stimulating discussions regarding the acceptance analysis and cross section measurement. I would also like to thank the other thesis student Rikki Roche for her hard work for this experiment.

I would like to thank the Hall A staff members: Drs. Kees de Jager, Jian-Ping Chen, Bob Michaels, Bogdan Wojsekhowski, John LeRose, Ole Hansen, Eugene Chudakov, Arun Saha, Joe Mitchell, Michael Kuss and the members of the Hall A collaboration for their commitment and shift efforts for this experiment. I would also like to thank the Jefferson Lab accelerator crew for delivering high quality polarized beam for this experiment.

I would like to thank my fellow graduate students whose friendship made my graduate study and work an enjoyable experience. They are: Marat Rvachev, Xiaochao Zheng, Bin Zhang, Alex Skabelin, Peter Monaghan, Bin Ma, Yuan Xiao, Chi Zhang, Lu Wan, Yi Qiang, Feng Xiong, Wang Xu, Lingyan Zhu. Special memories to Jindong Zhou for his dedication to physics, who was a graduate student with our group and passed away half a year after his work at Jefferson Lab.

I would like to thank my parents, my brother and my sister for their persistent support to my pursuit of the better education. Finally my thanks is to my wife Shuhua Wang for all her love and support to me through both easy and difficult times during all the years together.