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Two-Photon Exchange Effects in Elastic Electron-Proton Scattering

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ABSTRACT

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Two methods, Rosenbluth separation and polarization transfer, can be used to extract the proton form factor ratio $\mu_p G_{Ep}/G_{Mp}$, but they do not yield the same results. It is thought that the disagreement is due to two photon exchange corrections to the differential cross sections. High precision proton Rosenbluth extractions were carried out at 102 kinematics points spanning 16 values of momentum transfer Q^2 , from 0.40 to 5.76 GeV^2 . Reduced cross sections were found to 1.1% or better for Q^2 less than 3 GeV^2 , increasing to 4% at 5.76 GeV^2 . The form factor ratios were determined to 1.5 – 3% for $Q^2 < 1.5 GeV^2$, increasing to 9% by 3 GeV^2 and rapidly above. Our data agrees with prior Rosenbluth, improving upon it the 1.0 – 2.0 GeV^2 range to conclusively show a separation from polarization transfer where it had not been certain before. In addition, reduced cross sections at each Q^2 were tested for nonlinearity in the angular variable. Such a departure from linearity would be a signature of two photon exchange effects, and prior data had not been sufficiently precise to show nonzero curvature. Our data begins to hint at negative curvature but does not yet show a significant departure from zero.

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CHAPTER 1

Introduction

1.1. Historic Scattering

Scattering experiments began with Ernest Rutherford and the famous gold foil experiment. Prior to Rutherford, it was assumed that the atom was a 'plum pudding' of negatively charged electrons floating in a positively-charged 'pudding'. Rutherford scattered alpha particles (from a radon source) off of a thin sheet of gold foil, and found that rather than the small deflections expected from a charged pudding, most of the particles were hardly deflected at all, while about one in eight thousand were deflected at high angles, more than ninety degrees. As Rutherford said,

It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration, I realized that this scattering backward must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greater part of the mass of the atom was concentrated in a minute nucleus. It was then that I had the idea of an atom with a minute massive center, carrying a charge.

Calculating the size of the positively-charged center required to produce the observed scattering, Rutherford found an upper limit on the size of the nucleus at 3.4×10^{-14} meters, less than 1/4000 the radius of a gold atom (and about five times the actual radius of a gold nucleus). He also determined that the scattering obeyed the equation

$$(1.1) \quad \frac{d\sigma}{d\Omega_{Rutherford}} = \frac{(Z\alpha)^2}{4k^2 \sin^4\left(\frac{\theta_e}{2}\right)},$$

Where Z is the atomic number of the scattering nucleus, α is the fine structure constant, k is the momentum of the incident electron and θ_e is its scattering angle. Rutherford performed the calculation classically, though it turns out that the quantum-mechanical treatment gives the same result.

When electrons are scattered instead of alpha particles, the process is called Mott Scattering. Since electrons are far lighter than alpha particles they can penetrate further into the nucleus; since they have nonzero spin, this couples to the nuclear magnetic field. These effects result in the term in brackets, and the relativistic recoil of the nucleus gives the final term,

$$(1.2) \quad \frac{d\sigma}{d\Omega_{Mott}} = \frac{(Z\alpha)^2}{4k^2 \sin^4\left(\frac{\theta_e}{2}\right)} \left[1 - \beta^2 \sin^2\left(\frac{\theta_e}{2}\right) \right] \frac{E'}{E}.$$

This cross section is still incomplete; it describes a point particle scattering from another point particle. If one or both particles have some internal structure, then further terms are required in order to describe that structure.

1.2. Electron-Proton Scattering

In the case of electron-proton scattering, the electron is (so far as we have observed) a point particle, but the proton is composed of quarks and gluons. Even before the development of the quark model, it was known that it and the neutron must both have some internal structure, as their magnetic moments are respectively $\mu_p = 2.79\mu_N$ and $-1.91\mu_N$, where $\mu_N = \frac{e\hbar}{2M_pc}$ is the nuclear magneton, rather than the $1.0\mu_N$ and $0.0\mu_N$ expected for spin-1/2 charged and neutral point particles. The lowest-order (Born approximation) scattering diagram then looks like Figure 1.1, where the electron enters

Figure 1.1. Feynman diagram for elastic electron-proton scattering. Figure from [1].

with four-momentum k and exchanges a single virtual photon of momentum q with a

proton of initial momentum p . The electron exits with momentum $k' = k - q$ and the proton with $p' = p + q$. For elastic scattering, we find that $q^2 < 0$, and thus define the positive quantity $Q^2 = -q^2 = 4EE' \sin^2(\frac{\theta_e}{2})$ (assuming negligible electron mass).

In this diagram the point-particle electron has the simple vertex γ_μ , and its current takes the form $j_\mu = -e\bar{u}(k')\gamma_\mu u(k)$. The proton vertex Γ^μ is not simple, and must be expressed using form factors parameterizing its internal structure. The proton current must still be a Lorentz-invariant four vector that satisfies the parity and current conservation of the electromagnetic interaction. Thus, in the Born approximation of single-photon-exchange, only two form factors are allowed and the proton current J^μ is given by

$$(1.3) \quad J^\mu = e\bar{v}(p')\Gamma^\mu v(p)$$

$$(1.4) \quad = e\bar{v}(p') \left[F_1(q^2)\gamma^\mu + \frac{i\kappa_p}{2M_p} F_2(q^2)\sigma^{\mu\nu}q_\nu \right] v(p),$$

where κ_p is the proton anomalous magnetic moment, $1.793 \mu_N$; M_p is the proton mass; $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$; and $F_1(q^2)$ and $F_2(q^2)$ are form factors, with F_1 corresponding to the helicity-conserving current and known as the Dirac form factor, and F_2 corresponding to the helicity-flip current and referred to as the Pauli form factor. In the non-relativistic limit $q^2 \rightarrow 0$, $F_1(0) = F_2(0) = 1.0$ and the proton current simplifies to that of a point particle, the virtual photon becoming insensitive to the proton's internal structure.

With these two form factors, the differential cross section becomes

$$(1.5) \quad \frac{d\sigma}{d\Omega} = \left[\frac{\left| j_\mu \frac{1}{q^2} J^\mu \right|^2}{4((k\dot{p})^2 - m_e^2 M_p^2)} (2\pi)^4 \delta^4(k' - k + p - p') \frac{d^3k' d^3p'}{(2\pi)^3 2E' (2\pi)^3 2(M_p + \omega)} \right].$$

Integrating over the relevant variables (with delta functions ensuring momentum conservation and elastic scattering), averaging initial spin states and summing over final ones, the cross section as a function of angle becomes

$$(1.6) \quad \frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2\left(\frac{\theta_e}{2}\right)}{4k^2 \sin^4\left(\frac{\theta_e}{2}\right)} \frac{E'}{E} \left[\left(F_1^2 + \frac{\kappa_p^2 Q^2}{2M_p^2} F_2^2 \right) + \frac{Q^2}{2M_p^2} (F_1 + \kappa_p F_2)^2 \tan^2\left(\frac{\theta_e}{2}\right) \right],$$

with the form factor term simply multiplying the no-structure Mott cross section $\sigma_{ns} = \frac{d\sigma}{d\Omega}_{Mott}$ from equation 1.2, under the conditions $Z = 1$ for scattering from protons and $1 - \beta^2 \sin^2\left(\frac{\theta_e}{2}\right) \rightarrow \cos^2\left(\frac{\theta_e}{2}\right)$ for relativistic electrons.

Performing the replacement

$$(1.7) \quad G_{Ep}(Q^2) = F_1(Q^2) - \kappa_p \tau F_2(Q^2)$$

$$(1.8) \quad G_{Mp}(Q^2) = F_1(Q^2) + \kappa_p F_2(Q^2),$$

where $\tau = \frac{Q^2}{4M_p^2}$, we can rewrite the structure-dependant part of the cross section in terms of the electric and magnetic form factors $G_{Ep}(Q^2)$ and $G_{Mp}(Q^2)$ [2]. Then,

$$(1.9) \quad \frac{d\sigma}{d\Omega} = \sigma_{ns} \left[\frac{G_{Ep}^2(Q^2) + \tau G_{Mp}^2(Q^2)}{1 + \tau} + 2\tau G_{Mp}^2(Q^2) \tan^2\left(\frac{\theta_e}{2}\right) \right].$$

Defining the angular variable $\varepsilon = [1 + 2(1 + \tau) \tan^2\left(\frac{\theta_e}{2}\right)]^{-1}$ we may rewrite the cross section as

$$(1.10) \quad \frac{d\sigma}{d\Omega} = \sigma_{ns} \frac{1}{1 + \tau} \left[G_{Ep}^2(Q^2) + \frac{\tau}{\varepsilon} G_{Mp}^2(Q^2) \right].$$

In the non-relativistic ($Q^2 \rightarrow 0$) limit, these form factors are simply the fourier transforms of the charge and magnetization distributions [3],

$$(1.11) \quad F_{NR}(Q^2) = \int \rho(\vec{r}) e^{-i\vec{Q}\vec{r}} d^3\vec{r}.$$

In particular, if the charge distribution is exponential ($\rho(r) = \rho_0 e^{-r/a}$), the resulting form factor is known as the dipole form factor $G_D(Q^2) = (1 + a^2 Q^2)^{-2}$. More generally, if the charge and magnetic moment distributions are the same then their transforms will be as well, and we will have

$$(1.12) \quad \frac{\mu_p G_{Ep}(Q^2)}{G_{Mp}(Q^2)} = 1.0,$$

which is known as form factor scaling.

1.3. Rosenbluth Extractions Method

By factoring the no-structure cross section and some kinematics terms out of the total cross section, we find a component that is defined only in terms of the electric and magnetic form factors, multiplied respectively by the angular variable ε and the unitless momentum variable τ .

$$(1.13) \quad \sigma_R = \frac{d\sigma}{d\Omega} \frac{(1 + \tau)\varepsilon}{\sigma_{ns}} = (\tau G_{Mp}^2(Q^2) + \varepsilon G_{Ep}^2(Q^2))$$

Since this equation is linear in ε , plotting σ_{ns} as a function of ε at fixed Q^2 gives $G_{Ep}^2(Q^2)$ as the slope and $\tau G_{Mp}^2(Q^2)$ as the intercept. The Rosenbluth technique of extraction, developed by Marshall Rosenbluth in 1950 [4], is to measure the cross sections of elastically

scattered electrons at several angles at fixed momentum transfer, and then to perform a linear regression on the reduced cross sections, as shown in Figure 1.2.

Figure 1.2. Sample Rosenbluth extraction. Data from [5].

Comparing data for $G_{Ep}(Q^2)$ to $G_{Dp}(Q^2)$, the electric form factor follows the dipole form reasonably well (Figure 1.3) for a value of a , the scale of the proton radius, given by $a^2 = (0.71 \text{ GeV}^2)^{-1}$. The uncertainties in the data increase at high Q^2 — in the reduced cross section (Eq. 1.13), $G_{Ep}^2(Q^2)$ is multiplied by $\epsilon \in [0, 1]$ while $G_{Mp}^2(Q^2)$ is multiplied by the energy variable τ , which increases with Q^2 . Thus as the momentum transfer becomes larger $G_{Mp}^2(Q^2)$ dominates the cross section by increasingly large amounts, making extraction of $G_{Ep}^2(Q^2)$ more and more difficult.

The magnetic form factor $G_{Mp}(Q^2)$ also approximately follows the dipole form factor (Figure 1.4), though deviating at high Q^2 . Scatter and uncertainties remain reasonable

Figure 1.3. Rosenbluth G_{Ep} normalized to G_D , with $a^2 = (0.71 \text{ GeV}^2)^{-1}$. Data from [5–9].

even at high momentum transfer, so Rosenbluth cross sections can be used to extract $G_{Mp}(Q^2)$ even as measurements of $G_{Ep}(Q^2)$ become ineffective.

The ratio $\frac{\mu_p G_{Ep}}{G_{Mp}}$ also stays consistent with 1.0 (Figure 1.5), implying that the charge and magnetic moment distributions inside the proton are the same. Like the electric form factor alone, scatter and uncertainties in the ratio become large at large Q^2 . This was part of the motivation behind developing the polarization transfer technique, detailed in the next section.

Figure 1.4. Rosenbluth G_{Mp} normalized to G_D , with $a^2 = (0.71 \text{ GeV}^2)^{-1}$. Data from [5–9].

Figure 1.5. Rosenbluth $\mu_p G_{Ep}/G_{Mp}$ world data. Data from [5–9].

1.4. Polarization Transfer Method

Another method for measuring the form factor ratio is through polarization transfer. A longitudinally polarized electron beam is scattered from an unpolarized proton target (Figure 1.6). For electron-proton scattering in the single-photon-exchange approximation, it can be shown that the normal component of polarization vanishes, and the transverse and longitudinal components P_t and P_l follow [10]

$$(1.14) \quad I_0 P_t = -2\sqrt{\tau(1+\tau)} G_{Ep}(Q^2) G_{Mp}(Q^2) \tan\left(\frac{\theta_e}{2}\right)$$

$$(1.15) \quad I_0 P_l = \frac{E + E'}{M_p} \sqrt{\tau(1+\tau)} G_{Mp}^2(Q^2) \tan^2\left(\frac{\theta_e}{2}\right),$$

Figure 1.6. Schematic diagram for elastic e-p scattering with polarization transfer. Figure from [1].

with $I_0 = (G_{Ep}^2 + \frac{\tau}{\varepsilon} G_{Mp}^2)$. In order to measure the longitudinal polarization of scattered protons, they are scattered again from a secondary target inside a Focal Plane Polarimeter (FPP); the azimuthal angular distribution then gives transverse and normal components of polarization (at the FPP) simultaneously, and knowing how the spin processes through the magnetic fields inside the detector allows extraction of the transverse and longitudinal components at the point of scattering [10]. By taking the ratio of transverse and longitudinal polarizations, the factors of I_0 cancel and the ratio of G_{Ep} to G_{Mp} is given by

$$(1.16) \quad \frac{G_{Ep}}{G_{Mp}} = \frac{P_t}{P_l} \frac{E + E'}{2M_p} \tan\left(\frac{\theta_e}{2}\right).$$

Unlike the Rosenbluth separation method, this method gives the ratio G_{Ep}/G_{Mp} directly with no need to measure cross-sections. As a result, the effects of detector efficiencies and the analyzing power of the polarimeter are much reduced.

Results from polarization transfer measurements are shown along with Rosenbluth data in Figure 1.7 — these measurements show the ratio $\mu_p G_{Ep}/G_{Mp}$ decreasing with increasing Q^2 , not in agreement with Rosenbluth measurements. Polarization results are well fit by [11],

$$(1.17) \quad \frac{\mu_p G_{Ep}}{G_{Mp}} = 1 - 0.13(Q^2 - 0.04).$$

Notably, since this method only gives G_{Ep}/G_{Mp} , it still relies on cross section extractions to provide $G_{Mp}(Q^2)$ (where that method is still accurate), and then measurements of $G_{Ep}(Q^2)$ with better uncertainties can be obtained by combining the two sets of results.

Figure 1.7. $\mu_p G_{Ep}/G_{Mp}$ results from both Rosenbluth and polarization transfer experiments. Rosenbluth global analysis from [12], polarization transfer data from [10, 11].

1.5. Comparison of Results

The disagreement between the two methods of extraction is illustrated in Figure 1.7. Because of the higher precision in the polarization transfer data and the large uncertainties and scatter of G_{Ep} as extracted from L-T separations, it was suggested that perhaps the Rosenbluth extractions were inaccurate. Unfortunately, this would throw into question any prior experiments which used those form factors as input to calculations, and since polarization transfer measurements provide only the ratio of G_{Ep} to G_{Mp} they could not be used as a replacement. It was necessary to validate or reject the Rosenbluth measurements, and to search for any missing corrections that might affect one or the other set of measurements.

Figure 1.8. Fractional contribution of $G_{Ep}(Q^2)$ to the reduced cross section σ_R as a function of ε for several values of Q^2 from 1 GeV^2 to 7 GeV^2 , assuming form factor scaling. Figure from [1].

1.6. Super-Rosenbluth Method

Figure 1.8 illustrates the decreasing effect of G_{Ep} on the total cross section; as the contribution becomes smaller, the effects of any ε -dependent uncertainties are magnified. In order to perform Rosenbluth extractions with smaller angular-dependent corrections, it was determined that protons should be detected rather than electrons. For a stationary proton target, protons scattered at any fixed Q^2 have the same momentum, meaning there is no angular dependence to the momentum-dependent corrections (Figure 1.9). This includes such corrections as particle absorption, detector efficiency, multiple scattering, and particle identification. In addition, the proton cross section varies much less with ε , and with it the rate-dependent corrections. The sensitivity to offsets in beam energy and detector angle show less angular variation as well [1]. Radiative corrections for both

Figure 1.9. Proton and electron momenta as functions of ε at $Q^2 = 2.64 \text{ GeV}^2$. Figure from [1].

protons and electrons show approximately linear dependencies on ε , but those for protons vary about half as much in magnitude ($\sim 8\%$ vs. $\sim 17\%$).

1.7. Continued Disagreement

The results of the super-Rosenbluth experiment JLab E01-001 [13] are shown in Figure 1.10, along with the previously shown Rosenbluth and polarization transfer results. The super-Rosenbluth results are in good agreement with previous electron Rosenbluth results, and have uncertainties comparable to the polarization transfer results. This confirms the reliability of the electron Rosenbluth measurements (and of calculations based on those cross sections), strongly suggesting that some missing corrections in the analysis were responsible for the discrepancy.

Figure 1.10. $\mu_p G_{Ep} / G_{Mp}$ results from Rosenbluth, polarization transfer, and super-Rosenbluth experiments. Data from [5–11, 13].

Figure 1.11. Difference in observed σ_R and that predicted from polarization transfer, $Q^2 = 2.64$ and 4.1 GeV^2 . Figure from [14].

Figure 1.11 shows the size of the discrepancy between Rosenbluth and polarization transfer data at 2.64 and 4.1 GeV^2 . Data and linear fits (solid lines) are from JLab E01-001; dashed lines are from taking the same value of $\tau G_M^2(Q^2)$ as the intercept and deriving the slope $G_E^2(Q^2)$ from polarization transfer measurements. Note that the y-axis scales do not start at zero. The difference between the two is about half of the Rosenbluth slope at 2.64 GeV^2 and increases with Q^2 to around $6/7$ at 4.1 GeV^2 . This appears to be the result of an ϵ -dependent effect on the order of $5 - 8\%$ of the reduced cross sections.

1.8. Two Photon Exchange

With the consistency of the Rosenbluth measurements confirmed, the discrepancy between its results and those from polarization transfer is expected to come from some missing correction which would affect the two methods differently. Any correction would need to be approximately linear in ϵ , in order to modify the reduced cross sections in a way that would change the value of G_E^2 and also to have not already been seen as a nonlinearity in any Rosenbluth extraction, and on the order of $5 - 8\%$ of the reduced cross section, as described above.

The leading candidate for such a correction is two-photon exchange (TPE) [15, 16], illustrated in the box and crossed-box feynman diagrams in Figure 1.12. The standard

Figure 1.12. Box and crossed-box diagrams of two photon exchange in elastic electron-proton scattering. Figure from [16].

radiative correction procedure used in both methods of extraction includes all corrections in the single-photon-exchange approximation, but does not fully include two photon exchange — only the infrared-divergent term is included, which cancels another infrared divergence in the interference between electron and proton bremsstrahlung.

Figure 1.13. Handbag and Cat's Ears diagrams of two-photon exchange with the intermediate proton states.

Since they include intermediate, excited proton states and QCD effects, TPE effects are difficult to calculate. Figure 1.13 shows the “handbag” and “cat's ears” diagrams of

the proton interactions, where in the former both photons interact with the same quark and in the latter they interact with different quarks. A third form factor, $F_3(Q^2)$ becomes possible, due to the availability of another independent vector, and the form factors $F_1(Q^2)$ and $F_2(Q^2)$ take on complex values, modifying Eq. 1.3 into

$$(1.18) \quad (J^\mu)_{TPE} = e\bar{v}(p') \left[\tilde{F}_1 \gamma^\mu + \frac{i\kappa_p}{2M_p} \tilde{F}_2 \sigma^{\mu\nu} q_\nu + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M_p^2} \right] v(p),$$

where q is still the total momentum transfer, $K = (k + k')/2$, $P = (p + p')/2$, and the $\tilde{F}_i$ are complex-valued functions of Q^2 and ε .

Several attempts have been made to calculate the TPE contribution to the elastic electron-proton cross section. Blunden, Melnitchouck, and Tjon [16] use an intermediate proton model that neglects excited states, finding a $\sim 2\%$ dependence on ε with small nonlinearities at low ε and little Q^2 dependence. Including improved form factors [17] and the Δ resonance [18] increase the epsilon dependence to $\sim 3\%$. Chen et al. [19] use a generalized parton distribution to describe the diagrams shown in Figure 1.13, finding significant ε dependence and nonlinearity and weak Q^2 dependence. Their correction is only about half the size needed to explain the discrepancy, however, and their calculation is not expected to be valid at low Q^2 . Afanasev et al. [20] calculate at the quark-parton level in the double logarithm approximation, finding an ε dependence that shows nonlinearities at the high end. This model predicts only the ε and Q^2 dependencies and not the overall magnitude, and so must be externally scaled to match the discrepancy. Rekalo and Tomasi-Gustafsson [21] use symmetry arguments to argue that the TPE correction will depend on $x = \sqrt{1 + \varepsilon}/\sqrt{1 - \varepsilon}$, but do not calculate the size or momentum dependence. This calculation predicts nonlinearities at large ε (diverging as $\varepsilon \rightarrow 1$) but does not predict

the size or Q^2 dependence and is inconsistent with the requirement that $\delta \rightarrow 0$ as $\varepsilon \rightarrow 1$ as explained in [22]. Figure 1.14 shows the ε -dependence of the various TPE calculations, rescaled as necessary to match the size of the discrepancy.

Figure 1.14. ε dependence of various TPE calculations, scaled to the necessary magnitude. Figure from [14].

1.9. Searching for Two-Photon Effects

All of the TPE calculations in the previous section predict nonlinearities in the reduced cross section with respect to ε , which are not present in the Born approximation. Current measurements demand that the correction be approximately linear over most of the range in ε , but generally do not include much data at the high and low extremes. Searching for such nonlinearities, especially at those extremes, would provide evidence of two-photon exchange effects.

Figure 1.15. NE11 linearity test at 2.5 GeV^2 . Data from [6].

Figures 1.15 and 1.16 show linearity tests for experiments SLAC NE11 (electron Rosenbluth) and JLab E01-001 (proton Rosenbluth), respectively. For these tests, in addition to the linear fit (solid line) used in the form factor extraction, the data was fit to the second-degree polynomial (dashed line)

$$(1.19) \quad \sigma_R = P_0 \left(1 + P_1(\epsilon - 0.5) + P_2(\epsilon - 0.5)^2 \right),$$

such that P_2 becomes a curvature parameter measuring the nonlinearity as a fraction of the average reduced cross section. Both the SLAC and JLab data show P_2 terms consistent with zero, with the proton Rosenbluth measurement setting better limits on P_2 . Both plots also show that any nonlinearity would have the most effect at the extremes

Figure 1.16. E01-001 linearity test at 2.64 GeV^2 . Data from [13].

of epsilon. Any cross-section measurements looking for TPE effects should then show special attention to the previously-neglected extremes of ϵ .

The magnitude of the linear portion of the ϵ dependence can be extracted by requiring that Rosenbluth results become consistent with those from polarization transfer [23, 24]. The main limiting factor is then the precision of the Rosenbluth data [25]; improving the Rosenbluth data in overlapping Q^2 regions will allow better constraints on the magnitude of the linear portion of two photon exchange.

CHAPTER 2

Experimental Setup

2.1. Goals

For this experiment, we wished to perform high-precision proton Rosenbluth extractions, covering a large range in Q^2 focusing on the particularly on the extremes of ε where the largest two-photon exchange effects are expected. This experiment reached higher momentum transfers than the previous proton Rosenbluth experiment and provided higher precision at low momentum transfer. At low momentum transfer, we are sensitive only to the nonlinear portion of the two-photon exchange effects, while at high momentum the suppression of G_{Ep} means measurements there are sensitive to the full TPE correction. In addition, the difference between values given by L-T extractions and polarization transfer show the size of the linear portion of the TPE correction.

Figure 2.1 shows the kinematics points taken, with a total of 102 points at 16 different values for Q^2 from 0.40 to 5.76 GeV^2 . In particular, two extractions were performed with many ε points, 13 ε at 0.983 GeV^2 and 10 at 2.28 GeV^2 , in order to make highly detailed measurements of any nonlinearities that may appear. Special attention was paid to the extremes of ε at all value of momentum transfer, up to the limit of available beam energies and forward rotation of the spectrometer.

Figure 2.1. Kinematics points taken, by Q^2 and ε . Each solid line represents a single beam energy. Horizontal dashed lines are two Q^2 values chosen for particularly detailed extraction.

2.2. Beamline

For this experiment, beam energies from 0.849 to 5.157 GeV were provided by the Continuous Electron Beam Accelerator Facility (CEBAF) at the Thomas Jefferson National Accelerator Facility (TJNAF, or more commonly, JLab). The accelerator (Figure 2.2) consists of a 50 MeV injector and two antiparallel 0.6 GeV linear accelerators (linacs). Each linac consists of 20 radio frequency cryomodules, with each cryomodule containing

Figure 2.2. Schematic diagram of the CEBAF beamline.

eight superconducting neodymium cavities, cooled to a temperature of 2 K by liquid helium. Recirculating arcs at both ends of the accelerator (five on the east end, four on the west) allow for up to five beam passes through the linac pairs, providing beam energies E_{Beam} at

$$(2.1) \quad E_{Beam} = n * E_L + E_0,$$

where $n \in \{1, 2, 3, 4, 5\}$ is the number of passes, E_L is the sum of the energies of the north and south linacs, and E_0 is the injector energy. Electrons are delivered to the halls in 1.67 ns packets at a rate of 1497 MHz , with 100 μA split between Halls A and C and 100 nA available to Hall B, each receiving one third of the electron pulses, for a rate of 499 MHz to each hall.

Figure 2.3. Schematic view of the Hall C Beamline and Spectrometers. Figure from [26].

2.2.1. Position Measurement

Beam position in the hall is monitored by three Beam Position Monitors (BPMs), upstream from the target. Each BPM contains four wires, equally spaced at $\pm 45^\circ$ from the horizontal and vertical axes and tuned to resonate with the beam. The relative strength of the signal in each antenna is proportional to its distance from the beam, so knowing the coefficients of proportionality for each antenna allows the beam position at the BPM to be calculated. For more information on the BPMs see [27].

Superharp monitors at various positions along the beamline are used to calibrate the BPM coefficients and determine the absolute beam position. Each superharp consists of a moveable frame containing two vertical and one horizontal wires, each measuring the beam profile in the perpendicular direction, attached to a stepper motor. The current readings from the three wires combined with the position of the stepper motor allow for a very precise (better than $20\ \mu m$) but destructive determination of the beam position [28].

2.2.2. Energy Measurement

The field gradients of the beam turning magnets are well known, so the path of the beam through the magnets can be used to determine beam momentum and energy. Beam focusing magnets are turned off in order that beam deflection is exclusively from the turning magnets and Superharp monitors along the beam line — two before the turning magnets and two after, giving position and direction information — are used to precisely map the beam path. The total turning angle ϕ is then related to the beam momentum p

by

$$(2.2) \quad \phi p = c \int \mathbf{B} \cdot d\mathbf{l},$$

where c is the speed of light, $\mathbf{B}$ is the magnetic field, and $\mathbf{l}$ is the path through the magnets. This measurement is taken once per beam energy setting as it requires a modified tuning of the beam through the arc.

2.2.3. Current Monitoring

The beam current is monitored by two resonance chambers (BCMs), located near where the beam enters the hall (shown at the bottom of Figure 2.3). Each chamber is a cylindrical wave guide, tuned to the beam frequency, containing wire loop antennae that couple to its resonance modes. Passage of the beam excites the chamber, producing signals in the antennae proportional to the beam current.

2.2.4. Rastering

The CEBAF beam is capable of providing high current in a very small area, $\lesssim 200 \mu m$ FWHM, which would be capable of melting a solid target or boiling a liquid one. To prevent this, a fast raster system is installed in the beamline, 25 meters upstream of the target (second from the bottom in Figure 2.3), which provided a $2 \text{ mm} \times 2 \text{ mm}$ square spot at the target.

2.3. Hall Layout

The experiment was performed in Hall C at JLab. The electron beam from the accelerator is incident upon a target positioned in the center of the hall, and scattered particles can be detected by the High Momentum Spectrometer (HMS) on the right side of the beamline and the Short Orbit Spectrometer (SOS) on the left side. Both spectrometers pivot around the target platform to reach varying angles. Only the HMS was used for this experiment — though we had hoped to use the SOS for coincidence measurements of both scattered particles simultaneously, it was not available during our run time.

2.4. Target

The Hall C target system is mounted on a vertical ‘ladder’ at the center of the hall, inside a vacuum enclosure. Vertical motion of the target ladder allows different targets to be inserted and removed from the beamline. The target ladder contains three 4 *cm* cylindrical cells which can be filled with cryogenic liquid targets, aluminum plate targets for optics calibration and dummy subtraction, and several solid targets of various elemental composition.

Two targets were used for this experiment: one of the cylindrical cells filled with liquid hydrogen, and the 4 *cm* aluminum ‘dummy’ target, which consists of two plates spaced 4 *cm* apart. A solid carbon target was also used for some calibrations.

The liquid hydrogen target was a cylinder 40.13 ± 0.0013 *mm* in outer diameter, with the cell wall 0.1270 ± 0.0013 *mm* in the upstream direction, 0.1384 ± 0.0013 *mm* downstream, and 0.1359 ± 0.0013 *mm* at 90° to the right of the beamline. The cylinder axis is oriented vertically, so the target provides a circular cross section in the scattering

plane. The liquid hydrogen was held at a temperature of 19 K and density 0.0723 g/cm^3 by the target cooling system [29].

The dummy target consists of two aluminum foils aligned perpendicular to the beam line, 2 cm upstream and downstream of the target center. Both foils are 0.8 cm wide in the scattering plane, with thicknesses of $0.984 \pm 0.013 \text{ mm}$ for the upstream foil and $0.944 \pm 0.013 \text{ mm}$ for the downstream [29]. Data from the dummy target is used to subtract off the effects of the hydrogen cell walls. Since the dummy target foils are thicker than the walls of the hydrogen cell, dummy running requires less time to get acceptable yields but the radiative corrections are somewhat different.

2.5. High Momentum Spectrometer

The HMS is a vertical bend spectrometer in QQD configuration — three superconducting quadrupole magnets that focus the incident particles, followed by a dipole magnet that bends the particles upwards by 25° at the central momentum. The magnets and detector package are mounted on a common carriage that pivots about the target, keeping them in line with each other and with the target center (Figure 2.4). The detector shielding hut is mounted on a separate carriage and moves along with the detector package.

The spectrometer is capable of reaching central momenta between 0.5 and 7.5 GeV , with an acceptance of $\pm 9\%$ of the central momentum. The carriage can rotate to lab-frame angles from 10.5° to 90° , with $\pm 0.8 \text{ mr}$ acceptance in-plane and a total 6 msr solid angle [26].

Figure 2.4. HMS Magnets and Carriage.

2.5.1. HMS Coordinate System

The HMS coordinate system defines the $\hat{z}$ axis as the direction of particle travel, such that it points towards the back end of the detector hut and is 25° above parallel with the floor. The $\hat{x}$ (dispersive) direction is defined as the direction of increasing momentum, which is downwards (again, at a 25° angle) for a vertical bend spectrometer. The $\hat{y}$ (nondispersive) direction is then to the left when viewing the spectrometer from the target, in the direction of decreasing angle. The origin of the coordinate system is placed in the center of the focal plane, which is perpendicular to the central trajectory at a point approximately halfway between the drift chambers. The slopes $x'_{fp} = \frac{dx_{fp}}{dz}$ and $y'_{fp} = \frac{dy_{fp}}{dz}$ give the trajectory of the particle at the focal plane; these are taken as derivatives but generally referred to as angles in milliradians, effectively the same at such small angles. They are still treated as slopes when doing calculations.

When the particle trajectory is projected back towards the target a similar coordinate system is used, with x_{tar} vertically downwards, y_{tar} perpendicular to the spectrometer

Figure 2.5. HMS Detector Package. Figure from [30].

angle, and z_{tar} pointing towards the spectrometer. The origin is at the target center, and the slopes x'_{tar} and y'_{tar} are defined as with the focal plane coordinates.

2.5.2. Detector Package

The HMS detector package is shown in Figure 2.5. The detector consists of a pair of wire drift chambers (DC1 and DC2), providing particle tracking; aerogel (not pictured) and gas Cherenkov detectors, providing particle identification; a pair of hodoscope scintillator planes (S1X, S1Y, S2X, and S2Y), providing triggering and time-of-flight information; and a lead-glass calorimeter, which was not used in this experiment.

2.5.3. Magnets

The HMS quadrupoles are cold iron superconducting magnets — soft iron around the coil improves the quality of the central field. Multipole windings are present for multipole corrections, but field measurement indicates that they are not needed in the standard tuning. Q1 and Q3 focus in the dispersive direction, while Q2 (otherwise identical to Q3)

	Q1	Q2 & Q3
Gradient (G/cm)	605	445
Pole Tip Field (T)	1.5	1.56
Radius to Pole (cm)	25	35
Magnetic Length (cm)	189	210
Stored Energy ($10^6 J$)	0.25	0.5
Weight (tons)	20	30

Table 2.1. HMS Quadropole Parameters.

focuses in the transverse direction. Table 2.1 shows the parameters for the quadropoles (at maximum current, where relevant).

The HMS dipole is a 470-ton magnet, creating a 25° vertical bend with a 12.06 m bending radius and a 42 cm gap width, over a effective length of 5.26 m . The tip field reaches a maximum of 1.66 T , corresponding to a momentum of $6\text{ GeV}/c$. The bend introduced by this magnet is dependant upon the momentum of the incident particles, providing momentum discrimination to the spectrometer [30, 31].

2.5.4. Wire Chambers

Tracking inside the spectrometer is provided by two wire drift chambers, spaced 81.2 cm apart. Each drift chamber consists of both field and sense wires, as shown in Figure 2.6, and is filled with gas; a charged particle passing through the chamber will induce ionization in the gas, causing electrons to drift along the field lines to the sense wires. Vertex resolution at the two wire drift chambers provides particle tracking; tracking in the dispersive (X) direction reveals the particle momentum, while tracking in the non-dispersive (Y) direction gives the precise angle.

In each drift chamber, there are two planes of sense wires in the dispersive (X) direction, two in the non-dispersive (Y) direction, and two (U,V) at $\pm 15^\circ$ from the X planes.

Figure 2.6. HMS Drift Chamber Wire Configuration. Figure from [30]

The planes are ordered X, Y, U, V, Y', X', and are separated by a 1.8 *cm* inter-plane spacing; in-plane, sense wires are parallel and separated by 1 *cm* [30].

2.5.5. Aerogel Cherenkov

The aerogel Cherenkov detector was positioned between the front and back planes of hodoscope scintillators. It has an index of refraction of 1.015 — particles which are superluminal in this material will produce Cherenkov radiation, which is then detected by photomultiplier tubes. At most momentum settings, pions would be moving fast enough to trigger the detector but protons would not be, though at low momenta ($p < 0.817 \text{ GeV}$, $Q^2 < 0.574 \text{ GeV}^2$) neither will cause scintillation.

The aerogel was used for particle identification in some calibrations, but was not used in final measurements. It was also removed during a break in data taking and replaced

before data collection restarted. This required several calibrations to be done twice, once for each section of the run time.

2.5.6. Hodoscope

The hodoscope consists of four planes in two groups of two, each with one horizontal and one vertical set of scintillators, covering the dispersive x and nondispersive y directions. When a charged particle passes through a scintillator it causes a flash of light, which is picked up by photomultiplier tubes (PMTs) at either end of the scintillator. Scintillators in the same plane overlap at the edges, to prevent particles passing through without hitting at least one scintillator.

The hodoscope provided two important functions, event triggering and time-of-flight measurement. Events were triggered when at least three of the four planes recorded a particle; this trigger rate was compared to the rate of all four planes triggering in order to determine the scintillator efficiency. Time-of-flight was determined from the difference between front plane and back plane timings.

2.5.7. Gas Cherenkov

A gas Cherenkov detector is located between the two hodoscope planes. It was not used in this experiment, but must still be considered in the particle absorption and multiple scattering calculations.

CHAPTER 3

Calibrations and Efficiencies

3.1. Data Analysis Overview

The first step in converting raw detector output to physical spectra occurs in the replay code, which analyzes the individual hits, finds the particle tracks, and calculates angles, momenta, and times of flight. This process is the same for hydrogen and aluminium dummy runs. Calibrations such as Time of Flight, Aerogel, Spectrometer Alignment, Beam Current Monitoring, and Drift Chamber Offset are applied in the replay code.

The replay code looks at the hits in the wire drift chambers to attempt to reconstruct a track, generating the focal plane coordinates and trajectory; these are projected back to the target to find the point of scattering and initial trajectory. Momentum is determined from the vertical deflection caused by the bending dipole and angle from the trajectory in the scattering plane. Time of flight is determined from the timing difference between the hodoscope planes. The beam current is integrated over the run to determine the charge incident on the target and various hardware cuts are applied.

With the physics variables determined, we can reconstruct the Δp and $\Delta\theta$ spectra — the first is the difference between the momentum that would be observed for an elastically scattered proton at the same angle and the momentum that is actually observed, and the second is a similar comparison for the calculated and measured angles. Both spectra are scaled by the total charge incident on the target. This total charge is where the efficiencies

are applied, in order to determine the final effective yield; proton absorption, target boiling, β no-track fraction, particle identification efficiency, target length correction, and several scaler values that are given as output from the replay all apply as factors that reduce the amount of beam current capable of generating events in the spectrometer.

3.2. Calibrations

3.2.1. ToF Calibration

Prior to the experiment, care was taken to gain-match the photomultiplier tubes. Because the time of flight code looks for when the signal crosses a certain threshold, it is desirable that each hodoscope paddle produce the same height and width pulse when exposed to the same signal. For this calibration, a radioactive source was used to generate a consistent signal and the voltage applied to each photomultiplier tube was adjusted to ensure that the same source produced the same signal at any point in the hodoscope. With the gains calibrated, it was also necessary to calibrate the overall timing. Each paddle and each photomultiplier tube has its own characteristic delay, in addition to the variable delays produced by the wire runs carrying signals from the photomultipliers to the electronics. This procedure is described in [30].

Figure 3.1 shows the $\Delta\beta$ spectrum before and after the final calibration. $\Delta\beta$ is the difference between the speed that would be found for a proton (elastic or not) at the observed momentum and the speed that is actually observed; protons are the central, largest peak, pions are travelling faster and deuterons and tritons are travelling more slowly. After calibration, all of the peaks become sharper, despite only the proton peak being used in the fitting.

Figure 3.1. $\Delta\beta$ Before (solid)/After (dashed) ToF Calibration, for runs at $p = 1.36 \text{ GeV}$.

For the time of flight calibration, the efficiency of particle detection was not important, but it was necessary to have only protons and no pions or deuterons, which travel at different velocities. Thus for this calibration an aggressive aerogel cut was used to exclude pions, as well as a wide preliminary ToF cut in order to exclude deuterons.

Unsurprisingly, the calibrating the Time of Flight at a particular momentum gives the best results at that momentum, and worsening results as the protons get faster or slower. It was decided to use three calibration regions for the hodoscope, corresponding to momentum transfers of $Q^2 \leq 0.983 \text{ GeV}$, $0.983 < Q^2 < 1.90 \text{ GeV}$, and $Q^2 \geq 1.90 \text{ GeV}$.

In addition, each region had to be calibrated twice, for the sets of runs before the aerogel was removed and after it was replaced.

3.2.2. Aerogel Calibration

When histogrammed over all events, the Aerogel Detector output shows a number of peaks, one “pedestal” at 0 and each higher corresponding to one more photoelectron produced at a photomultiplier tube (Figure 3.2).

Figure 3.2. Sample aerogel output histogram, with fit to first photoelectric peak.

This calibration involved fitting the first photoelectric peak (the red curve in Figure 3.2), and then the total number of photoelectrons produced in an event could be determined by dividing the aerogel VDC signal by the fit value.

As with the Time of Flight, separate aerogel calibrations were needed for the sets of runs before it was removed and after it was replaced. An aerogel cut at 0.5 photoelectrons was used in other calibrations (particularly the Time of Flight) in order to remove pions.

3.2.3. Spectrometer Alignment

The HMS spectrometer rotates about the target chamber, so the spectrometer mispointing can never get very large, typically around 2 mm downstream of the target center. This is well known from repeated measurements (Figure 3.3) and was surveyed again for this experiment.

Figure 3.3. HMS mispointing as a function of angle.

3.2.4. Beam Current Monitors

The BCMs are known to have linear and stable response over a wide current range, but calibration can drift over long periods and thus they must be calibrated against a more stable monitor. Between the two BCMs there is another current monitor, called an Unser

Monitor, which can be calibrated to external current sources but has an offset that varies over time.

The BCMs are calibrated in comparison to the Unser by stepping the beam current over a range of values (Figure 3.4). The variation in the Unser offset is minimised by doing this quickly relative to the drift time, and also the offset is tracked by returning the beam current to zero between each calibration value. Cyan bars indicate the regions of positive beam current used to calibrate; grey bars indicate the regions of no current used to track the unser offset.

Figure 3.4. BCM Calibration Run; Current vs. Time.

3.2.5. Drift chamber offset

When looking at the angular distribution (y'_{tar}) of the detected particles in comparison to simulation, it was noticed that they didn't match (top plot in Figure 3.5 — triangles

Figure 3.5. Histogram of y'_{tar} against simulation, before and after correcting drift chamber shift.

are data, filled yellow is simulation), particularly because of a sharp shoulder in the data above 20 *mrad*. Using the sieve slit and optics targets, it was determined that the drift chambers had experienced a 0.45 *mm* shift relative to each other in the non-dispersive (y) direction, most likely due to flexing of the frame as some parts were replaced. This was corrected in the replay, bringing the data and simulation into accord (Figure 3.5, bottom plot) and also providing a slight increase in resolution. Details of this analysis can be found in Reference [32].

3.3. Efficiency Corrections

3.3.1. Proton Absorption

While multiple scattering is handled in the simulation, particle absorption is not. We needed to correct the efficiency for any proton absorption that may occur between the

point of scattering in the target and the third plane of the hodoscope, where the event trigger is generated. Any protons which are absorbed before reaching the third plane will not trigger events, reducing the measured yield of the interaction.

In a single material, proton absorption is determined by the thickness X and the mean free path between nuclear collisions $\bar{\lambda}$, with the absorption then given by $1 - e^{-X/\bar{\lambda}}$. For multiple materials in sequence, the total absorption is then given by

$$(3.1) \quad 1 - \prod_i e^{-X_i/\bar{\lambda}_i} = 1 - e^{-\sum_i X_i/\bar{\lambda}_i}.$$

To determine the mean free path $\bar{\lambda}$, it is necessary to know the mean free path between nuclear collisions λ_T and the mean free path between inelastic collisions λ_I . Since we do not know what fraction of elastic interactions cause events to be lost, two estimations were used for the total scattering length: $\lambda_{act} = 2\lambda_T\lambda_I/(\lambda_T + \lambda_I)$, assuming each component contributes half to the absorption length, and $\lambda_{avg} = (\lambda_T + \lambda_I)/2$, the simple average.

The relevant values for the absorbers in the HMS flight path are given in Table 3.1. The total absorption comes to about $7.5 \pm 2\%$ using either method, independent of ε , with the uncertainty covering lack of precise knowledge of material thicknesses and possible variation with Q^2 . Because this corresponds to an overall normalization correction at each Q^2 , the uncertainty will cancel in the extraction of the ratio G_{Ep}/G_{Mp} .

3.3.2. Target Boiling

Target boiling can occur as a result of heat delivered by the beam. Overall target heating is managed by the target cooling system, but local changes in density can still occur along the beam path.

Material	Density (gm/cm^3)	Thickness (cm)	X (gm/cm^2)	λ_T (gm/cm^2)	λ_I (gm/cm^2)	X/λ_{avg}	X/λ_{act}
LH2 Target	$7.08e-2$	$2.04e+0$	$1.44e-1$	$4.33e+1$	$5.08e+1$	$3.07e-3$	$3.09e-3$
Al Target wall	$2.70e+0$	$1.37e-2$	$3.70e-2$	$7.06e+1$	$1.06e+2$	$4.19e-4$	$4.36e-4$
Al Chamber Window	$2.70e+0$	$4.06e-2$	$1.10e-1$	$7.06e+1$	$1.06e+2$	$1.24e-3$	$1.29e-3$
Air	$1.21e-3$	$1.50e+1$	$1.82e-2$	$6.20e+1$	$9.00e+1$	$2.39e-4$	$2.47e-4$
Kevlar Mag. Entrance	$7.40e-1$	$3.81e-2$	$2.82e-2$	$7.00e+1$	$7.00e+1$	$4.03e-4$	$4.03e-4$
Mylar Mag. Entrance	$1.39e+0$	$1.27e-2$	$1.77e-2$	$6.25e+1$	$8.57e+1$	$2.38e-4$	$2.44e-4$
Al Mag Exit	$2.70e+0$	$2.79e-2$	$7.54e-2$	$7.06e+1$	$1.06e+2$	$8.52e-4$	$8.89e-4$
Air	$1.21e-3$	$2.56e+2$	$3.10e-1$	$6.20e+1$	$9.00e+1$	$4.08e-3$	$4.22e-3$
Mylar Wire Chamber Ent.	$1.39e+0$	$2.54e-3$	$3.53e-3$	$6.25e+1$	$8.57e+1$	$4.76e-5$	$4.88e-5$
Sense Wires	$1.93e+1$	$4.90e-6$	$9.46e-5$	$1.10e+2$	$1.85e+2$	$6.41e-7$	$6.84e-7$
Field Wires	$5.40e+0$	$1.77e-4$	$9.56e-4$	$6.98e+1$	$1.08e+2$	$1.08e-5$	$1.13e-5$
W.C. Gas	$1.54e-3$	$1.80e+0$	$2.77e-3$	$6.86e+1$	$1.01e+2$	$3.26e-5$	$3.39e-5$
Mylar W.C. Exit	$1.39e+0$	$2.54e-3$	$3.53e-3$	$6.25e+1$	$8.57e+1$	$4.76e-5$	$4.88e-5$
Al Aerogel Ent.	$2.70e+0$	$1.50e-1$	$4.05e-1$	$7.06e+1$	$1.06e+2$	$4.58e-3$	$4.77e-3$
Aerogel	$7.14e-2$	$9.00e+0$	$6.43e-1$	$6.55e+1$	$9.57e+1$	$7.97e-3$	$8.26e-3$
Al Aerogel Exit	$2.70e+0$	$1.00e-1$	$2.70e-1$	$7.06e+1$	$1.06e+2$	$3.05e-3$	$3.18e-3$
Polystyrene Scintillators	$1.03e+0$	$2.13e+0$	$2.20e+0$	$5.84e+1$	$8.20e+1$	$3.13e-2$	$3.22e-2$
Al Cherenkov Ent.	$2.70e+0$	$1.02e-1$	$2.74e-1$	$7.06e+1$	$1.06e+2$	$3.10e-3$	$3.23e-3$
Cherenkov Mirror Support	$5.00e-2$	$2.00e+0$	$1.00e-1$	$7.00e+1$	$7.00e+1$	$1.43e-3$	$1.43e-3$
SiO2 Cherenkov Mirror	$2.20e+0$	$3.00e-1$	$6.60e-1$	$6.52e+1$	$9.78e+1$	$8.10e-3$	$8.44e-3$
Al Cherenkov Exit	$2.70e+0$	$1.02e-1$	$2.74e-1$	$7.06e+1$	$1.06e+2$	$3.10e-3$	$3.23e-3$
Polystyrene Scintillators	$1.03e+0$	$2.67e-1$	$2.75e-1$	$5.84e+1$	$8.20e+1$	$3.91e-3$	$4.03e-3$
Sum						$7.72e-2$	$7.98e-2$
Transmission						0.926	0.923
Absorption						0.074	0.077

Table 3.1. Proton absorbers and their properties.

To measure any density changes that may happen, we collected data at several beam current values for fixed kinematics. The reduction in yield due to target boiling was found to scale linearly with current to a maximum of 1.5% at $100 \mu A$. This correction was applied on a run-by-run basis, to account for variations in beam current — ideally current would have been fixed at fixed Q^2 , but this was prevented by practical limitations such as the beam current requested by simultaneous running in Hall A.

3.3.3. Beta No-Track Fraction

Three of the four hodoscope planes are required to generate a trigger and record an event, but the time of flight code looks only for hits in an area around the ray projected from the wire chamber tracking. Sometimes stray hits from other particles, multiple scattering, or other effects will result in events where there are not hits along this path in either the front or back planes and thus no time of flight can be reconstructed. These events are given a β value of 0.

The percentage of events lost is easily counted in each run, and was found to be momentum dependent and uniform among elastic and background events. Fitting the events showed an approximate linear dependence from 1.80% at 0 *GeV* to 2.25% at 1.5 *GeV*, and a constant 2.25% above (Figure 3.6).

The lowest epsilon point at each Q^2 shows deviation from this trend, due to large background at the edges of the acceptance; these points are not considered in the rate fitting. Elastic events remain in the center of the acceptance and would be subject to the fitted rates.

3.3.4. Scaler Values

Several well-understood and easily calculated efficiencies are included in the scaler files output by the replay. These include the Prescale Factor, the Computer Deadtime, the Electronic Deadtime, the Trigger Efficiency, the Drift Chamber (DC) Cut Efficiency.

The Prescale Factor is a value n set in software, telling the data acquisition software to only record one in every n events. This serves to reduce the number of events that the computer is forced to drop because of processing capacity.

Figure 3.6. β No-Track Fraction as a function of Momentum Transfer.

The Electronic and Computer Deadtimes are the fractions of events (after Prescale) which are dropped because of insufficient capacity to record them, whether in the detector electronics package or at the computer recording the data. Electronic Deadtime increases with the rate of particles generating triggers, to a maximum of 0.8% (Figure 3.7a). Computer Deadtime scales with the rate of triggers after prescaling, to a maximum of 74% (Figure 3.7b). While this is high, the event loss is uniform across the acceptance and does not affect the quality of the data — tests have shown that runs with 90% deadtime (a factor of 10 correction to the cross section) yield $< 1\%$ uncertainty [30].

The Trigger Efficiency is a measure of how many events are lost because they did not generate signals in the three hodoscope planes necessary to produce a trigger. The per-plane efficiency and thus the Trigger Efficiency are calculated by comparing the fraction of events that generated signals in all four hodoscope planes to the total number of triggers. The hodoscope paddle efficiency is quite high, and this value is typically over 99% (Figure

3.7c), with the exception of particularly high-rate runs where some hodoscope panels at the outside edges of the acceptance were turned off in order to reduce the trigger rate. Elastic events remain in the center of the hodoscope, so this adjustment does not affect the relevant data.

The DC Cut Efficiency measures the fraction of events where there is enough data in the wire drift chambers and scintillator to conclude that there was probably a real event, and also the analysis code was able to reconstruct a track from the hits in the drift chambers (Figure 3.7d). Runs with high background near the edges of acceptance show a lower efficiency here as well.

(a) Electronic Deadtime
v. Trigger Rate.

(b) Computer Deadtime
v. Scaled Rate.

(c) Trigger Efficiency
v. Trigger Rate. Low eff. points due
to some paddles being turned off.

(d) DC Cut Efficiency
v. Trigger Rate. Low eff. points when
many events at the edge of acceptance.

Figure 3.7. Trigger Efficiencies.

3.3.5. PID Efficiency

Primary particle identification was provided by a $\Delta\beta$ cut. The $\Delta\beta$ spectrum, calculated from the Time of Flight, is the difference between the measured speed of the detected particle and the speed which would be seen for a proton at the measured momentum. Genuine protons, then, form a peak centered around zero. Faster pions form a peak on the positive side, and slower deuterons and tritons form peaks on the negative side.

Figure 3.8. $\Delta\beta$ High Side. The elastic peak at $\Delta p = 0$ disappears for the region in blue.

Figure 3.8 shows the Δp spectrum produced by particles moving faster than protons. Since Δp is the measured momentum minus the momentum of an elastic event with the same kinematics, events consistent with elastically scattered protons show a peak centered

on $\Delta p = 0$. The top plot shows the full $\Delta\beta$ spectrum in black, with the high region (including the pion peak) in blue. The bottom plot shows the full Δp spectrum in black, and the spectrum generated by the high $\Delta\beta$ region in blue. Notably, the elastic peak disappears when only the faster particles are considered. This provides a good measure of where to put the high-side particle identification cut — as long as there is no elastic peak in the excluded region, we are not removing protons.

At higher momentum transfer the pions and protons become closer in speed, and starting at $Q^2 = 1.7 \text{ GeV}^2$ the pion peak begins to merge into the proton peak. In addition, pions which lose energy in the detector will contribute a tail to the pion peak extending into the proton region. It is necessary to know how many pions are misidentified as protons.

Figure 3.9 shows the same $\Delta\beta$ spectrum, now with the slower particles, deuterons and tritons, highlighted in blue. In this case the Δp spectrum generated from the cut region does include an elastic peak — this is due to some protons scattering inside the detector after passing through the wire drift chambers, thus taking a longer or slower path between hodoscope planes than their measured momenta would indicate and forming a tail on the central proton $\Delta\beta$ peak. This means that when the low-side cut is applied some protons are lost, and the size of this loss must be applied as an efficiency correction to the cross section.

Electron-proton scattering does not produce deuterons, so any that are observed must come from the aluminium endcaps. The endcap contribution is subtracted off using the dummy target, which eliminates most of these deuterons; some remain, possibly due to the difference in thickness (and thus radiative length) between the dummy and the target

Figure 3.9. $\Delta\beta$ Low Side. The elastic peak is still present for this region.

endcaps. In order to measure the rate of proton loss due to the particle identification cuts, we adjusted the dummy subtraction to allow a fit of the proton tail in the regions between peaks. Comparing the size of the tail to that of the peak then allows extraction of the PID inefficiency.

Since particle identification is based upon time of flight, which is constant for protons at a constant momentum transfer, the PID efficiency was expected to be independent of ε . This turned out not to be true — a small dependence was observed, increasing from negligible at small momentum transfer to around 1.5% at the highest settings, possibly due to differences in the acceptance or the background at different spectrometer angles. Figure 3.10 shows the efficiencies plotted against Q^2 ; since the efficiency should not vary

Figure 3.10. Time of Flight Efficiency. Red dots are individual settings, black line is the applied correction as explained in the text.

with momentum transfer, a fixed value for the efficiency was applied to each ε point at fixed Q^2 (as determined by the fit in black). The observed variation was then treated as an uncertainty on that efficiency, with a 0.5% ε -dependent correction for $Q^2 \leq 2.0 \text{ GeV}^2$ and a 1.0% correction above. These corrections were used to obtain a slope correction to the LT separations, as described in Section 5.1.

3.3.6. Target Length

Because the target presents a curved face to the beam, an off-center beam will be exposed to less than the total target thickness. In order to determine how far off center the beam was during running, several data sets were taken at different beam positions. By fitting the normalized yield to the shape of the target (Figure 3.11), the beam offset from target

center can be determined. The run position can then be compared to the fit center to compute how much target thickness is reacting with the beam and adjust the efficiency accordingly. The target offset was found to be 2.34 ± 0.11 mm at BPMC (the beam position monitor closest to the target), and during running the beam was placed at about $+1$ mm, with a ± 1 mm raster. The target length correction was then found to be less than 0.5% for all runs, and is applied on a run-by-run basis in the analysis.

Figure 3.11. Target Position Scan; Normalized yield vs. Beam Position (bpmc).

CHAPTER 4

Data Analysis: Cross Section Extraction

4.1. Observed Spectrum

The hydrogen data (Figure 4.6, in black) shows three main features — a central peak from elastic e-p scattering, a broad, linear background from scattering off of the cell walls, and a low-energy contribution from the inelastic processes. Two simulations are performed, one modeling the elastic scattering (magenta) and another modeling the compton scattering and pion photoproduction background effects (green). These and the spectrum taken from the dummy target standing in for empty cell walls (blue) can be fit together to reproduce the full data. The total scattering cross section is then obtained by applying the normalization value used to fit the elastic simulation to the data to the cross section used to generate that simulation.

Figure 4.1. Sample Unpeeling Plot. Hydrogen data in black, dummy in blue, elastic simulation in magenta, inelastic simulation in green, summed simulations and dummy in dashed red. 80% of the dummy yield has been subtracted in order to reveal more detail.

4.2. Elastic Simulation

Elastic scattering was simulated using the SIMC package used extensively in Jefferson Lab Halls C and A. SIMC is a monte-carlo simulator designed to approximate various

processes — For this experiment, it was used to simulate elastic scattering of electrons and protons.

SIMC was extended to allow generation in terms of the scattered proton kinematics — previously it would only generate events in terms of the electron kinematics. When detecting protons, then, it would be necessary to generate the entire possible kinematic range for scattered electrons and project to see whether or not the corresponding proton would make it into the HMS detector. Generating protons dramatically reduced the phase space that had to be populated, with a corresponding decrease in run time.

The values that are needed to generate an event are the beam energy, scattering position, and scattered proton angle — for elastic scattering these fix the proton momentum and electron kinematics. Beam energy is well controlled, so it can be populated in a tight region (0.05%, specified in the input) around the central beam energy. Scattering position can be anywhere inside the hydrogen target, which provides a slight variation between events in multiple scattering and radiation length inside the target. Angle is generated in a window around the HMS central angle, with extra space on either side to ensure that the full region is covered. A few events outside the HMS angular acceptance may also scatter into the detector, the wider generation area captures these as well.

Energy loss and radiative corrections are applied at this step according to the methods in [33]. Possibilities of soft and real Bremsstrahlung, vertex corrections, and such are considered and adjusted for before the particles are propagated into the detectors. Particle absorption is not accounted for in the simulation, instead being applied as a correction to the total scattering yield.

Once the proton kinematics are known, the particle is propagated through the HMS magnets and into the detector hut. Each of the materials starting from the scattering position are considered in turn for multiple scattering effects and energy loss. Multiple scattering is applied in steps along the path from the target through the detector. This uses a small angle scattering approximation where the scattering angle follows a gaussian distribution with width in both dispersive and nondispersive directions given by

$$(4.1) \quad \theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} \sqrt{x/X_0} \left[1 + 0.038 \ln \frac{1}{\beta^2} \frac{x}{X_0} \right],$$

where x/X_0 is the thickness of the scattering material in radiation lengths [34].

The focussing quadrupoles and bending dipole have been field mapped, and the Monte-Carlo program COSY [35] was previously used to generate an optical model of the spectrometer magnets in both forward and backward directions, providing matrices that allow SIMC to propagate the scattered protons through the magnets. The particle's position is checked at each major limiting aperture within the spectrometer to determine whether the event would actually reach the detector hut.

Inside the hut, the particle is propagated through the various detector elements to determine the specific signals that such an event would generate. Once these are known, the scattering event is reconstructed using the same method as the replay, creating a final output that can be directly compared to the data.

4.2.1. Smearing and Non-gaussian Tails

In comparing the elastic simulation to the data, it was found that the elastic peaks in the data were broader than those obtained from the simulation. After several studies, it was

found that applying an additional gaussian smearing of 0.8 mrad in the angular variable y'_{tar} to the simulation best fit the observed discrepancy (Figure 4.2). In addition, non-gaussian tails were present in the data — these were corrected partially with additional angular smearing terms, and when comparing the measured and simulated yields, the elastic region was chosen to be wide enough to integrate over any remaining mismatch in the tails (as described in Section 4.4).

Figure 4.2. Comparison of simulated elastics to data, before and after angular smearing.

4.3. Background Simulation

There are two major background processes that contribute to the proton spectrum — Compton scattering and pion photoproduction. Both are caused by beam electrons radiating inside the cell wall or target, and the resulting photon interacting with the liquid

Figure 4.3. Schematic of π^0 photoproduction.

hydrogen (Figure 4.3). Compton scattering is when the photon and a proton interact elastically; pion photoproduction is an inelastic event where the photon is incident on a proton and the result is a nucleon and a pion. As we detect protons, we are concerned only with those from the $\gamma p \rightarrow \pi^0 p$ process.

In order to generate background spectra, we first need to know the bremsstrahlung spectra, which is dependent on the beam energy and thickness of the radiator. Figure 4.4 shows the calculated bremsstrahlung spectrum for a 3.362 GeV beam. The spectrum is generated in the thin target approximation [36] with the lower cutoff set below the acceptance for all angular settings. In this approximation, the spectrum scales proportionally to the radiation length — 0.143% for just the aluminium entrance endcap up to 0.611% for the endcap and full thickness of liquid hydrogen. The energy of the bremsstrahlung photon is then chosen according to this spectrum and the scattering depth in the target.

Figure 4.4. Fractional yield v. Bremsstrahlung energy, 3.362 GeV Electron Beam.

With the incident energy known, the scattering position and proton angle are generated as they are in the elastic simulation, and knowing these fixes the proton momentum as well as the kinematics of the outgoing photon or pion. Propagation of the proton through the HMS uses the same code as the elastic simulation, considering multiple scattering, energy loss, trajectory through the magnets, and so on. Background processes require the same smearing as does the elastic simulation, so that is also applied.

The photoproduction cross section, shown in Figure 4.5 (left plot), is used to scale the final event. Photoproduction cross sections approximately follow the form

$$(4.2) \quad \sigma_{\text{photo}} \propto s^{-7} (1 - \cos \theta^*)^{-5} (1 + \cos \theta^*)^{-4},$$

where s is the Mandelstam variable and θ^* is the pion angle in the center of momentum frame [37]. This equation is singular as $\theta^* \rightarrow \pm\pi/2$ and thus does not properly describe

Figure 4.5. π^0 and π^+ Photoproduction as a function of angle.

the scattering at high or low pion angles. Later rescaling to the observed data takes care of this scale error as well as any deviation from the described curve.

Because the photoproduction and compton scattering spectra overlap very heavily, they are not fit separately. Instead, the compton spectrum is rescaled to $(8.07 \times 10^{-3} * E_{beam} - 7.05 \times 10^{-3})$ (E_{beam} in GeV) of the yield of the pion photoproduction spectrum, as determined from the yields in [38], a range of about 0.5% to 4.2% of the photoproduction yield.

4.4. Unpeeling and Cross Section Extraction

The observed spectrum consists of three main regions, two of which are fit by the simulations and the last by the dummy spectrum (Figure 4.6a). Elastic protons populate the central peak, so that is where the elastic simulation dominates. Above the peak are

superelastic particles with more momentum than an elastic proton, which can only come from the target endcaps — the dummy dominates in this region, as it captures only the endcap effects. Below the center are inelastic particles with less momentum than would be seen for an elastic proton — these protons result from inelastic background processes, so those simulations dominate. Rather than fitting by each histogram bin individually, an ‘unpeeling’ procedure was used, in which these three larger regions were taken and the individual components scaled to the total spectrum. This method minimizes the effects of any mismatch in peak widths or tails by setting the edges of the scaling regions outside of any mismatch in shape. The boundaries of each region depend upon the kinematics, varying with momentum and angle.

Since the endcaps contribute events across the entire kinematics window and are the sole source in the superelastic region, unpeeling begins with a preliminary dummy subtraction. The dummy spectrum is scaled such that the counts in its superelastic fitting region are equal to those in the same region of the data. The elastic simulation always has a tail resulting from energy loss effects that extend underneath the inelastic background and frequently also spreads somewhat into the superelastic region used to find the dummy subtraction, so next a preliminary elastic subtraction is found by setting the counts in its central elastic fitting region equal to those in the dummy-subtracted data. In no case does the spectrum produced by the inelastic simulation extend into the superelastic fitting region, so the final dummy fitting value can then be found (Figure 4.6b). With this final dummy and preliminary elastic fit, the background simulation fitting value is found by comparing counts in the inelastic fitting region (Figure 4.6c). Last, the final elastic fit

(a) Initial scales.

(b) Dummy rescaled to simulation-subtracted data.

(c) Inelastics rescaled to simulation-and-dummy-subtracted data.

(d) Final scale.

Figure 4.6. The unpeeling process. Hydrogen data in black, dummy in blue, elastic simulation in magenta, inelastic simulations in green, final scaled sum in dashed red.

value R is determined by comparing the unscaled elastic simulation to the data with final scaled subtractions of the dummy and inelastic simulation spectra (Figure 4.6d).

Q^2 varies slightly across the finite momentum and angle acceptance of the spectrometer, so the elastic simulation uses a model for $G_{Ep}(Q^2)$ and $G_{Mp}(Q^2)$ from Bosted's global fit of previous Rosenbluth data [39] (Q^2 in GeV^2):

$$(4.3) \quad G_{Ep}(Q^2) = \frac{1}{1.0 + 0.62Q + 0.68Q^2 + 2.80Q^3 + 0.83Q^4}$$

$$(4.4) \quad \frac{G_{Mp}(Q^2)}{\mu_p} = \frac{1}{1.0 + 0.35Q + 2.44Q^2 + 0.50Q^3 + 1.04Q^4 + 0.34Q^5}.$$

This ensures that the elastic simulation is close (within a few percent) to the final cross section. The Bosted fit values of the form factors are then used with Equation 1.13 and the scale factor R found from the unpeeling to determine the reduced cross section σ_R of the data, which can then be used to perform L-T separations. Values of R and σ_R are given in Table 4.1.

Table 4.1. Reduced cross section values

$Q^2(GeV^2)$	ε	R	σ_R
0.40	0.568	0.963(10)	2.210(24) $E-01$
	0.883	0.962(10)	2.692(29) $E-01$
	0.984	0.945(10)	2.795(30) $E-01$
	0.985	0.937(10)	2.772(30) $E-01$
0.50	0.444	0.971(10)	1.712(18) $E-01$
	0.565	0.990(10)	1.880(20) $E-01$
	0.847	0.986(10)	2.188(23) $E-01$
	0.884	0.987(10)	2.231(24) $E-01$
	0.949	0.976(10)	2.278(24) $E-01$
	0.979	0.968(10)	2.292(24) $E-01$
	0.989	0.970(10)	2.306(25) $E-01$
0.60	0.318	0.991(10)	1.375(15) $E-01$
	0.462	1.005(11)	1.514(16) $E-01$
	0.590	0.994(10)	1.601(17) $E-01$
	0.808	0.993(10)	1.778(19) $E-01$
	0.881	0.983(10)	1.820(19) $E-01$
	0.966	0.982(10)	1.887(20) $E-01$
	0.981	0.996(11)	1.925(20) $E-01$
0.70	0.190	0.990(10)	1.103(12) $E-01$
	0.355	1.004(11)	1.220(13) $E-01$
	0.767	0.998(11)	1.465(15) $E-01$
	0.823	1.003(11)	1.508(16) $E-01$
	0.922	1.006(11)	1.574(17) $E-01$
	0.968	1.001(11)	1.594(17) $E-01$
0.80	0.066	1.004(11)	9.162(97) $E-02$
	0.247	1.006(11)	1.003(11) $E-01$
	0.416	1.002(11)	1.079(11) $E-01$
	0.504	0.983(10)	1.099(12) $E-01$
	0.722	1.001(11)	1.220(13) $E-01$
	0.827	0.987(10)	1.252(13) $E-01$
	0.952	0.992(11)	1.316(14) $E-01$
	0.973	1.020(11)	1.363(14) $E-01$

Table 4.1. Reduced cross section values (continued)

$Q^2(GeV^2)$	ε	R	σ_R
0.98	0.051	0.994(11)	$7.167(76)E-02$
	0.251	1.004(11)	$7.841(83)E-02$
	0.359	0.989(10)	$8.050(85)E-02$
	0.635	1.003(11)	$8.996(95)E-02$
	0.723	0.997(11)	$9.201(97)E-02$
	0.773	0.976(10)	$9.153(97)E-02$
	0.822	0.972(10)	$9.255(99)E-02$
	0.879	1.008(11)	$9.768(103)E-02$
	0.911	1.005(11)	$9.837(104)E-02$
	0.937	0.992(11)	$9.781(104)E-02$
	0.951	1.019(11)	$1.011(11)E-01$
	0.965	1.021(11)	$1.016(11)E-01$
	0.974	1.007(11)	$1.005(11)E-01$
1.19	0.057	0.951(10)	$5.293(57)E-02$
	0.525	1.008(11)	$6.528(69)E-02$
	0.637	0.994(11)	$6.645(71)E-02$
	0.841	0.984(10)	$6.960(74)E-02$
	0.936	0.994(11)	$7.205(76)E-02$
	0.941	0.996(11)	$7.232(77)E-02$
1.34	0.057	0.964(10)	$4.501(48)E-02$
	0.440	0.992(11)	$5.183(55)E-02$
	0.570	0.992(11)	$5.364(57)E-02$
	0.646	0.981(10)	$5.414(58)E-02$
	0.724	0.960(10)	$5.403(58)E-02$
	0.903	0.975(10)	$5.730(61)E-02$
	0.959	0.981(10)	$5.846(63)E-02$
1.70	0.222	0.990(11)	$3.239(35)E-02$
	0.394	0.984(10)	$3.353(36)E-02$
	0.892	0.963(10)	$3.643(40)E-02$
1.90	0.090	0.972(10)	$2.510(27)E-02$
	0.285	0.961(11)	$2.588(28)E-02$
	0.531	1.005(12)	$2.847(34)E-02$
	0.835	0.949(10)	$2.853(31)E-02$
	0.882	0.956(10)	$2.899(31)E-02$
	0.932	0.980(10)	$2.999(32)E-02$

Table 4.1. Reduced cross section values (continued)

$Q^2(GeV^2)$	ε	R	σ_R
2.28	0.078	0.975(11)	$1.745(19)E-02$
	0.222	0.976(11)	$1.798(20)E-02$
	0.382	0.974(11)	$1.867(20)E-02$
	0.571	0.988(11)	$1.938(21)E-02$
	0.685	0.964(11)	$1.928(21)E-02$
	0.746	0.952(10)	$1.927(21)E-02$
	0.779	0.956(11)	$1.945(22)E-02$
	0.829	0.969(11)	$1.986(22)E-02$
	0.882	0.977(11)	$2.021(22)E-02$
	0.909	0.951(10)	$1.976(22)E-02$
2.95	0.087	0.972(11)	$9.902(114)E-03$
	0.351	0.999(11)	$1.059(12)E-02$
	0.518	0.950(13)	$1.033(14)E-02$
	0.661	0.966(12)	$1.073(13)E-02$
	0.758	0.971(11)	$1.093(13)E-02$
	0.861	0.950(11)	$1.088(13)E-02$
3.61	0.107	1.016(16)	$6.338(98)E-03$
	0.323	1.013(18)	$6.523(113)E-03$
	0.447	1.010(13)	$6.654(87)E-03$
	0.519	0.981(12)	$6.525(83)E-03$
	0.743	0.976(13)	$6.675(87)E-03$
	0.811	0.975(13)	$6.734(87)E-03$
4.25	0.116	1.016(17)	$4.180(71)E-03$
	0.269	1.031(21)	$4.333(88)E-03$
	0.360	0.998(15)	$4.254(63)E-03$
	0.538	0.978(15)	$4.273(66)E-03$
	0.655	0.973(15)	$4.318(65)E-03$
	0.747	0.981(15)	$4.403(66)E-03$
5.08	0.132	1.053(18)	$2.683(45)E-03$
	0.520	1.016(18)	$2.734(49)E-03$
	0.646	1.004(17)	$2.741(48)E-03$
5.76	0.144	0.978(23)	$1.752(41)E-03$
	0.202	1.034(22)	$1.809(39)E-03$
	0.393	0.939(38)	$1.736(70)E-03$
	0.551	1.011(21)	$1.903(39)E-03$

CHAPTER 5

Data Analysis: Form Factors**5.1. Rosenbluth Extractions**

Figure 5.1 shows linear least squares fits of the reduced cross section to ε at four momentum transfer settings. From equation 1.13, this gives $G_{Ep}^2(Q^2)$ as the slope and $\tau G_{Mp}^2(Q^2)$ as the intercept. Fits were also taken in the form

$$(5.1) \quad \sigma_R = (G_{Mp}(Q^2))^2 \left(\tau + \varepsilon \left(\frac{G_{Ep}(Q^2)}{\mu G_{Mp}(Q^2)} \right)^2 \right),$$

in order that the error on the form factor ratio would be given directly by the fit.

In addition to the uncertainty δ_{FIT} taken from the fit, a second fit was performed with a small change in the slope ($\delta_{PID} = 0.5\%$ for $Q^2 < 2.0 \text{ GeV}^2$, $\delta_{PID} = 1.0\%$ above, see Section 3.3.5) over the covered epsilon range ε_{min} to ε_{max} , introduced by scaling each point by $1 - \delta_{PID} * \varepsilon'$ where $\varepsilon' = \frac{\varepsilon - \varepsilon_{max}}{\varepsilon_{min} - \varepsilon_{max}}$, ranging from 0 at ε_{max} to 1 at ε_{min} . The difference between the fit values with and without this term are used to determine the uncertainty δ_{SLOPE} which corrects for uncertainty in the portion of the systematic corrections that depend linearly on epsilon. Such corrections change the value of G_E/G_M but do not contribute to the point-to-point uncertainty used to determine the size of non-linear contributions. The total uncertainty is then found by adding the fit and slope uncertainty terms in quadrature,

Figure 5.1. Reduced cross sections vs. ε .

$$(5.2) \quad \delta_{TOT} = \sqrt{\delta_{FIT}^2 + \delta_{SLOPE}^2}.$$

In each subfigure of 5.1 the coloured dashed line is the linear least squares fit to the data and the black dotted line indicates the slope determined from the global fit to polarization transfer data at that Q^2 . Table 5.1 gives the dipole-normalized form factors and the form factor ratio at each momentum transfer investigated.

$Q^2(GeV^2)$	$G_M/\mu_p G_D$	$\mu_p G_E/G_M$	n	χ_ν^2
0.40	$0.979 \pm 0.022 \pm 0.010$	$0.914 \pm 0.024 \pm 0.010$	4	0.81
0.50	$0.974 \pm 0.012 \pm 0.006$	$0.959 \pm 0.017 \pm 0.006$	7	0.77
0.60	$0.994 \pm 0.008 \pm 0.006$	$0.959 \pm 0.016 \pm 0.006$	7	0.43
0.70	$0.996 \pm 0.006 \pm 0.006$	$0.989 \pm 0.015 \pm 0.006$	6	0.18
0.80	$1.010 \pm 0.005 \pm 0.005$	$0.973 \pm 0.014 \pm 0.005$	8	1.20
0.98	$1.017 \pm 0.004 \pm 0.007$	$0.994 \pm 0.013 \pm 0.007$	13	1.85
1.19	$1.009 \pm 0.005 \pm 0.009$	$1.052 \pm 0.020 \pm 0.009$	6	1.79
1.34	$1.023 \pm 0.005 \pm 0.007$	$0.999 \pm 0.025 \pm 0.007$	7	1.47
1.70	$1.047 \pm 0.007 \pm 0.019$	$0.892 \pm 0.057 \pm 0.019$	3	0.03
1.90	$1.035 \pm 0.005 \pm 0.015$	$0.979 \pm 0.035 \pm 0.015$	6	3.08
2.28	$1.040 \pm 0.005 \pm 0.035$	$0.954 \pm 0.038 \pm 0.035$	10	1.09
2.95	$1.034 \pm 0.006 \pm 0.049$	$0.939 \pm 0.071 \pm 0.049$	6	1.72
3.61	$1.042 \pm 0.007 \pm 0.076$	$0.820 \pm 0.134 \pm 0.076$	6	0.47
4.25	$1.027 \pm 0.008 \pm 0.104$	$0.793 \pm 0.194 \pm 0.104$	6	0.40
5.08	$1.024 \pm 0.011 \pm 0.170$	$0.724 \pm 0.372 \pm 0.170$	3	0.01
5.76	$0.964 \pm 0.013 \pm 0.108$	$1.443 \pm 0.299 \pm 0.108$	4	0.97

Table 5.1. Form Factor Ratio Values. First uncertainty is δ_{FIT} , second is δ_{SLOPE} .

5.2. Form Factor Ratios

Figure 5.2 shows the ratio $\frac{G_M}{\mu_p G_D}$ as a function of Q^2 . The solid line is a global fit to previous cross section data, and our data is consistent with prior cross section measurements. The dashed line shows how the fit would move after correction for two-photon exchange effects.

Figure 5.3a shows the form factor ratio $\mu_p \frac{G_E}{G_M}$ as a function of Q^2 , and Figure 5.3b compares our data to prior Rosenbluth and polarization transfer data. The solid line is a global fit to previous cross section data, without no TPE corrections; the dashed line is a combined fit to polarization transfer and TPE-corrected cross section data [12].

Figure 5.2. Comparison of Magnetic Form Factor to Dipole. 1% normalization uncertainty (not shown). This experiment in black, previous proton Rosenbluth in green. Solid line shows is a global fit to previous data, dashed shows how that fit moves with TPE included.

Our data improves upon previous Rosenbluth extractions to show a significant separation from polarization transfer in the $1.0 - 2.0 \text{ GeV}^2$ range where prior data was inconclusive — these improved Rosenbluth extractions will allow for an improvement in the extraction of TPE contributions based on the difference between cross section and polarization data in this range [24, 25].

Figure 5.4 shows the effect of correcting the data for the linear portion of the TPE correction, via the method in Ref. [12]. Black circles show the uncorrected data, and the black line is the global Rosenbluth fit. The red data is corrected by a term that increases with Q^2 and is linear in ε , going to zero at $\varepsilon = 1$. The red dashed line shows a fit to both polarization transfer and TPE-corrected prior Rosenbluth — the correction significantly improves agreement at high Q^2 , and at low Q^2 data shifts downward but does not yield good agreement.

(a) Our results and previous experiment.

(b) Global results.

Figure 5.3. Form Factor Ratio, Global Results. This experiment in black, previous proton Rosenbluth in green, global Rosenbluth (inc. proton Rosenbluth) in red, polarization transfer in blue. Solid line is a global fit to all prior Rosenbluth data, dashed line is a fit to polarization transfer and TPE-corrected Rosenbluth.

Figure 5.4. Comparison of data with and without TPE correction. Uncorrected ratios and global fit in black; TPE-corrected ratios and polarization transfer fit in red. Uncertainties associated with the TPE correction not shown. Slight shift in plotted Q^2 to make the points more distinct.

5.3. Linearity Measurements

In addition to the linear fit, the reduced cross section data was fit at each Q^2 value to Equation 1.19, where P_2 giving a measure of the deviation from nonlinearity — two such fits are shown in Figure 5.5. Extracted values of the curvature parameter P_2 are shown in Figure 5.6, our data in red and global analysis of prior Rosenbluth data in black.

Our data hints at a slightly negative value for P_2 but not to a significant degree. For points in the 2 to 5 GeV^2 range we achieved limits on the magnitude of nonlinearities comparable to previous global analysis, but using a single extraction at each point and with typically better coverage in the low- ε region. For $Q^2 < 2 \text{ GeV}^2$, we significantly improved upon prior limits, again using only a single extraction at each point. Overall we have attained a better limit upon the average value $\langle P_2 \rangle$, though while negative

Figure 5.5. Linearity tests.

Figure 5.6. Linearity Plot.

curvature would be consistent with the hadronic (Blunden) or GPD (Chen) models of two-photon exchange (described in Section 1.8) we do not yet see nonzero signature to a significant degree.

For Figure 5.7, a cross section σ_{PT} was determined from the polarization transfer ratio and the constraint that it meet our fit at $\varepsilon = 1$, where the TPE correction goes to

Figure 5.7. Comparison of data to TPE calculations at $Q^2 = 2.28 \text{ GeV}^2$. An additional 1 – 2% normalization uncertainty could shift our data up or down.

zero. The δ values for our data were obtained by subtracting σ_{PT} from the measured cross section: $\delta = (\sigma_R - \sigma_{PT})/\sigma_R$. The difference between the measured cross section and that expected in the Born approximation (based on the PT data) should correspond to the TPE contribution. Note that these points can all be shifted up or down 1 – 2%, due to the uncertainty in the extrapolation to $\varepsilon = 1$. This allows comparison with the various TPE models; while the data seems to agree with Blunden, both the Afanasev and Chen models curve inwards to meet zero at $\varepsilon = 1$, which would have the effect of shifting our δ value up or down, and the distance between the curves is comparable to the uncertainties on our data.

5.4. Future Work

Additional improvements may be possible to the uncertainties on the efficiencies at high Q^2 in order to rein in the large uncertainties, and to simulation statistics at low Q^2 . If so, this might reduce the uncertainties on $\mu_p G_E/G_M$ by $\approx 20 - 40\%$ (larger values at high Q^2) and P_2 limits by $\approx 20\%$, which would make the high Q^2 extractions of both comparable to the global analysis, without the complication of determining relative normalization factors. It may further be possible to obtain additional data on pion photoproduction cross sections from the background scaling, using the unpeeling scale factor and inelastic simulation cross sections in a similar manner as used here to determine the reduced cross section for elastic scattering (Section 4.4), though that is beyond the scope of this thesis.

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