Studying the Information Content of TMDs using Monte Carlo Generators

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Abstract. Theoretical advances in studies of the nucleon structure have been spurred by recent measurements of spin and/or azimuthal asymmetries worldwide. One of the main challenges still remaining is the extraction of the parton distribution functions, generalized to describe transverse momentum and spatial distributions of partons from these observables with no or minimal model dependence. In this topical review we present the latest developments in the field with emphasis on requirements for Monte-Carlo event generators, indispensable for studies of the complex 3D nucleon structure, and discuss examples of possible applications.

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1. Introduction

In the modern Standard Model of particle physics the nucleon is a complex object composed of interacting partons (quarks and gluons). The study of the partonic structure of nucleons is one of the main goals of nuclear physics. In a frame where the nucleon has very large momentum, the momenta of partons are almost collinear with the nucleon momentum (Fig.1, left panel), so that the nucleon can be approximated by a stream of partons, each carrying a fraction x of the nucleon momentum. The number densities of partons carrying a momentum fraction x are described by Parton Distribution Functions (PDFs). At large values of x PDFs are dominated by “valence” quarks, which determine the quantum numbers of hadrons, such as electric charge and isospin. In the last decades Deep Inelastic Scattering (DIS) of leptons off nucleons, emerged as a major tool for studying PDFs. Quantum Chromodynamics (QCD), the theory describing the strong interactions, provides crucial guidance for the phenomenology of PDFs in the shape of factorization theorems and evolution equations describing the dependence of PDFs on the hard scale Q , which in DIS is associated with the negative four-momentum transfer to the nucleon. However, QCD is not (yet) able to make definitive predictions of the x dependence of PDFs at a given scale due to their non-perturbative nature, which eventually have to be extracted from the data. Standard global fitting methodologies and error analysis have been developed during the decades of PDF measurements [1–4]. Recent studies have shown that especially in the region of large x , where little or no direct experimental information is available, the uncertainty related to the choice of parametrization and methodology may be as large or larger than the statistical uncertainty [5, 6]. Studies by HERAPDF [5] show that already for $x > 0.2$ the parametrization uncertainty for unpolarized PDFs is bigger than the statistical uncertainty. Similar conclusion was derived from the studies of polarized PDFs using different input parametrizations [6]. The bias due to the choice of the fit Ansatz can in principle be avoided using neural networks [7–9], but that technique requires a lot of data and has not yet been used for systematic studies of the multidimensional structure relevant for the understanding of the spin-orbit correlations. This indicates that understanding the dynamics is of importance already for the studies

Figure 1. The proton in 1D (left) and 3D (right).

of 1D PDFs, and more so for the studies of the 3D nucleon structure (Fig.1, right panel). 3D information on the nucleon is encoded in the transverse momentum dependent parton distributions (TMDs) and the generalized parton distributions (GPDs). As a generalization of PDFs, these functions have initiated a new phase in our exploration of the nucleon, and help to address long-standing questions concerning the orbital motion of partons inside the nucleon, and their spin and spatial distributions. The recent past has witnessed enormous progress in theoretical and experimental methods for the description of these functions. In this review, we summarize the latest developments, and discuss the highest-priority tasks to be addressed in the near future to advance the understanding of the transverse momentum and spatial structure of hadrons. Among these we will discuss the importance of developing Monte Carlo event generators capable of treating transverse and spin degrees of freedom, mainly focusing on the region of relatively large x , dominated by valence quarks and non-perturbative sea.

2. MC Generators

Monte Carlo event generators are essential components of almost all experimental analyses, and are widely used to make predictions and preparations for future experiments. The models used for underlying event generation and their reliability are important for the understanding of different systematic effects in studies of a variety of observables. The generated final-state particles can be fed into the detector simulation, allowing a precise prediction and verification for the entire experimental setup. However, as the detector simulation is usually a complex and computationally expensive task, simple event analysis techniques are also applied directly to event generator results.

So far simulations of high energy processes were focused primarily on the general features of parton showers, multiparton interactions and hadronization. There are two main classes of event generators, serving different purposes. Main applications include estimating rates of certain types of events, minimizing signal to background conditions, planning and optimizing detector performances, unfolding measured data for acceptance and detector resolution effects, and developing analysis frameworks. Most of the applications typically require full event generators where sets of outgoing particles are produced in the interactions between two incoming particles and a complete event is generated attempting to reproduce the raw data. In certain cases the single particle generators, where only the final state particles of interest are generated, can be used, providing fast tests of analysis procedures with relatively simple integration of different input models.

The most prominent examples of full event generators used by the high energy community include PYTHIA [10,11], HERWIG [12] and SHERPA [13]. Complete event generator programs are based on a combination of analytical results and various QCD-based models, and in general require a detailed modeling of the hadronization process in which colored partons are transformed into jets of colorless hadrons, photons, and leptons. PYTHIA in particular uses the so-called string fragmentation model developed

as an explicit and detailed framework, where the long-range confinement forces are allowed to distribute the energies and the flavors of a parton configuration among a collection of primary hadrons, which subsequently may decay further. In SHERPA the emission of additional QCD partons off the initial and final states is described through a parton-shower model. The fragmentation of partons into primary hadrons is described using a phenomenological cluster-hadronization model. HERWIG, a generator for simulation of hadronic final states in lepton-lepton, lepton-hadron and hadron-hadron collisions, incorporates important color coherence effects in the final state and initial state parton showers. A library of one dimensional PDFs, LHAPDF, has been developed to account for the latest advances in studies of PDFs [14].

The generators developed by the high energy community over the past decades describe well most of the features seen in experiments dedicated to PDF studies. Spin in hard scattering is a very challenging concept to implement in a consistent way. As a result, standard generators fail to describe the new class of measurements, which focuses on azimuthal distributions of final state hadrons and requires detailed understanding of the complex dynamics of spin-orbit correlations.

3. Generalizing partonic distributions

Recent measurements of large spin and azimuthal asymmetries worldwide [15–19] cannot be described in a 1D framework and are attributed to orbital motion of quarks, which requires an extension of PDFs to include transverse degrees of freedom. This is done in two complementary ways. GPDs [20–23] describe spatial distributions of partons, which carry the longitudinal momentum fraction x , in the transverse (“impact parameter”) plane [24]. TMDs [25–28] describe partons with longitudinal momentum fraction x and transverse momentum k_T . Model examples for each are shown in Fig. 2.

Studies of the distributions of partons in coordinate- and momentum-space as encoded in GPDs and TMDs respectively, have been widely recognized as key objectives of the JLab 12 GeV upgrade and the polarized pp program at RHIC, and are a key

Figure 2. Left panel: distributions of unpolarized up quark as function of x and impact parameter $b_\perp$ from Ref. [24]. Right panel: distribution of unpolarized up quark as function of x and the squared quark transverse momentum from Ref. [29].

part of the physics program of an Electron Ion Collider (EIC). Data on spin-azimuthal distributions of hadrons in semi-inclusive DIS (SIDIS) and polarized hadron-hadron collisions, which provide access to TMDs, have been accumulated in recent years by several collaborations, including HERMES, COMPASS, BELLE, BaBar, Halls A, B, C at JLab, and PHENIX and STAR at RHIC (BNL). After the first years of JLab12 and the upcoming polarized pp runs at RHIC, significantly larger data sets will be available. Although the interest in TMDs has grown enormously, we are still in need of a consistent theoretical and phenomenological description spanning the full kinematic regime covered by the (un)polarized world-data. Some TMDs have been already phenomenologically extracted, mainly from analyzing azimuthal distributions of single hadrons in SIDIS. To obtain a full picture about the 3D momentum structure of the partons in the nucleon from high to low x , it is important to connect the theoretical approaches to extract TMDs including evolution. The studies of 3D PDFs in general, and TMDs in particular, require a lot more attention to uncertainties due to input parametrizations, as more degrees of freedom and bigger number of input parameters may generate uncontrolled model uncertainties. This makes the development of a framework for testing different extraction procedures a high priority task for the community involved in studies of 3D PDFs. Development of realistic Monte-Carlo generators accounting for TMD evolution effects, spin-orbit and quark-gluon correlations will be a great support for a multifaceted effort to study the fundamental 3D structure of matter.

4. Azimuthal distributions of hadrons in electroproduction

Calculation of production cross sections rely upon knowledge of the distribution of the momentum fraction x and transverse momentum k_T of partons (quarks and gluons) in incoming hadrons in the relevant kinematic range. These parton densities cannot be calculated perturbatively, but rather are determined by global fits to data from SIDIS, Drell-Yan (DY), and, if applicable, from jet production at energy ranges of interest. In addition to having the best estimates for the values of the TMDs in a given kinematic range, it is important to understand their uncertainties. The partonic content of hadrons needs to be well understood and of sufficiently high precision if theoretical predictions are to match the experimental accuracy expected from future high precision measurements.

We consider the SIDIS process $\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X$ where ℓ denotes the lepton, N the nucleon, and h the produced hadron with four-momenta given in parentheses. The four-momentum transfer is $q = l - l'$ with $Q^2 = -q^2$. The relevant SIDIS variables are defined as $x = Q^2/(2P \cdot q)$, $y = (P \cdot q)/(P \cdot l)$, $z = (P \cdot P_h)/(P \cdot q)$. $P_{h\perp}$ is momentum component of the produced hadron transverse with respect to virtual photon with $P_{h\perp} = |P_{h\perp}|$. Assuming single photon exchange, the lepton-hadron cross section is expressed model-independently by a set of structure functions, see e.g. [30],

$$\frac{d\sigma^{e^-N \rightarrow e^-hX}}{dx dy d\phi_s dz d\phi_h dP_{h\perp}^2} = \frac{\alpha_{em}^2}{2xyQ^2} \frac{y^2}{1-\varepsilon} \left(1 + \frac{\gamma^2}{2x}\right)$$

$$\begin{aligned}
& \times \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
& + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \\
& + S_L [\sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h}] \\
& + \lambda_e S_L [\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h}] \\
& + S_T [\sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \\
& \quad + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
& \quad + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}] \\
& + \lambda_e S_T [\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \\
& \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right\}, \tag{1}
\end{aligned}$$

where α_{em} is the fine structure constant, and ϕ_h (ϕ_S) is the azimuthal angle of the produced hadron (azimuthal angle of transverse target polarization). On the structure functions $F_{XY,Z}^{\text{weight}} = F_{XY,Z}^{\text{weight}}(x, Q^2, z, P_{h\perp}^2)$, the subscripts $X, Y = \{U, L, T\}$ indicate the respective polarizations of beam and target, and $Z = \{L, T\}$ indicates polarization of the virtual photon where needed. The superscript “weight” reminds us of the kind of angular distribution of the produced hadrons. Furthermore, the ratio of longitudinal to transverse photon flux is $\varepsilon = (1 - y - \frac{1}{4}\gamma^2 y^2)/(1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2)$ with $\gamma = 2Mx/Q$. The polarization of the lepton is described by the helicity λ_e , and $(\mathbf{S}_T, S_L)$ is the nucleon polarization vector with $S_T = |\mathbf{S}_T|$.

Depending on the kinematics, the structure functions can be interpreted in terms of transverse-momentum dependent parton distribution and fragmentation functions, or in terms of the equally interesting but so far less studied fracture functions. By exploring different beam and target polarizations, and detecting a variety of final state hadrons (pions, kaons) one can access a wealth of novel information about the 3D spin structure of the nucleon, as well as spin-orbit effects in the target fragmentation region [30, 31].

Following the TMD factorization theorem one can write SIDIS structure functions in the current fragmentation region as a convolution of perturbatively calculable part $\mathcal{H}^a(Q^2)$, and universal TMD PDFs and TMD fragmentation functions [32–35]

$$\begin{aligned}
F_{UU,T}(x, z, P_{h\perp}^2, Q^2) &= \sum_a \mathcal{H}^a(Q^2) \int \int d\mathbf{k}_T d\mathbf{p}_T f_1^a(x, k_T^2; Q^2) \\
&\quad \times D_1^{a \rightarrow h}(z, p_T^2; Q^2) \delta(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{h\perp}) + Y(Q^2, P_{h\perp}^2) + \mathcal{O}(M/Q) \\
&= \sum_a \mathcal{H}^a(Q^2) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{P}_{h\perp}} \tilde{f}_1^a(x, b_T; Q) \tilde{D}_1^{a \rightarrow h}(z, b_T; Q) + \dots \tag{2}
\end{aligned}$$

and analog for polarized structure functions [36]. Here $f_1^a(x, k_T^2; Q^2)$ is the unpolarized TMD PDF describing an unpolarized parton of flavor a carrying longitudinal momentum fraction x and transverse momentum k_T at the renormalization scale chosen to be equal to Q^2 . $D_1^{a \rightarrow h}(z, p_T^2; Q^2)$ is the TMD fragmentation function for an unpolarized parton of flavor a fragmenting into an unpolarized hadron h which carries the longitudinal momentum fraction z and acquires the transverse momentum p_T (with respect to the fragmenting quark) during the fragmentation process. The term Y is introduced to

ensure a matching to the perturbative calculations at high transverse momentum. In the last step of Eq. (2) a 2D Fourier transform to express the TMDs in terms of the transverse coordinate b_T was used, which facilitates the discussion of evolution. At short distances $b_T \ll 1/\Lambda_{\text{QCD}}$ where $\alpha_s(1/b_T)$ is small, one can use perturbative QCD. But at large b_T one is sensitive to details of the hadron wave function and a perturbative treatment is not possible. The practical prescription to tame the growth of $\alpha_s(1/b_T)$ and match to a non-perturbative large distance description of the b_T -dependence consists in freezing the renormalization scale μ_b as

$$\mu_b = C_1/b_*(b_T) \quad \text{with} \quad b_*(b_T) = \begin{cases} b_T & \text{for } b_T \ll b_{\text{max}}, \\ b_{\text{max}} & \text{for } b_T \gg b_{\text{max}}, \end{cases} \quad (3)$$

which ensures a sufficiently large μ_b and applicability of perturbative QCD. The choice $C_1 = 2e^{-\gamma_E}$ for the arbitrary constant C_1 helps to minimize higher order corrections. One popular taming prescription (though other choices are also possible) is

$$b_*(b_T) = b_T / \sqrt{1 + b_T^2/b_{\text{max}}^2}. \quad (4)$$

Identifying renormalization and factorization scales and the scales regulating lightcone divergences with the hard scale Q of the process, the TMD evolved from μ_b to Q is

$$\tilde{f}_1(x, b_T; Q) = \tilde{f}_1(x, b_T; \mu_b) e^{-g_f(x, b_T; b_{\text{max}}) - g_K(b_T; b_{\text{max}}) \ln \frac{Q}{\mu_b}} \Phi, \quad (5)$$

where $\Phi = \Phi(b_*(b_T), \mu_b, Q)$ is perturbatively calculable. The function g_f is specific for a given TMD (or fragmentation function), while g_K is flavor-independent and describes universal soft-gluon radiation. The arbitrariness in choosing the taming prescription and the numerical value b_{max} is compensated by redefining correspondingly the non-perturbative functions $g_f(b_T; b_{\text{max}})$ and $g_K(x, b_T; b_{\text{max}})$. In practical applications it is nevertheless important to choose a physically well-motivated b_{max} in order to ideally exploit the physics output from a TMD analysis [37–44]. The scale μ_b could in principle be chosen as the initial scale of a model for TMDs. The MC generator accounting for spin-orbit correlations, which has been discussed recently on a dedicated workshop at INT Seattle [45], should account for latest developments in TMD theory and include all known scales, parameters and functions listed above as user defined input.

5. Nonperturbative properties of TMDs

The current knowledge of TMDs at relatively large x , where the spin-orbit correlations are significant, is based on theoretical studies in quark models, first lattices studies, and phenomenological extractions, which are reviewed below.

5.1. Information From First Principles

TMDs are subject to positivity constraints [46]. In the limit of a large number of colors N_c the $(u \pm d)$ and $(\bar{u} \pm \bar{d})$ flavor combinations of TMDs exhibit different behavior [47]. Such large- N_c scaling relations are well-supported by phenomenology [48] (see

below). First attempts to access information on Mellin moments of TMDs in Euclidean space-time lattice QCD simulations were reported [49–51]. One of the most exciting predictions from lattice studies were observations of strong spin and flavor dependences of k_T -distributions of partons [50, 51]. Recently a new approach was proposed to access also the x -dependence of PDFs and TMDs [52, 53] from lattice QCD. More work is needed [54], but the first calculations [55] indicate feasibility of the method.

5.2. TMD Models

TMDs were studied in various low-energy QCD-inspired models often based on complementary assumptions. Models oversimplify the complexity of QCD dynamics. Still, results from diverse models provide valuable insights to non-perturbative aspects. From the point of view of modeling, it is important to distinguish between T-even and T-odd TMDs according to their behavior under "naive time reversal", which is defined as usual time reversal but without interchange of initial and final states. The modeling of T-odd TMDs requires explicit gauge-field degrees of freedom, while for T-even TMDs it is sufficient to consider quark degrees of freedom.

Models of T-even TMDs can be classified as follows: (a) spectator models [56–61]; (b) light-front models [29, 62–64]; (c) chiral quark soliton models [65–67]; (d) bag models [68, 69]; (e) covariant parton models [70–72]; (f) quark-target models [73–77].

(a) In quark-diquark spectator models [56], one evaluates the correlation function that defines TMDs by truncating the sum over all intermediate states to a single on-shell spectator with quantum numbers of a scalar isoscalar or axial-vector isovector diquark. The diquark is understood as an effective particle which resumes non-perturbative effects related to sea quarks and gluons, and allows to treat the complicated many-particle system effectively by simple two-particle technique. A difficulty is that momentum and quark-number sum rules cannot be satisfied simultaneously, unless one considers the diquark as composed of two quarks [78]. Another approach free of these difficulties consists of describing scalar and axial-vector diquark correlations by a relativistic Faddeev equation in the Nambu–Jona-Lasinio model [79]. This model predicts transverse-momentum dependencies that are softer than a Gaussian fall-off, which was also recently seen in preliminary COMPASS data [80].

(b) Light-front models are built by expanding the nucleon state in the basis of free parton (Fock) states. One typically truncates the expansion to the minimal Fock state, which can correspond to three valence quarks in the light-front constituent quark model [29] or a valence quark and spectator diquark in the light-front quark-diquark model [62–64]. The probability amplitude to find these configurations in the nucleon is given by light-front wave functions which encode the bound state properties of hadrons, and have the important property to be eigenstates of the total quark orbital-angular momentum L_z^q [81, 82]. This allows one to map the multipole patterns in k_T associated with each TMD [29, 83, 84]. For phenomenological applications the k_T dependence in these approaches can be usefully approximated by a Gaussian form [63, 84–87].

(c) The chiral quark soliton model was applied to describe unpolarized [65,66] and helicity [66] TMDs of quarks and antiquarks. It is the only model so far including sea quarks, and predicts that sea-quark TMDs have significantly broader k_T -distributions than valence quarks, due to nonperturbative short-distance correlations caused by chiral symmetry breaking effects [66]. All T-even TMDs were predicted within the light-front version of this model restricted to the 3-quark light-front Fock state [67].

(d) The bag model is the only model discussed so far which incorporates confinement. It has been applied to calculate all twist-2 and twist-3 T-even TMDs in [68,69], and a complete set of linear and non-linear relations among them was derived (see also Ref. [88] for a discussion of the validity of such relations in quark models). Another remarkable insight is that the bag model strongly supports the Gaussian k_T -dependence of TMDs observed in phenomenology [89].

(e) In the covariant parton model [70–72] the partons are free, and described in terms of two covariant 3D momentum distributions which are spherically symmetric in nucleon rest frame: one describes unpolarized and the other polarized quarks. This model predicts the x - and k_T -dependence of TMDs from the x -dependence of PDFs [72] at large scales. Also this model supports an approximate Gaussian k_T -dependence. Other parton model approaches in the context of TMDs were discussed in [90–92].

(f) In quark-target models [73–77] the nucleon target is replaced by quark target. These are strictly speaking not nucleon models. But their QCD-type quark and gauge-field degrees of freedom make these models valuable theoretical test grounds, e.g. in [73] the limitation of the Lorentz-invariance relations was recognized (cf. [93] for a review).

In order to model T-odd TMDs gauge-boson degrees of freedom are required to describe initial/final-state interactions (ISI/FSI) between struck parton and target remnants. At leading twist there are two T-odd nucleon TMDs, the Sivers function and Boer-Mulders function. The first calculation which explicitly predicted a non-zero Sivers function within a scalar-diquark model [94] was followed by studies of T-odd TMDs in spectator models [57,58,60,74,95–98], bag models [99–101], non-relativistic model [102] and a light-front constituent quark model [103] (see Fig. 3 for a result from the latter). In these models the FSI/ISI are approximated by one-gluon exchange mechanism. The final expressions for T-odd functions are proportional to the strong coupling constant, which depends on the intrinsic hadronic scale of the model. Non-perturbative eikonal methods to calculate higher-order gluonic contributions from the gauge link were applied in spectator frameworks to improve this approximation [59,104,105]. A complementary approach is to construct augmented light-front wave functions which incorporate the rescattering effects by acquiring an imaginary (process-dependent) phase [106].

Although nucleon structure is the main topic, we include a brief review of TMD fragmentation functions which are of interest by themselves, besides their role for the interpretation of SIDIS data. Model studies of the T-odd Collins function were pioneered in [107] for pions using the Lund string mechanism, which was applied to vector mesons [108], and extended to include quark spin [109]. T-even pion and proton fragmentation functions were studied in [56] within a spectator model. The

Collins fragmentation function was studied in spectator-type frameworks with final state interactions modeled by rescattering among the quark fragments [110–114] or one-gluon exchange [115]. Twist-2 pion and kaon fragmentation functions were investigated in the Nambu–Jona-Lasinio–jet model using Monte Carlo simulations [79, 116], including favored and unfavored Collins functions [117, 118]. A review of fragmentation functions (including the collinear interference fragmentation functions not discussed here) can be found in [119].

5.3. First extractions of TMDs

Compared to PDFs, the status of TMD extractions is still in an “exploratory stage.” Admittedly, in the Collins-Soper-Sterman (CSS) framework [122] parametrizations of the k_T -dependence of unpolarized TMDs are routinely extracted from collider data on electroweak boson production [123–127] with important applications for Higgs production at LHC and jet physics, e.g. [128–130]. In these fits the entire richness of the 3D nucleon structure is “dumped” into 2 parameters assuming a fit form $g_f(x, b_T) = (g_1 + \ln \frac{Q}{2\mu_b}) b_T^2$ [123] (or 3 parameters [124, 125] using similar fit forms). This corresponds (roughly) to a Gaussian Ansatz, leaves no room for flavor dependence, and satisfactorily describes W - and Z -boson spectra, showing that these collider data have limited (though undeniable [131, 132]) sensitivity to the details of 3D nucleon structure.

There are open questions even in this rather well-established field. E.g. when using the $b_*(b_T)$ prescription (3) excellent fits to global DY and Z boson production data were obtained with very different choices of $b_{\max} = 0.1$ fm [126] and $b_{\max} = 0.3$ fm [127]. The first value is perhaps too small and in perturbative regime. The second value seems physically more appropriate to mark the transition between perturbative and non-perturbative regimes as argued in [40] and supported by chiral physics [66].

First efforts to extract information on novel TMDs concentrated on Sivers function [120, 121, 133–136], transversity [136–139] or Boer-Mulders function [140–144], and included tests of theoretical large- N_c predictions [121, 133]. Fig. 3 shows extractions

Figure 3. Extractions of (1)-transverse moments of the Sivers functions $x f_{1T}^{\perp(1)q}(x)$ by Anselmino et al [120] and Collins et al [121] at a scale of 2.5 GeV^2 . The theoretical calculation is from the light-front constituent quark model (LFCQM) [85, 103].

of the Sivers function by different groups (cf. [145]) from basically the same SIDIS data from HERMES. During the past decade the error bars have decreased, but discrepancies due to assumed fit functions and employed methodologies remain. In these studies TMD evolution was neglected or crudely estimated, with uncontrolled impact on DY [146–151]. With the advent of TMD evolution equations, advances towards more precise quantitative treatments became possible [37, 38, 41–43, 152–158]. Further progress will also be driven by data, such as the recent COMPASS data [159] analyzed in [160, 161]. Phenomenology efforts have been summarized recently by introduction of a library of fits and parameterisations for transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) together with an online plotting tool, TMDplotter [162].

6. MC in theory and practice

Further progress in the interpretation of the quantitative description of azimuthal, spin-orbit, and polarization phenomena in SIDIS at JLab 12 and EIC, and other processes (DY at RHIC, DY at COMPASS) will require novel MC technologies.

6.1. TMD-MC applications

Both full event generators and single-particle generators were extensively used for studies of different hard scattering processes and development of analysis frameworks. Several single-hadron and di-hadron generators were extensively used at HERMES, JLab and COMPASS.

Studies of the effects of limited energy on the available phase space to generate high P_T SIDIS events have been performed for JLab and COMPASS kinematics using a multidimensional generator [163] starting from the initial transverse momenta of quarks and accounting for the energy conservation across the production process. Fig. 4 (right panel) shows the final distribution, compared to the respective initial input Gaussian distribution. A possible effect of limited phase space may have been also observed by the Torino group [161] in attempts to describe P_T -distributions measured in JLab Hall-C experiment [164] by using parameters extracted from fits to HERMES multiplicities, as shown in left panel of Fig. 4.

One of the most commonly used MC applications are detector acceptance studies, which may have strong dependences on input distribution and fragmentation functions. Normally, experimental setups have a minimum polar angle for detection, which introduces some azimuthal modulation depending on the kinematical variables. An example of CLAS12 acceptance, defined as the ratio of reconstructed and generated events, for hadrons production in the range of P_T from 0.4 to 0.6 GeV is shown in Fig. 5.

In experiments, one measures a convolution of cross section and detector acceptance. One can describe the detector acceptance $A(x, Q^2, P_T, z, \phi)$ with Fourier

Figure 4. Left panel: JLab Hall-C data [164] on the cross section $d^5\sigma/d\Omega dE dz dP_T^2$ (in units of nb/GeV³sr) for π^+ production from deuterium in comparison to calculations of Ref. [161] based on simple Gaussian approximation, cf. [89]. Right panel: MC generated distributions implementing Gaussian function (solid line). The (blue) down triangles show the outcome for distributions from the MC at 6 GeV initial lepton energy, in comparison with the fit (dashed curve) with Gaussian function with a smaller width [163, 165].

series, where the constants A_m and B_n depend on the kinematical variables:

$$A(x, Q^2, P_T, z, \phi) = A_0 \left(1 + \sum_{m=1}^{\infty} A_m \cos m\phi + \sum_{n=1}^{\infty} B_n \sin n\phi \right). \quad (6)$$

With knowledge of the acceptance function provided by a realistic MC simulation, one can extract azimuthal moments from the complicated product using the following formulas

$$\begin{aligned} \sigma_{MN} = A(\phi)\sigma(\phi) = A_0\sigma_0 & \left\{ 1 + \frac{1}{2} \left(\sum_{i=1}^M A_i a_i + \sum_{i=1}^N B_i b_i \right) \right. \\ & + \cos \phi \left[a_1 + A_1 + \frac{1}{2} \left(\sum_{i=1}^{M-1} A_i a_{i+1} + \sum_{i=1}^{M-1} A_{i+1} a_i + \sum_{i=1}^{N-1} B_i b_{i+1} + \sum_{i=1}^N B_{i+1} b_i \right) \right] \\ & + \sin \phi \left[b_1 + B_1 + \frac{1}{2} \left(\sum_{i=1}^{M-1} B_{i+1} a_i - \sum_{i=1}^{M-1} B_i a_{i+1} + \sum_{i=1}^{N-1} A_i b_{i+1} - \sum_{i=1}^N A_{i+1} b_i \right) \right] \\ & \left. + \dots \right\}, \quad (7) \end{aligned}$$

where σ_{MN} is the measured yield parametrized in terms of respectively M and N cosine and sine modulations, and σ_0 , a_i and b_i correspond to azimuthal moments in the cross section, defined by the corresponding structure functions in Eq. (1).

The acceptance has weak dependence on the input cross sections, but for precision studies of azimuthal distributions this weak dependence may be crucial. In the CLAS12 example shown in Fig. 5, the two initial ϕ -distributions, ϕ -independent and ϕ -dependent with $\langle \cos \phi \rangle = -0.15$, yield $\cos \phi$ moments in the acceptance equal to 5.4% and 4.5%, respectively. This variation may be much stronger at higher z and P_T , indicating the importance of a precise description of spin-azimuthal terms for the extraction of different azimuthal moments in hard scattering processes. Accounting for all the relevant

Figure 5. Left panel: generated (flat) and reconstructed ϕ distributions in $ep \rightarrow e'\pi^+X$ events for CLAS12 detector and 11 GeV electron beam energy. Right panel: acceptance (ratio of reconstructed to generated events) fitted with the Fourier series in Eq. (6), including the first ten cosine moments and the first four sine moments.

Figure 6. Left panel: Azimuthal modulations in the final cross section generated by radiation of additional photon calculated from haprad2.0 [167] as a function of the transverse momentum of a pion in SIDIS in JLab12 kinematics at $x = 0.3$ and $z = 0.3$. Right panel: Sensitivity of the $\cos \phi$ -moment generated by radiative effects to ϕ -dependence of three different structure functions with $\cos \phi$ input amplitudes equal to -5% (squares), -10% (triangles up), and -15% (triangles down).

azimuthal moments in the MC generator, it will be crucial also to properly handle the radiative effects. As a matter of fact, the radiative effects in SIDIS also generate azimuthal moments which may couple with the moments in the Born cross section and produce more complex azimuthal structure in the observed cross section [166, 167]. Azimuthal moments generated by radiative effects, and their sensitivity to different input ϕ -dependent structure functions are shown on Fig.6.

A good agreement with some existing data on spin-azimuthal asymmetries in the current fragmentation region of SIDIS has been achieved using different TMD MCs. Implementation of spin-orbit correlations in a consistent way will, however, require development of a procedure for proper accounting of spin effects in the distribution and fragmentation processes.

6.2. Towards the complete TMD MC

Complete MC event generators, like LEPTO [168] and PYHTIA [10,11], are invaluable tools for various experimental and phenomenological studies, as mentioned in Sec. 2. Spin effects due to nucleon polarization or polarized partons in the nucleon and fragmentation process are not accounted for. Effects involving unpolarized partons, like Cahn and Sivers azimuthal modulations of the transverse momentum of the active quark before hard scattering and hadronization, can be incorporated in existing codes with small modifications, as done in mLEPTO [169,170] and LEPTO-PHI [171]. Moreover, mLEPTO has been recently used to study the Sivers single spin asymmetries in both single hadron and dihadron SIDIS production in COMPASS kinematics [172,173].

The incorporation of spin effects in the hadronization of polarized partons require much more efforts, and microscopic models of the fragmentation mechanism. Fragmentation of transversely polarized quarks was studied within the string fragmentation framework [174–176]. A stand-alone MC generator based on the NJL-jet model was developed in [117,118]. The development of a full MC event generator would require consistent models of the hadronization of polarized partons, including the polarization transfer in each hadron emission for cascade-type mechanisms, and the production and strong decays of vector mesons which is of particular importance for dihadron studies [177–179].

A new MC generator could be built extending existing MC generators, e.g., on the basis of the ThePEG framework [180] underlying the HERWIG++ generator. A desirable feature would be the separation of a user-friendly and easily modifiable physics layer from the underlying complex computational layer. The advantage of such a separation would be that developers could maintain and update the underlying algorithms and event generation software, without interfering with the customizations of the users. A realistic MC with solid fundamentals will be important part of the global 3D analysis framework, including the data analysis, phenomenology and extended 3D PDF library (see Fig.7). These and other features of the required MC event generators were vividly discussed at the dedicated INT workshop in Seattle [45].

Figure 7. Global analysis framework.

7. Conclusions

The 3D partonic structure accessible in hard scatterings is rich and complex. The understanding of the contributions to the final transverse-momentum dependence of different azimuthal moments in the cross section [181] will require detailed studies. Monte-Carlo event generators accounting for spin-orbit correlations will be crucial to study the dependence on different model inputs, as well as sensitivity of the extraction of underlying TMDs on various experimental uncertainties including acceptances, resolutions and radiative corrections. The future 3D nucleon structure extraction framework, should include in addition to extraction procedures a library of 3D PDFs[‡] and MC simulation frameworks using that library as input and allowing to check the quality of extracted PDFs for specific experimental conditions in a full range of accessible kinematics. This brief review was focused on MC applications to TMD studies, and those concepts that are of direct relevance such as model studies or extractions which provide input for MC generators. Our focus was primarily on spin-orbits effects expected to play a role in the region of not too small x . At small x unpolarized and polarized gluon TMDs become relevant, about which little is known and which can be studied in hadron colliders such as RHIC and LHC [182–193]. For these and other fascinating topics that could not be covered due to space limitations we refer to the excellent reviews [194–196].

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[‡] An important first step towards a library of 3D PDFs is the project “TMDlib and TMDplotter version 1.0.0” which recently became available [162] after this article was completed.

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