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The CST: its achievements and its connection to the light cone

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Abstract I review the applications of the Covariant Spectator Theory (CST) since its inception in 1969. Applications discussed here include calculations of NN scattering, 3N bound states, electromagnetic form factors of few-nucleon systems, and the recent successes in describing the dynamical generation of quark mass and the meson spectrum using a chirally invariant quark-antiquark interaction that includes confinement. The common origin of the Light Cone technique and the CST, which dates back to the 1970's, will be discussed.

Keywords Covariant Spectator Theory · nuclear force · three body force · linear confinement · meson spectrum

1 Background

1.1 Context: two ways to approach non-perturbative calculations

If the goal is to solve QCD non-perturbatively, and to treat the one and two-body problems consistently, then we can identify two major approaches:

- Approach #1 Start from the Dyson-Schwinger (DS) equation for the one-body self energy, and generalize to the two-body problem (c.f. Fig. 1), or
- Approach #2 Start from the two-body Bethe-Salpeter (BS) equation, and construct the one-body equation (c.f. Fig. 2).

If we had a theory where the vertices or kernels could be calculated exactly to all orders, the two approaches would be equivalent, but if the vertices or kernels are modeled using phenomenological expressions, each of them has disadvantages. The disadvantage of approach #1 (which starts from the one-body DS equation) is that its un-symmetric structure (only one of the vertices can be dressed if double counting is to be avoided) makes it less than straightforward how to extend it to the two-body sector without missing the dressing on the second vertex (new interaction terms are required). The

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Fig. 1 (Left panel) one-body Dyson-Schwinger equation. (Right panel) attempt to generalize the one-body DS equation to describe two-body bound states

Fig. 2 Two-body Bethe-Salpter equation with ladder and crossed ladder approximations.

disadvantage of approach #2 which starts from the two-body BS equation is that double counting may be introduced when the self-energy is calculated using the phenomenological two-body kernel. Spontaneous chiral symmetry breaking can be realized in both approaches, so this feature does not distinguish between them. The CST starts for the BS equation.

1.2 Origin of the CST – from 1964

Historical perspective In the early 1960's, field theory was not considered useful for non-perturbative calculations, and quarks and QCD had not yet been discovered. Strong interaction physics was the study of nucleon and meson (mostly pion) interactions, and dispersion theory (together with generalized unitarity) was the only non-perturbative technique available. (With the emergence of QCD and the development of dynamical quark models, dispersion theory fell into disfavor for a period of time. It has recently been “rediscovered” as a useful tool.)

It was in this context that CST was developed. At that time I calculated the deuteron form factors using dispersion theory [1; 2]. Each of the three form factors could be written in the form

$$F(Q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}F(s)}{s + Q^2} \quad (1)$$

where $Q^2 = -q^2$ with q the four-momentum transferred by the virtual photon. The threshold s_0 is normally determined by the threshold for the physical production of the intermediate $N\bar{N}$ system (at $s_0 = 4m^2 \simeq 181.4m_\pi^2$) that couples the photon to the deuteron-antideuteron final state (see the left panel of Fig. 3). This large threshold gives a very flat form factor that describes a very small deuteron,

Fig. 3 ((Left panel) triangle diagram showing the normal two-body $N\bar{N}$ “cut” and (right panel) the anomalous cut where the “spectator” is on-shell.

Fig. 4 (Left panel) anomalous region for the triangle diagram with a one-pion exchange (OPE) correction to the left vertex and (right panel) the anomalous region with OPE corrections at both vertices.

and is not in agreement with our picture of the deuteron as a weakly bound, extended system. To obtain the correct dispersion result for an extended deuteron with a mass $M_d \lesssim 2m$, one treats M_d as a complex variable and analytically continues the result for small M_d to large M_d , being careful to stay on the same sheet. A careful study of this continuation [3] shows that when $M_d > \sqrt{2}m$, the dispersion integral must be deformed with a new threshold that migrates to a smaller value. If the deuteron binding energy E_B is small, the new threshold is at $s_0 = 16mE_B \simeq 1.73m_\pi^2$, in complete agreement with our non-relativistic intuition for the behavior of a form factor of an extended, weakly bound system. Even more remarkable is the fact that the imaginary part of the form factor in the anomalous region (where $s_0 < s < 4m^2$) is given by the imaginary part of the triangle diagram with the *spectator on mass-shell*, illustrated in the right panel of Fig. 3.

When pion exchanges at the vertices are included, the cut structure becomes more complicated, and the anomalous thresholds and anomalous regions show a pattern illustrated in Fig. 4. The thresholds for the one and two pion exchanges shown in Fig. 4 are given in Table 1. Note that these contributions

Table 1 Thresholds s_0 for the pion exchange diagrams shown in Fig. 4.

diagram	normal threshold	anomalous threshold
one pion (left panel)	$(2m + \pi)^2 \simeq 209.3m_\pi^2$	$\simeq 11m_\pi^2$
two pions (right panel)	$(2m + 2\pi)^2 \simeq 239.2m_\pi^2$	$\simeq 28m_\pi^2$

can be summed by calculating a dressed $d\pi n$ vertex with one nucleon off-shell and the other nucleon restricted to its mass shell. When these contributions are summed to all orders, one has the spectator equation – this is the origin of the idea!

1.3 Quick derivation of the CST for two-body scattering

Start with the two-body Bethe-Salpeter equation for the scattering amplitude

$$\mathcal{M}(p, p'; P) = \mathcal{V}(p, p'; P) - i \int \frac{d^4k}{(2\pi)^4} \mathcal{V}(p, k; P) S(k) S(P - k) \mathcal{M}(k, p'; P) \quad (2)$$

Fig. 5 The ladder and crossed ladders to 4th order. The ladder can be separated into two parts represented by the restriction “c” on the two-body propagator, and its remainder, “1 – c”.

where $\mathcal{V}$ is the scattering kernel, S the dressed one body propagator, p, p', k are four-momenta of particle 1, and P the total four momentum. Unfortunately, the kernel is unknown and must be modeled. A common choice is to calculate the kernel from the infinite sum of all ladder and crossed ladder exchanges, illustrated to fourth order in Fig. 5. If the restriction “ c ” is chosen carefully, its remainder (“ $1 - c$ ”) and the crossed box will be very small. If this cancellation works to all orders, then the two-body equation using the restricted two-body propagator with a one boson exchange (OBE) kernel will give a very good approximation to the BS equation using the full *generalized ladder* (ladder plus crossed ladder) exchanges.

It is commonly assumed that the choice $c = 1$ (the BS in ladder approximation) is very good, but the crossed ladders are not small and this is not generally correct. In some cases, it can be proved that choosing c to restrict one particle to its mass-shell (the heavier one if they have unequal masses) is more efficient [3; 4], and in particular, this restriction is *exact* for scalar theories when the heavy (on shell particle) mass approaches infinity. This remarkable result provides a strong motivation for use of the CST.

1.4 Comparison of the two-body CST with the two-body light-cone equation

I close this background section by showing how the CST and the light cone equation are related. To see the connection, look at the 4th order box restricted to one of the two particles being on shell:

$$\mathcal{V}_4 = i \int \frac{d^4 k}{(2\pi)^4} \frac{\mathcal{V}(p, k; P) \mathcal{V}(k, p'; P)}{D(k) D(P - k)} \rightarrow - \int \frac{d^3 k}{(2\pi)^3} \frac{\mathcal{V}(p, \hat{k}; P) \mathcal{V}(\hat{k}, p'; P)}{2E_k D(P - \hat{k})} = \mathcal{V}_4^{\text{CST}} \quad (3)$$

where $D(k) = m^2 - k^2 - i\epsilon$, $\hat{k} = \{E_k, \mathbf{k}\}$, $E_k = \sqrt{m^2 + \mathbf{k}^2}$, and the decomposition is done in the rest frame $P = \{W, \mathbf{0}\}$, so that $D(P - \hat{k}) = W(2E_k - W)$.

The fourth order light cone result is computed in a similar way – by integrating over one of the components of k . Specifically, one decomposes k into light cone variables using $k_{\pm} = k_0 \pm k_z$, so that

$$\begin{aligned} k &= \{k_+, k_-, \mathbf{k}_{\perp}\} \\ P &= \{W, W, \mathbf{0}\} \end{aligned} \quad (4)$$

and integrates over k_+ keeping only the poles from the two particle propagators. Introducing the momentum fraction x defined through $k_- = xP_- = xW$, gives

$$\begin{aligned} \mathcal{V}_4 &= iW \int_{-\infty}^{\infty} dx \int \frac{dk_+ d^2 k_{\perp}}{2(2\pi)^4} \frac{\mathcal{V}(p, k; P) \mathcal{V}(k, p'; P)}{(E_{\perp}^2 - Wxk_+ - i\epsilon)(E_{\perp}^2 - W(1-x)(W - k_+) - i\epsilon)} \\ &\rightarrow - \int_0^1 dx \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \frac{\mathcal{V}(p, k_{\text{LC}}; P) \mathcal{V}(k_{\text{LC}}, p'; P)}{m^2 + \mathbf{k}_{\perp}^2 - x(1-x)W^2} = \mathcal{V}_4^{\text{LC}} \end{aligned} \quad (5)$$

where $k_{\text{LC}} = \{\frac{m^2 + \mathbf{k}_{\perp}^2}{xW}, xW, \mathbf{k}_{\perp}\}$. Note the famous result that the integral over x is restricted to the interval $[0, 1]$. This happens because outside of this interval the two poles from the propagator lie in the same half of the complex k_- plane, giving zero when the integral over k_- is evaluated using the Cauchy theorem.

At first the two results (3) and (5) appear to be unrelated, but introducing the variable y and the transformations

$$\begin{aligned} y &= \frac{E_k - k_z}{W} = \frac{\hat{k}_-}{P_-} & \hat{k} &= k'_{\text{LC}} = k_{\text{LC}}(x \rightarrow y) \\ W(2E_k - W) &= \frac{1}{y} [m^2 + \mathbf{k}_{\perp}^2 - y(1-y)W^2] \end{aligned} \quad (6)$$

we are able to transform $\mathcal{V}_4^{\text{CST}}$ into

$$\mathcal{V}_4^{\text{CST}} = - \int_0^{\infty} dy \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \frac{\mathcal{V}(p, k'_{\text{LC}}; P) \mathcal{V}(k'_{\text{LC}}, p'; P)}{m^2 + \mathbf{k}_{\perp}^2 - y(1-y)W^2} \quad (7)$$

Note that if we now change $y \rightarrow x$, (7) is *identical* to (5) except that the range of the x integration is now $[0, \infty)$ instead of $[0, 1]$ (a profound difference, of course). This similarity showing the close connection between CST and LC has been noticed by several people over many years.

Fig. 6 Variation of χ^2 from the fit to the two-body NN data (red line) and the triton binding energy E_t (blue line), with the off-shell coupling ν_σ defined in Eq. (8). This figure is for model WJC-2 discovered in 2008, but is qualitatively similar to the result first discovered in 1997.

2 Applications of the CST

The CST has been used to study many systems:

- Few-Quark Systems (discussed in Sec. 2.2)
- Few-Nucleon Systems (discussed in Sec. 2.1)
- Atomic Systems (S. J. Brodsky and G. P. Lepage [5] and M. I. Edies, et.al. [6])
- Inelastic scattering (W. P. Ford, et.al. [7])
- Nucleon-Nucleus Scattering (K. M. Maung, et.al. [8])
- Nuclear Matter (L. S. Celenza and C. Shakin [9])
- Pion-Nucleon Scattering (Y. Sura and FG [10])
- Relativistic Explanation of the EMC Effect (S. Liuti and FG [11])

In this talk I will only have time to briefly review the first two topics.

2.1 Applications of CST to Few-Nucleon Systems

A few key papers in the use of CST for the study of few-nucleon systems are:

- 1969 (FG) First CST paper deriving the cancellation of ladder and crossed ladder exchanges for scalar and electromagnetic interactions [4]
- 1987 (FG and D. O. Riska) Derivation of a two-body current that is conserved, even if the interaction contains phenomenological form factors [12]
- 1995 (J. W. Van Orden, N. Devine, FG) Calculation of deuteron form factors using the first realistic CST wave functions [13]
- 1997 (A. Stadler and FG) Observation that CST OBE models require *no* three-body forces. Demonstration that the fits to the NN data and the triton binding energy are very sensitive to the strength, ν_σ , of the off-shell coupling of the σ meson, and that the value of ν_σ that gives the best fit to the NN data also gives the correct triton binding energy, E_t (see more detailed discussion below) [14]
- 1997 (A. N. Kvinikhidze and B. Blankleider) Three-body CST currents fixed by the gauging method [15]
- 2004 (FG, A. Stadler, and M. T. Peña) For CST OBE models, a conserved three-body interaction current is derived that uses the same one-body and two-body interaction currents needed for the two-body system [16]
- 2008 (FG and A. Stadler) The CST OBE model (WJC-2) is found that gives a precise fit to np scattering data ($\chi^2/N_{\text{data}} \sim 1.1$) and requires only 15 parameters from the exchange of the photon and only 6 strongly interacting bosons $b[I(J^P)]$: $\sigma[0(0^+)]$, $\eta[0(0^-)]$, $\pi[1(0^-)]$, $\delta[1(0^+)]$, $\omega[0(1^-)]$, and $\rho[1(1^-)]$. These more accurate models show the same correlation between the quality of fit to the NN data and the three-body binding energy, first observed in 1997 [17]

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- 2014 (FG) derivation of a unique *isoscalar* two-body interaction current [18]

There is only space to focus on one of the most remarkable results of these CST OBE studies, illustrated in Fig. 6. The off-shell sigma coupling constant appears in the generalized σNN coupling

$$\Lambda^\sigma(p, p') = g_\sigma \mathbf{1} - \nu_\sigma [\Theta(p) + \Theta(p')] \quad (8)$$

where $p(p')$ is the four-momenta of the outgoing (incoming) nucleon and $\Theta(p) = (m - \not{p})/(2m)$ is the off-shell nucleon projection operator. Note that the interaction is *independent* of ν_σ if the nucleon is *on-shell*.

2.2 Applications of CST to Few-Quark Systems

The major results for the applications of CST to few-quark systems are:

- 1991–4 (FG and J. Milana) Initial studies of the meson spectrum with a relativistic generalization of confinement (for $L = 0$ states only) [19]
- 1999 (M. Uzzo and FG) Study of the stability of vector and scalar interactions using various equations [20]
- 2001 (C. Savkli and FG) Implications of the relativistic treatment of confinement [21]
- 2008 – (G. Ramalho, et.al.) Phenomenological models of hadronic form factors and static moments [22]
- 2014 – (E. Biernat, et.al.) Self energies and chiral symmetry breaking in a conjugation invariant CST model [23; 24; 25; 26]
- 2017 – (S. Leitão, et.al) Meson spectrum with $L \neq 0$ and spin (see the talk elsewhere in these proceedings) [27]

In this section I will highlight a few of these results. I start by emphasizing that here are two different ways to model confinement in QCD:

1. Require that the quark propagator have no poles on the real axis, *or*
2. Allow quark propagators to have poles on the real axis (i.e. quarks can be on shell), but construct an interaction that is zero when two or more quarks are on-shell (so that no physical scattering is possible)

The nonrelativistic and the relativistic CST approaches both assume the latter, while the well known Dyson Schwinger approach assumes the former. It may turn out that while the language used by the two approaches is quite different, they give equivalent results when properly carried out to all orders; future study may resolve these issues.

To understand the CST approach to confinement, first consider a screened definition of the linear confining potential in coordinate space

$$\tilde{V}_L(r) = -\lim_{\epsilon \rightarrow 0} \frac{\sigma}{\epsilon} (e^{-\epsilon r} - 1) = \sigma r. \quad (9)$$

Transforming this to momentum space gives

$$V_L(\mathbf{q}) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} = -8\pi\sigma \lim_{\epsilon \rightarrow 0} \left[\frac{1}{(\mathbf{q}^2 + \epsilon^2)^2} - \delta(\mathbf{q}) \int d^3q' \frac{1}{(\mathbf{q}'^2 + \epsilon^2)^2} \right] \quad (10)$$

Note that this satisfies the important constraint

$$0 = \int d^3q V_L(\mathbf{q}) \rightarrow \tilde{V}_L(0) \quad (11)$$

and leads immediately to the bound state momentum space Schrödinger equation for linear confinement

$$\left[\frac{\mathbf{p}^2}{2m_R} - E \right] \Psi(\mathbf{p}) = -\sigma \int \frac{d^3k}{\pi^2} \frac{\Psi(\mathbf{k}) - \Psi(\mathbf{p})}{(\mathbf{p} - \mathbf{k})^4}. \quad (12)$$

Fig. 7 (Left panel) the vertex function Γ with bound state mass μ and the momenta of the particles labeled (here μ is a shorthand notation for the four-momentum $P = \{\mu, \mathbf{0}\}$). (Right panel) the location of the particle poles (16) in the complex k_0 plane.

This equation is regular and has been solved numerically, giving solutions identical to the well known coordinate space solutions. Modifications are required for $L \neq 0$ states [27].

These result are easily generalized for use with the one-channel CST equations. Labeling the momenta as in Eq. (2), it is convenient to generalize (10), using the on-shell particle 1 four-momentum in place of the nonrelativistic three-momentum, giving

$$V_L(\hat{p} - \hat{k}) = -8\pi\sigma \lim_{\epsilon \rightarrow 0} \left\{ \frac{1}{[(\hat{p} - \hat{k})^2 - \epsilon^2]^2} - 2E_k \delta(\mathbf{p} - \mathbf{k}) \int \frac{d^3 k'}{2E_{k'}} \frac{1}{[(\hat{p} - \hat{k}')^2 - \epsilon^2]^2} \right\}. \quad (13)$$

Note that this reduces to (10) in the limit when the mass $m \rightarrow \infty$, and that it satisfies the relativistically invariant constraint

$$0 = \int d^3 k \frac{m}{E_k} V_L(\hat{p} - \hat{k}), \quad (14)$$

which reduces to (11) in the nonrelativistic limit. The covariant one channel CST equation becomes

$$[m^2 - (P - \hat{p})^2] \Psi(\mathbf{p}) = M_b(2E_p - M_b) = -2m\sigma \int \frac{d^3 k}{\pi^2} \frac{m}{E_k} \frac{\Psi(\mathbf{k}) - \Psi(\mathbf{p})}{(\hat{p} - \hat{k})^4}. \quad (15)$$

These equations provide a convenient relativistic generalization of the linear confining potential, and their solutions have been shown to give a good qualitative description of the low lying $L = 0$ states.

To study $q\bar{q}$ states which exhibit charge conjugation ($\mathcal{C}$) symmetric properties, it is necessary to generalize the one channel CST equation to two channels, and if the bound state mass (represented by μ in this discussion, so that in the rest system $P = \{\mu, \mathbf{0}\}$) is small, it must be extended to include all four possible channels. To understand how these coupled channel equations arise, examine the location of the four poles in the two-body propagator in the complex k_0 plane. In the rest system they are at:

$$\begin{aligned} S(k)^{-1} S(P - k)^{-1} &= (m^2 - k^2 - i\epsilon)(m^2 - (P - k)^2 - i\epsilon) \\ &= (E_k - k_0 - i\epsilon)(E_k + k_0 - i\epsilon)(E_k - \mu + k_0 - i\epsilon)(E_k + \mu - k_0 - i\epsilon) \end{aligned} \quad (16)$$

where the four-momenta of outgoing particle 1 is $k_1 = k$, the four-momenta of the the incoming particle 2 is $k_2 = k - P$ (or $-k_2 = P - k$ when reinterpreted as an outgoing antiparticle), as illustrated in the right panel of Fig. 7, and for simplicity I have used propagators for scalar particles since the pole structure is independent of spin. The location of these poles in the complex k_0 plane is shown in the left panel of Fig. 7. If μ is large and the system is not $\mathcal{C}$ invariant, it is sufficient to keep only the pole at $k_0 = E_k$ in the lower half plane, which restricts particle 1 to its positive energy mass shell. This is the one channel case I have discussed above.

When the system exhibits $\mathcal{C}$ invariance, it is necessary to construct CST equations that preserve this invariance. Since $\mathcal{C}$ transforms particles to antiparticles, the vertex function for a $q\bar{q}$ state is transformed from $\Gamma(k_1, k_2) = \Gamma(k, P)$ to $\Gamma(-k_2, -k_1) = \Gamma(P - k, P)$ (where, for simplicity, I write the result only for scalar particles). Hence, when particle 1 is on-shell the vertex function is $\Gamma(\hat{k}, P)$, and under $\mathcal{C}$ it is transformed to $\Gamma(P - \hat{k}, P)$. This amplitude has $k_2 = -\hat{k}$, and so corresponds to restricting the incoming particle 2 to is *negative* energy mass shell, and is *not* identical to $\Gamma(\hat{k}, P)$. The coupled equations shown in Fig. 8 couple these two different amplitudes together, ensuring that the

Fig. 8 The two-channel CST equation with exact charge conjugation symmetry. The particles marked with a $\times$ are on their positive energy mass shell and those with an $\times$ inside of a black square on their negative energy mass shell (see the discussion in the text). The triangles represent the vertex functions Γ .

system is $\mathcal{C}$ invariant. Operationally, the equations are obtained by averaging the contributions from the pole at $k_0 = E_k$ in the lower half plane and the contribution from the pole at $k_0 = -E_k + \mu$ in the upper half plane (hence a factor of $\frac{1}{2}$ is needed). Finally, if $\mu \rightarrow 0$, it is clear that none of the poles can be ignored, and in this limit a four channel equation is needed. For a detailed discussion, see Ref. [23].

Using the four channel equations, it has been shown what conditions must be met for spontaneous chiral symmetry breaking to occur [23]. If the kernel is a combination of scalar and vector interactions, so that it has the form $\mathcal{V} = V_S \mathbf{1} \otimes \mathbf{1} + V_V \gamma^\mu \otimes \gamma_\mu$, and the quark self energy is decomposed into $\Sigma(k) = A(k^2) + \not{k} B(k^2)$ (with $A_0 \equiv A(m^2)$, etc.), spontaneous chiral symmetry breaking can only occur if two conditions are simultaneously satisfied

$$1 = \frac{1 + m_0/A_0}{4m(1 - B_0)^2} \int \frac{d^3k}{(2\pi)^2} \frac{m}{E_k} [V_V(\hat{p}, \hat{k}) + V_V(\hat{p}, -\hat{k}) + V_S(\hat{p}, \hat{k}) + V_S(\hat{p}, -\hat{k})]$$

$$1 = \frac{1}{4m(1 - B_0)^2} \int \frac{d^3k}{(2\pi)^2} \frac{m}{E_k} [V_V(\hat{p}, \hat{k}) + V_V(\hat{p}, -\hat{k}) - V_S(\hat{p}, \hat{k}) - V_S(\hat{p}, -\hat{k})]. \quad (17)$$

The first condition is a consequence of the requirement that the mass function $M(k^2)$ have a solution for a spontaneously generated mass m (so that $M(m^2) = m$) and the second must be satisfied if the four-channel CST equations are to have a solution for a pion with zero mass. If the bare quark mass m_0 is zero, the equations are identical (signaling spontaneous chiral symmetry breaking) *provided* the constraint (14) is satisfied by the scalar part of the kernel. Hence, contrary to the normal intuition, a scalar interaction is *consistent* with the requirements of spontaneous chiral symmetry breaking provided it is *purely confining*.

3 Summary and Outlook

In conclusion, the CST

- is an efficient reorganization of the Feynman series,
- is manifestly covariant,
- has a smooth nonrelativistic limit which is easily interpreted
- provides a successful and efficient dynamical description of
 - few-quark systems including spontaneous chiral symmetry breaking, quark self energies, and understanding of the meson spectrum, and
 - few-nucleon systems including the ability to provide a precise fit to the np data base and the triton binding energy without the need for three-body forces .

Shortcomings of the CST (not discussed in this talk) include

- difficulties in extending it to new systems, where unexpected singularities can sometimes pose an obstacle,
- absence of a regularization scheme other than that based on the use of form factors, and
- the lack of a clear way to drive its equations from first principals.

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