$K\pi$ $I = 1/2$ S-wave from η_c decay data at BABAR and classic Meson-Meson scattering from LASS

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A recent analysis of data on the two photon production of the η_c and its decay to $K(K\pi)$ has determined the $K\pi$ S-wave amplitude in a "model-independent" way assuming primarily that the additional kaon is a spectator in this decay. The purpose of this paper is to fit these results, together with classic $K\pi$ production data from LASS, within a formalism that implements unitarity for the di-meson interaction. This fixes the $I = 1/2$ $K\pi \to K\pi$ S-wave amplitude up to 2.4 GeV. This resolves the Barrelet ambiguity in the original LASS analysis, and constrains the amount of inelasticity in $K\pi$ scattering, highlighting that this becomes significant beyond 1.8 GeV. This result needs to be checked by experimental information on the many inelastic channels, in particular $K\eta'$ and $K\pi\pi\pi$. Our analysis provides a single representation for the $K\pi$ S-wave from threshold, controlled by Chiral Perturbation Theory, through the broad κ, $K_0^*(1430)$ and $K_0^*(1950)$ resonances. There is no arbitrary sum of Breit-Wigner forms and random backgrounds for real $K\pi$ masses. Rather the form provides a representation that can be translated to other processes with $K\pi$ interactions with their own coupling functions, while automatically maintaining consistency with the chiral dynamics near threshold, with the LASS data and the new results on η_c decay.

I. INTRODUCTION

Electron-positron colliders have been a rich source of information about both hadron dynamics and the hadron spectrum. Decays of heavy flavors have given unprecedented access to the properties of light hadrons not so readily accessible in hadron machines. This is well illustrated by the classic meson-meson scattering studies of the 1970's and 80's. $\pi\pi$ production by pion beams [1, 2], or $K\pi$ production with kaon beams [3, 4] in peripheral reactions dominated by one-pion-exchange have given well-known results on meson-meson scattering reactions, both $\pi\pi$ and $K\pi$ in different charge configurations. The cross-sections for such processes are dominated by partial waves with the highest spin at any particular di-meson mass. Thus the peripheral cross-sections clearly feature the ρ, f_2, ρ_3, *etc.* resonances in $\pi\pi$ or their strange analogs, the $K^*(890)$, $K_2^*(1420)$, $K_3^*(1680)$, *etc.* in $K\pi$. The lower partial waves become more difficult to disentangle as the energy increases. In contrast, in e^+e^- collisions controlled by the production of vector mesons, the decay products automatically probe final state meson interactions in lower partial waves. A complementary set of primary quantum numbers is studied in two photon reactions.

Here we discuss the process initiated by $\gamma\gamma$ production of the η_c. Recently a model-independent partial wave analysis (MIPWA) of two decay channels $\eta_c \to \overline{K}K\pi$ has determined the $I = 1/2$ $K\pi$ S-wave amplitude well beyond 2 GeV, by studying two different charge modes: (1) $\eta_c \to K^+K^-\pi^0$ and (2) $\eta_c \to K_sK^\pm\pi^\mp$ [5]. S-wave $K\pi$ interactions are of particular interest because these are the outcome of so many reactions and decays involving a unit of strangeness. All the other interactions, both in the $K\pi$ and $\overline{K}K$ channels, were treated in a standard isobar picture with Breit-Wigner resonances, while the $K\pi$ S-wave is determined as a modulus and phase in 30 bins of 60 MeV from $K\pi$ threshold to 2.44 GeV. Thirty years ago the classic LASS experiment [3] was able to extract the $K^-\pi^+ \to K^-\pi^+$ partial waves starting from two hundred MeV above threshold at 0.825 GeV up to 1.7 GeV with one favored solution, and from 1.7 to 2.4 GeV with two Barrelet-related solutions. The universality of final state interactions imposed by unitarity means that the $K\pi$ system in each partial wave is related however it is produced. The best known of these relationships is given by Watson's theorem. When a final state of $(K\pi)X$ is produced where the particle X has no strong interaction with the $K\pi$ system in the energy region where $K\pi$ scattering is elastic, then the phase of the $K\pi$ interaction in each set of quantum numbers (isospin I, parity P and spin J) is the same as that in $K\pi \to K\pi$ scattering. Since amplitudes are analytic functions, the behavior of the phase plays a large part in determining the way its modulus varies. Above inelastic threshold relationships still hold, but these become a little more complicated. If the particle X does have strong interactions with K or π, these relations are further weakened and require a multi-channel treatment. Here we will assume, the third particle is a spectator. We know this is not exact, particularly in parts of the Dalitz plot with larger $K\pi$ mass.

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The aim of the present paper is to compare this $K\pi$ S-wave found in this Model Independent Partial Wave Analysis (MIPWA) with that in the classic meson-meson scattering reactions studied by LASS. With the η_c an isosinglet and the fact that its weak decay here converts a $\overline{c}c$ state into an $\overline{s}s$ system, this implies that the strong interaction that makes this materialize as $K\overline{K}\pi$ should preserve isospin. Then the $\overline{K}\pi$ or $K\pi$ sytem of whatever charge must have isospin 1/2. Thus the S-wave determined in the MIPWA is naturally assumed to have $I = 1/2$. On the other hand, $K^-\pi^+ \to K^-\pi^+$ scattering extracted by LASS from K^-p interactions involves both $I = 1/2$ and 3/2. Information on $K^-\pi^- \to K^-\pi^-$ scattering is then essential in separating out the pure $I = 1/2$ component of $K\pi$ scattering.

Here we presume that the K^+ or K_s in $K^+(K^-\pi^0)$ and $K_s(K^\pm\pi^\mp)$ decay modes, respectively, of the η_c are spectators. As already emphasized, these channels are of necessity related to $K\pi \to K\pi$ scattering by unitarity. As the data from peripheral production starts 200 MeV above $K\pi$ threshold, rather than use the entirely phenomenological effective range formula determined by LASS for this extrapolation [3], we input the results from a dispersive analysis which includes the constraints from Chiral Perturbation Theory as evaluated by Büttiker *et al.* [6].

In our combined analysis we treat the $K\pi$ process in terms of just two channels, $K\pi$ scattering itself, and an inelastic channel that starts at $K\eta$ threshold. We implement the constraints from unitarity in a K-matrix formalism, in a modification of the well-known P-vector approach: a modification that is physically appealing as it highlights what is common between the reactions and what is distinct, namely the particular coupling functions in the $\eta_c \to K(spectator)$. As we will see this provides an excellent description of the $I = 1/2$ S-wave $K\pi \to K\pi$ amplitude up to about 2.4 GeV. This is the main result of this paper. This representation has poles in the complex energy plane that can be identified with the κ (or $K_0^*(800)$) and the $K_0(1430)$ as required by the data. Importantly, our analysis is NOT a representation in terms of Breit-Wigner resonances and backgrounds, but a single representation of the entire S-wave up to 2.4 GeV. This can be used as a representation of the $I = 1/2$ $K\pi$ S-wave in many other reactions. It is important to recognize that unitarity is a property of the whole partial wave with no distinction between what is labelled "'resonance" and that called "background". Unitarity only knows about the sum. That is why our representation is for the whole partial wave. Above 2.4 GeV, inevitably other inelastic channels intrude and the assumption of no rescattering in the η_c decays is less trustworthy.

While our description of the $K\pi \to K\pi$ S-wave amplitude up to 2.2 GeV can be transported to other reactions with $K\pi$ final states, we are well aware that this analysis is performed within a *spectator* model. The third particle may not be a spectator and will inevitably rescatter with the other two particles. This is crucial for the complete implementation of both two and three body unitarity. While the required relationships are embodied in the long known Khuri-Treiman equations [7], their implementation in situations far outside the domain of elastic two body interactions is not yet practicable. While preliminary investigations of these equations in the simpler $\eta, \omega, \phi \to 3\pi$ and $D \to K\pi\pi$ have been completed [8–10], new techniques need to be developed to make such analyses possible within a maximum likelihood fit of the precision data to come in the years ahead. That is for the future. The present analysis goes as far as is possible at this time.

In Sect. 2 we set out the key elements of the K-matrix analysis we perform. Sect. 3 presents the results of our fit and in Sect. 4 we summarize.

II. S-MATRIX ELEMENTS DEFINED: K-MATRIX, P-VECTOR AND COUPLING FUNCTIONS

Resonances that feature in the $K\pi$ S-wave in η_c decay must be the same resonances that appear in any process with a $K\pi$ final state. Resonances are poles of the S-matrix in the complex energy plane on a nearby unphysical sheet (often with reflections on more distant sheets). It is a basic property of S-matrix theory that the same poles appear in all channels with the same quantum numbers at exactly the same positions in the complex energy plane. The residues of these poles will differ. These reflect the strength of coupling of the resonance to the particular production channel. Indeed, depending on how deep the pole is in the complex energy plane, its appearance in scattering or production can be quite different. A well-known effect in low energy pseudoscalar meson processes is generated by chiral symmetry breaking. This often places a zero in the amplitude, usually below threshold, that suppresses the near threshold production of S-wave final states. However, the appearance of such zeros is process dependent. This changes the shape of low energy scalar resonances in different reactions.

Importantly, low energy hadronic scattering is strongly constrained by unitarity, because of the small number of channels that are kinematically allowed, so we begin there. Let us consider $K\pi$ scattering with partial waves, $T(s)$, with given angular momentum J and isospin I, where s is the square of the c.m. energy. We restrict ourselves to just 2 possible final states: $K\pi$ and what we call $K\eta$, as the lowest inelastic threshold. Let us label the $K\pi$ channel by '1', and $K\eta$ by '2'. Then coupled channel unitarity requires:

$$\mathrm{Im}\, T_{11}(s) \;=\; \rho_1\, |T_{11}(s)|^2 \;+\; \rho_2\, |T_{12}|^2\ , \text{etc.}, \qquad (1)$$

where $\rho_i = 2k_i/\sqrt{s}$ with k_i the c.m. 3-momentum for final state 'i'. As is well-known these coupled channel relations can be simply satisfied by a K-matrix representation. While the amplitudes T_{ij} are complex functions even when s is real, the corresponding K-matrix elements, K_{ij}, are real for real s. As a matrix equation,

$$\mathbf{T} \;=\; \mathbf{K}\,[\mathbf{I}-\mathbf{i}\rho\mathbf{K}]^{-1}\;. \tag{2}$$

Then in our example with just two channels:

$$\begin{aligned} T_{11} &= \frac{K_{11}-\rho_2\,\det K}{\Delta}\;,\; T_{12} \;=\; \frac{K_{12}}{\Delta}\;,\\ T_{22} &= \frac{K_{22}-\rho_1\,\det K}{\Delta}\;, \end{aligned} \tag{3}$$

where $\det K \;=\; K_{11}\,K_{22}\;-\;K_{12}^2$ and

$$\Delta \;=\; 1\;-\;i\rho_1\,K_{11}\;-\;i\rho_2\,K_{22}\;-\;\rho_1\,\rho_2\,\det K\;.$$

Importantly, the ρ_i are to be analytically continued if the energy is below threshold 'i'.

The K-matrix elements themselves have no physical meaning; they are just a convenient way of parametrizing the constraint of coupled channel unitarity. For $K\pi \to K\pi$ scattering we have the benefit of the classic meson production, by one pion exchange, from LASS. For the $K^-\pi^+$ channel we have data above 825 MeV, and for $K^-\pi^-$ scattering from 730 to 1720 MeV. Down towards threshold the $I = 1/2, 3/2$ S-waves are better constrained by Chiral Perturbation Theory, known to next-to-leading order, rather then the simple effective range formula used originally by LASS [3, 11]. A similar fitting was performed as preparation for Dalitz plot analysis of $D \to (K\pi)\pi$ results from the FOCUS Collaboration [12]. However, here we will treat the inelastic channel, for which we have little information, as an effective channel starting at $K\eta$ threshold. Consequently, this channel is assumed to saturate the $K\pi$ inelastic cross-section inferred from the LASS data up to 2.4 GeV.

For the $I = 1/2$ $K\pi$ S-wave we represent the K-matrix elements by a sum of poles and a polynomial (with poles at $s = s_a, s_b, ...$):

$$K_{ij} \;=\; \frac{s-s_A}{s_{K\pi}}\left[\sum_{\alpha=a,b,..}\frac{g_i^\alpha g_j^\alpha}{s\;-\;s_\alpha}\;+\;\sum_n^N C_{ij,n}\,X^n\right]\;. \tag{4}$$

It is useful to define a scale $s_{K\pi} \;=\; (M_K^2 + m_\pi^2)$. Then, s_A is the Adler zero that at next-to-leading order in Chiral Perturbation Theory has been determined to be at $s_A \;=\; 0.87753\,s_{K\pi}$ for the $I = 1/2$ channel and $s_A \;=\; 1.0209\,s_{K\pi}$ for $I = 3/2$. The factor of $(s - s_A)$ is rendered dimensionless by dividing by the factor of $s_{K\pi}$ too. The variable X is linear in s chosen to be $-1 \le X \le +1$ in the range fitted from s_{bot} to s_{top}, so

$$X \;=\; \frac{2s-(s_{top}+s_{bot})}{s_{top}-s_{bot}}\;. \tag{5}$$

In our fits either one or two pole terms and a polynomial up to $N = 3$ is sufficient. How many K-matrix poles and the order of the polynomial is solely a matter of economy of parameters and not of physics. At least the pole at $s = s_a$ helps with this. Remember the poles of the K-matrix have no phyiscal significance, only the poles of the T-matrix do, *i.e.* the zeros of $[1 - i\rho K]$. Here the fitting covers $K\pi$ threshold to 2.4 GeV, with $s_{bot} = 0.36$ GeV^2 and $s_{top} = 5.832$ GeV^2.

For the $I = 3/2$ $K\pi$ S-wave we have little evidence about inelastic channels, so we simply use a one channel K-matrix representation which has no poles and is simply a polynomial, like Eq. (4), with terms up to $N = 2$ or 3. The data on $K^-\pi^+ \to K^-\pi^+$ S-wave come from Aston *et al.* [3], and $K^-\pi^- \to K^-\pi^-$ from Estabrooks *et al* [4]. Fitting will give us a representation of the T-matrix elements that respects two channel unitarity and is consistent with Chiral Perturbation Theory.

It will be helpful to define a reduced T-matrix, $\hat{T}$, from which the Adler zeros present in elastic $K\pi$ scattering have been removed, since these are process-dependent:

$$\hat{T}_{ij} \;=\; \frac{s_{K\pi}}{s-s_A}\,T_{ij}\quad. \tag{6}$$

Now the decay amplitude, here for $\eta_c \to K+$ channel 'i', $\mathbf{F}$, is represented by the vector equation

$$\mathbf{F} \;=\; \mathbf{P}\,[\mathbf{I}-i\rho\mathbf{K}]^{-1}\;, \tag{7}$$

where $\mathbf{F}$ and $\mathbf{P}$ are complex vectors, $\mathbf{P}$ being called the *production* vector [13]. Resonance poles transmit from scattering to production and decay through the common zeros in $[\mathbf{I} - i\rho\mathbf{K}]$ in Eqs. (2,7). It is helpful to rewrite this in a physically more transparent way, directly in terms of the relation of the decay amplitudes to the T-matrix elements [14]. This relationship is easily established by noting that as the P-vector must have the same poles as any that appear in the K-matrix elements (to ensure that at energies above threshold the amplitude $\mathbf{F}$ does not have poles on the real energy axis). In the two channel case we consider here, one can re-write the components of the vector $\mathbf{P}$ as

$$\begin{aligned} P_1 &= \alpha_1(s)\,(K_{11}\;-\;i\rho_2\det K)\;+\;\alpha_2(s)\,K_{12}\;,\\ P_2 &= \alpha_1(s)\,K_{12}\;+\;\alpha_2(s)\,(K_{22}\;-\;i\rho_1\det K)\;, \end{aligned} \tag{8}$$

In practice, as described in [14], we re-express Eq. (7), by first removing the reaction-dependent Adler zeros, so in the two channel case we have from Eqs. (2,6-8):

$$\begin{aligned} F_1 &= \alpha_1(s)\,\hat{T}_{11}\;+\;\alpha_2(s)\,\hat{T}_{12}\;,\\ F_2 &= \alpha_1(s)\,\hat{T}_{12}\;+\;\alpha_2(s)\,\hat{T}_{22}\;, \end{aligned} \tag{9}$$

where the functions $\alpha_i(s)$ represent the strength of coupling of $\eta_c \to K$ to channel 'i'. Though unitarity makes no constraint on the phase of the production amplitude, the final state interactions should have the same phase when only the lowest channel is open, provided the other kaon is indeed a spectator. Consequently, we constrain the $\alpha_i(s)$ to have a common phase for channels $i = 1, 2$. One would need precise enough information about the inelastic channels, like $K\eta$, $K\eta'$, and not just the $K\pi$

final state, to be able to check the spectator hypothesis. Consequently, we represent the coupling functions as

$$\alpha_i(s) \;=\; \exp(i\gamma)\,\sum_n^{N'} A_i^n\,X^n \quad , \qquad (10)$$

with real coefficients A_i^n. This is because these functions only have left hand cut singularities, and so other than the production phase γ, should be real along the right hand cut encoded in the $\hat{T}_{ij}$ of Eq. (9). It is further presumed that a polynomial approximation over a finite region of $K\pi$ energy, $\sqrt{s}$, studied in η_c decay is sufficient. Indeed, in our fits $N' \leq 3$. Our fits show that an Adler-like zero does occur in the η_c amplitudes, and so we refit by setting

$$\alpha_i(s) \;=\; \frac{(s - s_A^{\eta_c})}{s_{K\pi}}\,\hat{\alpha}_i(s) \quad , \qquad (11)$$

and write

$$\hat{\alpha}_i(s) \;=\; \exp(i\gamma)\,\sum_n^{N''} \hat{A}_i^n\,X^n \quad . \qquad (12)$$

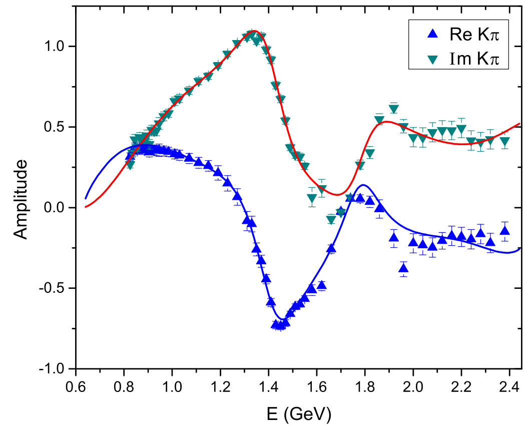

FIG. 1. The $K^-\pi^+ \to K^-\pi^+$ S-wave data as determined by LASS [3], plotted as the real and imaginary parts of the S-wave. The solid lines show the result of the fit to the combined data-sets: on $K\pi$ scattering from Aston *et al.* [3] and Estabrooks *et al.* [4] and next-to-leading order ChPT [6] below 825 MeV, as well as the η_c decay amplitudes.

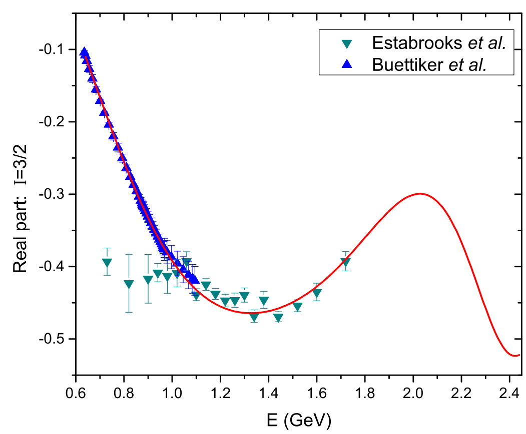

FIG. 2. The $K^-\pi^- \to K^-\pi^-$ S-wave data as determined by Estabrooks *et al.* [4] and at low energies by next-to-leading order ChPT [6]. The real part is plotted, as this wave is assumed to satisfy elastic unitarity in the whole energy range. However, this $I = 3/2$ wave is the least constrained above 1.8 GeV, and provides a significant uncertainty on this analysis. The solid line shows the result of the fit to the combined data-sets discussed in the text.

III. FIT TO η_c DECAY AND LASS $K\pi$ SCATTERING RESULTS

Having set up the formalism, we now introduce the data we are going to describe within this framework. As mentioned in the introduction, we have the $K\pi$ S-wave amplitude determined from a Dalitz plot analysis of η_c decay in two charge modes using a *spectator* model, in which only two body interactions are included. To distinguish the two modes we call the amplitudes F_1 for the channel $\eta_c \to K^+K^-\pi^0$ and G_1 for $\eta_c \to K_sK^\pm\pi^\mp$: both have the same structure and same coupling functions α_i, Eq. (8-12). Recall the subscript 1 indicates the $K\pi$ channel, while 2 the unmeasured effective inelastic channels KX. While the S-waves in $K\pi \to K\pi$ scattering are absolutely normalized by unitarity (with related phases) in the elastic region, that from η_c decay has arbitrary magnitude and phase, only the relative magnitudes and phases being determined. Consequently, in the MIPWA of Ref. [5] the amplitude has been chosen to be $F_1 = G_1 = \exp(i\pi/2)$ at 1450 MeV. The phase γ in Eqs. (9,11) takes care of this relative orientation. The statistical and systematic uncertainties tabulated in [5] are added in quadrature in the present analysis.

Below 1.7 GeV the LASS partial wave analysis is robust [3]. The partial waves are largely elastic. However, above that energy inelastic channels become increasingly important like $K\eta'$ and K + several pions. Partial wave (Barrelet) ambiguities are then removed by consideration of unitarity. Moreover, in $K\pi$ scattering the S-wave is a smaller and smaller fraction of the cross-sections as the energy increases above 2 GeV, making its accurate determination more difficult. This is where having data on channels with more limited quantum numbers, like that studied here from η_c decay, play a key role, since they naturally have a much larger $I = 1/2$ S-wave component. While unknown coupling functions intrude, nevertheless there are constraints on the $K\pi$ S-wave.

While $K\eta$ threshold is the onset of inelasticity, we will see that fitting the Aston *et al.* [3] and Estabrooks *et al.* [4] $K\pi$ data renders this onset weak. Only above 1.6 GeV does inelasticity become important. Using the formalism set out in Eqs. (3-6,8,9) in the two channel case, we fit the $K\pi$ scattering and η_c decay amplitudes. The quality of the fits is shown in Figs. (1-3) in each case for the real and imaginary parts. The exception is for the $K^-\pi^-$ results of Estabrooks *et al.* which are assumed to be elastic and for which we just give the real part. With no data above 1.8 GeV, this $I = 3/2$ wave is the least constrained, and provides a significant uncertainty on this analysis. The consistency between the η_c amplitudes and that of $K\pi$ scattering from LASS is good up to 1.7 GeV. Above that energy the η_c results impact particularly on the determination of the $I = 1/2$ $K\pi$ S-wave.

As described earlier, the overall phase of the $K\pi$ S-wave in the η_c decay amplitude is “corrected” in the fits by the angle γ shown in Eqs. (9,11). This is found to be $\sim 130^o$. In fitting the η_c-decay amplitudes we first assumed there to be no additional phase variation between the coupling functions α_1 and α_2, *i.e.* with coefficients $\hat{A}_i^n$ in Eq. (12) real. If a strong phase difference between these were required, this would signal the contribution from a strongly coupling third channel. To the extent that the fits are adequate (and even good), one can conclude that the second channel is satisfactorily playing the role of an *effective* inelastic channel. While the inelastic cross-section opens at $K\eta$ threshold, it contributes merely a few percent of the cross-section, in excellent consistency with other results from BaBar data [18]. The LASS Solution A is most consistent with the large inelasticity that is required above 1.9 GeV, so this is the solution shown. The behavior of the inelastic cross-section parametrized simply as $(1-\eta^2)/4$ illustrates this, Fig. 5, where η is the usual $K\pi$ partial wave inelasticity. The fact that below $K\eta'$ threshold this cross-section is small is consistent with the results from other data, including [18]. An independent check of the large inelasticity seen in Fig. 5 above 1.9 GeV would be a critical test of this analysis. Of course, in such an analysis, the large $I = 1/2$ inelasticity above 1.7 GeV may merely be parametrizing our ignorance. Consequently an independent determination of the inelastic amplitude, either $K\eta \to K\eta$, or in $\eta_c \to K(K\eta)$ decay mode would be essential. Reassuringly the first lattice calculation of S-wave $K\pi \to K\eta$ scattering finds the onset of inelasticity also to be weak [15], even though the masses in these studies are not yet those for physical pions. In the present analysis, $K\eta$ is mnemonic for any channel that is **not** $K\pi$. A check on the amplitudes shown would also be an independent determination of the $I = 3/2$ $K\pi$ S-wave, which is surely not elastic above 1.5 GeV, and is unconstrained by data above 1.8 GeV, Fig. 2.

The resulting Argand plot for the $I = 1/2$ $K\pi$ S-wave is shown in Fig. 6. The amplitude plotted is defined by $\rho_1 T_{11}$ from Eqs. (2,3). The corresponding η_c amplitude favors an Adler zero at $s_A^{\eta_c} = 1.517\ s_{K\pi}$, to be compared with the $I = 1/2$ $K\pi$ zero at $0.878\ s_{K\pi}$. Thus the Adler zero in the η_c amplitudes is very close to $K\pi$ threshold, as

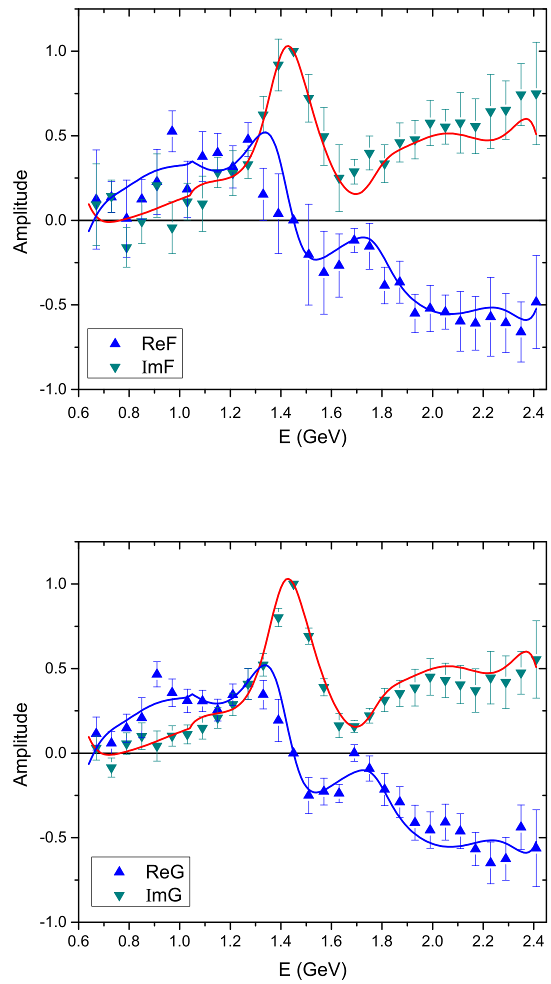

FIG. 3. The real and imaginary parts of the $K\pi$ S-wave in η_c decays. The solid lines are our combined fits to the $K\pi$ scattering and η_c decay amplitudes. The data in the upper plot are from the MIPWA analysis of the $K^{\pm}\pi^0K^{\mp}$ channel (ampl.itude $F_{(1)}$) and the lower from the $K^{\pm}\pi^{\mp}K_s$ mode (with amplitude $G_{(1)}$) [5]. In this MIPWA analysis the amplitudes at 1450 MeV in both channels have been normalized to have magnitude one and phase of $\pi/2$. Consequently, these datapoints have no errors associated with them. The fits are constrained to go through these points.

seen in Fig. 3. This partially accounts for the difference in shape of the κ and $K_0^*(1430)$ resonances in η_c decay and $K\pi$ scattering, *cf.* Fig. 1 with Fig. 3.

The simple isobar picture with nothing but resonance poles finds a $K_0^*(1430)$ and a $K_0^*(1900)$. Both are found here in the generalized spectator model. However, fitting the $I = 1/2$ $K\pi$ S-wave as one function contains also the κ. Though we know there is no $\kappa(900)$ [16] from earlier analysis of the LASS data, there is a $\kappa(650)$ as found from detailed dispersive analyses [17]. Our ampitudes are consistent with this.

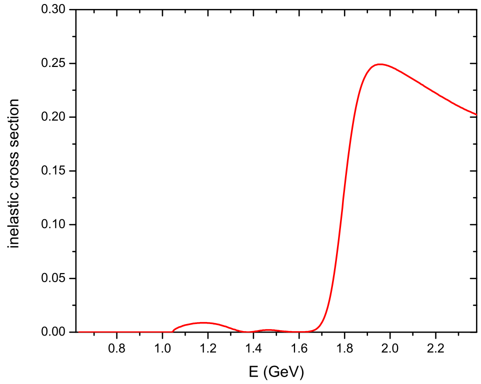

FIG. 4. The inelastic "cross-section" for the $I = 1/2$ S-wave defined from the partial wave amplitude by $(1-\eta^2)/4$, where η is the inelasticity. The result for this quantity from the fit. Though the K-matrix parametrization allows an inelasticity to start at $K\eta$ threshold, it is seen that below 1.7 GeV, the inelastic cross-section is consistent with zero. Note the unitarity limit is 1/4 for this quantity, which appears to be reached very rapidly at 1.9 GeV.

We now turn to the coupling functions of Eqs. (8,9), which are seen to be very smooth. Their variation with energy marks the change of coupling of each resonance and its final state interaction in $K\pi$ and effective $K\eta$ channels. Much of the overall energy variation comes from the appearance of an Adler-like zero in the $K\pi$ S-wave of the η_c decay amplitude. Consequently, we display $\hat{\alpha}_i(s)$, with this zero removed , see Eqs.(8-12), in Fig. 6. A check on their structure would of course be

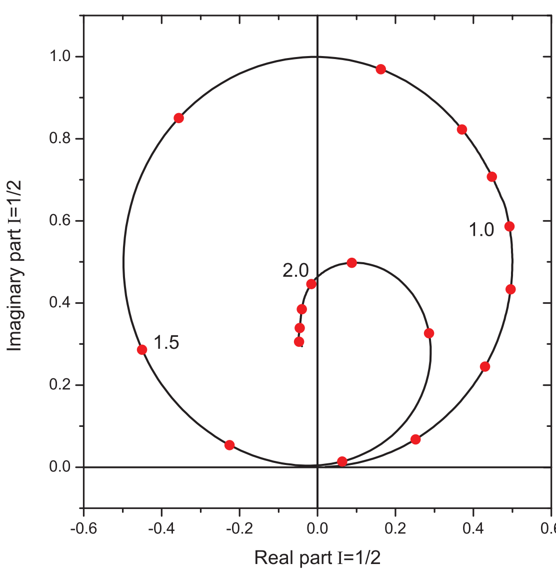

FIG. 5. The Argand plot of the $I = 1/2$ $K\pi$ S-wave amplitude from our fit. The numbers round the curve denote the energy in GeV at those points.

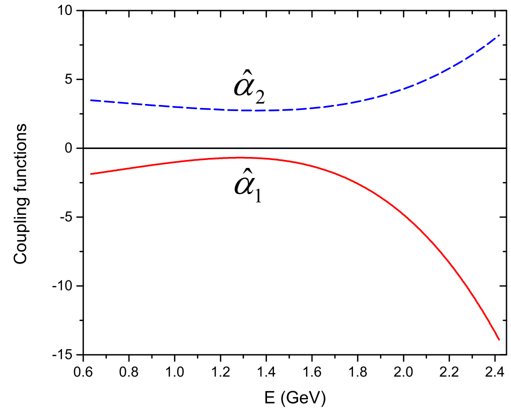

FIG. 6. The two channel η_c coupling functions of Eq. (12): $\hat{\alpha}_1$ gives the coupling to the $K\pi$ channel and $\hat{\alpha}_2$ the coupling to the *effective* $K\eta$ channel. These are each multipled by a common factor of $(s - s_A^{\eta_c})$, which places a zero very close to $K\pi$ threshold, Eq. (11), as seen in Fig. 3 plots.

to analyze a larger inelastic channel like $\eta_c \to (K\eta')K$, which would be predicted here to have the amplitude F_2 of Eq. (8) with all the parameters now determined. That the coupling function $|\alpha_2|$ is larger than that for $|\alpha_1|$ is to compensate for the fact that the amplitude T_{11} is larger than T_{12} below 1.7 GeV or so.

IV. DISCUSSION: POLES, AMPLITUDES

The simple K-matrix parametrization we use here does not have the right analytic properties having no left hand cut starting at $s = (M_K - m_\pi)^2$. Nevertheless, we can ask where are the poles of our fitted T-matrix for our $I = 1/2$ S-wave $K\pi$ scattering. In the neighborhood of the pole, the amplitudes are approximated by:

$$T(i \to j) = \frac{g_i g_j}{(E_P^2 - s)} \,, \tag{13}$$

where i, j are the $K\pi$ and "effective" inelastic channel $K\eta$. The poles are found to be at the following values of E_P in the complex plane:

$$\text{Pole 1} \quad E_{P1} = \ \ 659 - i302\,\text{MeV} \quad \text{on Sheet II}, \tag{14}$$
$$\text{Pole 2} \quad E_{P2} = 1409 - i128\,\text{MeV} \quad \text{on Sheet III}, \tag{15}$$
$$\text{Pole 3} \quad E_{P3} = 1768 - i107\,\text{MeV} \quad \text{on Sheet III}. \tag{16}$$

The corresponding residues are:

$$\text{Pole 1} \quad g(K\pi) \ = \ 0.64 \exp(i100^o) \,, \tag{17}$$
$$\text{Pole 2} \quad g(K\pi) \ = \ 0.56 \exp(i24^o) \,,$$
$$g(K\eta) = 0.125 \exp(i160^o) \,, \tag{18}$$
$$\text{Pole 3} \quad g(K\pi) \ = \ 0.45 \exp(i129^o) \,,$$
$$g(K\eta) = 0.35 \exp(i105^o) \,. \tag{19}$$

Isospin	parameter name	parameter value
1/2	s_a	1.7991 GeV^2
	g_1^a	0.3139 GeV
	g_2^a	-0.00775 GeV
	s_b	8.3627 GeV^2
	g_1^b	1.1804 GeV
	g_2^b	-0.22335 GeV
	$C_{11,0}$	-0.1553
	$C_{11,1}$	0.0909
	$C_{11,2}$	0.8618
	$C_{11,3}$	0.0629
	$C_{12,0}$	0.0738
	$C_{12,1}$	0.3866
	$C_{12,2}$	1.2195
	$C_{12,3}$	0.8390
	$C_{22,0}$	-0.0036
	$C_{22,1}$	0.2590
	$C_{22,2}$	1.6950
	$C_{22,3}$	2.2300
	γ	2.274 radians
3/2	$C_{11,0}$	-0.04046
	$C_{11,1}$	0.08143
	$C_{11,2}$	-0.08849

TABLE I. Parameters as defined by Eq. (4) used in forming the basic meson-meson scattering amplitudes, Eq. (3), of a typical fit, as shown in Figs. 1-4.

The uncertainties in the real and imaginary parts of the pole positions are estimated to be ± 20 MeV for Poles 1 and 2. For Pole 3 these are larger, typically $\pm(60-70)$ MeV, reflecting the sensitivity to the inelastic channel being merely *effective*, when clearly several inelastic channels should be included. Of course, their inclusion will only be possible once information on $K\pi \to K\eta', K\pi\pi\pi, \cdots$ is available. For pole 2, the $K_0^*(1430)$, we have a ratio of $K\eta$ to $K\pi$ decay of 0.05 ($\simeq g^2(K\eta)/g^2(K\pi)$ at the pole) quite consistent with the branching ratio of $(0.092 \pm 0.025^{+0.010}_{-0.025})$ determined from the Dalitz plot analysis of $\eta_c \to K^+K^-\eta/\pi^0$ decays [18].

Pole 1 is readily identified with the κ, the pole position of which was found to be at $[(658\pm7) - i\ (278\pm13)]$ MeV, in the dispersive analysis of [17]. An isobar model fit to the same η_c results assumes the pole of the $K_0^*(1430)$ is at $[(1438 \pm 8 \pm 4) - i\ (105 \pm 20 \pm 12)]$ MeV using a Breit-Wigner form [5, 18]. Pole 3 may be identified with the $K_0^*(1950)$ with a pole mass closer to that of the reanalysis of the Aston *et al.* result [3] by Anisovich *et al.* [19] with a pole at $E = (1820 \pm 20) - i(125 \pm 50)$ MeV. Let us stress that our representation has a seriously simplified analytic structure. Consequently continuing these amplitudes, away from the real axis where they are fixed by data, to find pole positions and residues, should not be regarded as certain. Nevertheless, they are reassuringly consistent with other treatments. One should also remember that the present analysis maintains a spectator picture and there is no rescattering. These assumptions inevitably impact on the pole positions and residues. As seen in Figs. 6,7, above 1.7 GeV, the inelastic channels become really important. Independent checks of this would be invaluable.

The $I = 1/2\ K\pi$ S-wave seen in Figs. 4,5, parametrized by Eqs. (2-5) with parameters listed in Table 1, provides an amplitude up to 2.4 GeV that can be translated to such $K\pi$ interactions in other processes.

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