**Measurement of Single Spin Asymmetry in ${}^3\text{He}^\uparrow(e, e'K^\pm)X$ from a
Transversely Polarized ${}^3\text{He}$ target**

A Dissertation

By

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This dissertation submitted by Yuxiang Zhao in partial fulfillment of the requirements for the degree of Doctor of Philosophy at University of Science and Technology of China, Hefei, Anhui, China is hereby approved by the committee under whom the work has been completed.

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ABSTRACT

Measurement of Single Spin Asymmetry in ${}^3\text{He}^\uparrow(e, e'K^\pm)X$ from a Transversely Polarized ${}^3\text{He}$ target. (May 2015)

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Spin-dependent observables are a powerful tool to probe the internal structure of the nucleon and to study the dynamics of the strong interaction. Experimental study of the nucleon spin structure has provided us with many exciting and often surprising results. The so-called “spin crisis” in the 1980s revealed the limitation of naive quark-parton models and led to a worldwide effort to study the nucleon spin structure. However, this effort has been focused mainly on the nucleon’s longitudinal spin structure. Recently, when the pioneer work revealed the significant role that transverse spin plays in understanding the full structure of the nucleon and in understanding the dynamics of the strong interaction, the study of the transverse spin structure became the new focus of the worldwide effort.

Jefferson Lab (JLab) is located at Newport News, VA, US. It is equipped with the continuous electron beam accelerator facility (CEBAF) and four experimental halls: A, B, C and D. The accelerator can provide a continuous electron beam (2 ns beam bunch) with high polarization (up to $\sim 90\%$) and high current (up to $\sim 200\mu\text{A}$) for fixed target experiments in all experimental halls. Hall A consists of two standard high-resolution spectrometers (HRS): left HRS (LHRS) and right HRS (RHRS). Another spectrometer, the BigBite spectrometer, can be installed on request by certain experiments.

The experiment E06-010 (“Transversity Experiment”) at JLab Hall A is the first measurement of the transverse spin structure of the neutron using a transversely

polarized ^3He target and a 5.89 GeV incident electron beam. The experiment measured target single spin asymmetries (SSA) and beam-target double-pin asymmetries (DSA) in semi-inclusive deep-inelastic scattering (SIDIS) and in deep-inelastic scattering (DIS) processes. It also collected inclusive hadron (pion, kaon and proton) production data parasitically. The scattered electrons were detected in the BigBite spectrometer with associated detectors while the hadrons were detected with the LHRS which had excellent particle identification capabilities.

In this dissertation, the first measurement of target single spin asymmetries of charged kaons produced in semi-inclusive deep inelastic scattering of electrons off a transversely polarized ^3He target will be discussed. Both the Collins and Sivers moments, which are related to the nucleon transversity and Sivers distributions, respectively, are extracted over the kinematical range of $0.1 < x_{bj} < 0.4$ for K^+ and K^- production. While the Collins and Sivers moments for K^+ are consistent with zero within the experimental uncertainties, both moments for K^- favor negative values. The Sivers moments are compared to the theoretical prediction from a phenomenological fit to the world data. While the K^+ Sivers moments are consistent with the prediction, the K^- results differ from the prediction at the 2-sigma level. The measurements of SSA and DSA in the inclusive hadron (pion, kaon, proton) production channels, with an average transverse momentum $\langle p_T \rangle = 0.64$ GeV/c, will also be discussed. The asymmetries strongly depend on the flavor of hadron. The π^+ and π^- asymmetries are opposite in sign, demonstrated a strong flavor dependency similar to what has been observed in polarized $p^\uparrow + p$ collisions. These measurements have provided important inputs to the world's data set on the investigation of transverse spin structure of the nucleon. They also laid the foundation for the future high precision study of the subject in the 12-GeV JLab and a future electron-ion collider.

Dedicated to my parents.

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Chapter 1

INTRODUCTION

Protons and neutrons (nucleons) are the basic building blocks of the visible matter. The internal structure of the nucleons determines their fundamental properties: mass and spin. While the Higgs mechanism is often invoked in response to the question “what is the origin of mass?”, David Gross, in his Nobel lecture [5] in 2004 stated, “This is incorrect. Most, 99% of the proton mass, is due to the kinematic and potential energy of the massless gluons and the essentially massless quarks, confined within the proton.” During his lecture, Gross at one point calls the understanding of the non-perturbative region of QCD, “... one of the most important areas of theoretical physics.” Understanding how the nucleon is built in terms of underlying quark and gluon degrees of freedom is one of the most important and challenging questions in the modern hadronic physics. In this dissertation, we will investigate the spin structure of the nucleon, especially the transverse momentum dependent spin structure.¹

1.1 Electromagnetic Probe

One of the most important tools to investigate the nucleon structure is the lepton-nucleon scattering, which utilizes the electromagnetic interaction. Quantum electrodynamics (QED) is clean and has been well studied. Moreover, the electromagnetic interaction is governed by the small fine structure constant $\alpha_{em} = e^2/4\pi \simeq 1/137$, and the one-photon approximation is good to an accuracy of about 1%. Hence, lepton-nucleon scattering is a well-defined electromagnetic probe which provides su-

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¹For a detailed QCD analysis of the nucleon mass structure, one can refer to [6].

Figure 1.1. Differential inclusive cross section of a electron scattering off a light nuclear target.

perior control over the kinematic properties of the exchanged virtual photon. Figure 1.1 shows a typical differential inclusive cross section spectrum of a electron scattering off a light nuclear target. The cross section depends on two kinematic variables (Q^2, ω). The spectrum includes four parts in general: elastic, quasi-elastic, resonance and deep inelastic (DIS) regions. In the elastic region, the light nucleus is recoiled by the incident electron, $e + A \rightarrow e' + A$; in quasi-elastic region, the scattering happens between the incident electron and a nucleon inside the light nucleus, $e + N \rightarrow e' + N$ ²; in the resonance region, the incident electron excites one of the nucleon into its resonance state, $e + N \rightarrow e' + N^{resonance}$; in DIS region, the incident electron interacts with a parton inside a nucleon and the nucleon is smashed by the incident electron, $e + N \rightarrow e' + X$.

Figure 1.2 illustrates a lepton-nucleon scattering in the DIS region in which the nucleon is smashed. The final state of the nucleon will be different in elastic and reso-

²There is no quasi-elastic peak for a single nucleon target.

Figure 1.2. Illustration plot for inclusive lepton-nucleon scattering. In the elastic scattering, the final state of the nucleon will be a recoiled nucleon. In the resonance region, the final state of the nucleon is in the excited resonance state $N^{resonance}$.

nance regions. The kinematic variables associated with the lepton-nucleon scattering are listed as the following:

- The lepton's energy loss in the nucleon rest frame, ν ³:

$$\nu = E_l - E_{l'} = \frac{q \cdot P}{M}, \quad (1.1.1)$$

- 4-momentum transfer squared of the virtual photon Q^2 :

$$Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}, \quad (1.1.2)$$

- the invariant mass of the recoiling system, W :

$$W = \sqrt{(P + q)^2}, \quad (1.1.3)$$

- the center-of-mass of the lepton-nucleon system, s :

$$s = \sqrt{(l + P)^2}, \quad (1.1.4)$$

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³The ω was used in some references.

- Bjorken scaling variable, x_{bj} , the fraction of the nucleon's momentum carried by the parton in the parton model:

$$x_{bj} = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}, \quad (1.1.5)$$

- the fraction of the lepton's energy transfer in the nucleon rest frame, y :

$$y = \frac{q \cdot P}{l \cdot P} = \frac{E_l - E_{l'}}{E_l}, \quad (1.1.6)$$

where $l(l')$ is the 4-momentum of the incoming (outgoing) lepton, θ is the scattering angle of the lepton in the rest frame, P is the 4-momentum of the initial state nucleon, M is the mass of the nucleon.

1.2 Form Factors

In the lepton-nucleon elastic scattering process, the final state of the nucleon remains the same, but with a finite recoil. This provides the opportunity to map the charge and density distributions in the nucleon. The physical observables characterizing the compositeness are form factors. Considering the unpolarized lepton-nucleon elastic scattering with one-photon exchange, one can write the differential cross section in the lab frame as:⁴

$$d\sigma = \frac{E'}{2EM^2} \frac{1}{1 + \frac{2E}{M} \sin^2(\theta/2)} |\mathcal{M}|^2 \frac{d\Omega}{(2\pi)^2}, \quad (1.2.1)$$

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⁴One can find the derivation of the formula in textbook.

where

$$\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{Q^4} \ell^{\mu\nu} W_{\mu\nu}, \\
\ell^{\mu\nu} &= 2(l'^\mu l'^\nu + l'^\nu l'^\mu - g^{\mu\nu} l' \cdot l), \\
W^{\mu\nu} &= \langle P | J^\nu | P' \rangle \langle P' | J^\mu | P \rangle.
\end{aligned} \tag{1.2.2}$$

The matrix elements of the current between the nucleon states define two form factors,

$$\langle P' | J^\mu(0) | P \rangle = \bar{U}(P') [F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2M}] U(P), \tag{1.2.3}$$

where $F_1(Q^2)$ is the Dirac form factor and $F_2(Q^2)$ is the Pauli form factor. Physically, people are more interested in using the Sachs electric and magnetic form factors,

$$\begin{aligned}
G_E(Q^2) &= F_1(Q^2) - \tau F_2(Q^2), \\
G_M(Q^2) &= F_1(Q^2) + F_2(Q^2),
\end{aligned} \tag{1.2.4}$$

where $\tau = \frac{Q^2}{4M^2}$.

The hadron tensor then becomes

$$W^{\mu\nu} = g^{\mu\nu} q^2 G_M^2 + 4P^\mu P^\nu \frac{G_E^2 + \tau G_M^2}{1 + \tau}. \tag{1.2.5}$$

Contracting the lepton and hadron tensors, and using $q^\mu \ell_{\mu\nu} = q^\mu W_{\mu\nu} = 0$, the elastic scattering cross section in the laboratory frame becomes

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \sigma_{mott} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2 \tan^2(\theta/2) \right], \\
\sigma_{mott} &= \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{1}{1 + \frac{2E}{M} \sin^2(\theta/2)}.
\end{aligned} \tag{1.2.6}$$

Experimentally, to extract the electric and magnetic form factors from elastic scattering, one can measure two cross sections at fixed Q^2 by varying the scattering

angle, and then plot the $\frac{d\sigma}{d\Omega}/\sigma_{mott}$ as a function of $\tan^2(\theta/2)$ to obtain the intercept and slope which provide information of electric and magnetic form factors. This method is called Rosenbluth separation method. A review of measurements of the nucleon electromagnetic form factors can be found in [7].

The $G_E(Q^2)$ can be interpreted as the Fourier transformation of the charge distribution in the Breit frame. Hence,

$$G_E(Q^2) = Q_e - \frac{1}{6}Q^2\langle r^2 \rangle + \dots \quad (1.2.7)$$

Then we can define the charge radius of the nucleon as

$$\langle r^2 \rangle = -6 \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0}. \quad (1.2.8)$$

The corresponding Fourier transform of $G_M(Q^2)$ could be interpreted as the magnetization density.

1.3 Deep-inelastic Scattering (DIS)

The process of deep-inelastic scattering (DIS), in which a quark (or a parton) in a nucleon is knocked out by the virtual photon, hence the nucleon gets smashed into many fragments, is another powerful tool to investigate the structure of the nucleon. The pioneer measurements through DIS process were first performed by Friedman, Kendall, and Tylor et al. at Stanford Linear Accelerator Center (SLAC) [8].

The formalism of the DIS process (see Figure 1.2) will be discussed here following the convention in [9] with the assumption of one-photon exchange and ignoring contributions from electro-weak processes. The inclusive DIS differential cross section

can be written in terms of the scattering tensors as ⁵

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4} \ell_{\mu\nu} W^{\mu\nu}, \quad (1.3.1)$$

where α is the fine structure constant and $q = l - l'$. The lepton tensor consists of two parts: symmetric (S) part under μ, ν exchange and anti-symmetric (A) part under μ, ν exchange,

$$\ell_{\mu\nu}(l, s; l') = \ell_{\mu\nu}^{(S)}(l; l') + i\ell_{\mu\nu}^{(A)}(l, s; l'), \quad (1.3.2)$$

where s is the spin of initial electron and ⁶

$$\begin{aligned} \ell_{\mu\nu}^{(S)}(l; l') &= 2(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu}(m^2 - l^\rho l'_\rho)), \\ \ell_{\mu\nu}^{(A)}(l, s; l') &= 2m\epsilon_{\mu\nu\alpha\beta} s^\alpha (l^\beta - l'^\beta), \end{aligned} \quad (1.3.3)$$

where $\ell^{(S)}$ is the electron spin independent part, $\ell^{(A)}$ is the electron spin dependent part, and $\epsilon_{0123} = +1$.

The hadronic tensor includes information about the structure of the nucleon and QED part. It can also be written in terms of symmetric and anti-symmetric parts as

$$W_{\mu\nu}(q; P, S) = W_{\mu\nu}^{(S)}(q; P) + iW_{\mu\nu}^{(A)}(q; P, S), \quad (1.3.4)$$

where S is the spin of the nucleon and P is the four momentum of the nucleon. In addition, the hadronic tensor can be expressed in terms of Lorentz-invariant inelastic

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⁵ $\frac{d^2\sigma}{dx dy d\phi} = (\frac{M\nu}{E'}) \frac{d^2\sigma}{dE' d\Omega} = (\frac{Q^2}{y}) \frac{d^2\sigma}{dx dQ^2 d\phi}$

⁶ Here the spin of the scattered electron has been summed over.

structure functions W_1, W_2, G_1, G_2 by using the conservation laws:

$$\begin{aligned}\frac{1}{2M}W_{\mu\nu}^{(S)}(q; P) &= (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})W_1(P \cdot q, Q^2) + \frac{1}{M^2}(P_\mu - \frac{P \cdot q}{q^2}q_\mu)(P_\nu - \frac{P \cdot q}{q^2}q_\nu)W_2(P \cdot q, Q^2), \\ \frac{1}{2M}W_{\mu\nu}^{(A)}(q; P, S) &= \epsilon_{\mu\nu\alpha\beta}q^\alpha \{MS^\beta G_1(P \cdot q, Q^2) + \frac{1}{M}[(P \cdot q)S^\beta - (S \cdot q)P^\beta]G_2(P \cdot q, Q^2)\}.\end{aligned}\tag{1.3.5}$$

Physically, one would like to define

$$\begin{aligned}F_1(x, Q^2) &= MW_1(P \cdot q, Q^2), \\ F_2(x, Q^2) &= \nu W_2(P \cdot q, Q^2), \\ g_1(x, Q^2) &= M^2 \nu G_1(P \cdot q, Q^2), \\ g_2(x, Q^2) &= M \nu^2 G_2(P \cdot q, Q^2).\end{aligned}\tag{1.3.6}$$

Hence, the symmetric and anti-symmetric parts of the hadronic tensor can be written in terms of F_1, F_2, g_1, g_2 as:

$$\begin{aligned}W_{\mu\nu}^{(S)}(q; P) &= 2(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})F_1(x, Q^2) + \frac{2}{P \cdot q}(P_\mu - \frac{P \cdot q}{q^2}q_\mu)(P_\nu - \frac{P \cdot q}{q^2}q_\nu)F_2(x, Q^2), \\ W_{\mu\nu}^{(A)}(q; P, S) &= \epsilon_{\mu\nu\alpha\beta} \frac{2Mq^\alpha}{P \cdot q} [S^\beta g_1(x, Q^2) + (S^\beta - \frac{S \cdot q}{P \cdot q}P^\beta)g_2(x, Q^2)].\end{aligned}\tag{1.3.7}$$

Contracting the leptonic and hadronic tensor, due to the parity conservation of the electromagnetic interaction, only the (S)(S) and (A)(A) terms survive in the cross section. The differential cross section of the inclusive DIS process is

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{2MQ^4} \frac{E'}{E} [\ell_{\mu\nu}^{(S)} W^{(S)\mu\nu} - \ell_{\mu\nu}^{(A)} W^{(A)\mu\nu}].\tag{1.3.8}$$

The first part of equation 1.3.8 represents the spin independent part of the cross section, the unpolarized inclusive DIS differential cross section after averaging over the spins of the incoming electrons and summing over the spins in the final state:

$$\frac{d^2\sigma^{unpolarized}}{dE'd\Omega} = \left(\frac{d^2\sigma}{dE'd\Omega}\right)_{Mott} \left[\frac{2}{M} F_1(x, Q^2) \tan^2(\theta/2) + \frac{1}{\nu} F_2(x, Q^2) \right], \quad (1.3.9)$$

where

$$\left(\frac{d^2\sigma}{dE'd\Omega}\right)_{Mott} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^2(\theta/2)}. \quad (1.3.10)$$

The second part of equation 1.3.8 is spin-dependent. Experimentally, asymmetry is measured by flipping the electron helicity or the nucleon spin direction to isolate the second part and it turns out that flipping the electron spin with a fixed target spin direction produces the same effect as flipping the target spin with a fixed electron spin. Let's consider the situation where the electron beam is longitudinally polarized with the target polarized longitudinally or transversely relative to the direction of the electron beam ⁷. The electron spin vector can be defined as

$$s_{\uparrow}^{\mu} = -s_{\downarrow}^{\mu} = \frac{1}{m}(E, \vec{l}), \quad (1.3.11)$$

and the polarized nucleon spin vector with arbitrary spin can be written as

$$S^{\mu} = (0, \hat{S}). \quad (1.3.12)$$

The cross section difference by flipping the target spin with the electron spin fixed now becomes

$$\frac{d^2\sigma^{\uparrow,S}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow,-S}}{d\Omega dE'} = -\frac{4\alpha^2}{Q^2} \frac{E'}{E} \left[\frac{(\vec{l}' \cdot \hat{S} + \vec{l} \cdot \hat{S})}{M\nu} g_1(x, Q^2) \right] - \frac{4\alpha^2}{Q^2} \frac{E'}{E} \left[\frac{2(E\vec{l}' \cdot \hat{S} - E'\vec{l} \cdot \hat{S})}{M\nu^2} g_2(x, Q^2) \right], \quad (1.3.13)$$

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⁷This is usually the case for JLab experiments

Figure 1.3. Coordinate system illustrating scattering plane and target plane in the beam-target double-spin asymmetry definition.

where $\uparrow$ ($\downarrow$) means the spin of the electron is parallel (anti-parallel) to its direction of motion. In order to explicitly write down the cross section, a coordinate system should be defined in order to calculate $\vec{l}' \cdot \hat{S}$ and $\vec{l} \cdot \hat{S}$. Figure 1.3 shows a possible coordinate system in which the incoming electron and the outgoing electron define the electron scattering plane, the incoming electron and the spin of the target define the target plane, ϕ is the azimuthal angle between the electron plane and the target plane and θ is the scattering angle of electron. In this coordinate system, when the target spin is oriented longitudinally (along $\vec{l}$), we have

$$\begin{aligned}\vec{l} \cdot \hat{S} &= |\vec{l}| = E, \\ \vec{l}' \cdot \hat{S} &= E' \cos(\theta).\end{aligned}\tag{1.3.14}$$

Then

$$\frac{d^2\sigma^{\downarrow,\uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow,\uparrow}}{d\Omega dE'} = \frac{4\alpha^2}{Q^2} \frac{E'}{E} \frac{1}{M\nu} [(E + E' \cos(\theta))g_1(x, Q^2) - \frac{Q^2}{\nu} g_2(x, Q^2)], \tag{1.3.15}$$

where $\uparrow$ ($\downarrow$) is the target spin parallel (anti-parallel) to the electron's momentum. When the target spin is oriented transversely, denoted as $\Rightarrow$ and $\Leftarrow$, we have

$$\begin{aligned}\vec{l} \cdot \hat{S} &= 0, \\ \vec{l}' \cdot \hat{S} &= E' \sin(\theta) \cos(\phi).\end{aligned}\tag{1.3.16}$$

Then

$$\frac{d^2\sigma^{\downarrow,\Rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow,\Rightarrow}}{d\Omega dE'} = \frac{4\alpha^2}{Q^2} \frac{E'^2}{E} \sin(\theta) \cos(\phi) \left[\frac{1}{M\nu} g_1(x, Q^2) + \frac{2E}{M\nu^2} g_2(x, Q^2) \right]. \tag{1.3.17}$$

With equations 1.3.15 and 1.3.17, one can separate the structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$. In practice, instead of measuring the cross section differences directly, the beam-target double-spin asymmetries (DSA) are often measured to investigate g_1 and g_2 ⁸.

So far, we have introduced the formalism of inclusive DIS cross sections in terms of the unpolarized and polarized structure functions without any physical interpretations. These structure functions are usually employed as experimental observables. In the following subsections, several theories and models will be introduced in order to provide insights into the physical meanings of these structure functions.

1.3.1 Unpolarized Structure Functions

The early SLAC data indicate that if the structure functions F_1 and F_2 are plotted as a function of x_{bj} , they are nearly independent of Q^2 . This behavior was named as Bjorken scaling [10]. Bjorken scaling occurs in what is known as the Bjorken limit: x_{bj} is fixed as $Q^2 \rightarrow \infty$ and $\nu \rightarrow \infty$. Figure 1.4 shows the world data of proton F_2 as a function of Q^2 for different x_{bj} . To explain Bjorken scaling, Feynman and Bjorken introduced simple quark-parton model (QPM) [10, 11], in which the nucleon is made

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⁸Some of the experiments at JLab do measure the difference of the differential cross section to extract g_1 and g_2 .

of non-interacting partons and the virtual photon scatters off these partons in the DIS process ⁹.

In QPM, the unpolarized structure functions are expressed by the unpolarized parton distribution functions (PDF) $f_1(x)$, which denotes the probability of finding a parton with certain momentum fraction x of the nucleon:

$$\begin{aligned} F_1(x) &= \frac{1}{2} \sum_q e_q^2 (f_1^q(x) + f_1^{\bar{q}}(x)), \\ F_2(x) &= x \sum_q e_q^2 (f_1^q(x) + f_1^{\bar{q}}(x)). \end{aligned} \tag{1.3.18}$$

Hence, the Callan-Gross relation [12] stands:

$$F_2(x) = 2xF_1(x). \tag{1.3.19}$$

Therefore, the parton model explains the scaling naturally. This is due to the fact that at Bjorken limit, where Q^2 and $\nu \rightarrow \infty$, the PDF $f_1(x, Q^2)$ becomes $f_1(x)$. In reality, Q^2 is finite in experiments and there are higher order corrections such as gluon radiations in the DIS processes, which give $\log(Q^2)$ dependence to the cross sections. The Q^2 dependence of the PDF can be calculated using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [13–15].

After three decades of efforts on the PDF study through DIS, Drell-Yan and direct photon production experiments etc., a lot of data have been accumulated. One can make general parameterizations of PDFs and fit the parameters to the experimental data. The result is a set of phenomenological parton distributions which characterize the structure of the nucleon. The well-known PDFs are available in CTEQ (USA), GRV (Germany), and MRST (England) packages.

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⁹To understand why the partons in a nucleon can be viewed as free in DIS, one can consider the nucleon in infinite-momentum frame (IMF), the interaction time is scaled by γ which is $\rightarrow \infty$. It is so long that the nucleon configuration can be considered essentially frozen.

Figure 1.4. Proton F_2 structure function measured by various collaborations with different x_{bj} .

1.3.2 Polarized Structure Functions

In QPM, as Q^2 is high and Bjorken limit stands, the g_1 structure function is independent of Q^2 to a very good approximation and only dependent on x_{bj} . It can be written as

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x), \quad (1.3.20)$$

where q is the flavor of a quark or anti-quark and

$$\Delta q(x) = q^\uparrow(x) - q^\downarrow(x), \quad (1.3.21)$$

where $\uparrow (\downarrow)$ means the spin of the quark is parallel (anti-parallel) to the spin of the nucleon. The $q^{\uparrow(\downarrow)}$ denotes the probability to find a quark of flavor q with certain momentum fraction x and spin polarized in the same (opposite) direction as the spin of the nucleon. Hence, $\Delta q(x)$, known as helicity distribution, measures the distribution of quark spin in the nucleon. By integrating the $\Delta q(x)$ over x , $\Delta q = \int_0^1 dx \Delta q(x)$, and summing over the up, down and strange quark, $\Delta u + \Delta d + \Delta s$, one can get the total fraction of the nucleon's spin carried by the spins of the quarks.

After 30 years' experimental efforts at CERN, SLAC, DESY, JLab and RHIC, we now know that $\Delta u + \Delta d + \Delta s \sim 0.35$ ¹⁰, which indicates that the spin of the nucleon is mostly carried by the gluon and orbital angular motion of the quarks. To explain the question “where is the remainder of the proton spin except for the quark spin?”, people came up with a set of nucleon spin sum rules. Jaffe and Manohar [16] suggested a sum rule in the Light-Cone Gauge:

$$\frac{1}{2} = \Delta L_q + \frac{1}{2} \Delta \Sigma + \Delta L_g + \Delta G, \quad (1.3.22)$$

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¹⁰This value was surprising from the view-point of early quark models and raised the “spin crisis”. In the modern view of the nucleon spin structure, this value is only representing the valence and sea quark spin effect.

where ΔL_q (ΔL_g) represents the total orbital angular momentum of quarks (gluons). $\Delta\Sigma$ is the contribution from quark spins. ΔG is the contribution from gluon spins. Xiangdong Ji proposed another sum rule [17, 18]:

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + J_g, \quad (1.3.23)$$

where J_q (J_g) is the total angular momentum of quarks (gluons). Ji's sum rule is gauge invariant and he also suggested a set of experiments to access J_q and J_g through the measurements of GPDs. To look at the latest reviews of the spin structure of the nucleon, including the current world measurements and the planned future experiments, one can refer to [19, 20].

Understanding the g_2 structure function in terms of PDFs is slightly tricky. Although the QPM can give intuitive pictures of F_1 , F_2 , g_1 structure functions in the Bjorken limit and the Q^2 dependence of these structure functions can be described using DGLAP evolutions by including NLO processes, it can not give an intuitive description of g_2 directly. This is due to the fact that the g_2 structure function involves higher-twist effects, which can be taken care of with the formalism of operator product expansion (OPE).

- Operator Product Expansion

The OPE, which is model independent and based on the general calculations from quantum field theory, was first introduced by Kenneth G. Wilson [21] to estimate the operator products. It can be employed to calculate the hadronic tensor $W_{\mu\nu} = \frac{1}{4\pi} \int d^4\xi e^{iq\cdot\xi} \langle P', S' | [J_\mu(\xi), J_\nu(0)] | P, S \rangle$. As ξ (4-D spatial vector) is close to 0, the Fourier transformation of the operators can be estimated as

$$i \int d^4\xi e^{iq\cdot\xi} J_\mu(\xi) J_\nu(0) = \sum_n C_{\mu\nu n} O^{(n)}. \quad (1.3.24)$$

Here $C_{\mu\nu n}$ is the Wilson coefficients which can be calculated perturbatively in different orders. The quark-gluon operator $O^{(n)}$ is associated with the non-perturbative contributions, distinguished in different twists:

$$O^{(n)} \sim x^s \left(\frac{M_p}{Q} \right)^{d-s-2}, \quad (1.3.25)$$

where d is the dimension of the operator $O^{(n)}$ and s is the spin of the operator. The twist τ is defined as $\tau = d - s$. Hence, the leading twist is the twist-2 contribution and one can conclude that the higher-twists play more important roles as Q decreases.

In 1977, Wandzura and Wilzeck showed that, in OPE framework, the g_2 structure function with twist-2 and twist-3 (quark-gluon correlations) contributions can be written as [22]:

$$g_2(x, Q^2) = g_2^{WW} + \bar{g}_2(x, Q^2). \quad (1.3.26)$$

Here g_2^{WW} is the twist-2 contribution, known as the Wandzura-Wilzeck term:

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(x, Q^2). \quad (1.3.27)$$

This part can then be described using g_1 . $\bar{g}_2(x, Q^2)$ includes twist-3 contributions and can be written as:

$$\bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left[\frac{m_q}{M} h_1(y, Q^2) + \xi(y, Q^2) \right] \frac{dy}{y}, \quad (1.3.28)$$

where $\xi(x, Q^2)$ is the twist-3 contribution, which arises from quark-gluon interactions; $h_1(x, Q^2)$ is the quark transverse polarization distribution (like $\Delta q(x)$ in longitudinal case), which is a twist-2 contribution.

1.3.3 $h_1(x)$

$h_1(x)$, which is so-called the transversity function, is the third class of PDFs. In the rest frame of the nucleon, the probability of finding a quark with its spin aligned in the same direction as that of a longitudinally polarized nucleon ($\Delta q(x)$) is obviously the same as the probability of finding a quark with its spin aligned in the same direction of a transversely polarized nucleon ($h_1(x)$). However, if we boost the nucleon in a direction transverse to its spin, in a relativistic context, Lorentz boosts can affect the spin distribution and cause the helicity and transversity to be different. This difference depends on the inner structure and dynamics of quarks inside the nucleon. By investigating $h_1(x)$, one has the opportunity to gain new information on the dynamics of quarks inside the nucleon, complementary to the helicity distribution.

The transversity function was first introduced by J. Ralston and D. Soper in 1979 [23] and then evaluated in different models [24–27] and lattice QCD calculations. It has several interesting features, such as valence-like behavior [28], Soffer’s inequality [29] ($|h_1(x)| \leq \frac{1}{2}(f_1(x) + \Delta q(x))$) and chiral-odd nature etc. Experimentally, one can hardly access the transversity function through polarized DIS process, since it is suppressed by the factor $\frac{m_q}{M}$. However, it was recognized that polarized Drell-Yan experiments [23, 30] and Semi-Inclusive Deep Inelastic Scattering (SIDIS) experiments can both access this function. Due to some practical difficulties, the polarized Drell-Yan experiments have not been carried out yet, and our current knowledge on this distribution is mainly from SIDIS process, which also accesses 3-D transverse-momentum-dependent parton distributions (TMDs) and will be described in the next chapter.

Chapter 2

PHYSICS MOTIVATION

In the previous chapter, we described the inclusive electron measurements in elastic and DIS regions, ending with the statement that polarized Drell-Yan and SIDIS processes would allow us to access the transversity distributions. In this chapter, we will focus on the SIDIS process, which is the main physics program in the experiment E06-010 (Transversity experiment). In the SIDIS process, one measures one of the final-state hadrons in coincidence with the scattered electron. Figure 2.1 illustrates a SIDIS process, in which the scattered electron ℓ' and a final-state hadron P_h are measured simultaneously. The SIDIS processes involve a set of transverse-momentum-dependent parton distribution functions (TMDs), which consist of not only the natural extensions of 1-D transversity distributions ($h_1(x)$) to 3-D distributions by considering transverse momentum (k_T) of partons, but also many other correlation functions such as the Sivers functions, the Worm-gear functions, and the Pretzelosity functions etc..

Figure 2.1. Illustration of the SIDIS measurement, in which the scattered electron ℓ' and a final state hadron P_h are measured simultaneously.

TMDs, or in general three dimensional imaging of the nucleon, is a hot and fast developing topic in recent years. A number of progresses from both theoretical and experimental efforts have been made and many questions are still open. Fortunately, we have experience in the study of 1-D PDFs through inclusive DIS processes and can basically follow similar steps in the investigation of the 3-D TMDs [31]:

- Define independent structure functions which contain information of the structure of the nucleon by using general symmetries such as Lorentz covariance, gauge invariance, and parity conservation, etc..
- Involve parton model without QCD interactions to interpret structure functions with leading order (LO) and Leading-Twist PDFs, which are Q^2 -independent.
- Involve parton model with QCD multiple gluon scattering after collinear expansion (quark-gluon correlation) to have beyond LO, beyond Leading-twist and Q^2 -dependent PDFs.

In the following sections of this chapter, we will first express the SIDIS differential cross section in terms of 18 independent structure functions, and then describe the relationship between the structure functions and TMDs at LO of the leading twist under the TMD factorization, and finally show how to access the TMDs experimentally.

2.1 Semi-inclusive Deep Inelastic Scattering (SIDIS)

The differential cross section of the SIDIS process, $\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X$, can be written as formula 2.1.1 using 18 structure functions with one-photon exchange approximation [32]:

Figure 2.2. Trento convention to define the azimuthal angles for SIDIS process.

$$\begin{aligned}
& \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = \\
& \frac{\alpha^2}{xy Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \{ \\
& F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos(\phi_h) F_{UU}^{\cos\phi_h} \\
& + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin(\phi_h) F_{LU}^{\sin\phi_h} \\
& + S_{\parallel} [\sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi_h) F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h}] \\
& + S_{\parallel} \lambda_e [\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi_h) F_{LL}^{\cos\phi_h}] \\
& + |S_{\perp}| [\sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \\
& \quad + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
& \quad + \sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi_S) F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}] \\
& + |S_{\perp}| \lambda_e [\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi_S) F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}] \\
& \},
\end{aligned}$$

(2.1.1)

where F , depending on x, y, z and P_h , represents structure functions. The first and second subscript of F indicate the polarization of beam and target, respectively, the third subscript indicates the polarization of virtual photon. ϕ_h and ϕ_S are the azimuthal angles defined following Trento convention as shown in Figure 2.2. λ_e is the electron beam helicity and $S_{\parallel(\perp)}$ is the target polarization ¹. The additional kinematical variables used in the formula 2.1.1 besides those used in inclusive DIS (see section 1.1) are:

- $z = \frac{P \cdot P_h}{P \cdot q}$, the fraction of the energy transfer carried by the observed hadron
- $W' = \sqrt{(l + P - l' - P_h)^2}$, the missing mass
- $\gamma = \frac{2Mx}{Q}$, inverse of energy transfer
- $\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}$, the ratio of longitudinal and transverse photon flux

2.2 Transverse-Momentum-Dependent Parton Distributions (TMDs) and Fragmentation Functions (FFs)

2.2.1 Interpreting SIDIS Process in the Parton Model

In the parton model, the virtual photon strikes a quark (or anti-quark), which fragments into a hadron h . The process can be represented by the handbag diagram in Figure 2.3. Taking transverse momentum of the quarks into account, the hadronic tensor is given by

$$W^{\mu\nu} = \sum_q e_q^2 \int d^4k \int d^4\kappa \delta^4(k + q - \kappa) Tr[\Phi^q(k)\gamma^\mu, \Xi^q(\kappa)\gamma^\nu], \quad (2.2.1)$$

where k is the four momentum of the initial quark, κ is the four momentum of the fragmenting quark, Φ is the quark-quark correlation matrix involving PDFs, and Ξ is the fragmentation analogue of Φ involving fragmentation functions.

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¹In this convention, $S_{\parallel} = -1$ as the target spin is parallel to the virtual photon momentum [32].

Figure 2.3. Diagram illustrating the SIDIS process in the parton model.

By doing the parameterizations of the correlators in the hadronic tensor ², the structure functions can be written in terms of TMDs and corresponding FFs. To have a compact notation for the results, we use the notation:

$$\mathcal{C}[wfD] = x \sum_q e_q^2 \int d^2 k_T \int d^2 \kappa \delta^2(k_T - \kappa_T - P_{h\perp}/z) w(k_T, \kappa_T) f^q(x, k_T^2) D^q(z, \kappa_T^2). \quad (2.2.2)$$

The non-vanishing structure functions at the leading twist are:

$$F_{UU,T} = \mathcal{C}[f_1 D_1], \quad (2.2.3)$$

$$F_{UU,L} = 0, \quad (2.2.4)$$

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²For the details of the parameterization, one can refer to [32]

$$F_{UU}^{cos2\phi_h} = \mathcal{C}[-\frac{2(\hat{h} \cdot k_T)(\hat{h} \cdot \kappa_T) - k_T \cdot \kappa_T}{MM_h} h_1^\perp H_1^\perp], \quad (2.2.5)$$

$$F_{UL}^{sin2\phi_h} = \mathcal{C}[-\frac{2(\hat{h} \cdot k_T)(\hat{h} \cdot \kappa_T) - k_T \cdot \kappa_T}{MM_h} h_1^\perp H_{1L}^\perp], \quad (2.2.6)$$

$$F_{LL} = \mathcal{C}[g_{1L} D_1], \quad (2.2.7)$$

$$F_{UT,T}^{sin(\phi_h - \phi_S)} = \mathcal{C}[-\frac{\hat{h} \cdot k_T}{M} f_{1T}^\perp D_1], \quad (2.2.8)$$

$$F_{UT,L} = 0, \quad (2.2.9)$$

$$F_{UT}^{sin(\phi_h + \phi_S)} = \mathcal{C}[-\frac{\hat{h} \cdot \kappa_T}{M_h} h_1 H_1^\perp], \quad (2.2.10)$$

$$F_{UT}^{sin(3\phi_h - \phi_S)} = \mathcal{C}[\frac{2(\hat{h} \cdot \kappa_T)(k_T \cdot \kappa_T) + k_T^2(\hat{h} \cdot \kappa_T) - 4(\hat{h} \cdot k_T)^2(\hat{h} \cdot \kappa_T)}{2M^2 M_h} h_{1T}^\perp H_1^\perp], \quad (2.2.11)$$

$$F_{LT}^{cos(\phi_h - \phi_S)} = \mathcal{C}[\frac{\hat{h} \cdot k_T}{M} g_{1T} D_1]. \quad (2.2.12)$$

where $\hat{h} = \frac{P_{h\perp}}{|P_{h\perp}|}$. For more information about the structure functions beyond leading twist (twist-2), one can refer to [33].

2.2.2 TMDs at Leading twist

At leading twist, as we can see from the expressions of the structure functions in the SIDIS process, there are eight TMDs. In the parton model, these TMDs

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ Boer-Mulders
	L		$g_{1L} =$ Helicity	$h_{1L}^\perp =$ Worm Gear
	T	$f_{1T}^\perp =$ Sivers	$g_{1T} =$ Worm Gear	$h_1 =$ Transversity $h_{1T}^\perp =$ Pretzelosity

Nucleon Spin
Quark Spin
: Survive Transverse Momentum Integration

Figure 2.4. Categorization of all eight TMDs at leading twist by nucleon and quark spins.

have probabilistic interpretations for different quark polarization in different polarized nucleon. Figure 2.4 shows the categorization by nucleon and quark spins.

Only three of the eight TMDs-diagonal terms f_1, g_{1L}, h_1 -survive after integration over transverse momentum k_T of initial quarks. The other five vanish when integrated over k_T and therefore provide novel information of the transverse motion of the quarks. The integration over k_T yields the relations between the TMDs and the 1-D PDFs in the QPM:

$$f_1(x) = \int d^2k_T f_{1T}(x, k_T^2), \quad (2.2.13)$$

$$\Delta q(x) = \int dk_T^2 g_{1L}(x, k_T^2), \quad (2.2.14)$$

$$h_1(x) = \int dk_T^2 h_1(x, k_T^2). \quad (2.2.15)$$

By using a transversely polarized nucleon target, it is possible to access the Sivers function ($f_{1T}^\perp$), Worm-Gear function (g_{1T}), Transversity function (h_1) and Pretzelosity function ($h_{1T}^\perp$).

- Sivers distribution function $f_{1T}^\perp$

Sivers function was first introduced by Sivers to explain the single-spin asymmetries observed in inclusive pion productions in $p\bar{p}$ collisions by E704 collaboration. It describes the probability of finding an unpolarized quark with non-zero transverse momentum in a transversely polarized nucleon. Originally it was demonstrated to be zero due to its T-odd nature [34]. Brodsky et al. [35,36] demonstrated that the single-spin asymmetries, associated with Sivers mechanism, could arise from gluon exchange between the struck quark and the nucleon remnant. Later, Collins pointed out that, time-reversal invariance does not imply a vanished Sivers function by taking into account gauge links correctly, but rather a sign difference between the Sivers function measured in SIDIS and the same distribution measured in Drell-Yan process [36,37],

$$f_{1T}^{\perp}(x, k_T^2)|_{SIDIS} = -f_{1T}^{\perp}(x, k_T^2)|_{Drell-Yan}. \quad (2.2.16)$$

- Worm-Gear function g_{1T}

Worm-Gear function describes the probability of finding a quark, with non-zero transverse momentum, polarized longitudinally in a transversely polarized nucleon. It provides the information of the quark spin-orbital correlations and represents the real part of an interference between nucleon wave functions differed by one unit of orbital angular momentum [38,39]. Non-zero g_{1T} would directly indicate that quarks carry orbital angular momentum.

- Transversity function h_1

Many properties have been mentioned in the inclusive DIS part in Section 1.3.3. It is a chiral-odd function which can be easily accessed in SIDIS process by employing the chiral-odd Collins fragmentation function. Besides the properties mentioned in Section 1.3.3, one of the fundamental properties of the nucleon, the “tensor charge”³, is also related to the transversity function.

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³It is related to the integration of transversity function over x , just like the “axial charge” in longitudinal case.

- Pretzelosity function $h_{1T}^\perp$

Pretzelosity function is also a Chiral-odd function. In SIDIS process, it is coupled with the Chiral-odd Collins fragmentation function as well. The intuitive picture of Pretzelosity can be interpreted as the difference between the helicity and the transversity distribution in the relativistic quark models [40, 41]. Another interesting feature, similar to g_{1T} , is that Pretzelosity emerges from the interference of quark wave-function components with a difference of two units of orbital angular momentum [42].

2.2.3 Fragmentation Functions

In SIDIS processes, each TMDs is accompanied by a fragmentation function (FF). The transversity function is associated with the so called Collins fragmentation function and the Sivers function is coupled with normal unpolarized fragmentation function. In general, fragmentation functions contain nonperturbative information required to describe a great variety of hard scattering processes with a given identified hadron in the final state within the framework of perturbative QCD. FFs describe the transition process of a specific quark fragmenting into a final state hadron, hence, they are categorized into “favored” and “unfavored” parts according to whether or not the fragmenting quark is the constituent quark of the final state hadron. Under the assumption of iso-spin symmetry and charge conjugation, one usually assumes

$$\begin{aligned} F_{favored}(z, Q^2) &= F_u^{\pi^+} = F_d^{\pi^+} = F_{\bar{u}}^{\pi^-} = F_d^{\pi^-}, \\ F_{unfavored}(z, Q^2) &= F_{\bar{u}}^{\pi^+} = F_d^{\pi^+} = F_u^{\pi^-} = F_{\bar{d}}^{\pi^-}. \end{aligned} \tag{2.2.17}$$

All the strange quarks ($s, \bar{s}$) fragment to $\pi^\pm$ through unfavored processes. For kaon final state, u and $\bar{s}$ quarks fragment to K^+ through favored processes while other quarks through unfavored processes, $\bar{u}$ and s quarks fragment to K^- through favored processes while other quarks through unfavored processes.

The extractions of FFs were originally from e^+e^- data [43–46], then the study was extended to the hadron-hadron collisions [47–49] as well as hadron multiplicities in SIDIS process [47]. Here some comments about the global fragmentation functions study are in order, on one hand, the universality of FFs in different hard processes need to be checked, on the other hand, the x-z separation in the SIDIS process is an assumption and has only been verified in a certain kinematic range experimentally. To fully understand the TMDs through SIDIS processes, understanding FFs is crucial.

2.2.4 TMD Factorization and Collinear Factorization

So far, we have presented the SIDIS cross section in the parton model which incorporates intrinsic transverse momentum of quarks. The interpretations have solid QCD foundation. There are at least two factorization theorems in QCD developed to interpret the cross section: TMD factorization and collinear factorization. They are characterized by two scales besides the confinement scale Λ_{QCD} ⁴-the momentum transfer Q and the transverse momentum of the final hadron $P_{h\perp}$ (or the transverse momentum Q_T of the virtual photon in the hN collinear frame).

TMD factorization has been proved to be valid [50–52] in the low transverse-momentum region $\Lambda_{QCD} \sim P_{h\perp}(Q_T) \ll Q$. In this framework, the structure functions can be written as

$$F \sim \sum_q e_q^2 \int d^2 k_T^2 \int d^2 \kappa_T \int d^2 l_T \delta^2(k_T - \kappa_T + l_T + q_T) H(Q^2) f^q(x, k_T^2) D^q(z, \kappa_T^2) U(l_T^2), \quad (2.2.18)$$

where H is a perturbative hard factor written as a series in powers of α_s , the soft factor U arises from the radiation of soft gluons with transverse momentum l_T , f is the parton distribution function, and D is the fragmentation function. The structure

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⁴ $\Lambda_{QCD} \sim 200$ MeV.

functions written in terms of TMDs in the parton model are recovered at tree level in the TMD factorization framework [53, 54].

At high transverse momentum, $Q_T \gg \Lambda_{QCD}$, the SIDIS structure functions can be described in collinear QCD. In the collinear factorization, the calculations are based on perturbative QCD, and transverse momenta are generated by gluon radiation. In this approach, which was proved at large transverse momentum [53, 55–59], the SIDIS cross section with a transversely polarized target has the general form

$$d\sigma \sim G_F(x, x') \otimes d\hat{\sigma} \otimes D_1(z) + h_1(x) \otimes d\hat{\sigma}' \otimes \hat{E}_F(z, z'), \quad (2.2.19)$$

where x, x' are the kinematic variables of the incident and fragmenting partons, respectively. The first term of the cross section contains a quark-gluon correlation function for the transversely polarized nucleon and the ordinary unpolarized fragmentation function for the final hadron, and the second term combines the transversity distribution with a twist-3 fragmentation function.

In the intermediate transverse-momentum region, $\Lambda_{QCD} \ll Q_T \ll Q$, one would expect that both TMD and collinear factorization should hold. This has been verified in [60, 61].

2.2.5 Model Calculations of TMDs

There has been considerable efforts in calculating the TMDs in various models of the nucleon and by lattice QCD. The first calculation of TMDs was performed in a quark-diquark spectator model [26]. In this model, T-odd Sivers function is predicted non-vanishing by including gluonic degrees of freedom [35]. Other models, such as the MIT bag model [62–65], the constituent quark model [65, 66] and a light-cone quark model [67], etc., were employed to calculate the T-odd TMDs. Models without gluonic degrees of freedom can be only used to compute T-even TMDs. These TMDs have

been estimated in a spectator model [68], light-cone quark models [69–71], covariant parton model with orbital motion [72] and in the bag model [73, 74], etc.

The models, which provide a dynamical picture of the nucleon, are at some fixed scales. To compare with the experimental data, TMDs should be evolved to the experimental scales. One can refer to Reference [75] for a brief overview of TMD evolution.

2.3 Probing the Collins and Sivers Effects

Experimentally, in order to access the TMDs, one usually measures an asymmetry. As one can see from equation 2.1.1, structure functions are coupled with the different target spin and beam helicity states and different structure functions have different azimuthal angular dependence in the SIDIS cross section. In the case target is transversely polarized and beam is unpolarized, we define a target single spin asymmetry as:

$$A_{UT}(\phi_h, \phi_S) = \frac{1}{|S_\perp|} \frac{d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi)}{d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi)}. \quad (2.3.1)$$

Here “U” means unpolarized target, and “T” means transversely polarized target. Taking into account the SIDIS cross section at the leading twist, we have:

$$A_{UT}(\phi_h, \phi_S) = A_{UT}^{Collins} \sin(\phi_h + \phi_S) + A_{UT}^{Sivers} \sin(\phi_h - \phi_S) + A_{UT}^{Pretzelosity} \sin(3\phi_h - \phi_S), \quad (2.3.2)$$

where Collins (Sivers, Pretzelosity) asymmetry is a combination of kinematic variables and structure function $F_{UT}^{\sin(\phi_h + \phi_S)}$ ($F_{UT,T}^{\sin(\phi_h - \phi_S)}$, $F_{UT}^{\sin(3\phi_h - \phi_S)}$), which contains the TMDs. Therefore, by doing the target single spin asymmetry measurements, one can access the Collins, Sivers and Pretzelosity moments through azimuthal angular dependence.

The pioneer measurements have been carried out by the HERMES, COMPASS and JLab collaborations in the past decade. A review of the HERMES and COMPASS measurements is in [76]. Results from the HERMES and COMPASS experiments have clearly shown the presence of the $\sin(\phi_h + \phi_S)$ and $\sin(\phi_h - \phi_S)$ modulations from proton targets [77–79]. In remarkable contrast, much smaller modulations were found from deuteron targets [80], suggesting that the process is flavor dependent. To shed new light on the flavor structure of the transversity and Sivers functions, it is important to extend SIDIS measurements to a polarized ^3He target, whose spin comes predominantly from the neutron.

The first polarized ^3He measurement was carried out in Hall A at the Jefferson Laboratory (E06-010, Transversity experiment) and results for the charged pion SIDIS production have already been reported [81,82]. In this thesis, we will focus on the azimuthal asymmetries in charged kaon SIDIS production. Since kaons contain strange quarks, the role of sea quarks in the nucleons with respect to the Collins and Sivers effects can be explored. The HERMES collaboration [78] observed that the Collins effect from the proton target for K^+ is larger than that for π^+ , while for K^- the Collins effect is small and consistent with zero. They also reported that the Sivers effect for K^+ from the proton target is large and positive, but very small for K^- [77]. The COMPASS collaboration reported that the Collins and Sivers effects for K^+ and K^- production from the polarized deuteron target are consistent with zero [80]. Results from our work using a polarized ^3He target will provide important additional information for the study of the flavor dependent behavior of the Collins and Sivers effects.

Chapter 3

EXPERIMENTAL SETUP

3.1 Overview

Experiment E06-010 (Transversity experiment) [83] was performed in Hall A at the Thomas Jefferson National Accelerator Facility (JLab) from Oct. 2008 to Feb. 2009 using a longitudinally polarized electron beam and a transversely polarized ^3He target. The main goal of the experiment was to measure the target single spin asymmetries (SSA) as well as beam-target double spin asymmetries (DSA) in $e(^3\text{He}^\uparrow, e'\pi^\pm/K^\pm)X$ semi-inclusive deep inelastic reactions. The beam energy was 5.9 GeV and the beam helicity was flipped at 30 Hz. The target spin direction was reversed every 20 minutes. One can either study SSA by summing two beam helicity states to achieve unpolarized beam or do DSA study. Two spectrometers were employed to achieve the coincidence measurements. The BigBite spectrometer was set at 30° on the beam-right, facing downstream, to detect the scattered electrons with a momentum range of 0.6-2.5 GeV and a solid angle of about 64 msr. The left HRS [1] was set at 16° on the beam-left, facing downstream, to detect the produced hadrons ($\pi^\pm$, $K^\pm$ and proton) with a central momentum of 2.35 GeV and a solid angle of about 6 msr.

Besides the coincidence measurements, by design of the experiment, inclusive data (electrons, hadrons) are also available from the experiment, which allow us to extract useful physics results from various parasitic inclusive measurements.

In this chapter, we will discuss the incident electron beam, the BigBite spectrometer and the HRS, the polarized ^3He target as well as the data acquisition (DAQ) system for the experiment.

3.2 Incident Electron Beam

3.2.1 The Continuous Electron Beam Accelerator

Jefferson Lab is recognized as a world leader in accelerator science. This expertise comes from the planning, building, maintaining and operating the continuous electron beam accelerator facility (CEBAF). CEBAF is based on superconducting radio frequency (SRF) technology. It can provide a continuous electron beam with high polarization (up to $\sim 90\%$) and high current (up to $\sim 200\mu\text{A}$) for three different experimental Halls: A, B and C ¹. The top view of the accelerator area is shown in Figure 3.1.

The accelerator consists of one injector, two super-conducting linear accelerators (the north linac and the south linac), and two recirculation arcs. Layout of the electron accelerator is shown in Figure 3.2. In the injector, polarized electrons are produced by shining a circularly polarized laser light on a strained gallium arsenide (GaAs) cathode. By tuning a left-handed circularly polarized laser to proper frequency, electrons from the $P_{\frac{3}{2}}(m = \frac{3}{2})$ state can be excited to the $S_{\frac{1}{2}}(m = \frac{1}{2})$ level of the conduction band. These polarized electrons from the conduction band are extracted by applying a voltage of -100 kV to the photo-cathode. These free electrons are accelerated to 45 MeV and then longitudinally compressed in bunching section to form 2 nanosecond bunches before being injected into the north linac. Each linac consists of 20 superconducting cryomodules and the beam can gain from 400 MeV to 600 MeV energy passing through the linac. At the end of the north linac, there are 5 re-circulation arcs to deflect the beam by 180° into the south linac. At the end of the south linac, there are 4 re-circulation arcs to deflect the beam by 180° into the north linac. Within the re-circulation arcs, electrons are not accelerated. Therefore, with 5 rounds of circulation, the maximum beam energy can reach 6 GeV. At the end of

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¹Hall A and C usually run with high currents, Hall B can only run at a current of a few nA.

Figure 3.1. Top view of JLab accelerator area. Three “bumps” in the bottom of the picture are 3 experimental halls (A, B and C respectively from left to right) in JLab 6 GeV era.

Figure 3.2. Layout of the JLab electron accelerator.

the accelerating process, beam is extracted and delivered to 3 different Halls by the radio frequency separator working with 1497 MHz frequency. In reality, beam can be extracted into experimental halls without finishing 5 passes (one pass means one full circulation in accelerator), so lower beam energies are also available. For example, 1.2306 GeV and 2.396 GeV beams were used for calibration purposes, and 5.89 GeV beam was used as production beam.

3.2.2 Hall A Beamline

Hall A is the largest experimental Hall among existing halls in the 6 GeV era of JLab. It is outfitted with two primary detector systems - both high-resolution spectrometers (HRS) with momentum resolution (FWHM) of 10^{-4} . Figure 3.3 shows an overview of Hall A. A relative large acceptance spectrometer (BigBite) can be installed on the request of different experiments.

Hall A beamline starts at the arc section (for beam energy measurement) and ends at the beam dump of Hall A. Along the beam line, there are energy measurement device, Compton and Moller polarimeters to measure beam polarization, beam

Figure 3.3. Overview of Hall A.

current monitors (BCM) to measure beam current, beam position monitors (BPM) to monitor beam position, beam raster to modify the effective beam size and shape.

3.2.2.1 Beam Energy Measurement

The beam energy in the E06-010 experiment was measured by the arc-energy method. When beam enters Hall A, it passes through a forty-meter arc section which is between the accelerator and the Hall A end station (see Figure 3.4). This arc contains eight dipole magnets and nine quadrupole magnets to bend and direct the beam. The arc-energy method is developed to measure the energy of the beam while it reaches Hall A having passed through the arc. The principle of the method is that the momentum of a charged particle is correlated to its bending angle as it moving through a magnetic field. The measured energy, as a function of the field integral of eight-dipole magnets and the bend angle of the arc, can be expressed as :

$$E = c \frac{\int \vec{B} \cdot d\vec{l}}{\phi} \quad (3.2.1)$$

where c is the speed of light, B is the strength of the magnetic field perpendicular to the particle's velocity direction, ϕ is the bending angle through the arc. Therefore,

Figure 3.4. The schematic of the arc-energy measurement.

the energy of the beam can be calculated by measuring its bending angle ϕ through a well determined magnetic field. The bending angle of the beam is measured by four wire scanners, referred to the “SuperHarps” in Figure 3.4, installed at both entrance and exit of the arc. Each pair of scanners is about 4 meters apart. The nominal bending angle ϕ for Hall A is 34.3° . The arc-energy measurement provides an absolute measurement to an accuracy (FWHM) of about 2×10^{-4} GeV. More information about the arc-energy method can be found in [84].

In the beginning of the experiment (Nov 17th, 2008), one full arc-energy measurement was performed and beam energy was determined to be $E = 5889.4 \pm 0.5(stat.) \pm 1.0(sys.)$ MeV. During the experiment, beam energy was also continuously monitored by the “Tiefenbach” method [85], which uses BPMs to measure the bending angle of the beam through the arc. The accuracy of this method is $\frac{\delta E}{E} \sim 5 \times 10^{-4}$.

3.2.2.2 Beam Position Monitor (BPM) and Raster

To determine the position and direction of the beam at the target location, two beam position monitors (BPM) were installed at about 7.5 (BPMA) and 1.3 (BPMB) meters upstream of the target cell. Each BPM is a cavity with four wire antenna

Figure 3.5. The schematic of a beam position monitor.

parallel to the direction of the beam and tuned to the beam RF frequency. These wires are placed at 45° relative to the horizontal and vertical direction. A schematic figure of a BPM is shown in Figure 3.5. As beam passes by, an antenna pair (marked as A in Figure 3.5) will pick up the induced signal. The difference in the induced signals will give the relative beam position. The absolute beam position could be determined by calibrating the BPMs using the harp scan, which is an invasive procedure where wires are scanned across the beam.

Once beam position is measured at BPMA and BPMB, the beam position and direction can be calculated at the target position:

$$x, y_{target} = \frac{x, y_{BPMA} \cdot \Delta_{z_{BPMB}} - x, y_{BPMB} \cdot \Delta_{z_{BPMA}}}{z_{BPMB} - z_{BPMA}} \quad (3.2.2)$$

$$\vec{x}_{beam} = \frac{\vec{x}_{BPMB} - \vec{x}_{BPMA}}{|\vec{x}_{BPMB} - \vec{x}_{BPMA}|},$$

where $\Delta_z = z_{BPM} - z_{target}$.

The target was designed in a way that ^3He gas was sealed in a super thin glass cell with window thickness of about 0.1 mm. High current beam with small beam spot will damage the target window due to overheating. Therefore, beam should be rastered when entering the target area. This was realized by a raster which was composed of two sets of (horizontal and vertical) air-core dipoles. During the experiment E06-010, a 3×3 mm square raster pattern was achieved by applying currents with specific frequencies and amplitudes to the coils of dipoles.

3.2.2.3 Beam Current Monitor (BCM)

Beam current was measured by the beam current monitor (BCM) which is designed for a stable, low-noise, non-interfering beam current measurement. It is composed of an Unser monitor, two RF cavities, associated electronics and a data-acquisition system. The cavities and the Unser monitor were enclosed in a temperature-stabilized box to shield magnetic field. The box was located 25 m upstream of the target location. The configuration of BCM is shown in Figure 3.6.

The Unser monitor is a parametric current transformer which provides an absolute reference [86]. It was calibrated by passing a well known current through a wire inside the beam pipe and had a nominal output of $4 \text{ mV}/\mu\text{A}$. It is sensitive to the magnetic field and temperature, therefore, it can not be used to monitor the beam current continuously. Instead, two RF cavity monitors were used to provide constant monitoring of the beam current.

The two RF cavity monitors on either side of the Unser monitor are stainless steel cylindrical high-Q (~ 3000) waveguides. During the operation, they were tuned to the frequency of beam (1.497 GHz). Voltage output of the cavity is proportional to the beam current. Each of the RF output signals from the two cavities is split into two parts, one is to be sampled and the other one is to be integrated. The

“to-be-sampled” data is sent to a high-precision digital AC voltmeter (HP 3458A) which provides a digital output every second. The digital number is proportional to the average beam current of that second. The “to-be-integrated” data is sent to an RMS-to-DC convertor, whose DC output signal is converted to a frequency by a Voltage-to-Frequency converter. The frequency signal is then counted by scalars, and thus provide a measurement of the total accumulated charge. The regular RMS to DC output is linear for currents from about $5 \mu\text{A}$ to above $200 \mu\text{A}$. A set of amplifiers is introduced with gain factors of 1, 3, 10 to lower currents at the expense of saturation at high current. Hence, there are total 6 BCM numbers (u1, u3, u10, d1, d3, d10) recorded for the beam current measurement. In the experiment E06-010, we used u3 and d3 signals to calculate the beam current. Please refer to [87] for more detailed study of beam current monitoring.

3.3 BigBite Spectrometer

The BigBite spectrometer was used to detect scattered electrons in the E06-010 experiment. It consists of a large-opening dipole magnet and a detector stack. The front face of the magnet was located at a distance of 1.5 m from the target center which provided an angular acceptance of about 64 msr. In the detector stack, there are 3 multi-wire drift chambers (MWDC) working as tracking system, a lead-glass calorimeter divided into preshower and shower sections for electron identification and a scintillator plane between preshower and shower providing the timing information of identified particles. A schematic view of the BigBite Spectrometer is shown in Figure 3.7.

Before the introduction of each sub-system of the BigBite spectrometer, the coordinate systems relevant to the BigBite spectrometer are defined below ².

Lab coordinate system:

- +z is along beam direction
- +y is against gravity, pointing up
- +x is to the left when looking from upstream to downstream

Magnet coordinate system:

- +x is to the right side, when looking at the front face of the magnet
- +y is against gravity, pointing up
- +z: $x \times y$

BigBite Detector coordinate system:

- The origin is specified by the center of first wire chamber.
- +z is pointing from the face of the chamber, towards the calorimeter
- +x is pointing to the bottom of the chamber
- +y: $z \times x$

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²The coordinate systems used in the experiment are also summarized in appendix A

Figure 3.7. The schematic view of the BigBite spectrometer. The Figure is from [3]. The Cherenkov detector is for the experiment E06-014. During the experiment E06-010, it was installed to take data for commissioning purposes.

Figure 3.8. Picture of the BigBite magnet. The picture was taken during the JLab “open house”.

3.3.1 BigBite Magnet

The magnet is a H-shaped dipole with a gap of 25 cm. The entrance face is perpendicular to the horizontal plane while the exit face has a rotation of 5° with respect to the entrance face. This enhances (reduces) the field integral for particles entering the upper (lower) region of the magnet, thereby making the dispersion more uniform across the acceptance of the spectrometer. Figure 3.8 is a picture of the BigBite magnet. During the experiment E06-010, the magnet was powered by the BigBox power supply to 710 A, which produced a 1.2 T magnetic field. In production runs, the nominal field direction was in +x direction so that positive particles will be bended down while negative particles will be bended up.

Figure 3.9. Illustration of the orientations for the wire planes of multi-wire drift chamber.

3.3.2 Multi-wire Drift Chamber (MWDC)

Three multi-wire drift chambers were used in the experiment E06-010 to provide tracking information for charged particles after passing through the BigBite magnet. The active area of the first chamber is about $140 \text{ cm} \times 35 \text{ cm}$, the active areas of the second and third one are both about $200 \text{ cm} \times 50 \text{ cm}$. Each chamber consists of 6 wire planes, two of them are oriented into one direction. Wire planes for each chamber are oriented into 3 directions: u , v and x , as illustrated in Figure 3.9. Within each plane, the “signal” wires are segmented by “field” wires and the distance between the “signal” wire (or “sense” wire) and “field” wire is 0.5 cm. For two wire planes with the same orientation, one of the planes (for example the two planes oriented in x direction) is shifted by 0.5 cm to the other to eliminate the “left-right” ambiguity of the hit.

Each wire plane is sandwiched by two cathode planes which are applied to a high voltage of -1600 V. The field wires are also applied to -1600 V while the signal wires are grounded. This configuration will provide a relatively uniform drift electric field for electrons. Working gas for the MWPC is a 50%/50% argon-ethane (C_2H_6) mixture with a flow rate of 10 liter/hour. When a charged particle passes through the chamber (eg. passes through “ x ” wire plane as illustrated in Figure 3.10), primary “electron-

Figure 3.10. Side view of “x” wire plane. Each wire plane is sandwiched by two cathode planes. In the Figure, a charged particle passes through the chamber in the z direction (detector coordinate), it will generate primary electron-ion pairs along the track, the electrons will drift to the adjacent sense wire where the avalanche happens.

ion pairs” are produced along the track of the charged particle due to ionization. Those electrons will drift to the nearest sense wire where avalanche happens, hence a signal is picked up by the sense wire. By measuring the drift time for electrons, the distance between the track of the charged particle and the sense wire is determined.

3.3.3 Lead-glass Calorimeter

A lead-glass calorimeter provides electron particle identification (PID) and trigger in the BigBite spectrometer. This calorimeter is segmented into preshower and shower detectors. The geometry of preshower and shower is illustrated in Figure 3.11. The preshower detector consists of 54 modules aligned in 2 columns and 27 rows. Each module is made of TF-5 lead-glass with a size of 8.5 cm (x) $\times$ 34 cm (y) $\times$ 8.5 cm (z). The overall active area of the preshower is 210 $\times$ 74 cm². Along the z direction (detector coordinate), the radiation length is 3 X_0 . The shower detector consists of 189 modules aligned in 7 columns and 27 rows. Each module has the same size as preshower module, while the shower module is made of TF-2 lead-glass material.

Figure 3.11. Geometry of BigBite preshower, scintillator and shower detectors.

Overall active area of the shower is $221 \times 85 \text{ cm}^2$ with radiation length of $13 X_0$ along z direction (detector coordinate).

3.3.4 Scintillator Plane

The BigBite scintillator plane consists of 13 plastic scintillator bars with the size of $17 \text{ cm (x)} \times 64 \text{ cm (y)} \times 4 \text{ cm (z)}$. The alignment is illustrated in Figure 3.11. Each bar is connected to two PMTs, one at each end. The entire scintillator plane is mounted between the preshower and shower detectors. The signal from the scintillator plane was used to provide the timing information of particles passing through the BigBite spectrometer.

3.4 Left High Resolution Spectrometer (HRS)

There are two high resolution spectrometers (left and right HRS) [1] working as standard equipments in Hall A. Each spectrometer consists of four magnets and a

detector package in the detector hut. Three of the four magnets are quadrupoles and the other one is a dipole. These magnets are mounted following the QQDQ sequence along the trajectory of accepted particles. A detector hut, which is made of steel and concrete to shield radiation from the Hall, is located at the end of the spectrometer. Detectors inside the detector hut can be different among different experiments. During the experiment E06-010, left HRS was employed to detect the produced hadrons (pions, kaons and protons). In the detector package, there were two vertical drift chambers (VDC), a scintillator plane (S1), an aerogel Cherenkov detector, a CO₂ gas Cherenkov detector, a ring imaging Cherenkov detector (RICH), a high resolution scintillator plane (S2m) and two layers of lead-glass detectors. Layout of the LHRS in E06-010 is shown in Figure 3.12. By applying the current with different polarity ($\pm$ polarity) to the dipole, the HRS will bend different charged particles (+ or -) up into the detector package. By changing the amplitudes of the current for the four magnets, the central momentum of the HRS can be modified. Main design characteristics of the LHRS are listed in table 3.1. During the experiment E06-010, the central momentum of the LHRS (p_0) was set to be 2.35 GeV/c.

3.4.1 Vertical Drift Chambers (VDC)

Two vertical drift chambers [88], which are placed horizontally at the entrance of the detector package, are used in the LHRS to provide tracking information. The side and top view of the VDCs is shown in Figure 3.13. The two VDCs are 335 mm apart from each other. Each VDC has two “sense” wire planes consisting of 368 sense wires spaced 4.24 mm apart. In each plane, wires are oriented at 45° or -45° with respect to the dispersive directions³. Therefore, wire orientations in two planes are perpendicular to each other. The nominal bending angle for the LHRS is 45°, hence, particles will pass through the VDCs at a nominal angle of 45°.

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³long edge of the VDC (2118 mm edge) is along the dispersive direction.

Figure 3.12. Schematic layout of the LHRS setup for the E06-010 experiment. This is a side-view of the LHRS. Charged particles will be deflected up by the dipole into the detector package. By applying the current with different polarity ($\pm$ polarity) to the dipole, the HRS will select different charged particles (positive charged particles or negative charged particles). The plot is from [3].

Descriptions	Design characteristics
Configuration	QQDQ vertical bend
Bending angle	45°
Optical length	24.2 m
Central Momentum range	0.3-4.0 GeV/c
Momentum acceptance	$\frac{ p-p_0 }{p_0} < 4.5\%$
Momentum resolution	2×10^{-4} (FWHM)
Horizontal angular acceptance	± 30 mrad
Vertical angular acceptance	± 60 mrad
Horizontal angular resolution	1.5 mrad
Vertical angular resolution	4.0 mrad
Central Solid angle	6 msr
Transverse length acceptance	± 5 cm
Transverse position resolution	2.5 mm

Table 3.1. Main design characteristics of the LHRS [1].

Figure 3.13. Side and top view of VDCs.

Each “sense” wire plane of the VDC is sandwiched by two cathode planes which are applied by a high voltage of -4 kV. The “sense” wires are grounded. During the operation, a 62%/38% argon-ethane (C_2H_6) gas mixture was flowed into the chambers. As illustrated in Figure 3.14, when a charged particle traverses the chamber with an angle ($\sim 45^\circ$), primary “electron-ion” pairs will be produced along the track due to ionization, electrons will drift vertically along the electric field to the nearest “sense” wire where avalanche happens and the signal is picked up by the “sense” wire. Different from MWDC in the BigBite spectrometer, where only one sense wire is fired for each track in each wire plane, VDC has up to five wires been fired for each track in each wire plane. Drift distance is calculated using the drift time recorded by each fired wire. The cross-over point that the track pass through the wire plane is determined by using a linear fit of the drift distances. The position resolution for VDCs is about $100 \mu\text{m}$ which corresponds to the angle resolution of $\sigma_{\theta(\phi)} \sim 0.5 \text{ mrad}$.

Figure 3.14. Configuration of wires in VDCs.

3.4.2 Two Scintillator Planes S1 and S2m

There are two scintillator planes S1 and S2m separated by about 2 meters in the LHRS detector package. The coincidence signal between the two planes was used to provide the singles trigger for the LHRS. The signal from S2m plane was used to provide the time of flight (TOF)⁴ for particles entering the LHRS. The S1 plane is made of 6 thin plastic (BC408) scintillators with two PMTs mounted at both ends of each scintillator, as shown in Figure 3.15. There is a small overlap between two adjacent scintillators to eliminate dead area. The active volume of each scintillator for S1 plane is $36 \text{ cm} \times 29.3 \text{ cm} \times 0.5 \text{ cm}$. The S2m plane is made of 16 fast plastic scintillator bars with size of $43.2 \text{ cm} \times 14 \text{ cm} \times 5.08 \text{ cm}$. Each scintillator of S2m plane is mounted with two PMTs at both ends.

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⁴TOF for the LHRS is the time for charged particles to fly from the reaction point to the S2m plane.

Figure 3.15. Layout of the S1 scintillator plane in the LHRS.

3.4.3 Cherenkov Detectors

Three Cherenkov detectors, Aerogel detector (A1) and CO₂ gas Cherenkov detector and RICH, were employed to do particle identifications based on Cherenkov effect during the experiment E06-010. The Cherenkov effect refers to the effect that, when a high energy charged particle travels through transparent materials with a velocity of v larger than the phase velocity of light in that material (c/n), a characteristic electromagnetic radiation is emitted. This radiation is called Cherenkov radiation or Cherenkov light. The Cherenkov light has a fixed angle to the particle trajectory:

$$\theta_c = \arccos(1/n\beta), \quad (3.4.1)$$

where n is the refraction index of the radiator, β is the relativistic velocity (v/c). The number of photons produced follows:

$$\frac{d^2 N}{dz d\lambda} \propto \frac{z^2}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right), \quad (3.4.2)$$

where z is the charge number of the radiator material element, λ is the wavelength of generated photons.

In the formula 3.4.1, there is a threshold β or momentum for a specific type of particle while n is fixed. From the formula 3.4.2, one can conclude that faster particles generate more photons. Aerogel and CO₂ gas Cherenkov detectors in the experiment E06-010 are threshold Cherenkov detectors, as will be discussed in the following text. RICH detector distinguishes particles by the θ angle of the Cherenkov light generated by different particles with momentum of ~ 2.35 GeV/c in the LHRs.

3.4.3.1 Aerogel Cherenkov Detector

Aerogel Cherenkov detector (A1) is made of a 9 cm thick Aerogel radiator with refraction index of 1.015, which provides a threshold of 2.84 and 0.803 GeV/c for kaons and pions, respectively. “GeV” level electrons will also fire the A1. A diffusion technique of light collection was used, as illustrated in Figure 3.16. Particles with momentum above threshold passing through the radiator will generate Cherenkov light, which is reflected by the diffusion reflector (Millipore paper) and then collected by the PMTs mounted on the two sides of the detector. Total area of the radiator is 32 cm $\times$ 170 cm. Diffusion reflector is 0.22 μ m thick. There are 24 PMTs in total to collect the light. The summed signal from all 24 PMTs gives information about the light generated by the particle. The A1 provided a kaon and proton rejection better than 10:1 for the pion identification [87].

Figure 3.16. Structure of the Aerogel Cherenkov detector A1. Particles enter the detector from the bottom of the figure.

3.4.3.2 CO₂ Gas Cherenkov Detector

The CO₂ gas Cherenkov detector is filled with CO₂ with one atmospheric pressure. The refractive index of the gas radiator is 1.00041 which gives a threshold momentum of 17 MeV/c and 4.8 GeV/c for electrons and pions, respectively. Only electrons within the LHRS acceptance will emit Cherenkov light.

The gas Cherenkov detector is made of steel with thin entry and exit window made of tedlar. Thickness of the gas tank is about 90 cm and surface area is about 250 cm × 80 cm. Ten mirrors positioned in a 2 × 5 array near the exit of the detector are used to reflect and focus the Cherenkov light. Each mirror is coupled to a PMT whose photocathode is facing downstream to collect the reflected and focused light from the mirror. A schematic view of the detector is shown in Figure 3.17.

Although pions don't produce any Cherenkov light directly, they can interact with the atoms in the detector and create secondary electrons (δ electrons). The probability of the δ electron production is less than 0.001. These electrons will produce Cherenkov light. However, these electrons don't necessary move in the same direction as pions, thus the emitted light is not efficiently focused by the mirrors. The summed ADC signals for these secondary electrons are mostly located around the single photo-

Figure 3.17. Schematic view of CO_2 gas Cherenkov detector in the LHRS.

electron peak. On the other hand, these pion events can be removed by the selection of coincidence events between LHRS and BigBite.

3.4.3.3 RICH detector

The ring imaging Cherenkov detector (RICH) in the experiment E06-010 is conceptually identical to one module of the ALICE-HMPID RICH [89]. It was first used in the experiment E94-107 “Hypernuclei Spectroscopy” [90] and has been upgraded to extend the momentum detection range to about 3 GeV/c. It is adapted in the experiment E06-010 to achieve the separation of pions, kaons and protons. The refraction index of the radiator is 1.28, therefore, electrons, pions, kaons and protons with momentum of ~ 2.35 GeV/c will generate Cherenkov lights with different angles. The expected angles for the Cherenkov light are 0.680, 0.677, 0.652, 0.583 rad for ~ 2.35 GeV electrons, pions, kaons and protons, respectively.

The RICH has a proximity focusing geometry (no mirrors involved) which makes the detector compact (total thickness less than 50 cm) and relatively thin ($18\% X_0$). Figure 3.18 shows the working principle of the RICH. The Cherenkov effect takes place in the liquid freon when a charged particle crosses it. The liquid radiator, 1.5

cm thick, is housed in a vessel made of NEOCERAM⁵ on all sides but the exit window which is made of pure quartz, 0.5 cm thick. The use of a liquid radiator has been imposed by the momentum range (around 2 GeV/c) of the particles to be identified. The Cherenkov photons, emitted along a conic surface, are refracted by the freon-quartz-methane interfaces and strike a pad plane after traveling a proximity gap of 10 cm filled with methane. The pad plane is covered by a thin substrate of CsI which acts as photon converter. The emitted photo-electron is accelerated by an electrostatic field (2200 V/ 2 mm) between the pad plane and an anode wire plane in front of the pads, forming a multi wire proportional chamber (MWPC). While the anode wires collect the electron avalanche, the counter part ions are collected by clusters of pads, each of which is connected to the input channel of a multiplexed sample and hold electronics, housed on the back of the pad plane. At the end of this process, the clusters of pads hit by the photons should be scattered on a ring (ellipse) while one cluster coming from the charged particle track should be located in the central region of the ring. A drift electrode operated at 200 V and located close to the quartz window, prevents electrons produced by ionization of the counting gas by charged particles in the proximity gap from reaching the MWPC. During the operation, the whole detector was put in a atmosphere filled with pure methane.

3.4.4 Lead-glass Detector

Two layers of lead-glass detectors were installed at the end of the detector package during the experiment E06-010. Both layers have the same geometry.⁶ Each layer consists of 17 short blocks and 17 long blocks, forming a 2 (column) $\times$ 17 (row) array. PMTs are mounted on the ends of each layer. The size of short modules is 14.5 cm $\times$ 14.5 cm $\times$ 30 cm while the long modules has the size of 14.5 cm $\times$ 14.5 cm $\times$

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⁵NEOCERAM is a glass-ceramic material with mechanical and thermal properties almost identical to quartz.

⁶The Lead-glass detector in the LHRs is not a “total-absorption” EM calorimeter.

Figure 3.18. Working principle of the freon CsI proximity focusing RICH.

35 cm. All modules are oriented transversely with respect to the nominal direction of accepted particles. The first and second layers are aligned in a way that the gap between blocks of the first layer is covered by a lead glass block in the second layer, as shown in Figure 3.19. The goal of the lead-glass detector is to provide additional electron rejection.

3.5 Polarized ^3He target

The polarized ^3He target consists of a 40-cm long glass cell containing ~ 10 atm of ^3He and a small amount of N_2 to reduce the depolarization [1, 91]. The ground state of ^3He nuclear wavefunction is dominated by the S-state, in which the proton spins cancel each other and the nuclear spin is mostly carried by the neutron [92]. Three pairs of Helmholtz coils were used to produce the holding magnetic field in any direction. During the experiment, the target spin direction was oriented to transverse and vertical directions in order to enlarge the azimuthal angular coverage ϕ_S . ^3He

Figure 3.19. Layout of the setup for the lead-glass detector in the LHRS.

nuclei were polarized by spin exchange optical pumping of a Rb-K mixture [93]. Nuclear Magnetic Resonance (NMR) measurements, calibrated by the known water NMR signal and the electron paramagnetic resonance (EPR) method, were performed to monitor the target polarization while the target spin direction was flipped every 20 minutes through adiabatic fast passage. An average in-beam target polarization of $(55.4 \pm 2.8)\%$ was achieved during the experiment. For a detailed description of the polarized ^3He target system, one can refer to chapter 4 of J. Huang's thesis [3] ⁷.

3.6 Data Acquisition (DAQ) and Trigger

3.6.1 CODA

The standard custom designed data acquisition system (DAQ) for physics experiments running at JLab is called CODA (CEBAF Online Data Acquisition). It contains a set of software and hardware packages.

The hardware package consists of commercial VME/Fastbus crates with corresponding digitization devices (ADC, TDC and scalers) and JLab custom designed

⁷There is also a thesis, written by Yi Zhang from Lanzhou University, including the details of the target system. His thesis is written in Chinese and can be found in our transversity home page [83]

control boards (crate controller, trigger interface board, trigger supervisor board). The crate controller has a CPU embedded running VxWorks operating system ⁸. Trigger interface board (TI) is used to synchronize the signals inside a crate, trigger signal for this crate should be fed into TI first, once TI accepts the trigger, it outputs a response signal (L1A signal), which can be used to gate ADCs/TDCs inside the crate. Trigger supervisor board (TS) is used to synchronize the signals among crates when more than one crate is used in a DAQ setup.

The software package components are a readout controller (ROC) running on the control board to communicate between DAQ and digitization devices, an event builder (EB) to collect the data from crates with certain header information, an event recorder to write data built by EB to disk, an event transfer (ET) which allows distributed access to the data stream from user processes and inserts additional data into the data stream every few seconds from the control system, a graphical user interface (GUI) to control the data taking.

3.6.2 Trigger Design

In the experiment E06-010, there are both singles triggers from LHRS/BigBite spectrometers and coincidental trigger between two arms. A summary of the trigger type is in Table 3.2. On the BigBite side, signals from two adjacent layers of preshower ($2 \times 2 = 4$ blocks) and shower ($2 \times 7 = 14$ blocks) modules are summed to provide the trigger (T1 or T6) after the discriminator (see Figure 3.20). The difference between T1 and T6 is the threshold of the discriminator. The logical diagram of the trigger on BigBite spectrometer is shown in Figure 3.21. On the HRS side, trigger (T3) is formed by the coincidence signal between S1 and S2 scintillator planes. The logical diagram of the trigger on LHRS is shown in Figure 3.22. The T5 trigger is formed by the coincidence of T1 and T3 signals. T1 signal was delayed by 40 ns second after

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⁸The latest developed controller for 12 GeV experiments is equipped with an Intel-CPU running Linux system.

Trigger type	Description
1	BB singles with low threshold on lead-glass
2	BB singles with gas Cerenkov detector
3	Left HRS singles (S1 .and. S2)
4	Left HRS trigger efficiency
5	Coincidence between BigBite and Left HRS (T1 .and. T3)
6	BB singles with high threshold on lead-glass

Table 3.2. Summary of physics triggers in the experiment E06-010.

Figure 3.20. Illustration of how signals from preshower and shower modules are summed over to form the BigBite trigger.

T3, hence, the T5 trigger was given by the leading edge of the T1 trigger. The timing information on how T1 and T3 formed T5 trigger is illustrated in Figure 3.23.

Figure 3.21. Logic diagram of the trigger algorithm for the BigBite singles trigger.

Chapter 4

DATA ANALYSIS

As described in the previous chapter, besides the coincidence trigger between LHRS and BigBite spectrometers, both LHRS and BigBite spectrometers have singles triggers. Hence, one can also perform inclusive (electron, $\pi^\pm$, $K^\pm$, proton) measurements in addition to the coincidence measurements. The main goal of this thesis is to extract the Collins and Sivers moments from the measurement of target single-spin asymmetry (SSA) through azimuthal angular dependence in the semi-inclusive deep inelastic scattering process, $^3\text{He}^\uparrow(e, e' K^\pm)X$. Target single spin asymmetry and beam-target double-spin asymmetry measurements in the inclusive hadron productions are also obtained. Beam-target double-spin asymmetry measurements in the inclusive DIS process as well as the extraction of the $g_2^{^3\text{He}}$ are performed as well.

Summaries of the “data-cooking” and detector calibrations are scattered in different dissertations [3, 87, 94, 95]. In this chapter, a brief summary of the detector calibration will be introduced in the beginning. Then the summary of data analysis including particle identification (PID), events selection and analysis strategy will be described.

4.1 Detector calibrations

Raw data (binary data) were first processed using the standard Hall A “analyzer” software [96], which is an object oriented framework to decode and process raw data. The decoded data were filled into “root trees” using a “run database” that stores the information of run configurations during the production run period. These rootfiles were then used for detector calibrations.

4.1.1 BigBite MWDC Calibration

Three sets of MWDC were used to determine the tracks of particles passing through the BigBite spectrometer. The tracks provided not only tracking information of the particles but also information for the momentum reconstruction. Therefore, precise measurements of tracks are critical. As described in section 3.3.2, hit positions of a track are recorded and determined by the arrival time of the signals. Calibration of the MWDC consists of three parts: t_0 , conversion function to convert drift time to drift distance, absolute positions of individual wires ¹.

- t_0 calibration

In practice, each channel of TDC recorded a time difference between BigBite spectrometer trigger ($t_{trigger}$) and the signal from one particular wire (t_{signal}). t_{signal} has two contributions ²: drift time (t_{drift}) of “primary electrons” drifting to the sense wire and propagation time ($t_{propagation}^{signal}$) for the signal been transported from the wire to TDC. $t_{trigger}$ consists of three parts: travel time (t_{travel}) of the track from the hitting point on the MWDC to calorimeter where the trigger is formed, propagation time ($t_{propagation}^{trigger}$) for the trigger signal been transported to TDC, trigger time-walk effect (t_{walk}) which needs to be corrected. One has:

$$t_{signal} = t_{drift} + t_{propagation}^{signal} \quad (4.1.1)$$

and

$$t_{trigger} = t_{travel} + t_{propagation}^{trigger} + t_{walk} \quad (4.1.2)$$

Therefore, the recorded time of TDC, after time-walk correction, can be linked to the real drift time of the “primary electrons” drifting from the hit point to the

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¹Detector mapping should be checked as well.

²Use the time point when a track hits MWPC as the reference time point ($t=0$ point).

Figure 4.1. Drift time spectrum for the 100_{th} wire in a x-plane of the 3_{rd} MWDC.

nearest sense wire by

$$t_{signal} - t_{trigger} = t_{drift} + (t_{propagation}^{signal} - t_{travel} - t_{propagation}^{trigger}) - t_{walk} \quad (4.1.3)$$

in which t_{walk} should be corrected and t_0 is defined as:

$$t_0 = t_{propagation}^{signal} - t_{travel} - t_{propagation}^{trigger} \quad (4.1.4)$$

By looking at the Equation 4.1.3, one can conclude that t_0 is an offset between TDC measurements and real drift time. In reality, The t_0 was extracted for individual wires by identifying the rising edge of the drift time spectrum of that wire. Figure 4.1 shows a drift time spectrum for the 100_{th} wire in a x-plane.

- Calibration of the conversion function to convert drift time to drift distance

The conversion function was obtained using reconstructed tracks. During the calibration, as shown in Figure 4.2, the drift distance VS the drift time was plotted and fitted, hence the conversion function was parameterized.

- Calibration of absolute positions of individual wires

Figure 4.2. Drift distance VS drift time plot (left) and parameterized conversion function (right).

The chamber position was surveyed before the experiment. Positions of individual wires in the chamber was calibrated by looking at the track residual, which is defined as the distance between the track projection on a wire plane from track reconstruction and the hit position on this plane calculated from drift distance with respect to the reported wire position. The central value of the track residual spectrum indicates the shift between the reported wire position and the absolute position while the variance indicates the spatial resolution for the tracking. The final resolution (σ) after calibration was about $180 \mu\text{m}$, as shown in Figure 4.3.

4.1.2 BigBite Spectrometer's Optics

After the calibration of MWDC, tracks can be fitted from the hit information from MWDCs using tracking software [97]. Kinematic information of the particles including momentum, interaction vertex and scattered angles when it left the target can then be determined by the BigBite optics. In general, optics is a matrix ³ to transport the quantities measured by MWDCs (associated with dipole) to the quantities in the target area, which were used to calculate kinematic variables. The goal of the optics calibration is to obtain the proper matrix.

³In practice, the transportation is performed by a polynomial expansion between two sets of variables.

Figure 4.3. The residual spectrum for the U-plane of the 1_{st} MWDC after calibration.

The procedure of BigBite optics calibration is described in detail in section 5.1.2 of [87]. The calibration was performed with two incident beam energies (1.2306 GeV and 2.3960 GeV) and checked by data taken with beam energy of 5.89 GeV. The interaction vertex reconstruction was calibrated using the multi-foils carbon target, whose locations were precisely known from the accurate survey conducted before the experiment. The scattering angles were calibrated using a sieve-slit (see Figure 3.7) placed in front of the BigBite magnet. The momentum was calibrated using the e-p elastic scattering, where the scattered electron momentum can be precisely calculated using the two-body kinematics. The quality of the calibration can be seen in Figure 4.4 and the reconstructed sieve pattern can be seen in Figure 4.5.

The calibration was further examined using e-p elastic scattering at incident electron energy of 5.892 GeV. The corresponding momentum of the scattered electron was about 3 GeV/c - 3.5 GeV/c in the BigBite spectrometer. The stability of the calibration is checked during the production running at incident electron energy of 5.892 GeV with ³He target. The final momentum resolution $\delta p/p$ was about 1%.

Figure 4.4. Demonstration the quality of the BigBite momentum reconstruction by reconstructing electrons scattered from a hydrogen target. The top (bottom) left panel shows the reconstructed invariant mass peaks for proton and resonances at an incident beam energy of 1.230 (2.396) GeV. The right two panels show the momentum vs. lab polar θ angle at two incident beam energies.

Figure 4.5. The left panel shows the reconstructed sieve pattern with first order optics model. The middle panel shows the sieve pattern after adding offsets. The right panel shows the sieve pattern after applying the higher order corrections. The red points label the location of sieve holes/slots.

The average vertex resolution was about 0.8 cm⁴. The angular resolution, which is determined by the design of sieve slit, was better than 10 mrad.

In addition, the BigBite positive optics model was also developed for calibrating the momentum of positively charged particle. This was achieved using the data from the E06-014 experiment which took data immediately following this experiment. The quality of the positive optics was slightly worse than the negative optics, due to the scattering angle difference between the E06-010 (30°) and the E06-014 (45°) experiments. The momentum resolution was about 1.2%. More details can be found in section 5.1.2 of [87].

4.1.3 BigBite Calorimeter Calibration

Calorimeter of the BigBite consists of a preshower and a shower as described in section 3.3.3. The calibration of calorimeter was realized by cosmic rays and elastic scattering on a hydrogen target. In the calibration process using cosmic rays, “gain match” was performed for all the preshower and shower modules by adjusting the high voltage on the PMTs. After that, the coefficients for converting the ADC amplitudes to the absolute energy deposition in individual modules were determined by using the data of elastic scattering on a hydrogen target, in which process 1.23 and 2.39 GeV electron beam were used. Firstly, the clusters in shower (1 central module + surround 8 modules) and preshower (6 modules) were reconstructed. Secondly, coefficients for individual modules were determined by minimizing the χ^2 defined as:

$$\chi^2 = \sum_{i=1}^N (E_e^i - \sum_{j=0}^M C_j A_j^i)^2 \quad (4.1.5)$$

where i is the i_{th} elastic event, N is the total number of elastic events, E_e^i is the energy calculated from elastic scattering, j is the j_{th} module including preshower and shower

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⁴The vertex resolution depends on the momentum.

Figure 4.6. Reconstructed energy of scattered electrons in the preshower + shower signal at two incident beam energies.

Figure 4.7. Distribution of the ratio between reconstructed energy of scattered electrons in the preshower + shower signal and momentum of scattered electrons reconstructed from BigBite optics at two incident beam energies

modules, C_j is the coefficient of the module to covert ADC amplitude to energy deposition, A_j is the pedestal subtracted ADC amplitude. After the calibration, the energy resolution of about $\sigma_{\frac{E}{P}} = 8\%$ was achieved. Figure 4.6 shows the reconstructed energy of electrons in the calorimeter at two incident beam energies. Figure 4.7 shows the distributions of E/P , where E is the reconstructed energy in the calorimeter, P is the momentum of scattered electrons reconstructed from BigBite optics.

4.1.4 LHRs Calibrations

The LHRs is a standard spectrometer in Hall A. The calibrations were performed using the standard techniques which were used in the past Hall A experiments.

- VDC Calibration

Figure 4.8. Sum of all PMT signals in Gas Cerenkov and A1 detectors. The single photo-electron peak for Gas Cerenkov is aligned to be 200 ADC channel, while 100 ADC channel for A1.

The calibration for VDC is similar to what was done for BigBite MWDC. Since VDC is a standard component whose performance is well known, only t_0 calibration for individual wires is needed. The calibration was done by looking at the rising edge of the drift time spectrum.

- Gas Cerenkov and A1 Aerogel Cherenkov Detector Calibration

Both Gas Cerenkov and A1 Cherenkov detectors are threshold Cherenkov detectors. The calibration was mainly focused on the gain alignment for individual PMTs by using single photo-electron peak. The single photo-electron peak was aligned to be located at ADC 200 channel for Gas Cerenkov detector, while ADC 100 channel for A1. The purpose of this alignment was to make sure that the distribution of the events which do not pass the threshold will be peaked at one specific ADC channel (single photo-electron peak position). In principle, events which were below the threshold will not generate Cherenkov light, however, they will generate secondary electrons, which have large probability to generate single photo-electrons. Figure 4.8 shows the sum of all PMT signals in Gas Cerenkov and A1 detectors after the calibration.

- LHRS Lead-glass Detector Calibration

The LHRS lead-glass detector calibration is similar to that of the BigBite calorimeter. The gain match was done by aligning the minimum ionization peak to channel 100 for each PMT. Then the conversion coefficients were obtained by using the cali-

Figure 4.9. E/P plot for the LHRS lead-glass detector.

bration runs. Energy deposition in the lead-glass detector was calculated by summing over signals from all the blocks in two layers. Figure 4.9 shows the energy divided by the reconstructed momentum (E/P) spectrum where a clear separation between pions and electrons is achieved. The electrons and pions are identified from Cherenkov detectors.

- LHRS RICH Calibration

The RICH detector is able to separate pions, kaons and protons with certain momentum. During the experiment E06-010, RICH detector was not used in the SIDIS analysis due to its low efficiency and the good pion/kaon separation in the coincidence time of flight spectrum. However, RICH is crucial in the analysis associated with inclusive hadron productions. The calibration was done by Y. Wang [95]. Basically, the process of RICH data follows the steps: 1) pedestal scan to get the baseline in order to do pedestal subtractions for all the wires; 2) identify noisy channels which were then masked; 3) adjust database in order to make the pion Cherenkov angle consistent with theoretical prediction; 4) reconstruct the Cherenkov angle and MIP angle for further data analysis. For more details about the work on the RICH detector during the experiment E06-010, please refer to section 5.1.3 of Reference [95].

The algorithm of reconstruction and particle identification in RICH is as following: The clusters were reconstructed from raw hits in MWPC, then a MIP was found within an area with radius “R” around the projected point from the track in drift chambers onto the RICH readout board. The particle going through RICH was assumed to be pion, kaon or proton whose ring size or fiducial zone is different. For each assumption (pion, kaon or proton), the χ^2 test was performed with all the clusters that occurred in the “logical or” area of the three fiducial zones. During the χ^2 test, with certain number of reconstructed “photons” or clusters, the χ^2 and χ^2 probability were calculated. If no χ^2 probability is acceptable (a mandatory number, 0.01), it means that “noise clusters” were involved in the χ^2 test, then χ^2 test was performed again with one cluster removed until one of the assumption stands (χ^2 probability was acceptable). By doing kaon (proton) selection, we require that the χ^2 probability for kaon (proton) assumption is acceptable while rejecting other assumptions at 10^{-4} level.

- LHRs Optics

The LHRs optics was used to reconstruct the θ_{tg} , ϕ_{tg} , δ_{tg} , y_{tg} , z_{react} variables of the scattered particle at the target area using the tracking information from VDCs and beam location. θ_{tg} and ϕ_{tg} are the tangent value of out-of-plane and in-plane angles relative to the LHRs central line, respectively; δ_{tg} is the relative momentum magnitude defined as $(p - p_0)/p_0$ (p_0 is the central momentum of LHRs; i.e. 2.35 GeV); y_{tg} and z_{react} are the horizontal track position in the target coordinate system and the vertex position along the beam, respectively. The optics matrix calibration of LHRs was performed by J. Huang [3].

In practice, the calibration was performed by minimizing the difference between the “true” values and the reconstructed values. The θ_{tg} and ϕ_{tg} angular calibration used the quasi-elastic scattering on the multi-carbon foil target with sieve-slit inserted between target and the entrance of LHRs. The “true” angles are the calculations using

survey numbers about positions of the holes in the sieve-slit and positions of target foils. The momentum calibration used the elastic scattering data and similar setup as that for the angular calibration. The “true” momentum was calculated using the scattered angle and taking into account the corrections due to the collision energy losses in the target material. A momentum resolution of better than 5×10^{-4} was achieved. The vertex calibration used DIS scattering data on the multi-carbon foil target. During the calibration, all reconstructed foil vertex peaks were aligned to their actual positions. An average z_{react} resolution of 6 mm was achieved.

4.1.5 Coincidence Time-of-Flight

In the SIDIS measurement, one particle (DIS electrons) in BigBite spectrometer and one particle (pions, kaons and protons) in LHRS were detected in coincidence with each other. A coincidence time-of-flight (CTOF) is defined as the time difference between the production of two particles at the reaction point as determined by these two spectrometers. In the calculation, the CTOF can be separated in to three parts:

$$CTOF = RF\ Time_{LHRS} - RF\ Time_{BigBite} - \Delta_{trigger\ time}, \quad (4.1.6)$$

where $\Delta_{trigger\ time}$ is the time difference between two single arm triggers which was measured by a TDC with a resolution of 60.2 ps, the $RF\ Time_{spectrometer}$ is defined as the amount of time between the vertex reaction and the single arm trigger. $RF\ Time_{spectrometer}$ includes contributions from:

- TOF: amount of time for a particle to travel from the reaction point to the timing detector.
- Response time of timing detector: including detector response time, cable delay and electronics process time, which is a constant for each scintillator bar.

Figure 4.10. RF Structures for LHRS and BigBite spectrometers after calibration.

- Time difference between timing detector signal and trigger signal. This was measured by high resolution TDCs (60.2 ps resolution for BigBite and 50 ps resolution for LHRS).

In addition, during the experiment E06-010, the beam RF signal, which characterizes the beam bunch time, was recorded in TDCs relative to the single arm trigger signal (noted as t_{RF}). Therefore, when the spectrometer's angle is small, $RF\ Time_{spectrometer} - t_{RF}$, also referred as “RF Structure”, would appear as sharp peaks every 2 ns (beam bunch spacing). By minimizing the width of these peaks, $RF\ Time_{spectrometer}$ was calibrated separately for each arm. The LHRS timing calibration was done by C. Dutta [98], a resolution of 140 ps was achieved. The BigBite timing calibration was done by J. Huang [3], a resolution of 270 ps was achieved. The “RF Structure” distributions for LHRS and BigBite are shown in Figure 4.10.

Once the $RF\ Time_{spectrometer}$ in individual spectrometers is calibrated, one can have the CTOF with the input of $\Delta_{trigger\ time}$. Due to the mass difference, different particles in LHRS with certain momentum will have different TOF, hence, the CTOF should be calculated with a certain particle species assumption whose CTOF will be peaked at zero while other particle species will be off zero. A final CTOF spectrum is shown in Figure 4.11 with pion assumption in LHRS. A resolution of 340 ps was achieved.

Figure 4.11. CTOF spectrum with pion assumption in the LHRS. Pion peak is centered at zero, kaon (proton) mass is larger than that of pion, the TOF is slower than pion with fixed momentum, hence its peak is located in the less-than-zero range.

4.2 Event Selection

4.2.1 Electron Selection from BigBite

Electron selection in BigBite spectrometer is relied on energy deposition in calorimeter (preshower+shower), tracking information from MWDCs and reconstructed momentum from optics.

- Track Quality Cut

The quality of reconstructed track is characterized by the quantity χ^2/N_{dof} where N_{dof} is the number of degree of freedom during track reconstruction and χ^2 is defined as

$$\chi^2 = \sum_i \frac{(x_{reconstruction} - x_{track})^2}{R_i^2}, \quad (4.2.1)$$

where $x_{reconstruction}$ is the MWDC reconstructed position in i^{th} plane and x_{track} is the projection of the track in the i^{th} plane, R_i is the intrinsic resolution used in the

Figure 4.12. χ^2/N_{dof} of all the tracks reconstructed in BigBite MWDC. The red line shows the position where the cut is applied.

tracking software. Figure 4.12 shows the distribution of χ^2/N_{dof} . In the analysis, $\chi^2/N_{dof} < 2.4$ was used.

- Optics Validity Cut

The magnetic field at top and bottom edges of BigBite magnet is weaker than that in the central region. Therefore, optics reconstruction in these regions is not reliable. A two dimensional cut was used for excluding these regions.

- Charge Type Cut

BigBite is an open geometry spectrometer, both negatively and positively charged particles will enter the detectors. Charged particles were distinguished by the bending directions when they were passing through the dipole magnet. During the production run, negatively charged particles were bent up while positively charged particles were bent down. Hence, by looking at the slop dx/dz (BigBite coordinate), one can easily select the negatively charged particles out. Figure 4.13 shows X versus dx/dz two-dimensional plot. Negatively charged particles are selected by a 2-D cut.

- Track Matching Cut

Calorimeter provided the central position of a reconstructed shower cluster. The cluster position should match the projected position on shower. Figure 4.14 shows the

Figure 4.13. X versus dx/dz (BigBite coordinate) plot for particles entering the BigBite acceptance. During the production run, negatively charged particles were bent upwards as marked out by the red line.

Figure 4.14. The distributions of the difference between the projected position from the momentum of the particle and the reconstructed position on shower.

distribution of the difference between the projected position and the reconstructed position on shower. In practice, a momentum dependent $3\text{-}\sigma$ cut was applied to this difference for SIDIS analysis.

- Momentum Acceptance Cut

The Momentum range cut is $0.6 \text{ GeV}/c < p < 2.5 \text{ GeV}/c$.

- Preshower and E/p Cut

PID on the calorimeter system (preshower + shower) was performed by a 2-D cut on the energy deposition in preshower versus the E/p, the ratio of total energy

Figure 4.15. Energy deposition in the preshower versus the E/p . Electrons and hadrons areas are labeled.

deposition in the calorimeter to the reconstructed momentum. Figure 4.15 shows a typical PID plot. Corresponding regions for electrons and hadrons are labeled. The preshower energy is required to be larger than 200 MeV. The E/p cut is performed by a momentum dependent $2.5\text{-}\sigma$ cut in practice.

- Vertex Cut

Both BigBite and LHRS optics reconstructed the interaction vertex (v_z). For SIDIS analysis, the vertex cut was not only applied to the BigBite and the LHRS reconstructed interaction vertex, separately, but also to the difference between these two, since a SIDIS event is supposed to be originated at the same vertex. Figure 4.16 shows the reconstructed vertex from BigBite and LHRS, respectively. Due to the two glass windows at the end of the 40 cm long target, there are two peaks located at -20 cm and 20 cm of the vertex distribution for LHRS. The events from the glass window area were blocked by two collimators at the BigBite side. For inclusive analysis, the vertex cut is $[-0.175 \text{ m}, 0.175 \text{ m}]$ for the LHRS vertex or the BigBite vertex, while the cut can be slightly loose, $[-0.185 \text{ m}, 0.185 \text{ m}]$ for SIDIS analysis, due to the additional requirement on these two vertex. The cut on the difference between the LHRS and

Figure 4.16. Reconstructed interaction vertex from the BigBite and the LHRs, respectively.

(2.35 GeV)	Electron	Pion	Kaon	Proton
CO_2	Y	N	N	N
Aerogel	Y	Y	N	N
RICH	Large ring	Large ring	Middle ring	Small ring
Lead-glass	Large signal	Small signal	Small signal	Small signal

Table 4.1. The responses of 2.35 GeV/c particles in three Cherenkov detectors and the lead-glass detector in the LHRs. “Y” means that the particle will fire the threshold detector, “N” means that the particle will not fire the threshold detector. Signal size in lead-glass detector means energy deposition. In practice, the E/p was used in lead-glass cut.

the BigBite vertex in SIDIS analysis was performed by a momentum dependent $3 - \sigma$ cut.

The contaminations of inclusive hadrons in inclusive electron sample from BigBite were studied in detail in [87].

4.2.2 PID in LHRs

During the experiment E06-010, central momentum of the accepted particles (electron, pion, kaon, proton) in LHRs was fixed to be 2.35 GeV/c. The responses of accepted particles in three Cherenkov detectors and lead-glass detector are summarized in table 4.1.

- Inclusive Pion Selection

Cut type	Applied cuts
Track number cut	L.tr.n==1
Vertex cut	abs(ReactPt_L.z)<0.175
Aerogel cut	L.a1.asum_c>250
CO ₂ Cherenkov cut	L.cer.asum_c<50
Lead-glass cut	(L.prl2.e+L.prl1.e)/1000./ExTgtCor_L.p<0.6
geometry cut	abs(L.prl1.try)<0.24&&L.prl1.trx>-1.5&&L.prl1.trx<1.

Table 4.2. Cuts for pion selection in the LHRS. Lead-glass and CO₂ Cherenkov detectors will reject electrons from hadrons. Aerogel will separate pion from kaon/proton.

Pion cuts are listed in table 4.2. The CO₂ Cherenkov and lead-glass detectors gave a combined electron rejection factor better than $10^4 : 1$. This enables electron contamination to be strongly suppressed to a negligible level. Using a very tight cut on the aerogel detector (>250 , single photo-electron peak is located at 100), the kaons and protons contamination in pions sample is controlled to be less than 1%. In LHRS, only single-track events were included in the analysis. Multi-track events are less than 2% of the single-track events, and the false asymmetry due to this cut is in the order of 1×10^{-4} .

- Inclusive Kaon Selection

Inclusive kaon selection is relied on RICH detector, the cuts are listed in table 4.3. When selecting kaon from RICH, we require that χ^2 probability for kaon assumption is acceptable while rejecting pion and proton assumptions at 10^{-4} level. The algorithm of RICH reconstruction is in section 4.1.4.

RICH cuts includes: 1) R cut, distance between the position of MIP cluster and the projected position from tracking information of VDCs. 2) Number of hits cut, number of hits which were used to reconstruct clusters for each event. If the event is noisy, the number of hits will be large. By applying the cut on this quantity, noisy events were removed. 3) MIP signal size cut, size of MIP signal in RICH readout board for ~ 2.35 GeV/c particles. The MIP position defines the center of a

Cut type	Applied cuts
Track number cut	L.tr.n==1
Vertex cut	abs(ReactPt_L.z)<0.175
Aerogel cut	L.a1.asum_c<10
CO ₂ Cherenkov cut	L.cer.asum_c<10
Lead glass cut	(L.prl2.e+L.prl1.e)/1000./ExTgtCor_L.p<0.6
Geometry cut	abs(L.prl1.try)<0.24&&L.prl1.trx>-1.5&&L.prl1.trx<1.
RICH cut	R < 10 mm L.rich.nhit<150.0 L.rich.mip.chrg>110.0 L.rich.mip.nphot[1]>4.0 L.rich.mip.n_chi2_corrected_phot[1]>4.0 L.rich.mip.chi2_corrected_prob[1]>0.01 (assume kaon) L.rich.mip.chi2_corrected_prob[0]<0.0001 (assume pion) L.rich.mip.chi2_corrected_prob[2]<0.0001 (assume proton) L.rich.mip.noise_cut_success==1 L.rich.mip.angles_corrected[1]>0.625&&L.rich.mip.angles_corrected[1]<0.66

Table 4.3. Cuts for inclusive kaon selection by using the RICH in the LHRs.

Cherenkov ring. 4) Number of photon cut, number of photons which were used to reconstruct Cherenkov ring. 5) χ^2 probability cuts, these are accepting and rejecting levels when reconstructing Cherenkov ring. 6) Noise removal validation cut, a flag describing whether the event passes the acceptable level of one assumption (pion, kaon or proton). 7) Cherenkov light angle cut.

The distribution of the difference between MIP position and projected position is shown in Figure 4.17 and the “R” cut is shown as well. The nhits distribution in RICH for a pion sample selected from the A1 Cherenkov detector is shown in Figure 4.18 and the nhits cut is illustrated as a red line. The small peak at low nhits range is due to efficiency fluctuation of RICH. The charge distribution of MIP cluster for a pion sample is shown in Figure 4.19.

A pion sample were also used to study the χ^2 probability distribution. As discussed in the section 4.1.4, there are three assumptions (pion, kaon and proton) during the reconstruction of each event in RICH. Therefore, each event has three χ^2 proba-

Figure 4.17. The difference between the position of the reconstructed MIP position in the RICH and the projected position on the RICH from tracking information from the VDCs. The “R” cut is illustrated as well.

Figure 4.18. The nhits distribution in the RICH of a pion sample selected from the A1 Cherenkov detector. The cut is illustrated as a red line.

Figure 4.19. The MIP charge distribution in the RICH of a pion sample. The cut is illustrated as a red line.

bility values corresponding to three assumptions. Figure 4.20 shows the calculated χ^2 probabilities of each assumption for a pion sample selected by A1 and gas Cherenkov detectors. One event corresponds to one point in the 3D plot. The axis corresponds to the χ^2 probability of a specific assumption. By looking at the plot, if one requires the accepting level (χ^2 probability is greater than a value) and the rejecting level (χ^2 probability is smaller than a value), a certain amount of pions will be reconstructed as kaons. The chance that a pion is reconstructed to be a proton is small. Figure 4.21 shows the ratio of the number of particles reconstructed as a pion to a kaon (N_π/N_k) by using a pion sample for different accepting and rejecting level. If one requires the accepting level to be 0.01 and rejecting level to be 0.0001, one get a pion to kaon identification ratio at the level of 10:1. Fortunately, the miss-identification was only happened in pion and kaon separation and the A1 Cherenkov will reject pion efficiently by requiring an extremely tight cut on it (< 10 , which is far away from the center of single photo-electron peak).

The kaon selection using RICH was further cross-checked by looking at the CTOF plot with SIDIS data. The contamination study of pion and proton sample in kaon sample was also performed using SIDIS data. Same RICH cuts were applied to SIDIS data and then the CTOF plot was drawn. The yields in pion, kaon and proton windows were calculated, hence the contaminations were estimated. Figure 4.22 (4.23) shows the CTOF plot for K^+ (K^-) selection after using RICH cuts. The proton contamination in K^+ sample was estimated to be less than 3%, the pion contamination in kaon sample was estimated to be less than 2%.

- Inclusive Proton Selection

Proton selection is also relied on RICH. The cuts are listed in table 4.4. Basically, it switches the RICH cuts from supporting kaon assumption to supporting proton assumption. we require that the χ^2 probability for proton assumption is acceptable while rejecting pion and kaon assumptions at 10^{-4} level.

Figure 4.20. The χ^2 probabilities of each assumption (pion, kaon and proton) in the RICH for a pion sample selected by the A1 and gas Cherenkov detectors

Figure 4.21. The ratio of the number of particles reconstructed as a pion to a kaon (N_π/N_k) in the RICH by using a pion sample for different accepting and rejecting level

Figure 4.22. CTOF plot after RICH cuts on SIDIS data to select K^+ . Kaon mass is assumed to reconstruct the CTOF, hence, kaon peak is centered at zero. Particles other than kaon will be off zero.

Figure 4.23. CTOF plot after RICH cuts on SIDIS data to select K^- . Kaon mass is assumed to reconstruct the CTOF, hence, kaon peak is centered at zero. Particles other than kaon will be off zero.

cut type	applied cuts
Track number cut	<code>L.tr.n==1</code>
Vertex cut	<code>abs(ReactPt_L.z)<0.175</code>
Aerogel cut	<code>L.a1.asum_c<10</code>
CO_2 Cherenkov cut	<code>L.cer.asum_c<10</code>
Lead glass cut	<code>(L.prl2.e+L.prl1.e)/1000./ExTgtCor_L.p<0.6</code>
Geometry cut	<code>abs(L.prl1.try)<0.24&&L.prl1.trx>-1.5&&L.prl1.trx<1.</code>
RICH cut	<code>R < 10 mm</code> <code>L.rich.nhit<150.0</code> <code>L.rich.mip.chrg>110.0</code> <code>L.rich.mip.nphot[2]>4.0</code> <code>L.rich.mip.n_chi2_corrected_phot[2]>4.0</code> <code>L.rich.mip.chi2_corrected_prob[2]>0.01 (assume proton)</code> <code>L.rich.mip.chi2_corrected_prob[0]<0.0001 (assume pion)</code> <code>L.rich.mip.chi2_corrected_prob[1]<0.0001 (assume kaon)</code> <code>L.rich.mip.noise_cut_success==1</code> <code>L.rich.mip.angles_corrected[2]>0.55&&L.rich.mip.angles_corrected[2]<0.6</code>

Table 4.4. Cuts for inclusive proton selection by using the RICH in the LHRS.

Figure 4.24. ${}^3\text{He}(e, e'h^+)X$ coincidence timing spectrum after a cut on the aerogel detector to remove pions, where h represents detected hadron. The kaon selection cuts are shown as the two vertical lines. The top right sub-plot shows only K^+ and π^+ peaks in a relatively small CTOF range.

4.2.3 (e' , kaon) SIDIS Event Selection

SIDIS events selection requires the cuts on both spectrometers and the cuts on CTOF. Due to the low efficiency of RICH detector and the good PID separation by CTOF, RICH detector is not used in the SIDIS event selection. For (e' , $\pi^\pm$) selection, please refer to [3, 87] for the details. The CTOF plot after all the cuts is shown in Figure 4.24. The two vertical lines (-1 ns, 1 ns) are the cuts on CTOF to select SIDIS kaons.

Since RICH detector was not used in the SIDIS analysis and pion/kaon yield ratio is about 100:1, the A1 cut is critical in the (e' , kaon) analysis. A loose cut on A1 will increase pion contamination in kaon sample, while a tight cut will decrease the statistics for the kaon sample. The A1 cut was optimized by looking at the quadrature combination of statistical uncertainty and systematic uncertainty due to contaminations. Figure 4.25 shows the combined uncertainties (systematic uncertainty, statistical uncertainty) versus A1 cuts. To be conservative, the final A1 cut was determined to be $L.a1.asum\_c < 60$ for SIDIS kaon analysis. Cherenkov cut was studied in the same way, it has very little effect on the results, so it was set to be $L.cer.asum\_c < 60.0$.

Figure 4.25. First plot: Quadrature combination of statistical uncertainty and systematic uncertainty due to contaminations versus the A1 cut. Second plot: Systematic uncertainty due to contaminations versus A1 cut. Third plot: Statistical uncertainty versus A1 cut. Data are from HRS positive polarity run.

After all the good PID cuts on both LHRS and BigBite, cuts on kinematic variables were applied to make sure that the events were in the DIS region, the cuts were: 1) four-momentum-transfer squared $Q^2 > 1 \text{ GeV}^2$, 2) virtual photon-nucleon invariant mass $W > 2.3 \text{ GeV}$, 3) the missing mass of undetected final-state particles $W' > 1.6 \text{ GeV}$. The kinematics coverage for K^+ is shown in Figure 4.26. The azimuthal angular coverage for ϕ_h and ϕ_S is shown in Figure 4.27. The total number of accepted SIDIS events were about 10k and 2k for K^+ and K^- , respectively. Due to the low statistics of the K^- sample, the data were binned in one kinematical bin, while for K^+ , the data were binned in four bins of x_{bj} . The data were analyzed by using an azimuthally unbinned maximum likelihood estimator to extract the Collins and Sivers moments, as will be described in Section 4.3.

Contaminations in the SIDIS kaon sample consist of two parts, one is the contamination for electrons (BigBite side), the other is the contamination for kaons (HRS side).

Electron selection is relied on the 2D cut on preshower versus E/p (Figure 4.15). Contamination of hadron (mostly pion) in the electron sample was estimated by looking at the preshower energy deposition. Figure 4.28 shows a spectrum on preshower

Figure 4.26. Correlation between x_{bj} and kinematics variables (Q^2 , P_t , z) for K^+ , where $x_{bj} = \frac{Q^2}{2P \cdot q}$, $P_t = \sqrt{\vec{P}_h^2 - (\vec{q} \cdot \vec{P}_h)^2}$, $z = \frac{P \cdot P_h}{P \cdot q}$, P is the four-momentum of the initial nucleon, q is the four-momentum of the virtual photon, P_h is the four-momentum of the detected hadron.

Figure 4.27. Azimuthal angular coverage of ϕ_h versus ϕ_S for the kaon sample.

energy deposition after all the PID cuts to get the kaon SIDIS sample. The hadron peak is fitted by a Gaussian distribution convoluted with a Landau distribution. The electron peak is fitted by a Gaussian distribution. Contamination of hadron in the electron sample was determined by

$$f_{cont} = \frac{\int_{400}^{\infty} f_{gt}(x) dx}{N_{>400channels}^{ele}}. \quad (4.2.2)$$

The hadron contamination in electron sample was estimated to be less than 1%. Another contamination in the electron sample arises from photon-induced charge-symmetric $e^{\pm}$ pairs, of which the e^{-} was detected in the BigBite. The yield of $(e^{+}, K^{\pm})$ coincidences was measured directly by reversing the magnetic field of BigBite. Since the e^{-} and e^{+} are from the same photon, one would expect that the yield of photon-induced $(e^{-}, K^{\pm})$ coincidences in BigBite negative polarity run is the same as that of $(e^{+}, K^{\pm})$ coincidences in BigBite positive polarity run. The contamination was estimated to be $14 \pm 7\%$ for K^{-} detection. Hardly any events were observed in the high x bins for K^{+} detection from BigBite positive polarity runs, which indicated that the contamination in these bins was small. To be conservative, the contaminations were given by a limit in these bins with assumption that the contamination decreases linearly through four bins. The photon-induced electron contamination for K^{+} was determined to be $18.6 \pm 8.3\%$, $< 10\%$, $< 5\%$, $< 3\%$, respectively for the four bins.

Contamination for the kaon sample by using CTOF consists of two parts: pion leakage in the kaon sample and the random coincidence in the (e' , kaon) sample. To estimate the contaminations, CTOF plot was drawn with A1 cut to select pion ($A1 > 200$), yield $Y_{[-1,1]}^1$ ($Y_{[1,3]}^1$) in $[-1\text{ns}, 1\text{ns}]$ ($[1\text{ns}, 3\text{ns}]$) window in the CTOF spectrum was calculated, random yield was also calculated by selecting certain areas away from pion, kaon and proton peaks, both up stream and down stream of CTOF spectrum, and then scaled into the yield Y_{random}^1 in a 2 ns window. The same process was done

Figure 4.28. Energy deposition in preshower of BigBite spectrometer. The hadron peak is fitted by a Gaussian distribution convoluted with a Landau distribution. The fitted result is extrapolated into the electron region ($> 200\text{MeV}$) to estimate the hadron contamination in electron sample.

with A1 cut to select kaon ($A1 < 60$), yield $Y_{[-1,1]}^2$, $Y_{[1,3]}^2$ and Y_{random}^2 were obtained.

The pion contamination in the kaon sample was estimated by

$$Cont_{pion \text{ in kaon}} = \frac{\frac{Y_{[-1,1]}^1 - Y_{random}^1}{Y_{[1,3]}^1 - Y_{random}^1} \times (Y_{[1,3]}^2 - Y_{random}^2)}{Y_{[-1,1]}^2}, \quad (4.2.3)$$

and random coincidence contamination was estimated by

$$Cont_{random \text{ in kaon}} = \frac{Y_{random}^2}{Y_{[-1,1]}^2}. \quad (4.2.4)$$

The contamination of π^+ (π^-) in the K^+ (K^-) sample was less than 2% (5%). The random coincidence contamination in the K^+ (K^-) sample was less than 4% (1%). Proton peak is far away from the kaon peak, its contamination in the kaon sample is negligible.

4.3 Analysis Method

In the experiment E06-010, two independent methods were used for obtaining asymmetries. The first one is called “local pair” method, the second method is maximum likelihood estimation (MLE). In this section, both methods will be introduced.

4.3.1 The Local Pair Method

In order to get the Collins and Sivers moments, target single spin asymmetry from ^3He was measured. The asymmetry is defined as:

$$A_{UT}^{raw} = \frac{1}{P_{target}} \frac{Y_+ - Y_-}{Y_+ + Y_-} = \frac{1}{P_{target}} \frac{\frac{N_+}{C_+L_+} - \frac{N_-}{C_-L_-}}{\frac{N_+}{C_+L_+} + \frac{N_-}{C_-L_-}}, \quad (4.3.1)$$

where P_{target} is the target polarization, $\pm$ means two opposite target spin directions, N is the number of events in each target spin state, C is the beam charge for each target spin state, and L represents the possible corrections including live time, target density etc.. In production target cell, as will be described in the following section, a small amount of N_2 gas was in the target cell to reduce the depolarization, which diluted the measured ^3He asymmetry and should be corrected by N_2 dilution factor f . Therefore, the raw asymmetry was corrected by:

$$A_{UT}^{^3\text{He}} = A_{UT}^{raw} / f, \quad (4.3.2)$$

After the raw asymmetries were obtained for each kinematic bin ⁵. The asymmetries were then fitted using azimuthal angular dependence to obtain the Collins and Sivers moments by

$$A_{UT}^{^3\text{He}}(\phi_h, \phi_S) = A^{Collins} \sin(\phi_h + \phi_S) + A^{Sivers} \sin(\phi_h - \phi_S). \quad (4.3.3)$$

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⁵Data were binned on kinematic variables, such as x_{bj} . In each kinematic bin, the raw asymmetries were calculated in different ϕ_h and ϕ_S two dimensional bins

In practice, as described in section 3.5, the target spin direction was reversed every 20 minutes in transverse and vertical directions to reduce systematic uncertainties. Therefore, there are thousands of target $+/-$ pairs, which are called “local pairs”. The asymmetries were combined by using the yields from two adjacent target spin states:

$$A_{raw} = \frac{\sum a_i \cdot (Y_{i+} - Y_{i-})/P_i}{\sum a_i \cdot (Y_{i+} + Y_{i-})}. \quad (4.3.4)$$

Here P_i is the target polarization for the i_{th} “local pair”, and

$$a_i = \frac{b_i}{Y_{i+} + Y_{i-}} \sim \frac{1}{\frac{1}{L_+} + \frac{1}{L_-}}. \quad (4.3.5)$$

In real analysis, each target spin state was further truncated into two, and each of these two would be half of one local pair ⁶ [87].

4.3.2 The Maximum Likelihood Method

In MLE, the Collins and Sivers moments can be directly extracted without measuring target single spin asymmetries. Furthermore, the corrections associated with beam charge, data acquisition livetime, target polarization, target density effects etc. can be realized simultaneously while the Collins and Sivers moments were extracted.

In the case that electron beam is not polarized and target is polarized, SIDIS yield can be expressed as:

$$yield(\phi_h, \phi_S) = \rho \cdot \sigma \cdot a_{\pm}(\phi_h, \phi_S) \left(1 + P \sum_{j=1}^2 \epsilon_j A_j(\phi_h, \phi_S)\right), \quad (4.3.6)$$

where ρ is the target density, σ is the unpolarized cross section, $a_{\pm}(\phi_h, \phi_S)$ is the acceptance for target spin state $\pm$, $A_j(\phi_h, \phi_S)$ is the j^{th} azimuthal angular modulation, $\sin(\phi_h + \phi_S)$ or $\sin(\phi_h - \phi_S)$, P is the target polarization, and ϵ_j is the amplitude of

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⁶This happens when the statistics is high.

each modulation, the Collins moment $A_{UT}^{sin(\phi_h+\phi_S)}$ or Sivers moment $A_{UT}^{sin(\phi_h-\phi_S)}$. The definition of ϕ_h and ϕ_S follows the Trento Convention [99]. Our goal is to extract ϵ_j as well as their uncertainties and correlations.

For a single SIDIS event, it has the probability of $yield(\phi_h, \phi_S)/Norm(\epsilon_j)$ to be recorded, where $Norm(\epsilon_j)$ is the normalization of the SIDIS yield:

$$\begin{aligned}
Norm(\epsilon_j) &= \int_0^{2\pi} \int_0^{2\pi} yield(\phi_h, \phi_S) d\phi_h d\phi_S \\
&= \sum_{V\pm, T\pm} \int_0^{2\pi} \int_0^{2\pi} \rho \cdot \sigma \cdot a_{\pm}(\phi_h, \phi_S) [1 + P \cdot A_{UT}^{sin(\phi_h+\phi_S)} sin(\phi_h + \phi_S) + P \cdot A_{UT}^{sin(\phi_h-\phi_S)} sin(\phi_h - \phi_S)] d\phi_h d\phi_S \\
&= \int_0^{2\pi} \int_0^{2\pi} \rho_{V+} \cdot \sigma \cdot a_{V+}(\phi_h, \phi_S) [1 + P_{V+} \cdot A_{UT}^{sin(\phi_h+\phi_S)} sin(\phi_h + \phi_S) + P_{V+} \cdot A_{UT}^{sin(\phi_h-\phi_S)} sin(\phi_h - \phi_S)] d\phi_h d\phi_S \\
&\quad + \int_0^{2\pi} \int_0^{2\pi} \rho_{V-} \cdot \sigma \cdot a_{V-}(\phi_h, \phi_S) [1 + P_{V-} \cdot A_{UT}^{sin(\phi_h+\phi_S)} sin(\phi_h + \phi_S) + P_{V-} \cdot A_{UT}^{sin(\phi_h-\phi_S)} sin(\phi_h - \phi_S)] d\phi_h d\phi_S \\
&\quad + \int_0^{2\pi} \int_0^{2\pi} \rho_{T+} \cdot \sigma \cdot a_{T+}(\phi_h, \phi_S) [1 + P_{T+} \cdot A_{UT}^{sin(\phi_h+\phi_S)} sin(\phi_h + \phi_S) + P_{T+} \cdot A_{UT}^{sin(\phi_h-\phi_S)} sin(\phi_h - \phi_S)] d\phi_h d\phi_S \\
&\quad + \int_0^{2\pi} \int_0^{2\pi} \rho_{T-} \cdot \sigma \cdot a_{T-}(\phi_h, \phi_S) [1 + P_{T-} \cdot A_{UT}^{sin(\phi_h+\phi_S)} sin(\phi_h + \phi_S) + P_{T-} \cdot A_{UT}^{sin(\phi_h-\phi_S)} sin(\phi_h - \phi_S)] d\phi_h d\phi_S.
\end{aligned} \tag{4.3.7}$$

Due to the exact spin direction flip, we have

$$a_+(\phi_h, \phi_S) = a_-(\phi_h, \phi_S + \pi), \tag{4.3.8}$$

$$\phi_S^- = \phi_S^+ + \pi. \tag{4.3.9}$$

Hence, the normalization factor can be written as:

$$\begin{aligned}
Norm(\epsilon_j) &= (\rho_{V+} \cdot \sigma + \rho_{V-} \cdot \sigma) \int \int a_{V+}(\phi_h, \phi_S) d\phi_h d\phi_S \\
&+ (\rho_{T+} \cdot \sigma + \rho_{T-} \cdot \sigma) \int \int a_{T+}(\phi_h, \phi_S) d\phi_h d\phi_S \\
&+ A_{UT}^{sin(\phi_h + \phi_S)} [(\rho_{V+} \cdot \sigma \cdot P_{V+} - \rho_{V-} \cdot \sigma \cdot P_{V-}) \int \int a_{V+}(\phi_h, \phi_S) sin(\phi_h + \phi_S) d\phi_h d\phi_S] \\
&+ A_{UT}^{sin(\phi_h + \phi_S)} [(\rho_{T+} \cdot \sigma \cdot P_{T+} - \rho_{T-} \cdot \sigma \cdot P_{T-}) \int \int a_{T+}(\phi_h, \phi_S) sin(\phi_h + \phi_S) d\phi_h d\phi_S] \\
&+ A_{UT}^{sin(\phi_h - \phi_S)} [(\rho_{V+} \cdot \sigma \cdot P_{V+} - \rho_{V-} \cdot \sigma \cdot P_{V-}) \int \int a_{V+}(\phi_h, \phi_S) sin(\phi_h - \phi_S) d\phi_h d\phi_S] \\
&+ A_{UT}^{sin(\phi_h - \phi_S)} [(\rho_{T+} \cdot \sigma \cdot P_{T+} - \rho_{T-} \cdot \sigma \cdot P_{T-}) \int \int a_{T+}(\phi_h, \phi_S) sin(\phi_h - \phi_S) d\phi_h d\phi_S].
\end{aligned} \tag{4.3.10}$$

Therefore, the log-likelihood function for all the events is

$$\begin{aligned}
L &= \log \prod_{all \ events} (yield(\phi_h, \phi_S) / Norm(\epsilon_j)) \\
&= \sum_{all \ events} [\log(yield) - \log(Norm(\epsilon_j))] \\
&= \sum_{all \ events} [\log(\rho \cdot \sigma \cdot a_{V\pm}(T_{\pm})) + \log(1 + P_{\pm} A_{UT}^{sin(\phi_h + \phi_S)} sin(\phi_h + \phi_S) + P_{\pm} A_{UT}^{sin(\phi_h - \phi_S)} sin(\phi_h - \phi_S)) - \log(Norm)],
\end{aligned} \tag{4.3.11}$$

where $\sum_{all \ events}$ is summing over all events in the experiment E06-010 with target transversely and vertically polarized. In order to extract the Collins and Sivers moments, maximum probability point for log-likelihood should be found:

$$\begin{aligned}
\frac{\partial L}{\partial A_{UT}^{sin(\phi_h + \phi_S)}} &= 0, \\
\frac{\partial L}{\partial A_{UT}^{sin(\phi_h - \phi_S)}} &= 0.
\end{aligned} \tag{4.3.12}$$

To calculate $\frac{\partial L}{\partial \epsilon_j}$ in the equation 4.3.12, one needs to calculate the following term (using the Collins moment $A_{UT}^{sin(\phi_h + \phi_S)}$ as an example):

$$\frac{\frac{\partial Norm}{\partial A_{UT}^{sin(\phi_h + \phi_S)}}}{Norm} = \frac{(\rho_{V+} \cdot \sigma \cdot P_{V+} - \rho_{V-} \cdot \sigma \cdot P_{V-}) \int \int a_{V+}(\phi_h, \phi_S) sin(\phi_h + \phi_S) d\phi_h d\phi_S + (\rho_{T+} \cdot \sigma \cdot P_{T+} - \rho_{T-} \cdot \sigma \cdot P_{T-}) \int \int a_{T+}(\phi_h, \phi_S) sin(\phi_h + \phi_S) d\phi_h d\phi_S}{Norm}. \tag{4.3.13}$$

We define two terms, $A_{cp,\oplus}$ and $A_{cp,\ominus}$, whose physical meaning will be explained later:

$$\begin{aligned}
A_{cp,\oplus} &= \frac{(\rho_{V+} \cdot \sigma \cdot P_{V+} - \rho_{V-} \cdot \sigma \cdot P_{V-}) \int \int a_{V+}(\phi_h, \phi_S) \sin(\phi_h + \phi_S) d\phi_h d\phi_S + (\rho_{T+} \cdot \sigma \cdot P_{T+} - \rho_{T-} \cdot \sigma \cdot P_{T-}) \int \int a_{T+}(\phi_h, \phi_S) \sin(\phi_h + \phi_S) d\phi_h d\phi_S}{(\rho_{V+} \cdot \sigma + \rho_{V-} \cdot \sigma) \int \int a_{V+}(\phi_h, \phi_S) d\phi_h d\phi_S + (\rho_{T+} \cdot \sigma + \rho_{T-} \cdot \sigma) \int \int a_{T+}(\phi_h, \phi_S) d\phi_h d\phi_S}, \\
A_{cp,\ominus} &= \frac{(\rho_{V+} \cdot \sigma \cdot P_{V+} - \rho_{V-} \cdot \sigma \cdot P_{V-}) \int \int a_{V+}(\phi_h, \phi_S) \sin(\phi_h - \phi_S) d\phi_h d\phi_S + (\rho_{T+} \cdot \sigma \cdot P_{T+} - \rho_{T-} \cdot \sigma \cdot P_{T-}) \int \int a_{T+}(\phi_h, \phi_S) \sin(\phi_h - \phi_S) d\phi_h d\phi_S}{(\rho_{V+} \cdot \sigma + \rho_{V-} \cdot \sigma) \int \int a_{V+}(\phi_h, \phi_S) d\phi_h d\phi_S + (\rho_{T+} \cdot \sigma + \rho_{T-} \cdot \sigma) \int \int a_{T+}(\phi_h, \phi_S) d\phi_h d\phi_S}.
\end{aligned} \tag{4.3.14}$$

Hence,

$$\begin{aligned}
\frac{\frac{\partial Norm}{\partial A_{UT}^{sin(\phi_h + \phi_S)}}}{Norm} &= \frac{A_{cp,\oplus}}{1 + A_{UT}^{sin(\phi_h + \phi_S)} A_{cp,\oplus} + A_{UT}^{sin(\phi_h - \phi_S)} A_{cp,\ominus}}, \\
\frac{\frac{\partial Norm}{\partial A_{UT}^{sin(\phi_h - \phi_S)}}}{Norm} &= \frac{A_{cp,\ominus}}{1 + A_{UT}^{sin(\phi_h + \phi_S)} A_{cp,\oplus} + A_{UT}^{sin(\phi_h - \phi_S)} A_{cp,\ominus}}.
\end{aligned} \tag{4.3.15}$$

Equation 4.3.12 can be written as:

$$\begin{aligned}
&\frac{\partial L}{\partial A_{UT}^{sin(\phi_h + \phi_S)}} \\
&= \sum_{all \ events} \left[\frac{P_{\pm} \cdot \sin(\phi_h + \phi_S)}{1 + P_{\pm} A_{UT}^{sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) + P_{\pm} A_{UT}^{sin(\phi_h - \phi_S)} \sin(\phi_h - \phi_S)} - \frac{A_{cp,\oplus}}{1 + A_{UT}^{sin(\phi_h + \phi_S)} A_{cp,\oplus} + A_{UT}^{sin(\phi_h - \phi_S)} A_{cp,\ominus}} \right] \\
&= 0, \\
&\frac{\partial L}{\partial A_{UT}^{sin(\phi_h - \phi_S)}} \\
&= \sum_{all \ events} \left[\frac{P_{\pm} \cdot \sin(\phi_h - \phi_S)}{1 + P_{\pm} A_{UT}^{sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) + P_{\pm} A_{UT}^{sin(\phi_h - \phi_S)} \sin(\phi_h - \phi_S)} - \frac{A_{cp,\ominus}}{1 + A_{UT}^{sin(\phi_h + \phi_S)} A_{cp,\oplus} + A_{UT}^{sin(\phi_h - \phi_S)} A_{cp,\ominus}} \right] \\
&= 0.
\end{aligned} \tag{4.3.16}$$

It is hard to obtain the explicit expression to calculate the Collins and Sivers moments from the equation 4.3.16. However, in case that asymmetries are small, one can do the expansion:

$$\frac{1}{1+x} = 1 - x + O(x^2). \quad (4.3.17)$$

Therefore, the equation 4.3.16 can be written as:

$$\begin{aligned} & A_{UT}^{sin(\phi_h+\phi_S)} [NA_{cp,\oplus}A_{cp,\oplus} - \sum_{all\ events} P^2 sin(\phi_h + \phi_S) sin(\phi_h + \phi_S)] \\ & + A_{UT}^{sin(\phi_h-\phi_S)} [NA_{cp,\oplus}A_{cp,\ominus} - \sum_{all\ events} P^2 sin(\phi_h + \phi_S) sin(\phi_h - \phi_S)] \\ & = NA_{cp,\oplus} - \sum_{all\ events} P sin(\phi_h + \phi_S), \end{aligned} \quad (4.3.18)$$

$$\begin{aligned} & A_{UT}^{sin(\phi_h+\phi_S)} [NA_{cp,\oplus}A_{cp,\ominus} - \sum_{all\ events} P^2 sin(\phi_h - \phi_S) sin(\phi_h + \phi_S)] \\ & + A_{UT}^{sin(\phi_h-\phi_S)} [NA_{cp,\ominus}A_{cp,\ominus} - \sum_{all\ events} P^2 sin(\phi_h - \phi_S) sin(\phi_h - \phi_S)] \\ & = NA_{cp,\ominus} - \sum_{all\ events} P sin(\phi_h - \phi_S). \end{aligned}$$

Define the folowing matrix:

$$F = \begin{pmatrix} \sum_{all\ events} P^2 sin(\phi_h + \phi_S) sin(\phi_h + \phi_S) - NA_{cp,\oplus}A_{cp,\oplus} & \sum_{all\ events} P^2 sin(\phi_h + \phi_S) sin(\phi_h - \phi_S) - NA_{cp,\oplus}A_{cp,\ominus} \\ \sum_{all\ events} P^2 sin(\phi_h - \phi_S) sin(\phi_h + \phi_S) - NA_{cp,\oplus}A_{cp,\ominus} & \sum_{all\ events} P^2 sin(\phi_h - \phi_S) sin(\phi_h - \phi_S) - NA_{cp,\ominus}A_{cp,\ominus} \end{pmatrix} \quad (4.3.19)$$

and

$$B = \begin{pmatrix} \sum_{all \ events} P \sin(\phi_h + \phi_S) - NA_{cp,\oplus} \\ \sum_{all \ events} P \sin(\phi_h - \phi_S) - NA_{cp,\ominus} \end{pmatrix} \quad (4.3.20)$$

and

$$\epsilon = \begin{pmatrix} A_{UT}^{\sin(\phi_h + \phi_S)} \\ A_{UT}^{\sin(\phi_h - \phi_S)} \end{pmatrix}. \quad (4.3.21)$$

Hence, the equation 4.3.18 becomes:

$$F\epsilon = B. \quad (4.3.22)$$

The results of the Collins and Sivers moments can be then obtained by:

$$\epsilon = F^{-1}B. \quad (4.3.23)$$

To get the error matrix, the second derivative of the log-likelihood was performed:

$$\begin{aligned} -\frac{\partial^2 L}{\partial A_{UT}^{\sin(\phi_h + \phi_S)} \partial A_{UT}^{\sin(\phi_h + \phi_S)}} &= \sum_{all \ events} P^2 \sin(\phi_h + \phi_S) \sin(\phi_h + \phi_S) - NA_{cp,\oplus} A_{cp,\oplus}, \\ -\frac{\partial^2 L}{\partial A_{UT}^{\sin(\phi_h + \phi_S)} \partial A_{UT}^{\sin(\phi_h - \phi_S)}} &= \sum_{all \ events} P^2 \sin(\phi_h + \phi_S) \sin(\phi_h - \phi_S) - NA_{cp,\oplus} A_{cp,\ominus}, \\ -\frac{\partial^2 L}{\partial A_{UT}^{\sin(\phi_h - \phi_S)} \partial A_{UT}^{\sin(\phi_h - \phi_S)}} &= \sum_{all \ events} P^2 \sin(\phi_h - \phi_S) \sin(\phi_h - \phi_S) - NA_{cp,\ominus} A_{cp,\ominus}. \end{aligned} \quad (4.3.24)$$

Hence, the error matrix M can be written as:

$$M^{-1} = F, \quad or \quad M = F^{-1}, \quad (4.3.25)$$

in which M is:

$$M = \begin{pmatrix} \sigma_{A_{UT}^{Collins}}^2 & cov(A_{UT}^{Collins}, A_{UT}^{Sivers}) \\ cov(A_{UT}^{Collins}, A_{UT}^{Sivers}) & \sigma_{A_{UT}^{Sivers}}^2 \end{pmatrix}. \quad (4.3.26)$$

- About $A_{cp,\oplus}$ and $A_{cp,\ominus}$

The defined $A_{cp,\oplus}$ and $A_{cp,\ominus}$ are correction factors associated to the luminosity fluctuation between different target spin states, non-symmetric acceptance of the spectrometer etc. In addition, if there is no asymmetry from luminosity between two target spin states, $A_{cp,\oplus(\ominus)}$ will be zero, which means acceptance will not affect the final results. Since the charge asymmetry in the experiment is well controlled (<1% for SSA and PPM level for DSA), acceptance effect is suppressed by the same ratio. In practice, to estimate the correction factor $A_{cp,\oplus(\ominus)}$, it is resonable to assume

$$\int \int a_{V+}(\phi_h, \phi_S) = \int \int a_{T+}(\phi_h, \phi_S). \quad (4.3.27)$$

Therefore,

$$A_{cp,\oplus(\ominus)} = \frac{(\rho_{V+} \cdot \sigma \cdot P_{V+} - \rho_{V-} \cdot \sigma \cdot P_{V-}) \frac{\int \int a_{V+}(\phi_h, \phi_S) \sin(\phi_h \pm \phi_S) d\phi_h d\phi_S}{\int \int a_{V+}(\phi_h, \phi_S)} + (\rho_{T+} \cdot \sigma \cdot P_{T+} - \rho_{T-} \cdot \sigma \cdot P_{T-}) \frac{\int \int a_{T+}(\phi_h, \phi_S) \sin(\phi_h \pm \phi_S) d\phi_h d\phi_S}{a_{T+}(\phi_h, \phi_S)}}{(\rho_{V+} \cdot \sigma + \rho_{V-} \cdot \sigma) + (\rho_{T+} \cdot \sigma + \rho_{T-} \cdot \sigma)}, \quad (4.3.28)$$

in which

$$\begin{aligned} & \frac{\int \int a_{V+(T+)}(\phi_h, \phi_S) \sin(\phi_h \pm \phi_S) d\phi_h d\phi_S}{\int \int a_{V+(T+)}(\phi_h, \phi_S)} = \\ & \frac{\int \int yield_{V+(T+)} \sin(\phi_h \pm \phi_S) d\phi_h d\phi_S - \int \int yield_{V-(T-)} \sin(\phi_h \pm \phi_S) d\phi_h d\phi_S}{\int \int yield_{V+(T+)} d\phi_h d\phi_S + \int \int yield_{V-(T-)} d\phi_h d\phi_S} = \\ & \frac{\sum_{all \ events} \sin(\phi_h \pm \phi_S) / \rho_{V+(T+)} - \sum_{all \ events} \sin(\phi_h \pm \phi_S) / \rho_{V-(T-)}}{N_{V+(T+)}/\rho_{V+(T+)} + N_{V-(T-)} / \rho_{V-(T-)}}. \end{aligned} \quad (4.3.29)$$

- Numerical calculations without $\frac{1}{1+x}$ expansion

One can also extract the Collins and Sivers moments without doing $\frac{1}{1+x}$ expansion.

In this method, named as numerical calculation, log-likelihood is calculated on a

Collins and Sivers grid with certain resolution, the maximum log-likelihood point yields the final Collins and Sivers moments, the $L_{max} - \frac{1}{2}$ curve yields the $1-\sigma$ contour.

As usual, the Log-likelihood is defined as:

$$\begin{aligned}
L &= \log \prod_{all\ events} (yield(\phi_h, \phi_S) / Norm(\epsilon_j)) \\
&= \sum_{events} \log(\rho \sigma a_{V_{\pm}(T_{\pm})}) + \sum_{events} \log(1 + P A_{UT}^{sin(\phi_h + \phi_S)} sin(\phi_h + \phi_S) + P A_{UT}^{sin(\phi_h - \phi_S)} sin(\phi_h - \phi_S)) - \sum_{events} \log(Norm) \\
&= \sum_{events} \log(1 + P A_{UT}^{sin(\phi_h + \phi_S)} sin(\phi_h + \phi_S) + P A_{UT}^{sin(\phi_h - \phi_S)} sin(\phi_h - \phi_S)) - N \cdot \log(1 + A_{UT}^{sin(\phi_h + \phi_S)} A_{cp, \oplus} + A_{UT}^{sin(\phi_h - \phi_S)} A_{cp, \ominus}) + Constant.
\end{aligned} \tag{4.3.30}$$

For each point on the $(A_{UT}^{sin(\phi_h + \phi_S)}, A_{UT}^{sin(\phi_h - \phi_S)})$ grid, L was calculated by using all the good events. After the calculations for all points on the grid, the maximum L point was found and the $1 - \sigma$ contour was found by identifying the $L_{max} - \frac{1}{2}$ points on the grid.

- MLE for multi-term fit

The formulas derived above are for the Collins and Sivers two-term fit. In reality, there are more azimuthal angular dependent terms. In the case of single spin asymmetries on a transversely polarized target, there are five terms: $A_{UT}^{sin(\phi_h + \phi_S)}$, $A_{UT}^{sin(\phi_h - \phi_S)}$, $A_{UT}^{sin(3\phi_h - \phi_S)}$ in leading twist (twist-2), $sin(\phi_S)$ and $sin(2\phi_h - \phi_S)$ in twist-3. With enough data and large azimuthal angular coverage, one can do five-term fit simultaneously. In this case, the F matrix should be expanded to 5 dimensions:

$$F = \begin{pmatrix} \sum_{all\ events} P^2 A_1 A_1 - N A_{cp,1} A_{cp,1} & \sum_{all\ events} P^2 A_1 A_2 - N A_{cp,1} A_{cp,2} & \dots \\ \sum_{all\ events} P^2 A_2 A_1 - N A_{cp,2} A_{cp,1} & \sum_{all\ events} P^2 A_2 A_2 - N A_{cp,2} A_{cp,2} & \dots \\ \dots & \dots & \dots \end{pmatrix}. \tag{4.3.31}$$

B matrix should be expanded to:

$$B = \begin{pmatrix} \sum_{all \ events} PA_1 - NA_{cp,1} \\ \sum_{all \ events} PA_2 - NA_{cp,2} \\ \sum_{all \ events} PA_3 - NA_{cp,3} \\ \sum_{all \ events} PA_4 - NA_{cp,4} \\ \sum_{all \ events} PA_5 - NA_{cp,5} \end{pmatrix}. \quad (4.3.32)$$

ϵ matrix should be expanded to:

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \end{pmatrix}. \quad (4.3.33)$$

where A_i is the azimuthal angle (eg, $\sin(\phi_h + \phi_S)$), ϵ_i is the moments to be extracted (eg, $A_{UT}^{\sin(\phi_h + \phi_S)}$).

- Obtaining yield asymmetry by using MLE

MLE can also be used to calculate the yield asymmetry as what was done in “local pair” analysis. In local pair analysis, the asymmetry is defined as (using SSA as an example):

$$A = \frac{1}{P} \frac{Y^+ - Y^-}{Y^+ + Y^-}. \quad (4.3.34)$$

Yield in each spin state was calculated and then combined together to extract the final asymmetry. Similarly, once event counts are counted for each spin state, Log-likelihood can be obtained by using Poisson distribution:

$$L(\epsilon) = \log \prod_{Spin \ states} \frac{(\sigma \rho_{i\pm} (1 \pm P_{i\pm}^{target} \epsilon))^{N_{i\pm}}}{N_{i\pm}!} \exp(-\sigma \rho_{i\pm} (1 \pm P_{i\pm}^{target} \epsilon)), \quad (4.3.35)$$

where σ is the average cross-section for spin + and - states, ρ includes luminosity, livetime and other correction factors, N is the good event counts in a spin state, P is the polarization ⁷, ε is the yield asymmetry to be extracted. By doing

$$\frac{\partial L}{\partial \varepsilon} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \sigma} = 0, \quad (4.3.36)$$

one can then extract the yield asymmetry ε .

- Check MLE at kaon statistical level using Monte Carlo (MC)

As the kaon SIDIS statistics is low, the analysis was done by the unbinned MLE method described above. Before the method was applied to the real data, it was checked by using MC which produced the events with similar statistics as the real experiment and also similar azimuthal angular acceptance. The events were generated with fixed Collins and Sivers inputs, and then the MLE was used to extract the Collins and Sivers moments as well as their uncertainties. One such process was called one run in MC. In reality, more than 50000 runs were done. Therefore, more than 50000 Collins and Sivers values (and their uncertainties) were estimated by the MLE. Figure 4.29 shows the distribution of estimated Collins and Sivers as well as their uncertainties for 50000 MC runs. The input Collins (Sivers) value for the event generation is -0.16 (-0.175). Taking Collins as an example (top two plots in Figure 4.29), the mean value (-0.1604) and sigma (0.05525) in the top-left plot reveal the “true” estimation from the MLE and they are consistent with our input (-0.16), which indicates the MLE will give us reliable Collins estimation; the sigma (-0.05525) in the top-left plot reveals the “true” variation of the data, the distribution of the uncertainty estimations for data of 50000 MC runs is in the top-right plot, the mean value (0.05611) and its sigma (0.001214) are consistent with the “true” variation, which means the MLE will give us reliable Collins error estimation. Due to the limited azimuthal angular coverage, the extracted Collins and Sivers moments were

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⁷In DSA case, P is the beam polarization times target polarization

Figure 4.29. The distributions of the extracted Collins and Sivers moments as well as their uncertainties for 50000 MC runs by using the MLE.

correlated to each other. The correlation estimation ($\text{cov}(\text{Collins}, \text{Sivers})$) of the MLE was also checked. Figure 4.30 shows the 2-D distribution for the extracted Collins and Sivers moments of 50000 MC runs. As described in the previous text, the MLE can extract error matrix, hence the correlation can be drawn clearly. In the figure, the ellipses illustrate the estimations of $1 - \sigma$, $2 - \sigma$, $3 - \sigma$ contours for a certain MC run by using the MLE. The ellipses can describe the correlation very well by looking at the plot. The check was also done with different Collins and Sivers inputs.

4.4 Prescale Factor Effects

The experiment E06-010 took both singles and SIDIS data simultaneously. Due to the high rate of singles trigger, the singles data were taken with certain prescale factors. In general, a counter in the trigger supervisor will count the accepted singles trigger, if the counter reaches the pre-configured prescale factor number, this event will be recorded by DAQ, otherwise events will be discarded. Prescale factor will regularly pick up events whose distribution follows Poisson distribution. Therefore,

Figure 4.30. The 2-D distribution of the extracted Collins and Sivers moments for 50000 MC runs by using MLE. The ellipses are drawn by using the error matrix from the MLE calculation for a certain MC run. The three ellipses are for $1 - \sigma$, $2 - \sigma$, $3 - \sigma$ areas, respectively.

it will affect the estimation of statistical uncertainty of event counts or yield. If the DAQ system is perfect (no deadtime) and the physics acceptance is 100%⁸, the estimation for the counts uncertainty is straight forward:

$$\begin{aligned} N_{after\ prescale} &= \frac{N_{before\ prescale}}{prescale\ factor}, \\ \sigma_{N_{after\ prescale}} &= \frac{\sqrt{N_{before\ prescale}}}{prescale\ factor}. \end{aligned} \tag{4.4.1}$$

In practice, this regular pick-up is convoluted with the DAQ lifetime and physics acceptance. In order to estimate the statistical uncertainty correctly, Yi Qiang introduced a correction factor [100]. The derivation for the correction factor includes the following quantities:

- ◇ Total number of triggers: N
- ◇ Prescale factor: P
- ◇ Physics acceptance: f

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⁸ $Physics\ acceptance = \frac{number\ of\ good\ events}{number\ of\ trigger}$. Number of good events means counts for certain type of particles (eg. electrons, pions, kaons or protons) pass through all the cuts.

◇ DAQ and detector system lifetime: L

Therefore, total number of good events (C) can be calculated as $C = N \cdot f \cdot L/P$. To evaluate the fluctuation of C , the origin of the fluctuation can be separated into two sources. The first one is from the Poisson distribution for the total number of triggers:

$$\begin{aligned}\sigma_1 &= \sqrt{N}, \\ \Delta\sigma_1 &= \frac{\sigma_1}{N} = \frac{1}{\sqrt{N}}.\end{aligned}\tag{4.4.2}$$

The second source is the fluctuation from $L \cdot f$. It is a Binominal selection of events, assuming N is fixed, then

$$\begin{aligned}\sigma_2 &= \sqrt{\frac{N}{P} \cdot L \cdot f \cdot (1 - L \cdot f)}, \\ \Delta\sigma_2 &= \frac{\sigma_2}{C} = \sqrt{\frac{1 - L \cdot f}{C}}.\end{aligned}\tag{4.4.3}$$

Combining the two fluctuations, we have:

$$\Delta\sigma_C = \sqrt{(\Delta\sigma_1)^2 + (\Delta\sigma_2)^2},\tag{4.4.4}$$

$$\sigma_C = \Delta\sigma_C \cdot C = \sqrt{C(1 - L \cdot f \cdot (1 - \frac{1}{P}))} = \sqrt{C} \cdot S.\tag{4.4.5}$$

So if we use $\sqrt{C}$ to estimate the statistical uncertainty of the counts, correction factor $S = \sqrt{1 - L \cdot f \cdot (1 - \frac{1}{P})}$ should be applied.

By looking at the correction factor, the larger $L \cdot f$ will result in larger correction for the statistical uncertainty. Please note that the correction factor is for the uncertainty estimation of counts, if the counts is normalized by the lifetime to calculate yield, the fluctuation of lifetime should be eliminated, the correction factor will become $S = \sqrt{1 - L \cdot f \cdot (\frac{1}{L} - \frac{1}{P})}$.

MC was employed to test the correction factor. During the MC, the lifetime and acceptance were artificial inputs when generating the events randomly for spin + and - states (Asymmetry is set to be zero for this study), then the asymmetry is calculated. One such process is called one run. In practice, 5000 runs were performed to get the “true” distribution of the asymmetry (“true” uncertainty of the asymmetry). Then the “true” uncertainty was compared to the uncertainty estimations for single runs with or without using correction factors. To enlarge the correction effects, a MC test for a large physics acceptance L is shown in Figure 4.31. The first plot shows the distribution of calculated asymmetries, sigma of the distribution gives the “true” distribution of the asymmetry. The second plot shows the asymmetries and error estimations with/without using correction factors for 10 single MC runs. The third plot shows the comparison of the “true” uncertainty of the asymmetry and the estimated uncertainties with/without using correction factors for 10 single MC runs. The test was also done for different inputs of L and f .

4.5 Dilution Study

4.5.1 N₂ Dilution

A small amount of N₂ gas, added to the target cell to reduce the depolarization effect [1], diluted the measured ³He asymmetry:

$$A_{^3\text{He}} = A_{\text{measure}}/f_{\text{N}_2}. \quad (4.5.1)$$

Here the asymmetry includes target polarization already and the nitrogen dilution factor is defined as

$$f_{\text{N}_2} = \frac{\rho_{^3\text{He}}\sigma_{^3\text{He}}}{\rho_{^3\text{He}}\sigma_{^3\text{He}} + \rho_{\text{N}_2}\sigma_{\text{N}_2}}, \quad (4.5.2)$$

Figure 4.31. First plot: The distribution of the calculated asymmetries for 5000 runs, sigma of the distribution gives the “true” distribution of the asymmetry. The second plot: The asymmetries and uncertainty estimations with/without using correction factors for 10 single MC runs. The third plot: The comparison of the “true” uncertainty of the asymmetry and the estimated uncertainties with/without using correction factors for 10 single MC runs.

Cell Name	N ₂ / ³ He (filling)	N ₂ / ³ He (pressure curve)
Astral	1.36%	1.48 ± 0.07%
Maureen	1.32%	1.6 ± 0.3%
Brady	1.4%	1.0 ± 0.29%
Combined for π^-	1.36%	1.32 ± 0.15%
Combined for π^+	1.36%	1.34 ± 0.21%

Table 4.5. The ratio of $\rho_{N_2}/\rho_{^3\text{He}}$ of all three target cells are compared with those obtained from data. The combined number for filling density ratio is obtained by taking an average. The combined number for density ratio determined by pressure curve analysis is combined by taking the average by the accumulated charge.

where ρ is the density of gas in the production target cell and σ is the unpolarized inclusive hadron (pion, kaon and proton) production cross section.

The density ratio of $\rho_{N_2}/\rho_{^3\text{He}}$ in the cell, as shown in Table 4.5, can be obtained from the filling density or dedicated data from pressure curve measurements [101]. As the pressure curve data was only taken for the “Astral”, the density ratios of N₂ and ³He of Maureen and Brady are determined by comparing the observed yield between different production cells. As shown in Table. 4.5, the ratios extracted in pressure curve analysis are consistent with those from filling density. The final N₂ to ³He density ratio was determined to be 1.36% with a relative 10% uncertainties.

The ratio of unpolarized cross sections $\sigma_{N_2}/\sigma_{^3\text{He}}$ was measured in dedicated runs on targets filled with known amounts of unpolarized N₂ or ³He gas. Figure 4.32 shows the yield of pion SIDIS production normalized by beam charge and target pressure. Once yields from reference cells were obtained, the ratio of cross sections, $\frac{Yield_{N_2}}{Yield_{^3\text{He}}}$, can be calculated. For kaon SIDIS analysis, due to the low cross section, data from reference runs were combined to calculate the yields.

Figure 4.32. The charge and pressure normalized yield of the reference cell data. The top panel shows the results of N_2 , and the bottom panel shows the results of ^3He . The yields are normalized to 135 psig pressure. The two periods of N_2 are corresponding to the negative and positive polarity of the left HRS setting. The middle section of the ^3He run is corresponding to the positive polarity of the left HRS, and the rest two sections are corresponding to the negative polarity of the left HRS. The yields are fitted with straight lines. The fit results together with the probability are listed in the plot.

4.5.2 Proton Dilution

Neutron asymmetries were obtained from ^3He asymmetries using the effective nucleon polarizations [102, 103]:

$$P_n = 0.86^{+0.036}_{-0.02}, \quad P_p = -0.028^{+0.009}_{-0.004} \quad (4.5.3)$$

Asymmetry from ^3He can be written as,

$$\begin{aligned} A_{^3\text{He}} &= \frac{\sigma_n^+ + 2\sigma_p^+ - \sigma_n^- - 2\sigma_p^-}{\sigma_n^+ + 2\sigma_p^+ + \sigma_n^- + 2\sigma_p^-} \\ &= \frac{\sigma_n^+ - \sigma_n^- + 2(\sigma_p^+ - \sigma_p^-)}{\sigma_{^3\text{He}}^+ + \sigma_{^3\text{He}}^-} \\ &= \frac{\sigma_n^+ - \sigma_n^- + 2(\sigma_p^+ - \sigma_p^-)}{2\sigma_{^3\text{He}}^U} \\ &= \frac{1}{2\sigma_{^3\text{He}}^U} \left[\frac{\sigma_n^+ - \sigma_n^-}{\sigma_n^+ + \sigma_n^-} 2\sigma_n^U + 2 \frac{\sigma_p^+ - \sigma_p^-}{\sigma_p^+ + \sigma_p^-} 2\sigma_p^U \right] \end{aligned} \quad (4.5.4)$$

where σ^U means unpolarized cross-section. With effective polarizations, formula 4.5.4 can be written as:

$$A_{^3\text{He}} = P_n(1 - f_p)A_n + P_p f_p A_p, \quad (4.5.5)$$

with the following definition:

$$f_p = \frac{2\sigma_p^U}{\sigma_{^3\text{He}}^U}. \quad (4.5.6)$$

f_p was measured by using dedicated reference runs with pure Hydrogen and ^3He gas target.

Chapter 5

RESULTS AND DISCUSSIONS

In this chapter, all the numbers used in the analysis as well as the results are summarized. It consists of four parts: (1) Extraction of the Collins and Sivers asymmetries in the ${}^3\text{He}^\uparrow(e, e'K^\pm)X$ process ¹. (2) Target single spin asymmetries in inclusive hadron production ². (3) Beam-target double-spin asymmetries in inclusive hadron production ³. (4) Beam-target double spin asymmetries in inclusive electron scattering ⁴. In part (2), we present target single spin asymmetries from $lN^\uparrow \rightarrow hX$ processes. An early theoretical study of this process was done by Anselmino *et al.*, under the assumption that the underlying mechanism that generates target single spin asymmetry is either Collins or Sivers effect [104]. More recently, this study was re-evaluated using newly available SIDIS data on the Sivers and Collins moments assuming that the TMD factorization is valid in $lp^\uparrow \rightarrow \pi^\pm X$ processes at large p_T values. Due to the presence of only one hard scale in this process, estimation of the asymmetries is generally done at large p_T values (typically >1 GeV). They predicted asymmetries between 5-10% for $\sqrt{s} \simeq 4.9$ GeV, $p_T=1.5$ GeV, and $x_F \leq 0.1$ with a contribution from the Sivers mechanism to the asymmetry, whereas the contribution from the Collins mechanism was negligible [105]. Non-zero SSAs were also estimated based on the twist-3 distribution and fragmentation functions in the framework of collinear factorization [106–108], and in the SIDIS process by integrating over the scattered electron's azimuthal angle [109]. In part (3), we present the beam-target double-spin asymmetries A_{LT} in $\vec{e} + N^\uparrow \rightarrow h + X$ processes. In the collinear factorization framework, A_{LT} in inclusive hadron production is an observable associated with

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¹Published in Physical Review C 90, 055201, 2014.

²Published in Physical Review C 89, 042201 (R), 2014.

³Published in Physical Review C 92, 015207, 2015

⁴Still in analysis when I am writing the thesis.

twist-3 effects. It can have twist-3 contributions from both the parton distributions inside the polarized nucleon and the parton fragmenting into final state hadrons. By measuring A_{LT} , one has the opportunity to investigate the “worm-gear”-type function $\tilde{g}(x)$ [4, 110] as well as the role of quark-gluon-quark correlations in the nucleon and twist-3 effects in the fragmenting hadron. The $\tilde{g}(x)$ is defined as an integration [4] over k_t^2 of $g_{1T}(x, k_t^2)$, which can be accessed by A_{LT} measurements in a SIDIS process [82]. In part (4), we present the beam-target double-spin asymmetries of inclusive electron scattering from a transversely polarized ^3He target. With the inputs of $g_1^{^3\text{He}}$ from the world data, we can extract the $g_2^{^3\text{He}}$ which provides access to the twist-3 effects.

5.1 Collins and Sivers Asymmetries in $^3\text{He}^\uparrow(e, e'K^\pm)X$ Processes

5.1.1 Results

As described in section 4.3, the kaon SIDIS data are limited by statistics, the analysis was done by using azimuthally unbinned MLE which can extract the Collins and Sivers moments simultaneously. Figure 5.1 shows the final Collins and Sivers asymmetries as well as their statistical and systematic uncertainties. The final results are also tabulated in Table 5.1. The central kinematical variables for each bin are tabulated in Table 5.2. In the following subsections, I will present how the final results were obtained.

5.1.2 Contaminations

The Kaon SIDIS sample has 4 types of contaminations: (1) π^- contamination in electron; (2) photon-induced electron contamination in electron; (3) pion contamination in kaon; (4) random coincidental contamination. The method to estimate these contaminations are described in section 4.2.3. The contamination numbers are sum-

Figure 5.1. The extracted Collins and Sivers moments on ^3He are shown together with their statistical errors and systematic error bands for both K^+ and K^- electroproduction. “Fit” systematic uncertainty means the uncertainty due to neglecting the other azimuthal angular dependent terms. The Sivers moments are compared to theoretical predictions from a phenomenological fit to the world data.

	x_{bj}	Collins moment	Sivers moment
K^+	0.137	$0.16 \pm 0.13 \pm 0.024(0.003)$	$0.078 \pm 0.13 \pm 0.017(0.005)$
K^+	0.190	$0.082 \pm 0.083 \pm 0.01(0.002)$	$-0.019 \pm 0.083 \pm 0.0065(0.004)$
K^+	0.250	$0.0009 \pm 0.063 \pm 0.003(0.002)$	$0.0074 \pm 0.063 \pm 0.006(0.003)$
K^+	0.324	$-0.075 \pm 0.074 \pm 0.006(0.002)$	$-0.019 \pm 0.07 \pm 0.006(0.002)$
K^-	0.210	$-0.21 \pm 0.10 \pm 0.03(0.009)$	$-0.25 \pm 0.10 \pm 0.039(0.01)$

Table 5.1. Tabulated ^3He results for the central kinematical variable x_{bj} . The format for the tabulated results follows “central value” $\pm$ “statistical uncertainty” $\pm$ “experimental systematic uncertainty (systematic uncertainty due to fit model, as will be explained in the following text)”.

	x_{bj}	y	z	Q^2 GeV ²	P_t GeV	W GeV	W' GeV
K^+	0.137	0.85	0.48	1.29	0.46	3.0	2.08
K^+	0.190	0.81	0.51	1.69	0.40	2.85	1.96
K^+	0.250	0.77	0.53	2.11	0.33	2.69	1.83
K^+	0.324	0.73	0.56	2.60	0.26	2.51	1.69
K^-	0.210	0.80	0.51	1.83	0.38	2.80	1.93

Table 5.2. Tabulated central values for kinematical variables x_{bj} , y , Q^2 , z , P_t , W , W' .

Bin number	h^- Cont. in e^-	photon-induced electron Cont.	Pion Cont.	Random Cont.
K^+ bin 1	$< 1\%$	$18.6 \pm 8.3\%$	$2.6 \pm 2.6\%$	$5.4 \pm 0.1\%$
K^+ bin 2	$< 1\%$	$< 7.9\%$	$1.6 \pm 1.6\%$	$2.8 \pm 0.1\%$
K^+ bin 3	$< 1\%$	$< 3.4\%$	$1.0 \pm 1.0\%$	$1.7 \pm 0.1\%$
K^+ bin 4	$< 1\%$	$< 2.4\%$	$0.9 \pm 0.0\%$	$1.5 \pm 0.1\%$
K^-	$< 1\%$	$13.9 \pm 6.9\%$	$4.5 \pm 4.5\%$	$0.4 \pm 0.1\%$

Table 5.3. Contaminations in the kaon SIDIS analysis.

marized in Table 5.3. Negative-charged hadron contamination in electron sample is very small. The photon-induced electron contamination was estimated by using the calibration runs with BigBite magnetic field reversed, only the first bin has positron-kaon SIDIS events for K^+ , and the following 3 bins are given a limit by the assumption that the contamination decreases linearly ⁵. To be conservative, pion contamination in kaon sample was assigned a 100% relative uncertainty due to the way we estimated the contaminations, in which process we assume that the pion CTOF shape doesn't depend on A1 cut. Random contamination in K^+ SIDIS sample is larger than that of K^- SIDIS sample, this is due to the large number of proton events in LHRS positive polarity runs.

5.1.3 N_2 Dilutions

N_2 dilution factors are tabulated in Table 5.4. In the dilution study, yields for N_2 and ^3He cells were calculated by using the same cuts as in the analysis. Filling density ratio of $N_2/^3\text{He}$ was $1.36 \pm 0.14\%$.

5.1.4 Systematic Uncertainties

- **Systematic uncertainty due to contaminations**

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⁵This assumption was verified by looking at the pion SIDIS data which has more statistics.

Bin number	N ₂ dilution factor
K^+ bin 1	0.889 ± 0.038
K^+ bin 2	0.909 ± 0.027
K^+ bin 3	0.956 ± 0.013
K^+ bin 4	0.891 ± 0.041
K^-	0.935 ± 0.031

Table 5.4. N₂ dilution factors in the kaon SIDIS analysis.

Once the Collins and Sivers asymmetries were obtained from the MLE, the asymmetries were corrected by the contaminations in first order:

$$A_{e-,e+,h-;kaon} = A_{measured} \frac{N_{total}}{N_{total} - N_{\pi} - N_{random}} - A_{e-,e+,h-;\pi} \frac{N_{\pi-}}{N_{total} - N_{\pi} - N_{random}} - A_{e-,e+,h-;random} \frac{N_{random}}{N_{total} - N_{\pi} - N_{random}}, \quad (5.1.1)$$

$$A_{e-;kaon} = A_{e-,e+,h-;kaon} \frac{N_{total}}{N_{total} - N_{e+} - N_{h-}} - A_{e+;kaon} \frac{N_{e+}}{N_{total} - N_{e+} - N_{h-}} - A_{h-;kaon} \frac{N_{h-}}{N_{total} - N_{e+} - N_{h-}}. \quad (5.1.2)$$

Where the equation 5.1.1 corrects the contaminations in LHRS (and random contamination) and the equation 5.1.2 corrects the contaminations in BigBite. N_a means number of “a” particles in the selected kaon SIDIS sample. $A_{measured}$ means the Collins or Sivers asymmetries obtained from the MLE with all the contaminations. $A_{a;b}$ means the asymmetry for (a,b) coincidental events. $A_{e-,e+,h-;\pi}$ was calculated by the raw Collins or Sivers asymmetries for (e-, pion) coincidental sample. $A_{e-,e+,h-;random}$ was calculated by the raw Collins or Sivers asymmetries for random coincidental sample. $A_{h-;kaon}$ was calculated by the raw Collins or Sivers asymmetries for (h,kaon) sample which was selected by reverse preshower cut on the BigBite calorimeter. $A_{e+;kaon}$ means asymmetries for (positron,kaon) coincidental events, since this contamination is primarily from photon-induced pair production, it carries the same asymmetry as photon production, in practice, the asymmetry was given by the Collins or Sivers

Figure 5.2. The Collins and Sivers asymmetries for photon-kaon coincident events as a function of photon energies. The first point is the combined asymmetry for all the latter 4 points.

asymmetries from high energy (γ , kaon) coincident events ⁶. Figure 5.2 shows the Collins and Sivers asymmetries for different photon energies. In the estimation of $A_{e^+;kaon}$, photon energy was required to be larger than 1.0 GeV/c.

If the contaminations and the asymmetries used to correct the measured asymmetry are well known, there will be no systematic uncertainty while doing the correction. In reality, these numbers has their own uncertainties which will give additional uncertainty to the final asymmetry, which were assigned as systematic uncertainties.

• Systematic uncertainty due to yield drift

As the BigBite lead-glass calorimeter accumulating more and more radiation, the light transparency will become worse, hence the light yield will decrease. PMTs will also drift due to the radiation damage. The calorimeter provided “electron” trigger in the BigBite spectrometer. Yield drift affected the trigger efficiency. Fortunately, our target spin direction was reversed every 20 minutes throughout the experiment regardless of what happened on the detector and to some extent, the target spin flip

⁶This is done for the first bin of K^+ and K^- , since in the other three bins of K^+ an up limit was given for the contamination. No asymmetry corrections due to $A_{e^+;kaon}$ were made to the latter three bins of K^+ data. However, systematic uncertainty to the asymmetry for this limit was still assigned.

Figure 5.3. Yield drift plot for T1 events. First plot: Weighting factors for ρ to study yield drift effects. Second plot: Relative yield versus target spin state id for T1 events.

was “random” vis-a-vis the yield drift. On the other hand, the yield drift is relatively slow compared to the spin flip frequency. The effect of the false asymmetries would be minimized by the frequent target spin flip.

The yield drift effect was studied by using singles electron events in the BigBite spectrometer (T1 events). As described in equation 4.3.6, ρ is the target density. In general, it includes all the correction factors, such as beam charge, livetime, target density, trigger efficiency etc.. Therefore, it can also be weighted by the yield drift factor. Yields of T1 events versus time were calculated, and then the relative trigger efficiency values were calculated for different period. ρ was weighted by these values in different period to extract the Collins and Sivers asymmetries. The difference before and after weighting was assigned as the systematic uncertainty due to yield drift. Figure 5.3 shows the relative yield versus time and the weighted factors to ρ .

• Systematic uncertainty due to target density fluctuation

The target density would fluctuate due to the changes in the temperature of the target. Therefore, the target density related information was added into the rootfiles⁷.

⁷The target density was calculated based on the measured target temperature information in each spin state.

The systematic uncertainty due to target density fluctuation was estimated by the difference of extracted asymmetries with and without considering target densities.

• **Systematic uncertainty due to other angular dependent terms**

For the asymmetry extraction from the MLE, we only included $\sin(\phi_h + \phi_S)$ and $\sin(\phi_h - \phi_S)$ modulations by neglecting other modulations, including $\sin(3\phi_h - \phi_S)$ modulation at twist-2, $\sin(\phi_S)$ and $\sin(2\phi_h - \phi_S)$ modulations at twist-3, Cahn $\cos(\phi_h)$ and Boer-Mulders $\cos(2\phi_h)$ modulations from unpolarized cross section. The leakage from the longitudinal polarized target single spin asymmetry (A_{UL}) due to the small longitudinal component of the target polarization was also neglected. These effects were estimated by varying each term within an allowed range derived from the HERMES proton data [111], assuming that the magnitude of each term for the neutron is similar to that of the proton. The maximum change on the extracted asymmetry by varying each term mentioned above was assigned as systematic uncertainty due to fit model.

• **Other systematic uncertainty sources**

Target polarization measurement: This will give a 5% overall relative ⁸ systematic uncertainty to the asymmetry.

Uncertainty of N₂ dilution factor: Uncertainty of the N₂ dilution will give additional uncertainty to the asymmetry while the extracted asymmetry was corrected by $A = A_{measured}/f_{N_2}$.

Bin centering effect: Possible false asymmetry raised by the binning. This was estimated by shifting the binning and checking the asymmetry difference.

HRS single track cut: Only single track events in HRS were analyzed, this will cause false asymmetry.

BigBite tracking quality cut.

All the systematic uncertainties discussed above are listed in APPENDIX B.

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⁸Relative to asymmetry

Figure 5.4. Results of one binning analysis for kaon SIDIS data. The top two plots are for K^+ data. The bottom two plots are for K^- data.

	x_{bj}	Collins moment	Sivers moment
K^+	0.2249	$0.023 \pm 0.040 \pm 0.01$	$-0.007 \pm 0.039 \pm 0.01$
K^-	0.2104	$-0.21 \pm 0.10 \pm 0.03$	$-0.25 \pm 0.10 \pm 0.04$

Table 5.5. Tabulated results for central kinematics x_{bj} and asymmetry results for the one-bin analysis.

5.1.5 One-Bin Analysis

The K^+ data were also analyzed in one x_{bj} bin. The final results for $K^\pm$ one-bin analysis are shown in Figure 5.4 and tabulated in Table 5.5.

5.1.6 Numerical calculation

The results obtained from the MLE by using $\frac{1}{1+x}$ expansion were cross-checked by the numerical calculations. In the one binning analysis, the Collins asymmetry was segmented by 0.0001 in step length and the Sivers asymmetry was segmented by 0.001 in step length while the Log-Likelihood was calculated across the Collins-Sivers grid. The $1-\sigma$ contour of the numerical calculation for $K^\pm$ data is shown in Figure 5.5.

Figure 5.5. The 1- σ contour of the numerical calculation for $K^\pm$ data. The top plot is for K^+ and the bottom plot is for K^- . The edges of the contour are illustrated by the red lines and labeled by the numbers.

The raw Collins and Sivers asymmetries (without any corrections) extracted from the MLE by using $\frac{1}{1+x}$ expansion are:

$$\begin{aligned}
 K_{\text{Collins}}^+ &= 0.0183 \pm 0.0335, K_{\text{Sivers}}^+ = -0.0064 \pm 0.0328; \\
 K_{\text{Collins}}^- &= -0.161 \pm 0.076, K_{\text{Sivers}}^- = -0.176 \pm 0.076.
 \end{aligned}
 \tag{5.1.3}$$

The results from numerical calculations are:

$$\begin{aligned}
 K_{\text{Collins}}^+ &= 0.0182 \pm 0.0335, K_{\text{Sivers}}^+ = -0.0064 \pm 0.0329; \\
 K_{\text{Collins}}^- &= -0.160 \pm 0.076, K_{\text{Sivers}}^- = -0.175 \pm 0.076.
 \end{aligned}
 \tag{5.1.4}$$

The results between two methods are consistent.

5.1.7 One-Term Fit

During the analysis, we also did the one-term (Collins or Sivers) fit besides the two-term (Collins and Sivers) fit. This is also a good way to check the results.

For two variables, x and y following Gaussian distribution, are correlated with correlation coefficient ρ , the 2-D probability distribution of (x, y) can be expressed as:

$$P(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \frac{1}{\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2} \frac{1}{1-\rho^2} \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} - \frac{2\rho xy}{\sigma_x\sigma_y}\right)\right]. \quad (5.1.5)$$

The conditional probability distribution of y for a fixed X is still a Gaussian distribution with central value $y_{mean} + \rho \frac{\sigma_x}{\sigma_y} (X - x_{mean})$ and sigma $\sigma_y \sqrt{1-\rho^2}$. When performing a one-term calculation (or fit), one actually looks at a variable following the conditional probability distribution from a 2-D correlated distribution. For Collins (Sivers) 1-term calculation, the condition is that Sivers (Collins) term equals to zero. Taking K^- data as an example, the results of two-term fit are:

$$Collins : -0.160 \pm 0.076,$$

$$Sivers : -0.175 \pm 0.076,$$

Error matrix :

$$\begin{pmatrix} \sigma_{Collins}^2 & cov(Collins, Sivers) \\ cov(Collins, Sivers) & \sigma_{Sivers}^2 \end{pmatrix} = \begin{pmatrix} 0.0058 & 0.0038 \\ 0.0038 & 0.0057 \end{pmatrix}, \quad (5.1.6)$$

Correlation Coefficient :

$$\rho = \frac{cov(Collins, Sivers)}{\sigma_{Collins}\sigma_{Sivers}} = 0.656.$$

One would expect that the 1-term Collins fit will yield $A_{Collins} = -0.0452 \pm 0.057$ and 1-term Sivers fit will yield $A_{Sivers} = -0.07 \pm 0.057$. The results of 1-term fit from the data are:

$$A_{Collins} = -0.044 \pm 0.058, A_{Sivers} = -0.071 \pm 0.057, \quad (5.1.7)$$

which are consistent with the expectation. The estimations from the 2-D fit including the central values, sigmas and correlations are reliable.

5.1.8 Discussions

In this study, we performed the first measurement of target single spin asymmetries of charged kaons produced in SIDIS using a transversely polarized ^3He target. Our data show that the Collins and Sivers moments for K^+ are consistent with zero within the experimental uncertainties, while the K^- results favor negative values. The Sivers moments from the ^3He target are compared to theoretical predictions from a phenomenological fit to the world data [112,113]. While the K^- results contain contributions from unfavored fragmentation processes, the K^+ results contain contributions from both favored and unfavored fragmentation processes. The theoretical calculations have included contributions from both favored and unfavored fragmentation processes, however the uncertainties due to the kaon fragmentation functions [114] are not fully estimated. The higher-twist contributions are not considered. While K^+ Sivers moments are consistent with the prediction, K^- results differ from the prediction at the 2-sigma level. Although the K^- Sivers asymmetry was observed to be non-zero at 2-sigma level, one has to be aware that the sea-quarks densities are small in our kinematical range. Due to the lack of information on the Collins fragmentation function for kaons, no theoretical predictions on the Collins moments are currently available. Our data on the Collins moments will provide independent in-

puts for future global analyses to extract flavor dependent transversity distributions. Although with large uncertainties for the K^- Collins and Sivers moments, the results are still surprising compared to our current knowledge of the effects of sea quarks and unfavored fragmentation functions. Current experimental and theoretical studies on the Collins and Sivers effects are limited in the leading twist formulism, however, higher-twist effects due to the strange quark mass effects or low Q^2 coverage could be important. Therefore, to fully understand the sea quark flavor dependence of the Collins and Sivers moments, high-precision kaon data are required for transversely polarized proton, deuteron and ^3He targets.

5.2 Beam-Target Double-Spin Asymmetries (DSA) in Inclusive Hadron (pions, kaons and protons) Production

5.2.1 Results

In this study, we performed a measurement of beam-target double-spin asymmetries in inclusive charged-hadron production using a longitudinally polarized electron beam scattered from a transversely polarized ^3He target. The measured asymmetry is defined as

$$A_{\text{LT}} = \frac{1}{|P_B P_{\text{target}}|} \frac{d\sigma^{\uparrow\rightarrow} - d\sigma^{\downarrow\rightarrow}}{d\sigma^{\uparrow\rightarrow} + d\sigma^{\downarrow\rightarrow}}, \quad (5.2.1)$$

where $d\sigma^{\uparrow(\downarrow)\rightarrow}$ is the differential cross-section for beam helicity $+$ ($-$) in a certain target spin direction. P_B is the beam polarization and P_{target} is the target polarization. The data used in this analysis is T3 singles trigger in LHRS. Two independent methods were used for obtaining asymmetries: “Local pair” and MLE.

In local pair analysis, the yields of two beam helicity states were calculated for a specific target spin state:

$$Y_{\pm} = \frac{N_{\pm}}{C_{\pm} L_{\pm}} \quad (5.2.2)$$

where $\pm$ means two beam helicity states, N is the good event counts, C is the beam charge for each state and L represents the possible corrections including livetime, target density etc.. The yield was calculated for a specific target spin state, then the asymmetry was obtained by combining the yields as:

$$A_{LT} = \frac{\sum a_i (Y_{i+}^{\rightarrow} - Y_{i-}^{\rightarrow}) / (P_i^{target} P_i^{beam})}{\sum a_i (Y_{i+}^{\rightarrow} + Y_{i-}^{\rightarrow})}, \quad (5.2.3)$$

Here $\rightarrow$ represent a specific target spin direction, $P_i^{target/beam}$ is target/beam polarization, and

$$a_i = \frac{b_i}{Y_{i+}^{\rightarrow} + Y_{i-}^{\rightarrow}} \sim \frac{1}{\frac{1}{L+C_+} + \frac{1}{L-C_-}} \quad (5.2.4)$$

In the MLE method, for each target spin direction, the log-likelihood function was formed as:

$$L(\varepsilon) = \log \prod_{states} \frac{(y \tilde{C}_{i\pm} (1 \pm P_{i\pm}^{target} P_{i\pm}^{beam} \varepsilon))^{N_{i\pm}}}{N_{i\pm}!} \exp(-y \tilde{C}_{i\pm} (1 \pm P_{i\pm}^{target} P_{i\pm}^{beam} \varepsilon)) \quad (5.2.5)$$

where $\pm$ means two beam helicity states, $N_{i\pm}$ is the good event counts for the i^{th} beam helicity $\pm$ state, ε is the double spin asymmetry to be extracted (A_{LT}), y is the average yield for beam helicity $+$ and $-$. We take the derivative of L with respect to ε and y respectively and equate them to zero to get the maximum point, hence extract the DSA.

Figure 5.6 shows the results from these two methods. The results are almost the same if we use the same set of data. During the experiment, the target spin direction was oriented to four directions ($T\pm$, $V\pm$ in figure 5.6). Kinematical configuration in the laboratory coordinate system of the measurement is shown in figure 5.7, in which ϕ_s is the azimuthal angle between hadron plane (formed by the incoming electron and outgoing hadron momentum) and target spin direction. $V+(-)$ direction is corresponding to $\phi_s=90^\circ$ (270°), $T+(-)$ direction is corresponding to $\phi_s=180^\circ$ (0°).

Figure 5.6. DSA results by using different methods. Trigger: T3 trigger, HRS singles. PID: $\pi^\pm$. Here we have three sets of points, the first two points are analyzed by using Xin's rootfiles. The first one was analyzed by the "local pair" method, the second one was analyzed by the "MLE" method. The third one is from Jin who did the analysis by using "MLE". The difference between Xin and Jin's rootfiles is that they used different data quality cuts when they cooked the data.

Figure 5.7. Kinematical configuration in the laboratory coordinate system for the $lN^\uparrow \rightarrow hX$ process. $\vec{l}$ ($\vec{P}_h$) represents the momentum direction of incident electron (produced hadron), and $\vec{S}$ is the spin vector of the nucleon. During the experiment, the target spin was oriented in $\phi_s = 0^\circ(+x), 90^\circ(+y), 180^\circ(-x), 270^\circ(-y)$ directions.

Figure 5.8. Beam-target double spin asymmetries on a ^3He target for $\pi^\pm$, $K^\pm$ and protons production in different target spin configurations.

The final beam-target double-spin asymmetries from ^3He are shown for different hadron species in Figure 5.8. The values are tabulated in Table 5.6. The pion data was separated into 3 p_T bins, the p_T dependent asymmetries are shown in Figure 5.9 and the values are tabulated in Table 5.8.

The spin-dependent part of the inclusive hadron production cross-section is proportional to the term $\vec{S} \cdot \vec{p}_T$ ($p_T = \sqrt{p_x^2 + p_y^2}$, the transverse momentum of the outgoing hadron), which gives rise to a $\cos(\phi_s)$ modulation in the definition of the asymmetry [4]. Hence, the asymmetry can be written as

$$A_{LT} = A_{LT}^{\cos(\phi_s)} \cos(\phi_s). \quad (5.2.6)$$

In Figure 5.8 and 5.9, it is clear that the beam-target double-spin asymmetry is consistent with zero with target spin direction oriented in y direction ($\phi_s = 90^\circ, 270^\circ$),

	$\phi_s = 90^\circ$	$\phi_s = 270^\circ$
π^+	$0.0001 \pm 0.0015 \pm 0.0008$	$0.0006 \pm 0.0015 \pm 0.0008$
π^-	$-0.0007 \pm 0.0011 \pm 0.0008$	$-0.0004 \pm 0.0011 \pm 0.0008$
K^+	$0.007 \pm 0.023 \pm 0.003$	$0.017 \pm 0.024 \pm 0.004$
K^-	$0.011 \pm 0.032 \pm 0.005$	$0.044 \pm 0.033 \pm 0.006$
P	$0.0073 \pm 0.0047 \pm 0.0009$	$-0.0083 \pm 0.0047 \pm 0.001$
	$\phi_s = 0^\circ$	$\phi_s = 180^\circ$
π^+	$-0.0058 \pm 0.0016 \pm 0.0009$	$0.0071 \pm 0.0016 \pm 0.0009$
π^-	$0.0044 \pm 0.0011 \pm 0.0008$	$-0.0056 \pm 0.0011 \pm 0.0009$
K^+	$0.055 \pm 0.030 \pm 0.006$	$-0.066 \pm 0.031 \pm 0.006$
K^-	$0.014 \pm 0.040 \pm 0.006$	$-0.045 \pm 0.041 \pm 0.007$
P	$0.0032 \pm 0.0067 \pm 0.0008$	$-0.0013 \pm 0.0069 \pm 0.0008$

Table 5.6. The beam-target double-spin asymmetries A_{LT} from ^3He for different hadron species. The data structure follows the format of asymmetry $\pm$ statistical uncertainty $\pm$ systematic uncertainty. The average p_T is 0.64 GeV/c.

Figure 5.9. Beam-target double-spin asymmetries A_{LT} for $\pi^\pm$ production from ^3He as a function of p_T for different ϕ_s . The left column is for the π^+ data, the right column is for the π^- data.

hadron	$p_T(\text{GeV})$	$\phi_s = 90^\circ$	$\phi_s = 270^\circ$
π^+	0.60	$0.0003 \pm 0.0024 \pm 0.0008$	$0.0022 \pm 0.0024 \pm 0.0008$
π^+	0.64	$-0.0042 \pm 0.0029 \pm 0.0008$	$0.0001 \pm 0.0029 \pm 0.0008$
π^+	0.68	$0.0033 \pm 0.0026 \pm 0.0008$	$-0.001 \pm 0.0026 \pm 0.0008$
π^-	0.60	$-0.0008 \pm 0.0017 \pm 0.0008$	$-0.0006 \pm 0.0017 \pm 0.0008$
π^-	0.64	$-0.0013 \pm 0.0021 \pm 0.0008$	$-0.0002 \pm 0.0021 \pm 0.0008$
π^-	0.68	$-0.0001 \pm 0.0019 \pm 0.0008$	$-0.0004 \pm 0.0019 \pm 0.0008$
hadron	$p_T(\text{GeV})$	$\phi_s = 0^\circ$	$\phi_s = 180^\circ$
π^+	0.60	$-0.0058 \pm 0.0026 \pm 0.0009$	$0.01 \pm 0.0026 \pm 0.001$
π^+	0.64	$-0.0084 \pm 0.0031 \pm 0.001$	$0.005 \pm 0.0032 \pm 0.0009$
π^+	0.68	$-0.0037 \pm 0.0028 \pm 0.0008$	$0.0049 \pm 0.0029 \pm 0.0009$
π^-	0.60	$0.0051 \pm 0.0017 \pm 0.0009$	$-0.0057 \pm 0.0016 \pm 0.0009$
π^-	0.64	$0.0035 \pm 0.002 \pm 0.0008$	$-0.006 \pm 0.002 \pm 0.0009$
π^-	0.68	$0.0042 \pm 0.0019 \pm 0.0008$	$-0.005 \pm 0.0019 \pm 0.0009$

Table 5.7. Tabulated results of p_T dependent beam-target double spin asymmetries on ^3He for $\pi^\pm$ production.

$\langle p_T \rangle$ (GeV/c)	π^+ ($A_{\text{LT}}^{\cos(\phi_s)} \pm \text{Stat.} \pm \text{Sys.}$)	π^- ($A_{\text{LT}}^{\cos(\phi_s)} \pm \text{Stat.} \pm \text{Sys.}$)
0.60	$-0.0081 \pm 0.0018 \pm 0.0009$	$0.0054 \pm 0.0012 \pm 0.0008$
0.64	$-0.0067 \pm 0.0022 \pm 0.0008$	$0.0048 \pm 0.0014 \pm 0.0008$
0.68	$-0.0043 \pm 0.0020 \pm 0.0008$	$0.0046 \pm 0.0013 \pm 0.0008$

Table 5.8. Tabulated results of p_T dependent $A_{\text{LT}}^{\cos(\phi_s)}$ for $\pi^\pm$ production from ^3He .

which agrees with the $\cos(\phi_s)$ dependence. The asymmetries for $\phi_s = 0^\circ$ and $\phi_s = 180^\circ$ of pion production were combined together to obtain $A_{\text{LT}}^{\cos(\phi_s)}$. The combination was weighted by the statistical uncertainties of the asymmetries. The final p_T -dependent $A_{\text{LT}}^{\cos(\phi_s)}$ asymmetries for $\pi^\pm$ production from ^3He are shown in Figure 5.10 and tabulated in Table 5.8.

Neutron asymmetries for pion production were obtained from the ^3He asymmetries using the effective polarizations of the proton and neutron in polarized ^3He using the equation:

$$A_{\text{LT}}^{^3\text{He}} = P_n(1 - f_p)A_{\text{LT}}^n + P_p f_p A_{\text{LT}}^p, \quad (5.2.7)$$

Figure 5.10. Beam-target double-spin asymmetries $A_{LT}^{\cos(\phi_s)}$ for $\pi^\pm$ production from ^3He as a function of p_T . The red (top) band is the systematic uncertainty band for π^- , and the black (bottom) band is the systematic uncertainty band for π^+ .

Since there were no A_{LT} experimental data from the proton, and the contribution to the final ^3He asymmetry from polarized protons in polarized ^3He is small due to the small P_p , the proton A_{LT}^p was treated as a systematic uncertainty while the neutron asymmetry was extracted from the ^3He asymmetry. The beam-target double-spin asymmetry from a polarized proton target was assumed to be no more than $\pm 5\%$ based on the calculations for a proton target in [4]. The final p_T -dependent asymmetries $A_{LT}^{\cos(\phi_s)}$ for $\pi^\pm$ production from the neutron are shown in Figure 5.11 and tabulated in Table 5.9.

5.2.2 Contaminations

The selection of inclusive hadron is described in the section 4.2.2. The contamination is well controlled. After all the particle identification cuts, the electron and positron contamination in hadron sample was negligible. The contamination of the pion sample from kaons and protons was estimated to be less than 1%. The contami-

Figure 5.11. Beam-target double-spin asymmetries $A_{LT}^{\cos(\phi_s)}$ for $\pi^\pm$ production from the neutron as a function of p_T . The systematic uncertainty is shown as a band. The red (top) band is the systematic uncertainty band for π^- , and the black (bottom) band is the systematic uncertainty band for π^+ . Predictions from collinear factorization by using two different scenarios [4] (Sivers function and Wandzura-Wilczek(WW)-type approximation) are shown as well. Please note that the prediction for π^+ by using the Sivers function is scaled by a factor of $\frac{1}{10}$.

$\langle p_T \rangle$ (GeV/c)	$\langle x_F \rangle$	π^+ ($A_{LT}^{\cos(\phi_s)} \pm \text{Stat.} \pm \text{Sys.}$)	π^- ($A_{LT}^{\cos(\phi_s)} \pm \text{Stat.} \pm \text{Sys.}$)
0.60	-0.269	$-0.063 \pm 0.014 \pm 0.012$	$0.024 \pm 0.005 \pm 0.006$
0.64	-0.263	$-0.049 \pm 0.016 \pm 0.011$	$0.020 \pm 0.006 \pm 0.006$
0.68	-0.254	$-0.032 \pm 0.015 \pm 0.011$	$0.019 \pm 0.005 \pm 0.005$

Table 5.9. Tabulated results of p_T dependent $A_{LT}^{\cos(\phi_s)}$ for $\pi^\pm$ production from the neutron. kinematical variable x_F is defined as $x_F = 2p^{CM}/\sqrt{s}$, where p^{CM} is the momentum of the outgoing hadron along the polarized nucleon's momentum direction in the e+N center-of-mass frame. A negative x_F indicates that the produced hadron is moving backwards with respect to the nucleon momentum direction in the center-of-mass frame of the e+N system.

	f_{N_2}
π^+	0.9233 ± 0.0047
π^-	0.9169 ± 0.0051
K^+	0.945 ± 0.006
K^-	0.902 ± 0.0027
P	0.919 ± 0.005

Table 5.10. N_2 dilution factors for 1 binning analysis in inclusive hadron (pion, kaon and proton) production.

$p_T(GeV)$	$f_{N_2}^{\pi^+}$	$f_{N_2}^{\pi^-}$
0.6	0.9196 ± 0.0026	0.9223 ± 0.0025
0.64	0.9178 ± 0.0026	0.9182 ± 0.0026
0.68	0.9173 ± 0.0026	0.9188 ± 0.0026

Table 5.11. N_2 dilution factors for 3 p_T bins.

nation of the kaon sample from other hadrons was estimated to be $\sim 3\%$ (proton) and $\sim 2\%$ (π^+) for positive kaons, and $\sim 2\%$ (π^-) for negative kaons. The contamination of the proton sample from other hadrons was estimated to be less than 1%. The final asymmetry was corrected by the contaminations whose uncertainties were assigned by 100% relative uncertainties, which contribute to the systematic uncertainties due to contaminations.

5.2.3 Dilutions

5.2.3.1 N_2 Dilution factors

The N_2 dilution values for the 1 binning analysis are tabulated in Table 5.10. The N_2 dilution values for the 3 p_T bins in pion production are tabulated in Table 5.11. The N_2 dilution values for 3 different cells in pion production were also studied, the results are shown in Figure 5.12.

Figure 5.12. N_2 dilution factors for 3 p_T bins in pion production. Each bin has 3 points for 3 production target cells. The first one is for the “astra” cell, the second one is for the “maureen” cell, the third one is for the “brady” cell.

Figure 5.13. Proton dilution factors for 3 p_T bins in pion production.

5.2.3.2 Proton Dilution factors

The proton dilutions⁹, $f_p = \frac{2\sigma_p}{\sigma_{^3\text{He}}}$, in ^3He were measured directly by measuring yields from unpolarized hydrogen and ^3He targets. The proton dilution values for 3 P_t bins in pion production are shown in figure 5.13 and the numbers are listed in table 5.12.

⁹We only extract neutron asymmetry from ^3He in pion production. The kaon and proton productions are lack of statistics.

$p_T(\text{GeV})$	$f_{N_2}^{\pi^+}$	$f_{N_2}^{\pi^-}$
0.6	0.8507 ± 0.0057	0.7354 ± 0.0037
0.64	0.8415 ± 0.0070	0.7227 ± 0.0045
0.68	0.8436 ± 0.0063	0.7196 ± 0.0041

Table 5.12. Proton dilution factors for 3 p_T bins.

5.2.4 Systematic Uncertainty Sources

The overall systematic uncertainty in the experiment was small due to frequent target spin and beam helicity flip. In order to estimate the false asymmetry from luminosity fluctuations, two methods are used: The first one is to look at the inclusive electron double spin asymmetries for target polarized in vertical direction. The second one is to look at the double spin asymmetries in inclusive pion production when target is vertically polarized. 0.07% false asymmetry due to luminosity fluctuations was determined. Another evidence for the small false asymmetry due to luminosity fluctuations lies in the pion double spin asymmetries from transversely polarized target, as the target spin direction is flipped, the amplitudes of the two asymmetries are consistent with each other at one sigma level. Systematic uncertainty due to contamination is well controlled to be less than 0.02% for pions, kaons and protons measurements. There are also 5% relative uncertainties (relative to asymmetries) due to target and beam polarization.

For kaons and protons measurements, there were two additional sources of systematic uncertainty associated with the RICH detector: 1) cut on the number of hits in the RICH detector; 2) detector inefficiencies in certain periods of the data taking. The first contribution was studied by varying the cut and checking the relative change in the asymmetry. Figure 5.14 and 5.15 show how the asymmetries change due to different cuts on number of hits. The systematic uncertainties were determined to be less than 15% for $K^\pm$, relative to statistical uncertainties. The same study was

Figure 5.14. K^+ double spin asymmetries due to different number of hits cuts on RICH.

Figure 5.15. K^- double spin asymmetries due to different number of hits cuts on RICH.

done for proton, and the systematic uncertainties was determined to be less than 3%, relative to statistical uncertainties. The effects of the second contribution should be small since the beam helicity was flipped at 30Hz randomly, the inefficiency effects in the detector is relatively slow compared to the beam helicity flip frequency. However, to be conservative, some inefficiency periods were removed to get the asymmetry variance which was signed as systematic uncertainty due to detector inefficiencies in certain periods of the data taking. The systematic uncertainties were determined to be <7%, <3%, and <1%, relative to the statistical uncertainties, for K^+ , K^- and protons. The K^+ relative systematic uncertainty is larger than that of K^- due to the fact that the yield for K^- is more stable than that of K^+ .

5.2.5 Discussions

In this study, the observed π^+ and π^- asymmetries from ^3He and effective neutron targets have opposite signs when the target is transversely polarized. The π^+ and π^- asymmetries for a vertically polarized target are consistent with zero within the experimental uncertainties. Although the uncertainty is large, the sign of the $K^\pm$ A_{LT} is flipped as the target spin direction is reversed transversely (in the x-direction). For the proton A_{LT} , the sign of the asymmetry is flipped as the target spin direction is reversed vertically (in the y-direction), while the asymmetry is consistent with zero within the experimental uncertainty with the target polarized transversely (in the x-direction). In the collinear factorization approximation, A_{LT} in inclusive pion production was estimated in the JLab 6 GeV kinematical region [4]. The estimations were done using two approximations to calculate the $\tilde{g}(x)$ while doing numerical predictions for A_{LT} in inclusive pion production. One is using the approximate relation, $\tilde{g}(x) \approx -f_{1T}^\perp(x)$, where $f_{1T}^\perp(x)$ is the Sivers function; the other one is using Wandzura-Wilczek(WW)-type approximation, $\tilde{g}(x) \approx x \int_x^1 \frac{dy}{y} g_1(y)$. Calculations based on the two approximations shown in Figure 5.11 give different predictions. Our data are consistent in sign with these two predictions, while the magnitude of the predictions is larger than that of our data. We point out that p_T in our experiment is around 0.64 GeV/c, which is lower than 1 GeV/c where the theoretical predictions are believed to be reliable. Moreover, the theoretical calculations only include the LO formula for the distribution term, one can't rule out the fact that the fragmentation term can give a sizable asymmetry and/or NLO corrections on the distribution term could be significant in the relative low p_T range. In addition, the A_{LT} measurements in inclusive hadron production and SIDIS processes are linked by the definition of $\tilde{g}(x)$. The behavior of the π^+ and π^- $A_{LT}^{\cos(\phi_s)}$ with opposite sign is similar to that in the SIDIS measurement in [3]. However, one has to be aware that the kinematical coverage for the non-detected electrons in the inclusive hadron production processes is larger than

that of the electrons in the SIDIS processes and the production mechanism can also be different. To fully interpret the data, one has to understand the mechanism of inclusive hadron production in different kinematical regions and the main contributions to the double-spin asymmetry.

5.3 Target Single Spin Asymmetries in Inclusive Hadron Production

5.3.1 Results

Besides the beam-target double-spin asymmetries in inclusive hadron production, target single-spin asymmetry measurements were also performed. The target SSA is defined as:

$$A_{UT} = \frac{1}{P} \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = A_N \sin\phi_S, \quad (5.3.1)$$

where $d\sigma^{\uparrow(\downarrow)}$ is the differential cross-section in the target “up” (“down”) state, and P is the target polarization. The spin-dependent part of the cross-section is proportional to the term $\vec{S} \cdot (\vec{l} \times \vec{P}_h)$, which gives rise to $\sin(\phi_S)$ modulation in the definition of the asymmetry. This term makes A_N parity-conserving, but T-odd under “naïve” time reversal, in which the initial and final states do not interchange. Kinematical configuration in the laboratory coordinate system of the measurement is shown in figure 5.7, which is the same as DSA measurements.

The PID cuts are the same with that of DSA analysis, hence the contaminations and the dilution factors are the same as that of the one binning analysis. The final ^3He asymmetry results are shown for different hadron species in Fig. 5.16. The kinematical variables and the asymmetry numbers are tabulated in Table 5.13.

For the high statistics pion sample, the SSA was analyzed in five bins in p_T . The SSAs for charged pions as a function of p_T from ^3He are shown in Figure 5.17 (left).

Figure 5.16. Inclusive SSA results on a ^3He target for $\pi^\pm$, $K^\pm$ and protons in the vertical target spin configuration ($\phi_S = \pm 90^\circ$). The error bars on the points represents the statistical uncertainty. The grey band shows the magnitude of the overall systematic uncertainty for each hadron channel.

Hadron	$\langle x_F \rangle$	$\langle p_T \rangle$ (GeV/c)	$A_N^{3\text{He}} \pm \text{Stat.} \pm \text{Sys.}$
π^+	-0.262	0.64	$0.0185 \pm 0.0007 \pm 0.0009$
π^-	-0.262	0.64	$-0.0109 \pm 0.0005 \pm 0.0005$
K^+	-0.215	0.64	$0.0665 \pm 0.0130 \pm 0.0038$
K^-	-0.215	0.64	$0.0122 \pm 0.0179 \pm 0.0027$
p	-0.087	0.64	$0.0038 \pm 0.0026 \pm 0.0002$

Table 5.13. Central kinematics for five types of hadrons along with the A_N results for a ^3He target. A negative x_F indicates that the produced hadron is moving backwards with respect to the nucleon momentum direction in the center-of-mass frame of the $e+N$ system.

Figure 5.17. Left: A_N results on a ^3He target for the $\pi^\pm$ channel as a function of p_T . Right: A_N results on a neutron target extracted from the measured ^3He asymmetries. The solid band on the bottom of each panel shows the magnitude of the systematic uncertainty for each momentum bin. The lower plot shows the x_F and p_T correlation in this measurement.

$\langle p_T \rangle$ (GeV/c)	$A_N^n(\pi^+) \pm \text{Stat.} \pm \text{Sys.}$	$A_N^n(\pi^-) \pm \text{Stat.} \pm \text{Sys.}$	R_{π^+/π^-}
0.58	$0.109 \pm 0.016 \pm 0.007$	$-0.044 \pm 0.006 \pm 0.003$	2.5 ± 0.5
0.61	$0.125 \pm 0.013 \pm 0.008$	$-0.051 \pm 0.005 \pm 0.003$	2.5 ± 0.4
0.63	$0.166 \pm 0.014 \pm 0.010$	$-0.055 \pm 0.006 \pm 0.004$	3.0 ± 0.5
0.66	$0.169 \pm 0.012 \pm 0.010$	$-0.056 \pm 0.005 \pm 0.004$	3.0 ± 0.4
0.69	$0.160 \pm 0.014 \pm 0.010$	$-0.053 \pm 0.006 \pm 0.003$	3.0 ± 0.5

Table 5.14. The extracted neutron A_N^n results for π^+ and π^- along with their ratio R_{π^+/π^-} in five different $\langle p_T \rangle$ bins.

The extracted neutron SSAs were shown in Figure 5.17 (right). During the study of extracting neutron asymmetry from ^3He , the SSA from a polarized proton target (A_N^p) was assumed to be no more than $\pm 5\%$ at $p_T \simeq 0.64$ GeV/c, which is consistent with the preliminary HERMES data on A_N^p [115, 116]. The neutron asymmetries are tabulated in Table 5.14.

Figure 5.18. SSAs of inclusive pion production as a function of ϕ_S . The SSAs in this plot were calculated as (T+) - (T-). “T+” (“T-”) means that the target spin direction is oriented transversely to the BigBite (HRS) side.

5.3.2 Control of False Asymmetry

As shown in the Equation 5.3.1, the SSA of inclusive hadron production has a $\sin(\phi_S)$ modulation. In the experiment E06-010, the target spin direction was also oriented horizontally in the transverse directions. Hence, by looking at the SSA with transversely oriented target, one can investigate the control of false asymmetry for the measurement. Figure 5.18 shows the SSAs of inclusive pion production as a function of ϕ_S . The SSAs clearly cross zero at $\phi_S \sim 0$.

5.3.3 Discussions

In this study, we performed the first measurement of SSAs in the inclusive hadron production reaction using unpolarized electrons on a transversely polarized ^3He target at $\langle p_T \rangle = 0.64 \text{ GeV}/c$. Clear non-zero asymmetries were observed for charged pions and positive kaons, showing a similar feature of flavor dependence to that observed in the Sivers asymmetry in SIDIS, and in A_N in $pp^\uparrow$ collisions. Currently there are no estimates or theoretical interpretations of these asymmetries at the relatively low p_T of $0.64 \text{ GeV}/c$ used for this measurement.

We can compare the observed behavior of our data with existing TSSAs in both proton-proton ($pp^\uparrow$) and lepton-nucleon ($lN^\uparrow$) reaction channels. Our results show that in the center-of-mass frame of the polarized neutron-electron collision, viewed along the direction of the neutron's momentum, π^+ favors the right side of the spin vector, whereas π^- favors the left side of the spin vector. Assuming isospin symmetry, this behavior is the same as that observed in $pp^\uparrow \rightarrow hX$ for the E704 [117, 118] and BRAHMS [119] experiments, and also the same as that observed in $lp^\uparrow \rightarrow hX$ at HERMES [115, 116]. In addition, this behavior is also the same as both the Collins and the Sivers asymmetries observed in SIDIS [120, 121]. We further note that the ratio of the π^+ to the π^- asymmetries is about $2.5 \sim 3.0$, which is rather close to the ratio of the u and d quark contributions (Eq. 29 in [122]) to the neutron's anomalous magnetic moment (2.4). This level of agreement is not very surprising if one assumes that the underlying process is dominated by the Sivers effect. Furthermore, the large and positive amplitude for the K^+ asymmetry compared to the measured π^+ asymmetries resembles the Sivers amplitude for π^+ and K^+ in the SIDIS reaction at HERMES [121].

Bin number	W (GeV)
1	$W < 2.0$
2	$2.0 \leq W < 2.3$
3	$2.3 \leq W < 2.6$
4	$2.6 \leq W < 2.8$
5	$2.8 \leq W < 3.0$

Table 5.15. W binning information for DSA electron analysis.

5.4 Beam-Target Double-Spin Asymmetries in Inclusive electron Production

While I am writing this dissertation, the analysis is still on going. The purpose of the analysis is to perform the beam-target double-spin asymmetries in inclusive electron scattering, which was detected in the BigBite spectrometer. The events are selected from T1, T2 and T6 singles triggers. The beam-target double-spin asymmetry is defined as:

$$A_{LT} = \frac{1}{|P_B P_{target}|} \frac{d\sigma^{\uparrow\rightarrow} - d\sigma^{\downarrow\rightarrow}}{d\sigma^{\uparrow\rightarrow} + d\sigma^{\downarrow\rightarrow}}, \quad (5.4.1)$$

where $d\sigma^{\uparrow(\downarrow)\rightarrow}$ is the differential cross-section for beam helicity $+$ ($-$) in a certain target spin direction. P_B is the beam polarization and P_{target} is the target polarization. Once the A_{LT} is obtained, we can extract the $g_2^{3\text{He}}$. In this section, I will summarize how we obtain the asymmetries and also tabulate all the numbers in the analysis. The analysis basically follows what was done in the “Ay DIS” analysis [123].

5.4.1 Binning Information and Kinematic Variables

The Binning information of the analysis is listed in Table 5.15. The Kinematic variables for different bins after unpolarized radiative corrections are listed in Table 5.16:

Bin number	W (GeV)	x	Q^2 (GeV ²)
1	1.72	0.65	3.98
2	2.17	0.46	3.24
3	2.46	0.34	2.65
4	2.70	0.24	2.08
5	2.89	0.17	1.58

Table 5.16. Average kinematics variables for each bin.

5.4.2 Obtaining Asymmetries

The dominant background in BigBite were photo-induced electron-positron pairs, in which electrons were detected as DIS electrons. A direct measurement of the pair-produced electron contamination was made by reversing the polarity of the BigBite magnet and calculating the positron yield under conditions identical to the normal data collection. Negative pions were also a source of contamination. Their contributions were estimated by fitting the pre-shower energy spectrum. Likewise, the positron data sample was contaminated by positive pions.

The final asymmetry was obtained by correcting the contaminations by ¹⁰:

$$A^{e^-} = \frac{Y_1 A_{raw}^{e^-} - Y_1 f_1 A^{\pi^-} - (Y_2 - Y_3) \frac{Y_2 A_{raw}^{e^+} - C Y_3 A^{\pi^+}}{Y_2 - C Y_3}}{Y_1 - Y_1 f_1 - (Y_2 - Y_3)} \quad (5.4.2)$$

where

- $A_{raw}^{e^-}$: Measured asymmetry from selected electron sample, which contains all the contaminations. (From production runs)
- A^{π^-} : Measured asymmetry from the π^- sample, which was selected by using < 150 MeV cut on preshower energy deposition as well as other cuts. (From production runs)

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¹⁰For the details, one can also refer to Transversity Elog entry 762

- $A_{raw}^{e^+}$: Measured asymmetry from the positron sample, which contains π^+ contamination. (From production runs, the positrons are bended down in the BigBite spectrometer)

- A^{π^+} : Measured asymmetry from the π^+ sample (including proton), which was selected by using < 150 MeV cut on preshower energy deposition and E/p 1 cut and positive charge cut as well as other cuts. (From production runs)

- Y_1 : raw e^- yields, including all the contaminations. (From production runs)

- f_1 : π^- contaminations in raw electron sample. (From production runs)

- Y_2 : raw e^+ yields, including π^+ contaminations. (From BB positive polarity runs)

- Y_3 : π^+ yields in the e^+ sample. (From BB positive polarity runs)

- C : $C = \frac{Y_{neg}^{\pi^+}/Y_{neg}^{raw\ e^+}}{Y_{pos}^{\pi^+}/Y_{pos}^{raw\ e^+}}$, where “neg” means BB negative polarity run and “pos” means BB positive polarity run. Those yields were obtained from the Geant3 simulation. C was determined to be 1.8 ± 0.4 .¹¹

f_1 (Y_1 , Y_2 , Y_3) for different triggers (T1, T2, T6) in different bins are tabulated in Table 5.17 (5.18, 5.19, 5.20).

After the corrections due to contaminations by using the Equation 5.4.2, A^{e^-} was obtained for different triggers with target spin oriented to different directions. Figure 5.19 shows the results.

5.4.3 Combination of Data

We don't have N_2 dilution data for T2 trigger, so data need to be combined before dilution correction. A^{e^-} from T1, T2 and T6 triggers were combined together by statistical weighting. The combination of systematic uncertainties due to the contamination corrections were treated as they were not correlated. Although there

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¹¹The reason for the existence of C is that the positive pion contamination in positron sample for BB negative polarity run (production runs) is not necessarily the same as the one for BB positive polarity run.

Bin number	T1	T1 stat. rel.	T1 sys. rel.
1	0.0071	0.069	1
2	0.0186	0.031	0.6
3	0.044	0.013	0.35
4	0.0843	0.008	0.35
5	0.1055	0.006	0.35

Bin number	T2	T2 stat. rel.	T2 sys. rel.
1	0.0034	0.14	1
2	0.008	0.053	0.6
3	0.0139	0.027	0.35
4	0.0165	0.021	0.35
5	0.0195	0.018	0.35

Bin number	T6	T6 stat. rel.	T6 sys. rel.
1	0.0038	0.031	1
2	0.0076	0.016	0.6
3	0.0126	0.0082	0.35
4	0.0157	0.0062	0.35
5	0.0229	0.0047	0.35

Table 5.17. Tabulated f_1 . π^- contaminations in raw electron sample. (From production runs)

Bin number	T1	T1 stat. rel.	T1 sys. rel.
1	3.90011	0.0208489	0
2	6.12833	0.016635	0
3	12.7835	0.0115138	0
4	16.2662	0.0102088	0
5	32.5485	0.00720998	0

Bin number	T2	T2 stat. rel.	T2 sys. rel.
1	2.73917	0.00893683	0
2	4.3142	0.00711736	0
3	8.72895	0.00499645	0
4	9.64662	0.00475074	0
5	11.2316	0.00440053	0

Bin number	T6	T6 stat. rel.	T6 sys. rel.
1	3.76158	0.00697739	0
2	5.88877	0.00557262	0
3	12.0426	0.00389147	0
4	13.3408	0.00369607	0
5	15.7127	0.00340379	0

Table 5.18. Tabulated Y_1 . Raw e^- yields, including all the contaminations. (From production runs)

Bin number	T1	T1 stat. rel.	T1 sys. rel.
1	0.0309573	0.999971	0.15
2	0.231985	0.378055	0.15
3	2.28691	0.118783	0.15
4	5.64271	0.076065	0.15
5	20.9482	0.0394273	0.15
Bin number	T2	T2 stat. rel.	T2 sys. rel.
1	0.0650373	0.123115	0.15
2	0.190781	0.0718096	0.15
3	1.00151	0.0313098	0.15
4	2.32077	0.0205591	0.15
5	5.81856	0.0129671	0.15
Bin number	T6	T6 stat. rel.	T6 sys. rel.
1	0.064532	0.150835	0.15
2	0.258812	0.0756232	0.15
3	1.43289	0.0320669	0.15
4	3.37303	0.0208995	0.15
5	8.24254	0.0133649	0.15

Table 5.19. Tabulated Y_2 . Raw e^+ yields, including π^+ contaminations. (From BB positive polarity runs)

Bin number	T1	T1 stat. rel.	T1 sys. rel.
1	0.0309573	0.185049	1
2	0.173355	0.125438	0.6
3	0.945335	0.0505703	0.35
4	2.41618	0.029815	0.35
5	7.0322	0.0165986	0.35
Bin number	T2	T2 stat. rel.	T2 sys. rel.
1	0.0154193	0.0734322	1
2	0.0569445	0.03935	0.6
3	0.223543	0.0191135	0.35
4	0.31242	0.014323	0.35
5	0.530259	0.00936543	0.35
Bin number	T6	T6 stat. rel.	T6 sys. rel.
1	0.0239452	0.0718147	1
2	0.0735998	0.0389539	0.6
3	0.290519	0.0185342	0.35
4	0.403321	0.0140581	0.35
5	0.904952	0.00857855	0.35

Table 5.20. Tabulated Y_3 . π^+ yields in the e^+ sample. (From BB positive polarity runs)

Figure 5.19. The results of A^{e-} for different triggers with target spin oriented to different directions.

are some correlations in the contaminations for different triggers, T6 dominates the statistics during the combination. The effect of correlation is negligible. Figure 5.20 shows the A^{e-} after combining the results for different triggers.

Double spin asymmetries of inclusive electron scattering in the DIS region follow $\cos(\phi_S)$ modulation [124]. Asymmetries in Figure 5.20 were further combined by statistical weighting for “T $\pm$ ” and “V $\pm$ ” data. The “T-” (“V-”) asymmetries were reversed by sign when doing the combination with “T+” (“V+”) data. Figure 5.21 shows the results after the combination for different target spin directions. In this combination, the systematic uncertainties due to contaminations were treated as they were fully correlated since the combined data were in the same kinematical bin.

Figure 5.20. The results of A^{e^-} after combining the results for T1, T2 and T6 triggers.

Figure 5.21. The results of A^{e^-} for vertical and transverse target after combining the T $\pm$ and V $\pm$ data.

Figure 5.22. N_2 dilution factors for T1 and T6 triggers.

5.4.4 N_2 Dilution Factors

N_2 dilution factors for T1 and T6 triggers are shown in Figure 5.22. They are consistent with each other. The T1 and T6 combined N_2 dilution factors will be applied to the N_2 dilution corrections for the asymmetries.

5.4.5 Extraction of g_2

In order to extract $g_2^{^3\text{He}}$, one needs the inputs of $g_1^{^3\text{He}}$ and unpolarized cross sections. The analysis was still ongoing while the dissertation was written.

5.5 Conclusion

In this dissertation, the transversity experiment (E06-010) was introduced. The Collins and Sivers moments for kaon SIDIS production were extracted from a ^3He target. Clearly non-zero target single-spin and beam-target double-spin asymmetries were observed in the inclusive hadron production. The preliminary results of beam-target double-spin asymmetries in the inclusive electron scattering were shown as

well. This is the first experiment to extract the Collins, Sivers, Pretzelosity, Worm-Gear asymmetries [81, 82, 125, 126] from a polarized ^3He (or effective neutron) target. Although with limited statistics and azimuthal angular coverage, it demonstrates the strong power of the measurement for potential new discover. Several new experiments, based on a solenoidal large intensity device (SoLID) in the 12 GeV period of JLab [127, 128], have been approved to investigate the TMDs systematically. The future electron-ion collider (EIC) [129] is also an idea place to perform precise measurements of TMDs. The new data from JLab 12 GeV projects and EIC will ultimately provide the multi-dimensional mapping of the TMDs.

APPENDICES

APPENDIX A

COORDINATE SYSTEM

Figure A.1. First plot: Lab coordinate system, looking from upstream to downstream. Second plot: BigBite magnet coordinate system, looking at the nominal direction of particle momentum. Third plot: LHRs and BigBite detector coordinate system, looking at the nominal direction of particle momentum.

APPENDIX B

SUMMARY OF SYSTEMATIC UNCERTAINTIES FOR DIFFERENT SOURCES

• K^+ Collins (1_{st} bin) ¹

The raw asymmetry is(A_m) : 0.0968563

The asymmetry after HRS contamination correction : 0.108588

The asymmetry after HRS + BB contamination correction : 0.139667

Statistic error for measured asymmetry: 0.085453

Statistic error for final asymmetry after correction: 0.115289

Systematic due to HRS pion contamination: 0.00423643

Systematic due to HRS random contamination: 0.0031803

Systematic due to BB positron contamination: 0.0183538

Systematic due to BB pim contamination: 0.000961567

Systematic due to bin centering effect: 0.00269815

Systematic due to fit model: 0.0027011

Systematics due to yield drift: 0.000474038

Systematics due to target density: 0.000205592

Systematics due to target polarization: 0.00698336

Systematics due to HRS single track: 0.0001

Systematics due to BB track quality: 0.00015

Systematics quadrature (before N2 dilution): 0.0205474

Systematics fit: 0.0027011

Systematics due to N2 dilution: 0.00662565

Final result : 0.157024 (asy.) $\pm$ 0.129616 (stat. err.) $\pm$ 0.0240323 (exp. sys. err.) $\pm$ 0.00303677 (fit sys. err.)

• K^+ Sivers (1_{st} bin)

The raw asymmetry is(A_m) : 0.0471676

The asymmetry after HRS contamination correction : 0.0522806

The asymmetry after HRS + BB contamination correction : 0.0691341

Statistic error for measured asymmetry: 0.0838766

Statistic error for final asymmetry after correction: 0.113158

Systematic due to HRS pion contamination: 0.00208435

Systematic due to HRS random contamination: 0.00311986

Systematic due to BB positron contamination: 0.0118576

Systematic due to BB pim contamination: 0.000913702

Systematic due to bin centering effect: 0.00674538

Systematic due to fit model: 0.00451412(0.00334608)

Systematics due to yield drift: 0.000834057

Systematics due to target density: 0.000733288

Systematics due to target polarization: 0.00345671

Systematics due to HRS single track: 0.0001

Systematics due to BB track quality: 0.00015

Systematics final(no N2 dilution): 0.0146366

Systematics due to N2 dilution: 0.00327964

Final result : 0.0777258 (asy.) $\pm$ 0.12722 (stat. err.) $\pm$ 0.0167792 (exp. sys. err.) $\pm$ 0.00507511 (fit sys. err.)

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¹I keep the original numbers here, ignoring the scientific notation.

• K^+ Collins (2_{nd} bin)

The raw asymmetry is(A_m) : 0.0687178
 The asymmetry after HRS contamination correction : 0.0712278
 The asymmetry after HRS + BB contamination correction : 0.0742504
 Statistic error for measured asymmetry: 0.0701838
 Statistic error for final asymmetry after correction: 0.0758614
 Systematic due to HRS pion contamination: 0.00107608
 Systematic due to HRS random contamination: 0.00152903
 Systematic due to BB positron contamination: 0.00790125
 Systematic due to BB pim contamination: 2.47822e-05
 Systematic due to bin centering effect: 0.00216179
 Systematic due to fit model: 0.00201117
 Systematics due to yield drift: 0.000480943
 Systematics due to target density: 0.000235077
 Systematics due to target polarization: 0.00371252
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.00920334
 Systematics fit: 0.00201117
 Systematics due to N2 dilution: 0.0024329

Final result : 0.0817201 (asy.) $\pm$ 0.0834932 (stat. err.) $\pm$ 0.0104173 (exp. sys. err.) $\pm$ 0.0022135 (fit sys. err.)

• K^+ Sivers (2_{nd} bin)

The raw asymmetry is(A_m) : -0.0170876
 The asymmetry after HRS contamination correction : -0.0175297
 The asymmetry after HRS + BB contamination correction : -0.017516
 Statistic error for measured asymmetry: 0.0695592
 Statistic error for final asymmetry after correction: 0.0751862
 Systematic due to HRS pion contamination: 0.000241049
 Systematic due to HRS random contamination: 0.00149942
 Systematic due to BB positron contamination: 0.00109158
 Systematic due to BB pim contamination: 2.89022e-05
 Systematic due to bin centering effect: 0.00540447
 Systematic due to fit model: 0.00327752(0.00303223)
 Systematics due to yield drift: 0.000816967
 Systematics due to target density: 0.000701766
 Systematics due to target polarization: 0.000875801
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.00588783
 Systematics due to N2 dilution: 0.000573932

Final result : -0.0192782 (asy.) $\pm$ 0.0827501 (stat. err.) $\pm$ 0.00650552 (exp. sys. err.) $\pm$ 0.00360725 (fit sys. err.)

• K^+ Collins (3_{rd} bin)

The raw asymmetry is(A_m) : 0.00186388
 The asymmetry after HRS contamination correction : 0.00163494
 The asymmetry after HRS + BB contamination correction : 0.000839495
 Statistic error for measured asymmetry: 0.0584648
 Statistic error for final asymmetry after correction: 0.060654
 Systematic due to HRS pion contamination: 0.00016644
 Systematic due to HRS random contamination: 0.000967255
 Systematic due to BB positron contamination: 0.000767419
 Systematic due to BB pim contamination: 0.000743597

Systematic due to bin centering effect: 0.00207489
 Systematic due to fit model: 0.00178514
 Systematics due to yield drift: 0.000412993
 Systematics due to target density: 0.00027753
 Systematics due to target polarization: 4.19748e-05
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.00258692
 Systematics fit: 0.00178514
 Systematics due to N2 dilution: 1.17511e-05

Final result : 0.000878308 (asy.) $\pm$ 0.0634582 (stat. err.) $\pm$ 0.00270655 (exp. sys. err.) $\pm$ 0.00186767 (fit sys. err.)

• K^+ Sivers (3_{rd} bin)

The raw asymmetry is(A_m) : 0.00773071
 The asymmetry after HRS contamination correction : 0.006767
 The asymmetry after HRS + BB contamination correction : 0.00706909
 Statistic error for measured asymmetry: 0.0578044
 Statistic error for final asymmetry after correction: 0.0599688
 Systematic due to HRS pion contamination: 0.000284978
 Systematic due to HRS random contamination: 0.000949892
 Systematic due to BB positron contamination: 0.000874396
 Systematic due to BB pim contamination: 0.000725713
 Systematic due to bin centering effect: 0.00518722
 Systematic due to fit model: 0.00264862(0.00255302)
 Systematics due to yield drift: 0.000917168
 Systematics due to target density: 0.00070049
 Systematics due to target polarization: 0.000353454
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.00553817
 Systematics due to N2 dilution: 9.89516e-05

Final result : 0.00739591 (asy.) $\pm$ 0.0627414 (stat. err.) $\pm$ 0.00579506 (exp. sys. err.) $\pm$ 0.00277107 (fit sys. err.)

• K^+ Collins (4_{th} bin)

The raw asymmetry is(A_m) : -0.0651285
 The asymmetry after HRS contamination correction : -0.0672392
 The asymmetry after HRS + BB contamination correction : -0.0672485
 Statistic error for measured asymmetry: 0.0641824
 Statistic error for final asymmetry after correction: 0.0657713
 Systematic due to HRS pion contamination: 0.000359393
 Systematic due to HRS random contamination: 0.000966379
 Systematic due to BB positron contamination: 0.0011047
 Systematic due to BB pim contamination: 2.33352e-05
 Systematic due to bin centering effect: 0.00204951
 Systematic due to fit model: 0.0015737
 Systematics due to yield drift: 0.000434679
 Systematics due to target density: 0.000308926
 Systematics due to target polarization: 0.00336242
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.00425526
 Systematics fit: 0.0015737
 Systematics due to N2 dilution: 0.00348677

Final result : -0.0754565 (asy.) ± 0.073799 (stat. err.) ± 0.00591225 (exp. sys. err.) ± 0.00176577 (fit sys. err.)

• K^+ Sivers (4_{th} bin)

The raw asymmetry is(A_m) : -0.0176493
 The asymmetry after HRS contamination correction : -0.017252
 The asymmetry after HRS + BB contamination correction : -0.0172398
 Statistic error for measured asymmetry: 0.0617023
 Statistic error for final asymmetry after correction: 0.0632298
 Systematic due to HRS pion contamination: $9.8689e-05$
 Systematic due to HRS random contamination: 0.000945343
 Systematic due to BB positron contamination: $2.77566e-05$
 Systematic due to BB pim contamination: $2.92479e-05$
 Systematic due to bin centering effect: 0.00512378
 Systematic due to fit model: 0.00207617 (0.00202602)
 Systematics due to yield drift: 0.000863198
 Systematics due to target density: 0.000648486
 Systematics due to target polarization: 0.00086199
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.00539438
 Systematics due to N2 dilution: 0.000893867

Final result : -0.019344 (asy.) ± 0.0709473 (stat. err.) ± 0.00611843 (exp. sys. err.) ± 0.00232958 (fit sys. err.)

• K^- Collins:

The raw asymmetry is(A_m) : -0.160149
 The asymmetry after HRS contamination correction : -0.169509
 The asymmetry after HRS + BB contamination correction : -0.19962
 Statistic error for measured asymmetry: 0.0764798
 Statistic error for final asymmetry after correction: 0.0939083
 Systematic due to HRS pion contamination: 0.00938659
 Systematic due to HRS random contamination: 0.000844181
 Systematic due to BB positron contamination: 0.0232531
 Systematic due to BB pim contamination: 0.00148506
 Systematic due to bin centering effect: 0.00301407
 Systematic due to fit model: 0.0081711
 Systematics due to yield drift: 0.000382399
 Systematics due to target density: $3e-05$
 Systematics due to target polarization: 0.00998102
 Systematics due to HRS single track: 0.0001
 Systematics due to BB track quality: 0.00015
 Systematics final(no N2 dilution): 0.0272143
 Systematics fit: 0.0081711
 Systematics due to N2 dilution: 0.00707854

Final result : -0.213498 (asy.) ± 0.100437 (stat. err.) ± 0.0299546 (exp. sys. err.) ± 0.00873915 (fit sys. err.)

• K^- Sivers

The raw asymmetry is(A_m) : -0.175361
 The asymmetry after HRS contamination correction : -0.184859
 The asymmetry after HRS + BB contamination correction : -0.233988
 Statistic error for measured asymmetry: 0.0757778
 Statistic error for final asymmetry after correction: 0.0930541
 Systematic due to HRS pion contamination: 0.0106348
 Systematic due to HRS random contamination: 0.000813172

Systematic due to BB positron contamination: 0.0318698

Systematic due to BB pim contamination: 0.001633

Systematic due to bin centering effect: 0.00398098

Systematic due to fit model: 0.00998678(0.00813388)

Systematics due to yield drift: 0.000378889

Systematics due to target density: 1.3e-05

Systematics due to target polarization: 0.0116994

Systematics due to HRS single track: 0.0001

Systematics due to BB track quality: 0.00015

Systematics final(no N2 dilution): 0.0358471

Systematics due to N2 dilution: 0.00829722

Final result : -0.250255 (asy.) $\pm$ 0.0995231 (stat. err.) $\pm$ 0.0392267 (exp. sys. err.) $\pm$ 0.0106811 (fit sys. err.)

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the glue that binds us all. *arXiv:1212.1701*, 2014.

Biography

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Awards and Honors

1. National Scholarship, USTC, China; 2014

2. Scholarship for the outstanding 1st year graduate students, USTC, China; 2010

3. First place for the competition of summer research projects, USTC, China; 2009

4. Outstanding students award, USTC, China; 2006, 2007, 2008, 2009

Refereed Journal Publications

1. “Single Spin Asymmetries in Charged Kaon Production from Semi-Inclusive Deep Inelastic Scattering on a Transversely Polarized ^3He Target” **Y.X. Zhao**, Y.Wang et al. Phys. Rev. C, 90.055201, 2014

2. “Measurement of the Target-Normal Single-Spin Asymmetry in Deep-Inelastic Scattering from the Reaction $^3\text{He}^\uparrow(e, e')X$ ”, J. Katich, X. Qian, **Y.X. Zhao** et al. Phys. Rev. Lett. 113,022502, 2014

3. “Single Spin Asymmetries of Inclusive Hadrons Produced in Electron Scattering from a Transversely Polarized ^3He Target”, K. Allada, **Y.X. Zhao** et al. Phys. Rev. C, 89.042201(R), 2014

4. “Double Spin Asymmetries of Inclusive Hadron Electroproductions from a Transversely Polarized ^3He Target”, **Y.X. Zhao** et al. Arxiv: 1502.01394, submitted to PRC, 2015.