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TESI DI DOTTORATO DI RICERCA IN FISICA

Search for Hybrid Baryons with CLAS12 experimental setup

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“I’m Winston Wolfe. I solve problems.”

The Wolf.

Introduction

Hybrid baryons are hypothetical three quark states with gluonic degrees of freedom. Their existence is allowed by Quantum Chromo Dynamics (QCD), and Lattice QCD (LQCD) calculations predict the existence of several hybrid states 1.3 GeV above the nucleon ground state. Hybrid and regular baryons are expected to have same quantum numbers, but the information from the electrocouplings can be exploited to identify their nature. In fact theoretical models predict a peculiar behavior of baryons electrocouplings at low Q^2 : a fast drop of the transverse helicity amplitude $A_{1/2}(Q^2)$ and a suppression of the longitudinal helicity amplitude $S_{1/2}(Q^2)$. The same studies permitted to understand that the Roper was not a hybrid state. For these reasons it is crucial to study the meson electroproduction in the kinematic region dominated by the formation of resonances.

CLAS12 setup in Hall B at Jefferson Lab (Newport News (VA, USA)) is particularly suitable for this task, since it is able to detect scattered electrons at low polar angles thanks to the Forward Tagger (FT) component. The FT detector is composed of a hodoscope, a calorimeter and a tracker to distinguish between electrons and photons scattered at low angles, measure their energy and determine their polar and azimuthal angles.

The process that we propose to study is $ep \rightarrow e'K^+\Lambda$, where the electron beam will be provided by the CEBAF accelerator with energies of 6.6, 8.8 and 11 GeV. The electrons will be detected by the FT for polar angles between 2.5 and 4.5 degrees, and by the CLAS12 detector for angles higher than 6° , while the hadrons will be detected by CLAS12 only. In order to properly design the experiment it is necessary to establish the sensitivity of the experimental setup to the presence of hybrid resonances, and to determine the dependence of the sensitivity on the resonance parameters values.

This thesis work describes the setup and calibration of the FT calorimeter and the studies related to the search of hybrid baryons through the measurement of the $K^+\Lambda$ electroproduction cross section.

Concerning the first main topic, I participated to the measurements of light transmission of the 332 lead tungstate crystals that compose the FT calorimeter, using the spectrophotometer that is available in the irradiation facility of Giessen University, in Germany, and to the subsequent data analysis. It is important to obtain the calibration constants of the calorimeter, in particular the corrections that are necessary to switch from the energy that is reconstructed by the calorimeter to the energy that was really deposited in it. To obtain these corrections I exploited the GEMC Geant Monte Carlo to simulate events in the FT, and then I analyzed the output to establish the corrections as a function of the polar angle and of the particles energy.

Regarding the studies about hybrid baryons, I participated to the simulation of $ep \rightarrow e'K^+\Lambda$ events using a FASTMC, in order to find the best run conditions for the experiment. Then I devoted the final part of my work

to study the experimental sensitivity to the presence of a hybrid resonance. For this purpose I added a hybrid resonance at the amplitude level to the theoretical model that was selected to describe the $K^+\Lambda$ electroproduction reaction, the Regge plus Resonance (RPR 2011) Ghent model, and then I performed a Legendre moments analysis and a blind analysis to identify the added resonance from the quasi-data.

This thesis work is divided as follows. In Chapter 1 hybrid baryons are defined, and the signature for a hybrid state is described. In Chapter 2 the procedure to implement a hybrid resonance at the amplitude level in the RPR-2011 model amplitude is shown. In Chapter 3 the experimental setup that will be used to search for hybrid baryons is explained, with a particular interest in the description of FT equipment, whose lead tungstate crystals assessment is reported in Chapter 4. In Chapter 5 the calibration procedure for FT electromagnetic calorimeter is described. In particular the procedure to obtain the calibration corrections to switch from the deposited to the real energy in the calorimeter is described. In Chapter 6 the generation and FASTMC reconstruction of $ep \rightarrow e'K^+\Lambda$ events is reported, and the optimal run conditions are achieved. Finally in Chapter 7 the theoretical studies reported in Chapter 2 are employed to perform sensitivity studies of the experimental setup to the presence of a hybrid resonance.

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Abbreviations

ADC	A nalog to D igital C onverter
APD	A valanche P hoto D iode
BST	B arrel S trip T racker
BTCP	B ogoriditsk T echnical C hemical P lant
CD	C entral D etector
CEBAF	C ontinuous E lectron B eam A ccelerator F acility
CLAS12	C EBAF L arge A cceptance S pectrometer 12 GeV
CM	C enter of M ass
CND	C entral N eutron D etector
CTOF	C entral T ime- O f- F light
DAQ	D ata A c Q uisition
d.o.f.	d egrees o f f reedom
DVCS	D eeply V irtual C ompton S cattering
EC	E lectromagnetic C alorimeter
EG	E vent G enerator
FASTMC	F AST M onte C arlo
FD	F orward D etector
FEC	F orward E lectromagnetic C alorimeters
FST	F orward S ilicon T racker
FT	F orward T agger
FT-Cal	F orward T agger F orward T agger
FT-Hodo	F orward T agger H odoscope
FT-Trck	F orward T agger T racker
FTOF	F orward T ime O f F light
GEMC	G Eant M onte C arlo
GUI	G raphical U ser I nterface
HPS	H eavy P hoton S earch
HTCC	H igh T hreshold C erencov C ounter
INFN	I stituto N azionale di F isica N ucleare

IC	I nn e r C alorimeter
JM	J lab- M oscow
LAB	L A B oratory
LAAPD	L arge A rea APD
LED	L ight E mitting D iode
LFRQM	L ight- F ront R elativistic Q uark M odel
LQCD	L attice Q uantum C hromo D ynamics
LTCC	L ow T hreshold C erencov C ounter
MSU	M oscow S tate U niversity
PCAL	P re-shower C A L orimeter
PMT	P hoto- M ultiplier T ube
PWO	PbWO ₄
QCD	Q uantum C hromo D ynamics
QED	Q uantum E lectro D ynamics
RPR-2011	R egge P lus R esonance- 2011
RTD	R esistance T emperature D etector
SICCAS	S hanghai I nstitute of C eramics, C hinese A cademy of S ciences
TDC	T ime to D igital C onverter

Physical Constants

Speed of Light	c	$=$	$2.997\,924\,58 \times 10^8 \text{ ms}^{-1}$ (exact)
Fine-structure constant	α	$=$	$\frac{e^2}{(4\pi\epsilon_0)\hbar c} = 0.0072973525664(17)$
Elementary charge	e	$=$	$1.6021766208(98) \times 10^{-19} C$
Irrational number Pi	π	$=$	3.1415
Electric constant	ϵ_0	$=$	$8.854187817... \times 10^{-12} F \cdot m^{-1}$
Reduced Planck constant	$\hbar$	$=$	$\frac{h}{2\pi}$
Planck constant	h	$=$	$6.626070040(81) \times 10^{-34} J \cdot s$

A Marco,

La mia scoperta più bella

Chapter 1

Hybrid Baryons

Introduction

The currently accepted theory of strong interactions among quarks is quantum chromodynamics (QCD).

According to QCD quarks are fermionic elementary particles that interact via the exchange of bosons called gluons. Like the electric charge in electromagnetic interactions, described by quantum electrodynamics (QED), gluons are massless vector particles of spin-parity $J^P = 1^-$, but, differently from photons, which are electrically neutral, gluons carry a charge called color. When the distance between quarks decreases, the force that binds them weakens, causing the *asymptotic freedom*. On the other hand, if the distance increases, the binding force gets stronger and above a certain threshold pairs of quarks and antiquarks form, a phenomenon known as *confinement*.

Since higher beam energies probe shorter distances, experiments performed at energies higher than 100 GeV imply the asymptotic freedom and allow the use of perturbation theory. On the contrary, a low energy of the beam may probe confinement effects (nucleon mass and its excited states, $\sim 1\text{-}2$ GeV) and the perturbation theory is not applicable anymore. In this situation a common approach is Lattice QCD (LQCD), which employs a grid of spacetime points (the lattice), tending to the continuum of QCD in the limit of a lattice infinitely large with infinitely close points. LQCD calculations have provided a proton mass reconstruction with a precision better than 2% [2].

The attraction between different quarks causes the formation of composite particles named hadrons: inside hadrons *valence quarks* determine their quantum numbers and spin properties, but other particles are also present that do not contribute, such as virtual quarks and antiquarks (called sea) and gluons. Two types of hadrons have been observed: quark and antiquark pairs, *mesons*, and three quark states, *baryons*.

Besides regular mesons and baryons a number of "exotic" hadrons have been conjecturized, states having more valence quarks (such as tetraquarks and pentaquarks), or valence gluons in their structure, *hybrid mesons* and *hybrid baryons*.

QCD not only allows for the existence of hybrid baryons, but LQCD predicts several hybrid states, with lowest

masses laying 1.3 GeV above the nucleon ground state. The search for hybrid baryons is more difficult than for hybrid mesons because of issues in distinguishing a hybrid from a regular baryon. In fact, differently from hybrid mesons, hybrid and regular baryons have same quantum numbers. A useful approach has been recently employed in the study of the Roper resonance $N(1440)\frac{1}{2}^+$, that has been considered as a candidate for the lowest mass hybrid baryon [3]. To dismiss this possibility [4] [5], meson electroproduction data were exploited, studying the photon virtuality (Q^2) evolution of the electrocouplings, that should present a different low Q^2 behavior for hybrids with respect to regular states. The ongoing program at the Thomas Jefferson National Accelerator Facility (Jefferson Laboratory) in Newport News (VA) employs photoproduction and electroproduction reactions to study the excitation of nucleons in the so-called nucleon resonance region and proved to be successful; moreover a program concerning the search for hybrid baryons is under development. This program is meant to vastly improve upon the available information regarding both hybrids and three-quark "missing" resonances. In particular the meson electroproduction kinematics can be extended to cover the full nucleon resonance mass range up to 3.5 GeV with a Q^2 range starting from values as low as 0.05 GeV^2 up to 7 GeV^2 , using electron beam energies of 6.6 GeV, 8.8 GeV and 11 GeV.

This Chapter focuses on the search for hybrid baryons, providing insights on the features that allow to identify them. Section 1.1 briefly describes the properties of strong interactions. Section 1.2 reports the theoretical models that describe hybrid baryons; a possible signature of their detection is described in Section 1.3; finally the experimental program that has been formulated to search for hybrid baryons, with the CLAS12 experimental setup in Hall B at the Thomas Jefferson National Accelerator Facility, is reported in Section 1.4.

1.1 Quarks and Gluons

Before describing in detail the studies regarding hybrid baryons the properties of the hadron spectrum are briefly summarized.

1.1.1 Quark Model

The study of the hadron spectrum is a very important tool to understand the properties of hadron constituents and of the forces acting among them.

At the beginning of the 1960s a plethora of hadron states had been observed leading to a theoretical impasse: were they all elementary particles? Large groups of hadrons presented some common properties, such as spin-parity, suggesting the presence of an underlying symmetry that would have explained the observation of these regularities. In 1964 M. Gell-Mann [6] and G. Zweig [7] came independently to the description of light hadrons via the $SU(3)$ flavor symmetry, or $SU(3)_f$. Gell-Mann and Zweig introduced a triplet of fermionic elementary particles, called quarks, characterized by a baryon number $B = \frac{1}{3}$ and suggested that the fundamental $SU(3)_f$

triplet consists of a doublet of 0 strangeness quarks with isospin $I = \frac{1}{2}$: the *up* quark u (having the third component of the isospin $I_z = +\frac{1}{2}$) and the *down* quark d ($I_z = -\frac{1}{2}$); and an isospin singlet, the *strange* quark s with -1 strangeness. The relationship between charge Q , hypercharge Y , baryon number B , strangeness S and the third component of the isospin I_z is given by the Gell-Mann Nishijima equation,

$$Q = \frac{1}{2}(B + S) + I_z = \frac{Y}{2} + I_z. \quad (1.1)$$

The conjugate triplet $\bar{3}$ consists of antiquarks with opposite sign of additive quantum numbers I_z , B and S . In Fig. 1.1 the fundamental triplet and its charge conjugate are represented.

There are six types of quark flavors: up, down, strange, charm, bottom and top. The first three flavors

FIGURE 1.1: Three quark triplet and its C conjugate [8].

correspond to light quarks, while the others to heavy quarks. In Table 1.1 the quantum numbers of light quarks and antiquarks are reported.

Light hadrons are built composing quarks and antiquarks following the $SU(3)_f$ algebra. When a quark is bound to an antiquark, a meson state is formed. When a quark couples to other two quarks, a baryon is produced.

1.1.1.1 Mesons

Mesons are hadrons that consist of a quark-antiquark couple. According to $SU(3)_f$ algebra (see [9] for details)

$$3 \otimes \bar{3} = 1 \oplus 8 \quad (1.2)$$

states composed by a quark-antiquark pair may be classified in multiplets consisting of a singlet plus an octet of fixed spin-parity values. The lowest energy octet of pseudo-scalar ($J^P = 0^-$) mesons is shown in Fig. 1.2.

Using the expressions holding for the total angular momentum J , the parity P and the charge conjugation parity C :

$$J = L \oplus S \quad (1.3)$$

$$P = (-1)^{L+1} \quad (1.4)$$

Flavour	Spin	Charge	I_z	Baryon number	Strangeness	Hypercharge
u	$\frac{1}{2}$	$+\frac{2}{3}$	$+\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$
d	$\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$
s	$\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	-1	$-\frac{2}{3}$
$\bar{u}$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$
$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{2}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$
$\bar{s}$	$\frac{1}{2}$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	+1	$+\frac{2}{3}$

TABLE 1.1: Quantum numbers of the light quarks and antiquarks.

$$C = (-1)^{L+S} \quad (1.5)$$

and the possible values for the orbital momentum L and the spin S , it is possible to see that not all the J^{PC} states are permitted. For instance if a state with $L = 0$ and $S = 0$ is considered, it will have $P = -1$ and $C = 1$, and, as a consequence, $J^{PC} = 0^{-+}$. States with $J^{PC} = 0^{+-}$ are not permitted. Hybrid mesons may, on the contrary, have exotic quantum numbers, i.e. J^{PC} values that are not allowed by the combination rules (1.3) (1.4) (1.5), due to the contribution of an extra gluon quantum numbers. This is the straightforward signature for a hybrid meson state. We will see that it is not possible to exploit this to identify hybrid baryons, since all J^P values can be populated by states built with three quarks.

1.1.1.2 Baryons

In the case of baryons, having a qqq structure, the following relation holds:

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10. \quad (1.6)$$

This means that all baryons could be arranged in a singlet, two octets and a decuplet with fixed J^P quantum numbers, but only the decuplet and the octet are compatible with the correct wave function symmetry with respect to the exchange of two quarks. In Figures 1.3 and 1.4 the $J^P = \frac{1}{2}^+$ octet and the $J^P = \frac{3}{2}^+$ decuplet are shown respectively.

Baryons are classified into groups according to their isospin values and quark content. There are six groups of baryons, nucleon (N), Delta (Δ), Lambda (Λ), Sigma (Σ), Xi (Ξ) and Omega (Ω), defined as follows [10],

FIGURE 1.2: The meson lowest mass nonet of pseudo-scalar mesons; in parentheses the masses in MeV [8].

- Baryons with three u and/or d quarks are N's ($I = \frac{1}{2}$) or Δ 's ($I = \frac{3}{2}$).
- Baryons with two u and/or d quarks are Λ 's ($I = 0$) or Σ 's ($I = 1$). If the third quark is heavy, its identity is given by a subscript.
- Baryons with one u or d quark are Ξ 's ($I = \frac{1}{2}$). One or two subscripts are used if one or both of the remaining quarks are heavy.
- Baryons with no u or d quarks are Ω 's ($I = 0$) and subscripts indicate any heavy quark content.

As already said, it is not possible to construct hybrid baryons states with J^{PC} different from the ordinary baryons, so, differently from the mesons case, it is not possible to identify hybrid baryons via their J^{PC} quantum numbers.

1.1.2 Color Charges

The bosons mediating the quark-quark interactions are called gluons. Color is the charge of the strong interactions. Each gluon carries a color and an anti-color and belongs to an octet of states; since gluons are charged they can self-interact. Quarks carry colors and anti-quarks carry anti-colors. There are three possible colors: red, green and blue. Only color singlets states may exist: quarks and gluons form colorless baryons and mesons. This phenomenon is called confinement.

Since gluons can self-interact, the expansion of a strong interaction vertex must include both fermion and gluon loops, as shown in Figure 1.5. QCD exhibits a very different behavior depending on the distance between quarks.

FIGURE 1.3: The $J^P = \frac{1}{2}^+$ baryon octet; in parentheses the masses in MeV [8].

FIGURE 1.4: The $J^P = \frac{3}{2}^+$ baryon decuplet; in parentheses the masses in MeV [8].

FIGURE 1.5: The lowest-order diagrams that contribute to the QCD vertex [8].

When the distance increases, quark–antiquark pairs come out of the vacuum and shield the color charge, reducing its value. On the contrary, gluons produce a smearing of the color charge, that induces an opposite effect, called ‘antiscreening’. The combination of these two components results in a decrease of color charge with decreasing distance. D. Politzer, D. Gross and F. Wilczek discovered this property theoretically in 1973.

The strong coupling is shown as a function of the distance d and of the four-momentum transfer q , in Figures 1.6(a) and 1.6(b), respectively.

FIGURE 1.6: Left: α_s evolution with distance ($1 \text{ fm} = 10^{-15} \text{ m}$) [11]. Right: α_s evolution with Q [12].

The α_s value is running, i.e. it changes depending on four-momentum transfer [8] and it can be expressed as

$$\alpha_s(q^2) \approx \frac{12\pi}{(33 - 2n_f)\ln(q^2/\Lambda_{QCD}^2)} \quad (1.7)$$

where n_f is the number of flavors and Λ_{QCD} is the QCD energy scale. The Λ_{QCD} parameter is important because it separates two energy regimes. At high q^2 , i.e. energies higher than Λ_{QCD} and short distances, the quarks behave as if they were free, a phenomenon known as asymptotic freedom, and the perturbation theory is applicable. At low q^2 , i.e. energies lower than Λ_{QCD} and large distances, α_s becomes large and perturbation theory becomes meaningless. In this regime we have the confinement.

1.2 Hybrid Baryons Theoretical Studies

Several groups devoted studies to hybrid baryons, providing predictions about quantum numbers and mass range of the lowest states. In this Section the most relevant theories, including the LQCD, that plays a prominent role, are briefly listed and described.

1.2.1 Model Predictions

Before LQCD several models predicted the existence of gluonic three quark states. They suggested experimental accessible mass ranges and quantum numbers for hypothetical lowest energy hybrid states, but because of the issues in distinguishing hybrid from regular baryons, both theoretical and experimental searches remained suspended for thirty years.

Among the theoretical models that were formulated we recall an extension of the MIT bag model of 1983 [13]. Hybrid baryons emerge as gluonic excitations of the nucleon to states where a constituent gluon in the lowest energy transverse electric mode combines with three quarks in a color octet state to form a colorless state. According to this model the lowest energy state should have a mass in the range 1.600 ± 0.100 GeV and $J^P = \frac{1}{2}^+$ as the proton or the Roper.

Another model that predicted same quantum numbers but higher masses was the gluon flux-tube applied to hybrid baryons [14] [15]. Its lowest mass hybrid baryon was 1.870 ± 0.100 GeV.

Reference [16] considered hybrid baryons in the Large N_c approximation of QCD for heavy quarks, also providing the justification of the constituent glue picture used in the models.

Finally hybrid baryons were described by [17] in a high energy regime.

1.2.2 Lattice QCD Predictions

First Lattice QCD predictions regarding hybrids were formulated in 2003 [18]. These calculations used the quenched approximation, i.e. the quantum loops of fermions in Feynman diagrams were neglected. According to them the lowest hybrid baryon mass was around 2 GeV, 1 GeV above the nucleon mass. In 2012 the first unquenched LQCD calculations were proposed [19]: they provided the full light-quark baryon spectrum including the projections of the hybrid nucleon N_G states and hybrid Δ_G states. As already said both N_G and Δ_G are baryons with three u and/or d quarks; the difference concerns their isospin status: $I = \frac{1}{2}$ for N_G and $I = \frac{3}{2}$ for Δ_G . Figure 1.7 shows the extracted spectrum of Nucleon and Delta states in the lowest mass range [19]. In this notation, the N^* and Δ^* denote excited states of the Nucleon and Delta fundamental states and are considered as different particles, or resonances. Grey boxes are regular states and blue boxes are hybrid states. The corresponding negative parity hybrid states should occur at higher masses.

Considering that in this spectrum the mass of the pion is almost 300 MeV above the real one, all masses should be decreased by 300 MeV to obtain a more realistic evaluation of the real values. This mass shift will be considered in all quantitative estimations of the hybrid spectrum in the work described in this thesis. Another characteristic

FIGURE 1.7: The light-quark baryon spectrum predicted in Lattice QCD at a pion mass of 396 MeV [19]. Grey boxes are conventional qqq states and the blue shaded boxes indicate hybrid baryons states. Note that both the mass of the nucleon ground state and of the $\Delta(1232)$ are shifted by nearly 300 MeV to higher masses.

of this spectrum is the position of the hybrid states: they are grouped in a restricted mass range of ~ 300 MeV in an energy band not occupied by ordinary baryons. The first hybrid baryon should be 1.3 GeV above the nucleon ground state, with $J^P = 1/2^+$, and if its width is not too broad it should be possible to distinguish it from ordinary states.

In this conditions we expect to find the lowest hybrid state in a mass range of 2.2 - 2.3 GeV and a first group of $J^P = 1/2^+$ and $J^P = 3/2^+$ states in a 2.2 - 2.5 GeV band.

1.3 Signature

Besides the mass range prediction that has just been described, the signature to distinguish a hybrid baryon from an ordinary one arises from the study of their electromagnetic couplings.

In this Section, after a brief description of hadronic couplings, the crucial role of electromagnetic couplings will be explained and finally the characteristic signatures, useful for the quest of hybrid baryons, will be listed.

1.3.1 Hadronic Couplings

Unfortunately there is a lack of information regarding possible hadronic couplings of hybrid baryons. From the knowledge of the hadronic couplings one could select the best channel for an experimental study, but without theoretical information any experimental approach must rely on a set of assumptions. Final states with significant

gluonic admixture, e.g. $B_G \rightarrow N\eta'$ [20], should be preferential as well as final states containing $s\bar{s}$ contributions due to the $G \rightarrow s\bar{s}$ coupling, e.g. $B_G \rightarrow K^+\Lambda$, $B_G \rightarrow N^*(1535)\pi \rightarrow N\eta\pi$, $B_G \rightarrow N\pi\pi$, $B_G \rightarrow \phi(1020)N$ and $B_G \rightarrow K^*\Lambda$. Among all possible candidate final states, the $K^+\Lambda$ has been selected to study the sensitivity of the electroproduction reaction to the formation of a hybrid resonance, since it is the easiest channel among the listed ones for which a theoretical description is available. Assuming hadronic couplings of a few percent, the task is to identify the $K^+\Lambda$ final state and to study the electromagnetic coupling dependence of the hybrid state formation as a function of the photon virtuality $Q^2 = -q^2$, where q is the virtual photon four-momentum.

1.3.2 Electromagnetic Couplings

The electroproduction is the production of particles from the interaction of high energy electrons with nucleons and, similarly, photoproduction is the production of particles from the absorption of high energy photons. Nucleon resonances may be formed by the absorption of a real or virtual photon by the nucleon as shown in Figure 1.8.

FIGURE 1.8: The absorption of a real or virtual photon by the nucleon induces the production of a composite state decaying in final state particles [21].

While photoproduction proved to be an efficient instrument to search for new baryon states, electroproduction allows to probe the interior of the formed excited states and to understand their intrinsic spatial and spin structures. Meson electroproduction in particular provides insights on the active degrees of freedom underlying the structure of the excited states and their scale dependencies. The electromagnetic couplings, the so called helicity amplitudes, for the electroexcitation of baryon resonances, $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ are calculated as the transition matrix elements of the transverse and longitudinal electromagnetic interaction between the nucleon and the resonance states. When plotting the electromagnetic couplings of the nucleon resonances as a function of Q^2 , two behaviours are recognized, for short and large distances, corresponding to virtual photons with high and low Q^2 , respectively. In the first Q^2 range the resonance electromagnetic couplings are sensitive to the momentum dependence of the light quark masses; in the second Q^2 range the role of quark-gluon dressing and meson-cloud effects may be explored.

At present, studies have been performed concerning the electrocouplings of several resonances, such as the $\Delta(1232)\frac{3}{2}^+$, the Roper $N(1440)\frac{1}{2}^+$, the $N(1520)\frac{3}{2}^-$, the $N(1535)\frac{1}{2}^-$, the $\Delta(1600)\frac{3}{2}^+$, the $N(1675)\frac{5}{2}^+$ and the $N(1680)\frac{5}{2}^-$. For instance, from the study of $\Delta(1232)\frac{3}{2}^+$ [22], a clear evidence was achieved that the resonance

is not excited from quark transitions alone, but meson-baryon interactions can significantly contribute as well. Both processes participate to the excitation of the same state. In fact for $\Delta(1232)\frac{3}{2}^+$ only 2/3 of the transverse helicity amplitudes at the photon point ($Q^2 = 0$) can be interpreted as the quark contribution of the state. For $Q^2 \geq 3 \text{ GeV}^2$ the quark content is nearly exhausting the measured strength. Other resonances, such as the $N(1440)\frac{1}{2}^+$ and the $N(1520)\frac{3}{2}^-$, present similar features.

The study of electromagnetic couplings evolution with Q^2 is expected to provide critical information in the search for hybrid baryons. The Roper $N(1440)\frac{1}{2}^+$ is a particularly important example since we expect the lowest mass hybrid state to be an isospin- $\frac{1}{2}$, $J^P = \frac{1}{2}^+$ resonance. The photoexcitation of the hybrid Roper resonance $N(1440)\frac{1}{2}^+$ was explored in Reference [23] and the electroproduction transition form factors of a hybrid Roper state were evaluated in Reference [24]. The aim was to understand whether the Roper was a hybrid baryon state or a regular three-quark state. Because of their different internal structure, as well as their different spin-flavor structure, studies of transition properties, especially of the electromagnetic transition form factors, may prove very effective in distinguishing between these states. Important phenomenological consequences for the electromagnetic transition, in particular for the Q^2 dependence of the electroproduction helicity amplitudes, are expected. The calculations in [24] in the hypothesis of a hybrid nature of the $N(1440)\frac{1}{2}^+$ describe a fast drop of transverse helicity amplitude $A_{1/2}(Q^2)$ at low Q^2 and a longitudinal helicity amplitude $S_{1/2}(Q^2) = 0$ at all Q^2 . They also foresee that the $A_{1/2}(Q^2)$ should behave in the same way both for the Roper and for the $\Delta(1232)$. Figure 1.9 shows the experimental information available up to 1992 for both the transverse $A_{1/2}(Q^2)$, Figure 1.9(a) and longitudinal $S_{1/2}(Q^2)$, Figure 1.9(b), electrocouplings for the transitions $\gamma p \rightarrow P_{11}(1440)$.

FIGURE 1.9: Q^2 dependence of the transverse $A_{1/2}$ and longitudinal $S_{1/2}$ electrocouplings [24]. Left: $A_{1/2}(Q^2)$ for the transition $\gamma p \rightarrow P_{11}(1440)$ for a three-quark state Roper [25] (long-dashed line), or for a hybrid baryon state (solid line), respectively. The short-dashed and the dash-dotted lines represent different results of the analysis by Gerhardt [26]. Note that the analysis was constrained by fixing the amplitude at the photoproduction point. The data points indicate the results of fits at fixed Q^2 . Gerhardt, diamond symbols; Boden and Kroen [27], cross symbols. Right: $S_{1/2}(Q^2)$ for the transition $\gamma p \rightarrow P_{11}(1440)$ for a three-quark state Roper (long-dashed line). The short-dashed and dash-dotted lines represent different results of the analysis by Gerhardt. $S_{1/2}(Q^2)$ is zero for a hybrid state. Data from Gerhardt [26] (squares) and from Boden and Kroen [27] (crosses).

Squares and crosses symbols are the results of the world data analyses by Gerhardt [26], and Boden and Kroen

[27], respectively. Data are compared with theoretical predictions for the cases when the Roper is described as a three-quark state [25] (long-dashed line), or a hybrid baryon state [24] (solid line), respectively. The short-dashed and the dash-dotted lines represent different results of the analysis by Gerhardt [26]. We note that $S_{1/2}(Q^2)$ is predicted to be zero for a hybrid state. Data available at the time were more consistent with a hybrid baryon description than with a radially excited three-quark state. Exploiting data with $Q^2 > 0.2 \text{ GeV}^2$ from more recent CLAS experiments [5] [28] and recalling the expectations regarding the electrocouplings evolution with Q^2 reported in [24], the Roper puzzle was solved. In fact, considering the comparison between data and predictions from [24] it is possible to eliminate the Roper from the candidates for a hybrid baryon, as shown in Figure 1.10 from Reference [5].

In this figure the full circles are the results obtained in [5] from CLAS data, the bands show the model uncertainties.

FIGURE 1.10: Helicity amplitudes for the $\gamma^* p \rightarrow N(1440)P_{11}$ transition. The full circles are the results obtained in [5] from CLAS data. The bands show the model uncertainties. The open boxes are the results of the combined analysis of CLAS single π and 2π electroproduction data [28]. The full triangle at $Q^2 = 0$ is the RPP estimate [29]. The thick curves correspond to the results obtained in the LF relativistic quark models assuming that $N(1440)P_{11}$ is a first radial excitation of the 3-quark ground state: [30] (dashed), [31] (solid). The thin dashed curves are obtained assuming that $N(1440)P_{11}$ is a gluonic baryon excitation (q^3G hybrid state).

uncertainties, the open boxes are the results of the combined analysis of CLAS single π and 2π electroproduction data [28], the full triangle at $Q^2 = 0$ is the RPP estimate [29], the thick curves correspond to the results obtained in the light-front relativistic quark models (LF RQM) assuming that $N(1440)P_{11}$ is a first radial excitation of the 3-quark ground state: [30] (dashed), [31] (solid). Finally the thin dashed curves are obtained assuming that $N(1440)P_{11}$ is a hybrid baryon. It is evident for both amplitudes that data points are inconsistent with the hybrid predictions, especially in the longitudinal helicity amplitude case, where in the hybrid case a suppression was expected and large values occur instead.

Similar results have been obtained for the other candidate as the lowest mass gluonic Δ_G hybrid baryon, the $\Delta(1600)\frac{3}{2}^+$. The theoretical predictions reported in [24] are similar to the ones obtained for the Roper, a fast falling transverse helicity amplitude $A_{1/2}(Q^2)$ and a suppression of the longitudinal helicity amplitude $S_{1/2}(Q^2)$.

In every case the region where the difference between hybrid and ordinary baryons is more evident is at low photon virtualities, $Q^2 < 2.0 \text{ GeV}^2$: to distinguish hybrid from ordinary states it is crucial to map their electrocoupling behavior in this region.

1.3.3 Helicity Amplitudes Extraction from $N^* \rightarrow K^+ \Lambda$ Process

This thesis work is dedicated to the study of the $K^+ \Lambda$ electroproduction on the proton, as displayed in Fig. 1.11.

FIGURE 1.11: Considered process: an electron is scattering, producing a photon. This photon is absorbed by a proton in the target and a composite state is produced, which then decays in a $K^+ \Lambda$ final state.

An electron is scattered, producing at first order of QED expansion one photon. This photon is absorbed by a proton target and a composite state is produced, which then decays into a $K^+ \Lambda$ final state. The hadronic part of the process can be decomposed in different terms, as it is possible to see in Fig. 1.12.

FIGURE 1.12: Components of the hadronic part of the process showed in Fig. 1.11 [21].

Following the approach of Reference [21] the resonant contribution in the helicity representation can in turn be expressed exploiting a relativistic Breit-Wigner (BW) ansatz for the N^* hadronic amplitudes; the resonance electroexcitation amplitudes may be then expressed in the form:

$$\langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle = \sum_{N^*} \frac{\langle \lambda_f | T_{dec} | \lambda_R \rangle \langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle}{M_r^2 - W^2 - i \Gamma_r(W) M_r} \quad (1.8)$$

where λ_f , λ_γ and λ_p are the helicities of the final hadrons, of the photon and of the proton respectively, $\Gamma_r(W)$ is the resonance width, M_r is the resonance mass and W is the center-of-mass energy.

The N^* hadronic decay amplitudes can be expanded in partial waves of total momentum J

$$\langle \lambda_f | T_{dec} | \lambda_R \rangle = \langle \lambda_f | T_{dec}^{J_r} | \lambda_R \rangle d_{\mu\nu}^{J_r}(\cos\theta^*) e^{i\mu\phi^*} \quad (1.9)$$

where

$$\langle \lambda_f | T_{dec}^{J_r} | \lambda_R \rangle = \frac{2\sqrt{2}\pi\sqrt{2J_r+1}M_r\sqrt{\Gamma_{\lambda_f}}}{\sqrt{\langle p_i^r \rangle}}. \quad (1.10)$$

On the other hand the resonance electroexcitation amplitudes can be related to the $\gamma_v NN^*$ electrocouplings $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$

$$\langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle = \frac{W}{M_r} \sqrt{\frac{8M_N M_r q_{\gamma_r}}{4\pi\alpha}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} A_{1/2,3/2}(Q^2) \quad (1.11)$$

with $|\lambda_\gamma - \lambda_p| = \frac{1}{2}, \frac{3}{2}$ for transverse photons and

$$\langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle = \frac{W}{M_r} \sqrt{\frac{16M_N M_r q_{\gamma_r}}{4\pi\alpha}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} S_{1/2}(Q^2) \quad (1.12)$$

for longitudinal photons.

Performing a partial wave analysis it is possible to obtain the electrocouplings coefficients $A_{1/2}$ and $S_{1/2}$ from the differential cross section, that allow us to distinguish ordinary baryons from hybrid baryons. This method is an extension of the Bonn-Gatchina approach [32] [33] [34] to the analysis of exclusive electroproduction channels to extract the resonance parameters and search for new baryon states. The Bonn-Gatchina approach consists in an advanced amplitude analysis for extraction of the nucleon resonance parameters from the global analysis of photoproduction data, via the decomposition of the production amplitudes over a set of partial waves. This method has been applied to world data, a large part of which measured with the CLAS apparatus at the Jefferson Laboratory and has provided information on masses, widths, photocouplings and hadronic decay parameters for most excited nucleon states in the mass range up to 3 GeV. The Bonn-Gatchina approach allowed the discovery on 8 new baryon states, as reported in PDG [10].

1.3.4 Signature for Hybrid Baryon States

Summarizing our understanding, based on available knowledge, we may state that the signature for the lower mass hybrid baryons may consist of :

- Extra resonances with $I = \frac{1}{2}$, $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, with masses from 1.8 GeV to 2.5 GeV and decays to $N\pi\pi$, $K^+\Lambda$ or $K^+\Sigma^0$ final states;
- A drop of the transverse helicity amplitudes $A_{1/2}(Q^2)$ and $A_{3/2}(Q^2)$ as a function of Q^2 faster than for ordinary three quark states, because of extra glue-component in valence structure;

- A suppressed longitudinal amplitude $S_{1/2}(Q^2)$ in comparison to transverse electro-excitation amplitudes.

A better comprehension of the mass range will arrive from new LQCD calculations, as masses may shift to lower values with a more realistic mass of the pion.

A new experiment requiring high statistics will allow to identify any new or poorly known state, whether it is a hybrid or not.

1.4 Strategy for a New CLAS12 Experiment

To plan a new experiment to search for hybrid resonances using the CLAS12 detector, it is important to verify that the expected data quality will allow to observe a signal from candidates for hybrid baryon states under realistic conditions including luminosity, efficiency, resolution, as well as accounting for the non-resonant mechanisms and regular three-quark N^* state contributions to the exclusive channels. Steps to be taken to study the $K^+\Lambda$ channel are:

- Development of a realistic event generator (EG) for $K^+\Lambda$ and $K^+\Sigma^0$ exclusive electroproduction channels, which incorporates/takes-off the contribution from a hybrid baryon candidate;
- Simulation of quasi-data events including all experimental effects from CLAS12 assuming that hybrid baryon states are contributing/taken-off;
- Extraction of quasi-measured cross sections and polarization asymmetries for $K^+\Lambda$ exclusive electroproduction from simulated quasi-data events;
- Evaluation of the hybrid electrocouplings from these quasi-measured $K^+\Lambda$ observables

The model employed to describe the KY electroproduction channel is the "Regge-Plus-Resonance" developed by the Ghent group [35]. It describes the reaction amplitude as the superposition of tree-level s-channel nucleon resonances on the non-resonant contribution. A set of 11 nucleon resonances has been considered to determine the optimum resonant contribution in the RPR $p(\gamma, K^+)\Lambda$ framework. The model with the highest Bayesian evidence, called RPR-2011, includes the resonances S_{11} (1535), S_{11} (1650), F_{15} (1680), P_{13} (1720), P_{11} (1900), P_{13} (1900), D_{13} (1900) and F_{15} (2000). The non-resonant component is the sum of t-channel exchanges of Reggeized K^+ (494) and K^{*+} (892). The model is able to properly reproduce photoproduction data, but provides only an approximate description of the electroproduction ones. The plan is to employ the RPR model to extract the resonance electrocouplings from KY electroproduction data once the model will be upgraded to properly describe data. The upgrade was announced in Reference [36]. In Chapter 2 the implementation of a hybrid resonance contribution to the RPR reaction amplitudes will be shown, to be used in Chapter 7 for the sensitivity studies.

1.4.1 Experimental Equipment

This thesis considers the possibility to perform a new experiment with the CLAS12 apparatus, which is at present under construction at Thomas Jefferson Laboratory. The facility will be described in more detail in Chapter 3; here we summarize the main experimental features.

CLAS12 detector, shown in Fig. 1.13, is designed to work with longitudinally polarized high energy electron beams scattering on a liquid-hydrogen target, with an operating luminosity of $\mathcal{L} = 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ which correspond to a total hadronic rate of $5 \times 10^6 \text{ s}^{-1}$.

The CLAS12 detector is highly hermetic, provides high multiplicity reconstruction, can run at high luminosity

FIGURE 1.13: Schetch of the CLAS12 detector: it consists of a central detector (CD) equipped with a superconducting 5T solenoid and a forward detector equipped with a Torus magnet, to cover polar angular ranges from 35° to 125° and from 5° to 35° , respectively.

and is best suited to carry out the experiment under study. An upgrade of the former CLAS detector, CLAS12 presents improved particle identification (id) with a front time of flight detector (FTOF) for charged particle id, improved Cherenkov counter for electron id and for π and K separation. It consists of a Forward Detector (FD), covering the forward polar angle range between 5° and 35° , and a Central Detector (CD), covering larger angles up to 125° . The first angular region is equipped with a Torus magnet, while the central polar angle range of 35° to 125° is provided with a 5T Solenoid magnet.

Charged hadrons may be detected in the whole polar angle range from 5° to 125° . To extend the detection of scattered electrons to even lower polar angles, CLAS12 includes the Forward Tagger (FT) experimental setup, shown in Fig. 1.14; it allows to detect electrons and photons emitted between 2.5 and 4.5 degrees.

The electron beam interacts with the target via the exchange of a virtual photon, whose four momentum $q = (\nu, \mathbf{q})$ is the difference between the beam and the scattered electron momenta. It is possible to express the photon

FIGURE 1.14: The Forward Tagger (FT) system. The FT provides electron and high energy photon detection in a range of polar angles $\theta = 2.5^\circ - 4.5^\circ$ and will be fully integrated into the operation of CLAS12.

virtuality Q^2 as a function of the electron scattering angle θ_e and of the energy of the final electron $E_{e'}$. To this aim, first we consider the photon four-momentum definition

$$q^2 = (p_e - p_{e'})^2 = m_\gamma^2 c^4 \quad (1.13)$$

where m_γ is the virtual photon mass and

$$p_e = \begin{pmatrix} E_e \\ \mathbf{p}_e \end{pmatrix}, \quad p_{e'} = \begin{pmatrix} E_{e'} \\ \mathbf{p}_{e'} \end{pmatrix} \quad (1.14)$$

are the four-momenta of the electron before and after the interaction with the target, respectively. We then recall the relationship between Q^2 and q^2 ,

$$Q^2 = -q^2, \quad (1.15)$$

and the relationship among energy, momentum and mass

$$E = \sqrt{c^2 p^2 + c^4 m^2}. \quad (1.16)$$

We obtain that:

$$Q^2 = -q^2 = -(p_e - p_{e'})^2 = 2E_e E_{e'} + m_e^2 + m_{e'}^2 - 2\mathbf{p}_e \mathbf{p}_{e'} = 2(E_e E_{e'} - |\mathbf{p}_e| |\mathbf{p}_{e'}| \cos \theta_e). \quad (1.17)$$

Considering that for high energies electrons, $E_e \gg m_e c^2$, we also neglected the electron mass. Using the proper prosthaphaeresis formula we finally obtain:

$$Q^2 = 4E_e E_{e'} \sin^2 \frac{\theta_e}{2}. \quad (1.18)$$

Events for which electron scattering occurs at very low angles, such as the ones detected by FT, also may have a low Q^2 (the electroproduction at this Q^2 can be considered for practical purposes photoproduction). To be precise by using electron beams of variable incoming energies $E_e = 6, 8$, or 12 GeV and detecting the scattered electron in the angular range from 2.5° to 135° it is possible to span Q^2 virtualities between 0.05 GeV^2 and 7 GeV^2 . The lowest Q^2 range is reachable using the lowest beam energy and detecting scattered electrons in the FT. Considering the importance of events at low Q^2 , the FT is therefore a crucial component for this study. In Chapter 3 both CLAS12 and FT apparatus will be described in detail.

Chapter 2

$K^+\Lambda$ Electroproduction

Introduction

$K^+\Lambda$ electroproduction is a promising technique for the hybrid baryon search. Although $K^+\Lambda$ photoproduction is easier to treat theoretically than $K^+\Lambda$ electroproduction and is thus more appropriate for a detailed quantitative analysis, the electroproduction reaction is potentially a much richer source of information concerning the electromagnetic excitation amplitude of hadronic resonances. The access to single and double polarization observables in meson photoproduction reactions has proved to be effective in assessing the existence of resonances weakly coupled to πN channels. A special role has been played by $K^+\Lambda$ photoproduction. One reason is that the final state polarization is measurable thanks to the analysing power of the parity-violating weak Λ hyperon decay. When Λ decays into $\pi^- p$ (see Table 6.1 for a complete list of Λ decays), the proton is produced with a preferred direction relative to the polarization direction of the Λ baryon. This allows to reconstruct the Λ polarization from the angular distribution of its decay products:

$$W(\cos\theta_P) = \frac{1}{2}(1 + \alpha|P_\Lambda|\cos\theta_P) \quad (2.1)$$

where θ_P is the proton decay angle with respect to the center of mass of the decaying lambda and $\alpha = 0.642 \pm 0.013$ [37].

Moreover, the creation of a $s\bar{s}$ pair in the final state has proved to be a powerful way of exciting baryon resonances, not accessible through πN scattering reactions. In the last four years ten new resonances have been identified and reported in the Review of Particle Physics [10] [38], where the analysis of $K^+\Lambda$ photoproduction data has been crucial.

Electroproduction of $K^+\Lambda$ is expected to provide information on the possible presence of hybrid resonances thanks to the Q^2 dependence of the electrocouplings.

The purpose of this Chapter is to describe our knowledge of $K^+\Lambda$ electroproduction both from a theoretical and an experimental point of view. The theoretical description of $K^+\Lambda$ electroproduction which will be used in this

work is the RPR-2011 [35] [39] model. The experimental $K^+\Lambda$ electroproduction has been already explored, for instance in [40], providing measurements of the separated structure functions σ_U , σ_{LT} , σ_{TT} and $\sigma_{LT'}$ for the $K^+\Lambda$ and $K^+\Sigma^0$ final states, at a beam energy of 5.499 GeV, spanning W from threshold to 2.6 GeV, Q^2 from 1.4 to 3.9 GeV^2 and nearly the full center of mass angular range of the kaon.

Since no experimental data are available in the low Q^2 region, a new experiment is needed.

To study the sensitivity of $K^+\Lambda$ process to the possible excitation of a hybrid resonance, it is necessary to study the cross section evolution with and without the contribution of the resonance at the amplitude level. While sensitivity studies will be performed in Chapter 7, here the implementation of a hybrid resonance will be described.

In this Chapter the hybrid resonance has been added at the amplitude level to the RPR-2011 model and the cross sections have been obtained.

In Section 2.1 the kinematics of $ep \rightarrow e'K^+\Lambda$ process is described. Section 2.2 shows the theoretical model employed in this thesis work for $K^+\Lambda$ electroproduction. In Section 2.3 the available experimental data for this channel are described. Section 2.4 is devoted to the achievement of the connection between cross section and amplitude. In Section 2.5 the hybrid baryon amplitude is calculated for both $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. The helicity tensors are computed in Section 2.6. Finally, the cross section with the implementation of a hybrid resonance at the amplitude level is presented in Section 2.7.

2.1 Kinematics of Electroproduction

As already said, the electroproduction is the production of particles from the interaction of high energy electrons with nucleons.

We are considering the electroproduction process shown in Figure 2.1: an incoming electron that scatters off a

FIGURE 2.1: Electroproduction process with a $K^+\Lambda$ final state via a virtual photon exchange.

nucleon through the exchange of a virtual photon, whose absorption by the nucleon results in a lambda hyperon and a kaon in the final state, along with the scattered electron.

The process can be modeled dividing it into two parts, a leptonic one, with the initial electron that scatters producing a virtual photon,

FIGURE 2.2: Leptonic part of the process shown in Figure 2.1.

and a hadronic one, with the same photon being absorbed by a proton in the target, and with this composite state decaying into a $K^+\Lambda$ final state.

FIGURE 2.3: Hadronic part of the process shown in Figure 2.1.

The common way to describe this double system is to represent the leptonic part in the laboratory frame and the hadronic one in its center of mass (CM) frame (where the proton-photon composite state is at rest), as shown in Fig. 2.4.

FIGURE 2.4: Kinematics for $K^+\Lambda$ electroproduction off the nucleon. The initial and final electrons define the leptonic or scattering plane. The propagation directions of the virtual photon and the outgoing kaon determine the hadronic or reaction plane.

The particles defined in Figure 2.4 in the hadronic CM frame have the following four-vectors ¹:

$$k^* = \begin{pmatrix} \omega^* \\ \mathbf{k}^* \end{pmatrix}, p_N^* = \begin{pmatrix} E_P^* \\ \mathbf{p}_P^* \end{pmatrix} = \begin{pmatrix} E_P^* \\ -\mathbf{k}^* \end{pmatrix}, p_K^* = \begin{pmatrix} E_K^* \\ \mathbf{p}_K^* \end{pmatrix}, p_\Lambda^* = \begin{pmatrix} E_\Lambda^* \\ \mathbf{p}_\Lambda^* \end{pmatrix} = \begin{pmatrix} E_\Lambda^* \\ -\mathbf{p}_K^* \end{pmatrix} \quad (2.2)$$

and in laboratory frame the particles four-momenta are:

$$\begin{aligned} p_e^{LAB} &= \begin{pmatrix} E_e^{LAB} \\ \mathbf{p}_e^{LAB} \end{pmatrix}, p_{e'}^{LAB} = \begin{pmatrix} E_{e'}^{LAB} \\ \mathbf{p}_{e'}^{LAB} \end{pmatrix}, k^{LAB} = \begin{pmatrix} \omega_{LAB} \\ \mathbf{k}_{LAB} \end{pmatrix} \\ p_N^{LAB} &= \begin{pmatrix} m_P \\ 0 \end{pmatrix}, p_K^{LAB} = \begin{pmatrix} E_K^{LAB} \\ \mathbf{p}_K^{LAB} \end{pmatrix}, p_\Lambda^{LAB} = \begin{pmatrix} E_\Lambda^{LAB} \\ \mathbf{p}_\Lambda^{LAB} \end{pmatrix}. \end{aligned} \quad (2.3)$$

A consideration concerning the number of degrees of freedom (d.o.f.) of the system is necessary. The initial state is completely determined, with the virtual photon going in the z-axis direction. All the masses in the final state are fixed, therefore we have $3 \times 3 = 9$ (electron, lambda and kaon three-momenta) variables. But we have four constraints, from the conservation of four-momenta,

$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}_{initial\ state} = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}_{final\ state} \quad (2.4)$$

therefore $9 - 4 = 5$ d.o.f to describe the system. The angle Φ fixes the azimuthal orientation of the leptonic plane in the laboratory system and coincides with the azimuthal scattering angle of the electron ϕ_e . The other 4 d.o.f. can be chosen to be the photon virtuality Q^2 , the invariant mass of the $p\gamma$ system W , the polar and azimuthal scattering angles of the kaon in the center-of-mass frame θ_K^* and ϕ_K^* . In the following this statement will be demonstrated. Summarizing, the system has 5 degrees of freedom and can be represented using Φ , Q^2 , W , θ_K^* and ϕ_K^* variables.

2.1.1 Lorentz Transformation

A Lorentz transformation is a coordinate transformation between two coordinate frames moving at constant velocity relative to each other.

For a transformation in a generic direction between two systems with a relative speed $\mathbf{v}$, with parallel axes, without rotations, we have

$$\begin{bmatrix} \omega^* \\ \mathbf{k}^* \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\boldsymbol{\beta}^T \\ -\gamma\boldsymbol{\beta} & \mathbf{I} + (\gamma - 1)\boldsymbol{\beta}\boldsymbol{\beta}^T/\beta^2 \end{bmatrix} \begin{bmatrix} \omega_{LAB} \\ \mathbf{k}_{LAB} \end{bmatrix} \quad (2.5)$$

¹Variables in CM are denoted by an asterisk

i.e.

$$\omega^* = \gamma\omega_{LAB} - \gamma\boldsymbol{\beta}^T \mathbf{k}_{LAB} \quad (2.6)$$

$$\mathbf{k}^* = -\gamma\boldsymbol{\beta}\omega_{LAB} + (\mathbf{I} + (\gamma - 1)\boldsymbol{\beta}\boldsymbol{\beta}^T/\beta^2)\mathbf{k}_{LAB} \quad (2.7)$$

with $\mathbf{I}$ the identity matrix, while $\boldsymbol{\beta} = \frac{\mathbf{v}}{c}$ is defined as

$$\boldsymbol{\beta} = \frac{\mathbf{v}}{c} = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} = \frac{1}{c} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \quad (2.8)$$

$\boldsymbol{\beta}^T = \frac{\mathbf{v}^T}{c}$ is its transpose, a row vector:

$$\boldsymbol{\beta}^T = \frac{\mathbf{v}^T}{c} = [\beta_x \quad \beta_y \quad \beta_z] = \frac{1}{c} [v_x \quad v_y \quad v_z] \quad (2.9)$$

and γ is

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}. \quad (2.10)$$

In the laboratory frame $P_{tot} = [\sum_k E_k, \sum_k \mathbf{p}_k]$ and in the CM frame $P_{tot}^* = [\sum_k E_k^*, \vec{0}] = [W, \vec{0}]$, with the invariant mass W , defined as

$$W = \sqrt{(E_{tot}^{LAB})^2 - (\mathbf{p}_{tot}^{LAB})^2} = \sqrt{(\omega_{LAB} + m_p)^2 - \mathbf{k}_{LAB}^2} = (E_K^* + E_\Lambda^*) \quad (2.11)$$

which is equal to either the invariant mass of the $K^+\Lambda$ system or the γ^*p system.

The Lorentz transformation from the leptonic to the hadronic reference frame is defined by the speed of the CM frame associated to the vector

$$\boldsymbol{\beta}_{CM} = \frac{\mathbf{p}_{tot}^{LAB}}{E_{tot}^{LAB}} = \frac{\mathbf{k}_{LAB}}{\omega_{LAB} + m_P} \quad (2.12)$$

from which we also obtain γ_{CM}

$$\gamma_{CM} = \frac{1}{\sqrt{1 - \beta_{CM}^2}} = \frac{1}{\sqrt{1 - \frac{\mathbf{k}_{LAB}^2}{(\omega_{LAB} + m_P)^2}}} = \frac{\omega_{LAB} + m_P}{\sqrt{(\omega_{LAB} + m_P)^2 - \mathbf{k}_{LAB}^2}} = \frac{E_{tot}^{LAB}}{W} = \frac{\omega_{LAB} + m_P}{W}. \quad (2.13)$$

The dependence of the photon energy in the laboratory frame ω_{LAB} on the independent variables can be obtained using the invariant mass definition

$$W = \sqrt{(\omega_{LAB} + m_p)^2 - \mathbf{k}_{LAB}^2} \rightarrow W^2 = \omega_{LAB}^2 + m_p^2 - \mathbf{k}_{LAB}^2 + 2m_p\omega_{LAB} = -Q^2 + m_p^2 + 2m_N\omega_{LAB} \quad (2.14)$$

where the Q^2 definition has been exploited $Q^2 = -q^2 = -\omega_{LAB}^2 + \mathbf{k}_{LAB}^2$ and finally

$$\omega_{LAB} = \frac{W^2 + Q^2 - m_N^2}{2m_N}. \quad (2.15)$$

To obtain the dependence of the photon energy in the CM frame on the independent variables, we exploit Equation 2.6

$$\begin{aligned} \omega^* &= \gamma_{CM}\omega_{LAB} - \gamma_{CM}\beta_{CM}^T \mathbf{k}_{LAB} = \gamma_{CM}\omega_{LAB} - \gamma_{CM} \frac{k_{LAB}^2}{\omega_{LAB} + m_p} = \gamma_{CM} \frac{\omega_{LAB}^2 + m_p\omega_{LAB} - k_{LAB}^2}{\omega_{LAB} + m_p} = \\ &= \frac{\omega_{LAB} + m_p}{W} \frac{-Q^2 + \frac{W^2 + Q^2 - m_p^2}{2}}{\omega_{LAB} + m_p} = \frac{W^2 - Q^2 - m_p^2}{2W} \end{aligned} \quad (2.16)$$

where

$$\beta_{CM}^T \mathbf{k}_{LAB} = \frac{k_{LAB}^2}{\omega_{LAB} + m_p}. \quad (2.17)$$

We finally achieve the relationship between the laboratory and CM photon three-vector using Equation 2.7:

$$\begin{aligned} \mathbf{k}^* &= -\gamma_{CM}\beta_{CM}\omega_{LAB} + (\mathbf{I} + \frac{(\gamma_{CM} - 1)\beta_{CM}\beta_{CM}^T}{\beta_{CM}^2})\mathbf{k}_{LAB} = \\ &= -\gamma_{CM} \frac{\mathbf{k}_{LAB}}{\omega_{LAB} + m_p} \omega_{LAB} + \mathbf{k}_{LAB} + \frac{(\omega_{LAB} + m_p)^2}{k_{LAB}^2} \left(\frac{\omega_{LAB} + m_p}{W} - 1 \right) \frac{\mathbf{k}_{LAB}}{\omega_{LAB} + m_p} \frac{k_{LAB}^2}{\omega_{LAB} + m_p} = \\ &= \mathbf{k}_{LAB} \left(-\frac{\omega_{LAB} + m_p}{W} \frac{\omega_{LAB}}{\omega_{LAB} + m_p} + 1 + \frac{\omega_{LAB} + m_p - W}{W} \right) = \mathbf{k}_{LAB} \left(\frac{m_p}{W} \right). \end{aligned} \quad (2.18)$$

2.1.2 Leptonic Plane

The leptonic plane, as we anticipated, is described in the laboratory frame. We use the approximation

$$m_e \ll |\mathbf{p}_e| \Rightarrow E_e = |\mathbf{p}_e|. \quad (2.19)$$

The relationship between the four-momenta of the initial state electron e , the final state electron e' and the virtual exchanged photon γ is:

$$p_\gamma = p_e - p_{e'}. \quad (2.20)$$

From this equation we can obtain the formula that links the photon virtuality Q^2 , to the energies of the electron before and after the scattering and to the scattering angle:

$$\begin{aligned} q^2 &= \omega_{LAB}^2 - \mathbf{k}_{LAB}^2 = (E_e - E_{e'})^2 - (\mathbf{p}_e - \mathbf{p}_{e'})^2 = -2E_e E_{e'} + 2E_e E_{e'} \cos\theta_e = \\ &= -2E_e E_{e'} (1 - \cos\theta_e) = -4E_e E_{e'} \sin^2 \frac{\theta_e}{2} \Rightarrow Q^2 = -q^2 = 4E_e E_{e'} \sin^2 \frac{\theta_e}{2}. \end{aligned} \quad (2.21)$$

But Q^2 can also be expressed as

$$Q^2 = -q^2 = -(\omega_{LAB}^2 - \mathbf{k}_{LAB}^2) = |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2. \quad (2.22)$$

In the laboratory frame the following relationship holds:

$$\begin{aligned} W^2 &= (m_P + \omega_{LAB})^2 - (\mathbf{k}_{LAB})^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - |\mathbf{k}_{LAB}|^2 \\ \Rightarrow |\mathbf{k}_{LAB}|^2 &= m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2. \end{aligned} \quad (2.23)$$

So, substituting 2.23 in 2.22, we have:

$$\begin{aligned} Q^2 &= |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2 - \omega_{LAB}^2 \\ &= m_P^2 + 2m_P\omega_{LAB} - W^2 \end{aligned} \quad (2.24)$$

and we can use this result to obtain the energy of the photon:

$$\omega_{LAB} = \frac{W^2 + Q^2 - m_P^2}{2m_P}. \quad (2.25)$$

In this way we also determine the value of $E_{e'}$ knowing the beam energy E_e since

$$p_\gamma = p_e - p_{e'} \Rightarrow \omega_{LAB} = E_e - E_{e'}. \quad (2.26)$$

So, knowing W and Q^2 , we also know $E_{e'}$ and the scattering angle θ_e :

$$\begin{aligned} E_{e'} &= E_e - \frac{W^2 + Q^2 - m_p^2}{2m_p} \\ \sin^2 \frac{\theta_e}{2} &= \frac{Q^2}{4E_e E_{e'}} = \frac{Q^2}{4E_e (E_e - \frac{W^2 + Q^2 - m_p^2}{2m_p})}. \end{aligned} \quad (2.27)$$

At this point we have determined all kinematical variables as a function of the independent variables Q^2 , W and Φ for a fixed beam energy E_e , where the components of the final electron are

$$p_{e'} = \begin{pmatrix} E_{e'} \\ |\mathbf{p}_{e'}| \sin \theta_e \cos \Phi \\ |\mathbf{p}_{e'}| \sin \theta_e \sin \Phi \\ |\mathbf{p}_{e'}| \cos \theta_e \end{pmatrix} \quad (2.28)$$

and from the four-momentum conservation we also know k^{LAB} .

2.1.3 Hadronic Plane

The hadronic plane, which contains the virtual photon, the proton, K^+ and Λ , is described in the CM frame, where K^+ and Λ are emitted back-to-back and the following relations hold

$$\begin{aligned}\theta_\Lambda^* &= \pi - \theta_K^* \\ \phi_\Lambda^* &= \pi + \phi_K^*.\end{aligned}\tag{2.29}$$

Both θ_K^* and ϕ_K^* may be considered the independent variables of the reaction. It is important to note that ϕ_K^* is, for construction, the angle between the leptonic and the hadronic plane.

We obtain the modulus of the three-momenta for K^+ and Λ using the formula for the two-body decay in the CM frame:

$$|\mathbf{p}_K^*| = |\mathbf{p}_\Lambda^*| = \frac{\sqrt{W^4 + (m_K^2 - m_\Lambda^2)^2 - 2W^2(m_K^2 + m_\Lambda^2)}}{2W}\tag{2.30}$$

where m_K is equal to $0.4937 \text{ GeV}/c^2$ and m_Λ is equal to $1.116 \text{ GeV}/c^2$.

At this point we have completely determined all the kinematics of variables in terms of the independent ones and we may use them to express the reaction amplitudes.

2.2 Theoretical Models for $K^+\Lambda$ Electroproduction

As already mentioned in Section 1.3.4, the model employed in this thesis for the theoretical description of the KY exclusive photo- and electroproduction channels is the ‘‘Regge-Plus-Resonance’’ (RPR-2011) approach developed by the Ghent group [35] [39]. In this model, which combines ingredients of Regge phenomenology with elements of a typical isobar approach, the full production amplitude is described by the superposition of eight resonances and a Reggeized t-channel background.

In open strangeness channels the background, or nonresonant contribution, is higher than for the pionic channels, thus demanding for a theoretical description that takes it into account. Regge phenomenology provides a good way to define the nonresonant amplitude [41] [42]. In the RPR framework, the adjustable parameters of a Reggeized background model are optimized to data obtained at sufficiently high energies so that the contribution of individual resonances is projected to become marginal; this allows to constrain the background amplitude [41] [43].

To evaluate the resonant contribution, the Bayesian evidence was employed; it is a quantitative measure that allows to establish the overall efficiency of the model under study, preferring simple descriptions of the available data. The basic principle is the Occam’s razor, stating that if one has to choose between two theories with the same predictive power, the more economical one should be selected. The overall result is a compromise between model complexity and accuracy. Exploiting the Bayesian evidence, eight resonances were selected for the resonant

contribution, namely $N(1535)\frac{1}{2}^+$, $N(1650)\frac{1}{2}^-$, $N(1680)\frac{5}{2}^+$, $N(1720)\frac{3}{2}^+$, $N(1900)\frac{1}{2}^+$, $N(2000)\frac{5}{2}^+$, $N(1900)\frac{3}{2}^-$ and $N(1900)\frac{3}{2}^+$.

The Ghent group also developed a computer code, `strangecal` [44], that exploits RPR model to calculate observables for the $p(\gamma,K)Y$ processes, where Y can be either a Λ or a Σ yperon. In this software the contributions to the reaction amplitude come from the diagrams shown in Figure 2.5. `strangecal` is able to calculate differential and total cross sections as well as single and double polarization asymmetries for polarized photons, nucleon targets and recoil hyperons. Beside cross sections it is also able to produce the probability amplitudes that are used in quantum mechanics to represent the behaviour of systems from which the cross section may be evaluated. Ghent group provided the dependence of the amplitudes $\mathcal{M}_{\lambda_\gamma}^{\lambda_p\lambda_Y}$ in the electroproduction process upon W , the invariant mass of the $p\gamma$ system, Q^2 , the photon virtuality and $\cos\theta_K^*$, with θ_K^* the polar scattering angle of the kaon in the CM frame.

Exploiting the description of a hybrid resonance amplitude $M_{\lambda_\gamma}^{\lambda_p\lambda_Y}$ from [21], we were able to add it to the reaction amplitudes $\mathcal{M}_{\lambda_\gamma}^{\lambda_p\lambda_Y}$, obtaining a set of total amplitudes which were used to calculate the reaction cross sections. We employed them to investigate the sensitivity of the cross section distribution to the excitation of hybrid baryons, described in Chapter 7.

2.3 Existing Experimental Data for $K^+\Lambda$ Electroproduction

The observable value from experiments is the cross section. The electroproduction differential cross section can be expressed as

$$\frac{d^5\sigma}{dE_{e'}d\Omega_{e'}d\Omega_K^*} = \Gamma' \left(\frac{d\sigma_T}{d\Omega_K^*} + \epsilon \frac{d\sigma_L}{d\Omega_K^*} + \epsilon \frac{d\sigma_{TT}}{d\Omega_K^*} \cos(2\phi_K^*) + \sqrt{\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{d\Omega_K^*} \cos(\phi_K^*) \right) \quad (2.31)$$

in which $E_{e'}$ and $\Omega_{e'}$ are the energy and solid angle of the final electron in laboratory frame ($E_{e'} = E_{e'}^{LAB}$), Ω_K^* is the solid angle of the kaon in the center-of-mass frame, Γ' is the virtual photon flux, ϵ is the virtual photon (transverse) polarization and the dependence on the kaon azimuthal angle ϕ_K^* has been made explicit.

The separated structure functions can be written as

$$\sigma_T = \frac{d\sigma_T}{d\Omega_K^*}, \quad \sigma_L = \frac{d\sigma_L}{d\Omega_K^*}, \quad \sigma_{LT} = \frac{d\sigma_{LT}}{d\Omega_K^*}, \quad \sigma_{TT} = \frac{d\sigma_{TT}}{d\Omega_K^*} \quad (2.32)$$

where the combination

$$\sigma_U = \sigma_T + \epsilon\sigma_L. \quad (2.33)$$

is called “unseparated” cross section.

Further details concerning the unpolarized differential cross section will be provided in 2.4.2.

An opportunity to measure the $\gamma_\nu NN^*$ transition helicity amplitudes (i.e. the N^* electrocoupling parameters) is given by the comprehensive experimental data sets obtained with the CLAS detector on single pseudoscalar meson

FIGURE 2.5: Feynman graphs of the reaction amplitude employed by `strangeCalc` software for the photoproduction of a kaon on a proton. k , p , K and Y denote the real photon, proton, kaon and hyperon, respectively. (a), (b) and (c) are the Born terms in the s -, t - and u -channel. In (d) the Y' hyperon is exchanged in the u -channel. In (e), a vector meson is exchanged in the t -channel. In (f) and (g) the intermediate particles are nucleon resonances in the s -channel. Finally in (h) and (i) the exchanged particles are hyperon resonances in the u -channel [45].

electroproduction, e.g. $p\pi^0$, $n\pi^+$, $p\eta$ and KY , and double charged pion electroproduction [46] [47]. Extensive precision data for these exclusive channels (i.e. for which the final states are fully measured) are available from existing CLAS data on differential cross sections and both single and double polarization observables, over a broad kinematic range from $Q^2 = 0.2$ to 4.5 GeV^2 .

In particular, considering the $K^+\Lambda$ and $K^+\Sigma^0$ electroproduction sector, the world's database was considerably modified with the publications from the CLAS Collaboration. The new data consist of:

- beam-recoil transferred polarization for $K^+\Lambda$ [48] and for $K^+\Sigma^0$ [49]
- induced recoil polarization [50]
- structure functions $\sigma_U = \sigma_T + \epsilon\sigma_L$, σ_{LT} and σ_{TT} for $K^+\Lambda$ and $K^+\Sigma^0$, as well as σ_T and σ_L [51] [52]
- polarized structure function $\sigma_{LT'}$ for $K^+\Lambda$ [53] [52]
- first Rosenbluth separation from CLAS data at beam energies of 2.5 and 4 GeV allowing for a separation of σ_T and σ_L [51] [54]

Moreover, the database was increased by Reference [40] with the largest data set yet acquired for polarized electrons on an unpolarized proton target. Measurements of the separated structure functions σ_U , σ_{LT} , σ_{TT} and $\sigma_{LT'}$ for the $K^+\Lambda$ and $K^+\Sigma^0$ final states at a beam energy of 5.499 GeV, spanning W from threshold to 2.6 GeV, Q^2 from 1.4 to 3.9 GeV^2 and nearly the full center-of-mass angular range of the kaon were provided by this work. The full set of differential cross sections $d\sigma/d\Omega_K^*$ and of structure functions σ_U , σ_{LT} , σ_{TT} and $\sigma_{LT'}$ included in this work for the $K^+\Lambda(K^+\Sigma^0)$ final state consists of 3840 (3600) data points in the first case and of 480 (450) data points in the second case. The full set of differential cross sections and of separated structure functions were expressed in bins of Q^2 , W , $\cos\theta_K^*$ and Φ and of Q^2 , W and $\cos\theta_K^*$, respectively. Figure 2.6 shows the W dependence of the structure functions σ_U , σ_{LT} , σ_{TT} and $\sigma_{LT'}$ for $K^+\Lambda$ final state at a beam energy of 5.5 GeV, $Q^2 = 1.80 \text{ GeV}^2$ and $\cos\theta_K$ values as shown from CLAS data [55]. The associated data from this analysis were all inserted in the CLAS physics database [56]. A global (Figure 2.7) multi-channel analysis of exclusive meson photoproduction data was carried out within the framework of Bonn-Gatchina approach [32] [33] [34] which allowed to observe the evidence of contributions from new baryon states. As many as N^* baryon states and 1 Δ^* new states have been established, which are shown in green boxes in Figure 2.7 proving that access to polarization observables of final states different from (πN) is the key of further understand the baryon spectrum [59].

2.4 Relationship between Reaction Amplitude and Cross Section

A connection is needed between the observable information and the theoretical description. The first consists of a cross section, while the second is based on reaction amplitudes.

FIGURE 2.6: Structure functions $\sigma_U = \sigma_T + \epsilon\sigma_L$, σ_{LT} , σ_{TT} and $\sigma_{LT'}$ (nb/sr) for $K^+\Lambda$ production as a function of W (GeV) for $E_b = 5.5$ GeV for $Q^2 = 1.80 \text{ GeV}^2$ and $\cos \theta_K$ values as shown from CLAS data [52] [56]. The error bars represent the statistical uncertainties only. The red curves are from the hadrodynamical KY model of Maxwell [57] and the blue curves are from the hybrid RPR-2011 KY model from Ghent [58].

FIGURE 2.7: Spectrum from CLAS physics database [56]. A global multi-channel analysis of exclusive meson photoproduction data carried out within the framework of the Bonn-Gatchina approach allowed to observe signals from the states shown in the green boxes [32] [33] [34]

2.4.1 Probability Amplitude

We want to describe a transition that may occur either by the excitation of intermediate resonant states or by other processes that may be interpreted as background. If we simply consider the resonant contributions (R_1 and R_2), the most straightforward logic would suggest that the probability to have a system with two resonances is given by the sum of the probabilities to find each resonance R_1 and R_2 , i.e. to have a *incoherent* sum of the two contributions. However, the reality is different because in Quantum Mechanics, the object that describes a quantum state is the probability amplitude ψ and the resonances should be described and added at the amplitude level, to provide a proper physical scenario.

According to the quantum superposition principle two (or more) wavefunctions ψ_1 and ψ_2 , representing quantum states, can be added and their sum or *superposition*

$$\psi_{TOT} = \psi_1 + \psi_2 \quad (2.34)$$

is a quantum state as well. ψ_1 and ψ_2 are complex numbers and can be expressed as

$$\psi = |\psi| e^{i\phi} = |\psi| (\cos\phi + i \sin\phi). \quad (2.35)$$

The probability is the squared modulus of the probability amplitude,

$$\begin{aligned}
P &= |\psi_{TOT}|^2 = |\psi_1 + \psi_2|^2 = ||\psi_1|(\cos\phi_1 + i\sin\phi_1) + |\psi_2|(\cos\phi_2 + i\sin\phi_2)|^2 = \\
&= (|\psi_1|\cos\phi_1 + |\psi_2|\cos\phi_2 + i(|\psi_1|\sin\phi_1 + |\psi_2|\sin\phi_2))^2 = \\
&= (|\psi_1|\cos\phi_1 + |\psi_2|\cos\phi_2)^2 + (|\psi_1|\sin\phi_1 + |\psi_2|\sin\phi_2)^2 + \\
&= |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1||\psi_2|\cos(\phi_1 - \phi_2)
\end{aligned} \tag{2.36}$$

where the trigonometry relationship $\cos^2\phi + \sin^2\phi = 1$ and the prosthaphaeresis formulas were employed. The probability of an event is not given by the sum of the probabilities of the two separate events to occur only, but their *interference term* $2|\psi_1||\psi_2|\cos(\phi_1 - \phi_2)$ must be taken into account. So, on the basis of quantum mechanics superposition principle, to compute the probability of an event to occur, you have to start from the probability amplitude and to compute it as the addition of all the components, in a *coherent* sum.

We describe the transition from an initial γ^*p state to the final KY state exploiting the Quantum Mechanics formalism in which the initial state is described by the *ket* $|\rangle$, the interaction T is described by a transition operator and the final state by the *bra* $\langle|$. The virtual photoproduction amplitude of the final $K^+\Lambda$ state is a complex number and its dependence upon the spin degrees of freedom may be expressed in terms of the helicity of the particles λ_i , which are defined as the projection of the spin along the particle's momentum direction.

We indicate the transition amplitude provided by the Regge Plus Resonance (RPR) model [35] as

$$\mathcal{M}_{\lambda_\gamma}^{\lambda_p\lambda_Y} = \langle\lambda_Y|T|\lambda_\gamma\lambda_p\rangle \tag{2.37}$$

where λ_p , λ_γ and λ_Y represent the proton, photon and hyperon helicities.

The description of the virtual photon production of a hybrid baryon, which decays into a KY final state will be indicated as

$$M_{\lambda_\gamma}^{\lambda_p\lambda_Y} = \langle\lambda_f|T_r|\lambda_\gamma\lambda_p\rangle \tag{2.38}$$

where T_r is the corresponding transition operator.

The determination of the $M_{\lambda_\gamma}^{\lambda_p\lambda_Y}$ amplitudes as a function of the kinematic variables introduced in paragraph 2.1 will be performed in Section 2.5 via an analytical approach.

2.4.2 Unpolarized Cross Section

As already said, the electroproduction cross section can be expressed as

$$\frac{d^5\sigma}{dE_{e'}d\Omega_{e'}d\Omega_K^*} = \Gamma' \left(\frac{d\sigma_T}{d\Omega_K^*} + \epsilon \frac{d\sigma_L}{d\Omega_K^*} + \epsilon \frac{d\sigma_{TT}}{d\Omega_K^*} \cos(2\phi_K^*) + \sqrt{\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{d\Omega_K^*} \cos(\phi_K^*) \right) \tag{2.39}$$

in which $E_{e'}$ and $\Omega_{e'}$ are the energy and solid angle of the final electron in laboratory frame ($E_{e'} = E_{e'}^{LAB}$), Ω_K^* is the solid angle of the kaon in the hadronic center-of-mass frame and the dependence on the kaon azimuthal

angle ϕ_K^* has been made explicit.

Following the formalism of [35], the factor Γ' represents the virtual photon flux and may be expressed as:

$$\Gamma' = \frac{\alpha}{2\pi^2} \frac{E_{e'}}{E_e} \frac{(\omega_{LAB} - \frac{Q^2}{2m_N})}{Q^2} \frac{1}{1 - \epsilon} \quad (2.40)$$

where α is the fine-structure constant and E_e is the energy of the incoming electron in the laboratory frame ($E_e = E_e^{LAB}$). ϵ is the virtual photon (transverse) polarization, defined in terms of the electron scattering angle θ_e and of the virtual photon three-momentum in the lab frame $\mathbf{k}_{LAB}$ as:

$$\epsilon = \frac{1}{1 + 2 \frac{|\mathbf{k}_{LAB}|^2}{Q^2} tg^2 \frac{\theta_e}{2}}. \quad (2.41)$$

Exploiting the kinematic relationships,

$$\begin{aligned} |\mathbf{k}_{LAB}|^2 &= Q^2 + \omega_{LAB}^2 \\ tg^2 \frac{\theta_e}{2} &= \frac{\sin^2 \frac{\theta_e}{2}}{1 - \sin^2 \frac{\theta_e}{2}} = \frac{\frac{Q^2}{4E_e E_{e'}}}{1 - \frac{Q^2}{4E_e E_{e'}}} \\ \omega_{LAB} &= E_e - E_{e'} \rightarrow E_{e'} = E_e - \omega_{LAB} \end{aligned} \quad (2.42)$$

ϵ can be also written in terms of the incoming and scattered electron energies E_e and $E_{e'}$ and of the photon virtuality Q^2 , as:

$$\epsilon = \frac{4E_e E_{e'} - Q^2}{2(E_e^2 + E_{e'}^2) + Q^2}. \quad (2.43)$$

We notice that in electroproduction, although the electron beam is not polarized, it is possible to produce a polarized virtual photon. Beside the transverse polarization that has just been defined, the longitudinal polarization characterizing the virtual photon is given by

$$\epsilon_L = \frac{Q^2}{\omega_{LAB}^2} \epsilon. \quad (2.44)$$

If we want to express the dependence of the cross section in terms of the invariant quantities (W , Q^2), instead of the laboratory variables ($E_{e'}$, $\Omega_{e'}$), we need to compute the appropriate Jacobian as follows:

$$\frac{d^5\sigma}{dE_{e'} d\Omega_{e'} d\Omega_K^*} = \frac{d^5\sigma}{dE_{e'} d\phi_e d\cos\theta_e d\Omega_K^*} = \frac{1}{d\phi_e d\Omega_K^*} \frac{d^5\sigma}{dW dQ^2} \frac{dW dQ^2}{dE_{e'} d\cos\theta_e} = |det I| \frac{d^5\sigma}{d\phi_e d\Omega_K^* dW dQ^2} \quad (2.45)$$

where $det I$ is the determinant of the Jacobian:

$$I = \begin{pmatrix} \frac{\partial Q^2}{\partial E_{e'}} & \frac{\partial W}{\partial E_{e'}} \\ \frac{\partial Q^2}{\partial \cos\theta_e} & \frac{\partial W}{\partial \cos\theta_e} \end{pmatrix} = \begin{pmatrix} 2E_e(1 - \cos\theta_e) & -\frac{[m_P + E_e(1 - \cos\theta_e)]}{W} \\ -2E_e E_{e'} & \frac{2E_e E_{e'}}{W} \end{pmatrix} \quad (2.46)$$

where each element of the Jacobian was obtained as follows:

$$\begin{aligned}
\frac{\partial Q^2}{\partial E_{e'}} &= \frac{\partial(4E_e E_{e'} \sin^2 \frac{\theta_e}{2})}{\partial E_{e'}} = 2E_e(1 - \cos\theta_e) \\
\frac{\partial W}{\partial E_{e'}} &= \frac{\partial(\sqrt{m_P^2 + 2m_P \omega_{LAB} - Q^2})}{\partial E_{e'}} = \frac{\partial(\sqrt{m_P^2 + 2m_P \omega_{LAB} - 4E_e E_{e'} \sin^2 \frac{\theta_e}{2}})}{\partial E_{e'}} = -\frac{[m_P + E_e(1 - \cos\theta_e)]}{W} \\
\frac{\partial Q^2}{\partial \cos\theta_e} &= \frac{\partial(4E_e E_{e'} \sin^2 \frac{\theta_e}{2})}{\partial \cos\theta_e} = \frac{\partial(4E_e E_{e'} \frac{1}{2}(1 - \cos\theta_e))}{\partial \cos\theta_e} = -2E_e E_{e'} \\
\frac{\partial W}{\partial \cos\theta_e} &= \frac{\partial(\sqrt{m_P^2 + 2m_P \omega_{LAB} - Q^2})}{\partial \cos\theta_e} = \frac{\partial(\sqrt{m_P^2 + 2m_P \omega_{LAB} - 4E_e E_{e'} \frac{1}{2}(1 - \cos\theta_e)})}{\partial \cos\theta_e} = \frac{2E_e E_{e'}}{W}
\end{aligned} \tag{2.47}$$

and the following relationships were used:

$$\begin{aligned}
(\sin \frac{\theta_e}{2})^2 &= \frac{1}{2}(1 - \cos\theta_e) \\
\omega_{LAB} &= \frac{W^2 + Q^2 - m_P^2}{2m_P} \rightarrow W = \sqrt{m_P^2 + 2m_P \omega_{LAB} - Q^2} \\
Q^2 &= 4E_e E_{e'} \sin^2 \frac{\theta_e}{2} \\
\omega_{LAB} &= E_e - E_{e'} \rightarrow E_{e'} = E_e - \omega_{LAB}.
\end{aligned} \tag{2.48}$$

The Jacobian determinant results:

$$|detI| = |2E_e(1 - \cos\theta_e) \frac{E_e E_{e'}}{W} - 2E_e E_{e'} \frac{(m_P + E_e(1 - \cos\theta_e))}{W}| = |-\frac{2E_e E_{e'} m_P}{W}| = \frac{2E_e E_{e'} m_P}{W}. \tag{2.49}$$

Now it is possible to invert Equation 2.45 and obtain:

$$\begin{aligned}
\frac{d^5\sigma}{d\phi_e dW dQ^2 d\Omega_K^*} &= \frac{d^5\sigma}{d\Omega_e dE_{e'} d\Omega_K^*} \cdot \frac{1}{|detI|} = \frac{\Gamma'}{|detI|} [\sigma_U + \epsilon \sigma_{TT} \cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)} \sigma_{LT} \cos \phi_K^*] = \\
&= \Gamma [(\sigma_T + \epsilon \sigma_L) + \epsilon \sigma_{TT} \cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)} \sigma_{LT} \cos \phi_K^*]
\end{aligned} \tag{2.50}$$

with

$$\sigma_T = \frac{d\sigma_T}{d\Omega_K^*}, \quad \sigma_L = \frac{d\sigma_L}{d\Omega_K^*}, \quad \sigma_{LT} = \frac{d\sigma_{LT}}{d\Omega_K^*}, \quad \sigma_{TT} = \frac{d\sigma_{TT}}{d\Omega_K^*} \tag{2.51}$$

where we define

$$\Gamma = \frac{\Gamma'}{detI} = \frac{\alpha}{2\pi^2} \frac{E_{e'}}{E_e} \frac{\omega_{LAB} - \frac{Q^2}{2m_P}}{Q^2(1 - \epsilon)} \cdot \frac{W}{2E_e E_{e'} m_P}. \tag{2.52}$$

Substituting

$$\omega_{LAB} - \frac{Q^2}{2m_P} = \frac{W^2 + Q^2 - m_P^2}{2m_P} - \frac{Q^2}{2m_P} = \frac{W^2 - m_P^2}{2m_P} \tag{2.53}$$

it is possible to rewrite Γ as

$$\Gamma = \frac{\Gamma'}{detI} = \frac{\alpha}{2\pi^2} \frac{W}{E_e^2} \frac{W^2 - m_P^2}{Q^2(1 - \epsilon)} \cdot \frac{1}{4m_P^2}. \tag{2.54}$$

Finally, if we consider unpolarized electron beam and we integrate upon the scattered electron azimuthal angle, we obtain:

$$\begin{aligned} \frac{d^4\sigma}{dW dQ^2 d\Omega_K^*} &= \int \frac{d^5\sigma}{d\phi_e dW dQ^2 d\Omega_K^*} d\phi_e = 2\pi\Gamma[(\sigma_T + \epsilon\sigma_L) + \epsilon\sigma_{TT}\cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)}\sigma_{LT}\cos\phi_K^*] = \\ &= \Gamma_v[(\sigma_T + \epsilon\sigma_L) + \epsilon\sigma_{TT}\cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)}\sigma_{LT}\cos\phi_K^*] \end{aligned} \quad (2.55)$$

where

$$\Gamma_v = 2\pi\Gamma = \frac{\alpha}{4\pi} \frac{W}{E_e^2} \frac{W^2 - m_P^2}{Q^2(1-\epsilon)} \frac{1}{m_P^2}. \quad (2.56)$$

So

$$\frac{d^4\sigma}{dW dQ^2 d\Omega_K^*} = \Gamma_v[(\sigma_T + \epsilon\sigma_L) + \epsilon\sigma_{TT}\cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)}\sigma_{LT}\cos\phi_K^*] = \Gamma_v\sigma_4 \quad (2.57)$$

where

$$\sigma_4 = (\sigma_T + \epsilon\sigma_L) + \epsilon\sigma_{TT}\cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)}\sigma_{LT}\cos\phi_K^*. \quad (2.58)$$

Note that it is not possible to disentangle σ_L from σ_T via the dependence of the cross section as a function of the variable ϕ_K^* . For this reason the two contributions appear in the same parenthesis and in the rest of this thesis work we will often refer to them as the *unseparated* cross section term $\sigma_U = \sigma_T + \sigma_L$.

2.4.3 Cross Sections Computation

The relation between each of the cross section terms or structure functions and the transition amplitudes $M_\lambda^{\lambda_p\lambda_Y}$ may be easily expressed introducing the helicity tensors which are bi-linear combinations of the total transition amplitudes $M_\lambda^{\lambda_p\lambda_Y}$ and are defined as follows,

$$H_{\lambda\lambda'} = \sum_{\lambda_p\lambda_Y} M_\lambda^{\lambda_p\lambda_Y} (M_{\lambda'}^{\lambda_p\lambda_Y})^\dagger. \quad (2.59)$$

The structure functions are given by

$$\frac{d\sigma_L}{d\Omega_K^*} = 2\chi \frac{1}{(4\pi)^2} (H_{0,0}) \quad (2.60)$$

$$\frac{d\sigma_T}{d\Omega_K^*} = \chi \frac{1}{(4\pi)^2} (H_{1,1} + H_{-1,-1}) \quad (2.61)$$

$$\frac{d\sigma_{LT}}{d\Omega_K^*} = -\chi \frac{1}{(4\pi)^2} (H_{0,1} + H_{1,0} - H_{-1,0} - H_{0,-1}) \quad (2.62)$$

$$\frac{d\sigma_{TT}}{d\Omega_K^*} = -\chi \frac{1}{(4\pi)^2} (H_{1,-1} + H_{-1,1}) \quad (2.63)$$

where Ω_K^* is the final state kaon solid angle in the CM frame and the factor χ will be detailed in the next paragraph. The structure functions do not depend on the kaon azimuthal angle ϕ_K^* .

The total transition amplitude is expressed in this thesis as the sum:

$$M_{\lambda}^{\prime\lambda_p\lambda_Y} = M_{\lambda}^{\lambda_p\lambda_Y} + \mathcal{M}_{\lambda}^{\lambda_p\lambda_Y} \quad (2.64)$$

where the $\mathcal{M}_{\lambda}^{\lambda_p\lambda_Y}$ are the numerical values from the RPR-2011 model and the $M_{\lambda}^{\lambda_p\lambda_Y}$ are the hybrid baryon excitation transition amplitudes. A detailed study of these amplitudes will be performed in 2.5.

The expression of the χ term is reported in the following.

2.4.3.1 χ factor

The χ term is defined as:

$$\chi = \frac{1}{16Wm_P} \frac{|\mathbf{p}_K^*|}{K_H} \quad (2.65)$$

where W is the invariant mass of the $K^+\Lambda$ system, m_P is the proton mass, equal to $0.938 \text{ GeV}/c^2$, $|\mathbf{p}_K^*|$ is the modulus of kaon three-momentum in the center-of-mass frame and K_H is the equivalent real photon lab energy. As already obtained in Equation 2.30, $|\mathbf{p}_K^*|$ is expressed using the formula for the two-body decay in the CM frame:

$$|\mathbf{p}_K^*| = |\mathbf{p}_Y^*| = \frac{\sqrt{W^4 + (m_K^2 - m_Y^2)^2 - 2W^2(m_K^2 + m_Y^2)}}{2W} \quad (2.66)$$

where m_K is $0.4937 \text{ GeV}/c^2$ and m_{Λ} is $1.116 \text{ GeV}/c^2$.

The equivalent real photon lab energy K_H is defined as

$$K_H = \omega_{LAB} - \frac{Q^2}{2m_P} \quad (2.67)$$

where Q^2 has been already defined in Eq. 2.22 as

$$Q^2 = -q^2 = -(\omega_{LAB}^2 - \mathbf{k}_{LAB}^2) = |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2. \quad (2.68)$$

In the laboratory frame the relationship 2.23 holds:

$$\begin{aligned} W^2 &= (m_P + \omega_{LAB})^2 - (\mathbf{k}_{LAB})^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - |\mathbf{k}_{LAB}|^2 \\ \Rightarrow |\mathbf{k}_{LAB}|^2 &= m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2. \end{aligned} \quad (2.69)$$

So, substituting 2.69 in 2.68, we have:

$$\begin{aligned} Q^2 &= |\mathbf{k}_{LAB}|^2 - \omega_{LAB}^2 = m_P^2 + \omega_{LAB}^2 + 2m_P\omega_{LAB} - W^2 - \omega_{LAB}^2 \\ &= m_P^2 + 2m_P\omega_{LAB} - W^2 \end{aligned} \quad (2.70)$$

and we can use this result in 2.67 obtaining

$$K_H = \omega_{LAB} - \frac{Q^2}{2m_P} = \omega_{LAB} - \frac{m_P^2 + 2m_P\omega_{LAB} - W^2}{2m_P} = \frac{W^2 - m_P^2}{2m_P}. \quad (2.71)$$

In conclusion χ factor can be written only as a function of W and the particle masses m_P , m_K and m_Y .

$$\begin{aligned}\chi &= \frac{1}{16Wm_P} \frac{|\mathbf{p}_K^*|}{K_H} = \frac{1}{16Wm_P} \frac{\frac{\sqrt{W^4 + (m_K^2 - m_Y^2)^2 - 2W^2(m_K^2 + m_Y^2)}}{2W}}{\frac{W^2 - m_P^2}{2m_P}} \\ &= \frac{1}{16Wm_P} \frac{\sqrt{W^4 + (m_K^2 - m_Y^2)^2 - 2W^2(m_K^2 + m_Y^2)}}{2W} \frac{2m_P}{W^2 - m_P^2}\end{aligned}\quad (2.72)$$

2.5 Hybrid Baryon Excitation Amplitude

The task of this Section is to achieve a full description of the $M_{\lambda_\gamma}^{\lambda_p \lambda_Y} = \langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle$ transition amplitude, where a single hybrid resonance is excited, since this is the term we want to add to the numerical values of $\mathcal{M}_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ of Eq. 15 of article [35] to simulate the presence of a hybrid resonance in an electroproduction process. To properly sum the two terms, the $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ must be represented as a function of the degrees of freedom of the system, that have been introduced in Section 2.1.

The model we use for the $\langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle$ resonant amplitude of a single hybrid resonance in the helicity representation is a relativistic Breit-Wigner (BW) ansatz, following the formalism of [21]:

$$M_{\lambda_\gamma}^{\lambda_p \lambda_Y} = \langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle = \frac{\langle \lambda_f | T_{dec} | \lambda_R \rangle \langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle}{M_r^2 - W^2 - i\Gamma_r M_r}, \quad (2.73)$$

where M_r and Γ_r are the resonance mass and total width respectively.

The matrix elements $\langle \lambda_f | T_{dec} | \lambda_R \rangle$ and $\langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle$ are the electromagnetic production and the hadronic decay amplitudes of the hybrid resonance N^* with helicity $\lambda_R = \lambda_\gamma - \lambda_p$, in which λ_γ and λ_p stand for the helicities of the photon and proton in the initial state and λ_f represents the helicity of final-state hadrons after the N^* decay. The hadronic decay amplitude $\langle \lambda_f | T_{dec} | \lambda_R \rangle$ is given by the expression:

$$\langle \lambda_f | T_{dec} | \lambda_R \rangle = \langle \lambda_f | T_{dec}^{J_r} | \lambda_R \rangle d_{\mu\nu}^{J_r}(\cos\theta_K^*) e^{i\mu\phi_K^*}, \quad (2.74)$$

where $d_{\mu\nu}^{J_r}$ is the Wigner rotation matrix with $\mu = \lambda_R$ and $\nu = -\lambda_Y$, and

$$\langle \lambda_f | T_{dec}^{J_r} | \lambda_R \rangle = \frac{2\sqrt{2\pi}\sqrt{2J_r+1}M_r\sqrt{\Gamma_{\lambda_f}}}{\sqrt{p_i^r}} \sqrt{\frac{p_i^r}{p_i}}, \quad (2.75)$$

where J_r is the hybrid resonance spin, and p_i^r and p_i are the magnitudes of the three-momenta of the final state K for the $N^* \rightarrow K\Lambda$ decay ($i=1$) or for the $N^* \rightarrow K\Sigma$ decay ($i=2$), evaluated at $W = M_r$ and at the running W , respectively. The variables θ_K^* , ϕ_K^* are the CM polar and azimuthal angles for the final kaon (already defined among the independent variables of the system) and J_r stands for the N^* spin.

The final state Λ or Σ baryons can only be in the helicity states $\lambda_f = \pm\frac{1}{2}$. The hadronic decay amplitudes $\langle \lambda_f | T_{dec}^J | \lambda_R \rangle$ with $\lambda_f = \pm\frac{1}{2}$ are related by P-invariance, which imposes the absolute values for both amplitudes to be the same. We have assumed that the hybrid state partial decay widths to the $K\Lambda$ and $K\Sigma$ final states Γ_{λ_f}

can be estimated as:

$$\Gamma_{\lambda_f} = \frac{1}{2}\Gamma_r 0.03, \quad (2.76)$$

where the factor 0.03 corresponds to a 3% BF for hybrid baryon decays to the KY final state. The following relationships between the transition amplitudes $\langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle$ and the $\gamma_\nu NN^*$ electrocouplings $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ were obtained in the paper [21]:

$$\begin{aligned} \langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle &= \frac{W}{M_r} \sqrt{\frac{8M_N M_r q_{\gamma_r}}{4\pi\alpha}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} A_{1/2,3/2}(Q^2), \\ \text{with } |\lambda_\gamma - \lambda_p| &= \frac{1}{2}, \frac{3}{2} \text{ for transverse photons } (\lambda_\gamma = \pm 1), \text{ and} \\ \langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle &= \frac{W}{M_r} \sqrt{\frac{16M_N M_r q_{\gamma_r}}{4\pi\alpha}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} S_{1/2}(Q^2), \\ &\text{for longitudinal photons } (\lambda_\gamma = 0), \end{aligned} \quad (2.77)$$

where q_γ is the absolute value of the initial photon three-momentum of virtuality $Q^2 > 0$ with $q_\gamma = \sqrt{Q^2 + \omega^{*2}}$ and ω^* the photon energy in the CM frame at the running W

$$\omega^* = \frac{W^2 - Q^2 - M_N^2}{2W}. \quad (2.78)$$

as already saw in Eq. 2.16.

The $q_{\gamma,r}$ value is then computed from Eq. 2.78 with $W=M_r$. We will investigate hybrid baryon states with spin-parities $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$. Electroexcitation of the former state can be described by two electrocouplings $A_{1/2}$ and $S_{1/2}$, while the latter should be described by three electrocouplings, $A_{1/2}$, $S_{1/2}$ and $A_{3/2}$. Information on the expected Q^2 -evolution of the electrocouplings for hybrid states is, to the best of our knowledge, currently not available. We will vary the hybrid baryon electrocouplings to determine their minimal absolute values above which the signal from the hybrid baryon can be observed in the difference between the angular distributions with and without hybrid baryon contributions. These studies will be done as a function of Q^2 and the results are reported in paragraph 7.2.3.

To take into account the different formalisms used by Reference [21] and by the Ghent group [35], one must consider a normalization factor equal to $\sqrt{4\pi\alpha}$.

If we substitute Eq. 2.74 and Eq. 2.77 in Eq. 2.73, we have

$$\begin{aligned} \langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle &= \frac{\langle \lambda_f | T_{dec} | \lambda_R \rangle \langle \lambda_R | T_{em} | \lambda_\gamma \lambda_p \rangle}{M_r^2 - W^2 - i\Gamma_r M_r} = \\ &= \frac{\langle \lambda_f | T_{dec}^{J_r} | \lambda_R \rangle}{M_r^2 - W^2 - i\Gamma_r M_r} d_{\mu\nu}^{J_r}(\cos\theta_K^*) e^{i\mu\phi_K^*} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma_r}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \begin{cases} A_{1/2,3/2}(Q^2), & c = 8 \\ S_{1/2}(Q^2), & c = 16 \end{cases} \end{aligned} \quad (2.79)$$

and if we also substitute Eq. 2.75,

$$= \frac{\sqrt{8\pi}\sqrt{2J_r+1}M_r\sqrt{\Gamma_{\lambda_f}}}{\sqrt{p_i^r}} \sqrt{\frac{p_i}{p_i^r}} d_{\mu\nu}^{J_r}(\cos\theta_K^*) e^{i\mu\phi_K^*} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma_r}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \left\{ \begin{array}{l} A_{1/2,3/2}(Q^2), c=8 \\ S_{1/2}(Q^2), c=16 \end{array} \right. \quad (2.80)$$

and Eq. 2.76,

$$\langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle = \frac{\sqrt{8\pi}\sqrt{2J_r+1}M_r\sqrt{\frac{1}{2}\Gamma_r 0.03}}{\sqrt{p_i}} d_{\mu\nu}^{J_r}(\cos\theta_K^*) e^{i\mu\phi_K^*} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma_r}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \left\{ \begin{array}{l} A_{1/2,3/2}(Q^2), c=8 \\ S_{1/2}(Q^2), c=16 \end{array} \right. \quad (2.81)$$

As already said, we are interested at the cases with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, therefore J_r can be $\frac{1}{2}$ or $\frac{3}{2}$.

2.5.1 Transition Amplitude for a $J^P = \frac{1}{2}^+$ Hybrid Baryon Excitation

In this case we have

$$\langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle = \frac{\sqrt{8\pi}\sqrt{2}M_r\sqrt{\Gamma_r 0.015}}{\sqrt{p_i}} d_{\mu\nu}^{1/2}(\cos\theta_K^*) e^{i\mu\phi_K^*} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma_r}} \sqrt{\frac{q_{\gamma_r}}{q_\gamma}} \left\{ \begin{array}{l} A_{1/2}(Q^2), c=8 \\ S_{1/2}(Q^2), c=16 \end{array} \right. \quad (2.82)$$

Now we need to calculate $\lambda_R = \lambda_\gamma - \lambda_p = \mu$ and $\nu = -\lambda_Y$ for all the possible values of λ_P , λ_Y and λ_γ . The results are reported in Table 2.1.

At this point we can express the $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ transition amplitude in terms of a factor that doesn't change with λ_P ,

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$\mu = 1/2$ $\nu = -1/2$	$\mu = 1/2$ $\nu = 1/2$	$\mu = 3/2$ $\nu = -1/2$	$\mu = 3/2$ $\nu = 1/2$
0	$\mu = -1/2$ $\nu = -1/2$	$\mu = -1/2$ $\nu = 1/2$	$\mu = 1/2$ $\nu = -1/2$	$\mu = 1/2$ $\nu = 1/2$
-1	$\mu = -3/2$ $\nu = -1/2$	$\mu = -3/2$ $\nu = 1/2$	$\mu = -1/2$ $\nu = -1/2$	$\mu = -1/2$ $\nu = 1/2$

TABLE 2.1: Values of $\lambda_R = \lambda_\gamma - \lambda_p = \mu$ and $\nu = -\lambda_Y$ for all the possible values of λ_P , λ_Y and λ_γ .

λ_Y and λ_γ . We introduce the kinematical factor:

$$F_{J=1/2} = \frac{4\sqrt{\pi}M_r\sqrt{\Gamma_r 0.015}}{\frac{\sqrt{p_i}}{M_r^2 - W^2 - i\Gamma_r M_r}} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma r}} \sqrt{\frac{q_{\gamma r}}{q_\gamma}} \quad (2.83)$$

where

$$\begin{cases} c = 8 \text{ for } A_{1/2}(Q^2) \\ c = 16 \text{ for } S_{1/2}(Q^2) \end{cases} \quad (2.84)$$

and we explicit the polar angular dependence of the reaction amplitude imposing $\phi_K^* = 0$, obtaining the expressions summarized in Tab. 2.2 for the different λ_Y and λ_γ helicity values.

The $d_{\mu\nu}^{1/2}(\cos\theta_K^*)$ coefficients are Wigner rotation matrix elements, for whom the general relationship holds:

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{1/2}A_{1/2}d_{1/2 \ -1/2}^{1/2}$	$F_{1/2}A_{1/2}d_{1/2 \ 1/2}^{1/2}$	0	0
0	$F_{1/2}S_{1/2}d_{-1/2 \ -1/2}^{1/2}$	$F_{1/2}S_{1/2}d_{-1/2 \ 1/2}^{1/2}$	$F_{1/2}S_{1/2}d_{1/2 \ -1/2}^{1/2}$	$F_{1/2}S_{1/2}d_{1/2 \ 1/2}^{1/2}$
-1	0	0	$F_{1/2}A_{1/2}d_{-1/2 \ -1/2}^{1/2}$	$F_{1/2}A_{1/2}d_{-1/2 \ 1/2}^{1/2}$

TABLE 2.2: Expressions of the transition amplitudes $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ for the excitation of a hybrid baryon with $J^P = \frac{1}{2}^+$, for all the possible values of λ_p , λ_Y and λ_γ . $\lambda_\gamma = \pm 1$ for transverse photons and $\lambda_\gamma = 0$ for longitudinal photons. Note that the terms with $\mu = 3/2$ are not present.

$$d_{m',m}^J = (-1)^{m-m'} d_{m,m'}^J = d_{-m,-m'}^J. \quad (2.85)$$

The explicit functions for $J = \frac{1}{2}$ are:

$$\begin{aligned} d_{1/2 \ 1/2}^{1/2}(\cos\theta_K^*) &= \cos\frac{\theta_K^*}{2} \\ d_{1/2 \ -1/2}^{1/2}(\cos\theta_K^*) &= -\sin\frac{\theta_K^*}{2} \\ d_{-1/2 \ -1/2}^{1/2}(\cos\theta_K^*) &= d_{1/2 \ 1/2}^{1/2}(\cos\theta_K^*) = \cos\frac{\theta_K^*}{2} \\ -d_{-1/2 \ 1/2}^{1/2}(\cos\theta_K^*) &= d_{1/2 \ -1/2}^{1/2}(\cos\theta_K^*) = -\sin\frac{\theta_K^*}{2} \end{aligned} \quad (2.86)$$

We can substitute these values in Table 2.2, obtaining the results in Tab. 2.3.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{1/2} A_{1/2}(-\sin \frac{\theta_K^*}{2})$	$F_{1/2} A_{1/2}(\cos \frac{\theta_K^*}{2})$	0	0
0	$F_{1/2} S_{1/2}(\cos \frac{\theta_K^*}{2})$	$F_{1/2} S_{1/2}(\sin \frac{\theta_K^*}{2})$	$F_{1/2} S_{1/2}(-\sin \frac{\theta_K^*}{2})$	$F_{1/2} S_{1/2}(\cos \frac{\theta_K^*}{2})$
-1	0	0	$F_{1/2} A_{1/2}(\cos \frac{\theta_K^*}{2})$	$F_{1/2} A_{1/2}(\sin \frac{\theta_K^*}{2})$

TABLE 2.3: Expressions of the transition amplitudes $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ for the excitation of a hybrid baryon with $J^P = \frac{1}{2}^+$, for all the possible values of λ_P , λ_Y and λ_γ . The exact expressions of Wigner rotation matrix elements have been inserted.

Finally it is convenient to express our complex amplitudes $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ separating the real and the imaginary part, as $M_{\lambda_\gamma}^{\lambda_p \lambda_Y} = \text{Re}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y}) + i \text{Im}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$. So, first of all, we rewrite the complex factor in the $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ amplitude as:

$$\frac{1}{M_r^2 - W^2 - i\Gamma_r M_r} = \frac{(M_r^2 - W^2) + i(\Gamma_r M_r)}{(M_r^2 - W^2)^2 + (\Gamma_r M_r)^2}. \quad (2.87)$$

We obtain the values for $\text{Re}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ reported in table 2.4 and for $\text{Im}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ in table 2.5 for $J^P = \frac{1}{2}^+$.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{1/2} A_{1/2}(-\sin \frac{\theta_K^*}{2})(M_r^2 - W^2)$	$F_{1/2} A_{1/2}(\cos \frac{\theta_K^*}{2})(M_r^2 - W^2)$	0	0
0	$F_{1/2} S_{1/2}(\cos \frac{\theta_K^*}{2})(M_r^2 - W^2)$	$F_{1/2} S_{1/2}(\sin \frac{\theta_K^*}{2})(M_r^2 - W^2)$	$F_{1/2} S_{1/2}(-\sin \frac{\theta_K^*}{2})(M_r^2 - W^2)$	$F_{1/2} S_{1/2}(\cos \frac{\theta_K^*}{2})(M_r^2 - W^2)$
-1	0	0	$F_{1/2} A_{1/2}(\cos \frac{\theta_K^*}{2})(M_r^2 - W^2)$	$F_{1/2} A_{1/2}(\sin \frac{\theta_K^*}{2})(M_r^2 - W^2)$

TABLE 2.4: Expressions of the transition amplitudes real part $\text{Re}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ for the excitation of a hybrid baryon with $J^P = \frac{1}{2}^+$, for all the possible values of λ_P , λ_Y and λ_γ . The exact expressions of Wigner rotation matrix elements have been inserted.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{1/2}A_{1/2}(-\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{1/2}A_{1/2}(\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	0	0
0	$F_{1/2}S_{1/2}(\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{1/2}S_{1/2}(\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{1/2}S_{1/2}(-\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{1/2}S_{1/2}(\cos\frac{\theta_K^*}{2})\Gamma_r M_r$
-1	0	0	$F_{1/2}A_{1/2}(\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{1/2}A_{1/2}(\sin\frac{\theta_K^*}{2})\Gamma_r M_r$

TABLE 2.5: Expressions of the transition amplitudes imaginary part $Im(M_{\lambda_\gamma}^{\lambda_p\lambda_Y})$ for the excitation of a hybrid baryon with $J^P = \frac{1}{2}^+$, for all the possible values of λ_P , λ_Y and λ_γ . The exact expressions of Wigner rotation matrix elements have been inserted.

2.5.2 Transition Amplitude for a $J^P = \frac{3}{2}^+$ Hybrid Baryon Excitation

In this case we have

$$\begin{aligned} \langle \lambda_f | T_r | \lambda_\gamma \lambda_p \rangle = & \frac{\sqrt{8\pi} 2M_r \sqrt{\Gamma_r 0.015}}{M_r^2 - W^2 - i\Gamma_r M_r} d_{\mu\nu}^{3/2}(\cos\theta_K^*) e^{i\mu\phi_K^*} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma r}} \sqrt{\frac{q_{\gamma r}}{q_\gamma}} \left\{ \begin{array}{l} A_{1/2}(Q^2), c = 8 \\ A_{3/2}(Q^2), c = 8 \\ S_{1/2}(Q^2), c = 16 \end{array} \right. \end{aligned} \quad (2.88)$$

Exploiting the results in Table 2.1 for $\lambda_R = \lambda_\gamma - \lambda_p = \mu$ and $\nu = -\lambda_Y$ with respect to all the possible values of λ_P , λ_Y and λ_γ , we obtain the different values of the $d_{\mu\nu}^{3/2}(\cos\theta_K^*)$ Wigner rotation matrix elements for $J^P = \frac{3}{2}^+$ hybrid baryon spin with respect to all the possible values of the helicities of all particles λ_P , λ_Y and λ_γ (Table 2.6).

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$d_{1/2 \ -1/2}^{3/2}(\cos\theta_K^*)$	$d_{1/2 \ 1/2}^{3/2}(\cos\theta_K^*)$	$d_{3/2 \ -1/2}^{3/2}(\cos\theta_K^*)$	$d_{3/2 \ 1/2}^{3/2}(\cos\theta_K^*)$
0	$d_{-1/2 \ -1/2}^{3/2}(\cos\theta_K^*)$	$d_{-1/2 \ 1/2}^{3/2}(\cos\theta_K^*)$	$d_{1/2 \ -1/2}^{3/2}(\cos\theta_K^*)$	$d_{1/2 \ 1/2}^{3/2}(\cos\theta_K^*)$
-1	$d_{-3/2 \ -1/2}^{3/2}(\cos\theta_K^*)$	$d_{-3/2 \ 1/2}^{3/2}(\cos\theta_K^*)$	$d_{-1/2 \ -1/2}^{3/2}(\cos\theta_K^*)$	$d_{-1/2 \ 1/2}^{3/2}(\cos\theta_K^*)$

TABLE 2.6: Expressions of $d_{\mu\nu}^{3/2}(\cos\theta_K^*)$ for all the possible values of λ_P , λ_Y and λ_γ .

The $d_{\mu\nu}^{3/2}(\cos\theta_K^*)$ Wigner rotation matrix elements, for whom the relationship $d_{m',m}^J = (-1)^{m-m'} d_{m,m'}^J = d_{-m,-m'}^J$ is valid, are:

$$\begin{aligned} d_{3/2 \ 3/2}^{3/2}(\cos\theta_K^*) &= \frac{1 + \cos\theta_K^*}{2} \cos\frac{\theta_K^*}{2} = d_{-3/2 \ -3/2}^{3/2}(\cos\theta_K^*) \\ d_{3/2 \ 1/2}^{3/2}(\cos\theta_K^*) &= -\sqrt{3} \frac{1 + \cos\theta_K^*}{2} \sin\frac{\theta_K^*}{2} = -d_{-3/2 \ -1/2}^{3/2}(\cos\theta_K^*) \\ d_{3/2 \ -1/2}^{3/2}(\cos\theta_K^*) &= \sqrt{3} \frac{1 - \cos\theta_K^*}{2} \cos\frac{\theta_K^*}{2} = (-1)^{-3/2-1/2} d_{-3/2 \ 1/2}^{3/2}(\cos\theta_K^*) = d_{-3/2 \ 1/2}^{3/2}(\cos\theta_K^*) \\ d_{3/2 \ -3/2}^{3/2}(\cos\theta_K^*) &= -\frac{1 - \cos\theta_K^*}{2} \sin\frac{\theta_K^*}{2} \\ d_{1/2 \ 1/2}^{3/2}(\cos\theta_K^*) &= \frac{3\cos\theta_K^* - 1}{2} \cos\frac{\theta_K^*}{2} = d_{-1/2 \ -1/2}^{3/2}(\cos\theta_K^*) \\ d_{1/2 \ -1/2}^{3/2}(\cos\theta_K^*) &= -\frac{3\cos\theta_K^* + 1}{2} \sin\frac{\theta_K^*}{2} = -d_{-1/2 \ 1/2}^{3/2}(\cos\theta_K^*) \end{aligned} \quad (2.89)$$

Also in this case we can express the $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ transition amplitude in terms of a factor that doesn't change with λ_P , λ_Y and λ_γ ,

$$F_{J=3/2} = \frac{2\sqrt{2\pi} 2M_r \sqrt{\Gamma_r 0.015}}{M_r^2 - W^2 - i\Gamma_r M_r} \frac{W}{M_r} \sqrt{cM_N M_r q_{\gamma r}} \sqrt{\frac{q_{\gamma r}}{q_\gamma}} \quad (2.90)$$

where

$$\left\{ \begin{array}{l} c = 8 \text{ for } A_{1/2}(Q^2) \\ c = 8 \text{ for } A_{3/2}(Q^2) \\ c = 16 \text{ for } S_{1/2}(Q^2) \end{array} \right. \quad (2.91)$$

multiplied for terms that present a dependence. Imposing that $\phi_K^* = 0$, we obtain the results for the transition matrix presented in Table 2.7.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{3/2}A_{1/2}d_{1/2 \ -1/2}^{3/2}$	$F_{3/2}A_{1/2}d_{1/2 \ 1/2}^{3/2}$	$F_{3/2}A_{3/2}d_{3/2 \ -1/2}^{3/2}$	$F_{3/2}A_{3/2}d_{3/2 \ 1/2}^{3/2}$
0	$F_{3/2}S_{1/2}d_{-1/2 \ -1/2}^{3/2}$	$F_{3/2}S_{1/2}d_{-1/2 \ 1/2}^{3/2}$	$F_{3/2}S_{1/2}d_{1/2 \ -1/2}^{3/2}$	$F_{3/2}S_{1/2}d_{1/2 \ 1/2}^{3/2}$
-1	$F_{3/2}A_{3/2}d_{-3/2 \ -1/2}^{3/2}$	$F_{3/2}A_{3/2}d_{-3/2 \ 1/2}^{3/2}$	$F_{3/2}A_{1/2}d_{-1/2 \ -1/2}^{3/2}$	$F_{3/2}A_{1/2}d_{-1/2 \ 1/2}^{3/2}$

TABLE 2.7: Expressions of $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ for $J^P = \frac{3}{2}^+$ for all the possible values of λ_P , λ_Y and λ_γ . Note that now the terms with $\mu = 3/2$ are present.

Again it is convenient to express our $M_{\lambda_\gamma}^{\lambda_p \lambda_Y}$ as $M_{\lambda_\gamma}^{\lambda_p \lambda_Y} = \text{Re}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y}) + i \text{Im}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ using relation 2.87 and we finally obtain values for $\text{Re}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ reported in table 2.8 and for $\text{Im}(M_{\lambda_\gamma}^{\lambda_p \lambda_Y})$ in table 2.9.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{3/2}A_{1/2}(-\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}A_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}A_{3/2}(\sqrt{3}\frac{1-\cos\theta_K^*}{2}\cos\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}A_{3/2}(-\sqrt{3}\frac{1+\cos\theta_K^*}{2}\sin\frac{\theta_K^*}{2})(M_r^2-W^2)$
0	$F_{3/2}S_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}S_{1/2}(\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}S_{1/2}(-\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}S_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})(M_r^2-W^2)$
-1	$F_{3/2}A_{3/2}(\sqrt{3}\frac{1+\cos\theta_K^*}{2}\sin\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}A_{3/2}(\sqrt{3}\frac{1-\cos\theta_K^*}{2}\cos\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}A_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})(M_r^2-W^2)$	$F_{3/2}A_{1/2}(\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})(M_r^2-W^2)$

TABLE 2.8: Expressions of the transition amplitudes real part $Re(M_{\lambda_\gamma}^{\lambda_p\lambda_Y})$ for the excitation of a hybrid baryon with $J^P = \frac{3}{2}^+$, for all the possible values of λ_P , λ_Y and λ_γ . The exact expressions of Wigner rotation matrix elements have been inserted.

λ_γ	$\lambda_p = 1/2$ $\lambda_Y = 1/2$	$\lambda_p = 1/2$ $\lambda_Y = -1/2$	$\lambda_p = -1/2$ $\lambda_Y = 1/2$	$\lambda_p = -1/2$ $\lambda_Y = -1/2$
1	$F_{3/2}A_{1/2}(-\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}A_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}A_{3/2}(\sqrt{3}\frac{1-\cos\theta_K^*}{2}\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}A_{3/2}(-\sqrt{3}\frac{1+\cos\theta_K^*}{2}\sin\frac{\theta_K^*}{2})\Gamma_r M_r$
0	$F_{3/2}S_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}S_{1/2}(\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}S_{1/2}(-\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}S_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})\Gamma_r M_r$
-1	$F_{3/2}A_{3/2}(\sqrt{3}\frac{1+\cos\theta_K^*}{2}\sin\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}A_{3/2}(\sqrt{3}\frac{1-\cos\theta_K^*}{2}\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}A_{1/2}(\frac{3\cos\theta_K^*-1}{2}\cos\frac{\theta_K^*}{2})\Gamma_r M_r$	$F_{3/2}A_{1/2}(\frac{3\cos\theta_K^*+1}{2}\sin\frac{\theta_K^*}{2})\Gamma_r M_r$

TABLE 2.9: Expressions of the transition amplitudes imaginary part $Im(M_{\lambda_Y}^{\lambda_p\lambda_Y})$ for the excitation of a hybrid baryon with $J^P = \frac{3}{2}^+$, for all the possible values of λ_P , λ_Y and λ_γ . The exact expressions of Wigner rotation matrix elements have been inserted.

2.6 Helicity Tensors

Summarizing, the cross sections or structure functions will be achieved exploiting their dependence on the helicity tensors, bi-linear combinations of the total transition amplitudes, expressed in Equations 2.60, 2.61, 2.62 and 2.63.

Considering the definition of the helicity tensors of Equation 2.59,

$$H_{\lambda\lambda'} = \sum_{\lambda_p\lambda_Y} M_{\lambda}^{'\lambda_p\lambda_Y} (M_{\lambda}^{'\lambda_p\lambda_Y})^{\dagger} \quad (2.92)$$

and using the total transition amplitude expression of Equation 2.64,

$$M_{\lambda}^{'\lambda_p\lambda_Y} = M_{\lambda}^{\lambda_p\lambda_Y} + \mathcal{M}_{\lambda}^{\lambda_p\lambda_Y} \quad (2.93)$$

we obtain the explicit expressions for the helicity tensors:

$$\begin{aligned} H_{0,0} = & (M_0^{++} + \mathcal{M}_0^{++})(M_0^{++} + \mathcal{M}_0^{++})^{\dagger} + (M_0^{--} + \mathcal{M}_0^{--})(M_0^{--} + \mathcal{M}_0^{--})^{\dagger} + \\ & + (M_0^{+-} + \mathcal{M}_0^{+-})(M_0^{+-} + \mathcal{M}_0^{+-})^{\dagger} + (M_0^{-+} + \mathcal{M}_0^{-+})(M_0^{-+} + \mathcal{M}_0^{-+})^{\dagger} \end{aligned} \quad (2.94)$$

$$\begin{aligned} H_{1,1} = & (M_1^{++} + \mathcal{M}_1^{++})(M_1^{++} + \mathcal{M}_1^{++})^{\dagger} + (M_1^{--} + \mathcal{M}_1^{--})(M_1^{--} + \mathcal{M}_1^{--})^{\dagger} + \\ & + (M_1^{+-} + \mathcal{M}_1^{+-})(M_1^{+-} + \mathcal{M}_1^{+-})^{\dagger} + (M_1^{-+} + \mathcal{M}_1^{-+})(M_1^{-+} + \mathcal{M}_1^{-+})^{\dagger} \end{aligned} \quad (2.95)$$

$$\begin{aligned} H_{-1,-1} = & (M_{-1}^{++} + \mathcal{M}_{-1}^{++})(M_{-1}^{++} + \mathcal{M}_{-1}^{++})^{\dagger} + (M_{-1}^{--} + \mathcal{M}_{-1}^{--})(M_{-1}^{--} + \mathcal{M}_{-1}^{--})^{\dagger} + \\ & + (M_{-1}^{+-} + \mathcal{M}_{-1}^{+-})(M_{-1}^{+-} + \mathcal{M}_{-1}^{+-})^{\dagger} + (M_{-1}^{-+} + \mathcal{M}_{-1}^{-+})(M_{-1}^{-+} + \mathcal{M}_{-1}^{-+})^{\dagger} \end{aligned} \quad (2.96)$$

$$\begin{aligned} H_{1,-1} = & (M_1^{++} + \mathcal{M}_1^{++})(M_{-1}^{++} + \mathcal{M}_{-1}^{++})^{\dagger} + (M_1^{--} + \mathcal{M}_1^{--})(M_{-1}^{--} + \mathcal{M}_{-1}^{--})^{\dagger} + \\ & + (M_1^{+-} + \mathcal{M}_1^{+-})(M_{-1}^{+-} + \mathcal{M}_{-1}^{+-})^{\dagger} + (M_1^{-+} + \mathcal{M}_1^{-+})(M_{-1}^{-+} + \mathcal{M}_{-1}^{-+})^{\dagger} \end{aligned} \quad (2.97)$$

$$\begin{aligned} H_{-1,1} = & (M_{-1}^{++} + \mathcal{M}_{-1}^{++})(M_1^{++} + \mathcal{M}_1^{++})^{\dagger} + (M_{-1}^{--} + \mathcal{M}_{-1}^{--})(M_1^{--} + \mathcal{M}_1^{--})^{\dagger} + \\ & + (M_{-1}^{+-} + \mathcal{M}_{-1}^{+-})(M_1^{+-} + \mathcal{M}_1^{+-})^{\dagger} + (M_{-1}^{-+} + \mathcal{M}_{-1}^{-+})(M_1^{-+} + \mathcal{M}_1^{-+})^{\dagger} \end{aligned} \quad (2.98)$$

$$\begin{aligned} H_{0,1} = & (M_0^{++} + \mathcal{M}_0^{++})(M_1^{++} + \mathcal{M}_1^{++})^{\dagger} + (M_0^{--} + \mathcal{M}_0^{--})(M_1^{--} + \mathcal{M}_1^{--})^{\dagger} + \\ & + (M_0^{+-} + \mathcal{M}_0^{+-})(M_1^{+-} + \mathcal{M}_1^{+-})^{\dagger} + (M_0^{-+} + \mathcal{M}_0^{-+})(M_1^{-+} + \mathcal{M}_1^{-+})^{\dagger} \end{aligned} \quad (2.99)$$

$$\begin{aligned} H_{1,0} = & (M_1^{++} + \mathcal{M}_1^{++})(M_0^{++} + \mathcal{M}_0^{++})^{\dagger} + (M_1^{--} + \mathcal{M}_1^{--})(M_0^{--} + \mathcal{M}_0^{--})^{\dagger} + \\ & + (M_1^{+-} + \mathcal{M}_1^{+-})(M_0^{+-} + \mathcal{M}_0^{+-})^{\dagger} + (M_1^{-+} + \mathcal{M}_1^{-+})(M_0^{-+} + \mathcal{M}_0^{-+})^{\dagger} \end{aligned} \quad (2.100)$$

$$\begin{aligned} H_{-1,0} = & (M_{-1}^{++} + \mathcal{M}_{-1}^{++})(M_0^{++} + \mathcal{M}_0^{++})^{\dagger} + (M_{-1}^{--} + \mathcal{M}_{-1}^{--})(M_0^{--} + \mathcal{M}_0^{--})^{\dagger} + \\ & + (M_{-1}^{+-} + \mathcal{M}_{-1}^{+-})(M_0^{+-} + \mathcal{M}_0^{+-})^{\dagger} + (M_{-1}^{-+} + \mathcal{M}_{-1}^{-+})(M_0^{-+} + \mathcal{M}_0^{-+})^{\dagger} \end{aligned} \quad (2.101)$$

$$\begin{aligned}
H_{0,-1} = & (M_0^{++} + \mathcal{M}_0^{++})(M_{-1}^{++} + \mathcal{M}_{-1}^{++})^\dagger + (M_0^{--} + \mathcal{M}_0^{--})(M_{-1}^{--} + \mathcal{M}_{-1}^{--})^\dagger + \\
& + (M_0^{+-} + \mathcal{M}_0^{+-})(M_{-1}^{+-} + \mathcal{M}_{-1}^{+-})^\dagger + (M_0^{-+} + \mathcal{M}_0^{-+})(M_{-1}^{-+} + \mathcal{M}_{-1}^{-+})^\dagger
\end{aligned} \tag{2.102}$$

2.7 Results from Implementation

From the knowledge of the hybrid baryon amplitude evolution, a study can be performed of the model with and without one hybrid resonance. These studies will be employed to evaluate the sensitivity of our system to the presence of a hybrid resonance.

2.7.1 Differential Cross Section Dependence on $\cos\theta_K^*$ and W

The dependence of the structure functions σ_U , σ_{LT} and σ_{TT} on the W, $\cos\theta_K^*$ and Q^2 variables has been plotted in Figures 2.8(a), 2.10(a) and 2.9(a) for $Q^2 = 0.01 \text{ GeV}^2$, and in Figures 2.8(b), 2.10(b) and 2.9(b) for $Q^2 = 1.002 \text{ GeV}^2$. In these plots the RPR-model distribution has been reproduced. In all plots higher cross sections arise in the forward direction, for $\cos\theta_K^* = 1$. σ_{TT} plots present a deeper structure in the forward direction, as well as a smaller backward contribution. Considering the total cross section definition expressed by Equation 2.39, σ_U contribution is the one that does not disappear when an integration in ϕ_K^* occurs.

2.7.2 Differential Cross Section Dependence on $\cos\theta_K^*$ for Different W values

The dependence of the differential cross section on the $\cos\theta_K^*$ independent variable, for five different W values and for $Q^2 = 0.01 \text{ GeV}^2$ is shown in Figure 2.12. In these plots the cross section from RPR-2011 has been implemented with a hybrid resonance with the following parameters:

M_{res}	Γ_{res}	$A_{1/2}$	$S_{1/2}$	J
2.2 GeV	0.2 GeV	$2.2 \text{ GeV}^{1/2}$	$0 \text{ GeV}^{1/2}$	0.5

TABLE 2.10: Parameters of the resonance added to the RPR-2011 model at the amplitude level.

The RPR-2011 model distribution, with and without the addition of a hybrid resonance at the amplitude level, is represented by the red and black curves, respectively.

The dependence of the differential cross section on the $\cos\theta_K^*$ independent variable, for five different W values and for $Q^2 = 1.002 \text{ GeV}^2$ is shown in Figure 2.12

2.7.3 Differential Cross Section Dependence on W , for Different $\cos\theta_K^*$ Values

The dependence of the differential cross section on the W independent variable, for five different $\cos\theta_K^*$ values and for $Q^2 = 1.002 \text{ GeV}^2$, is shown in Figure 2.13. A structure in correspondence with the mass of the implemented hybrid resonance, 2.2 MeV, emerges for all the $\cos\theta_K^*$ values. Plots presenting a higher discrepancy between the two curves, such as the $\frac{d\sigma_{TT}}{d\Omega_K^*}$ for $\cos\theta_K^* = -0.46341$, allow to identify conditions for whom the system is particularly sensible to the presence of a hybrid resonance.

2.7.4 Differential Cross Section Dependence on W , Integrated in $\cos\theta_K^*$

The dependence of the differential cross section on the W independent variable, integrated in $\cos\theta_K^*$, and for $Q^2 = 1.002 \text{ GeV}^2$ is shown in Figure 2.14. A structure in correspondence with the mass of the implemented hybrid resonance, 2.2 MeV, emerges.

FIGURE 2.8: σ_U tridimensional representation of Ghent RPR-2011 model for $Q^2 = 0.01 \text{ GeV}^2$ (top plot) and $Q^2 = 1.002 \text{ GeV}^2$ (bottom plot). Differential cross section is plotted as a function of W and $\cos\theta_K^*$.

FIGURE 2.9: σ_{LT} tridimensional representation of Ghent RPR-2011 model for $Q^2 = 0.01 \text{ GeV}^2$ (top plot) and $Q^2 = 1.002 \text{ GeV}^2$ (bottom plot). Differential cross section is plotted as a function of W and $\cos\theta_K^*$.

FIGURE 2.10: σ_{TT} tridimensional representation of Ghent RPR-2011 model for $Q^2 = 0.01 \text{ GeV}^2$ (top plot) and $Q^2 = 1.002 \text{ GeV}^2$ (bottom plot). Differential cross section is plotted as a function of W and $\cos\theta_K^*$.

FIGURE 2.11: Differential cross section dependence on $\cos\theta_K^*$ for five different W values and for $Q^2 = 0.01 \text{ GeV}^2$. The three columns, from left to right, correspond to the σ_U , σ_{TT} and σ_{LT} differential cross sections. The five rows correspond to five different W values, from bottom to top: $W = 2.6442$ GeV, $W = 2.4471$ GeV, $W = 2.25$ GeV, $W = 2.0036$ GeV and $W = 1.8065$ GeV. The RPR model distribution and the RPR with the addition of a hybrid resonance with the parameters listed in Table 2.10 are represented by the black and red curves, respectively.

FIGURE 2.12: Differential cross section dependence on $\cos\theta_K^*$ for five different W values and for $Q^2 = 1.002 \text{ GeV}^2$. The three columns, from left to right, correspond to the σ_U , σ_{TT} and σ_{LT} differential cross sections. The five rows correspond to five different W values, from bottom to top: $W = 2.6442 \text{ GeV}$, $W = 2.4471 \text{ GeV}$, $W = 2.25 \text{ GeV}$, $W = 2.0036 \text{ GeV}$ and $W = 1.8065 \text{ GeV}$. The RPR model distribution and the RPR with the addition of a hybrid resonance with the parameters listed in Table 2.10 are represented by the black and red curves, respectively.

FIGURE 2.13: Differential cross section dependence on W for five different $\cos\theta_K^*$ values and for $Q^2 = 1.002 \text{ GeV}^2$. The three columns, from left to right, correspond to the σ_U , σ_{TT} and σ_{LT} differential cross sections. The five rows correspond to five different $\cos\theta_K^*$ values, from top to bottom: $\cos\theta_K^* = 0.90244$, $\cos\theta_K^* = 0.46341$, $\cos\theta_K^* = 0.02439$, $\cos\theta_K^* = -0.46341$ and $\cos\theta_K^* = -0.90244$. The RPR model distribution and the RPR with the addition of a hybrid resonance with the parameters listed in Table 2.10 are represented by the black and red curves, respectively.

FIGURE 2.14: Differential cross section dependence on W integrated in $\cos\theta_K^*$ for $Q^2 = 1.002 \text{ GeV}^2$. The RPR model distribution and the RPR with the addition of a hybrid resonance with the parameters listed in Table 2.10 are represented by the black and red curves, respectively.

Chapter 3

CLAS12 and the Forward Tagger

Introduction

The CLAS12 detector (CEBAF Large Acceptance Spectrometer at 12 GeV) is a large angular acceptance spectrometer under installation at the Thomas Jefferson National Laboratory (JLab) of Newport News in Virginia. The laboratory, whose experimental activity started in 1994, has carried out an extensive experimental program in the field of nuclear and subnuclear physics using an electron beam of 6 GeV energy. The primary mission of JLab is to conduct fundamental research on the structure of matter and in particular on the atomic nucleus using a particle accelerator, known as the Continuous Electron Beam Accelerator Facility (CEBAF) [60] [61] [62]. CEBAF is composed of two linear accelerators connected by recirculation arcs, that provides a high-quality, continuous-wave polarized electron beam: the electromagnetic interaction is employed to probe the structure of the nucleon, the production and decay of light mesons and the interactions of nucleons in the atomic nucleus. The experiments are conducted using electron scattering or the production of high energy real photons. In addition, highly polarized electron and photon beams can be obtained, allowing exploration of spin degrees of freedom.

A major upgrade of the entire laboratory started in 2004 leading to the the increase of the electron beam energy from 6 GeV to 12 GeV, to the renewal of the instrumentation of the three existing experimental halls (Hall A, B, C) and the construction of a new one, Hall D. In particular Hall B will house the CLAS12 detector, whose mission is to investigate the spatial and angular momentum structure of the nucleon, the quark confinement, the hadron excitations and the strong interaction in nuclei. CLAS12 is optimized for the measurement of exclusive and semi-inclusive scattering processes induced by the electron beam on a fixed target of protons or deuterons. As a consequence of the 12 GeV energy of the electron beam, the kinematics of these processes is characterized by the production of low-momenta baryons emitted at large angles and energetic mesons produced in the forward direction. For this reason the CLAS12 detector is divided into two parts: the Forward Detector, covering polar angles between 5 and 35 degrees and the Central Detector, for angles larger than 35°. In addition, the CLAS12 apparatus has been equipped with the Forward Tagger (FT) device, to identify electrons that are scattered in the polar angle range from 2.5 up to 4.5 degrees. The combination of the large acceptance of the detector and of

the nominal luminosity ($\mathcal{L} = 10^{35} \text{ cm}^{-2}\text{s}^{-1}$) will allow for the measurement of reactions characterized by cross sections of the order of the nanobarn. The experimental program described in this thesis work will be performed with CLAS12 exploiting in particular the performances of one of its subsystems, the Forward Tagger.

In this Chapter the experimental setup for CLAS12 and the Forward Tagger are illustrated. In Section 3.1 the Jefferson Laboratories are introduced and the Continuous Electron Beam Accelerator Facility (CEBAF) is briefly described. The 12 GeV upgrade of the CLAS detector, CLAS12, is reported in Section 3.2, with a summary of its components. In Section 3.3 the Forward Tagger equipment is shown.

3.1 The Jefferson Laboratory

The Thomas Jefferson National Accelerator Facility (Newport News, VA, USA) was founded in 1984 to study the fundamental structure of nuclear matter and the mechanisms of QCD at the GeV scale. The facility allows to conduct research on the atomic nucleus at quark level with electromagnetic probes, provided by the Continuous Electron Beam Accelerator Facility (CEBAF) [60] [61] [62]. From 1994 to 2012 this accelerator has provided a high-quality, continuous-wave polarized electron beam with energy up to 6 GeV and current up to 200 μA simultaneously to three experimental halls (Hall A, Hall B and Hall C). Inside each hall a spectrometer is employed to study the interaction of the electron beam (or of real photons in Hall B) with a stationary target. In 2004 an upgrade program started to increase the maximum energy to 12 GeV and to provide the beam in four experimental halls (the new Hall D was build). Hall A, Hall B and Hall C will receive a 11 GeV electron beam, Hall D will use 12 GeV to produce an energy tagged photon beam. The experimental equipment of the three existing halls has been upgraded, as well. The CEBAF upgrade is performed using the framework of the existing accelerator, reaching 2.2 GeV/pass (1.1 GeV/linac) and expanding the beam transport system. With the upgrade the CEBAF consists of a polarized electron source, an injector and a pair of superconducting RF linear accelerators that are 7/8-mile (1400 m) long and are connected to each other by ten arcs sections that contain steering magnets.

3.1.1 CEBAF Accelerator

The Continuous Electron Beam Accelerator Facility (CEBAF) [62] [60] [61] is the electron accelerator of Jefferson Laboratory. Figure 3.1 shows a schematic view of the accelerator with the modifications that are necessary to increase the maximum value of reachable energy from 6 GeV to 12 GeV.

As already mentioned, the CEBAF is composed by two LINACs connected by recirculation arcs. Each LINAC consists of a set of superconducting Niobium accelerating cavities, permeated with a stationary magnetic field oscillating at a frequency of 1.5 GHz. The recent upgrade consists of the addition of five cryomodules in the preexisting linacs, of a recirculation arc, in the increase of the cryo-capacity of the Central Helium Liquifier

FIGURE 3.1: Schematic layout of the Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab and of the required facility modifications to realize the 12 GeV Upgrade.

(CHL) refrigerating system and in the improvement of the curving magnets. Electrons in the CEBAF are injected with a starting energy of 45 MeV at the same frequency of the cavities. They perform several orbits inside the accelerator and are accelerated by the LINAC at each pass before being extracted: 5.5 orbits to reach 12 GeV energy in Hall D and 5 orbits for a final energy of 11 GeV in Halls A, B and C. Electrons are divided in bunches with a timing separation of 0.7 ns. Each hall receives a particle bunch every 2.1 ns. The electrons enter in a specific arc which has a different magnetic field, depending on the pulse value of the electrons: each magnetic field inside the arcs imposes the electrons to follow a fixed central trajectory. The beam can be considered quasi continuous because of the high operating frequency. The electron beam reaches a maximum intensity of $200 \mu\text{A}$ and a longitudinal polarization up to 90%.

3.2 The CLAS12 Detector

In coincidence with the 12 GeV upgrade of CEBAF, the CLAS detector in Hall B, a spectrometer projected around a torus magnet, has been upgraded to CLAS12 [63]. CLAS12 has been designed to work with a high energy electron beam, with polarized and unpolarized targets and with a nominal luminosity of $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$. The geometry of the apparatus is suited to detect the forward-going particles expected from the interaction. The electron beam is extracted by CEBAF and impinges on a polarized or unpolarized target: the final products from exclusive and semi-inclusive reactions, that will be studied with CLAS12, will be baryons scattered at large angles and mesons of high momentum. As a consequence of this task, the detector is suited to detect forward-going particles and covers two regions: polar angles, between 5 and 35 degrees, will be covered by the Forward Detector (FD) while large angles, up to 125° , will be covered by the Central Detector (CD). The performances of the CLAS12 detector are summarized in Tab. 3.1. Both the Forward Detector and the Central Detector, described in Sec. 3.2.1 and 3.2.2 respectively, are composed of several components. The components of Forward Detector are:

Parameter	Forward Detector	Central Detector
Charged tracks:		
Polar angular range (θ)	5° to 35°	35° to 125°
Polar angular resolution ($\delta\theta$)	< 1 mr	< 1 mr to 20 mr
Azimuthal angular resolution ($\delta\phi$)	< 4 mr	< 5 mr
Momentum resolution ($\delta p/p$)	< 1% at 5 GeV/c	< 5% at 1.5 GeV/c
Neutral particles:		
Polar angular range (θ)	5° to 40°	40° to 125° (neutrons only)
Polar angular resolution ($\delta\theta$)	< 4 mr	< 10 mr
Energy/momentum resolution	< 0.1/ $\sqrt{E}$	< 5%
Particle identification:		
e/ π	In full momentum range	NA
π/p	In full momentum range	< 1.25 GeV/c
K/ π	< 3 GeV/c	< 0.65 GeV/c
K/p	< 4 GeV/c	< 1 GeV/c

TABLE 3.1: CLAS12 detector performances.

- [Superconducting torus magnet](#)
- [Drift chambers](#)
- [Cherenkov counter \(HTCC\) \(LTCC\)](#)
- [Forward Time-of-flight system \(FTOF\)](#)
- [Electromagnetic calorimeter \(EC\)](#)

The components of Central Detector are:

- [Superconducting Solenoid](#)
- [Silicon tracker: Barrel strip tracker \(BST\) and Forward silicon tracker \(FST\)](#)
- [Time-of-flight \(CTOF\)](#)

The exact position of the components is shown in Figure 3.3.

3.2.1 The CLAS12 Forward Detector

The Forward Detector (FD) is the part of the CLAS12 detector devoted to the measurement and identification of charged and neutral particles scattered between 5 and 35 degrees, in the full azimuthal angle and momentum range. It consists of six identical magnetic spectrometers working in conjunction with a toroidal field, generated by a six coil superconducting magnet. Each spectrometer, corresponding to a sector of the Forward Detector, will be composed of a high threshold Cherenkov counter (HTCC), a set of drift chambers (FDC), a low threshold Cherenkov counter (LTCC), scintillation counters (FTOF) and an electromagnetic calorimeter (FEC). The

FIGURE 3.2: CLAS12 subsystems overview. The FD components: the Torus magnet, the three drift chambers (DC), the Cherenkov counters (HTCC and LTCC), the forward time-of-flight system (FTOF) and the electromagnetic calorimeter (EC). The CD components: the Solenoid, the Silicon tracker vertex (SVT) and the central time-of-flight system (CTOF).

working principle of FD is the following: charged particles are deflected from their trajectory due to the Torus magnet and their momentum is reconstructed using the drift chambers. The Cherenkov counters and the TOF detectors provide information useful for the discrimination among particles and the electromagnetic calorimeter gives the information about the energy.

Superconducting Torus Magnet The main task of the Torus magnet is to deflect charged particles, without changing their azimuthal angle, to reconstruct their momentum from the curvature radius of the tracking in the drift chambers. It is composed of six superconducting coils arranged symmetrically around the beam line to generate a 3.6 T peak field along in the azimuthal (ϕ) direction: the result is an approximate toroidal field distribution around the beam axis.

The configuration of the Torus magnet is driven by the requirements determined by the physics program. The principal requirements are:

- uniform coverage, in large momentum and angle ranges, and symmetry around the beam axis,
- an open structure that allows for long path lengths for charge and neutral particles, resulting in good particle identification through precise time-of-flight measurements,

- low background from electromagnetic processes to reach high luminosity.

The toroid configuration provides a central field-free zone that allows the use of polarized targets.

Considering that for fixed target experiments the particles with higher momentum are emitted at low polar angles (θ), the six coils have been shaped to produce a higher field in their central part, providing the requested resolution as a function of θ .

The temperature of the coils is maintained at 4.5 K exploiting a forced flow of supercritical liquid helium.

Drift Chambers The drift chambers reconstruct the charged particles trajectories, by measuring their track and providing their momentum. The requirements are:

- momentum resolution of 0.5-1% at 5 GeV
- efficient performance at high luminosity.

The 18 drift chambers have a triangular shape and are assembled in three exagonal regions, each divided in six sectors, the first in front of the torus coils, the second between the magnet coils and the third outside the torus coils. In this geometry the first region will be placed outside the torus field region, the second inside the high field region and the third in the low field region.

The drift chambers contain a mixture of 90% Argon and 10% CO_2 . The drift chambers work exploiting the ionization principle. The volume inside the chamber is filled with high voltage parallel wires with alternately opposite charges. When a charged particle crosses the chamber, an ion-electron pair is produced by ionization. The electric field forces the electron to accelerate to the anode and the ion to the cathode. The charges that are collected constitute an electronic signal that is amplified and recorded by a computer. From the knowledge of the wires that gave a signal and completing the information with the time that the particle employed to reach the wires, it is possible to estimate the trajectory of the particles.

Each region is made by two *superlayers*, with parallel wires placed at a stereo angle of $\pm 6^\circ$ respect to the direction of the toroidal field, to measure polar and azimuthal angle of the trajectory.

FIGURE 3.3: Drift chamber working principle [64].

Cherenkov Counter (HTCC) (LTCC) The Cherenkov counters exploit the Cherenkov effect to estimate the velocity of a particle. According to Cherenkov effect, if a charged particle moves in a transparent material with a speed v larger than the velocity of light, namely if $v > c/n$ where n is the refractive index, it generates light. Changing the n refractive index it is also possible to modify the threshold of light emission. It is possible to distinguish two particles having same energy and momentum thanks to Cherenkov counters: in fact a heavier particle will not produce Cherenkov light, a lighter will.

CLAS12 will be equipped with a high threshold Cherenkov counter (HTCC) and a low threshold Cherenkov counter (LTCC). The HTCC will be installed between the Silicon Vertex Tracker (SVT) and the first DC region. It shall introduce minimal amount of materials and, at the same time, will provide efficient coverage of the CLAS12 acceptance. The HTCC will be filled with CO_2 , whose refraction index is $n=1.00041$. With this value only pions with energy higher than 4.6 GeV produce light: therefore, if the particle energy is lower than the 4.6 GeV threshold and Cherenkov light is produced, we can deduce that the particle is an electron.

The LTCC system will be filled with C_4F_{12} gas and will be used for pions/kaons discrimination from 3.5 to 9 GeV/c. In fact the refraction index in this case is $n=1.00153$ and it allows to distinguish pions from kaons since for energies lower than 2.6 GeV only pions produce Cherenkov light.

Forward Time-Of-Flight System The task of the Forward Time-of-Flight System (FTOF) will be to measure the time-of-flight of charged particles emerging from the target. Two particles having same momentum will reach the FTOF at different times according to their mass: a lighter particle will arrive before a heavier one. In particular the FTOF will allow the separation of pions from kaons for momenta lower than 3 GeV/c (the separation for higher energies is provided by the Cherenkov counters). Considering that higher momentum particles are emitted at lower angles and will reach the FTOF at earlier times, it is constructed to provide a better timing resolution at lower angles.

The FTOF system consists of 6 sectors of plastic scintillators with doublesided PMT readout. Each sector includes three arrays of counters:

- Panel-1a – 23 counters
- Panel-1b – 62 counters
- Panel-2 – 5 counters

The system is required to have excellent timing resolution for particle identification (the system specifications call for a time resolution of $\sigma = 80$ ps at the more forward angles of CLAS12 and 150 ps at angles larger than 36 degrees) and will also provide a measurement of the energy lost by the particles in the plastic scintillators.

Forward Electromagnetic Calorimeters The Forward Electromagnetic Calorimeters (FEC) are devoted to

- measurement and identification of electrons with energies higher than 0.5 GeV

- measurement of photons with energies higher than 0.2 GeV, that allows π^0 and η reconstruction through their neutral decays
- neutrons/photons separation also exploiting the TOF information.

The FEC are composed of the same modules forming the CLAS electromagnetic calorimeter (EC) and a pre-shower calorimeter (PCAL) installed in front of it.

They cover polar angles from 10 to 45 degrees. The PCAL, as well as the EC, is composed of six triangular sectors arranged around the beam line to form an exagonal structure. Each PCAL module is a sampling calorimeter and is composed of 15 layers of 1-cm thick scintillators, segmented into 4.5-cm wide strips, sandwiched between lead sheets of 2.2-mm thickness, corresponding to about 5.5 radiation lengths.

The neutral particles are identified requiring the anti-coincidence with the drift chambers. π^0 and η are reconstructed from the two photon invariant mass.

3.2.2 The CLAS12 Central Detector

The Central Detector is located around the target to detect particles emitted at low momentum, < 1.5 GeV/c, with large polar angles, between 35 and 125 degrees, in the full azimuthal range. It is composed of a 5T superconducting solenoid magnet, a silicon vertex tracker (SVT), that surrounds the target, a time-of-flight detector (CTOF) and a Central Neutron Detector (CND). Similarly to FD, the solenoid and the trackers provide the information about the particle momentum and the particle is identified completing the information with the time-of-flight information. Moreover the CD shields the FD from Möller electrons.

Superconducting Solenoid The Solenoid is a 5-T super-conducting magnet that surrounds the target and produces a field along the beam direction. The Solenoid satisfies several requirements:

- it provides the magnetic field necessary to track central particles
- it functions as a shielding of the FD from Möller electrons
- it provides a uniform magnetic field for polarized target operations.

The solenoid produces a field that causes the charged particles to bend in spiralizing trajectory, without changing the polar angle. The drift chambers measure the curvature radius of the particle trajectories from which the momentum is obtained from the relation:

$$p = erB \quad (3.1)$$

where p , e and r are the particle momentum, charge, mass and curvating radius and B is the magnetic field.

Central Tracking System The central tracking system is a silicon vertex tracker (SVT); it is the part of the CLAS12 central detector devoted to the tracking of particles in the angular region from 35 to 125 degrees. It will be used to measure the momentum and determine the vertex of charged particles emerging from the target. The SVT system includes 4 regions with 10, 14, 18 and 24 sectors of double-sided modules (silicon sensors on both sides of the backing structure), respectively. The system is designed to operate at a luminosity of $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ and to have a momentum resolution of 5% for 1 GeV particles emerging from the target at a θ of 90° .

Central Time-Of-Flight System and Central Neutron Detector The Central Time-of-Flight (CTOF) system provides the time-of-flight of particles from the reaction vertex. By comparing the time of flight with the momentum of charged particles measured by the drift chambers it is possible to identify the particle. The CTOF is placed inside the 5-T superconducting solenoid. It has the structure of a barrel aligned with the beam axis and it is formed by an array of 48 plastic scintillators with doublesided photomultiplier (PMT) readout via 1 m-long upstream and 1.6 m-long downstream focusing light guides, respectively. A 60 ps timing resolution is required to the CTOF for particle identification.

An important instrument for a complete understanding of nucleon structure in terms of Generalized Parton Distributions is the detection of the recoil neutron in nDVCS and in other exclusive reactions of electroproduction on deuterium targets. For this task, the CLAS12 Central Detector will be equipped with a scintillator barrel, the Central Neutron Detector (CND), placed between the CTOF and the magnet, which will allow the detection of neutrons between 0.2 and 1 GeV/c of momentum, identifying them via the measurement of their time of flight and hit position. The CND has a geometry similar to the CTOF one. It is composed of 4 cylindrical layers, each composed by 30 trapezoidal scintillator counters coupled to PMTs for light readout via light guides. The design momentum resolution for neutron detection is 5% with 10% detection efficiency.

3.3 The Forward Tagger

As already said in Section 1.4.1, while CLAS12 experimental setup will be devoted to the detection of final state hadrons, the Forward Tagger (FT) device will be dedicated to the measurement of low-angle scattered electrons, in the polar angle range $2.5^\circ < \theta_e < 5^\circ$, outside CLAS12 acceptance. An electron scattering on a target with a polar angle in this range can be considered for practical purposes a quasi-real photon. In fact the virtuality of the emitted photon Q^2 is related to the scattered electron energy $E_{e'}$ and polar angle θ_e through the relation

$$Q^2 = 4E_{beam}E_{e'}\sin^2\frac{\theta_e}{2}. \quad (3.2)$$

The identification of these low-angle scattered electrons allows to select processes that occurred in the low Q^2 region, that is crucial in the search for hybrid baryons.

The FT is made of three components: an electromagnetic calorimeter (FT-Cal), to measure the energy of the electromagnetic showers, produced by electrons and photons and to provide a fast trigger signal; a hodoscope

(FT-Hodo) to discriminate between electrons and photons that enter in the calorimeter; and a tracker (FT-Trck) to measure the polar θ_e and azimuthal ϕ_e scattering angles of the electrons. The system composed of the three described elements is shown in Figure 3.4.

The FT is located between the High Threshold Cerenkov Counter and the torus support, at about 190 cm

FIGURE 3.4: The Forward Tagger FT composed of (from the top): FT-trk, FT-hodo and FT-Cal.

downstream of the target nominal position: at this position the distance from the beam line, corresponding to 2.5 degrees polar angles, is only ~ 8 cm. The crystals are therefore expected to receive high radiation doses during normal operations. Moreover the FT must occupy a small volume, to avoid possible shadowing effects on CLAS12 with a subsequent loss of information. Finally it is located in a high electromagnetic field region. As a consequence, several requirements to the setup are imposed: its components must present a high radiation hardness, a small radiation length and Moliere radius and the capability to operate inside high magnetic fields. All the technical details concerning Forward Tagger can be found in [65].

3.3.1 Forward Tagger Hodoscope

The Forward Tagger Hodoscope, FT-Hodo, allows to separate electrons from photons, both producing an electromagnetic shower in the FT-Cal. The hodoscope is located in front of FT-Cal and is composed by two layers of segmented arrays of plastic scintillator tiles. Inside the tiles, the light is collected by wavelength shifting (WLS) fibres, that transport the scintillation light from the detector elements and convert its wavelength from UV to green. For a better matching with the optimal quantum efficiency of the Silicon photomultipliers (SiPMs) that read out the optical signal. These sensors have been selected because of the high magnetic field and the low light: their performance is poorly affected by the magnetic field. A schematic view of the hodoscope is shown in Figure 3.5(a).

Figure 3.6 shows the photographs of one paddle embedded with optical fibers (left) and a Silicon photomultiplier (SiPM) that transforms the optical information into an electronic signal (right). Electrons are identified by a hit in the hodoscope that is correlated in position and time with a signal in the calorimeter. In Figure 3.7 the

FIGURE 3.5: Left: Schematic view (from the top) of the device. Right: the FT-Hodo.

(a) One paddle embedded with optical fibers.

(b) Silicon photomultiplier.

FIGURE 3.6: One paddle embedded with optical fibers (left) and Silicon photomultipliers (Sipms) that read the signal (right).

comparison between the energy spectrum released in the hodoscope by an impinging electron (Figure 3.7(a)) and photon (Figure 3.7(b)) is shown. Only a small fraction of the photons leave a signal in the hodoscope, while all electrons deposit some energy. The photons mainly convert into an electron-positron couple and the two peaks in Figure 3.7(b) correspond to the detection of one of two particles. This is an example of false electron event inside the hodoscope. Considering the ratio between the counts integration for energies higher than 1 GeV for photon and electron events, we have a misidentification of the photon of the order of percent. The time resolution of the hodoscope must match the one of the calorimeter in order to provide a trigger signal. The time resolution value is required to be comparable with the FT-Cal one, of the order of 1 ns.

3.3.2 Forward Tagger Tracker

The role of the Forward Tagger Tracker (FT-Trck), shown in Figure 3.8, is to reconstruct the polar and the azimuthal scattering angles of electrons between 2.5 and 4.5 degrees.

FIGURE 3.7: Discrimination between electrons and photons: energy released in the detector by an electron (left) and by a photon (right).

It will be placed in front of the FT-Cal, at few centimeters from the front face of the crystals. The FT-Trck is a

FIGURE 3.8: The Forward Tagger Tracker

Micromegas detector, exploiting the ionization of the gas when a charged particle crosses it, see Figure 3.9. The working principle of the Micromegas detector consists of several steps. A charged particle crosses the detector and a electron/ion pair is created from the ionization, if an electric field is applied, the electron migrates toward the amplification electrode (Mesh) and the ion toward to the cathode. When the electron arrives close to the Mesh it enters in a high electric field, which causes the production of an avalanche of electrons and ions. The electronic signal can be read on the readout electrode by a charge amplifier. The readout electrode is usually segmented in strips and/or pixels in order to obtain the position of the impinging particle in the detector.

The peculiarity of these detectors is the metallic grid (Micromesh) that have a high potential at a distance of the order $\approx 100\mu m$ from the electrode: this allows a high amplification gain, generally in the order of 10^4 and the generation of a fast signal. Micromegas detectors were used for the reconstruction of the electron point of impact and path.

The FT-Trck is composed of two double layers of Micromegas detectors located respectively at 177 cm and at 179 cm downstream of the target and centered on the beam axis. The two different Micromegas in the same layer display strips which are orthogonal to produce a (x,y) couple. In Figure 3.10 the two double layers of the

FIGURE 3.9: Working principle of the Micromegas detector: (1) a charged particle crosses the detector and a electron/ion pair is created from the ionization, (2) if an electric field is applied, the electron migrates toward the amplification electrode (Mesh) and the ion toward to the cathode. When the electron arrives close to the Mesh (3) it enters in a high electric field, which causes the production of an avalanche of electrons and ions (4). The electronic signal can be read on the readout electrode (5) by a charge amplifier. The readout electrode is usually segmented in strips and/or pixels in order to obtain the position of the impinging particle in the detector [66].

FT-Trck are schematically shown. The two layers will cover polar angles from 2.5 to 4.5 degrees.

FIGURE 3.10: The FT-Trck is composed of two layers, each composed of two Micromegas detectors with perpendicular strips. Its task is to reconstruct the polar and the azimuthal scattering angles of electrons emitted between 2.5 and 4.5 degrees [65].

The angular resolution provided by the FT-Trck depends on the size of the readout strips: a spatial resolution of $144 \mu\text{m}$ is expected for $500 \mu\text{m}$ strips.

3.3.3 Forward Tagger Electromagnetic Calorimeter

The Forward Tagger Calorimeter (FT-Cal) is a subsystem of the Forward Tagger device that provides energy reconstruction of electrons that scatter with a polar angle from 2.5 to 4.5 degrees. Its fast response is used to form the trigger signal for the acquisition of all FT detectors. A schematic view of the calorimeter is presented in Figure 3.11.

FIGURE 3.11: Forward Tagger Calorimeter Layout [1].

The FT-Cal has been designed to fulfill several requirements:

- High radiation hardness
- Small radiation length and Moliere radius
- Fast recovery time
- High light yield
- Good energy and time resolution.

These properties are required since the FT-Cal will be placed in a restricted region with a significant magnetic field and will be subject to a high radiation dose. Moreover it is supposed to provide a trigger signal, therefore a very fast response is required.

The final FT-Cal design consists of 332 lead tungstate ($PbWO_4$, PWO) crystals, with a parallelepiped shape (15 mm $\times$ 15 mm $\times$ 200 mm each), displayed according to a circular distribution, as shown in Figure 3.11, and light is converted into electronic signals by avalanche photodiodes (APDs) attached on the back of each crystal.

Lead Tungstate Crystals PWO is the material that has been chosen for the FT-Cal inorganic scintillating crystals that will convert electrons and photons in visible light thus allowing the measurement of their energy via photo-sensors.

Lead Tungstate crystals present the following characteristics:

- High density (8.28 g/cm^3)
- Small radiation length (0.92 cm)
- Very fast decay time (6.5 ns (97 %) 30.4 ns (3 %))
- Very high radiation hardness

fulfilling the constraints of a compact geometry, resistance to high radiation dose (0.05 Gy/h) and fast response. The same crystal choice has been employed in a variety of experiments including CMS [67] [68], PANDA [69], CLAS [70]. In Figure 3.12 a PWO crystal wrapped with Wifi2000, that provides the reflectivity properties, is shown.

However the LY of PWO is only a fraction of percent compared to other inorganic crystals such as NaI and it

FIGURE 3.12: One PWO crystal wrapped with Wifi2000.

is strongly dependent upon temperature, as shown in Figure 3.13.

The LY rapidly increases at lower temperatures: it rises by a factor 2.5 reducing the temperature from 18°C to -25°C . This imposes a number of technical problems to be solved. First of all the whole calorimeter must be cooled to maximize the LY; the operating temperature must be controlled to avoid variations in response amplitude. However operations at low temperatures present additional technical issues:

- Reduced radiation hardness
- Longer decay time
- Thicker thermal insulation

FIGURE 3.13: Dependence of light yield on temperature, obtained in [1].

- More sophisticated cooler system

Therefore the experiment will be performed at 0 °C that represents a good compromise between the low temperature necessary to increase LY of crystals and the risk to have moisture condensation upon the electronic components. Accurate temperature dependence tests have been performed at the INFN laboratories in Genova on FT-Cal to study the stability of the FT-Cal response and the results are reported in 3.3.3.1.

Avalanche Photodiodes Avalanche photodiodes (APDs, Hamamatsu LAAPD s8664-1010) have been chosen to convert the photons produced by the scintillators in electric charge, because they are weakly sensitive to the presence of high magnetic fields. APDs are reverse-biased diodes, operating in the proportional zone, with an internal high electric field employed to multiply the charge carriers through an avalanche mechanism.

APDs present several good characteristics:

- Small sizes
- High quantum efficiency ($\cong 70\%$ in the region of the PWO emission)
- Unsensitivity to magnetic fields

Besides this, the issues in the employment of APDs are:

- Low gain that requires the use of an additional amplification circuit
- Temperature must be controlled to avoid variations in gain and noise

FIGURE 3.14: APD attached on the back of a crystal.

Figure 3.14 shows an APD glued to a PWO crystal, while Figure 3.15(a) presents a schematic view of an APD. When scintillation photons impinge on the diode, they cross the SiO_2 layer, the N layer, and they finally enter in the depletion region where they may produce electron-hole pairs. When a reverse-bias voltage V_B is applied, the high electric field created in the depletion region multiplies the charge-carriers through ionization effect, resulting in an avalanche process. Electrons then reach the N layer, while holes are collected in the anode. The output current provides information concerning the intensity of the incident light, since it is proportional to the number of photons that hit the photo-detector.

The APD gain G is defined as:

FIGURE 3.15: APD schematic view (a) and gain dependence on applied voltage at room temperature (b).

$$G(V_B) = \frac{I(V_B) - I_{\text{dark}}(V_B)}{I(G=1) - I_{\text{dark}}(G=1)} \quad (3.3)$$

where I is the photo-current, I_{dark} is the dark current, V_B is the reverse bias voltage, and $I_{\text{on}}(G=1)$ and $I_{\text{off}}(G=1)$ are, respectively, the values of the photo-current and the dark current when the internal gain of the

APD is equal to one, i.e. when the bias voltage is sufficiently low so that the avalanche mechanism is not active. These values have been independently measured using a LED to simulate the response to the scintillation light and calculate the gain. The result is shown in Figure 3.15(b). The gain of an APD not only strongly depends on the bias voltage but also on the temperature.

The gain of each of the 332 APDs and the LY of each the 332 PWO crystals have been measured as a function of temperature using a dedicated apparatus equipped with a cooling system. Crystals with higher LY have been coupled with APDs with lower gain to approximately equalize the final signal response of each detector. Moreover the APDs were grouped in batches presenting same gain properties to provide them with the same HV.

Cooling System Both the light yield performances of lead tungstate crystals and the APD gains are very sensitive to the operating temperature which has been chosen to be at $0\text{ }^{\circ}\text{C}$. Therefore temperature stabilization and monitoring is very important for precise calorimeter measurements. A dedicated cooling system has been developed at INFN Genova which consists of a copper thermal shield composed of two coaxial cylinders and planar endcaps encapsulating the crystals connected to cooling pipes. In Figure 3.16(a) the external cylinder of the thermal shield with cooling pipes surrounding the crystals is shown. The system is closed with an insulator, shown in Figure 3.16(b), and a carbon fiber skin enclosure, presented in Figure 3.16(c).

FIGURE 3.16: Left: FT-Cal cooling system, the cooling pipes surrounding crystals on the outside are visible. Center: insulator. Right: carbon fiber skin.

The temperature of the cooling fluid that flows in the copper pipes is maintained stable with an external LAUDA refrigerator [71] that allows to set the operating temperature with an accuracy of $0.1\text{ }^{\circ}\text{C}$. Since the operating temperature has been chosen to be $0\text{ }^{\circ}\text{C}$, nitrogen was flushed in the calorimeter space to prevent moisture formation. Temperature monitoring is accomplished by temperature sensors installed at different locations in the calorimeter enclosure. PT100 ("Pt" is the symbol for platinum, "100" for the resistance in ohms at 0°C) sensors were chosen to measure the temperature outside the detector region and in between crystals. 12 commercial PT100 detectors were located at different heights on the external surface of the copper shield, while eleven custom made flex PT100 having a flat shape were inserted between crystals. Their placement is shown in Figure 3.17.

FIGURE 3.17: Thermocouple custom made flex PT100 placement between crystals.

Preamplifiers, High Voltage Supplier and Data Acquisition The APDs output signal is too low to be directly acquired and it needs to be amplified by a proper circuit: the electrical signal from each APD is amplified by preamplifiers, its charge and arrival time are acquired by a fADC (flash Analog to Digital Converter) module. Each readout channel is composed by a combination of a PWO crystal, an APD and a preamplifier, as it is shown in Figure 3.18.

FIGURE 3.18: PWO crystal, APD and pre-amplifier

In the assembled calorimeter the preamplifiers are connected together on a motherboard which also provides high voltage supply channels for the APDs bias and low voltage supply for the preamplifiers, as shown in Figure 3.19(a). The power supply system is composed of high voltage (HV) CAEN boards, CAEN crate and 15 m long cables. The typical voltage applied to APDs is ~ 400 V. The low voltages (LV) supply is the WIENER MPOD,

providing ± 5 V through a 15 m long cable. The cables length, 15 m, is determined by the distance between the experimental hall and the electronics zone, outside CLAS12 acceptance. The signals from the preamplifiers connected to the FT-Cal motherboard (see Figure 3.19(b)) are carried to the fADC boards by coaxial cables and lemo cables merged by patch panels.

FIGURE 3.19: Preamplifiers. On the left, top view of FT-Cal, with the 332 preamplifiers visible, as well as the four wings with HV, LV and signal connectors. Center: the preamplifiers on the motherboard. On the right, detail of preamplifiers.

LED System Finally, a LED system was developed for the stability evaluation of the light yield and gain of the crystals and the APDs. It is made of three parts: one controller, six driver boards and one LED board (see Figure 3.20), on which the 332 blue LEDs, one for each crystal, are located. Verifying the response to the

FIGURE 3.20: LED board.

LED light provides also the fastest way to check crystals during the assembly procedure. In Figure 3.21 a LED housing attached on the back of a crystal is shown. The LED are also important for the optical annealing of the crystals, as it will be explained in next Chapter (see subsection 4.4.2).

FIGURE 3.21: LED housing on the PWO crystal.

3.3.3.1 Temperature Dependence Test

CLAS12 FT-Cal was mounted and tested at the INFN laboratory in Genova [72] for the first time during summer 2015. The primary mission of the test was to study the detector performances at different temperatures using

FIGURE 3.22: Cosmic ray set up.

cosmic rays, since both the PWO crystals and the APDs efficiency are affected by the internal temperature of the device. As a final result, data presented a $(27 \pm 1)\%$ signal increase from 19°C to 0°C : this is consistent with the light yield increase of the $PbWO_4$ crystals in the same range of temperatures. Figure 3.22 shows the cosmic rays setup of FT-Cal used for the measurements.

The 332 PWO crystals composing the FT-Cal were acquired by two different companies. A first batch with few crystals was produced by the Russian Bogoroditsk Technical Chemical Plant (BTCP), a second group by the Shanghai Institute of Ceramics Chinese Academy of Sciences (SICCAS). All the PWO crystals were characterized evaluating transparency, sensitivity to radiations and recovery using LED light [73] (all these tests are fully described in next Chapter), as well as light yield. The crystals that fulfilled the requirements were accepted for FT-Cal and arranged according to their radiation hardness, with the more resistant crystals placed closest to the beam line, where the expected radiation dose is 0.05 Gy/h . Also APDs present gain differences (ΔG), therefore

photo-sensors with higher gain were associated to crystals with lower light yield. The correspondent variation was estimated to be less than 3% [74].

The light yield characterization of the crystals was achieved in 2014 with two campaigns. In the first one a ^{60}Co source at 18° C was employed to measure the absolute light yield of 9 BTCP crystals at the B3 CERN facility. In the second one a ^{22}Na source was used for a comparative measurement of all the SICCAS crystals. This measurement was allowed by the experimental setup, the ACCOS facility [75] at CERN, developed to test the PWO crystal of the CMS experiment.

The light yield has been evaluated for each crystal in terms of the number of photo-electrons produced in the crystal for 1 MeV of deposited energy (N_{pe}/MeV). The values of the obtained light yield for all the crystals are shown in Figure 3.23 (left) in a color code as a function of the transverse position in the FT-Cal. Light yield values range from 11 to 18 N_{pe}/MeV . The light yield distribution of the crystals is shown in Figure 3.23 (right): it has a mean value $\langle LY \rangle = 14.47 \pm 0.08 N_{pe}/\text{MeV}$ with a standard deviation $\sigma_{\langle LY \rangle} = 1.23 \pm 0.07 N_{pe}/\text{MeV}$.

The signals from the different crystals S_i (pC) may be normalized to take into account the variations in light

FIGURE 3.23: Light yield values of the FT-Cal calorimeter (left) and its distribution (right).

yield values:

$$S_{i,corrected}(pC) = S_i(pC) \frac{\langle LY \rangle}{LY_i} \quad (3.4)$$

where $S_{i,corrected}$ is the normalized signal value of the crystal i having an associated light yield value LY_i , $\langle LY \rangle$ being the average light yield of the 332 crystals batch. When the temperature inside the crystal decreases, its light yield increases [65] causing an increase of the electrical signal and a better signal over noise ratio. APD gain is very sensitive to the temperature, therefore the applied high voltage (HV) was optimized to maintain a constant gain of $G = 150$: two HV values were evaluated, at 0° and 20° C, and were employed for the tests temperatures. The analysis involved only a quadrant of FT-Cal, Q1. A Landau and an exponential functions were used to fit the typical charge histogram released by a cosmic muon in a crystal to represent the signal and the noise, respectively. In Figure 3.25 the distribution, LY corrected, for quadrant Q1 at 19° C is shown on the left with signal peaked at (10.3 ± 0.3) pC, the same distribution at 0° is reported in the middle, where the

FIGURE 3.24: Fit of the crystal charge distribution for the 2nd temperature configurations. Clear difference of the Landau peak position between 19° C, on the left, and 0° C, on the right are observed.

average signal increases to 13.6 ± 0.1 pC. The right plot represents the ratio of the signals at 0° and 19° as a function of the position of the crystal in the FT-Cal. The histogram of the same ratios is plotted in Figure 3.26, showing a peaked distribution, centered at $R = 1.270 \pm 0.007$ with a small standard deviation of $\sigma_R = 0.056 \pm 0.005$. After the test in Genova the FT-Cal was disassembled and send to JLab where longer measurements were

FIGURE 3.25: 19° C signal distribution of the left, 0° C signal distribution in the center and the ratio on the right plot. All signals are LY corrected.

performed, confirming that a uniformity of corrected yields of the crystals better than 5% may be obtained at 0°.

FIGURE 3.26: Ratio of the signals obtained at 0° and 19° C.

Chapter 4

PWO Crystals Radiation Hardness Tests

Introduction

A quality assessment program has been carried out for the selection of the Lead Tungstate crystals for the Forward Tagger electromagnetic calorimeter. The crystals have been purchased from SICCAS, the Shanghai Institute of Ceramics, Chinese Academy of Sciences. The quality assessment is based on the performance under irradiation: the crystals light transmission is measured before and after gamma ray irradiation. The measurement of the light transmission is used to calculate the induced radiation absorption coefficient (dk) at 420 nm, the peak of the Lead Tungstate emission spectrum. These results have been published in [73]. The task is to obtain the performances for the Forward Tagger within the practicalities of SICCAS lead tungstate crystals. The results obtained for SICCAS crystals are compared with the equivalent measurements for the BTCP (Bogoriditsk Technical Chemical Plant) crystals since the BTCP crystals have been successful in several experiments, but their production is not available on the market anymore.

Tests have been also performed to study the possibility to recover the light transmission property of crystals using the optical annealing with a laser. The recovery may be performed on a regular basis in the final assembly within CLAS12 since, as already mentioned in paragraph 3.3.3, each lead tungstate crystal of the FT will be equipped with a LED.

In this Chapter the radiation harness tests on PWO crystals for the FT-Cal are presented. In Section 4.1 the radiation absorption coefficient is defined and the procedure for the tests is explained. Section 4.2 contains a description of the irradiation tests on the lead tungstate crystals. In Section 4.3 the procedure to recovery the radiation damage by LED illumination is illustrated. Section 4.4 is devoted to the analysis and the results. In Section 4.5 the conclusions are briefly reported.

4.1 Induced Radiation Absorption Coefficient and Test Procedure

Lead tungstate crystals ($PbWO_4$ or PWO) are employed for the Forward Tagger calorimeter FT-Cal and have already been used in several experiments, [67], [68], [69], [70], for EM calorimetry. The principal producer of these crystals was the Bogoriditsk Technical Chemical Plant (BTCP) in Russia, but after its closure the Shanghai Institute of Ceramics, Chinese Academy of Sciences (SICCAS) is one of the most practical remaining facilities. The crystals for the FT-Cal have been produced by SICCAS using the Bridgman method [76].

The FT-Cal is expected to be subject to a high radiation dose because of its proximity to the beamline, with an average radiation rate equal to 5 rad/h and up to ten times this rate for crystals closest to the beamline. Moreover the calorimeter will remain installed also for runs that do not require its data and for which higher rates may be required. For all these reasons the lead tungstate has been selected. In fact this material presents a very high resistance to radiation damage [77] together with a small radiation length and a fast decay time. The characteristics of $PbWO_4$ satisfy the demanding specifications of the FT-Cal as mentioned in paragraph 3.3.3.

The FT-Cal is composed of 332 $PbWO_4$ crystals, each measuring $200 \times 15 \times 15$ mm. The technical specifications required for the FT-Cal crystals are summarized in Table 4.1.

Property	Value
Length (mm)	200.00 ± 0.15
Width (mm)	15.00 ± 0.15
Height (mm)	15.00 ± 0.15
Longitudinal Transmission (360 nm)	$\geq 25\%$
Longitudinal Transmission (420 nm)	$\geq 60\%$
Longitudinal Transmission (620 nm)	$\geq 70\%$
Light Yield ($T = 18^\circ\text{C}$)	≥ 13 phe/MeV
Radiation Hardness (Induced absorption coefficient at 420 nm, dk)	$\leq 1.0 \text{ m}^{-1}$

TABLE 4.1: Specifications required to SICCAS for the $PbWO_4$ crystals of the FT-Cal

The tests reported in this chapter regard the assessment of the crystals radiation hardness properties. In order to quantify the resistance of crystals to the dose, the radiation induced absorption coefficient, dk, defined by the following relation, is employed:

$$dk = \frac{1}{L} \ln \frac{T_{bef}}{T_{irr}} \quad (4.1)$$

where T_{bef} is the light transmission at 420 nm, measured before irradiation, and T_{irr} the light transmission at the same light wave-length after irradiation. Crystals that present a higher radiation damage to light transmission also have a higher dk value. The purpose of the program is to verify that the light transmission (LT) and the dk value of all the 332 crystals fall within Table 4.1 specifications.

The procedure that has been followed to establish the dk value for each crystal consists of three steps. In the first part the light transmission of the crystal is studied: a spectrophotometer is employed to assess the LT of the crystal for wavelengths from the visible near-ultraviolet to the near-infrared spectrum. In the second part

the crystal is removed from the spectrophotometer and it is inserted in an irradiation chamber that provides a 30 Gy dose, equivalent to the dose expected in the FT over one month of operation of CLAS12 in Hall B. Then the crystal, after a period of recovery under a black cloth, is placed again inside the spectrophotometer to quantify its level of radiation hardness after irradiation and recovery and its dk value is calculated according to Equation 4.1. In the third part the recovery of the crystal is performed by optical annealing. The crystals are exposed, after irradiation, to the visible light produced by a LED and then inserted in the spectrophotometer to obtain a new value of the dk coefficient. The FT-Cal will be equipped with LEDs to apply the same optical procedure for the recovery of the light yield in Hall B. A total of 370 lead tungstate crystals were bought from SICCAS, including spares. For each crystal the induced absorption coefficient dk was measured and 51 crystals were rejected, because the value obtained for dk was too high. SICCAS replaced these crystals and the assessing procedure was repeated for these new crystals. The 332 crystals that presented better values of radiation hardness were selected for FT-Cal, the others were held as spares.

Tests with LEDs were performed on a subset of these crystals, to assess the suitability of LEDs for crystal recovery by optical annealing.

A comparison was also performed with a set of typical BTCP crystals. This batch included 8 crystals used for the prototype of the FT-Cal [65] and a single crystal of a slightly different geometry (160 mm long, tapering from 16×16 mm at one end to 13×13 mm at the other), employed in several previous calorimeter devices at Jefferson Lab, for example in the ECal subsystem of the Heavy Photon Search (HPS) experiment [78].

4.2 Irradiation Tests

Lead tungstate crystals have been used for a variety of experiments including CMS and ALICE at CERN [67] [68], PANDA at FAIR [69] and the CLAS Inner Calorimeter (CLAS-IC) at Jefferson Lab [70]. This large employment led to intense studies of their properties, such as the radiation hardness. Thanks to these studies a good understanding of the mechanisms of radiation-damage in $PbWO_4$ crystals and its effects on the crystal light yield [79] is available: under irradiation, impurities and defects in the crystal structure and traps for electrons and holes lead to the formation of color centers, the overall effect of which is a decrease in light transmission of the crystal at optical wavelengths. After exposure, these color centers spontaneously relax, with a fast thermal component at room temperature acting over a timescale of around 30 minutes [80]. In Figure 4.1 the fast recovery is shown for a sample SICCAS crystal: the dk coefficient is measured at 420 nm immediately after irradiation. Due to the fast recovery component of $PbWO_4$, both creation and elimination of color centers will take place during crystal irradiation and the total damage induced will depend on the equilibrium between these two effects, determined by the dose rate used for irradiation [81] [82]. After this fast recovery, crystals continue to recover on a longer timescale, typically several weeks and restore their light transmission property unless some dose rate dependent damage takes place. A complete recovery may be obtained performing a thermal annealing procedure at 200 °C [83]. A full description of LT performances has been developed for BTCP lead tungstate crystals, but they are no more commercially available. The studies must be repeated for SICCAS-type $PbWO_4$ crystals, since

FIGURE 4.1: Radiation induced absorption coefficient, dk , at 420 nm for a sample SICCAS-type $PbWO_4$ crystal, measured at three-minute intervals following irradiation with 30 Gy. After around 30 minutes, the fast recovery component is complete.

they present similar behaviour, but with consistent discrepancies with respect to BTCP ones. A preliminary study to quantify these differences was performed exploiting the Automatic Crystal quality Control System (ACCOS) [75] on a subset of the crystals produced by SICCAS: the results showed significant variations of the crystals parameters with respect to the nominal values and several crystals failed to satisfy the specifications of Table 4.1. For these reasons a program was undertaken to assess the light transmission properties of all the crystals acquired for the FT calorimeter. The crystals have been irradiated with a 30 Gy dose, equivalent to one month of operation inside CLAS12, but at a higher radiation dose to keep the test timing at a reasonable rate.

4.2.1 Experimental Setup for Irradiation Studies

The facility that has been exploited to perform the radiation hardness tests is the Strahlencentrum, used by researchers in a variety of disciplines, at the biotechnical operating unit of the University of Gießen. The Strahlencentrum allows to perform gamma ray irradiation studies thanks to its ^{60}Co sources, which have already been used by the II Physikalisches Institut of the university in studies of $PbWO_4$ crystals for the PANDA EM calorimeter. The irradiation station at the Strahlencentrum is equipped with a Cary 4000 spectrophotometer, which uses a Hamamatsu R928 PMT as a photo-detector with a customized enclosure into which crystals are loaded for measurements of the light transmission along their longitudinal axis. In Figure 4.2 a crystal loaded inside the spectrophotometer is shown. The spectrophotometer is monitored by a computer running Cary WinUV software that allows the display and the storage of spectra and it has options for exporting data in various formats for offline analysis.

The irradiation chamber is shown in Figure 4.3. It allows to impart the expected dose to the entire crystal, thanks to six sources whose irradiation volume cover the whole space occupied by the crystal. The dose is delivered with a rate of 120 Gy/hr corresponding to a 30 Gy dose in 15 minutes. The dependance on the irradiation position of

FIGURE 4.2: One lead tungstate crystal loaded inside the spectrophotometer

the crystals are negligible as already seen in multiple irradiation cycles of FT crystals and in previous work by the PANDA collaboration at the Strahlencentrum.

FIGURE 4.3: The irradiation chamber

4.2.2 Crystal Testing Procedure

For all the 370 SICCAS crystals, the 51 spare crystals and the BTCP crystals from both the HPS ECal and the FT-Cal prototype, the following procedure has been used at least once. The light transmission was measured along the longitudinal direction of the crystal after loading the crystal in the spectrophotometer. The crystal was then inserted in the irradiation chamber where the 30 Gy dose was delivered. After the irradiation the crystal was removed from the chamber and placed in a dark environment for the fast recovery component to take place. The dark was chosen to avoid an optically stimulated recovery. Finally the crystal transmission was measured again in the spectrophotometer, placing the crystal in the same position of the first measurement to minimize changes in the light transmission due to crystal positioning. The presence of dust, dirt and imperfections on the

crystal surface were avoided as well. The described sequence will be referred to as "once-through" irradiation procedure, in the following.

4.2.3 Detailed Studies of Crystal Properties

Besides the tests that have just been described, further studies were performed on a sample of specific crystals. These studies include the exposure to multiples of the 30 Gy dose, as well as repeat measurements of the "once-through" irradiation procedure after thermal annealing of the crystals, the latter being performed as a consistency check of the reproducibility of measurements.

The light yield of the crystals was studied in a separate facility at the University of Gießen to check the measurements already performed with ACCOS described in section 4.2: the new results were consistent with the previous ones and with the quoted values from the manufacturer and all the crystals meet the requirements for light yield for FT-Cal. Another test that was performed regards the possibility to recover LT with an optical annealing technique and it will be described in the next section.

4.3 Optical Annealing by LED Illumination

Lead tungstate crystals are expected to have their light transmission property restored if they are at room temperature over an extended timescale except in cases of prolonged high dose, where high levels of radiation damage will accumulate, such as in experiments with high energy colliders like CMS. The color centers formed around defects in the crystal release their captured electrons and/or holes and the crystal exhibits again similar light transmission properties to those it had before irradiation. Another technique to recover crystals LT is by thermal annealing, a process that requires several hours at temperatures of around 200 °C: the probability of releasing charge carriers from color centers depends exponentially on the temperature.

In addition to this methods, another way to restore the LT of a $PbWO_4$ crystal is by optical annealing: illumination from visible light is able to induce crystal recovery by optically-stimulating the restoration of the color centers in the crystals. This process has been observed on several types of scintillating crystals [80] [84] [85] [86] [87]. The studies of optical annealing were performed in Gießen both with LEDs and with different sources at various intensities of illumination, such as fluorescent and incandescent lamps. Both SICCAS and BTCP irradiated examples of $PbWO_4$ crystals were used. The results of this procedure using LEDs, which have been chosen for the gain monitoring system of the FT-Cal, are reported in section 4.4.

4.4 Analysis and Results

The analysis on light transmission measurements before and after irradiation, as well as all multiply-irradiated, annealed and re-irradiated crystals, and LED illumination studies was performed using the data saved as .csv text

Property	Mean	σ
LT Before Irradiation (360 nm, %)	27.1 ± 0.2	4.3
LT After Irradiation (360 nm, %)	23.7 ± 0.2	4.4
LT Before Irradiation (420 nm, %)	61.5 ± 0.2	3.2
LT After Irradiation (420 nm, %)	50.8 ± 0.2	4.9
LT Before Irradiation (620 nm, %)	74.7 ± 0.1	2.7
LT After Irradiation (620 nm, %)	68.3 ± 0.2	3.7

TABLE 4.2: Average and Standard Deviation of light transmission values before and after irradiation of the SICCAS $PbWO_4$ crystals used for the FT-Cal

files by the spectrophotometer control software. The offline analysis collated light transmission measurements at three wavelengths, 233 nm, 360 nm and 620 nm, computed dk at 420 nm for all irradiated crystals and examined the LED-stimulated recovery properties of the SICCAS and BTCP crystals. Possible systematic effects and data normalization issues were also considered. The first systematic effect regards the change of the baseline. The baseline measurement is a reference measurement taken with no crystal sample installed and no illumination of the photodetector within the device. Variations of the baseline are primarily due to thermal fluctuations, therefore a stable room temperature in the Strahlencentrum should be preserved to avoid them. The baseline spectrum has been regularly updated, especially after longer periods where the spectrophotometer was not in use. The spectrophotometer is automatically set up to subtract baseline measurements. The second systematic effect regards the reproducibility of the crystal positioning: even small misalignments of the crystal have an appreciable effect on the measured values of light transmission. To minimize the misalignment of the crystals, a small identifying mark allowed to measure the crystals light transmission always with the same orientation inside the spectrophotometer. In order to quantify the reproducibility of a spectrum measurement, several crystals were measured multiple times. The total systematic error has been finally estimated at the level of 1-2 %, resulting in an error in dk between 0.1 and 0.2 m^{-1} . As a consequence, the requirement for dk was increased to accept crystals with $dk \leq 1.3 m^{-1}$.

4.4.1 SICCAS $PbWO_4$ Crystal Results

Figure 4.4 shows a comparison between the spectrum of a crystal that was accepted for use in the FT-Cal, on the right, and the spectrum of a crystal that was rejected, on the left. The red line is the light transmission of the crystal before the 30 Gy irradiation and the black line is the spectrum after irradiation and recovery. The radiation damage is present in both crystals but is clearly more evident in the spectrum on the left. Figures 4.5 4.6 4.7 show the light transmission measurements at 360 nm, 420 nm and 720 nm, before and after irradiation and recovery for all the SICCAS crystals tested, demonstrating the spread in physical properties of the crystals. The corresponding mean and standard deviation values of light transmission are given in Table 4.2.

The overall statistics of SICCAS-type $PbWO_4$ crystals with respect to dk at 420 nm are shown in Figure 4.8. Crystals with a radiation induced absorption coefficient higher than 1.3 m^{-1} were rejected.

Two crystals, referred to here as A and B, were subject to multiple exposures to the 30 Gy irradiation,

FIGURE 4.4: Longitudinal Light Transmission for SICCAS-type $PbWO_4$ crystals, before and after 30 Gy gamma irradiation and fast recovery. The irradiated spectra exhibits lower light transmission, particularly in the 400 - 600 nm region. The plot on the right represents good response of the SICCAS crystal to irradiation, while the plot on the left is typical of a very poorly performing crystal.

FIGURE 4.5: Light Transmission measurements at 360 nm for all SICCAS-type $PbWO_4$ crystals tested in Gießen. Filled red squares are measurements before irradiation and open squares from measurements made after irradiation with 30 Gy. “Crystal ID” is an internal numbering scheme used to track the crystals.

totalling 1, 2, 4 and 51 times a single dose, allowed 30 minutes in a dark environment for fast recovery, then re-measured in the spectrophotometer. The results of these studies are shown in Figure 4.9. The two crystals show different behaviours under irradiation: crystal B presents less reduction in light transmission as a result of radiation damage. However they both experience a severe degradation in light transmission after the $51\times$ dose, meaning that, under sufficient gamma irradiation, all SICCAS- type crystals will suffer appreciable levels of light transmission damage.

FIGURE 4.6: Light Transmission measurements at 420 nm for all SICCAS-type $PbWO_4$ crystals tested in Gießen. Filled red squares are measurements before irradiation and open squares from measurements made after irradiation with 30 Gy. “Crystal ID” is an internal numbering scheme used to track the crystals.

FIGURE 4.7: Light Transmission measurements at 620 nm for all SICCAS-type $PbWO_4$ crystals tested in Gießen. Filled red squares are measurements before irradiation and open squares from measurements made after irradiation with 30 Gy. “Crystal ID” is an internal numbering scheme used to track the crystals.

4.4.1.1 Comparison with BTCP Crystal from HPS Calorimeter

In Figure 4.10 a comparison is presented between the spectra for BTCP crystal from the HPS ECal and an example FT-Cal SICCAS crystal, both before and after irradiation. The BTCP crystal presents differences with respect to the SICCAS one: the “wobble” feature, clearly visible around 600 nm in the SICCAS spectra, is less pronounced in the BTCP, the light transmission is higher at all wavelengths and the degree of radiation damage across the visible spectrum is far lower, resulting in a lower value of $dk = 0.6 \text{ m}^{-1}$ being computed from the light transmission measurements at 420 nm.

FIGURE 4.8: Histogram of the radiation induced absorption coefficient, dk , for all SICCAS-type $PbWO_4$ crystals tested in Giessen. Crystals with a dk greater than $1.3 m^{-1}$ were rejected.

FIGURE 4.9: Light Transmission measurements for SICCAS crystals irradiated multiple times, referred to as A (left) and B (right). Each crystal was subject to sequential doses of 30 Gy totalling 1 (30 Gy, black line), 2 (60 Gy, green line) and 51 (1530 Gy, blue line) times, the 30 Gy dose representing one month of expected FT-Cal operational. Labels show intersections of the curves with a line at 440 nm.

4.4.1.2 Comparison with BTCP Crystals from FT-Cal Prototype

The BTCP crystals from FT-Cal prototype present a narrower range of values of dk than the SICCAS crystals. Moreover they all show a dk value less than $1.0 m^{-1}$, the originally-specified value for the FT-Cal. In Figure 4.11 the induced radiation absorption coefficients for the 8 BTCP sample crystals are shown, overlapped to the SICCAS crystals which have dk less than $1.0 m^{-1}$. Because of their higher performances, also in terms of light transmission at visible wavelengths, when compared to the SICCAS crystals, the employment of the available

FIGURE 4.10: Light Transmission measurements for a single SICCAS $PbWO_4$ crystal, before and after irradiation (left), alongside similar measurements performed for a BTCP crystal (right). The irradiated spectra exhibit lower light transmission, particularly in the 400 - 600 nm region, however the BTCP type crystal exhibits a greater resistance to light transmission damage under irradiation.

FIGURE 4.11: Induced absorption coefficient, dk , for the BTCP sample crystals from FT-Cal prototypes (squares), superimposed to all FT-Cal SICCAS crystals with dk less than 1.0 m^{-1} (crosses), the originally-specified value for the FT-Cal. Both crystal types shown have the same $200 \times 15 \times 15 \text{ mm}$ geometry, allowing these BTCP samples to be used together with SICCAS crystals in the Forward Tagger.

samples of BTCP crystals has been considered for use in the regions of the FT-Cal expected to receive the highest rates of radiation.

4.4.2 LED Recovery Results

As shown in Figure 4.9 and in the previous section, all the SICCAS-type crystals, if subject to sufficient irradiation, suffer a degradation of their light transmission. As already explained, the LT can be restored with a thermal annealing process. However it is not practical to perform this process on the crystal that compose the FT-Cal because the required high temperatures impose to dismount the 332 lead tungstate crystals, to prevent a damage to the surrounding components of the FT inside CLAS12. A thermal annealing process could be applied during the shutdown periods of CLAS12 operations, but an alternative method to reduce crystals radiation damage between these available maintenance windows is advisable. The optical annealing has been selected to mitigate the effects of radiation damage to the crystals, via the employment of LED illumination, available in the FT-Cal from the light monitoring system. Both the SICCAS and the BTCP type crystals have experienced this use of LED illumination to recover radiation damage. The light transmission recovery effect was observed on irradiated samples of the two crystal types when illuminated with blue light from a single-color NICHIA NSBP500AS LED [88].

4.4.2.1 Results for SICCAS Crystals

Three SICCAS crystals were used for LED recovery studies: A and B, from section 4.4.1 and a third crystal, here referred to as crystal C. The three crystals were all subject to the “once-through” irradiation procedure described in section 4.2.2. As already discussed in section 4.4.1, crystals A and B were also subject to multiple irradiations. Therefore in the complex crystal C presented a 1×30 Gy dose, A and B a 51×30 Gy dose, when they were exposed to LED illumination for the optical annealing to take place. The recovery effects in light transmission were observed and reported in Figures 4.12 and 4.13 for crystal C and for crystals A e B respectively. Figure 4.12 shows the recovery of crystal C, that was subject to a 30 Gy dose irradiation: 30 seconds of LED illumination is sufficient to start observing recovery in light transmission, with significant recovery observed after several minutes of exposure to illumination from the LED. Figure 4.13 shows the recovery of crystals A and B which were subject to a 51×30 Gy dose after 4 hours illumination: having received a higher dose, a much longer illumination time, of order of several hours, is necessary to see any significant recovery effect. The light monitoring system of the FT-Cal is expected to be used to optically anneal the crystals, on a regular basis usually between experiments. A few minutes illumination should be sufficient for the optical recovery after one month of irradiation so that the effects of accumulated doses over long periods of time may be prevented. There is also the possibility to illuminate crystals between runs during an experiment if necessary: the opportunity to do so arises during beam operations or configuration changes in the experimental hall.

4.4.2.2 Results for HPS BTCP Crystal

Similarly to SICCAS type crystals, also the BTCP crystal from the HPS calorimeter was subject to LED illumination. This crystal followed a “once-through” irradiation procedure, like SICCAS crystal C, and then it was

FIGURE 4.12: Light Transmission measurements for SICCAS-type $PbWO_4$ crystal C, irradiated with 30 Gy ('After' curve) and recovered using LED illumination. Initial recovery of light transmission can be seen after just 30 seconds illumination (green curve). Successive 30-second illuminations show that significant recovery of light transmission is possible after a few minutes (further curves shown representing 60 seconds accumulated illumination and 90 seconds). Labels show intersections of these curves with a line at 440 nm.

exposed to varying degrees of LED illumination used to obtain light transmission recovery. The light transmission recovery is almost complete after 35 minutes of illumination, as it is possible to observe in Figure 4.14.

4.5 Conclusions

The demanding requirements for FT-Cal led to some concern in acquiring lead tungstate crystals with sufficient performance for their use in the device: the degradation in light transmission under irradiation was an important issue. The program of crystal testing undertaken at the University of Gießen allowed to identify the crystals that had to be rejected because of low radiation hardness and to decide the most appropriate positioning of the accepted crystals in the calorimeter, according to the value of the induced radiation absorption coefficient dk . The optical annealing procedure demonstrated to be effective to recover light transmission properties of the crystals subject to radiation damage, therefore the LED-based light monitoring system of the FT-Cal, designed for preliminary gain equalization, can also be applied to the optically-stimulated recovery of radiation damage to the crystals, in substitution of the time-consuming alternative of dismounting the calorimeter for thermal annealing.

FIGURE 4.13: Light Transmission measurements for multiply irradiated SICCAS crystals A (left) and B (right), showing pre-irradiated measurements, irradiated spectra for a 51 times 30 Gy dose and the effect of 4 hours LED illumination on the recovery of light transmission. After such heavy doses, where light transmission has been severely decreased, crystal recovery via LED illumination is more difficult.

FIGURE 4.14: Light Transmission measurements for BTCP-type $PbWO_4$ crystal, before and after irradiation with 30 Gy, and recovered using LED illumination totalling 35 minutes. After this illumination, almost complete recovery of the light transmission is observed.

Chapter 5

Forward Tagger Calorimeter Calibration

Introduction

The energy measurement provided by the electromagnetic calorimeter of the FT will be crucial. As already explained in Chapter 3, the detected energy comes from the electromagnetic shower of electrons and photons inside the FT-Cal: PWO crystals are devoted to the conversion of the energy deposited by the shower in visible light that is then transformed by APDs in an electronic signal. The integrated current is then proportional to the deposited energy and it is digitalized by analog to digital conversion modules (ADC).

The final goal of the calibration procedure is to reconstruct the energy of the incoming electron or photon from the ADC information. A first step is to convert back the integer numbers provided by the ADC (n_{ADC}) into the corresponding energy released by the shower in the crystal (E_i). Assuming that non-linear effects are negligible the energy E_i is given by

$$E_i = c_i \cdot n_{ADC} + c_{offset} \quad (5.1)$$

where c_{offset} and c_i are the calibration constants that may be determined through dedicated calibration runs. As a consequence of the electromagnetic shower, the energy of the incoming photon or electron will be not totally deposited on a single crystal, but, more realistically, will be spread on a set of crystals, forming a *cluster*. The best determination of the energy of the incoming particle is therefore given by the sum of the detected energies of all the adjacent crystals that have been involved by the electromagnetic shower. In many cases the electromagnetic shower will be not fully contained in the calorimeter, with a subsequent loss of energy. Moreover, for practical reasons, data will be not recorded if the energy collected by a single crystal is too low, i.e. a threshold on the energy reconstructed by a crystal in a single event will be employed. A second step in the calibration procedure is therefore to determine the relation function and its parameters that allows to obtain the energy of the incoming particle from the cluster energy. This correction is expected to be dependent on both the energy

and the position of the cluster. This Chapter describes the procedure that has been followed for this second calibration step studying the correlation between the reconstructed and the incoming particle energy.

To perform this task, events have been simulated with a photon entering the FT having energies in the range from 0.1 up to 8 GeV, with steps of 50 MeV, and polar angles from 2.5 up to 4.5 degrees, with steps of 0.2 degrees. The detected energy of the event has been reconstructed identifying the cluster of adjacent crystals that have been involved in the electromagnetic shower and summing over the deposited energies in each of them; this value has been compared with the actual energy of the simulated photon. The difference between the generated and the reconstructed energy has been plotted for every angle and for every energy bin. Thus we obtained a set of gaussians (for each of the 11 angles 160 plots were needed to cover all the energy range) that were fitted obtaining a mean and a standard deviation. These values were used for the subsequent analysis. Eleven plots were produced with the dependence of the mean values on the energy of the beam; the plots were then fitted to obtain the energy corrections.

In this Chapter the simulation and reconstruction codes used for the reconstruction of the events in the FT calorimeter are described, as well as the calibration procedure and the calibration corrections results.

In Section 5.1 the simulation and reconstruction codes are described; in Section 5.2 the π_0 invariant mass reconstruction approach to calibrate the detector is illustrated; in Section 5.3 the same calibration procedure that has been used for a previous experiment (CLAS-DVCS) and that will be also employed for the calibration of FT-Cal is briefly shown. In Section 5.4 the energy corrections necessary to switch from the energy that is read by the device to the actual one are obtained.

5.1 Simulation and Reconstruction Codes

In this Section the chain of software codes used for the simulation and the reconstruction of the events in the FT calorimeter will be described.

The interaction of particles with all CLAS12 detectors may be simulated using a GEANT4-based code called GEMC. Part of GEMC code describes the geometry of all CLAS12 subsystems shown in Chapter 3. It is possible to run the GEMC code in the so-called interactive mode, that allows to visualize the CLAS12 apparatus with all its subsystems and all the primary and secondary particles tracks originated by the interaction of the particles with the detectors. The whole CLAS12 geometry is shown in Figure 5.1(a) and the detail of the FT position around the beam pipe is reported in Figure 5.1(b). Once events have been simulated with GEMC, an output file is produced including all the information about particles in CLAS12 and a software is employed to reconstruct

FIGURE 5.1: CLAS12 experimental apparatus. Forward Tagger detector.

final state particles. The logical procedure is illustrated:

and will be fully described in the following paragraphs. Therefore, for instance, an event with an electron entering the FT-Cal can be simulated with GEMC, see Figure 5.2(a), and then a reconstruction algorithm can be used to obtain all the information concerning the final clusters, including energy, polar and azimuthal angles: Figure 5.2(b) has been produced with the reconstruction algorithm and displays a section of FT-Cal highlighting crystals that collected energy.

5.1.1 GEMC Event Simulation

As already explained, GEMC is a Geant4 Monte Carlo that simulates the interaction of incoming particles with the CLAS12 experimental setup. To make GEMC run with a particular configuration selected by the user, two modalities are available. The first one is the interactive mode, where a graphical interface (GUI, Graphical User Interface) is employed and the interactions inside the detectors are visualized online event by event. An example of one event simulating an electron entering the FT is illustrated in Figure 5.3: to be noticed that the primary

FIGURE 5.2: Event with one electron (left). Section of FT-Cal where the crystals that collected energy are highlighted (right).

vertex with the generation of the electron is visible. Despite this modality allows to visualize the interaction

FIGURE 5.3: Event with one electron: the primary vertex is visible.

of particles in the setup, it is not the most useful in practice, since it is very time-consuming. In fact for the high statistics required for simulations the non-interactive mode is employed. In this modality the user gives the instructions to GEMC with the support of a gcard, a file where all the requirements are reported. A complete description of gcards is reported in [89]. An example of a simple gcard is :

```

1 <gcard>
2     <option name="geometry" value="900x600"/>
3     <detector name="experiments/clas12/ft/ft" factory="TEXT" variation="test1"/>
4     <option name="INTEGRATEDDRAW" value="ft_cal ft_hodo ft_trk"/>
5     <option name="BEAM_P" value="e-, 5.0*GeV, 4*deg, 0*deg"/>
6     <option name="SPREAD_P" value="2.0*GeV, 4*deg, 180*deg"/>
7     <option name="BEAM_V" value="(0, 0, -5)cm"/>
8     <option name="SPREAD_V" value="(0.1, 10)cm"/>
9     <!-- <option name="INPUT_GEN_FILE" value="LUND, input.dat"/>-->
10    <option name="OUTPUT" value="evio, output.ev"/>
11    <option name="N" value="1000"/>
12    <option name="USE_GUI" value="0"/>
13    <option name="PRINT_EVENT" value="2"/>
14 </gcard>

```

Every line corresponds to a command given to GEMC. The first command line (line 2) specifies the size of the display window. In this gcard the FT detector has been loaded (line 3) using a variation: a variation can

be a different material, or dimensions, or positioning with respect to the original configuration. In the same command line the name (with path) of the detector system and its variation are specified. To remove a detector, its corresponding line has to be removed or commented.

The digitized and smeared information provided by each channel of the selected detectors is the default output result of the simulation. The corresponding information before the smearing and the digitalization steps are called *true information* and the corresponding output is disabled by default. To activate it for a given system, the INTEGRATEDDRAW flag has to be employed. For example, to save FT-Cal, FT-Hodo, and FT-Trk true information, line 4 has been used. In this gcard the internal generator of GEMC has been employed (lines 5 and 6) to produce electrons with energies between 3.0 GeV and 7.0 GeV, with polar angles between 0 and 8 degrees and with azimuthal angles between 0 and 360 degrees. The position of the vertex along z axis will be between -5 and 5 cm (line 7), while vertex radius will be between 0 and 0.1 cm (line 8). Line 9 is commented: it gives the instruction to use as incoming particles the ones described by a lund file named *input.dat*. The lund extension is a text format supported by GEMC: its header defines quantities such as the number N of generated particles for each event and other kinematic properties. After the header there are N lines (one per particle) describing each particle property as reported in the Tables below. A bold line indicates a quantity used by GEMC. Quantities not

(a) LUND header		(b) LUND particles	
LUND header		LUND particles	
column	quantity	column	quantity
1	Number of particles	1	index
2	Number of target nucleons	2	charge
3	Number of target protons	3	type (1 is active)
4	Target Polarization	4	particle ID
5	Beam Polarization	5	parent index
6	x	6	daughter index
7	y	7	momentum x [GeV]
8	W	8	momentum y [GeV]
9	Q^2	9	momentum z [GeV]
10	nu	10	E
		11	mass
		12	vertex x [cm]
		13	vertex y [cm]
		14	vertex z [cm]

TABLE 5.1: LUND format: structure of files.

used by GEMC are kept in the output stream (user defined meanings could be assigned to them). An example of lund file describing the final state of reaction $e^-p \rightarrow e^-K^+\Lambda$ is reported in the following:

```

3 1 1 0 0 0 0 2.33883 0.233241 2.57016
1 -1 1 11 0 0 0 0.142619 0.348942 4.01217 4.02984 0.0005 0 0 0
2 1 1 321 0 0 -0.213297 -0.563672 2.15342 2.29001 0.4936 0 0 0
3 0 1 3122 0 0 0.0706788 0.21473 0.434418 1.21843 1.115 0 0 0

```

To be noticed that 11, 321 and 3122 are the particle IDs of the electron, K^+ and Λ , according to the PDG [10] standard notation.

The output will be called *output.ev*, as specified in line 10. The 1000 events (line 11) will be generated in non-interactive mode (line 12). Event number will be printed every two events (line 13).

5.1.2 Photon and Cluster Reconstruction

Once the events are simulated with GEMC, they are read and reconstructed by a C++ algorithm that can be compiled with ROOT [90]. To allow the algorithm to read evio files they must be converted in .root files.

Following the interaction of a high energy electron or photon with the electromagnetic calorimeter, an electromagnetic shower is produced, which distributes the energy of the initial particle among several crystals. When the energy collected by a crystal exceeds a certain fixed threshold, we have a hit: each detected shower element is a hit. The energy of the impinging particle can be reconstructed from the energies of the hits. All the adjacent calorimeter modules or hits, that were involved in the reconstruction of the electromagnetic shower, compose the cluster.

The primary task of the reconstruction algorithm is to properly associate the crystals with a hit from the same electromagnetic shower into a cluster. Reconstructed clusters have position and energy: the position is obtained by the weighted sum of the positions of the crystals that compose the cluster and the energy is the sum of the energies from each hit in the cluster, according to the relations

$$\begin{aligned} E_{cluster} &= \sum_i E_{hit}^i \\ x_{cluster} &= \frac{\sum_i E_{hit}^i \cdot x_{hit}^i}{E_{cluster}} \\ y_{cluster} &= \frac{\sum_i E_{hit}^i \cdot y_{hit}^i}{E_{cluster}} \end{aligned} \tag{5.2}$$

where x_{hit}^i and y_{hit}^i are the coordinates of the center of the crystal.

A photon or electron interacting with the calorimeter deposits most of its energy on the crystals closest to the position of incidence; the crystal with the highest hit energy is called the *seed*. The detected hits are sorted according to their energy, the central module of the first cluster is the first most energetic hit. The cluster finding algorithm scans all other hits and associates surrounding crystals, whose energies are assumed to be connected to the same electromagnetic shower and are above the required threshold, into a cluster. The loop ends when the algorithm finds all the hits that compose the cluster, that is not limited by some specific number of elements.

After finding the first cluster, the procedure is repeated until all hits above the threshold are associated to the appropriate cluster.

5.2 π^0 Invariant Mass

The calibration of calorimeter elements is required for a proper energy reconstruction. A common approach to calibrate a module is to direct a particle with known energy on it, reconstruct its ADC spectrum and associate

the peak channel to the value in MeV that corresponds to the particle energy.

If a monochromatic beam is not available an alternative method is to exploit the invariant mass reconstruction of a particle, for instance the π^0 , via its decay into two photons, which can be detected and reconstructed by an electromagnetic calorimeter. As it is shown in the following, this second approach is more complex since it requires an iterative procedure, changing the calibration parameters at each step; however it may conveniently be applied to ordinary experimental runs not requiring dedicated runs. In Table 5.2 the decay modes and branching fractions of π^0 are reported [10]. In Figure 5.4 the GEMC simulation of the π^0 decay into two photons is shown.

π^0 decay modes	Fraction (Γ_i/Γ)
2γ	$(98.823 \pm 0.034) \%$
$e^+e^-\gamma$	$(1.174 \pm 0.035) \%$
γ positronium	$(1.82 \pm 0.29) \times 10^{-9}$
$e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
e^+e^-	$(6.46 \pm 0.33) \times 10^{-8}$
4γ	$< 2 \times 10^{-8}$
$\nu\bar{\nu}$	$[e] < 2.7 \times 10^{-7}$

TABLE 5.2: π^0 decay modes and branching fractions.

The two electromagnetic showers produced by the photons in FT-Cal are reconstructed as described in the

FIGURE 5.4: Decay of the π^0 into a couple of photons which is detected by the FT-Cal (left). Calorimeter section with two clusters corresponding with the photon couple (right).

previous section, from which the four-momenta of the two photons are determined. The invariant mass of the

pion, W_{π^0} can be calculated using the formula

$$W_{\pi^0} = \sqrt{E^2 - p_x^2 - p_y^2 - p_z^2} \quad (5.3)$$

where

$$\begin{aligned} E &= E^{\gamma_1} + E^{\gamma_2} \\ p_x &= p_x^{\gamma_1} + p_x^{\gamma_2} \\ p_y &= p_y^{\gamma_1} + p_y^{\gamma_2} \\ p_z &= p_z^{\gamma_1} + p_z^{\gamma_2} \end{aligned} \quad (5.4)$$

and γ_1 and γ_2 are the two photons. The four-momentum of the π^0 is equal to the sum of the four-momenta of the two photons from its decay. The value of the π^0 invariant mass W_{π^0} is calculated and it is used to appropriately fill the 332 calibration histograms, one for each crystal. For each π^0 event the reconstructed W_{π^0} value is assigned to the histograms of all the crystals that took part to the clusters.

From the knowledge of the theoretical π^0 invariant mass, $M_{\pi^0} = 134.9766 \pm 0.0006 \text{ MeV}/c^2$ [10] and performing a gaussian fit to the W_{π^0} distribution, it is possible to check that these two values are compatible within the errors. A first set of calibration constants can be initialized via dedicated cosmic rays runs and small adjustments may be iteratively applied using the reconstruction of the π^0 mass.

At this simulation stage it has been verified that, starting from correct calibration constants, the invariant mass spectrum of all the 332 crystals was peaked at the nominal π^0 mass value.

In Figure 5.5 the fitted histograms for each of the 332 crystals are shown as a function of the crystals coordinates.

This analysis is useful also to notice that the crystals participate differently to the energy reconstruction. As already mentioned, the crystals on the edge are involved in clusters that partially escape the calorimeter, therefore the reconstructed energy of the cluster differs more from the actual photon energy, with a subsequent worst reconstruction of the π^0 invariant mass. This results in a broadening of the gaussian, i.e. in a larger value of the standard deviation σ . In Figure 5.6(a) the invariant mass distribution from a crystal located in the center of the calorimeter is fitted with a Gaussian function and in Figure 5.6(b) the same procedure is accomplished for a crystal mounted on the edge. From a comparison between the two figures it is possible to see that the peak width in the second case is almost doubled, for the reasons that have been previously explained.

A global view of these effects is shown in Figure 5.7 where the standard deviation of the gaussians from Figure 5.5 are shown. Crystals in the central part present sharper peaks with respect to the ones on the edge.

5.3 Cluster Energy Dependence upon Position

A part of the calibration studies have been dedicated to study the cluster energy dependence upon the distance R of the shower from the beamline. This effect has been studied in analogy with the CLAS/DVCS Inner Calorimeter Calibration [91] procedure. Figure 5.8 represents the dependence of the leakages in the FT-Cal detector on the

FIGURE 5.5: Each of the 332 crystals of the FT-Cal has a corresponding histogram where the invariant mass distribution of the π^0 is displayed. The invariant mass value is inserted in all the crystals that took part to the cluster. The result has been obtained simulating $10^5 \pi^0 \rightarrow \gamma\gamma$ events with GEMC.

polar angle θ for single photons of energy 2.5 GeV. If $\theta < 2.5^\circ$, as in Figure 5.8(a), the photon collapses against the Møller cone shielding; if $2.5^\circ < \theta < 4.5^\circ$, as in Figure 5.8(b), the electromagnetic shower is almost fully contained in the calorimeter; in the case of Figure 5.8(c) the scattering polar angle is $\theta = 4.85^\circ$, which is out of the fiducial volume of FT and the leakages are significant.

Systematic simulations of single photon events have been carried on for $E_{gen}^\gamma = 2.5 \text{ GeV}$, with polar range $2.0^\circ < \theta < 5.0^\circ$ and azimuthal range $-180^\circ < \phi < 180^\circ$, generating 10^5 events. Figure 5.9 shows the reconstructed energy of the cluster E_{clust} as a function of the distance from the beamline R. We recall that the polar ranges $2.0^\circ < \theta < 5.0^\circ$ and $2.5^\circ < \theta < 4.5^\circ$ (fiducial volume) correspond to range distances R $6.5 < R < 16 \text{ cm}$ and $8 < R < 15 \text{ cm}$ from the beamline, respectively.

The reconstructed cluster energy E_{clust} (MeV) dependence on the distance from the beamline R (cm) is displayed in Figure 5.9, showing that leakages increase for $R > 14.3 \text{ cm}$. E_{clust} distributions, sliced in R bins of 1 cm width, are plotted in Figure 5.10 and are fitted with gaussian functions. It is possible to note that only the R bins corresponding to the fiducial volume are covered and that the first and last plots present fewer events.

FIGURE 5.6: Example of π^0 invariant mass reconstruction. Histogram relative to one crystal is reported as an example. The distribution is shown on the left, in the middle the gaussian fit for a crystal in the central part of the detector is shown, presenting a better reconstruction of π^0 invariant mass distribution (better statistics, sharper peak), on the right the example of a crystal from the edge is reported, which presents a worst reconstruction (lower statistics, broader peak).

FIGURE 5.7: FT-Cal section highlighting the gaussian fits standard deviation in MeV.

Mean values of the fits as a function of R are presented in Figure 5.11(a): mean values are stable and close to the generated photon energy value up to 14.5 cm but decrease for higher distances departing from the actual energy. This result confirms that the chosen fiducial volume corresponds to the geometry region where the FT response is optimized. Finally, the standard deviation σ of the fits as a function of R is plotted in Figure 5.11(b): as expected, the standard deviation values are lower and stable up to 13.5 cm and then increase. There are only eight points in the last two plots since the fit for the last bin in R was not reliable because of the lack of statistics. Although the calorimeter covers angles from 2.0° , the range $2.0^\circ < \theta < 2.5^\circ$ is shielded by the structures in

(a) Photon event with $\theta = 2.15^\circ$. (b) Photon event with $\theta = 3.5^\circ$. (c) Photon event with $\theta = 4.85^\circ$.

FIGURE 5.8: Comparison between different events: in the case of Figure 5.8(a) the photon is produced with theta value less than 2.5 but higher than 2, and collapses against the tube, in the case of Figure 5.8(b) the scattering angle of the photon is 3.5 degrees and the electromagnetic shower is almost fully contained in the calorimeter, while in case of Figure 5.8(c) the scattering angle is 4.85 degrees, out of the fiducial volume of FT and the leakages are significant.

front of it.

5.4 Energy Corrections

The final task is to obtain cluster energy corrections that are necessary to compensate for the difference between the reconstructed and the generated energy, in the analysis of the events in the Forward Tagger calorimeter, because of the threshold and the leakage effects, seen in the previous section.

To achieve these corrections, events have been generated using GEMC, with one photon produced with a θ polar angle value ranging from 2.5 up to 4.5 degrees (corresponding to the fiducial volume for the FT verified in the previous section) with steps of 0.2° , with a ϕ azimuthal angle value from -180° up to 180° and with an energy from 100 MeV up to 8 GeV.

The energy of the clusters has been reconstructed using four different techniques, based on *true* or *digitalized* data, with or without the application of a threshold on the single hit.

FIGURE 5.9: Single photon reconstruction. Reconstructed cluster energy slices for $E_{gen}^{\gamma} = 2.5 \text{ GeV}$. Reconstructed cluster energy E_{clust} (GeV) as a function of radius R (cm).

FIGURE 5.10: Single photon reconstruction. Reconstructed cluster energy slices for $E_{gen}^{\gamma} = 2.5 \text{ GeV}$. E_{clust} sliced with R bins and fitted with Gaussian.

The difference between reconstructed and generated energies (E_{rec} and E_{gen} , respectively) has been plotted for 50 MeV bins of the reconstructed energy, in the range from 100 MeV up to 7 GeV. It is important to note that the events were generated in the energy range 0.1 - 8.0 GeV, but that the corresponding reconstructed energy range reaches only 7 GeV, since a part of the energy is lost by leakages. Although the dependence on E_{rec} restricts the covered energy range, this is the more appropriate choice since in the experiment we will deal with reconstructed values rather than with theoretical generated ones. Extrapolation of the reconstructed to the actual energy values can be done once the calibration curves are available.

The procedure employed for the analysis is the following:

FIGURE 5.11: Single photon reconstruction. Reconstructed cluster energy slices for $E_{gen}^{\gamma} = 2.5 \text{ GeV}$. On the left mean values of the fit as a function of R . On the right sigma of the fit as a function of R .

- Generation of events with GEMC: $2 \cdot 10^5$ events are generated with random energy in the range $[0.1, 8] \text{ GeV}$ and with a fixed value of θ equal to $2.5^\circ, 2.7^\circ, 2.9^\circ, \dots, 4.5^\circ$.
- For a fixed θ value, the algorithm reconstructs the energy for each event and divides events in 50 MeV energy bins.
- For each bin the difference between the reconstructed (with the four different techniques) and the generated energies is displayed.
- The distributions are fitted with gaussian functions whose means and standard deviations are saved for the analysis.

The procedure is repeated (from the second step) for a new set of events characterized by a different polar angle θ .

In Figure 5.12 one of the bins is reported as example.

The cluster energy has been reconstructed using two options:

- reading the *digitized* adc or the *true* totEdep bank
- with/without threshold

where the threshold is the minimum value that is acceptable for a single hit and is fixed at 50 MeV. As a consequence, four distributions are displayed. The most realistic distribution is the adc information with threshold since it corresponds to the experimental output. The distributions are

- Black: adc with threshold
- Red: totEdep with threshold
- Green: totEdep without threshold

FIGURE 5.12: Gaussian fit for a single bin. The single photon has been generated with $\theta = 3.5^\circ$, in this bin the events with a reconstructed energy in the range are collected. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

- Blue: adc without threshold

There is no significant difference between *true* totEdep and *digitalized* adc data, while the discrepancy increases between data with and without threshold, the overall effect resulting in a shift on the left of the red and black gaussians.

The distributions have been fitted using a gaussian function

$$f(x) = Ae^{\frac{x - x_0}{\sigma}} \quad (5.5)$$

whose x_0 means and σ standard deviations have been saved for further analysis. It is interesting to note that the structures present a tail on the right side that has been cut using an upper limit on the range to be fitted. The upper limit has been chosen as 1.2 standard deviations above the mean value.

In Figure 5.13 the whole set of plots of the $E_{rec} - E_{gen}$ distribution for 140 energy bins at a fixed value of $\theta = 2.7^\circ$ is shown. It is evident that peak position shifts to higher values at increasing energies. In Appendix A the plots corresponding to all the angle values are reported, showing a similar behavior at all angles.

Data obtained from these fits have been analyzed to determine the cluster energy corrections. In Figure 5.14 the dependence of the fitted mean values of the difference between generated and reconstructed energies as a function of E_{rec} is displayed and fitted. Also in this case there are four curves corresponding to the previous conditions

FIGURE 5.13: Gaussian fits for every energy bin from 0.1 up to 8 GeV, with a step of 50 MeV, for $\theta = 2.7^\circ$. Four different distributions are fitted, corresponding to the options with/without threshold, *true*/*digitized* data.

- Red: adc with threshold
- Black: totEdep with threshold
- Green: adc without threshold
- Empty circle: totEdep without threshold

The curves without threshold have been fitted using linear functions

$$f(x) = p_0 + p_1x, \quad (5.6)$$

while those with threshold have been fitted using a third order polinomyal function

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3. \quad (5.7)$$

A complete list of the fit results is reported in [B](#). As anticipated, although the photons have been generated in an energy range from 0.1 GeV up to 8 GeV, the reconstructed energies are lower as a part of the cluster escapes the calorimeter with a subsequent loss in energy. In Figures [5.16](#) and [5.15](#) the final calibrations are shown for all

FIGURE 5.14: Fits to obtain energy corrections for $\theta = 3.5^\circ$. The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

the θ polar angles. Complete fit results are available in [Appendix B](#).

It is evident that the four curves are superimposed two by two showing that the smearing effects of the *digitized* values with respect to the *true* ones are negligible. As already noticed, photons that reach the central part of the detector produce clusters that are fully contained and the energy collected is higher; on the contrary photons on the edge have clusters that partially escape thus lowering the reconstructed energy. As a consequence, plots corresponding to the central part of the calorimeter present lower values for the difference between generated and reconstructed energy, while plots corresponding to the edge of the calorimeter have higher discrepancy between E_{gen} and E_{rec} . Finally the threshold effects, which causes the displacement of the black and red curves with respect to the blue and green ones, assume less relative importance as the energy and angle increase. The energy correction is not symmetric for the lower and the higher angle extremes of the fiducial volume: leakages are more important at larger angles.

For each cluster energy and angle experimentally reconstructed it is possible to determine the actual incoming photon/electron energy by adding the measured value to the difference obtained by the fitted curves.

FIGURE 5.15: Fits to obtain the energy corrections for angles from 2.5 up to 3.5 degrees. The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

FIGURE 5.16: Fits to obtain the energy corrections for angles from 3.5 up to 4.5 degrees. The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

Chapter 6

Simulations for the $K^+\Lambda$ Final State

Introduction

The simulations of $K^+\Lambda$ electroproduction events detected by CLAS12 were performed with an event generator to produce quadri-momenta of final state particles and a FASTMC (FAST Monte Carlo) to simulate CLAS12 acceptance effects and provide quasi-data for further analysis. The $ep \rightarrow eK^+\Lambda$ event generator is based on the cross sections of a model [35] that describes KY electroproduction in the framework of a Regge-Plus-Resonance (RPR) approach, developed by the Ghent group. The resonance contributions in the s-channel are described with the help of an effective-Lagrangian approach and the background part of the amplitude is modeled in terms of t-channel Regge-trajectory exchanges. Studies have been performed [59] to assess the capability of this model to reproduce data from CLAS and a coherency has been found for $0.65 \text{ GeV}^2 < Q^2 < 1.5 \text{ GeV}^2$, while the cross section is underestimated for $Q^2 > 1.5 \text{ GeV}^2$: this is not a crucial issue since the kinematic region of interest for the analysis is the low Q^2 one. The output of the event generator and FASTMC software chain contains a full description of the particles in the final state, including their identity, angular distribution, energy and momentum. Exploiting the simulations from the event generator and FASTMC and testing different configurations for the electron beam energy and the magnetic field from the torus, the best run conditions for the future experiment were obtained.

In Section 6.1 the generation of $K^+\Lambda$ electroproduction events is reported, while Section 6.2 is devoted to the description of the FASTMC that is used to simulate acceptance effects. Section 6.3 defines the trigger and briefly describes the event selection. In Section 6.4 the best run conditions, in terms of beam energy and magnetic field strength, are achieved. The count rates and the total event rates are calculated in Sections 6.5 and 6.6, respectively. Finally, the software that has been developed for future event reconstruction is described in 6.7.

6.1 Event Generator

The event generator offers the possibility to choose the electron beam energy, the intervals in (W, Q^2) to be spanned, the number of events and the theoretical model (the possible options are RPR-2011 model [35] for the description of $K^+\Lambda$ and RPR-2007 model [41] [39], for the description of $K^+\Lambda$ and $K^+\Sigma$), to produce events in the form of LUND files. These choices may be listed by the user in an input file which is read by the C++ event generator algorithm. Starting from this information, the algorithm creates an `evGenerator` object which takes the channel, the electron beam energy and the ranges in W and Q^2 as given values and extracts the final state quadri-momenta of e^- , K^+ , Λ and also of the proton and π^- from the Λ decay in the laboratory frame. The `evGenerator` class is provided with a function named `getEvent` which executes the following steps to apply the accept/reject method for Monte Carlo event extraction:

- it extracts random Q^2 and W values in the range provided by the user, and random $\cos\theta_K^*$ and ϕ_K^* values in the ranges $(-1,1)$ and $[0,2\pi]$, respectively.
- it evaluates the fifth-fold differential cross section corresponding to the previously extracted independent kinematic quantities on the basis of the chosen model.
- it accepts the event if a new random number, chosen in the $[0,1]$ interval, is lower than the ratio between the evaluated cross section and the maximum value that the cross section may reach in the selected kinematic region.
- relations reported in Chapter 2 are used to compute the final state electron quadri-momentum in the laboratory frame and to transform the final state hadrons quadri-momenta from the CM to the laboratory frame.

The algorithm calls `getEvent` in a `for` loop that stops when the number of events requested by the user is reached. A complete description of the final state particles for the whole set of events is printed in an output LUND file.

6.2 FASTMC

A full GEANT-based Monte Carlo and reconstruction software code is still under development for CLAS12, and its main features are described in Section 6.7. In the meanwhile a fast Monte Carlo program has been developed to estimate CLAS12 acceptance and event production rates. This code contains geometrical acceptances and parameterized resolution functions for the tracking systems, the electromagnetic calorimeters, the scintillation counters and the Forward Tagger. It also uses realistic field maps for the CLAS12 torus and solenoid, that can be scaled to study the effect of different magnet currents and polarities. The sensitive volumes of the tracking and detection regions have been designed to closely match the CLAS12 detector design. The high efficiency fiducial

areas of the detector have been taken into account in the event reconstruction, which also provides a smearing of the final state charged particle momenta. Preliminary comparisons of the CLAS12 response using the FASTMC code suite and the full CLAS12 GEANT-4 code suite [92] have shown reasonably good correspondence.

The FAST MC is used to simulate CLAS12 acceptance effects, to select the trigger and run conditions for channel

$$e + p \rightarrow e' + K^+ + \Lambda \quad (6.1)$$

producing events with different electron beam energies and torus currents to maximize the scattered electron acceptance in the Q^2 and W kinematic plane.

For a realistic trigger definition the decays of the Λ must be taken into account, which are reported in Table 6.1 from Particle Data Group [10].

Λ decay modes	Fraction (Γ_i/Γ)
$p\pi^-$	$(63.9 \pm 0.5) \%$
$n\pi^0$	$(35.8 \pm 0.5) \%$
$n\gamma$	$(1.75 \pm 0.15) \times 10^{-3}$
$p\pi^-\gamma$	[n] $(8.4 \pm 1.4) \times 10^{-4}$
$pe^-\bar{\nu}_e$	$(8.32 \pm 0.14) \times 10^{-4}$
$p\mu^-\bar{\nu}_\mu$	$(1.57 \pm 0.35) \times 10^{-4}$

TABLE 6.1: Λ decay modes.

6.3 Trigger Definition and Event Selection

Both the Forward Tagger and the FD will be devoted to the detection of electrons: if their scattering angle is above 5° , they will be reconstructed by FD, in the High Threshold Cherenkov Counter (HTCC) and in the PCAL and EC calorimeters; on the other hand, as already explained, FT allows to identify electrons scattered between 2.5° and 4.5° and a very high electron rate is expected, fully dominating the event rate in the FT calorimeter and hodoscope. It is necessary to keep the trigger rate low enough to be compatible with the data acquisition capabilities. Therefore, in addition to the main CLAS12 trigger, where at least two charged particles are detected, a coincidence between a signal in the FT and two hadrons in CLAS12 may be used. An electron in either the FT or CLAS12 is requested, plus two hadrons in CLAS12, one of which should be the kaon.

Charged hadrons ($\pi^\pm$, $K^\pm$ and p) will be tracked in the silicon tracker and barrel micromegas at large angles in

the CLAS12 Central Detector (CD), and in the drift chambers and micromegas in the CLAS12 Forward Detector (FD). Charged particle identification is provided in the CLAS12 time-of-flight detector system at all angles. Photons and neutrons scattered at forward angles are detected in the electromagnetic calorimeters (PCAL, EC, FT) and neutrons at large angles are detected in the Central Neutron Detector (CND).

6.4 Selection of the Best Run Conditions

Events were simulated exploiting the chain of event generator and FASTMC to determine the best run conditions. Plots were produced, showing the comparison between accepted and generated events and using the conditions:

- $E_{beam} = 6.6 \text{ GeV}, 8.8 \text{ GeV}, 11 \text{ GeV}$.
- Torus current = $\pm 1500 \text{ A}, \pm 2950 \text{ A}, \pm 3375 \text{ A}$.

in the range $1.6 \text{ GeV} < W < 3.5 \text{ GeV}$ and $0.05 \text{ GeV}^2 < Q^2 < 2 \text{ GeV}^2$. The W lower limit coincides with the threshold for the $K^+\Lambda$ production, the Q^2 one is dictated by the FT fiducial volume and by the forward hole, where high energy electrons cannot be detected.

Magnetic Field In order to choose between a positive and a negative value for the torus current, events have been simulated with both the configurations and from the comparison, shown in Figure 6.1, it is possible to notice that the negative one, which corresponds to outbending electrons, provides a smaller gap between the acceptances of the FT and of CLAS12. The top plots correspond to a positive value of torus current, the bottom plots to a negative one. On the left the Q^2 vs. W plots reflect the areas covered by detectors, the gaps are related with the empty area between FT and CLAS12. On the right the efficiency curves matching the two different configurations are displayed: the definition employed for the efficiency is the following

$$efficiency = \frac{\text{number of events that pass the selection}}{\text{number of produced events}}. \quad (6.2)$$

The efficiency curve provides the information concerning the number of events that, for a fixed value of Q^2 , pass the selection determined by the trigger.

Outbending electrons generated in a W interval from the $K^+\Lambda$ threshold at 1.6 GeV to 3.5 GeV and scattering angles $\theta_e \geq 2.5^\circ$ are detected in the FT and then in the CLAS12 Forward Detector calorimeters starting at $\sim 5^\circ$. The acceptance studies that were pursued showed that the acceptance for outbending electrons is a factor of two larger than the inbending field configuration. Additionally a polar-angle band at $\theta_e \approx 35^\circ$ exists where the acceptance is depleted due to the partially blind transition region between the CLAS12 Forward and Central Detector systems. Another task was to identify the best strength of the torus current. In Figure 6.2 the comparison between two different strengths for the torus current is shown, highlighting that the best energy

FIGURE 6.1: (Left) Q^2 vs. W scatterplots of the detected electrons in CLAS12 for the $ep \rightarrow eK^+ \Lambda$ reaction for torus currents $I = +3375$ A (top) and $I = -3375$ A (bottom) for $E_e = 11$ GeV. (Right) The corresponding electron acceptances as a function of Q^2 .

resolution is obtained for high values of the torus current.

Beam Energy Finally, to select the best energy for the electron beam the efficiency curve has been studied for three different configurations, $E_{beam} = 6.6$ GeV, $E_{beam} = 8.8$ GeV and $E_{beam} = 11$ GeV. The trigger condition employed is the detection of one electron in the final state. Figures 6.3 report the Q^2 dependence on W of the generated and accepted events (left and central plots, respectively), with the corresponding efficiency plot on the right. Both the positive and the negative value of the torus current have been used for a final comparison. As already said, the low Q^2 region plays a crucial role in the search for hybrid baryons. As a consequence, it will be very important to have both a high statistics and a high efficiency in the low Q^2 range. On the other hand, the lower statistics for high values of Q^2 , which makes the efficiency curve less reliable in that range, will be not a issue. It is possible to note that high values of efficiency occur at low values of Q^2 for $E = 6.6$ GeV. However, the information from this configuration can be integrated with the ones using the other two energies to cover the whole low Q^2 range. In Figure 6.4 the three efficiency curves for a torus current equal to -3375 A are overlapped, highlighting the possibility to cover the whole $[0.05, 0.55]$ GeV² range exploiting different beam energies. Table

FIGURE 6.2: Simulations at $E_b=6.6$ GeV. The left column shows the W versus Q^2 distributions and the next three columns show the $e \rightarrow K^+$ missing mass. The top row of plots use a torus current setting of $I = -1500$ A and the bottom row of plots use a torus current setting of $I = -3375$ A. The right column shows the combined $K^+\Lambda$ and $K^+\Sigma^0$ overlap histograms (no background included) demonstrating the advantage of high CLAS12 torus currents..

6.2 summarizes the minimum Q^2 value that may be reached for the K^+Y final states at $E_b = 6.6$ GeV and 8.8 GeV for a torus current of $I = -3375$ A.

E_b , GeV	Torus Current, A	Q_{min}^2 , GeV^2
6.6	-3375	0.05
8.8	-3375	0.1

TABLE 6.2: Minimal achievable Q^2 (Q_{min}^2) for $E_b = 6.6$ GeV and 8.8 GeV and a torus current of $I = -3375$ A for the $K^+\Lambda$ final state.

Final State Particle Acceptance Plots The azimuthal versus polar angles of generated and accepted events for electrons, kaons, protons and pions have been plotted, see Figures 6.5, 6.6, 6.7, 6.8. For these plots the trigger condition is two charged particles plus an electron in the final state, where the charged particle can be either a proton, a π^- or a K^+ . The twisting pattern occurring in the accepted p and K^+ plots is due to the azimuthal motion of the charged tracks in the strong central solenoid field that generates an azimuthal “kick” that depends on the production angle and particle momentum. The pattern for the π^- is different as they have on average much lower momenta and their migration in ϕ is larger and more diffuse.

FIGURE 6.3: Number of events that pass the selection as a function of Q^2 and W . One electron in the final state is used as trigger.

6.5 Count Rates for $K^+\Lambda$

The expected total number of $K^+\Lambda$ electroproduction events in the reaction $ep \rightarrow eK^+\Lambda$ can be expressed as:

$$N = \mathcal{L} \cdot t \cdot \int \frac{d^5\sigma}{dE_e d\Omega_e d\Omega_K^*} dE_e d\Omega_e d\Omega_K^* \quad (6.3)$$

where

- $\mathcal{L} = 1 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ is the expected CLAS12 operating luminosity
- t is the expected run time and

FIGURE 6.4: Superposition of the efficiency curves for torus current equal to -3375 A. The blue, green and magenta curves correspond to a beam energy equal to 6.6 GeV, 8.8 GeV and 11.0 GeV, respectively.

- $\frac{d^5\sigma}{dE_e d\Omega_e d\Omega_K^*}$ is the cross section from Eq. 2.39.

The model cross sections for $\frac{d^2\sigma}{d\Omega_K^*}$ are the same of the ones used in the event generator (see Section 6.1). The event rate R is defined as $\frac{N}{t}$. The integration in Eq. 6.3 is performed over the entire kinematic space and can be done numerically. To obtain the minimum achievable Q^2 , the minimum electron polar scattering angle θ_e must be employed, as already explained. Since both for $E_b = 6.6 \text{ GeV}$ and for $E_b = 8.8 \text{ GeV}$ beam energies Q^2 is greater than 0.05 GeV^2 (see Table 6.2), it is possible to integrate in Eq. 6.3 given that $Q^2 > 0.05 \text{ GeV}^2$. The calculated event rate R_Λ is reported in Table 6.3. A detailed simulation is needed to account for the acceptance

E_b [GeV]	R_Λ [Hz]
6.6	1500
8.8	1400

TABLE 6.3: Estimated $K^+\Lambda$ production rates for events with $Q^2 > 0.05 \text{ GeV}^2$ for $E_b = 6.6 \text{ GeV}$ and 8.8 GeV .

of CLAS12. It is necessary to generate events in the entire kinematic space with $Q^2 > 0.05 \text{ GeV}^2$, therefore the ratio of reconstructed to generated events gives the averaged acceptance. The event rates that account for the CLAS12 acceptance are obtained multiplying the event rates from Table 6.3 by that ratio. An $ep \rightarrow e K^+\Lambda \rightarrow e K^+ p\pi^-$ event is considered to be reconstructed if the electron and at least two hadrons have been detected. A trigger condition requiring at least two charged hadrons and an electron would select our channels of interest. The accepted event rate calculations are presented in Table 6.4 for $E_b = 6.6 \text{ GeV}$ and 8.8 GeV and a torus current $I = -3375 \text{ A}$, taking into account the branching ratio for the Λ decay into $p\pi^-$ (64%). A rough estimate suggests

FIGURE 6.5: Number of electron events that pass the selection as a function of Q^2 and W . One electron and two charged particles in the final state are required as trigger.

FIGURE 6.6: Number of proton events that pass the selection as a function of Q^2 and W . One electron and two charged particles in the final state are required as trigger.

FIGURE 6.7: Number of π^- events that pass the selection as a function of Q^2 and W . One electron and two charged particles in the final state are required as trigger.

FIGURE 6.8: Number of K^+ events that pass the selection as a function of Q^2 and W . One electron and two charged particles in the final state are required as trigger.

E_b [GeV]	Torus Current [A]	R_Λ [Hz]
6.6	-3375	70
8.8	-3375	50

TABLE 6.4: Estimated event rates for the $ep \rightarrow e' K^+ \Lambda \rightarrow e' K^+ p \pi^-$ channel for $E_b = 6.6$ GeV and 8.8 GeV for a torus current $I = -3375$ A for the trigger condition where the electron and two hadrons (one of which is the K^+) are required to be detected in CLAS12.

that the KY exclusive channel contribution is about 1% with respect to the two and three pion events, which are expected to dominate the events that have an electron and at least two charged hadrons in the final state. The maximal total event rate is therefore expected to be ≈ 25 kHz for the trigger condition described above.

6.6 Expected $K^+\Lambda$ Total Event Rates

The positive identification of a hybrid signal necessitates of high statistics. Therefore, high event rates are required and the transverse and longitudinal virtual photon polarization induced in the electroproduction, see Equations 2.43 and 2.44 respectively, is expected to enhance the resonant signal due to amplitude interference. Thanks to the presence of the FT, the electron rates at the very forward electron scattering angles will be very high, thus allowing an increase of the statistics. However, the rates are so high that may exceed the capabilities of the data acquisition system. As a consequence, the total acquisition rate must be decreased defining an optimal trigger configuration, with constraints on the final state particles that can be expected for a hybrid baryon candidate. The trigger that was considered for the initial running program has at least two charged particles as final state: $K^+\Lambda \rightarrow K^+ p \pi^-$, $p \pi^+ \pi^-$, $p \phi \rightarrow p K^+ K^-$ and $p \eta \rightarrow p \pi^+ \pi^- \eta$ final states are covered. To estimate realistically the rate, projections of the hadronic coupling strengths of the hybrid baryons are needed, but they are not available at present. Moreover, a trigger with a single charged hadron will be provided with a pre-scaling factor for the FT, which in parallel will collect events with a single charged hadron in the final state, i.e. $\pi^+ n$, $p \pi^0$ and $K^+\Lambda$, among others. Considering a beam energy $E_b = 6.6$ GeV, a torus current of $I = -3375$ A, a Q^2 range from 0.05 GeV^2 to 2.0 GeV^2 and an expected operating luminosity for CLAS12 of $\mathcal{L} = 1 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$, a 70 Hz production rate is computed for $K^+\Lambda$ electroproduction channel. This corresponds to a total number of $K^+\Lambda$ events to be 3.0×10^8 for a 50 days run time. The lowest statistics is predicted for high W and Q^2 . A total number of 3.3×10^5 $K^+\Lambda$ events is awaited for the kinematics with the lowest event rate, e.g. $2.675 \leq W \leq 2.700$ GeV and $2.0 \leq Q^2 \leq 2.5 \text{ GeV}^2$.

6.7 CLAS12 Full Simulation Software

The reconstruction of events will be achieved with a Service Oriented Architecture framework for data processing, named *Clara*, designed to deploy the event reconstruction software and which is at present under development. The complete CLAS12 software release package *coatjava*, whose documentation is available on the CLAS12

webpage [93], include the services for each detector component, which are compiled as java archive (JAR) plugins. Raw data produced by either simulation or the detectors are reconstructed, event by event, by the CLAS12 reconstruction software to provide physics analysis output such as track parameters and particle identification. Scripts that can be launched on the JLab computing farm to process the data, as well as a framework to do event analysis using the flexible scripting language *Groovy* are contained in this package. As already mentioned, the GEANT-4 Monte Carlo simulation application GEMC [92] is employed to generate events considering CLAS12 acceptance effects. The studies for the proposal [59], reported in this thesis, were carried out using FASTMC. The first extension of the works presented in this Chapter is to repeat the simulations using the GEMC Monte Carlo with the full CLAS12 event reconstruction package [93] when they have been fully developed and optimized. However a reasonably good match has been found from the comparison of FASTMC data with the ones from the current reconstruction software package, showing the reliability of the work already performed.

Chapter 7

Sensitivity Studies to Hybrid Baryons Contributions

Introduction

The effect of implementing the contribution of the lightest hybrid state in the differential cross sections for exclusive $K^+\Lambda$ reaction channel has been studied, to prove the feasibility of observing hybrid baryons. LQCD evaluations [19] have been used to establish the hybrid baryon mass range and spin-parity. On the basis of these predictions the lightest hybrid baryons is located in a mass range from 2.4 GeV to 2.6 GeV with spin-parity either $J^P = \frac{1}{2}^+$ or $J^P = \frac{3}{2}^+$, as it can be seen in Figure 1.7. However, these computations were carried out with a pion mass of 396 MeV and the whole spectrum is shifted at higher masses by approximately 300 MeV. To take this approximation into account lower masses for hybrid baryons in the energy range from 2.1 GeV to 2.3 GeV are considered. In particular we considered a value of 2.24 GeV, which is simply the sum of the nucleon mass (0.938 GeV) and the gluon mass from Reference [19] (1.3 GeV). According to the RPP14 results [10] on the resonance parameters in the mass range around 2.0 GeV, a range from 200 MeV to 300 MeV has been used for the hybrid baryon total decay width.

It is important to understand if, given the existence of a hybrid resonance, the CLAS12 experimental setup is able to detect it, within the expected statistical accuracy. This possibility will be studied varying the resonance parameters, that are currently unknown, using variable running masses, total decay widths and electrocouplings. The approach that has been used is to compare between cross section values including or not a hybrid resonance with running parameters, taking into account the expected statistical uncertainty on the measured values. This study has been applied to both the cross sections and their Legendre moments, which are expected to be particularly sensitive to the multipole decomposition of the cross section. Finally a blind analysis aimed at extracting unknown hybrid baryons parameters from cross section quasi-data has been attempted to prove the capability of

identifying new hybrid baryons contributions from future measurements.

In this Chapter the sensitivity studies to hybrid baryons contributions are presented. In Section 7.1 the study of sensitivity is reported, while the Legendre moments analysis is showed in Section 7.2. In Section 7.3 the data sensitivity to the presence of two hybrid resonances, whose masses and electrocouplings were unknown, was explored. Finally in Section 7.4 a blind analysis is performed to check our capability to determine the correct resonance parameters of hybrid baryon resonance.

7.1 Determination of Minimum Detectable Electrocouplings

To search for a hybrid baryon state, first of all a new resonance must be identified, whose mass and spin-parity could be consistent with a hybrid candidate (see Chapter 1). Then a study of this state is needed to assess whether it is a hybrid or not. As already said, its identity relies on the Q^2 dependence of the $\gamma_\nu p N^*$ electrocouplings that can be extracted employing advanced reaction models, as well as multi-channel partial wave fits that incorporate advanced coupled-channel approaches.

It is not certain that a hybrid state can be revealed, it will depend on its electrocouplings values: only states with sufficiently high electrocouplings will be detected with the sensitivity provided by the experimentally available setup.

In order to determine the feasibility to observe the signal from a hybrid baryon in exclusive KY electroproduction, the two-fold virtual photon differential cross sections $\frac{d^2\sigma}{d\Omega_K^*}$ has been computed according to the equations 2.58, 2.60, 2.61, 2.62 and 2.63, where $d\Omega_K^*$ is the differential of the solid angle for K^+ emission in the CM frame. These cross sections were computed using the model without the hybrid state included (Model A) and with the hybrid state included (Model B). Representative examples of the two models have already been plotted in Figures 2.11, 2.12, 2.13 and 2.14 in Chapter 2. For the hybrid resonance parameters, a mass of 2.2 GeV, total decay width 0.25 GeV, and a 3% branching fraction for decays to the $K^+\Lambda$ final state has been used. The contributions from hybrids of spin-parities $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ were simulated. In order to identify the candidate hybrid state, the difference between two-fold differential virtual photon $K^+\Lambda$ cross sections $\frac{d^2\sigma_A}{d\Omega_K^*}$ and $\frac{d^2\sigma_B}{d\Omega_K^*}$ computed in Model A and Model B, respectively, was examined. The statistical significance of this difference can be quantified in terms of the following χ^2 /d.p. value (d.p. being the total number of kinematic bins):

$$\chi^2/d.p. = \frac{1}{N_{d.p.}} \sum_{W, \cos(\theta_K^*), \phi_K} \frac{(\frac{d^2\sigma_A}{d\Omega_K^*} - \frac{d^2\sigma_B}{d\Omega_K^*})^2}{\delta^2}. \quad (7.1)$$

The sum includes 24 bins over both θ_K^* and ϕ_K , in each bin of W and fixed values of Q^2 . The W value in Equation 7.1 runs from 2.1 GeV to 2.3 GeV with a bin width of 20 MeV. Only statistical contributions were

taken into account in the evaluation of the uncertainties δ in Equation 7.1:

$$\delta^2 = p^2 \left(\frac{d^2 \sigma_A}{d\Omega_K^*} + \frac{d^2 \sigma_B}{d\Omega_K^*} \right)$$

$$p = \frac{\sqrt{N(W, Q^2, \theta_K^*, \phi_K)}}{N(W, Q^2, \theta_K^*, \phi_K)} \quad (7.2)$$

δ were estimated using the percentage error p computed from the statistical uncertainty in the cross section measurement. The values of p in the four dimensional bin of $(W, Q^2, \theta_K^*, \phi_K)$ were determined by the number of $K^+\Lambda$ exclusive events $N(W, Q^2, \theta_K^*, \phi_K)$ collected in the bin. The bin populations were computed starting from the rates and the number of total collected $K^+\Lambda$ events, estimated for the run conditions, as described in Section 6.5, also taking into account the CLAS12 acceptance evaluated using FASTMC.

To determine the minimal hybrid electrocoupling values providing a statistically significant signal from the new state, the χ^2 /d.p. has been computed varying the values of the hybrid-candidate electrocouplings for the spin-parity assignments $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ in the following way. Each of three couplings $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$, and $S_{1/2}(Q^2)$ were varied in the range from $0.01 \text{ GeV}^{-1/2}$ to $0.03 \text{ GeV}^{-1/2}$, while the two others were set equal to zero. With the variable hybrid electrocouplings, the χ^2 /d.p. values were computed independently in 10 Q^2 bins of 0.1 GeV^2 width in a range of photon virtualities from 0.05 GeV^2 to 1.05 GeV^2 . The signal from a candidate hybrid baryon state is considered as statistically significant if χ^2 /d.p. > 4 . This threshold has been chosen following the experience in the two pion electroproduction analysis within the CLAS collaboration according to which a systematic fluctuation in cross section values leads to a χ^2 /d.p. value of at least 2. An additional signal is required to change the cross section leading to a χ^2 /d.p. of at least 4.

The minimal values of $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ found in this way are presented in Tables 7.1 and 7.2 for $E_b = 6.6 \text{ GeV}$ and 8.8 GeV .

Q^2, GeV^2	$J_R=1/2$		$J_R=3/2$		
	$A_{1/2}$	$S_{1/2}$	$A_{1/2}$	$A_{3/2}$	$S_{1/2}$
0.1	12	12	16	11	10
0.5	17	19	18	19	12
1.0	16	21	16	18	10

TABLE 7.1: The minimal values of the electrocouplings for $E_b=6.6 \text{ GeV}$ and torus current $I = -3375 \text{ A}$ for the resonances with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ are in units of $10^{-3} \cdot \text{GeV}^{-1/2}$. When determining the minimal value of $A_{1/2}$, only $A_{1/2}$ was varied, while setting the other electrocouplings to zero. The minimal values of $A_{3/2}$ and $S_{1/2}$ were obtained in the same way.

It was found that the minimal electrocoupling values have no pronounced dependence on the beam energy. The minimal electrocoupling values listed in Tables 7.1 and 7.2 for the observation of a hybrid baryon turn out to be smaller than the photo-/electrocouplings of most regular nucleon resonances extracted from the photo- [94] and electroproduction data at Q^2 up to 1.0 GeV^2 [4] measured with CLAS. Note that the studies of exclusive photoproduction with CLAS have already revealed signals of eight new N^* states. Therefore, the estimates of

Q^2, GeV^2	$J_R=1/2$		$J_R=3/2$		
	$A_{1/2}$	$S_{1/2}$	$A_{1/2}$	$A_{3/2}$	$S_{1/2}$
0.3	12	12	14	12	9
0.5	19	20	19	21	12
1.0	16	21	16	18	9

TABLE 7.2: The minimal values of the electrocouplings for $E_b=8.8$ GeV and torus current $I = -3375$ A for the resonances with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ are in units of $10^{-3} \cdot GeV^{-1/2}$. When determining the minimal value of $A_{1/2}$, only $A_{1/2}$ was varied, while setting the other electrocouplings to zero. The minimal values of $A_{3/2}$ and $S_{1/2}$ were obtained in the same way.

the minimal electrocoupling values needed for hybrid baryon observation suggest very encouraging prospects for a hybrid baryon search in exclusive KY electroproduction.

7.2 Hybrid Baryon Search from Legendre Moment Expansion

The expansion of the structure functions $\sigma_U = \sigma_T + \epsilon\sigma_L$, σ_{LT} and σ_{TT} in terms of Legendre moments is believed to be an alternative way to probe the sensitivity to a hybrid baryon contribution, since the appearance of a structure in a single Legendre moment at the same value of W for each Q^2 point is likely a signal from a resonance contribution.

The Legendre moments are defined as:

$$C_m = \frac{2m+1}{2} \int_{-1}^1 L_m \sigma_i(x) dx \quad (7.3)$$

where

$$\begin{aligned} x &= \cos\theta_K^* \\ L_m(x) &= \sum_{j=0}^m a_{mj} x^j \\ a_{mj} &= (-1)^{(m-j)/2} \frac{1}{2^m} \frac{(m+j)!}{(\frac{m-j}{2})! (\frac{m+j}{2})! j!} \\ m-j &= \text{even} \end{aligned} \quad (7.4)$$

Here, $\sigma_i(x)$, with $i = T, L, LT$ and TT , represent the transverse, longitudinal and interference structure functions that depend on the variables W , Q^2 and θ_K^* .

The Legendre moments may be used to expand the structure functions in terms of the orthogonal Legendre polynomials:

$$\sigma_i(x) = \sum_m C_m(Q^2, W) L_m(\cos\theta_K^*). \quad (7.5)$$

The first seven Legendre polynomial functions are:

$$\begin{aligned}
 L_0 &= 1 \\
 L_1 &= \cos\theta_K^* \\
 L_2 &= \frac{1}{2}(3(\cos\theta_K^*)^2 - 1) \\
 L_3 &= \frac{1}{2}(5(\cos\theta_K^*)^3 - 3\cos\theta_K^*) \\
 L_4 &= \frac{1}{8}(35(\cos\theta_K^*)^4 - 30(\cos\theta_K^*)^2 + 3) \\
 L_5 &= \frac{1}{8}(63(\cos\theta_K^*)^5 - 70(\cos\theta_K^*)^3 + 15\cos\theta_K^*) \\
 L_6 &= \frac{1}{16}(231(\cos\theta_K^*)^6 - 315(\cos\theta_K^*)^4 + 105\cos\theta_K^* - 5)
 \end{aligned} \tag{7.6}$$

For instance, such an approach was carried out for the $K^+\Lambda$ and $K^+\Sigma^0$ electroproduction separated structure functions from CLAS data in Reference [40]. Figure 7.1 reports the first four Legendre moments for $K^+\Lambda$ at $Q^2 = 1.8 \text{ GeV}^2$. A Legendre moment analysis highlighted two structures at $W = 1.7 \text{ GeV}$ and $W = 1.9 \text{ GeV}$, that

FIGURE 7.1: Legendre polynomial fit coefficients for $m = 0 \rightarrow 3$ as a function of W for $d\sigma_U$ for CLAS $K^+\Lambda$ electroproduction data at $Q^2 = 1.8 \text{ GeV}^2$ [40].

were identified in the Bonn-Gatchina partial wave analysis as coming from the $N(1710)\frac{1}{2}^+$ and a new $N(1900)\frac{3}{2}^+$ state based on fits including the CLAS $K^+\Lambda$ photoproduction data. Of course, only a full amplitude partial wave analysis will constitute a definite proof for the presence of a resonance, but the study of the Legendre moments of the separate structure functions may provide the first hint of their appearance.

A study of the first seven Legendre moments of the $d\sigma_U$, $d\sigma_{LT}$ and $d\sigma_{TT}$ structure functions has been performed for Models A and B for $K^+\Lambda$ electroproduction, described previously. An example evaluated for both Model A with no hybrid and Model B, where a $J^P = \frac{1}{2}^+$ hybrid with a fixed electrocoupling of $A_{1/2} = 40 (10^{-3}) GeV^{-1/2}$ has been added, was considered. Results will be shown in paragraph 7.2.2.

7.2.1 Estimation of Uncertainties on Structure Functions Separation

The available observable in $K^+\Lambda$ electroproduction experiments is the total four-fold cross section, defined by Equation 2.57,

$$\frac{d^4\sigma}{dWdQ^2d\Omega_K^*} = \Gamma_v[(\sigma_T + \epsilon\sigma_L) + \epsilon\sigma_{TT}\cos 2\phi_K^* + \sqrt{\epsilon(\epsilon+1)}\sigma_{LT}\cos\phi_K^*] = \Gamma_v\sigma_4. \quad (7.7)$$

As a consequence it will be not possible to measure directly the three structure functions σ_U , σ_{LT} and σ_{TT} that will be used to evaluate the Legendre moments, but they will be computed by fitting the differential cross section dependence on ϕ_K^* . The statistical uncertainty on the four-fold differential cross section will propagate on the uncertainties of the three structure functions, that are associated to the errors of the fitting parameters. To estimate the structure functions errors, $\frac{d^4\sigma}{dWdQ^2d\Omega_K^*}$ has been plotted with fixed bins in W , Q^2 , ϕ_K^* and $\cos\theta_K^*$, smearing each point according to a gaussian distribution whose standard deviation will be $\sigma_{\Gamma_v\sigma_4}$. To calculate $\sigma_{\Gamma_v\sigma_4}$, we start from the relative error in Equation 7.2:

$$p = \frac{\sqrt{N(W, Q^2, \theta_K^*, \phi_K)}}{N(W, Q^2, \theta_K^*, \phi_K)} = \frac{\sigma_{\Gamma_v\sigma_4}}{\Gamma_v\sigma_4} \quad (7.8)$$

As a consequence, the absolute error will be:

$$\sigma_{\Gamma_v\sigma_4} = \Gamma_v\sigma_4 \cdot \frac{1}{\sqrt{N(W, Q^2, \theta_K^*, \phi_K)}} \quad (7.9)$$

where the number of events in a certain bin of W , Q^2 , ϕ_K^* and $\cos\theta_K^*$ is defined as

$$N(W, Q^2, \theta_K^*, \phi_K) = \Gamma_v\sigma_4 \cdot \mathcal{L} \cdot T \cdot \mathcal{E} \cdot \Delta W \cdot \Delta Q^2 \cdot \Delta\cos\theta_K^* \cdot \Delta\phi_K^* \quad (7.10)$$

and consequently

$$\Gamma_v\sigma_4 = \frac{N(W, Q^2, \theta_K^*, \phi_K)}{\mathcal{L} \cdot T \cdot \mathcal{E} \cdot \Delta W \cdot \Delta Q^2 \cdot \Delta\cos\theta_K^* \cdot \Delta\phi_K^*} \quad (7.11)$$

where $\mathcal{L}$ is the CLAS12 luminosity, T is the run time, $\mathcal{E}$ is the average detection efficiency and $\Delta W \cdot \Delta Q^2 \cdot \Delta\cos\theta_K^* \cdot \Delta\phi_K^*$ are the bins in invariant mass, photon virtuality and angles of the scattered kaon in CM frame.

The numerical values that were used are the following:

$$\begin{aligned}
\mathcal{L} &= 10^{35} \text{ cm}^{-2} \text{ s}^{-1} \\
T &= 50 \text{ d} = 50 \times 24 \times 3600 \text{ s} = 432 \cdot 10^4 \text{ s} \\
\mathcal{E} &= 0.4 \\
\Delta W &= 0.05 \text{ GeV} \\
\Delta Q^2 &= 0.1 \text{ GeV}^2 \\
\Delta \cos \theta_K^* &= 0.04878 \\
\Delta \phi_K^* &= 2\pi/20
\end{aligned} \tag{7.12}$$

The chosen run time is fifty days, coherently with the assumptions used in Section 6.6.

Defining a quantity L as

$$L = \mathcal{L} \cdot T \cdot \mathcal{E} \cdot \Delta W \cdot \Delta Q^2 \cdot \Delta \cos \theta_K^* \cdot \Delta \phi_K^* \tag{7.13}$$

that is equal to $130.52 \cdot 10^5 \mu\text{b}^{-1} \text{ GeV}^3$, the number of events in each bin $N(W, Q^2, \theta_K^*, \phi_K)$ can be rewritten as

$$N(W, Q^2, \theta_K^*, \phi_K) = \Gamma_v \sigma_4 \cdot L \tag{7.14}$$

and replacing this value in Equation 7.9, the absolute error on the four-fold cross section may be expressed as

$$\sigma_{\Gamma_v \sigma_4} = \Gamma_v \sigma_4 \cdot \frac{1}{\sqrt{N(W, Q^2, \theta_K^*, \phi_K)}} = \Gamma_v \sigma_4 \cdot \frac{1}{\sqrt{\Gamma_v \sigma_4 \cdot L}} = \sqrt{\frac{\Gamma_v \sigma_4}{L}}. \tag{7.15}$$

Figure 7.2 shows the differential cross section for Model B, considering a mass of 2.2 GeV and a 0.25 GeV total decay width for the hybrid resonance, as a function of the kaon azimuthal angle ϕ_K^* for a representative bin at $Q^2 = 1.0 \text{ GeV}^2$, $W = 2.2 \text{ GeV}$ and $\cos \theta_K = 0.90$. Three curves are displayed in this plot, showing the $\Gamma_v \sigma_4$ distribution as a function of azimuthal angle ϕ_K^* . The black line was obtained from Model A, the red line corresponds to the hybrid baryon excitation alone assuming $J^P = \frac{1}{2}^+$, 2.2 GeV mass and electrocoupling strengths $A_{1/2} = 40 (10^{-3}) \text{ GeV}^{-1/2}$ and $S_{1/2} = 0 \text{ GeV}^{-1/2}$; and the blue line corresponds to the Model B where quasi-data were smeared according to the statistical uncertainty defined in Equation 7.15.

To extract the structure functions contributions σ_U , σ_{LT} and σ_{TT} , a fit has been performed using a function of the form:

$$f(\phi) = p_0 + p_1 \cos 2\phi_K^* + p_2 \cos \phi_K^* \tag{7.16}$$

where

$$p_0 = \Gamma_v \sigma_U \tag{7.17}$$

$$p_1 = \epsilon \Gamma_v \sigma_{TT} \tag{7.18}$$

$$p_2 = \sqrt{\epsilon(\epsilon + 1)} \Gamma_v \sigma_{LT} \tag{7.19}$$

Repeating this procedure for different values of W , Q^2 and $\cos \theta_K^*$, a strong dependence on the θ_K^* angle has been

FIGURE 7.2: Total cross section as a function of azimuthal angle. $d\sigma_U$, $d\sigma_{LT}$ and $d\sigma_{TT}$ structure functions for $K^+\Lambda$ electroproduction extracted from the simulated dependence of the differential cross section vs. ϕ_K^* for a representative bin in Q^2 , W and θ_K^* . The cross section $\frac{d^2\sigma}{d\Omega_K^*}$ has been evaluated from Model B where a $J^P = \frac{1}{2}^+$ hybrid baryon of 2.2 GeV mass has been added with electrocoupling strengths $A_{1/2} = 40 (10^{-3}) \text{ GeV}^{-1/2}$ and $S_{1/2} = 0 \text{ GeV}^{-1/2}$. The cross section points have been smeared according to the expected statistical uncertainty.

found. Final relative errors for the separate structure functions have been estimated as an average of the results for different W , Q^2 and $\cos\theta_K^*$ values, and are equal to $\frac{\Delta\sigma_U}{\sigma_U} = 2\%$, $\frac{\Delta\sigma_{LT}}{\sigma_{LT}} = 20\%$ and $\frac{\Delta\sigma_{TT}}{\sigma_{TT}} = 15\%$. This values have been used for the final evaluation of the Legendre momenta reported in the following paragraph.

7.2.2 Legendre Moments Plots

Figures 7.3 and 7.4 show the first seven Legendre moments calculated for the σ_{LT} and σ_{TT} cross sections as a function of W . The Legendre moments have been numerically evaluated exploiting Equation 7.3 summing over 41 bins in $\cos\theta_K^*$ and the uncertainties have been determined applying the statistical error propagation formula and the relative errors on the cross sections computed in the previous paragraph. The black curves have been obtained from Model A and the blue curves from Model B. The hybrid resonance has been implemented using $J^P = \frac{1}{2}^+$, $M_{res} = 2.2 \text{ GeV}$ and electrocoupling strengths $A_{1/2} = 40 (10^{-3}) \text{ GeV}^{-1/2}$ and $S_{1/2} = 0 \text{ GeV}^{-1/2}$; it is possible to note that significant structures appear in most of the Legendre moments at the value of $W = 2.2 \text{ GeV}$, corresponding to its mass. As expected, the difference between Model A and Model B Legendre moments is more striking for increasing orders of the momentum. However, the statistical uncertainties become larger than the difference, so that the significant information is obtainable only for the first three-four Legendre moments orders.

FIGURE 7.3: Legendre polynomial fit coefficients for $m = 0 \rightarrow 6$ as a function of W for the $d\sigma_{LT}$ structure functions for $K^+\Lambda$ electroproduction at $Q^2 = 1 \text{ GeV}^2$, extracted for Model A (black curve) and Model B (blue curve) with an added $J^P = \frac{1}{2}^+$ hybrid baryon of $M = 2.2 \text{ GeV}$ with electrocoupling strengths $A_{1/2} = 40$ (10^{-3}) $\text{GeV}^{-1/2}$ and $S_{1/2} = 0 \text{ GeV}^{-1/2}$.

FIGURE 7.4: Legendre polynomial fit coefficients for $m = 0 \rightarrow 6$ as a function of W for the $d\sigma_{TT}$ structure functions for $K^+\Lambda$ electroproduction at $Q^2 = 1 \text{ GeV}^2$, extracted for Model A (black curve) and Model B (blue curve) with an added $J^P = \frac{1}{2}^+$ hybrid baryon of $M = 2.2 \text{ GeV}$ with electrocoupling strengths $A_{1/2} = 40$ (10^{-3}) $\text{GeV}^{-1/2}$ and $S_{1/2} = 0 \text{ GeV}^{-1/2}$.

7.2.3 Threshold Electrocouplings from Legendre Moments

A sensitivity test using the Legendre moments has been performed as an independent check of the results reported in Tables 7.1 and 7.2 for the threshold values of the electrocouplings. To this task, Legendre moments have been computed for Model A and for Model B at fixed Q^2 values, setting the resonance mass at 2.2 GeV, the spin and parity J^P , keeping $S_{1/2}$ fixed at 0 and varying the $A_{1/2,3/2}$ electrocoupling values of the resonance. The $\chi_m^2(A_{1/2,3/2})$ value has been evaluated for the first seven Legendre moments of the three structure functions σ_U , σ_{LT} and σ_{TT} . For each $A_{1/2,3/2}$ value a $\chi_m^2(A_{1/2,3/2})$ has been calculated as

$$\chi_m^2(A_{1/2,3/2}) = \frac{1}{N_{d.p.}} \sum_W \frac{(C_m^{Model\ B} - C_m^{Model\ A})^2}{\delta^2} \quad (7.20)$$

where δ is the error associated to $C_m^{Model\ B}$ and the sum has been performed on $N_{d.p.} = 27$ bins in W , from threshold up to 2.9 GeV, in 0.05 GeV steps.

Different configurations have been used:

- $J^P = \frac{1}{2}^+, J^P = \frac{3}{2}^+$
- $Q^2 = 0.1 \text{ GeV}^2, 0.562 \text{ GeV}^2, 1.002 \text{ GeV}^2$
- $E_b = 6.6 \text{ GeV}, 8.8 \text{ GeV}$

The results for $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ are reported in Figures 7.5 and 7.6, respectively, where the dependence of the $\chi_m^2(A_{1/2,3/2})$ on the running $A_{1/2,3/2}$ electrocoupling value is plotted for fixed Q^2 and for the three separate structure functions. No dependence upon the beam energy is observed.

The signal from a candidate hybrid baryon state is considered as statistically significant if $\chi_m^2(A_{1/2,3/2}) > 4$, therefore for each curve the value of $A_{1/2,3/2}$ for which the $\chi_m^2(A_{1/2,3/2})$ exceeds 4 has been evaluated and are shown in the legend. Intersection values for the first order Legendre moment are reported in Table 7.3 for the three cross sections. The σ_{LT} and σ_{TT} Legendre moments show a higher sensitivity than the unseparated σ_U moments. In fact, their curves intersect the $\chi_m^2(A_{1/2,3/2}) = 4$ threshold for lower values.

It is possible to note that the threshold values showed in Figure 7.5 present a good agreement with the minimal electrocoupling values listed in Tables 7.1 and 7.2 that have been obtained independently by the Moscow State University (MSU) group.

7.2.4 Sensitivity to the Hybrid Resonance Mass

In this paragraph the possibility of constraining the hybrid baryon mass is studied, employing an approach similar to the one developed by the Bonn-Gatchina group for the identification of new states. This method consists in a comparison between the measured observables and simulations with a running mass for the candidate resonance M_{res} , extracting for each variable mass value the $\chi^2(M_{res})/d.p(W)$ and identifying the correct mass by the

FIGURE 7.5: $\chi_m^2(A_{1/2})$ as a function of $A_{1/2}$: for each plot the seven curves correspond to the first seven Legendre moments $C_0, \dots, C_6$. The red dashed line indicates the $\chi_m^2(A_{1/2}) = 4$ threshold. Curves intersecting the red dashed line for lower values of $A_{1/2}$ identify the polynomials with a higher sensitivity to the presence of the hybrid resonance.

Q^2, GeV^2	$J_R=1/2$			$J_R=3/2$		
	$A_{1/2}$			$A_{3/2}$		
	σ_U	σ_{LT}	σ_{TT}	σ_U	σ_{LT}	σ_{TT}
0.1	9	43	17	7	-	22
0.5	12	19	12	8	34	12
1.0	21	9	3	7	13	3

TABLE 7.3: The minimal values of the electrocouplings for $E_b = 6.6 \text{ GeV}$ and torus current $I = -3375 \text{ A}$ for the resonances with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. $A_{1/2}$ and $A_{3/2}$ are in units of $10^{-3} \cdot \text{GeV}^{-1/2}$. When determining the minimal value of $A_{3/2}$, only $A_{3/2}$ was varied, while setting the other electrocouplings to zero. The minimal values of $A_{1/2}$ have been obtained in the same way.

minimum $\chi^2(M_{res})$. Bonn-Gatchina approach was successfully used in the analyses of the exclusive CLAS $K^+\Lambda$ photoproduction data [95] that allowed the identification of the new baryon states reported in the recent PDG

FIGURE 7.6: $\chi_m^2(A_{3/2})$ as a function of $A_{3/2}$: for each plot the seven curves correspond to the first seven Legendre moments $C_0, \dots, C_6$. The red dashed line indicates the $\chi_m^2(A_{3/2}) = 4$ threshold. Curves intersecting the red dashed line for lower values of $A_{3/2}$ identify the polynomials with a higher sensitivity to the presence of the hybrid resonance.

edition [10]. A similar approach is followed in [59] and is described in the following. Cross section and Legendre moments have been evaluated for Model B implementing a hybrid resonance with fixed parameters, $M_{res} = 2.2 \text{ GeV}$, for different configurations:

- $J^P = \frac{1}{2}^+, J^P = \frac{3}{2}^+$
- $J^P = \frac{1}{2}^+ : A_{1/2}=20, 40 (10^{-3}) \text{ GeV}^{-1/2}$
- $J^P = \frac{3}{2}^+ : A_{1/2}=0 (10^{-3}) \text{ GeV}^{-1/2}, A_{3/2}=20, 40 (10^{-3}) \text{ GeV}^{-1/2}$
- $Q^2 = 0.1 \text{ GeV}^2, 0.562 \text{ GeV}^2, 1.002 \text{ GeV}^2$
- $E_{beam} = 6.6 \text{ GeV}, 8.8 \text{ GeV}$

FIGURE 7.7: $\chi^2(M_{res})$ as a function of M_{res} for $J^P = \frac{1}{2}^+$ and σ_U : for each plot the seven curves correspond to the first seven Legendre moments $C_0, \dots, C_6$. Note that the curves reach a minimum for the correct value of M_{res} and that sharper minima correspond to polynomials with higher sensitivity to the presence of a hybrid resonance.

Cross sections and Legendre moments have then been calculated running the resonance mass W_R from 1.9 up to 2.5 GeV and $\chi^2(M_{res})$ values have been computed for each Legendre moment and fixed Q^2 bins according to the relation:

$$\chi^2(M_{res}) = \frac{1}{N_{d.p.}} \sum_W \frac{(C_m^{Model B, M_{res}=2.2\text{GeV}} - C_m^{Model B, variable M_{res}})^2}{\delta^2} \quad (7.21)$$

where δ is the error associated with $C_m^{Model B, variable M_{res}}$. The $\chi^2/d.p.$ value has been computed for the first seven Legendre moments of the three structure functions σ_U , σ_{LT} and σ_{TT} . The results for the hybrid spin-parity assignments $J^P = \frac{1}{2}^+$ are reported in Figures 7.7, 7.8 and 7.9 for different Q^2 bins, 0.1 GeV^2 , 0.562 GeV^2 and 1.002 GeV^2 , in the range of photon virtualities. In all cases pronounced minima in $\chi^2(M_{res})$ denote the presence of a hybrid resonance at the nominal mass of $M_{res} = 2.2 \text{ GeV}$. Similar results have been obtained for $J^P = \frac{3}{2}^+$.

The higher sensitivity to the presence of the hybrid resonance is obtained for the C_0 and C_1 (black and red curves, respectively). The zero values of $\chi^2(M_{res})$ are due to the fact that no experimental smearing has been introduced in the evaluation of the Legendre moments. The observed behavior of $\chi^2(M_{res})$ strongly suggests the capability to identify the presence of new states from the data of the proposed experiment.

FIGURE 7.8: $\chi^2(M_{res})$ as a function of M_{res} for $J^P = \frac{1}{2}^+$ and σ_{LT} : for each plot the seven curves correspond to the first seven Legendre moments $C_0, \dots, C_6$. Note that the curves reach a minimum for the correct value of M_{res} and that sharper minima correspond to polynomials with higher sensitivity to the presence of a hybrid resonance.

7.3 Sensitivity to the Presence of Two Resonances

The data sensitivity to the presence of two hybrid resonances, whose masses and electrocouplings were unknown, was explored. To this extent cross section values in different kinematic bins have been compared changing the two resonance parameters. A χ^2 is calculated as:

$$\chi^2 = \frac{1}{N_{d.p.}} \sum_{W, \cos\theta_K^*, \phi_K^*} \frac{(\sigma_{fixed} - \sigma_{variable})^2}{\delta^2} = \frac{1}{N_{d.p.}} \sum_{W, \cos\theta_K^*, \phi_K^*} \frac{(\sigma_{fixed} - \sigma_{variable})^2}{\sigma_{variable}} N_{ev}. \quad (7.22)$$

where the σ is the cross section defined in Equation 2.39 and:

- σ_{fixed} : is calculated from Model B with two resonances of fixed parameters
 - Resonance 1: $J^P = \frac{1}{2}^+$, $A_{1/2} = 20 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $M_{res}^1 = 2.1 \text{ GeV}$
 - Resonance 2: $J^P = \frac{3}{2}^+$, $A_{3/2} = 20 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $A_{1/2} = 20 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $M_{res}^3 = 2.2 \text{ GeV}$
- $\sigma_{variable}$: is calculated from Model B with two resonances of variable parameters
 - Resonance 1: $J^P = \frac{1}{2}^+$, $A_{1/2} = 0-50 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $M_{res}^1 = 1.8-2.5 \text{ GeV}$

FIGURE 7.9: $\chi^2(M_{res})$ as a function of M_{res} for $J^P = \frac{1}{2}^+$ and σ_{TT} : for each plot the seven curves correspond to the first seven Legendre moments $C_0, \dots, C_6$. Note that the curves reach a minimum for the correct value of M_{res} and that sharper minima correspond to polynomials with higher sensitivity to the presence of a hybrid resonance.

– Resonance 2: $J^P = \frac{3}{2}^+$, $A_{3/2}=0-50 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $A_{1/2}=0-50 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $M_{res}^3=1.8-2.5 \text{ GeV}$

- N_{ev} = number of expected events in 50 days beamtime for each bin in Q^2 , W , $\cos\theta_K^*$, ϕ_K^*

χ^2 has been estimated for variable M_{res}^1 , M_{res}^3 in steps of 20 MeV and $A_{1/2}^{1/2^+} = A_{1/2}^{3/2^+} = A_{3/2}^{3/2^+}$ (for simplicity the three parameters were the same) in steps of $1 \cdot 10^{-3} \text{ GeV}^{-1/2}$ for $E_{beam} = 6.6 \text{ GeV}$, $Q^2 = 0.562 \text{ GeV}^2$ and $\Gamma_{res} = 0.25 \text{ GeV}$.

Figure 7.10 shows the results of the scanning of the χ^2 values as a function of the running masses and electrocouplings. The bidimensional plots report the values of the χ^2 in a color code as a function of the running masses of the two resonances, in the range from 1.8 up to 2.5 GeV, for the value of the electrocouplings indicated in the upper right pad. The central plot corresponds to the correct value of the electrocouplings $A_{1/2}^{1/2^+} = A_{1/2}^{3/2^+} = A_{3/2}^{3/2^+} = 20 \cdot 10^{-3} \text{ GeV}^{-1/2}$ for $\sigma_{variable}$, while on the top and on the bottom a $A_{1/2}^{1/2^+} = A_{1/2}^{3/2^+} = A_{3/2}^{3/2^+} = 10 \cdot 10^{-3} \text{ GeV}^{-1/2}$ and $A_{1/2}^{1/2^+} = A_{1/2}^{3/2^+} = A_{3/2}^{3/2^+} = 30 \cdot 10^{-3} \text{ GeV}^{-1/2}$ are displayed, respectively. It is possible to note that a sharp minimum is obtained for the correct combination of the resonance masses and electrocouplings, while χ^2 values higher than 15 are obtained for wrong electrocouplings values.

FIGURE 7.10: Scanning in mass and electrocouplings with two resonances. In this case the correct value of the parameters is: $A_{1/2}^{1/2+} = A_{1/2}^{3/2+} = A_{3/2}^{3/2+} = 20 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $M_{res}^1 = 2.1 \text{ GeV}$, $M_{res}^3 = 2.2 \text{ GeV}$. The bidimensional plots show the $M_1 - M_2$ couple on the x-y axes and the electrocoupling value is written in the pad. A color code is used to represent the χ^2 value obtained for that combination of masses and electrocouplings. The three plots, from top to bottom, correspond to electrocouplings values equal to $10 \cdot 10^{-3} \text{ GeV}^{-1/2}$, $20 \cdot 10^{-3} \text{ GeV}^{-1/2}$ and $30 \cdot 10^{-3} \text{ GeV}^{-1/2}$, respectively.

7.4 Blind Analysis

In order to check our capability to determine the correct resonance parameters of hybrid baryon resonance, a blind analysis of quasi-data has been performed. The Moscow State University group produced a grid of cross sections values inserting a hybrid resonance at the amplitude level and the task for us was to find its parameters scanning χ^2 value from Equation 7.22. Two resonances of $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ have been implemented in Model B, fixing $S_{1/2} = 0 \text{ GeV}^{-1/2}$, and $Q^2 = 0.516 \text{ GeV}^2$. The other seven parameters of the resonances, namely the masses $M_{res}^{J=1/2}$ and $M_{res}^{J=3/2}$, the total widths $\Gamma_{res}^{J=1/2}$ and $\Gamma_{res}^{J=3/2}$, and the electrocouplings $A_{1/2}^{J=1/2}$, $A_{1/2}^{J=3/2}$ and $A_{3/2}^{J=3/2}$ were fixed, but unknown during the procedure. The goal of the blind analysis was then to determine the hybrid parameters from the evaluation of the χ^2 in a seven-dimensional space in the search range:

$$\begin{aligned}
 2.00 \text{ GeV} < W < 2.5 \text{ GeV} \\
 0.1 \text{ GeV} < \Gamma < 0.4 \text{ GeV} \\
 -0.05 \text{ GeV}^{-1/2} < A_{1/2} < 0.05 \text{ GeV}^{-1/2} \\
 -0.05 \text{ GeV}^{-1/2} < A_{3/2} < 0.05 \text{ GeV}^{-1/2}.
 \end{aligned} \tag{7.23}$$

A strategy has been developed to obtain this result. In a first round all the search range has been studied.

The algorithm tries all the combinations with the parameters:

- Γ : 0.1 - 0.4 GeV with step 0.1 GeV (4 steps $\times$ 4 steps)
- M : 2.0 - 2.5 GeV with step 0.1 GeV (6 steps $\times$ 6 steps)
- A : $-0.05 \text{ GeV}^{-1/2}$ - $0.05 \text{ GeV}^{-1/2}$ with step $0.02 \text{ GeV}^{-1/2}$ (6 steps $\times$ 6 steps $\times$ 6 steps)

For a total of $4 \times 4 \times 6 \times 6 \times 6 \times 6 = 124416$ combinations.

Table 7.4 reports the χ^2 value obtained for each combination. The combinations with $\chi^2 < 10$ are printed below:

$M_{res}^{J=1/2}$	$M_{res}^{J=3/2}$	$\Gamma_{res}^{J=1/2}$	$\Gamma_{res}^{J=3/2}$	$A_{1/2}^{J=1/2}$	$A_{1/2}^{J=3/2}$	$A_{3/2}^{J=3/2}$	χ^2
0.1	0.4	2.3	2.5	0.01	-0.01	0.05	9.8267
0.2	0.4	2.3	2.5	0.01	-0.01	0.05	8.14997
0.3	0.4	2.3	2.5	0.01	-0.01	0.05	8.11312
0.4	0.4	2.3	2.5	0.01	-0.01	0.05	8.5098
0.2	0.4	2.4	2.5	0.01	-0.01	0.05	9.10313
0.3	0.4	2.4	2.5	0.01	-0.01	0.05	8.48565
0.4	0.4	2.4	2.5	0.01	-0.01	0.05	8.47921
0.4	0.4	2.5	2.5	0.01	-0.01	0.05	9.83554

TABLE 7.4: First run

This result is exploited to restrict the scanning region for the parameters using a limited region with finer bins.

Therefore, on the basis of the results of the first round the analysis has been repeated in the region:

- $\Gamma_{res}^{J=1/2}$: 0.2 - 0.35 GeV with step 0.05 GeV (4 steps)
- $\Gamma_{res}^{J=3/2}$: 0.31 - 0.40 GeV with step 0.03 GeV (4 steps)
- $M_{res}^{J=1/2}$: 2.3 - 2.45 GeV with step 0.05 GeV (4 steps)
- $M_{res}^{J=3/2}$: 2.45 - 2.5 GeV with step 0.05 GeV (2 steps)
- $A_{1/2}^{J=1/2}$: 0 - 0.02 $GeV^{-1/2}$ with step 0.01 $GeV^{-1/2}$ (3 steps)
- $A_{1/2}^{J=3/2}$: -0.02 - 0 $GeV^{-1/2}$ with step 0.01 $GeV^{-1/2}$ (3 steps)
- $A_{3/2}^{J=3/2}$: 0.04 - 0.05 $GeV^{-1/2}$ with step 0.01 $GeV^{-1/2}$ (2 steps)

For a total of $4 \times 4 \times 4 \times 2 \times 3 \times 3 \times 2 = 2304$ combinations.

The conditions corresponding to $\chi^2 < 1.5$ are printed below:

$\Gamma_{res}^{J=1/2}$	$\Gamma_{res}^{J=3/2}$	$M_{res}^{J=1/2}$	$M_{res}^{J=3/2}$	$A_{1/2}^{J=1/2}$	$A_{1/2}^{J=3/2}$	$A_{3/2}^{J=3/2}$	χ^2
0.25	0.31	2.3	2.45	0.02	-0.01	0.04	1.38365
0.3	0.31	2.3	2.45	0.02	-0.01	0.04	1.19933
0.35	0.31	2.3	2.45	0.02	-0.01	0.04	1.22970
0.3	0.31	2.35	2.45	0.02	-0.01	0.04	1.38856
0.35	0.31	2.35	2.45	0.02	-0.01	0.04	1.22506
0.25	0.34	2.3	2.45	0.02	-0.01	0.04	1.42131
0.3	0.34	2.3	2.45	0.02	-0.01	0.04	1.26096
0.35	0.34	2.3	2.45	0.02	-0.01	0.04	1.32852
0.35	0.34	2.35	2.45	0.02	-0.01	0.04	1.41246

TABLE 7.5: Second run

The third and last round is used to identify the minimum in the range selected in the previous iteration:

- $\Gamma_{res}^{J=1/2}$: 0.29 - 0.31 GeV with step 0.01 GeV (3 steps)
- $\Gamma_{res}^{J=3/2}$: 0.30 - 0.32 GeV with step 0.01 GeV (3 steps)
- $M_{res}^{J=1/2}$: 2.31 - 2.33 GeV with step 0.01 GeV (3 steps)
- $M_{res}^{J=3/2}$: 2.44 - 2.46 GeV with step 0.01 GeV (3 steps)
- $A_{1/2}^{J=1/2}$: 0.018 - 0.020 $GeV^{-1/2}$ with step 0.001 $GeV^{-1/2}$ (3 steps)
- $A_{1/2}^{J=3/2}$: -0.015 - -0.013 $GeV^{-1/2}$ with step 0.001 $GeV^{-1/2}$ (3 steps)
- $A_{3/2}^{J=3/2}$: 0.037 - 0.039 $GeV^{-1/2}$ with step 0.001 $GeV^{-1/2}$ (3 steps)

For a total of $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187$ combinations.

The parameters corresponding to $\chi^2 < 0.18$ are printed below:

$\Gamma_{res}^{J=1/2}$	$\Gamma_{res}^{J=3/2}$	$M_{res}^{J=1/2}$	$M_{res}^{J=3/2}$	$A_{1/2}^{J=1/2}$	$A_{1/2}^{J=3/2}$	$A_{3/2}^{J=3/2}$	χ^2
0.29	0.31	2.31	2.45	0.019	-0.014	0.038	0.171621
0.3	0.31	2.31	2.45	0.019	-0.014	0.038	0.178127
0.29	0.31	2.32	2.45	0.019	-0.014	0.038	0.166922
0.3	0.31	2.32	2.45	0.019	-0.014	0.038	0.166424
0.31	0.31	2.32	2.45	0.019	-0.014	0.038	0.174443

TABLE 7.6: Last run

The global minimum is found for:

$$M_{res}^{J=1/2} = 2.32 \text{ GeV}, M_{res}^{J=3/2} = 2.45 \text{ GeV}, \Gamma_{res}^{J=1/2} = 0.3 \text{ GeV}, \Gamma_{res}^{J=3/2} = 0.31 \text{ GeV}, A_{1/2}^{J=1/2} = 0.019 \text{ GeV}^{-1/2}, \\ A_{1/2}^{J=3/2} = -0.014 \text{ GeV}^{-1/2}, A_{3/2}^{J=3/2} = 0.038 \text{ GeV}^{-1/2}, \chi^2 = 0.166424.$$

To be compared with the correct blind parameters:

$$M_{res}^{J=1/2} = 2.30 \text{ GeV}, M_{res}^{J=3/2} = 2.45 \text{ GeV}, \Gamma_{res}^{J=1/2} = 0.3 \text{ GeV}, \Gamma_{res}^{J=3/2} = 0.35 \text{ GeV}, A_{1/2}^{J=1/2} = 0.020 \text{ GeV}^{-1/2}, \\ A_{1/2}^{J=3/2} = -0.015 \text{ GeV}^{-1/2}, A_{3/2}^{J=3/2} = 0.04 \text{ GeV}^{-1/2}.$$

Only small differences have been found proving that the expected statistical accuracy from CLAS12 measurement for $K^+\Lambda$ electroproduction is sufficient to assess the existence of unknown resonances if they exist.

Conclusions

The work reported in this thesis was finalized to the approval of a program for the search of hybrid baryons with CLAS12 experimental setup in Hall B at Jefferson Laboratories (VA, USA).

This program implies the extensive study of $K^+\Lambda$ ($K^+\Sigma^0$) and $p\pi^+\pi^-$ electroproduction reactions, in the kinematic region that covers the mass range up to 3 GeV and in the Q^2 range from 0.05 GeV^2 to 2.0 GeV^2 . CLAS12, and in particular the Forward Tagger facility, is suitable to study this range, thanks to its capability to detect electrons scattered at angles between 2.5 and 4.5 degrees, which provides a minimum Q^2 of 0.05 GeV^2 . The low Q^2 region study is crucial to identify hybrid resonances, thanks to the peculiar behavior that their electrocouplings $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ show in that range.

To obtain the approval of the experiment, a detailed study has been carried on, determining acceptance, resolution and events rates for 100 days runs.

In my thesis work I studied the $K^+\Lambda$ electroproduction channel. Exploiting the event generator provided by the Moscow State University (MSU), I produced events using different run conditions, simulating CLAS12 acceptance selection employing a FASTMC. This procedure allowed to select the best run conditions: 6.6 GeV and 8.8 GeV and -3375 A torus current. Also the run time of 50 days for each decay channel, $K^+\Lambda$ and $K^+\Sigma$, was established on the basis of the simulations.

Another task of my thesis was to study the sensitivity of the experimental setup to the presence of hybrid resonances. The theoretical description of the hybrid resonances has been employed to implement them in the RPR model 2011 distribution at the amplitude level. I then performed a Legendre moment analysis of the model with and without the hybrid state included, finding from the comparison that a structure arises for each Legendre polynomial in coincidence with the mass value of the added resonance. The collaboration with MSU resulted in a blind analysis to test our capability to identify two hybrid resonances with unknown mass and spin.

In parallel I participated to the calibration of the Forward Tagger electromagnetic calorimeter, to the tests on its 332 lead tungstate crystals and to the following analysis.

The measurements of light transmission on the scintillating crystals have been performed using the spectrophotometer available in the irradiation facility, the Strahlenzentrum, of the University of Gießen (Germany). The crystals that didn't pass the quality control were replaced.

I performed a calibration of the FT electromagnetic calorimeter to obtain the corrections to switch from the energy read by the calorimeter to the one that was really deposited. The corrections have been obtained simulating photons with energies up to 8 GeV and with polar scattering angle between 2.5° and 4.5° exploiting the Geant

based Monte Carlo of CLAS12, GEMC. The corrections have been determined as a function of polar angle and energy of the generated particle.

At present the FT setup has been tested and mounted in Hall B. CLAS12 is under construction, the first data taking will be in 2017.

The Hybrid Baryons program has been submitted for approval to Jefferson Laboratory PAC44 in June 2016 and was approved on July 2016. The 100 requested days for measurements have been approved as well.

When data will be collected, they will be subjected to a full amplitude partial wave analysis, which can constitute a definite proof for the existence of a resonance.

Appendix A

FT Energy Corrections Distributions

In this appendix the plots that have been used to obtain the energy corrections as a function of energy and polar angle are shown. There are eleven sets of plots, corresponding to the eleven angles that have been used, from 2.5° up to 4.5° with a step of 0.2° . For each angle $2 \cdot 10^5$ single photon events have been generated with fixed angle in the energy range 100 MeV - 8 GeV. The energy of the electromagnetic shower in the calorimeter has been reconstructed with four different techniques:

- with/without threshold
- reading the *digitized* adc or the *true* totEdep bank.

Therefore, four different energy values for the reconstructed energy occur. Each of these E_{rec} values is used to calculate the difference between generated and reconstructed energies (that we expect to be positive), to select the correct energy bin of reconstructed energy (from 100 MeV up to 8 GeV, 50 MeV step) and a corresponding plot where the $E_{gen} - E_{rec}$ distribution is displayed. Figure 5.12 has been used as example: the title “ $E_{min} = 4.175$ GeV $E_{max} = 4.225$ GeV” indicates that all data composing the four histograms had $4.175 \text{ GeV} < E_{rec} < 4.225 \text{ GeV}$. Depending on the options employed to reconstruct the energy value, it is inserted in a different histogram:

- with threshold, reading the *digitized* adc bank: black histogram
- with threshold, reading the *true* totEdep bank: red histogram
- without threshold, reading the *digitized* adc bank: blue histogram
- without threshold, reading the *true* totEdep bank: green histogram.

Counts were displayed as a function of the difference between generated and reconstructed energies, producing a gaussian distribution with a slight asymmetry. For every plot the four histograms were fitted with a gaussian function (in Figure 5.12 it is possible to see the overlapping curves), thus obtaining four means and four standard

deviations. The fits were repeated using an upper limit of $mean_{adc\ with\ thr.} + 1.2\ \sigma_{adc\ with\ thr.}$ to cut the tail. The four means were associated to the mean value of the energy bin (for instance in [5.12](#), the mean value of the energy bin is 4.2 GeV) and were used to plot the difference between generated and reconstructed energies as a function of reconstructed energy (see for example [Figure 5.14](#)).

FIGURE A.1: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 2.5^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.2: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 2.7^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.3: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 2.9^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.4: Gaussian fits for every energy bin from 0.1 up to 8 GeV, with a step of 50 MeV, for $\theta = 3.1^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.5: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 3.3^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.6: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 3.5^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.7: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 3.7^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.8: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 3.9^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.9: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 4.1^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.10: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 4.3^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

FIGURE A.11: Gaussian fits for every energy bin from 0.1 up to 7 GeV, with a step of 50 MeV, for $\theta = 4.5^\circ$. Four distributions are visible corresponding to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively). The tails have been cut.

Appendix B

FT energy correction fits

In this appendix the fit results for the energy corrections are reported for all θ polar angles. As already explained, the points in each plot are obtained from the gaussian fits in Appendix A. Recalling the description of the distributions from Chapter 5, the cluster energy has been reconstructed using two options:

- reading the *digitized* adc or the *true* totEdep bank
- with/without threshold

where the threshold is the minimum value that is acceptable for a single hit and is fixed at 50 MeV.

$\theta = 2.5^\circ$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \tag{B.1}$$

where

$$\begin{aligned} p_0 &= 0.02037 \pm 0.00071 \text{ GeV} \\ p_1 &= 0.04776 \pm 0.00017 \end{aligned} \tag{B.2}$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \tag{B.3}$$

FIGURE B.1: Fits to obtain the energy corrections for angle 2.5° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

where

$$\begin{aligned} p_0 &= 0.01265 \pm 0.00079 \text{ GeV} \\ p_1 &= 0.04729 \pm 0.00019 \end{aligned} \tag{B.4}$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \tag{B.5}$$

where

$$\begin{aligned} p_0 &= 0.0300 \pm 0.0016 \text{ GeV} \\ p_1 &= 0.0951 \pm 0.0018 \\ p_2 &= -0.00911 \pm 0.00059 \text{ GeV}^{-1} \\ p_3 &= 0.00063 \pm 5.5 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \tag{B.6}$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3 \quad (\text{B.7})$$

where

$$\begin{aligned} p_0 &= 0.02912 \pm 0.0016 \text{ GeV} \\ p_1 &= 0.0951 \pm 0.0020 \\ p_2 &= -0.00922 \pm 0.00064 \text{ GeV}^{-1} \\ p_3 &= 0.00064 \pm 5.9 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (\text{B.8})$$

FIGURE B.2: Fits to obtain the energy corrections for angle 2.7° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

$\theta = 2.7^\circ$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1x \quad (\text{B.9})$$

where

$$\begin{aligned} p_0 &= 0.01893 \pm 0.00060 \text{ GeV} \\ p_1 &= 0.04140 \pm 0.00015 \end{aligned} \tag{B.10}$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \tag{B.11}$$

where

$$\begin{aligned} p_0 &= 0.01117 \pm 0.00071 \text{ GeV} \\ p_1 &= 0.04113 \pm 0.00017 \end{aligned} \tag{B.12}$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \tag{B.13}$$

where

$$\begin{aligned} p_0 &= 0.0311 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.0930 \pm 0.0019 \\ p_2 &= -0.01039 \pm 0.00060 \text{ GeV}^{-1} \\ p_3 &= 0.00070 \pm 5.5 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \tag{B.14}$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \tag{B.15}$$

where

$$\begin{aligned} p_0 &= 0.0292 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.0929 \pm 0.0018 \\ p_2 &= -0.01044 \pm 0.00060 \text{ GeV}^{-1} \\ p_3 &= 0.00071 \pm 5.5 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \tag{B.16}$$

$$\theta = 2.9^\circ$$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \tag{B.17}$$

FIGURE B.3: Fits to obtain the energy corrections for angle 2.9° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

where

$$\begin{aligned} p_0 &= 0.01976 \pm 0.00063 \text{ GeV} \\ p_1 &= 0.03617 \pm 0.00015 \end{aligned} \quad (\text{B.18})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \quad (\text{B.19})$$

where

$$\begin{aligned} p_0 &= 0.01082 \pm 0.00067 \text{ GeV} \\ p_1 &= 0.03604 \pm 0.00016 \end{aligned} \quad (\text{B.20})$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \quad (\text{B.21})$$

where

$$\begin{aligned}
 p_0 &= 0.0311 \pm 0.0015 \text{ GeV} \\
 p_1 &= 0.0921 \pm 0.0018 \\
 p_2 &= -0.01118 \pm 0.00058 \text{ GeV}^{-1} \\
 p_3 &= 0.00076 \pm 0.00053 \text{ GeV}^{-2}
 \end{aligned}
 \tag{B.22}$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \tag{B.23}$$

where

$$\begin{aligned}
 p_0 &= 0.0307 \pm 0.0015 \text{ GeV} \\
 p_1 &= 0.0911 \pm 0.0018 \\
 p_2 &= -0.01093 \pm 0.00060 \text{ GeV}^{-1} \\
 p_3 &= 0.00074 \pm 0.00055 \text{ GeV}^{-2}
 \end{aligned}
 \tag{B.24}$$

FIGURE B.4: Fits to obtain the energy corrections for angle 3.1° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

$$\theta = 3.1^\circ$$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \quad (\text{B.25})$$

where

$$\begin{aligned} p_0 &= 0.01916 \pm 0.00058 \text{ GeV} \\ p_1 &= 0.03467 \pm 0.00014 \end{aligned} \quad (\text{B.26})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \quad (\text{B.27})$$

where

$$\begin{aligned} p_0 &= 0.00988 \pm 0.00066 \text{ GeV} \\ p_1 &= 0.03485 \pm 0.00016 \end{aligned} \quad (\text{B.28})$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \quad (\text{B.29})$$

where

$$\begin{aligned} p_0 &= 0.0320 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.0901 \pm 0.0018 \\ p_2 &= -0.01083 \pm 0.00058 \text{ GeV}^{-1} \\ p_3 &= 0.00072 \pm 5.4 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (\text{B.30})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \quad (\text{B.31})$$

where

$$\begin{aligned} p_0 &= 0.0306 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.0911 \pm 0.0018 \\ p_2 &= -0.01112 \pm 0.00058 \text{ GeV}^{-1} \\ p_3 &= 0.00075 \pm 5.4 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (\text{B.32})$$

FIGURE B.5: Fits to obtain the energy corrections for angle 3.3° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

$\theta = 3.3^\circ$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \quad (\text{B.33})$$

where

$$\begin{aligned} p_0 &= 0.01889 \pm 0.00054 \text{ GeV} \\ p_1 &= 0.03457 \pm 0.00013 \end{aligned} \quad (\text{B.34})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \quad (\text{B.35})$$

where

$$\begin{aligned} p_0 &= 0.00827 \pm 0.00070 \text{ GeV} \\ p_1 &= 0.03506 \pm 0.00017 \end{aligned} \quad (\text{B.36})$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3 \quad (\text{B.37})$$

where

$$\begin{aligned} p_0 &= 0.0307 \pm 0.0014 \text{ GeV} \\ p_1 &= 0.0927 \pm 0.0016 \\ p_2 &= -0.01160 \pm 0.00053 \text{ GeV}^{-1} \\ p_3 &= 0.00079 \pm 4.9 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (\text{B.38})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3 \quad (\text{B.39})$$

where

$$\begin{aligned} p_0 &= 0.0302 \pm 0.0014 \text{ GeV} \\ p_1 &= 0.0931 \pm 0.0017 \\ p_2 &= -0.01197 \pm 0.00056 \text{ GeV}^{-1} \\ p_3 &= 0.00083 \pm 5.2 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (\text{B.40})$$

$$\theta = 3.5^\circ$$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1x \quad (\text{B.41})$$

where

$$\begin{aligned} p_0 &= 0.01771 \pm 0.00050 \text{ GeV} \\ p_1 &= 0.03596 \pm 0.00012 \end{aligned} \quad (\text{B.42})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1x \quad (\text{B.43})$$

where

$$\begin{aligned} p_0 &= 0.00758 \pm 0.00065 \text{ GeV} \\ p_1 &= 0.03641 \pm 0.00016 \end{aligned} \quad (\text{B.44})$$

FIGURE B.6: Fits to obtain the energy corrections for angle 3.5° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3 \quad (\text{B.45})$$

where

$$\begin{aligned} p_0 &= 0.0343 \pm 0.0017 \text{ GeV} \\ p_1 &= 0.0857 \pm 0.0021 \\ p_2 &= -0.00890 \pm 0.00068 \text{ GeV}^{-1} \\ p_3 &= 0.00053 \pm 6.3 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \quad (\text{B.46})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1x + p_2x^2 + p_3x^3 \quad (\text{B.47})$$

where

$$\begin{aligned}
 p_0 &= 0.0336 \pm 0.0017 \text{ GeV} \\
 p_1 &= 0.0856 \pm 0.0020 \\
 p_2 &= -0.00904 \pm 0.00066 \text{ GeV}^{-1} \\
 p_3 &= 0.00055 \pm 6.2 \cdot 10^{-5} \text{ GeV}^{-2}
 \end{aligned}
 \tag{B.48}$$

FIGURE B.7: Fits to obtain the energy corrections for angle 3.7° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

$\theta = 3.7^\circ$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \tag{B.49}$$

where

$$\begin{aligned}
 p_0 &= 0.01546 \pm 0.00055 \text{ GeV} \\
 p_1 &= 0.04060 \pm 0.00013
 \end{aligned}
 \tag{B.50}$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \quad (\text{B.51})$$

where

$$\begin{aligned} p_0 &= 0.00785 \pm 0.00061 \text{ GeV} \\ p_1 &= 0.04050 \pm 0.00015 \end{aligned} \quad (\text{B.52})$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 + p_4 x^4 \quad (\text{B.53})$$

where

$$\begin{aligned} p_0 &= 0.0181 \pm 0.0016 \text{ GeV} \\ p_1 &= 0.1217 \pm 0.0031 \\ p_2 &= -0.0285 \pm 0.0018 \text{ GeV}^{-1} \\ p_3 &= 0.00452 \pm 3.9 \cdot 10^{-5} \text{ GeV}^{-2} \\ p_4 &= -0.00026 \pm 2.7 \cdot 10^{-5} \text{ GeV}^{-3} \end{aligned} \quad (\text{B.54})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \quad (\text{B.55})$$

where

$$\begin{aligned} p_0 &= 0.0167 \pm 0.0016 \text{ GeV} \\ p_1 &= 0.1231 \pm 0.0032 \\ p_2 &= -0.0295 \pm 0.0018 \text{ GeV}^{-1} \\ p_3 &= 0.00472 \pm 3.9 \cdot 10^{-5} \text{ GeV}^{-2} \\ p_4 &= -0.00027 \pm 2.7 \cdot 10^{-5} \text{ GeV}^{-3} \end{aligned} \quad (\text{B.56})$$

$$\theta = 3.9^\circ$$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \quad (\text{B.57})$$

FIGURE B.8: Fits to obtain the energy corrections for angle 3.9° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

where

$$\begin{aligned} p_0 &= 0.01424 \pm 0.00049 \text{ GeV} \\ p_1 &= 0.04612 \pm 0.00012 \end{aligned} \tag{B.58}$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \tag{B.59}$$

where

$$\begin{aligned} p_0 &= 0.00603 \pm 0.00058 \text{ GeV} \\ p_1 &= 0.04623 \pm 0.0001468 \end{aligned} \tag{B.60}$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \tag{B.61}$$

where

$$\begin{aligned}
 p_0 &= 0.0308 \pm 0.0016 \text{ GeV} \\
 p_1 &= 0.0916 \pm 0.0020 \\
 p_2 &= -0.00100 \pm 0.00063 \text{ GeV}^{-1} \\
 p_3 &= 0.00073 \pm 5.8 \cdot 10^{-5} \text{ GeV}^{-2}
 \end{aligned}
 \tag{B.62}$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 \tag{B.63}$$

where

$$\begin{aligned}
 p_0 &= 0.0309 \pm 0.0016 \text{ GeV} \\
 p_1 &= 0.0904 \pm 0.0019 \\
 p_2 &= -0.00971 \pm 0.00061 \text{ GeV}^{-1} \\
 p_3 &= 0.00071 \pm 5.6 \cdot 10^{-5} \text{ GeV}^{-2}
 \end{aligned}
 \tag{B.64}$$

FIGURE B.9: Fits to obtain the energy corrections for angle 4.1° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

$$\theta = 4.1^\circ$$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \quad (\text{B.65})$$

where

$$\begin{aligned} p_0 &= 0.0111 \pm 0.00050 \text{ GeV} \\ p_1 &= 0.05844 \pm 0.00012 \end{aligned} \quad (\text{B.66})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \quad (\text{B.67})$$

where

$$\begin{aligned} p_0 &= 0.00369 \pm 0.00059 \text{ GeV} \\ p_1 &= 0.05840 \pm 0.00014 \end{aligned} \quad (\text{B.68})$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x + p_2 x^2 \quad (\text{B.69})$$

where

$$\begin{aligned} p_0 &= 0.02866 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.07708 \pm 0.00095 \\ p_2 &= -0.00156 \pm 0.00013 \text{ GeV}^{-1} \end{aligned} \quad (\text{B.70})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x + p_2 x^2 \quad (\text{B.71})$$

where

$$\begin{aligned} p_0 &= 0.0396 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.07713 \pm 0.00096 \\ p_2 &= -0.00159 \pm 0.00013 \text{ GeV}^{-1} \end{aligned} \quad (\text{B.72})$$

$$\theta = 4.3^\circ$$

FIGURE B.10: Fits to obtain the energy corrections for angle 4.3° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1 x \quad (\text{B.73})$$

where

$$\begin{aligned} p_0 &= 0.00956 \pm 0.00066 \text{ GeV} \\ p_1 &= 0.07594 \pm 0.00016 \end{aligned} \quad (\text{B.74})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1 x \quad (\text{B.75})$$

where

$$\begin{aligned} p_0 &= 0.00328 \pm 0.00069 \text{ GeV} \\ p_1 &= 0.07575 \pm 0.00017 \end{aligned} \quad (\text{B.76})$$

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1x + p_2x^2 \quad (\text{B.77})$$

where

$$\begin{aligned} p_0 &= 0.0363 \pm 0.0016 \text{ GeV} \\ p_1 &= 0.0928 \pm 0.0011 \\ p_2 &= -0.00144 \pm 0.00014 \text{ GeV}^{-1} \end{aligned} \quad (\text{B.78})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1x + p_2x^2 \quad (\text{B.79})$$

where

$$\begin{aligned} p_0 &= 0.0350 \pm 0.0017 \text{ GeV} \\ p_1 &= 0.0932 \pm 0.0011 \\ p_2 &= -0.00153 \pm 0.00015 \text{ GeV}^{-1} \end{aligned} \quad (\text{B.80})$$

$$\theta = 4.5^\circ$$

- Intrinsic no threshold (green points, green line)

$$f(x) = p_0 + p_1x \quad (\text{B.81})$$

where

$$\begin{aligned} p_0 &= -0.0021 \pm 0.0012 \text{ GeV} \\ p_1 &= 0.11990 \pm 0.00029 \end{aligned} \quad (\text{B.82})$$

- Reconstructed no threshold (blue points, blue line)

$$f(x) = p_0 + p_1x \quad (\text{B.83})$$

where

$$\begin{aligned} p_0 &= -0.0071 \pm 0.0011 \text{ GeV} \\ p_1 &= 0.11950 \pm 0.00027 \end{aligned} \quad (\text{B.84})$$

FIGURE B.11: Fits to obtain the energy corrections for angle 4.5° . The four data sets correspond to *digitalized* data with or without the use of a threshold (black and blue distributions, respectively), *true* data with or without a threshold (red and green distributions, respectively).

- Intrinsic threshold (red points, red line)

$$f(x) = p_0 + p_1 x \quad (\text{B.85})$$

where

$$\begin{aligned} p_0 &= 0.0358 \pm 0.0014 \text{ GeV} \\ p_1 &= 0.12760 \pm 0.00035 \end{aligned} \quad (\text{B.86})$$

- Reconstructed threshold (black points, black line)

$$f(x) = p_0 + p_1 x \quad (\text{B.87})$$

where

$$\begin{aligned} p_0 &= 0.0351 \pm 0.0015 \text{ GeV} \\ p_1 &= 0.12760 \pm 0.00037 \end{aligned} \quad (\text{B.88})$$

Appendix C

KY photoproduction

In the case of photoproduction the following reaction (the corresponding four-momenta of the particles are given between brackets) is considered:

$$p(p) + \gamma(k) \rightarrow K(p_K) + Y(p_Y). \quad (\text{C.1})$$

To calculate the unpolarized differential cross section, the following well known expression is employed [96]:

$$d\sigma = \int \frac{1}{v_{rel} 2\omega 2E_p} \frac{d^3\mathbf{p}_K}{(2\pi)^3} \frac{1}{2E_K} \frac{d^3\mathbf{p}_Y}{(2\pi)^3} \frac{1}{2E_Y} (2\pi)^4 \delta^{(4)}(p + k - p_K - p_Y) \frac{1}{4} \sum_{\lambda, \lambda_N, \lambda_Y} |\mathcal{M}_\lambda^{\lambda_N \lambda_Y}|^2, \quad (\text{C.2})$$

where λ , λ_p and λ_Y are the photon, proton and hyperon polarization, respectively, and v_{rel} is the relative photon-nucleon energy. The factor 1/4 emerges from the averaging over the initial state's polarizations λ and λ_N .

C.1 Center of mass system

The kinematic conventions are shown in the hadronic plane of Figure 2.4, since in the case of photoproduction the interaction takes place in this plane. We describe the system in the center of mass (CM) frame: the $\mathbf{z}$ -axis is the propagation direction of the incident photon, and the $\mathbf{xz}$ -plane is the reaction plane.

In the CM system, the particles' four-momenta are defined as:

$$k^* = \begin{pmatrix} \omega^* \\ \mathbf{k}^* \end{pmatrix}, \quad p_N^* = \begin{pmatrix} E_P^* \\ \mathbf{p}_P^* \end{pmatrix} = \begin{pmatrix} E_p^* \\ -\mathbf{k}^* \end{pmatrix}, \quad p_K^* = \begin{pmatrix} E_K^* \\ \mathbf{p}_K^* \end{pmatrix}, \quad p_\Lambda^* = \begin{pmatrix} E_\Lambda^* \\ \mathbf{p}_\Lambda^* \end{pmatrix} = \begin{pmatrix} E_\Lambda^* \\ -\mathbf{p}_K^* \end{pmatrix} \quad (\text{C.3})$$

with $|\mathbf{k}^*| = \omega^*$ because of the mass relation for real photons.

Inserting the CM momenta from C.3 into Equation C.2, and fixing ($s = W^2 = (k^* + p_N^*)^2$, $t = (p_K^* - k^*)^2$), the

expression for the unpolarized differential cross section becomes:

$$\frac{d\sigma}{d\Omega_K^*} = \frac{1}{64\pi^2} \frac{|\mathbf{p}_K^*|}{\omega^*} \frac{1}{(E_p^* + \omega^*)} \frac{1}{4} \sum_{\lambda, \lambda_N, \lambda_Y} |\mathcal{M}_\lambda^{\lambda_N \lambda_Y}|^2, \quad (\text{C.4})$$

where E_p^* , E_K^* and E_Y^* are determined by the mass relations:

$$E_p^* = \sqrt{M_p^2 + |\omega_K^*|^2}, E_K^* = \sqrt{M_K^2 + |\mathbf{p}_K^*|^2}, E_Y^* = \sqrt{M_Y^2 + |\mathbf{p}_K^*|^2}. \quad (\text{C.5})$$

$|\mathbf{p}_K^*|$ is uniquely given by the energy conservation relation:

$$\omega^* + \sqrt{M_p^2 + (\omega^*)^2} = \sqrt{M_K^2 + |\mathbf{p}_K^*|^2} + \sqrt{M_Y^2 + |\mathbf{p}_K^*|^2}. \quad (\text{C.6})$$

C.2 Transition amplitude

The transition amplitude in Equation C.4 can be written as:

$$\mathcal{M}_\lambda^{\lambda_N \lambda_Y} = \epsilon_\lambda^\mu J_\mu^{\lambda_N \lambda_Y}, \quad (\text{C.7})$$

where ϵ_λ^μ is the photon polarization vector, and the hadronic current can be expressed as

$$J_\mu^{\lambda_N \lambda_Y} = \bar{u}_{\lambda_Y}^Y(p_Y) T_\mu u_{\lambda_N}^N(p_N). \quad (\text{C.8})$$

Here, $u_{\lambda_Y}^Y(p_Y)$ and $u_{\lambda_N}^N(p_N)$ are the hyperon and nucleon spinors. Substituting C.8 in C.7 we obtain the complete expression for the transition amplitude

$$\mathcal{M}_\lambda^{\lambda_N \lambda_Y} = \bar{u}_{\lambda_Y}^Y(p_Y) \epsilon_\lambda^\mu T_\mu u_{\lambda_N}^N(p_N). \quad (\text{C.9})$$

The polarization λ determines the photon polarization four-vector ϵ^λ .

For circularly polarized real photons, two transverse polarizations $\lambda = +1$, $\lambda = -1$ occur

$$\epsilon^{\lambda=\pm 1} = \mp \frac{1}{\sqrt{2}} (0, 1, \pm i, 0). \quad (\text{C.10})$$

This equation gives the expression for ϵ_λ in the “unprimed” reference frame. This coordinate system corresponds to the black axes in Figure 2.4. The z-axis of this system is defined by the propagation direction of the incident photon, and the x-axis is determined to lie in the reaction plane and in the hemisphere of the outgoing kaon.

On the other hand, for linearly polarized photons, the polarization vectors are

$$\epsilon^{\lambda=x} \equiv \epsilon^\parallel = (0, 1, 0, 0) \quad (\text{C.11})$$

$$\epsilon^{\lambda=y} \equiv \epsilon^\perp = (0, 0, 1, 0). \quad (\text{C.12})$$

C.3 Single polarization observables

The nucleon and hyperon polarizations λ_N and λ_Y , are the nucleon and hyperon spin projections on the $\mathbf{y}$ - ($\equiv \mathbf{y}'$)-axis, see Figure 2.4. Once the polarization has been achieved, the asymmetry in the cross section with regard to this polarization can be measured.

The target (T) and recoil (P) asymmetries are defined according to equation,

$$A = \frac{d\sigma^{\lambda_X=+1/2} - d\sigma^{\lambda_X=-1/2}}{d\sigma^{\lambda_X=+1/2} + d\sigma^{\lambda_X=-1/2}} \quad (\text{C.13})$$

where λ_X is either the nucleon (T asymmetry) or the hyperon (P asymmetry) polarization and $d\sigma$ is a shorthand notation for $\frac{d\sigma}{d\Omega_K^*}$.

The photon beam asymmetry with circularly polarized photon vanishes because of the parity conserving nature of the interaction,

$$\Sigma^{circ.} = \frac{d\sigma^{\lambda=+1} - d\sigma^{\lambda=-1}}{2d\sigma} = 0. \quad (\text{C.14})$$

Therefore, the beam asymmetry is defined in terms of linearly polarized photons, as follows

$$\Sigma^{lin.} = \frac{d\sigma^\perp - d\sigma^\parallel}{2d\sigma}. \quad (\text{C.15})$$

In this definition $\sigma^\perp(\sigma^\parallel)$ refers to a linear photon polarization along the $\mathbf{y}(\mathbf{x})$ axis.

C.4 Double polarization observables

Double polarization observables are important to determine the complete transition amplitude $\mathcal{M}_\lambda^{\lambda_N \lambda_Y}$, as defined in Equation C.9. A significant effort is required for their achievement since the information about the spin of two particles is needed.

They are defined as

$$\frac{d\sigma^{(++)} + d\sigma^{(--) - d\sigma^{(+-)} - d\sigma^{(-+)}}{d\sigma^{(++)} + d\sigma^{(--) + d\sigma^{(+-)} + d\sigma^{(-+)}} \quad (\text{C.16})$$

where $(+-)$ is a shorthand notation for $(\lambda_A = +s_A, \lambda_B = -s_B)$, the polarizations of the particles A and B that determine the asymmetry. Three types of double polarization observables, referred to as beam-recoil (γ and Y are polarized), beam-target (γ and N) and target-recoil (N and Y) asymmetries arise from the possible combinations of particles. Beam-recoil $p(\gamma, K^+)\Lambda$ double polarization data are available for circularly (C_x, C_z) and obliquely (O_x, O_z) polarized photon beams. A common way to express them employs the “primed” coordinate system, which is rotated about the $\mathbf{y} = \mathbf{y}'$ -axis over an angle θ_K^* , with θ_K^* the angle between the incoming photon and the outgoing kaon momentum in the CM frame, see the red system in Figure 2.4.

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