The quark-confinement-induced pressure distribution in the proton

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The first experimental investigation of the quark confinement in the proton is presented. Data taken with the CEBAF Large Acceptance Spectrometer (CLAS) at the Jefferson Laboratory electron accelerator were used to determine the pressure and force distribution of the confined quarks in the proton. The results shown in Fig. 1 reveal the quark contributions to the radial pressure distribution. The characteristic feature is the strong repulsive pressure at radial distances below 0.3 femtometers from the center, and the wider distribution of the confining pressure at larger radial distance. In the experiment spin-polarized electrons of 6 GeV energy were scattered off a liquid hydrogen target. The scattered electrons and recoil protons as well as the generated high-energy photons were measured in coincidence and ensured the exclusivity of the reaction. The analysis combined the cross section and spin-polarized beam asymmetry measurement with a dispersion relation that allowed for the extraction of the D-term, which is a fundamental property of matter. Knowledge of the D-term of the proton is of similar significance as its much better known properties, mass and spin. It encodes the $d_1^q(t)$ form factor of the Energy-Momentum Tensor (EMT) and is key to the understanding of confinement.

The formation of protons occurs during the evolution of the microsecond old universe from its state of chiral symmetric, de-confined and non-interacting colored quarks and gluons to the state of color neutral protons and neutrons. In this process chiral symmetry is spontaneously broken and the full complement of excited baryons of all flavors drive the transition to the stable state. The phenomenon of color confinement is at the core of what makes the proton a stable particle and hence provides the stability of the visible universe. The formation of confinement in protons is still out of reach of theory, and no analytic proof exists that Quantum-Chromo-Dynamics (QCD), the theory of the strong interaction of quark and gluons, leads to confinement.

While the confinement of quarks in the proton is an unassailable reality (*the proton does not decay*) very little is known about the intrinsic forces and pressure distribution that provides its stability, and only models have so far been used to obtain some understanding of the phenomenon.

A direct determination of the pressure distribution requires measurements of the form factors of the nucleon matrix element of the energy-momentum tensor (EMT) [1]. That matrix element contains 3 scalar form factors that only depend on the momentum transfer t to the proton, one of them, $d_1(t)$, encodes the shear forces and pressure distribution in the proton. The other two encode the mass and the angular momentum distribution in the proton. The form factor $d_1(t)$ can in principle be measured directly only through elastic graviton-proton scattering, which in practice is an impossible task. The development of the framework of generalized parton distribution (GPDs) [2], and the discovery of deeply virtual Compton scattering (DVCS) as a means of measuring GPDs [3], and to model them [4], showed a promising way forward to determine $d_1(t)$. Recently, methods have been developed to extract information about the GPDs and the related Compton form factors (CFFs) from data on DVCS [5–8].

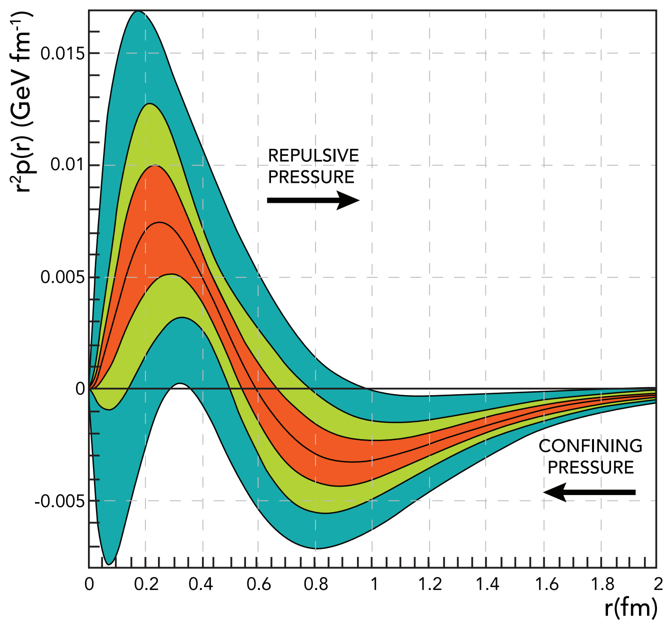

FIG. 1: Radial pressure distribution determined from interactions on the quarks in the proton versus radial distance from the center. See text for details.

Absent of beams of gravitons, we can exploit the remarkable correspondence of the gravitational form factors with the second Mellin moments of the chiral-even GPDs. The most suitable process to access the chiral-even GPDs is DVCS. $ep \to e'p'\gamma$. The important feature of this process is that the proton is left intact, while a

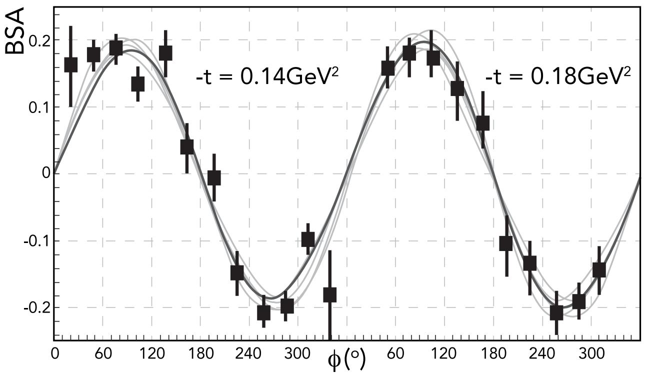

FIG. 2: Examples of the fits to Beam Spin Asymmetry from CLAS. The beam spin asymmetries are shown as square markers as functions of ϕ. The fit is shown as the thick curve. The light thin curves correspond to the variations of the fit parameters for the imaginary part of CFF $\mathcal{H}$ from the KM parameterization adjusted to the CLAS data.

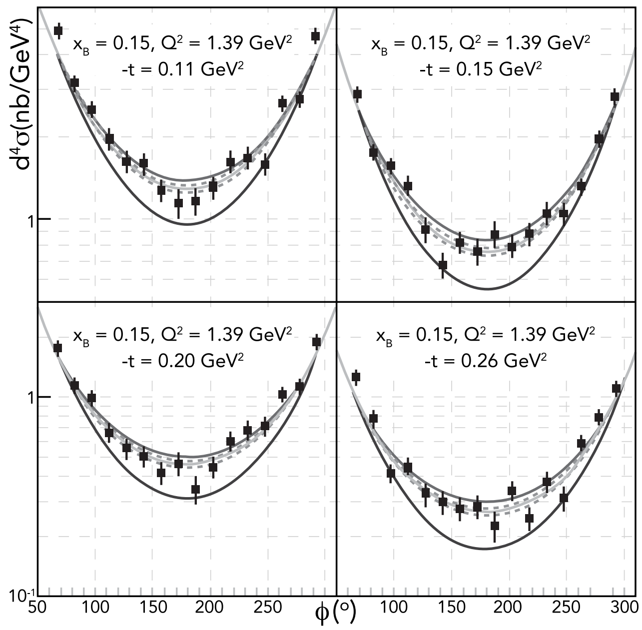

FIG. 3: Examples of the fits to unpolarized cross-sections from CLAS. The unpolarized cross-sections are shown as the black square points as functions of ϕ. The grey curves show results of fits with the parameter $d_1^q(t)$ at fixed $-t$. The upper black curves correspond to the result of our global fits to the $-t$ dependence of $d_1^q(t)$ with the real part of the DVCS amplitude calculated form the dispersion relations and the subtraction constant evaluated from the d_1^q contribution. The pure Bethe-Heitler (BH) contribution is shown with the lower black curve. The BH contribution largely dominates the extreme regions of ϕ in the unpolarized cross-sections.

highly virtual photon is exchanged with the elementary quarks, and at the same time the momentum transfer to the proton is controlled by the kinematics of the emitted photon.

In order to determine the pressure distribution in the proton from the experimental data we begin with the known parameterization of the Mellin moments of (chiral-even) GPDs. Their second moments are parameterized as :

$$\int \mathrm{d}x\, x\, [H(x,\xi,t) + E(x,\xi,t)] \;=\; 2J(t)$$

$$\int \mathrm{d}x\, x H(x,\xi,t) \;=\; M_2(t) + \frac{4}{5}\xi^2 d_1(t)$$

where $M_2(t)$ and $J(t)$ respectively correspond to the time-time and time-space components of the EMT, and give access to the mass and total angular momentum distributions carried by the quarks in the proton, and where the quantity $d_1(t)$ corresponds to the space-space components of the EMT, and encodes the shear forces and pressure acting on the quarks. We have some constraints on $M_2(t)$ and $J(t)$, notably at $t = 0$ they are fixed to the proton mass and spin. By contrast, rather little is known on the equally fundamental quantity $d_1(t)$.

Since $d_1(t)$ encodes the shear forces and pressure distribution in the proton, we can expect the existence of a zero sum rule ensuring the total pressure and forces to vanish, thus preserving the stability of the dynamics. This was studied in the limit of a large number of colors $N_c \to \infty$, within the framework of the chiral quark-soliton model in [9]. Those studies suggest that the repulsive forces in the form of pressure acting on the quark core are compensated by the existence of attractive forces and the corresponding confining pressure acting on the quarks at the periphery of the proton in what is often referred to as the "pion cloud".

The observables are parameterized by the Compton Form Factors (CFFs), which for the GPD H are the real quantities $\mathrm{Re}\mathcal{H}$ and $\mathrm{Im}\mathcal{H}$ defined by :

$$\mathrm{Re}\mathcal{H}(\xi,t) + i\mathrm{Im}\mathcal{H}(\xi,t) = \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H(x,\xi,t)$$

and similarly for the other GPDs. The average momentum fraction x is not observable in the process, it is integrated over with the quark propagators. Analytical properties of the amplitude in the Leading Order (LO) approximation lead to the dispersion relation :

$$\mathrm{Re}\mathcal{H}(\xi,t) \overset{\mathrm{LO}}{=} D(\xi,t) + \mathcal{P}\int_{-1}^{1} dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right) \mathrm{Im}\mathcal{H}(\xi,t)$$

where the subtraction constant is the so-called D-term. The dispersion relation allows us trading-off the two CFFs as unknowns with one CFF and the D-term. For our purpose we recover the EMT Form Factor d_1 as the first coefficient in the Gegenbauer expansion of the D-term. Here, we will truncate this Gegenbauer expansion to d_1 only.

$$D(\xi, t) = (1 - \xi^2) \left[d_1(t) C_1^{3/2}(\xi) + \cdots \right]$$

Our starting points are the global fits presented in [5, 6], referred to as KM parameterization. The imaginary part of the amplitude is calculated from a parameterization of the GPDs along the diagonal $x = \xi$. The real part of the amplitude is then reconstructed assuming LO dominance and applying the dispersion relation. The ξ dependence of the D-term is completely generated by the Gegenbauer expansion, restricted to the d_1 term only. Finally, the momentum transfer dependence of the d_1 term is given as a functional form, with two parameters $d_1(0)$ and M :

$$d_1^q(t) = d_1^q(0) \left(1 - \frac{t}{M^2}\right)^{-3},$$

where we indicate the quark contribution as the superscript in $d_1^q(t)$. The form of $d_1^q(t)$ is consistent with the asymptotic behavior required by the dimensional counting rules in QCD [15]. We adjust and fix the central values of the model parameters to the CLAS data at 6 GeV [10, 11]. They include unpolarized and beam polarized cross-sections over a wide phase space in the valence region, and support the model indicating that the GPD H largely dominates these observables. Illustration of the fit results on the Beam Spin Asymmetries is provided in figure 2, and to the unpolarized cross sections in figure 3. An illustration of a fit to the $d_1^q(t)$ dependence is provided in figure 4. The data points correspond to the values extracted from the fit to the unpolarized cross section data in figure 3.

The experimental analysis shows that $d_1(0)$ has a negative sign consistent with the theoretical studies. In the chiral quark soliton model calculation of Ref. [9] the negative sign was found as condition for internal stability of the proton, and recently in [13] the negative sign was found as a result of a dispersion relation analysis.

The form factor $d_1^q(t)$ is related to the pressure distribution *via* the spherical Bessel integral :

$$d_1^q(t) \propto \int \mathrm{d}^3\mathbf{r} \, \frac{j_0(r\sqrt{-t})}{2t} \, p(r)$$

The main results of our analysis are illustrated the results in figure 1. The black central line corresponds to pressure p extracted from the D-term parameters fitted to the published CLAS data at 6 GeV [10].The corresponding estimated uncertainties are shown as the shaded area shown in light green. There is a positive core and a negative tail of the pressure distribution as a function of the radial distance from the proton's center with a zero-crossing near 0.6 fm from that center. We also note that confinement is not locally realized but globally, and

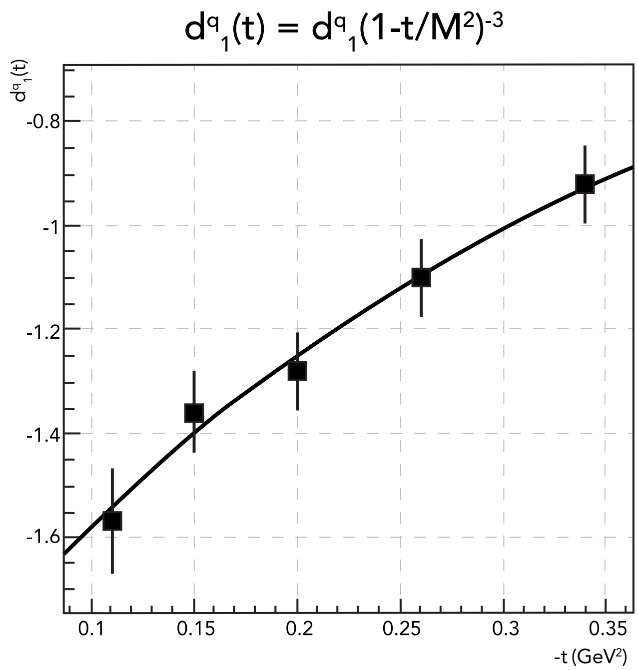

FIG. 4: Example of a fit to $d_1^q(t)$.

the regions where repulsive and compressive (confining) pressures dominate are separated in radial space, with the repulsive pressure peaking near $r = 0.15$ fm, and the maximum of the confining pressure occurring near $r = 0.75$ fm. The overall shape of the radial pressure distribution extracted from these data mimics closely the results obtained within the chiral quark soliton model [9].

The outer shaded area shown in dark green in figure 1 corresponds to the D-term uncertainties obtained in the global fit results from [5, 6]. Qualitatively they exhibit a shape similar to the light green area and confirm the robustness of the analysis procedure to extract the D-term.

Looking to the future, the new generation of precision experiments will have substantially more data available on hard exclusive reactions, both in accuracy as well as channels other than DVCS [14]. This will reduce and control the systematic uncertainties in the GPD extraction procedures from DVCS. We project that the combination of beam energies with the upgraded CLAS apparatus will allow us to map $d_1^q(t)$ in much finer steps and in a much larger $-t$ range. We also expect that this work will spawn new theoretical efforts to understand the fundamental characteristics underlying the stability of the proton from first principles.

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