THE CATHOLIC UNIVERSITY OF AMERICA

Exploring Hadron Structure Through
Exclusive Kaon Electroproduction

A DISSERTATION

Submitted to the Faculty of the
Department of Physics
School of Arts and Sciences
Of The Catholic University of America
In Partial Fulfillment of the Requirements
For the Degree
Doctor of Philosophy

By

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Washington, D.C.

2017

Exploring Hadron Structure Through Exclusive Kaon Electroproduction

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The kaon electroproduction cross section was extracted from data from the E93-018 and the E01-004 (FPI-2) experiments taken at the Thomas Jefferson National Accelerator Facility in the $p(e,e'K^+)\Lambda$ channel. The cross section was fully separated into longitudinal, transverse, and two interference components at four-momentum transfers Q^2 of 1.0 (GeV/c)^2 (with center of mass energy $W=1.81 \text{ GeV}$), 1.36 and 2.07 (GeV/c)^2 ($W=2.31 \text{ GeV}$). The kaon form factor (F_K) was extracted from the longitudinal cross section in these kinematics using the Regge model by Vanderhaeghen, Guidal, and Laget. Results show F_K to be systematically lower than the empirical monopole form, although still compatible with this form within the estimated uncertainties. The resulting kaon form factor values were combined with the world pion and kaon form factor data to extract the transverse charge densities of the pion and kaon. These were compared to that of the proton, showing a possible experimental glimpse of the transition between the proton core and the meson cloud in terms of transverse densities.

This dissertation by Marco Antonio Pannunzio Carmignotto fulfills the dissertation requirement for the doctoral degree in physics approved by Tanja Horn, Ph.D., as Director, and by Daniel I. Sober, Ph.D., Ian L. Pegg, Ph.D., and David Gaskell, Ph.D. as Readers.

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Dedication

To my wife, Adriana Rocha-Lima.

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List of Symbols

α	Fine structure constant.
b	Impact parameter.
B	Magnetic field.
β	Particle velocity in units of the speed of light.
β_{coin}	Kaon velocity (in units of the speed of light) reconstructed from the time difference between the triggers of the HMS and SOS spectrometers.
β_K	Kaon velocity (in units of the speed of light) reconstructed from its TOF in the hadron arm.
cointime	Coincidence time, i.e., time difference between the triggers from the SOS and the HMS.
$\Delta_{model}(b)$	Incompleteness error of the charge density.
δ	Fractional momentum of the particle with respect to the central momentum of spectrometer.
ϵ	Longitudinal polarization of the virtual photon.
e	Electron charge.
E	Energy of the incident electron.
E'	Energy of the scattered electron.
$F(Q^2)$	Form factor.
F_K	Kaon form factor.
F_π	Pion form factor.
$F_\pi^{monopole}$	Pion form factor as described by the monopole model.
F_π^{LF}	Pion form factor as described by the light front model.
Γ_ν	Virtual photon flux.

$hcer_npe$	Number of photo-electrons measured in the Cherenkov detector of the HMS.
H_{int}	Interaction operator between electron and the scattering center.
J	Jacobian of a transformation.
$J_n(x)$	Bessel function of the first kind.
$J_0(x)$	Bessel function of the first kind (with $n=0$).
λ	Absorption length of kaons in a material.
Λ	Lambda baryon, with rest mass 1.116 GeV/c ² .
Λ_K^2	Cutoff parameter of the Regge trajectory for K ⁺ .
$\Lambda_{K^*}^2$	Cutoff parameter of the Regge trajectory for K*.
K ⁺	Kaon (positively charged).
K*	Meson K*, with rest mass 0.892 GeV/c ² ..
$\mathbf{k}$	Three-momentum of the incident electron.
$\mathbf{k}'$	Three-momentum of the scattered electron.
k	Four-momentum of the incident electron, $k \equiv (E, \mathbf{k})$.
k'	Four-momentum of the scattered electron, $k' \equiv (E', \mathbf{k}')$.
m_K	Mass of the kaon.
M	Mass of the nucleus.
M_x	Missing mass.
n	Refractive index (real part) of a material.
Ω^*	Solid angle in the center of mass frame.
w	Energy difference between incoming and scattered electron.
$\mathbf{p}_K$	Three-momentum of the kaon.
$\mathbf{p}_\Lambda$	Three-momentum of the Λ .
p_t	Minimum particle three-momentum required for radiating Cherenkov radiation in a material.
π^+	Pion (positively charged)
ϕ	Azimuthal angle between scattering and reaction planes.
$\Phi(\mathbf{x})$	Potential describing the scattering center.
ψ_i	Wave-function of the incoming electron.

ψ_f	Wave-function of the outgoing electron.
r_K	Root-mean square radius of the kaon.
r_π	Root-mean square radius of the pion.
R	Distance parameter of the finite radius approximation.
ρ_V	Density of the material.
$\rho(b)$	Charge density at the impact parameter b .
ρ_{in}	Charge density within a volume.
σ_A	Kaon absorption cross section in the material.
σ_L	Longitudinal part of the separated cross section.
σ_T	Trasverse part of the separated cross section.
σ_U	Unseparated cross section.
saer_npe	Number of photo-electrons measured in the aerogel detector of the SOS.
sshtrk	Tracked events from the SOS calorimeter.
ssshsum	Sum of the signals from the elements of the SOS calorimeter, in units of p/E.
$\mathbf{q}$	Three-momentum transver by the exchanged photon.
Q^2	Squared four-momentum transfer by the exchanged photon, $Q^2 = -q^2 \equiv -(\mathbf{k}-\mathbf{k}')^2 = 4EE' \sin^2(\theta/2)$.
Q_{max}^2	Maximum value of Q^2 in which the form factor was measured.
τ_K	Mean lifetime of the kaon, 12.4 ns.
θ_{arc}	Arc bend angle at the entrance of Hall C.
θ_e	Scattering angle of the electron.
θ_{pq}	Angle between the direction of the produced kaon and the virtual photon.
t	Four-momentum squared to the nucleon.
v_t	Minimum particle velocity required for radiating Cherenkov radiation in a material.
W	Total energy of the system in the center of mass frame.
$\mathbf{x}$	Three-vector of the spatial coordinate system.
X_n	n^{th} zero of the Bessel function $J_0(x)$.
x'	Out-of-plane angular acceptance of the spectrometer.

Y	Yield (number of events per μC) of the experiment or the simulation.
y'	In-plane angular acceptance of the spectrometer.
Z	Atomic number

Acknowledgments

I feel honored when thinking about how many people contributed in so different ways to make this work possible!

I would like to start by acknowledging my adviser, Tanja Horn, who has supported all of my work in an incredible way. She gave me the opportunity to work on amazing projects, guiding me in exploring paths that led to important accomplishments in my life and in my career. I will miss the times we went to VSL to have coffee (tea for her!), a good excuse to break the academic routine for some minutes and brainstorm about different ideas and projects for our group! Thank you for all, Dr. Horn.

My special thanks to Ian Pegg, Daniel Sober, and Dave Gaskell, the members of my committee. They provided very important feedback regarding my research and my performance, which inspired me to accomplish my PhD.

I also want to acknowledge two amazing researchers who helped me debug some details of the analysis. Thank you John Arrington and Betsy Beise for dusting off all of those 15 year-old files and helping me set up the analysis of this data. Your support was extremely

valuable to me.

I met great people while working at JLab, and I would like to thank them for their support and inspiration during the time I was there. My experience at JLab was fantastic, thanks to all of the JLab staff and the amazing scientific community I was in contact with during these years! My special thanks to Carlos Munoz-Camacho, Alex Camsonne, Julie Roche, and Charles Hyde for letting me participate in the preparation and initial data taking of the DVCS-3 experiment in Hall A. Thank you also to researchers from Armenia, who I worked with while building the aerogel detector: Arthur Mkrtchyan, Arshak Asaturyan, Hamlet Mkrtchyan, Vardan Tadevosyan, Simon Zhamkochyan, and Mariana Khachatryan. I am also grateful to have had the chance to work with Rolf Ent, Pawel Nadel-Turonski, Ibrahim Albayrak, Mark Jones, Steve Wood, Thia Keppel, Paul King, Kijun Park, Jerry Miller, and so many other outstanding people there!

I also had an astonishing experience at Jülich Forschungszentrum and at CERN, and I would like to express here my sincere words of thankfulness to Dieter Grzonka for allowing me (and supporting me) to be part of the P349 experiment.

As a graduate student at CUA, I had the opportunity to interact with nice professors! They taught me inside and outside the classrooms, and I am thankful for all of their support. Thank you Abhijit Sarkar, Biprodas Dutta, Duilia de Mello, Franz Klein, Grzegorz Kalicy, John Philip, Lorenzo Resca, Steve Kraemer, and Tommy Wiklind for being part of my journey at CUA! Thank you also for the staff at the Physics Department, who helped

preparing tons of paperwork to keep me at the graduate program during all these years: Gail Hershey, Patrick Burke, and Sam Boucher. I also had a chance to work with some people from VSL/CUA, and I will take good memories from them: Patrick Gibbons, Spooner Underwood, Miguel Penafiel, Miguel Hung, Marek Brandys, Dave Horton, Cathy Paul, Nicholas Mecholsky, Rich Cecil, and Paul and Michael from the machine shop. Ms. Reyna Rivas, thank you for supporting us all!

I also studied and worked with many students from different groups at CUA! It was great to work with those from the nuclear physics group: Nathaniel, Mike, Laura, Indra, Bijay, Joel, Justin, Johann, Andres, Salina, Rishab, Richard and all of the high school summer interns! Thanks also to those other graduate students who were in the same boat as I was, sharing the rows to move our lives as graduate students forward: Trevor, Keshab, Emmy Soto, Jordan, Heather, Robbie, Ryan, Rafael, Andreza, Jullianna, Rajendra, Bishnu, Bryce, Sandra, Meriem.

I will never forget my friends from the Institute for Technological Research. They were remarkable in my professional and personal life. Thank you Zanutto, Padovezzi, James, Hélio, Ana, Erics, Mário, David, Felipe, Fábio, Daniel, Renato, Bruno, Maria, Adriano, Luis, Luiz, Jorge, Militão, Silas, Nascimento, Maricelia and many others.

I will also never forget the GFAA at the University of São Paulo, especially my former adviser. Thank you Nemilata Added for guiding me on my first steps in scientific research. And thanks also to Manfredo Tabacniks and Márcia Rizzutto for all of their support!

There are not enough words that I could find to express all my gratefulness to my family. Adriana Rocha-Lima, my deepest words of gratitude to you. I would not have reached this milestone without your support. Atilio, Ana, Gabriela, and Thiago... such an amazing family I have! It has been difficult to live far away, but I knew you were always there ready to support me whenever I needed. Thank you!

There are also so many other people who contributed to my work that I do not even know. As an example, the shift crews at both the E93-018 and the FPI-2 experiments that collected the data I analyzed for my PhD. In particular the spokespeople of those experiments, O. Baker and R. Whitney for the E93-018 experiment and Henk Blok, Dave Mack, and Garth Huber for the FPI-2 experiment, that envisioned and worked hard to approve and run these experiments. And an uncountable number of other people behind the scenes at The Catholic University of America and at Jefferson Lab who built and have been maintaining these institutions. I want to thanks all of those whose efforts supported me in a variety of ways so I could develop this research.

Finally, I would like to acknowledge the National Science Foundation (NSF) and the Jefferson Science Associates (JSA) for funding my research.

Chapter 1

Introduction

A brief introduction to the concept of electroproduction reactions involving meson production and the possibility to access the meson form factor from these reactions are explored in this chapter. Special attention is given to the study of the kaon, which is the simplest existing hadron containing strangeness. The tools for assessing the transverse charge density of the pion and the kaon are also presented.

1.1 The building blocks of matter and their interaction

The first ideas suggesting that matter is made up of elementary and discrete constituents come from a long time ago, around 400 B.C. [Rocha, 2006]. Different cultures have records along this philosophical line, but the ancient Greek Democritus is recognized

as the “father of modern science” by developing the first hypothesis of atomism with his mentor, Leucippus. Their basis for formulating the concept of the indivisible unit of matter (“atom”) was philosophical and theological, and it was necessary to wait more than two thousand years for measurements to show the first evidence of the inner structure of matter [Berryman, 2008].

In the 18th century, scientists such as Lavoisier, Proust, and Dalton started to recognize some properties of the chemical elements in combining them in fixed proportions to build up compounds and found that the weights of the primary elements would always be conserved in a chemical reaction [Leicester, 1968]. Later in 1808, Dalton realized that the theory of atomism would greatly explain these observations [Nash, 1956]. Among other propositions, he described the elements as being composed of atoms. These atoms were very small particles equal to one another for atoms of the same element, but different in size, mass, and other properties for different elements. The theory of building blocks of matter had been realized experimentally for the first time, marking the opening of a new era in science.

After one more century of research, experimental evidences suggested that the atoms were not the elementary particles, but that a more fundamental constituent could exist. Henri Becquerel discovered radioactivity, a first glimpse that the atoms could somehow break apart. One year after Becquerel’s discovery, Sir Joseph John Thomson found the electron, an electrically negative particle much lighter than the atom, indicating the existence of subatomic structures. J. J. Thomson also found the first indications of the existence of isotopes, elements with the same atomic number (similar chemical properties) but different

masses. At the time, Dmitri Mendeleev had organized the periodic table according to symmetries observed based in the atoms' properties. J. J. Thomson also worked in the direction that there should be an inner structure of matter, which became known as the first atomic model named the “plum pudding” model.

After a few years, still in the beginning of the 20th century, the team of Ernest Rutherford used the emerging studies of radioactivity to probe the inner structure of the atom. They concluded that the atom should be composed of a dense positively charged nucleus orbited by an electron cloud in order to explain the observation of wide angle scattering of alpha particles off the nuclei, from the experiments of Geiger and Marsden [1913]. That marked the beginning of a new era of Nuclear Physics, just a little more than 100 years ago, when we started to dive deep into studies to understand the building blocks of matter and their interaction. At the same time, the usage of particle accelerators was emerging for studying nuclear physics, a great benefit to the development and progress of the field.

A neutral massive particle, the neutron, was earlier visualized by Rutherford as a necessary ingredient in the nucleus of the atom to explain that its typical weight is twice its atomic number. Walther Bothe, Herbert Becker, Irène Joliot-Curie, and Frédéric Joliot first observed some kind of neutral particle coming out of reactions and knocking out protons from paraffin. That was one of the first experimental hints of the detection of the free neutron. James Chadwick performed a series of experiments to establish the first observation of the neutron. The proton-neutron model of the nuclei was born. Just after that, Werner Heisenberg applied the emerging theory of Quantum Mechanics to propose the nucleus as

a quantum mechanical system. In addition, the symmetries observed between protons and neutrons led Heisenberg to describe these particles as being the same “particle”, but with two quantum states (isospin): the nucleon. At about the same time, Wolfgang Pauli proposed the existence of the neutrino to explain the apparent violation of energy and momentum conservation in beta decays (that was later detected by Clyde Cowan, Frederick Reines, F. Harrison, H. Kruse, and A. McGuire). A new level of knowledge was reached, and once again, the most elementary particles were thought to have been defined. Electrons, protons, neutrons, and neutrinos were the building blocks of matter.

However, how would these nucleons be connected in a dense nucleus at the center of atoms? There should be a force to bind the nucleons, and this force should be stronger than the electromagnetic force that would result in protons repelling each other. Heisenberg is known to be the first to introduce the theory of nuclear exchange forces that bind the nucleons together. A few years after Heisenberg’s publication, Hideki Yukawa proposed the existence of a particle with mass between that of the electron and that of the proton masses, which would be the force carrier for this nuclear force. He called this particle the “meson”, and 10 years later Lattes et al. [1947] published the discovery of the “pi-meson”, or the pion. Once again in history, the so far known building blocks of the existing matter appear to have an inner structure.

The pace for the development of physics from this point on in history increased dramatically. The development of particle accelerators allowed for controlled study of reactions in the laboratory and removed limitations from observations of events created with cosmic

rays on the top of mountains. A few years after the discovery of the pion from cosmic rays, pions were being identified from reactions in the laboratory.

It did not take long for other particles to be observed in experiments using cloud-chambers and accelerators, especially the observations by Rochester and Butler [1947] of a particle that appeared to be neutral and which decayed into two pions, and another charged particle decaying into a pion and something neutral. With their masses measured to be between the pion and the proton masses, these particles had a lifetime (on the order of 0.1 ns) much longer than what was expected for particles with their masses [Wróblewski, 2004]. The neutral counterpart seemed to have two distinct lifetimes [Gell-Mann and Pais, 1955], and the production of all of these particles was always observed to be in pairs. This observation intrigued the scientific community, marking the discovery of the “K-mesons” – or “kaons” (K^+) as they are known today. The production of these particles always in pairs (or in association with heavy particles called hyperons) would be explained by a new conserved quantum mechanical quantity in reactions involving them, *strangeness* [Gell-Mann, 1956].

The kaon has played a very important role in the development of modern physics. It was essential in shining the first light on the existence of a more general symmetry within the family of particles that were being discovered, giving rise to the *quark model* [Gell-Mann, 1956; Zweig, 1964]. These quarks are the most elementary particles, and they are held together by means of the *strong force* to form “hadrons”. The number of quarks in a hadron would redefine the concept of “meson”, with two quarks, and the existence of “baryons”, with three quarks. Years later, studies with kaons were also very important

for the understanding of matter-antimatter asymmetries observed in nature (charge-parity symmetry violation) [Christenson et al., 1964], the development of the *quark mixing matrix* explaining the origin of the broken symmetry for the existence of at least three families of quarks [Cabibbo, 1963; Kobayashi and Maskawa, 1973], and more recent interest of the scientific community to understand the origin of mass studying the pion and the kaon in the context of dynamical chiral symmetry breaking [Hutauruk et al., 2016].

The theories of fundamental forces underlying the interaction of the fundamental particles were also further developed over time. Quantum Electrodynamics (QED), first envisioned by Paul Dirac, would come to fully describe the interaction of charged particles by means of photon exchange, adding important ingredients such as special relativity and quantum mechanics to the classical theory of electromagnetism using field theory [Dirac, 1927]. Quantum Chromodynamics (QCD), the quantum field theory describing the interaction of quarks and gluons by the strong force, would come to connect the symmetries observed in the quark model [Ne’eman, 1961] and form the base for the development of the *Standard Model*, where all known fundamental subatomic particles of electromagnetic, weak, and strong nuclear interactions are classified.

QED has been called “the jewel of physics” [Feynman and Zee, 2012], modeling very accurately observations such as the anomalous magnetic moment of the electron and the Lamb shift of the energy levels of the hydrogen atom. QED has shown consistency in treating problems in a very wide range of the electromagnetic spectrum. On the basis of this precise theory, several experiments have been designed since the last half of the 20th century to use

charged *leptons* (fundamental particles that do not interact by strong force) as probes of the hadronic structure. In particular, experiments using Deep Inelastic Scattering (DIS), where electrons were used to probe deep inside the proton, were essential for the observation of the proton structure and to relate to the quark model [Bloom et al., 1969]. QED has been extremely important to the development of QCD, allowing, for instance, the study of space and momentum distributions of *partons* (quarks and gluons) inside the hadron, modeled by the Generalized Parton Distribution (GPD) [Diehl, 2003].

Measuring and understanding meson structure has been of great interest for understanding the intermediate energy QCD. As mesons are hadrons formed by two quarks only, a lower number of degrees of freedom are in principle required for the description of these systems. This has been subject of intense study at the Thomas Jefferson National Accelerator Facility (JLab), where an electron beam accelerator has been used to probe the hadronic structure with energies of up to 6 GeV prior to 2012, and upgraded to 12 GeV in 2016.

1.2 Assessing the structure of hadrons

The investigation of the structure of the subatomic world using electron scattering has been driven on the basis of the precise theory of QED. Being a point-like particle, electrons are a clean probe of the hadronic structure, since their interaction with subatomic particles is well understood from the first principles of QED. The first measurements realizing that nuclei explicitly appear not to be point-like particles were made more than five decades ago, and since then more studies have investigated the building blocks and the structure of

matter in great depth.

1.2.1 Kinematics and cross sections

Particle scattering at low energies is well described by the Rutherford formula from a simple treatment of the scattering process based on the principles of classical electrodynamics. In this approach, the kinetic energy of the electron is not enough to deeply interact with the constituents of the nucleon, and the recoil of the target nucleus is negligible. In addition, spin effects are not taken into account by this classical calculation. In this scenario, the scattering of an electron with charge e and kinetic energy E off a nucleus (with charge Ze) at an angle θ_e is described by

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{(Ze^2)^2}{(4\pi\epsilon_0)^2(4E)^2 \sin^4(\theta_e/2)}. \quad (1.1)$$

Effects of the electron spin and magnetic moment and the recoil of the nucleus are important in the elastic scattering process at high energy regimes, where relativity plays an important role. These effects are considered with the use of additional factors in Eq. 1.1, and hence the cross section describing the scattering of an electron off a point-like nucleus is given by the Mott cross section as

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \left(1 - \beta^2 \sin^2 \frac{\theta_e}{2}\right) \cdot \frac{E'}{E}, \quad (1.2)$$

where E' is the energy of the scattered electron off the nucleus of mass M and velocity β (in units of speed of light) given by

$$E' = \frac{E}{1 + \frac{E}{Mc^2}(1 - \cos \theta_e)}. \quad (1.3)$$

Corrections for nuclear recoil and the spin of the electron reduce the cross section calculated by the classical Rutherford formalism and, in principle, account for all of the known effects of the elastic scattering of two point-like particles. If the nucleus has no dimension, i.e., it is a point-like particle with no internal structure, the experimental cross section for the scattering of electrons off this object should be directly described by the Mott cross section from Eq. 1.2. However, experiments show that the observed cross sections in elastic scattering of electrons off nuclei are lower than the Mott cross section. Therefore, nuclei and consequently nucleons should have a spatial extent and, as a result, an inner structure. The parametrization of this non-point-like charge distribution can be described by an additional quantity to the scattering process, the *form factor*.

1.2.2 Form factor

The deviation of the measured cross section from the calculated Mott cross-section (considering electron spin and target recoil) was one important hint that subatomic matter has an inner structure. The charge of the nucleons and nuclei are actually distributed in space, observed when the scattering process was found to differ from point-like scattering between the electron and the target nucleus, as shown by Hofstadter and McAllister [1955] and McAllister and Hofstadter [1956] and reproduced here in Fig. 1.1. Even the effects of the spin of the proton considered by the Dirac magnetic moment and the anomalous

contributions calculated by Rosenbluth [1950] would not explain these differences, as shown in this figure. Other independent experiments also corroborated the conclusions about the non-pointlikeness of the nucleus, but it was electron scattering that provided the most precise quantification of its structure.

Fig. 1.1: The first experimental results confirming a structure in the proton, as measured by Hofstadter and McAllister [1955] and McAllister and Hofstadter [1956]. (a) Mott cross section calculated considering a spinless and point-like proton. Effects of the spin in the cross section due to the Dirac magnetic moment are considered in (b), and calculation of both the Dirac magnetic moment and the anomalous contribution in (c) — (b) and (c) are from [Rosenbluth, 1950]. The experimental data is not explained by any of these models, indicating the existence of a structure inside the proton. The estimate of the charge radius of the proton from this work was 0.70 fm, while the accepted value at the date of this thesis is 0.8751 ± 0.0061 fm [Patrignani et al., 2016]. Figure extracted from McAllister and Hofstadter [1956].

In a simplified view, the higher the energy of the virtual photon exchanged between the particles during the scattering process, the higher the resolution due to its reduced wavelength. Therefore, only a fraction of the target charge distribution is seen by the electron with higher virtual photon energy and the scattering off an extended charge distribution is expected to be smaller than that of a point-like particle (after effects due to spin are considered). The ratio of the measured cross section to the Mott cross section defines the magnitude of the “form factor” of the scattering center:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{experimental}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot |F(Q^2)|^2. \quad (1.4)$$

The form factor $F(Q^2)$ encodes the information about the spatial distribution of the object being studied. It can be understood as the diffraction pattern from the scattering of the electron off the target particle, and therefore gives a direct way to precisely study how the object is extended in space.

Studying objects with the use of diffraction, as used here to study hadron structure from measurements of form factors, is one the most accurate tools in modern science for quantifying structures. Diffraction is used in many fields, most often with electromagnetic radiation. As exemplified in Fig. 1.2, from a simple experiment to measure the thickness of a hair [Hu et al., 2010; Rocha-Lima et al., 2004] to the more complex experiments where the DNA structure is unveiled with x-ray diffraction [Watson and Crick, 1953], measurements with diffraction have provided important results to science.

Fig. 1.2: Examples of experiments using diffraction of electromagnetic radiation (visible light and x-ray) for probing the structure of objects, such as a simple hair or a complex double helix pattern of the DNA. In (a), an experimental setup designed for measuring the dimension of a human hair using light diffraction by shining visible light on it, producing the diffraction pattern shown in (b). The structure of a DNA schematized in (c) was first measured by diffraction of x-rays, producing the diffraction pattern shown in (d).

1.3 Meson form factor

The structure of mesons can also be measured with electron scattering. The main difficulty in probing mesons is that these particles are very unstable, and forming a fixed target with them is not possible.

Experiments in the late 70's and early 80's showed, using elastic scattering of charged pions and kaons off atomic electrons, that mesons are also not point-like particles [Adylov et al., 1977; Amendolia et al., 1984, 1986; Dally et al., 1981]. The pion and kaon elastic electromagnetic structure is parametrized by a single charge form factor $F(Q^2)$. It describes its non-point-like structure and therefore the spatial distribution of its charged constituents. The measurement of light-meson form factors has recently received much interest in the scientific community to study properties of the hadronic structure, such as understanding the origin of mass. For example, Hutaurok et al. [2016] discuss the role of light quarks in the pion and the kaon mass in the context of dynamical chiral symmetry breaking, where QCD binding energy forms the hadron.

Measurements of the pion and kaon form factors from elastic scattering at very low values of Q^2 provided the first glimpses of the spatial distribution of these particles. However, kinematic limitations of these experiments such as the cross sections reduction at steep rates ($1/Q^4$) and very small scattering angles would not allow access to the form factor of these mesons at higher values of Q^2 using the same technique. With new ideas emerging in the 90's, it was possible to extend these studies and access the mesons form factors by probing

them inside the nucleon. Meson electroproduction data at forward kinematics is now used for the determination of the form factor [Blok et al., 2008; Huber et al., 2008]. This is done by measuring the separated cross sections, in which the longitudinal cross section contains the pion exchange process. More details about these results and the method of probing the meson inside the nucleon using electroproduction data is given in the next sections.

1.3.1 The pion

The pion form factor F_π is well determined up to values of Q^2 of 0.28 GeV^2 by elastic $\pi - e$ scattering [Adylov et al., 1977; Amendolia et al., 1984, 1986; Dally et al., 1981], from which the mean charge radius of the pion was extracted. F_π at larger values of Q^2 was obtained with the use of pion electroproduction data, where Blok et al. [2008] and Huber et al. [2008] present a comprehensive review on the extraction of F_π from pion electroproduction data.

The pion form factor from pion electroproduction data was obtained for values of Q^2 of 0.18 to $9.8 (\text{GeV}/c)^2$ at the Cambridge Electron Accelerator (CEA), at Cornell [Bebek et al., 1976, 1978] and at the Deutsches Elektronen-Synchrotron (DESY) [Ackermann et al., 1978; Brauel et al., 1976, 1977, 1979]. However, most of the high Q^2 form factor data from these measurements were extracted with assumptions on the cross sections, that led to important model-dependent uncertainties on the resulting form factor. Pion electroproduction data were also obtained at DESY [Ackermann et al., 1978; Brauel et al., 1976, 1977] for values of Q^2 of 0.35 and 0.7 GeV^2 . Finally, the pion form factor was also extracted at Jefferson Laboratory with high precision in the regime of $Q^2 = 0.6\text{-}2.45 (\text{GeV}/c)^2$ Horn et al. [2006b,

2008]; Horn [2012]; Tadevosyan et al. [2007]; Volmer et al. [2001].

Different empirical fits and model calculations have emerged in the last decades to describe the pion form factor. Huber et al. [2008] summarize some of these models, such as those based on Perturbative Quantum Chromodynamics (pQCD), lattice QCD, dispersion relations with QCD constraint, QCD sum rules, the Bethe-Salpeter equation, local quark-hadron duality, the constituent quark model, and holographic QCD. Masjuan et al. [2008] also describe a method using rational approximants for calculating F_π . A new method has recently been developed to calculate F_π on the entire region of Q^2 using the Dyson-Schwinger equation framework [Chang et al., 2013]. The results are in very good agreement with the world F_π data. The low- Q^2 behavior is well described by the monopole form

$$F_\pi^{monopole}(Q^2) = \frac{1}{(1 + Q^2 r_\pi^2/6)}, \quad (1.5)$$

where r_π denotes the RMS radius of the pion, obtained from the slope of F_π at $Q^2 \rightarrow 0$. The global fit for r_π is given by Patrignani et al. [2016], where $r_\pi = 0.672 \pm 0.008$ fm.

Figure 1.3 shows the world data for the pion form factor together with the monopole and a Light front model (LF) calculation based on that described by Hwang [2001]. Both the monopole and the LF models are in very good agreement with the data up to values of $Q^2 \approx 2.5 \text{ GeV}^2$. Above that, $Q^2 F_\pi^{monopole}(Q^2)$ and $Q^2 F_\pi^{LF}(Q^2)$ deviate from each other with very distinct asymptotic behaviors. $Q^2 F_\pi^{monopole}(Q^2)$ tends to a constant value while $Q^2 F_\pi^{LF}(Q^2)$ decreases as $Q^2 \rightarrow \infty$. All other models of the pion form factor fall between these two curves. No distinction can be made between the models based on the current

data due to their large uncertainties in particular at values of Q^2 between 3 and 10 GeV^2 .

Fig. 1.3: Global data of the pion form factor from pion-electron scattering and pion electroproduction on the nucleon. The error bars represent the statistical and systematic uncertainties of the individual measurements. The monopole (blue line) and the Light-Front model (dashed-gray line) both describe the existing F_π data well. These models are constrained by the existing data, but differ in their asymptotic behavior.

1.3.2 The kaon

The kaon form factor was first measured through the scattering of high energetic kaons off atomic electrons in fixed target experiments at Fermilab [Dally et al., 1980] and at CERN [Amendolia et al., 1986], as shown in Fig. 1.4. In these experiments, F_K was measured directly from the deviation of the measured cross section to the Mott cross section (point-like charge assumed). These experiments provided F_K in a very low Q^2 regime, ranging between 0.02 and 0.12 GeV^2 , and they allowed the extraction of RMS radius of the kaon, $r_K = 0.560 \pm 0.031$ fm as of today [Patrignani et al., 2016].

Fig. 1.4: Low- Q^2 F_K data from elastic experiments, measured by elastic scattering of high energetic kaons off electron in a fixed target [Amendolia et al., 1986; Dally et al., 1980].

As with the pion form factor, the extraction of F_K to higher Q^2 values through elastic scattering is very challenging experimentally, perhaps even unfeasible with the existing tech-

nologies. The limitations in this technique arise mostly from the availability of accelerators to produce high-energy and high-current beams of kaons (unstable particles) and detectors for identifying and measuring the scattered particles at very forward angles. Furthermore, the cross section of elastic scattering decreases rapidly (proportional to $1/Q^4$) and, therefore, extending the measurements of F_K to higher Q^2 with reasonable statistics would require experimental luminosity higher than that currently achievable in this type of experiment.

In this work, the kaon form factor is extracted from electroproduction cross section data, where the kaon is probed in the nucleon meson cloud. The longitudinal part of the cross section from meson electroproduction (σ_L) encodes the meson exchange process, in which the virtual photon couples to a virtual meson inside the nucleon. This process is expected to dominate at small values of the four-momentum transfer $-t$, allowing for the determination of the kaon form factor using a technique similar to that used for the pion. More details about this technique follow in the next sections.

1.4 Meson form factor and electroproduction experiments

To extract the meson form factor from electroproduction data, a target of protons (liquid hydrogen) is used. The treatment used is based on that by Horn [2006a], that has been successfully used for the measurement of the pion. Electroproduction of kaons is highlighted here instead, and a discussion of the differences for measuring the pion and the kaon follows with the formalism.

To study kaon electroproduction, an electron beam is incident on a proton target

and the outgoing particles of the reaction $p(e,e'K^+)\Lambda$ are studied. The reaction can be summarized by the diagram of Fig. 1.5, where the incoming and outgoing electron three-momenta are $\mathbf{k}$ and $\mathbf{k}'$ respectively. The electron scattering angle between $\mathbf{k}$ and $\mathbf{k}'$ is θ_e , and these two vectors define the *scattering plane*. The four-momentum of the virtual photon exchanged with the nucleon is given by $q \equiv (\omega, \mathbf{q}) = (k - k')$, where ω is the energy difference between the incoming and outgoing electron ($\omega = E - E'$) and $\mathbf{q}$ is the three-momentum transfer. Here, the invariant quantity $q^2 = \omega^2 - |\mathbf{q}|^2$ is always negative as in any space-like electron scattering. For convenience, the quantity $Q^2 \equiv -q^2 > 0$ is defined. Finally, the kaon is produced with three-momentum $\mathbf{p}_K$ emitted at an angle θ_{pq} relative to the direction of the virtual photon (γ_v), while the remaining baryon (Λ) has its momentum $\mathbf{p}_\Lambda$. The *reaction plane* is then defined as the plane that contains both $\mathbf{p}_K$ and $\mathbf{p}_\Lambda$, and the angle between scattering and reaction planes is defined by the angle ϕ .

The kinematics of the electroproduction reaction are fully described in terms of the Lorentz-invariant quantities Q^2 , W , and t . Q^2 is the negative invariant mass of the exchanged virtual photon, while W represents the total energy of the system in the photon-nucleon center of mass frame, and t is the four-momentum transfer to the nucleon squared. In addition, as the remaining baryon (Λ) is not detected in the experiment due to its very short lifetime, so the missing mass M_x of the recoil system is defined by conservation of energy and momentum in the reaction. All of these quantities are obtained in the experiment by measuring angles and momenta of the particles. To summarize, these invariant quantities can be expressed as

Fig. 1.5: Scattering and reaction planes diagram of the kaon electroproduction reaction. Figure adapted from Mohring [1999].

$$\begin{aligned}
 W &= \sqrt{M^2 + 2M\omega - Q^2}, \\
 t &= (q - p_K)^2 = (E_K - \nu)^2 - |\mathbf{p}_K|^2 - |\mathbf{q}|^2 + 2|\mathbf{p}_K||\mathbf{q}| \cos \theta_{pq}, \\
 M_x^2 &= (E_e - E_{e'} - E_K)^2 - |\mathbf{q} - \mathbf{p}_K|^2.
 \end{aligned} \tag{1.6}$$

In Eq. 1.6, M is the mass of the target (proton). From Eq. 1.6, the minimum value of $|t|$ can be achieved when the kaon is produced at parallel kinematics, e.g., it is produced in the direction of the virtual photon. For the reaction studied here, t is always negative and henceforth it will denote the value of $-t$.

The electroproduction cross section can be defined using the plane wave Born approximation in terms of the invariant variables from Eq. 1.6. In this approximation, the incident

and scattered electrons are considered Dirac plane waves and only a single virtual photon is exchanged. The differential cross section for this process can then be expressed as

$$\frac{d^5\sigma}{d\Omega_e dE'_e d\Omega_K} = \Gamma_\nu \frac{d^2\sigma}{dt \cdot d\phi} \cdot J(t, \phi \rightarrow \Omega_K^*), \quad (1.7)$$

where Γ_ν is a virtual photon flux factor, $\frac{d^2\sigma}{dt \cdot d\phi}$ is the virtual photon production differential cross section, and J is a Jacobian used to transform the production cross section in terms of t and ϕ . The virtual photon flux can be described as

$$\Gamma_\nu = \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{1}{Q^2} \frac{1}{1-\epsilon} K, \quad (1.8)$$

where α is the fine structure constant, K is the equivalent real photon energy required to excite a target of mass M to an excited state of mass W ($K = (W^2 - M^2)/(2M)$), and ϵ is the longitudinal polarization of the virtual photon

$$\epsilon = \frac{1}{1 + 2\frac{|\mathbf{q}|^2}{Q^2} \tan^2(\theta_e/2)}. \quad (1.9)$$

The information about the hadronic system is encoded in the virtual photon cross section. This cross section is expressed in terms of four unpolarized response functions, directly connected to the components of the hadronic current, as

$$2\pi \frac{d^2\sigma}{dt d\phi} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi, \quad (1.10)$$

where L and T correspond to the polarization states of the virtual photon (longitudinal and transverse polarization, respectively), while LT and TT are the interference terms. The functional form of each of these separated cross sections can be expressed in terms of Q^2 , W , and t only.

By measuring the separated cross sections from Eq. 1.10 for a given kinematic point (represented by Q^2 , W , and t) integrated over the azimuthal angle ϕ , the integrated cross section differential in t can be written

$$\frac{d\sigma}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}. \quad (1.11)$$

The L/T/LT/TT separation of the cross section can be done using the Rosenbluth method. This method consists of measuring the cross section at different values of ϵ , while keeping Q^2 , W , and t constant. These *unseparated* cross sections are then fitted as a function of ϵ and, as seen in Eq. 1.11, the longitudinal (σ_L) and the transverse (σ_T) cross sections can be obtained from the slope and the intercept of the fitted curve. If the experiment measures the unseparated cross section (σ_U) at two values of ϵ , e.g. $\sigma_U(\epsilon_H)$ and $\sigma_U(\epsilon_L)$ at “high” (ϵ_H) and “low” (ϵ_L) respectively, the separated cross sections from Eq. 1.11 can be also expressed as

$$\begin{aligned} \sigma_L &= \frac{\sigma_U(\epsilon_H) - \sigma_U(\epsilon_L)}{\epsilon_H - \epsilon_L} \\ \sigma_T &= \frac{\epsilon_H \sigma_U(\epsilon_L) - \epsilon_L \sigma_U(\epsilon_H)}{\epsilon_H - \epsilon_L} \end{aligned} \quad (1.12)$$

In addition to the measurements of form factors, the separated cross sections from

exclusive reactions play an important role in the interaction of elementary particles and their dynamics on the basis of QCD. At sufficiently high values of Q^2 , hard and soft physics are expected to factorize opening an opportunity for probing quarks and gluons through the GPDs. These are connected to nucleon structure, which is being probed by experiments at 12 GeV Jefferson Lab. If GPDs can be extracted from meson electroproduction, these studies would give information on the flavor structure of hadrons.

A large program for probing the transition from hadronic to partonic degrees of freedom on the basis of QCD and the onset of hard-soft factorization has been performed during the 6 GeV era of JLab. The results from this era have brought the highest precision pion form factor data to date and a first glimpse at experimental signatures for hard-soft factorization necessary to access GPDs. Compared to data for pion electroproduction, kaon electroproduction data are scarce, especially at higher Q^2 . This is due to the lack of facilities providing sufficiently high energies and detection capability at very forward angles. The kaon is the simplest system available for studies offering a way to probe the production mechanism involving strangeness physics. Recent theoretical progress has resulted in renewed interest in meson form factor measurements and meson cross section data over a wide kinematic range for hard-soft factorization tests. These topics are part of the future of nuclear physics as detailed in the 2014 QCD and Hadron Physics White Paper and the 2015 U.S. Nuclear Science Advisory Committee (NSAC) Long Range Plan.

1.4.1 Meson electroproduction models

As described by [Horn, 2006a], the meson form factors such as that of the pion or kaon can be studied from electroproduction data. Studying the meson form factor requires the study of the longitudinal meson cross section at small values of t , in particular with t at the physical meson mass pole ($t = m_K$ for the kaon). In the actual measurement, one cannot measure at the kaon pole and so the lowest possible values of t are used.

The electroproduction process can be described in terms of the contributing covariant Born term diagram, using the first order Born approximation. The t-channel pole process, represented for the case of kaons by the diagrams in Fig. 1.6 (a), corresponds to the knockout of a virtual meson by a virtual photon. The t-channel pole is therefore of interest for studying the electromagnetic structure of the mesons. However, other channels may also compete with the meson pole, such as those shown in Fig. 1.6 (b) and (c). These are the nucleon pole and the crossed nucleon pole respectively. In the simplest Born term model, (b) and (c) are suppressed in forward kinematics and the longitudinal cross section (σ_L) encodes the Born exchange amplitude for values of W above the resonance region.

(a) t-channel mechanism

(b) u-channel mechanism

(c) s-channel mechanism

Fig. 1.6: Feynman diagrams for kaon electroproduction mechanisms.

The Born exchange process is related to the product of the probability of a virtual meson to interact with a virtual photon of a particular momentum and the probability to encounter a virtual meson with that momentum. These correspond to the meson form factor $F(Q^2)$ and the vertex coupling $g(t)$ respectively, so that in the pole approximation, the form factor of the meson with mass m is expressed as

$$\sigma_L \approx \frac{-tQ^2}{(t - m_K^2)^2} g_{KNN}^2(t) F_K^2(Q^2, t). \quad (1.13)$$

1.4.2 VGL Regge model

A more detailed approach to describe meson electroproduction reactions is given in terms of the Regge theory, whose fundamental aspects are reviewed by Eden [1971]. In this formalism, electroproduction is treated in terms of a family of particles by replacing the pole-like Feynman propagator of the Born term, $1/(t - m^2)$, with Regge propagators for particles with the same quantum numbers. Vanderhaeghen, Guidal, and Laget [Guidal et al., 1997; Vanderhaeghen et al., 1998] developed the VGL model in the context of the Regge theory to describe pion and kaon photoproduction data. There is no free parameter in the photoproduction model, since the coupling constants at the vertices are well constrained by studies of the resonance region. They use π and ρ Regge trajectories to describe pion production reactions, which give a good agreement with pion photoproduction data at a wide range of momentum transfer (up to $-t \approx 2 \text{ (GeV/c)}^2$). For kaon production, the K^+ and K^* Regge trajectories are considered. The VGL model formalism was also extended

to describe electroproduction data, treating both the pion and the kaon electroproduction reactions.

In the case of the kaon, the extension to electroproduction is done by introducing a parametrization of the K^+ form factor following the monopole form factor

$$F_K(Q^2) = (1 + Q^2/\Lambda_K^2)^{-1}, \quad (1.14)$$

where Λ_K^2 is a cutoff parameter of the Regge trajectory for the K^+ , taken as input to the model. Another monopole-like form factor is used to parametrize the K^* trajectory with another cutoff parameter $\Lambda_{K^*}^2$, although the model is less sensitive to this K^* trajectory. As the cutoff parameter for K^* is unknown, it was considered to be equal to Λ_K^2 , and a study of the dependence of the results to changes in Λ_K^2 was considered for the evaluation of the uncertainties on the extracted form factor.

The VGL Regge model used in this work is therefore dependent on a single input parameter only, the cutoff parameter Λ_K^2 . The model was evaluated for different values of Λ_K^2 , and the best value was taken by minimizing the differences between the model longitudinal cross section to that from the experiment. Once the best value of Λ_K^2 is found, the kaon form factor is calculated using Eq. 1.14.

It is important to note that the VGL Regge model has been shown to be a successful model in describing pion electroproduction data. Tests of the pole dominance for pions show that the t-channel pole is dominant over a wide range in t from the data by Volmer [2000]. This allows for the extraction of the pion form factor at Q^2 of up to 2.5 GeV^2 with

high precision [Horn, 2006a]. There, F_π is extracted by careful studies of σ_L at low t with the VGL Regge model. Extraction at different distances from the pion pole shows moderate model dependence. Existing and planned kaon data are at similar distances from the kaon pole, so in principle it is possible to extract F_K .

Data from two experiments were analyzed in this work, the E93-018 and the E01-004 (FPI-2). Details of these experiments are discussed in the next sections. Table 1.1 summarizes the kinematics of the analyzed data.

Tab. 1.1: Kinematics of the data analyzed in this work for the extraction of the separated kaon cross sections.

Experiment	Q^2 GeV ²	W GeV
E93-018	1.00	1.81
FPI-2	1.360	2.31
FPI-2	2.067	2.31

1.5 The E93-018 experiment

The E93-018 experiment was designed to measure the L/T separated kaon electroproduction cross section in both the Λ and Σ^0 channels, aiming primarily to give information of the reaction mechanism based on the behavior of the structure functions.

Four Q^2 points were measured during this experiment: 0.5, 0.75, 1.0, and 2.0 GeV². At each Q^2 , the cross section was measured at three values of the virtual photon polarization, allowing for the extraction of the L/T separated cross sections at the invariant mass $W \approx 1.84$ GeV.

A first analysis of the kaon electroproduction data was presented by Niculescu [1998a],

where the separated cross sections were analyzed. Using these cross sections, Niculescu [1998b] determined the kaon form using the Chew-Low extrapolation method [Chew and Low, 1959]. In this method, the cross section σ_L is extrapolated to the mass pole by fitting different polynomials to the data. Mohring [1999] also analyzed the cross sections of the E93-018 experiment and found differences mostly from some simplifications done in the previous analysis. Mohring et al. [2003] published updated cross section results of E93-018 based on the reanalysis of the data.

Having a reasonably low t (0.4 GeV^2), the kinematics of $Q^2 = 1.0 \text{ GeV}^2$ was reanalyzed in this work with the goal of providing the t -dependence. The extracted cross section was compared to the world data and also allowed for extraction of the kaon form factor.

1.6 The FPI-2 experiment

The primary goal of the Experiment E01-004 from the Thomas Jefferson National Accelerator Facility (FPI-2) [Mack et al., 2000] was to measure the separated cross section of pion electroproduction from hydrogen and deuterium, and extract the pion form factor at Q^2 of 1.6 and 2.5 GeV^2 and $W = 2.22 \text{ GeV}$ [Horn, 2006a; Horn et al., 2006b]. The main outcomes of the FPI-2 experiment regarding pion electroproduction are discussed in detail by Horn [2006a] and Huber et al. [2008].

Events from kaon electroproduction reaction were also detected during this experiment. This gives a great opportunity to further analyze the FPI-2 data and extract information on the kaon production mechanism at high values of Q^2 . A description of the

instrumentation used in this experiment and details of the data analysis for the extraction of the kaon electroproduction cross section follows in the next chapters of this work.

1.7 Charge distribution

A simplified way to describe how electrons scatter off an extended nucleus and the connection between the concepts of form factor and charge distribution is with the use of Fermi's golden rule, as described by Povh et al. [2008]. Considering a negligible recoil of the target, the wave functions of the incoming and scattered electron can be written as plane waves as a function of the incoming and outgoing electrons three-momenta, $\mathbf{k}$ and $\mathbf{k}'$ respectively. In this case, the incoming plane wave, $\psi_i = (1/\sqrt{V})e^{i\mathbf{k}\mathbf{x}/\hbar}$, and the outgoing plane wave, $\psi_f = (1/\sqrt{V})e^{i\mathbf{k}'\mathbf{x}/\hbar}$, would describe the scattering, where V is a finite volume considered large enough for the purpose of the normalization of the wave functions.

The interaction operator of an electron with charge e in a potential $\Phi(\mathbf{x})$ describing the scattering center is given by $H_{int} = e\Phi(\mathbf{x})$. The matrix element of the scattered process in this potential is therefore

$$\langle \psi_f | H_{int} | \psi_i \rangle = \frac{e}{V} \int e^{-i\mathbf{k}'\mathbf{x}/\hbar} \Phi(\mathbf{x}) e^{i\mathbf{k}\mathbf{x}/\hbar} d^3x. \quad (1.15)$$

By defining the three-momentum transfer $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, this matrix element can be written as

$$\langle \psi_f | H_{int} | \psi_i \rangle = \frac{e}{V} \int \Phi(\mathbf{x}) e^{i\mathbf{q}\mathbf{x}/\hbar} d^3x. \quad (1.16)$$

Considering that the potential Φ falls off at large distances, the use of the identities of Green's theorem for two arbitrary scalar fields u and v is possible¹. With the use of this theorem and considering that $\nabla^2 e^{i\mathbf{q}\mathbf{x}/\hbar} = -(|\mathbf{q}|^2/\hbar^2)e^{i\mathbf{q}\mathbf{x}/\hbar}$, the matrix element from Eq. 1.16 can be re-written as

$$\langle \psi_f | H_{int} | \psi_i \rangle = -\frac{e\hbar^2}{V|\mathbf{q}|^2} \int \nabla^2 \Phi(\mathbf{x}) e^{i\mathbf{q}\mathbf{x}/\hbar} d^3x. \quad (1.17)$$

Using Poisson's equation, $\nabla^2 \Phi(\mathbf{x}) = -\rho_{in}(\mathbf{x})/\epsilon_0$, the matrix element for the scattering process in Eq. 1.17 turns to be

$$\langle \psi_f | H_{int} | \psi_i \rangle = \frac{e\hbar^2}{\epsilon_0 V |\mathbf{q}|^2} \int \rho_{in}(\mathbf{x}) e^{i\mathbf{q}\mathbf{x}/\hbar} d^3x. \quad (1.18)$$

Finally, by defining a normalized charge distribution function $\rho(\mathbf{x}) = \rho_{in}(\mathbf{x})/(Ze)$, with Ze being the total charge of the target nucleus (therefore satisfying the normalization condition $\int \rho(\mathbf{x}) d^3x = 1$), the matrix element of the scattering is given by

$$\langle \psi_f | H_{int} | \psi_i \rangle = \frac{Z \cdot 4\pi\alpha\hbar^3 c}{V|\mathbf{q}|^2} \int \rho(\mathbf{x}) e^{i\mathbf{q}\mathbf{x}/\hbar} d^3x. \quad (1.19)$$

The integral $F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\mathbf{x}/\hbar} d^3x$ is defined as the *form factor* of the charge distribution. It is the Fourier transform of the charge distribution, normalized to the total charge of the target particle. It is often more convenient to express the form factor as a function of Lorentz invariant variables, such as Q^2 , and the form factor is then represented

¹ The second Green's identity for two scalar fields u and v that falls off fast enough at large distances can be written as $\int (u\nabla^2 v - v\nabla^2 u) d^3x = 0$. In this case, u and v play the role of $\Phi(\mathbf{x})$ and $e^{i\mathbf{q}\mathbf{x}/\hbar}$ in Eq. 1.16.

by $F(Q^2)$.

1.7.1 Charge distribution - relativistic treatment

The concept of charge distribution defined in Sec. 1.7 can only be applied in a non-relativistic approximation. As explored by Miller [2010], the three-dimensional Fourier transform of the charge density is not a rigorous definition of the form factor for systems in which the constituents are moving relativistically. Miller [2010] revisited the formalism of relativistic electron scattering presented here and defined the concept of *transverse charge density*, that is the charge distribution as seen in a reference frame moving with infinite momentum. This new quantity is therefore invariant under Lorentz transformation, and it is directly connected to the hadron form factor by a two-dimensional Fourier transform with respect to the impact parameter b :

$$\rho(b) = \frac{1}{(2\pi)^2} \int F(Q^2) e^{-i\mathbf{q}\mathbf{b}} d^2q. \quad (1.20)$$

In fact, the description of electron scattering off protons (spin 1/2) requires the consideration of an additional magnetic form factor due to the proton's magnetic moment. In contrast, the pseudoscalar mesons (K^+ and π in this work) are spin zero particles, so the electromagnetic structure is parameterized by a single charge form factor. For this reason, the discussion of magnetic form factors will not be extended here.

Examples of the relationship between some charge distributions and their respective nonrelativistic form factors are shown in Fig. 1.7. These are pictorial examples of charge

distributions, but they represent existing particles in nature. For instance, the electron is a pointlike particle and therefore its form factor is a constant as a function of Q^2 . The proton has been measured to have a dipole-like form factor, and consequently its charge distribution is described by an exponential curve. Most of the heavier nuclei were measured to have an oscillating form factor, corresponding to a spherical shape generally with a diffuse surface.

Fig. 1.7: Schematic representation of form factors and the respective charge distribution of particles. Some of these examples are found in nature, as listed in the last column. Figure extracted from Povh et al. [2008].

The extraction of form factors has shown to be a powerful tool for assessing some

of the basic structures of matter. It was in the context of this formalism that Hofstadter [1956] probed the form factor of nuclei by elastic scattering over a wide range of Q^2 . The corresponding charge density of these particles revealed important information about the nuclear structure of matter. This work was recognized with a Nobel Prize in Physics. As an example of his results, the oscillating cross section of two isotopes of calcium is shown in Fig. 1.8.

Fig. 1.8: Example of the scattering cross section in function of the scattering angle (related to Q^2). The oscillatory scattering pattern observed in this scattering is evidence for a spherical shape of the measured nuclei. Figure extracted from Povh et al. [2008], originally published by Hofstadter [1957].

With these extensive measurements of form factors, the charge distribution of a variety of nuclei has been obtained. Examples of a few of them are illustrated in Fig. 1.9.

Fig. 1.9: Retrieval of the charge density of several nuclei from the measurement of their form factor. Figure extracted from Povh et al. [2008].

The charge densities shown in Fig. 1.9 are mostly modeled by a Fermi distribution, $\rho(r) = \rho(0)/(1 + e^{(r-c)/a})$. This is a model with approximately a constant charge density in the inner core of the nucleus with a diffuse surface.

1.7.2 Meson charge distribution

The extraction of the form factor of light mesons (such as pion and kaon) can, in principle, be done using a formalism similar to that mentioned for nuclei. However, the measurement of the form factor does not extend to values of Q^2 as high as those from measurements with nuclei, as shown in Fig. 1.3 and Fig. 1.4. The method for calcula-

ting the pion and kaon transverse charge densities is therefore modified, based on that by Venkat et al. [2011]. The transverse charge density $\rho(b)$ is the matrix element of the light-front density operator integrated over longitudinal distance [Miller, 2010] and is given by the two-dimensional Fourier transform of the space-like form factor $F(Q^2)$ as shown in Eq. 1.20. The transverse density $\rho(b)$ denotes the probability that a charge is located at a transverse distance b from the transverse center of momentum, following the normalization condition $\int d^2b \rho(b) = 1$. If azimuthal symmetry of $\rho(b)$ is considered, Eq. 1.20 reduces to a one-dimensional integral

$$\rho(b) = \frac{1}{2\pi} \int_0^\infty Q dQ J_0(Qb) F(Q^2). \quad (1.21)$$

Intuitively it is expected the charge of the meson to be localized within a finite volume of radius R . This assumption is called the *finite radius approximation* [Venkat et al., 2011] and it is used to simplify Eq. 1.21. For values of b less than the chosen distance parameter R , the function $\rho(b)$ can be expanded in a series of the Bessel function J_0 as

$$\rho(b) = \sum_{n=1}^{\infty} c_n J_0\left(X_n \frac{b}{R}\right), \quad (1.22)$$

where X_n is the n -th zero of J_0 and c_n , as obtained from the orthogonality of the Bessel functions over the range $0 \leq b \leq R$ is given by the expression

$$c_n = \frac{1}{2\pi} \frac{2}{R^2 (J_1(X_n))^2} F(Q_n^2), \quad (1.23)$$

with Q_n defined as

$$Q_n \equiv \frac{X_n}{R}. \quad (1.24)$$

Combining Eqs. 1.22 and 1.23 yields the following expression for $\rho(b)$:

$$\rho(b) = \frac{1}{\pi R^2} \sum_{n=1}^{\infty} F(Q_n^2) \frac{J_0(X_n \frac{b}{R})}{(J_1(X_n))^2}. \quad (1.25)$$

This expansion provides the transverse density for values of $b < R$ for measurements of the form factor up to momentum transfers of Q_{max}^2 . This formalism is used in Chapter 6 for the extraction of the pion and the kaon transverse charge densities.

Uncertainties due to lack of knowledge of form factor for values of $Q^2 > Q_{max}^2$, where no measurements exist, must be estimated [Venkat et al., 2011]. The first step is understanding the necessary truncation. Low Q^2 form factor data are readily available and as a result the value of the transverse charge density is well defined and its uncertainty is small for large impact parameters b . For values of Q^2 above Q_{max}^2 , which corresponds to shorter distances, the effects of this truncation in the calculation is estimated in the *incompleteness error*. Models describing the high Q^2 behavior of the form factor are used to estimate the incompleteness error, as denoted by $F^{model}(Q_n^2)$. The incompleteness error for the chosen models are estimated using

$$\Delta_{model}(b) = \left| \frac{1}{\pi R^2} \sum_{n=n_{max}+1}^{\infty} F^{model}(Q_n^2) \frac{J_0(X_n \frac{b}{R})}{(J_1(X_n))^2} \right|, \quad (1.26)$$

as a function of b , where n_{max} is the last term of the transverse charge density series where $F(Q_{max}^2)$ has been measured, i.e., the $(n_{max})^{th}$ term of the series is evaluated just below the last currently available data point of the form factor. To apply this method, the use of models to describe F_K at higher Q^2 ranges is necessary and a selection of models describing the kaon form factor follows:

- He et al. [2005] employ a simple representation of the mass operator with single-quark currents to calculate the kaon form factor using relativistic kinematics. It is interesting to highlight the $1/Q^4$ fall-off of their results for high- Q^2 ; however, the RMS radius of the kaon was found to be 0.632 fm with this model, 13% above the global value compiled by Patrignani et al. [2016].
- Using the quark Dyson-Schwinger equation, the kaon form factor was calculated by Maris and Tandy [2000] in the impulse approximation with the ladder Bether-Salpeter equation to describe the meson amplitude. Up to first order in their expansion of the amplitudes, their F_K curve describes the low Q^2 data and the Root Mean Square (RMS) radius of the kaon reasonably well. In addition, their model is interesting because it follows closely the monopole form for up to $Q^2 = 3.0 \text{ GeV}^2$, and it converges to the pQCD power-law behavior for high- Q^2 .
- The kaon form factor is assessed by da Silva et al. [2012] within the framework of light-front field theory, where the quark masses and the model's regulator mass are adjusted to best fit the weak decay constants and the radii of the pion and the kaon.

- At sufficiently large Q^2 , perturbative QCD can be used to simplify the fundamental equations of interaction between the hadron constituents, allowing for numerical evaluation of the system properties. In this regime, the elastic form factor of the kaon can be calculated directly from the Parton Distribution Amplitude (PDA) of its valence quarks. Horn and Roberts [2016] evaluate F_K in the pQCD regime using PDAs from Dyson-Schwinger equations (DSE) and lattice QCD analyses.
- The monopole form factor is a result of the Vector Meson Dominance (VMD) model. Although VMD has shown to fail in describing different features of the hadronic structure, VMD has still been often used to represent the form factor of mesons nowadays. As described by O’Connell et al. [1995], VMD assumes the dominance role of the exchanged photon with the hadronic structure to formulate the Lagrangian of the photon-hadron coupling, and it has shown to describe relatively well the available form factor data of the mesons in both the time- and space-like domains. An empirical monopole form factor is often used in literature, where the cutoff parameter (Eq. 1.14) of the monopole is taken from the fit of the low- Q^2 data of the meson.

From the models listed above, the monopole and the perturbative QCD description were chosen to define an upper and lower limit, respectively, to represent the behavior of F_K at high Q^2 . The evaluation of the uncertainties on the charge density of the kaon considered these two models as representatives. A discussion of the charge densities are presented for both the pion and the kaon in Chapter 6.

Chapter 2

Measurements at JLab Hall C

Aspects of the focusing spectrometers used for measuring the kaon electroproduction cross sections in Hall C/JLab are presented in this chapter. Details of the detectors that compose these spectrometers and the components of the trigger and data acquisition systems are discussed, giving an overview on how particles were detected during the E93-018 and FPI-2 experiments.

The Thomas Jefferson National Accelerator Facility (JLab), a national laboratory operated by SURA/PAE Applied Technologies for the U.S. Department of Energy, was proposed in the long-range plan of the Nuclear Science Advisory Committee (NSAC), back in 1979 and 1983, to provide forefront research of the atomic nucleus and its fundamental constituents, including precise tests of their interactions [JLab Missions, 2015]. In particular,

the foundation of JLab in 1984 represented the solidification of ideas for the construction of a laboratory that could allow the investigation of the largely unexplored transition between the hadronic and partonic pictures of the nuclear system [Leemann et al., 2001]. Since then, great efforts from country-wide researchers supported by the National Science Foundation (NSF) and the Department of Energy (DOE) (and also the collaboration of the international community) have allowed important advances in particle accelerator and detector technology at JLab, providing high-quality tools to the scientific community to explore properties and help to address questions about the inner structure of the nuclear matter.

All the experiments presented in this work were carried out at the Continuous Electron Beam Accelerator Facility (CEBAF), the main accelerator facility currently running at JLab. CEBAF was originally envisioned to generate a high-intensity beam of electrons for high-precision measurements of processes with low cross section, with beam energy in the GeV scale necessary to resolve the spatial structure of the nucleus in the regime of the transition from nucleon-meson to quark-gluon degrees of freedom, and with large duty cycle ideal for coincidence experiments. The combination of all these requirements led to the creation of the world's first large superconducting accelerator for generating a continuous GeV-scale electron beam. In addition, a complementary set of high resolution spectrometers were designed to instrument the experimental halls.

2.1 The accelerator: CEBAF

CEBAF is a high intensity electron accelerator designed to provide a “continuous wave” (CW) beam to the experimental halls. The electron beam at CEBAF is extracted from a three-beam chopping injector that can provide unpolarized or highly polarized electron beams ($>85\%$) to the main accelerator. This three-beam concept of the injector allows simultaneous delivery of a beam with different currents to three experimental halls. By producing and pre-accelerating these beams in short co-linear bunches of electrons, the injector delivers to the main accelerator an already ultra-relativistic electron beam with energy of about 45 MeV ($(1 - \beta/c) \approx 10^{-5}$; $\gamma \approx 90$) .

The main accelerator at CEBAF is built of a pair of linear accelerators (North and South linacs) connected by recirculating beam lines, as shown in Fig. 2.1. Refrigerated by a 2 K helium plant, superconducting niobium cavities in the linacs resonate at the fundamental frequency of 1497 MHz to accelerate the beam. As the electrons are already ultra-relativistic at the entrance of the main accelerator, bunches from different passes of the recirculation will have practically the same speed. CEBAF takes advantage of this and puts beams with different energies in phase in the main accelerator’s cavities, therefore using the same linacs multiple times for accelerating the recirculated beam. At the end of the South linac, the beam can be recirculated or sent to the Beam Switch Yard after any of the passes, and extracted simultaneously to the different experimental halls. If the maximum beam energy is desired, the beam is circulated around all of the recirculating lines and extracted to the

experimental halls.

Fig. 2.1: Schematic drawing of the CEBAF accelerator during the 6 GeV era of JLab. The electron beam produced at the injector is accelerated by the North and South linacs and recirculated up to four times, to finally be extracted for the three experimental Halls simultaneously. Figure from Alcorn et al. [2004].

Existing linear accelerators based on resonant cavities produce a beam with spatial structure, the beam bunches, which are “packages” of particles accelerated in the oscillating electromagnetic fields of the cavities. The very high frequency of the cavities’ first oscillating mode at CEBAF allows the delivery of bunches with very small longitudinal structure, with a period of 2 ns (or ≈ 60 cm long bunches for an ultra-relativistic beam). This beam structure, one order of magnitude smaller than the typical length of the spectrometers in the experimental halls (like those shown in Fig. 2.5 and Fig. 2.6), is practically a “continuous” beam (100% duty cycle) for the purpose of the experiments. This high duty cycle feature of CEBAF allows the operation of the machine with relatively high beam currents while

minimizing effects such as accidental coincidence events during the experiments.

After almost thirty years since the beginning of its construction, the successful history of CEBAF is associated with continuous improvements and upgrades. At the time of the E93-018 experiment (1996), the accelerator was operating with 5 passes delivering only an unpolarized beam at a maximum energy of ≈ 4 GeV to the experimental Halls A, B, and C. Later, at the time of the FPI-2 experiment (2003), the improvements on the machine and upgrades in the injector allowed the extraction of unpolarized and polarized beams with energies of up to 6 GeV. With the major recent upgrade of the machine to 12 GeV, with the addition of new cryomodules and a complete redesign of the recirculating lines and their magnets, the machine has already started in 2014 to deliver beam to the physics program with up to 12 GeV of beam energy. Moreover, this last upgrade provided an additional experimental hall (Hall D) for housing the GlueX experiment, with additional upgrades to most spectrometers of the existing halls.

2.2 Beam line instrumentation

A long pipe starting at the Beam Switch Yard, the exit of the main accelerator, brings the accelerated electron beam to the experimental Hall. Several elements of beam diagnostic instruments are used in this section of CEBAF to measure some properties of the beam delivered to the experiment. Figure 2.2 summarizes the distribution of instruments along the beam line used for these measurements.

As the beam reaches the target, its energy, profile, position, and direction are known

Fig. 2.2: Diagram of the beam line elements used to measure some beam properties at the entrance of Hall C, with their positions along the beam line with respect to the target. Image extracted from Gaskell [2001].

with high precision. The following subsections summarize the elements used in these measurements.

2.2.1 Superharps: beam energy and profile

The basic elements used to measure the beam energy and profile are the superharps, placed at several locations along the beam line. The superharps are composed of sets of thin wires connected to current monitors, and they provide a signal proportional to the beam current that is measured as a function of the wire position as these wires are moved across the beam direction.

A section of the accelerator connecting CEBAF to the experimental Hall has the form of an arc, where magnetic dipoles bend the beam towards the entrance of Hall C. Superharps were installed to measure the position and the transverse size of the beam at three sections of the arc: the entrance, the middle, and the exit. In addition, the magnetic field of the dipoles used in the arc are mapped in order to have a precise determination of the fields along the beam path. Therefore, knowing the position of the beam at the sections of the arc and also the magnetic field along the beam path, it was possible to determine the electron beam momentum by

$$p = \frac{e}{\theta_{arc}} \int B \cdot dl, \quad (2.1)$$

where e is the electron charge, θ_{arc} is the arc bend angle measured to be 34.3° , and B is the magnetic field provided by the dipoles along the beam path. A precision on the order of 0.05% on the measurement of the beam energy is achieved using the superharps at the arc. It is important to note that this measurement requires some optics elements along the arc to be turned off. Therefore, the measurement of beam energy with superharps cannot be done during the data taking. This was done periodically though, allowing for checks of the beam energy along the experiments duration.

Finally, with a very fine resolution provided by step motors to position the wires and encoders to read back the position, the superharps could also determine the beam profile. It was found that the typical FWHM of the beam was on the order of $200 \mu m$ during both E93-018 and FPI-2 experiments.

2.2.2 BCM: beam current

The current of the beam was continuously measured throughout the experiments using Beam Current Monitors (BCM). These are cylindrical wave guides placed in the path of the beam with geometry designed to resonate at the frequency of the beam. The BCMs were generally excited in the TEM_{010} mode, and their output voltages were proportional to the beam current when tuned to the frequency of the beam.

The BCMs are sensitive to temperature variation, since expansion or contraction in their dimensions change their resonant frequency and, therefore, their output signal for the measured current. For that reason, special care was taken for stabilizing the BCMs during the experiments, always keeping their temperature stable to 0.2 °C. In addition, both BCM1 and BCM2 as shown in Fig. 2.2 were regularly calibrated in reference to an absolute current measurement device. The BCM calibrations are performed with the use of the Unser current monitor, as described by Unser [1992].

The Unser is a parametric DC current transformer with high stability in its gain, although it suffers from drifts of the offset at short time scales. The Unser was used only at some specific times during the experiments, where special runs were taken for quantifying the gain of the BCMs. In these runs the beam current was varied in steps of 10 μA , from 0 to about 100 μA , and back to 0 μA again. With that, a cross calibration of the BCMs with respect to the Unser was obtained. The measurements with no beam at the beginning and at the end of the calibration were used to remove any drift of the offset of the Unser during the calibration run. Horn [2006a] estimated that this procedure of calibrating the

BCMs with the Unser has an uncertainty on the order of 0.4%. Considering an estimated uncertainty from the long term drift of the BCs on the order of 0.2%, the uncertainty in the charge measurement during both E93-018 and FPI-2 experiments was 0.5%.

2.3 Target

The main target used for both E93-018 and FPI-2 experiments was composed of liquid hydrogen (LH2). This target is prepared by refrigerating super pure hydrogen gas below its condensation point, at cryogenic temperatures. With the density of the liquid being much higher than the gas density, the number of scattering centers available for the reaction is optimized, providing a higher luminosity for the experiments.

The designs of the target cells used for the LH2 target are the “beer can” for the E93-018 experiment, and the “tuna can” for the FPI-2 experiment. These are cylindrical cells made of aluminum that allow LH2 to flow continuously. Besides the LH2 itself, a $71 \pm 3 \mu\text{m}$ thick entrance window and a $137 \pm 5 \mu\text{m}$ thick exit window (both made of aluminum 6061) are the components of the target also in the path of the beam.

A set of three “loops” was usually available for experiments in Hall C, making possible the use of multiple cryogenic targets depending on the requirements of the experiments. Each loop of the “beer can” contains a 4 cm long cell and another 15 cm long cell, that makes it possible to choose from these two lengths. The “tuna can” has 4 cm long cells only. Both E93-018 and FPI-2 experiments used the 4 cm long cells, from loop 1 (E93-018) and loop 2 (FPI-2). Figure 2.3 shows how the three loops are connected. The whole stack

could be moved upwards or downwards to select which target would be irradiated during a run.

Fig. 2.3: Schematic drawing of the cryogenic target used for the E93-018 experiment, known as “beer can”. In (a), the stack of three cryogenic loops for circulating LH2 in the path of the beam and the “dummy” targets used for accounting the contributions of the aluminum walls to the experiment. In (b), details on one of the loops where two cells of 4 cm and 15 cm are available. Figures are from Mohring [1999].

A Proportional, Integral, and Derivative (PID) feedback system controls specific heaters placed on the cryogenic circuit of the target, balancing the cooling power provided by the main heat exchanger to keep the LH2 at constant conditions (19 K and 24 psi) during the experiment. This is an important control of the target system that compensates for changes of the power dissipated in the target, mostly due to variations of the beam current.

To study and further correct for effects of the aluminum entrance and exit windows of

the cell on the measurements of cross sections, a “dummy” target was available to be used during the experiment (as seen in Fig. 2.3). This special target is composed of aluminum foils placed 4 cm (or 15 cm) apart from each other, allowing the quantification of the effects of the target walls only. To speed up the measurement with the dummy target, the foils were made thicker than the actual windows of the LH2 target (about seven times thicker during the FPI-2 experiment, and about ten times thicker during the E93-018 experiment). With that, the beam time required to get reasonable statistics with the dummy target was reduced significantly. For the data analysis of the dummy target, a normalization of the event rates to the actual foil thickness was done before subtracting the contribution from the target windows from the LH2 data.

In addition to LH2, other solid targets were used during the experiments to support the beam conditioning. Five carbon foils positioned along the beam direction ($z=0$, $z=\pm 2$ cm, $z=\pm 4$ cm) and two additional aluminum foils ($z=\pm 7$ cm) composed the “quintar” target, which was used for the calibration and checks of the spectrometer optics. Finally, a target composed of a simple carbon foil or a simple aluminum foil was also available for the purpose of general checking of the spectrometer conditions, e.g., when low event rates were required.

Finally, beam rastering was used during the experiments to reduce the effects of LH2 boiling and/or possible damage to the target can or the beam dump. A rastering system is composed of dipoles aimed to slightly move the beam around the center of the target, at frequencies of 17 kHz (vertical direction) and 24.2 kHz (horizontal direction) and amplitude on the order of 2 mm. The rastering pattern followed a sinusoidal form for the E93-018

experiment, while it was changed to triangular form for FPI-2. The rastering reduces local heating of any component of the target or dump due to the high power density of energy deposition in the target – about $5 \text{ MeV}/(\text{g}/\text{cm}^2.\text{s})$, contained in a beam spot of $200 \text{ }\mu\text{m}$ of diameter. However, effects of local reduction of the target density due to the heat deposition of the beam into the LH2 are observed even with the use of the rastering system. To account for these effects in the extractions of cross sections, a study of target boiling and target length contraction (due to temperature variation) was performed during both experiments. A correction factor to the effective target length was used for accounting these effects in the experiments.

2.4 Focusing spectrometers in experimental Hall C

At the time of the E93-018 and the FPI-2 experiments, the experimental Hall C was equipped with two magnetic focusing spectrometers for the identification and characterization of particles of interest: the Short Orbit Spectrometer (SOS) and the High Momentum Spectrometer (HMS). The Short Orbit Spectrometer (SOS) was primarily designed for detecting short-lived particles, such as pions and kaons, by having a relatively short distance from the target to the focal plane of the spectrometer ($\approx 10 \text{ m}$). The High Momentum Spectrometer (HMS) has a greater distance from target to its focal plane, making it more suitable for measurements of particles with higher momenta. Figure 2.4 shows a schematic drawing of the SOS and the HMS positioned at Hall C.

The feasibility of using both spectrometers in either positive or negative polarity makes

Fig. 2.4: Diagram of the SOS and HMS as positioned at the Hall C during the 6 GeV era. Figure from Mohring [1999].

it possible the choice to use one as electron arm and the other as hadron arm during an electroproduction experiment. This choice depends mostly on the kinematics of the reaction being studied and the limitations of the spectrometers. In particular, for the measurement of pion or kaon electroproduction reactions, the shorter pathway of the SOS was more suitable for being used as the hadron arm (measuring the desired mesons) when these particles have

lower momenta, reducing the losses of statistics due to particle decay. This was the condition used for the E93-018 experiment. On the other hand, the hadron arm during the FPI-2 experiment was the HMS due to the requirement of measuring mesons with angles and momenta that could be only detected by this spectrometer. For this reason, it is important to note that the reference to electron arm and hadron arm in this work refers to distinct spectrometers depending on the experiment. Table 2.1 shows the different spectrometer assignments for the experiments analyzed here.

Tab. 2.1: Assignment of Hall C spectrometers for the measurement of the scattered electrons and the produced mesons from the electroproduction reactions for the experiments analyzed in this work.

Experiment	e arm	Hadron arm
FPI-2	SOS	HMS
E93-018	HMS	SOS

More details of the SOS and the HMS spectrometers follow in the next subsections.

2.4.1 The Short Orbit Spectrometer - SOS

The Short Orbit Spectrometer was used during both E93-018 and FPI-2 experiments in Hall C. It has now been decommissioned for the 12 GeV upgrade of JLab. This spectrometer consisted of a set of resistive magnets in a $QD\overline{D}$ configuration (one quadrupole and two dipoles) designed to bend the particle trajectory vertically by about 18° out of plane, focusing them to the particles' detectors inside its detector hut. A schematic drawing of the SOS is shown in Fig. 2.5.

The SOS was a “compact” spectrometer, with a distance from the target to the focal

Fig. 2.5: Schematic drawing of the SOS, with the quadrupole (Q) and the dipoles (D and $\bar{D}$) used as optics elements to focus particles into the detector hut of the spectrometer. Figure from Mohring [1999].

plane of its magnets of 10 m. This spectrometer had a relatively “large” solid angle (7 msr) defined by an octagonal collimator made of 2.5 inches thick HEVIMET. Its design allowed for a very large momentum acceptance of particles (compared to other focusing spectrometers), on the order of $\pm 20\%$ relative to the set central momentum of the spectrometer. It should be noted that the axis of the SOS lay 0.15° below the horizontal plane, a fact accounted for in the data analysis.

As most magnets with ferromagnetic cores, the magnets of the SOS spectrometers presented magnetic hysteresis. The magnetic field strength is not strictly proportional to the current in the coils, i.e., the magnets will keep “memory” of the magnetic fields once they are magnetized and they will stay magnetized even if the currents are ceased.

Special care was taken for handling the SOS magnets during the experiments, making sure the magnets were demagnetized every time the magnetic fields needed to be decreased in this spectrometer. In order to demagnetize the SOS magnets, their currents were first ramped up to the region of magnetic saturation in the desired polarity of the magnets, by applying a high enough current to reach the regime where the magnetic field strength is an injective function of the magnetization. Their currents were then ramped down to zero, and sequentially raised to the region of magnetic saturation in the opposite polarity of the magnets. Finally, the currents in the magnets were zeroed again and raised up to the desired current. This procedure guaranteed accuracy and reproducibility of the magnetic fields in the SOS, an extremely important quantity for properly selecting the particles with momenta of interest during the experiments.

In addition to the hysteresis, the fields of SOS magnets were found to suffer from saturation effects for momentum settings of the spectrometer above 1.0 GeV/c. The linearity of the magnetic field with the set current was compromised, causing a deviation from the set configuration to the actual momentum as large as 1% for the maximum setting of 1.7 GeV/c. Moreover, it was observed that the spectrometer optics varied with the reconstructed fractional momentum δ , an effect addressed by a refit of the matrix elements of this spectrometer for higher momentum settings [Horn, 2006a]. All of these effects were considered during the data analysis, by applying corrections during the event reconstruction.

2.4.2 The High Momentum Spectrometer - HMS

The HMS is the second magnetic spectrometer used during the E93-018 and FPI-2 experiments, and it is still operational as of the date of this work (2017) being readied for the 12 GeV era of JLab in Hall C. Designed to detect particles with high momenta, the HMS can reach a central momentum of up to 7.4 GeV/c with a moderate momentum acceptance of $\pm 10\%$. In order to handle such high momenta with a “compact” design, the HMS is built of superconducting magnets (three quadrupoles and one dipole) to focus and bend the particles into its detector hut with a focal plane about 25 m from the target, tilted by 25° in the dispersive direction. A schematic drawing of the HMS is shown in Fig. 2.6.

Fig. 2.6: Schematic drawing of the HMS, where four optics elements are used to focus particles into its detector hut: three quadrupoles (Q) and one dipole (D). Figure from Mohring [1999].

Similarly to the SOS, the HMS collimator is made of 2.5 inch thick HEVIMET material positioned before the first quadrupole, defining the HMS acceptance to be on the

order of 6-7 msr. Although the HMS magnets are superconducting, hysteresis effects were present (although smaller than the SOS) and could be noticed when ramping up/down the currents of the magnets. Even though the magnetic field of the dipole was controlled by a closed loop circuit based on the reading of a Nuclear Magnetic Resonance (NMR) probe, a special systematic procedure was always followed for minimizing hysteresis effects during the experiments. Whenever the setting of the spectrometer was changed, the currents of the quadrupoles were raised to a value exceeding the desired current by 100 A and then ramped down to the configuration of the experiment. If in addition the polarity of the spectrometer needed to be reversed, the currents of the magnets were always ramped up to 800 A before setting the experimental condition. This cycling method made possible a reproducibility of the experimental conditions in one part in 10^{-4} .

Some other details of the HMS are worth mentioning here. First, a small offset in the set current of the third quadrupole (Q3) was known to be present, as discussed by Gaskell [2001]. Although a tentative correction of this offset was applied in subsequent experiments, a residual offset still remained. A new correction factor was determined and the routines in the analyzer were adapted to account for this effect in the data analysis, as discussed by Horn [2006a]. Secondly, the standard exit window of the HMS was replaced by the 0.02 inch Titanium window prior to FPI-2 (the E93-018 used the originally designed windows). A more complete discussion about this spectrometer is given by Arrington [1998]. It is important to mention that there was a change in the position of the HMS quadrupoles in between the experiments E93-018 and FPI-2, therefore resulting in different HMS optics for

these experiments.

2.5 Particle detectors in the SOS and HMS huts

Shielded inside the detector hut of the spectrometers, a series of detectors were used to identify the particles and track their trajectories. The concept of the detectors in the SOS and in the HMS are very similar, so this section summarizes their general features.

The detector packages in both spectrometers was composed of a pair of drift chambers for particle trajectory tracking, four planes of scintillators for defining the trigger and timing, and threshold Cherenkov detectors and lead-glass calorimeters for particle identification. Schematic drawings of the particle detectors in the detector huts of the spectrometers are shown in Fig. 2.7.

A brief description of the fundamental aspects of these detectors follows in the next subsection, highlighting their main features regarding the design and operation necessary for the understanding of the data analysis done in the present work. A more detailed description of the detectors can be found at Arrington [1998].

2.5.1 Drift chambers: particle trajectory tracking

The drift chambers are detectors designed to track the trajectory of particles in the (field free) region of the detector hut. The resulting tracks of these detectors allow for the reconstruction of the particles' momentum and direction as they were at the target, by using the optical properties of the spectrometer. This provides essential information for the reconstruction of kinematics variables of the reaction being studied.

(a) Particle detectors in the detector hut of the SOS

(b) Particle detectors in the detector hut of the HMS

Fig. 2.7: Schematic drawings of the particle detectors in the detector hut of the SOS and HMS spectrometers. Their conceptual designs are similar, providing full capability for particle tracking and particle identification during the E93-018 and the FPI-2 experiments. Note: the HMS aerogel detector was not yet available during the E93-018 experiment. Figures from Mohring [1999] and Horn [2006a] respectively.

The basic principle of operation of a drift chamber relies on the collection of ionized charges (electrons) created by a fast moving particle traversing the gaseous medium of the chamber. Sets of sense and field wires are placed transversely to the direction of propagation of the incoming particle for detecting the ionization of the gas. High voltage is applied between the field and the sense wires to create a high electric field in the medium, accelerating

the ionized charges towards the closest sense wires and producing an avalanche of electrons in the chamber gas to be collected by the sense wires.

The drift chamber of each spectrometer is composed of two chambers, each with six parallel planes of wires aligned in different directions. There are a few differences in the design of the HMS and SOS chambers, although they operate with the same principle.

Each drift chamber of the HMS covers an active area of about 113 cm by 52 cm, containing six planes of sense wires with a separation of 18 mm between each plane. These planes, labeled as x, y, u, v, y', x' , contain sense wires separated by 10 mm, filling the whole active area. Field wires were intercalated between the sense wires for the application of high voltage. Additionally, planes of “guard” wires were built between the six active planes for also holding high voltage. With this design, the sense wires of the HMS chambers were completely surrounded by high voltage wires optimizing the gradient of electric field for charge collection and reducing the interference between planes.

The label of the planes of the HMS drift chambers represents the direction that they are sensitive for. The x and x' planes are designed for measuring the particles position along the dispersive direction, while y and y' are perpendicular to those. The wires in the planes u and v are tilted by plus and minus 15° respectively with respect to the x plane, completing the necessary set of information for tracking the position where the traveling particle hit that chamber. As there are two similar chambers in the spectrometer separated by a drift distance, it is then possible to track the particle trajectory between them given that the particle traveled in a straight line between those planes.

Fig. 2.8: Direction of the sense wires of the HMS drift chamber, used to measure particles' position and direction in the spectrometer. Figure from Mohring [1999].

The design of the SOS chambers differs slightly different from the HMS, in the orientation of the wires in the planes and the use of cathode foils in place of the guard wires for holding high voltage between the sense planes. For the SOS chambers, the planes are labeled in order as u, u', x, x', v, v' . The x and x' planes measure the position in the dispersive direction, while the v and v' (u and u') planes are rotated by 60° counterclockwise (clockwise) from x . A schematic drawing of the SOS drift chamber is shown in Fig. 2.9.

The signals collected by the sense wires are read by specialized electronics, allowing for the identification of the wires that had triggered in each event (done with discriminators)

Fig. 2.9: Schematic drawing of the SOS drift chamber. Figure from Mohring [1999].

and measuring the time difference between the main trigger of the event and the signal from the fired wire (measured with TDCs). The TDCs were multihit for the FPI-2 experiment, i.e., the timing of more than one particle hitting the spectrometer within a very short time window can be saved to the same event for later offline processing for choosing the best candidate track to the event reconstruction. With high resolution time information of the TDCs (on the order of 500 ps) and a proper calibration of the chambers, it is possible to determine the position of the particles traversing the chambers with a resolution on the order of $280 \mu m$ for the HMS chambers, and $180 \mu m$ for the SOS chambers.

2.5.2 Hodoscopes: triggering units

The hodoscopes of both HMS and SOS are responsible for the lowest level trigger of the data acquisition. Composed of sets of fast scintillator paddles (bars) connected on both ends to a PMT (interfaced with light guides), the hodoscopes also provide a reference start time for the measurement of times (by TDCs) in the different detectors of the spectrometers.

The basic principle of operation of the hodoscopes relies on the detection of photons created in their scintillator paddles when particles cross them. The incident particles initially ionize atoms of the scintillator material, releasing fast moving electrons in it. These electrons may both ionize other atoms and/or excite molecules of the scintillators. The relaxation of these molecules to their ground state radiates photons in the visible wavelength range that can travel all along the scintillator length reflecting on its walls by internal reflection and reach the PMTs.

The hodoscopes of the HMS and the SOS were very similar, differing basically by the area they cover. They are made of scintillator paddles BC404 with 1 cm of thickness and 8 cm wide, placed side-by-side with 0.5 cm of overlap to avoid gaps between the elements. Each paddle is wrapped individually with Aluminized Mylar and Tedlar, to improve the efficiency of reflection of radiated photons inside the scintillators and to make them light tight for external light. At the end of each paddle, a lucite light guide drives the photons into PMTs.

Each spectrometer contains four planes of hodoscopes, as shown in Fig. 2.7. The SX planes of the hodoscopes have the scintillator paddles aligned along the dispersive direction,

while the SY planes are along the non-dispersive direction. A pair of SX/SY planes are placed at about 220 cm apart from the other pair in the HMS (180 cm in the SOS), allowing for the measurement of the time-of-flight of the particles between the planes. With a timing resolution on the order of 100 ps, this timing can also be used for particle identification when at lower spectrometer momentum ($\lesssim 1 - 2$ GeV/c).

2.5.3 Gas Cherenkov detectors: particle identification

In order to provide a particle identification feature capable of separating electrons from pions in the spectrometer, a gas Cherenkov detector was installed in both HMS and SOS. This is a threshold Cherenkov detector and, as such, it distinguishes particles by measuring whether or not these particles radiate light when traversing its gaseous medium.

Cherenkov radiation is electromagnetic radiation created when a charged particle passes through a medium with speed greater than the phase velocity of light in that medium. The minimum velocity (v_t) necessary for the emission of this radiation is given by

$$v_t = c/n, \tag{2.2}$$

where c is the speed of light in vacuum and n is the real part of the refractive index of the medium. Using the relativistic momentum (p) of a particle

$$p = \gamma m v, \tag{2.3}$$

where m and v are the mass and velocity of the particle, the momentum threshold (p_t) of

the particle to radiate Cherenkov radiation is given by

$$p_t = \frac{m \cdot c}{\sqrt{n^2 - 1}}. \quad (2.4)$$

Therefore, to identify electrons and pions with similar momenta (selected by the setting of the magnetic spectrometer), the medium of the gas Cherenkov detector is chosen to have a refractive index in which the electrons would radiate light in the momentum range of the spectrometer operational range, while the pion would not radiate. The gas Cherenkov detector of the SOS was filled with Freon-12 (refractive index of 1.00108), providing a momentum threshold for Cherenkov radiation of 11 MeV/c for electrons and 3.0 GeV/c for pions (well above the maximum momentum setting of this spectrometer, 1.7 GeV/c). For the HMS, the gas and pressure were different. For example, during FPI-2 the gas was C_4F_{10} and the pressure was 0.47 atm. This gas at this pressure has a refractive index of 1.00066, providing a higher momentum threshold for pion (3.8 GeV/c) given the high momentum capability of this spectrometer.

The design of the gas Cherenkov detector is based on a chamber to hold the gas in it. The detector in the HMS has a special structure capable of supporting pressures lower or slightly higher than the atmospheric pressure.

Special attention is necessary when using threshold Cherenkov detectors since particles with momentum below the threshold can sometimes create a signal in the detector. The main cause for this is the possibility of particles to create δ -electrons (also known as knock-on electrons) when passing through the material of other detectors prior to the gas Cherenkov.

The δ -electrons can have enough energy to create a signal that cause a mis-identification of the particle of interest. A study of this effect was necessary to properly quantify the correction factor to account for this effect.

2.5.4 Lead Glass calorimeter: particle identification

The lead glass calorimeter is an additional detector used to separate electrons from hadrons in the spectrometers. As the last detector in the detector hut, the calorimeter is composed of a thick layer of lead-glass segmented in light-tight blocks, with radiation length of ≈ 15 .

The signal from lead glass is due to Cherenkov light, which is approximately proportional to the total path length of the relativistic charged particles in the glass. This light is collected by PMTs connected to one end of each block, creating a proportional electric signal for further signal processing and acquisition by the electronics.

Electrons, positrons, and photons create a shower in the lead glass. As the calorimeter thickness is greater than ≈ 10 radiation lengths, this shower is absorbed within the detector. The ultimate mechanism of absorption is that the electrons and positrons in the shower lose energy by ionization, at approximately the minimum ionization rate ($\approx 2 \text{ MeV}/(\text{g}/\text{cm}^2)$). Thus the total charged particle path length is approximately equal to the incident energy divided by $(2 \text{ MeV}/(\text{g}/\text{cm}^2))$, and the Cherenkov light output is approximately proportional to the incident energy.

For a hadron like a pion or a kaon, the path length in the lead glass calorimeter is essentially the detector thickness. A hadron with energy lower than $\approx 10 \text{ GeV}$ loses

energy primarily by ionization. Therefore, the light created in the detector for these heavier particles is smaller than for those that develop a shower in the calorimeter.

A typical plot of the ratio between the energy measured in the calorimeter (E_{cal}) and the incident momentum of the particle has a two peak structure, as shown in Fig. 2.10.

Fig. 2.10: Distribution of E_{cal}/p of the electron arm for one experimental setting of the FPI-2 experiment. The histogram shows the signal measured for all the events captured by the spectrometer without any selection of events. In blue, a detail of the $p(e,e'K^+)\Lambda$ events selected for analysis using all the cuts but the SOS calorimeter cut. For the cross section analysis, a cut was placed at 0.7 as indicated by the arrow.

The first peak seen in Fig. 2.10 located at $E_{cal}/p \approx 0.3$ GeV/p is due mostly to pions (or other hadrons), while the peak at unity is due to electrons. Together with the gas Cherenkov detector, the electromagnetic calorimeter allows for a proper identification of electrons in the electron arm of the experiments and also for studies of the electron identification efficiencies.

2.5.5 Aerogel detector: particle identification

The aerogel detector is a special threshold Cherenkov detector that can be added or removed from the spectrometers depending on the requirements of the experiment. This detector is very useful when kaons are measured in the spectrometers, providing a clear separation of kaons from pions depending on the selected refractive index of the aerogel.

Aerogel is an extremely light solid material built from a supercritical drying process of a gel. Its composition is mostly air (95%) trapped inside the structural nanopores of the aerogel. There are different compounds that can be used to make aerogel, but silica oxide is the most useful one for building a Cherenkov detector since it has a very low light absorption, allowing the Cherenkov radiation to reach the photodetectors. Besides its translucence, the remarkable property of aerogel for building a Cherenkov detector is the intermediate refractive index that it can have (1.01 to 1.1); not as low as the refractive index of gases (typically lower than 1.01), and not as high as those of other liquids and solids (typically higher than 1.1). This provides a threshold momentum for Cherenkov radiation on the order of several GeV/c which is ideal for kaon/pion separation (and protons in some cases).

The E93-018 experiment had an aerogel detector installed in the SOS for kaon identification. The HMS also had an aerogel detector installed during FPI-2, whose refractive index was optimized for pion/proton separation. The role of the HMS aerogel detector in the present analysis of kaon electroproduction will be discussed in Chapter 3.

Conceptually similar to the design of the gas Cherenkov detector described in Sec. 2.5.3,

the aerogel threshold Cherenkov detector is composed of a chamber containing stacked tiles of aerogel. Again, Cherenkov radiation is created only when the incoming particle has a velocity higher than the phase velocity of the light in that medium (Eq. 2.2), and the identification of particles is done by detecting whether or not the particle radiates Cherenkov light in it. As aerogel scatters light more significantly as compared to gas, the whole chamber of the detector is covered with a diffusive reflector to reflect the light. PMTs are placed along the side walls of the detector for light collection. A schematic drawing of the SOS aerogel detector is shown in Fig. 2.11.

δ -electrons are also important for the aerogel detector. A particle that would not create Cherenkov radiation in the detector by itself may knock out an electron from the materials of detectors before the aerogel, and this electron can be above the threshold velocity radiating light in the detector. These δ -electrons result in mis-identification of particles, and special studies were necessary to correct the experimental yields.

2.6 Trigger logic

The trigger components of the experiments in Hall C were designed to optimize the data acquisition of the spectrometers, keeping a reduced electronic deadtime that could be produced by high volumes of data while maintaining a high trigger efficiency. The trigger system in Hall C was designed in three separated units, called the HMS pretrigger, the SOS pretrigger, and the coincidence logic. The first two systems, the pretriggers of each spectrometer, are responsible for forming triggers when particles are detected within that

Fig. 2.11: Schematic drawing of the aerogel detector of the SOS as an example of the layout of threshold Cherenkov detectors used for pion/kaon separation during the E93-018 experiment. The HMS aerogel detector design is similar, but has 8 PMTs on either sides and the aerogel volume is $9.5 \times 6.7 \times 11.7 \text{ cm}^3$. Figure from Mohring [1999].

spectrometer. The coincidence logic is then formed when the two spectrometers' pretriggers are activated within a certain time window, signaling to the data acquisition in order to record the data. More details of the trigger components of the spectrometers at Hall C are described in the next subsections.

2.6.1 HMS pretrigger

As described in Sec. 2.4, the HMS was used as the hadron arm during the E93-018 experiment, while it was the electron arm during FPI-2. The circuit for forming the pretrigger of the HMS is very comprehensive, and basically the same logic could be used for running both experiments. Small differences existed though, as will be highlighted at the end of this section.

Several detectors can be used to form the pretrigger of the HMS, such as the hodoscopes, the gas Cherenkov, and the calorimeter. A diagram with the logic of the HMS pretrigger is shown in Fig. 2.12.

The core of the trigger unit for the HMS is the existence of a signal in at least three (out of four) planes of the hodoscope (**S1X**, **S1Y**, **S2X**, and **S2Y**), creating the **SCIN** signal of the logic shown in Fig. 2.12. Moreover, if at least one plane from each pair of hodoscope planes (**S1** and **S2**) is triggered, a **STOF** signal is also created. These are the two signals that flag the existence of a particle in the spectrometer, where **STOF** is a more relaxed trigger requiring only two hodoscope planes (from different sets) to trigger.

Although the efficiency of a single hodoscope plane is in general quite high, the use of two or three planes only to form the trigger is an interesting approach, allowing the study of the efficiencies of individual planes and reducing the dependence of the final trigger to any possible region of a particular plane with lower efficiency. At this point, the signal from the particle identification detectors are used for the different legs of the trigger. **PRHI** and **PRLO** are signals created from the calorimeter, when the energy deposited exceeds a high

Fig. 2.12: Diagram of the HMS pretrigger logic. Details of the components of this diagram are described in the text. The gates width for studies of the electronic deadtime (as marked by the dashed box) were for the E93-018 experiment; the FPI-2 experiment used gates width of 60, 100, and 150 ns instead. Adapted from Mohring [1999].

or low threshold respectively. Also from the calorimeter, **SHLO** is a signal created if the total energy deposited in the calorimeter exceeds a (low) threshold. Finally, $\check{C}$ flags the existence of signal in the gas Cherenkov detector.

There are three main logical components for the HMS, that can be summarized as follows:

- **ELLO**: this signal is formed when two out of the following three signals are present: **STOF**, **PRLO**, and **SCIN**. Moreover, **NOT $\check{C}$** (the not of the logic signal $\check{C}$) is used to veto this function, i.e., $\check{C}$ must be present for the formation of **ELLO**. Therefore,

ELLO is primarily designed to create the pretrigger of the HMS relying mostly on the information of the gas Cherenkov detector as the PID detector.

- **ELHI**: to form this signal, all **SCIN**, **PRHI**, and **SHLO** must be present. The requirement of **PRHI** uses the fact that electrons are expected to deposit more energy than hadrons in the calorimeter, so **PRHI** should be active for electrons only. **ELHI** is then created to identify electrons in the HMS, relying mostly on the information of the calorimeter for PID.
- **PIPRE**: this third element of the main HMS logic was originally designed to give prescaled pion sample, being active for detecting hadrons in general. Just requiring the presence of the hodoscope signal through the requirement of **SCIN**, the first part of this trigger circuit (**PIONHI**) is only vetoed by signals in the gas Cherenkov detector, therefore inhibiting electrons to trigger it. After that, if the spectrometer is designed to be an electron arm (E93-018 experiment), **PIONHI** can be prescaled to provide more time for the measurements of electrons.

ELLO and **ELHI** were most important for the E93-018 experiment, where the HMS was used as the electron arm. These two signals were then combined with an “OR” logic, creating the real electron signal (**ELREAL**). To provide a small sample of the hadrons reaching the HMS during the E93-018 experiment, the prescaled **PIPRE** was used only for controlling the experimental conditions.

For the FPI-2 experiment, the HMS was used as the hadron arm. Given the rates of

particles reaching the HMS spectrometer during this experiment, it was possible to accept all of the hadrons hitting the spectrometer by getting the **SCIN** signal directly as a fourth branch of the main trigger.

Finally, to complete the HMS pretrigger, an “OR” of all of the logic described above was used to create the pretrigger signal of the HMS (**PRETRIG**). Therefore, any of the **ELLO**, **ELHI**, or **PIPRE** (also **SCIN** for FPI-2 only) signals would create a valid pretrigger in the HMS. This signal would then fire the four different gate generators with distinct gate widths, used for determining the electronic deadtime (see Sec. 3.4.3). Additionally, the HMS pretrigger was sent to the coincidence logic as will be described in Sec. 2.6.3. For FPI-2, **ELLO** and **ELHI** were disabled.

2.6.2 SOS pretrigger

The general SOS pretrigger is very similar to the HMS pretrigger system, allowing for adjustments depending on the requirement of each experiment. The SOS was used as the hadron and electron arm for the E93-018 and FPI-2 experiments respectively, therefore some differences between the two experiments existed.

The core of the SOS pretrigger also relies on the formation of the **SCIN** signal from the coincidence of 3 out of 4 hodoscope planes. From there, the electron PID components **ELLO** and **ELHI** are formed in the same way as in the HMS, where **ELLO** is mostly related to the presence of a signal from the gas Cherenkov detector (plus some signal in the whole calorimeter) and **ELHI** requires a high energy deposition in the calorimeter. The existence of either of these two creates **ELREAL**, triggering the **PRETRIG** of the SOS.

This branch of the **PRETRIG** formation was most important for the FPI-2 experiment, as electrons were detected in the SOS.

For detecting hadrons in the SOS, such as the detection of K^+ during the E93-018 experiment, the creation of the **SCIN** signal (from the hodoscopes) was required with the aerogel detector used to veto this signal. This veto eliminated events where pions were present, releasing the data acquisition from much higher pion rates compared to kaons. Finally, this branch of the trigger system would fire the **PRETRIG** of the SOS for further processing by the coincidence unit.

Similarly to the HMS, the SOS pretrigger also generated gates with different widths for determining the electronic deadtime of the detector, as explained in Sec. 3.4.3.

2.6.3 Coincidence trigger logic

Once the pretrigger of both the SOS and HMS spectrometers are formed within their trigger logic, they are sent to the 8LM LeCroy programmable logic unit and trigger supervisor (TS) for the formation of the actual trigger for the data acquisition system. The intent of this unit is to monitor whether the data acquisition is not busy saving events and, when pretriggers from the SOS and HMS arrive, trigger the acquisition of an event for the experiment. The basic logic of the coincidence trigger unit is shown in Fig. 2.13.

In addition to the spectrometer pretriggers, the 8LM unit also receives a random pretrigger (**PED PRETRIG**). This input is used only up to the accumulation of 1000 events in the active run. **PED PRETRIG** therefore triggers the spectrometers at the beginning of each run, allowing for checks of the pedestals of all of the detectors during the

Fig. 2.13: Diagram of the coincidence trigger logic, that receives the pretriggers of the SOS and the HMS spectrometers to form the main trigger with necessary details for all of the modules of the data acquisition system.

experiments. The three other inputs of the 8LM unit are reserved for communication with the TS unit. They are the **TS GO**, which flags the data taking is ongoing so that the 8LM unit can create outputs; the **TS EN1** is the enable from the TS, flagging the normal data taking condition (active only after the first 1000 events containing **PED PRETRIG** was taken) and the **TS BUSY**, that is active whenever the TS is busy processing events and cannot accept any other event.

The outputs of the 8LM unit are given by a logic performed with the input signals. **HMS PRE** and **SOS PRE** are copies of the **HMS PRETRIG** and **SOS PRETRIG** whenever a run is ongoing, i.e., **TS EN1** is active. Also, a coincidence between the spectrometers is identified and **COIN PRE** is formed when both **HMS PRETRIG** and **SOS**

PRETRIG are active within 120 ns, again requiring **TS EN1** to be active. All three “**PRE**” signals are then sent to scalers, to count the number of hits identified in the spectrometers and the respective identified coincidences. Finally, if the TS is alive (able to accept a trigger, as flagged by **TS BUSY**), the creation of the actual triggers is done by sending the “**PRE**” signals to the TS, identified now as “**TRIG**”. The accepted pretriggers are also sent to scalers for counting the corresponding numbers of accepted hits, for the calculation of deadtimes.

The TS unit receives the “**TRIG**” signals, as illustrated in Fig. 2.13, and forms the gates for the ADCs and TDCs of both spectrometers whenever it receives a **COIN TRIG**. Very few events with a single spectrometer trigger (**HMS TRIG** only, or **SOS TRIG** only) are accepted for checking purpose. It is important to note that a retiming of the gates is done within the TS, ensuring that time differences between the pretriggers of the spectrometers are properly accounted when forming the gates.

2.6.4 Data acquisition

The creation of a trigger by the coincidence logic fires a series of readout electronics (TDCs, ADCs, and scalers) that read and digitized the different components of the detectors. These electronics were located in FASTBUS and VME crates, properly distributed to account for the many hundred channels being read simultaneously during the experiments. At the end of each event, all of the information collected by the electronics was read by ROC (Read Out Controller) CPUs and sent to the central workstation of the experiment to be saved on disk.

This chain of processes for the data acquisition is managed by the CODA (CEBAF Online Data Acquisition) software, designed to handle all of the electronics related to data acquisition used at Jefferson Laboratory. CODA was responsible for bringing the data from all of the different modules using a parallel link through Ethernet protocol and saving them to disk. The synchronization between the ROCs being used in the experiments was always checked as well, to make sure the data from distinct events were never mixed up. In addition, information about the beam position, magnets settings, target status, and accelerator status were read out every 30 seconds using an auxiliary software (EPICS). Details of the acquired data during the experiment and their analysis follow in the next chapter.

Chapter 3

Data Analyses Framework

The framework for extracting the kaon electroproduction cross section from the $p(e,e'K^+)\Lambda$ reaction is presented in this chapter. The analysis to retrieve physics quantities from the raw data is discussed, with further creation of distributions of these quantities for the deconvolution of the final cross section of kaon production out of the experimental data.

3.1 General overview

For the first part of the kaon electroproduction analysis, the data taken during the experiments were analyzed with the standard Hall C analysis code, “ENGINE”. In this code, the raw data collected with the different ADCs, TDCs, and scalers are processed to form the higher level information for each event registered by the DAQ. Results of the data processing in ENGINE can be summarized as follows:

- Particle trajectories in the spectrometers are reconstructed from the signals of the drift

chambers. From the magnetic optics models of the spectrometers, these trajectories are used to reconstruct the interaction vertex of the reaction, allowing the calculation of physics quantities (such as Q^2 , W , t , ϕ , ...) for each event.

- Signals registered for the different detectors are calibrated, allowing for particle identification analysis.
- Signals from TDCs are processed together with particle trajectories, making possible the quantification of real coincidences of scattered electrons and kaons from the $p(e,e'K^+)\Lambda$ reaction.
- Efficiencies of the many components of the hardware for event detection and reconstruction are calculated from scalers and other detector signals, allowing for the evaluation of correction factors that are applied to the measured yields.

All the decoded physical quantities are saved by the ENGINE into files for every event. With these files, a later selection of the events of interest can be accomplished using a set of cuts for the analysis of the $p(e,e'K^+)\Lambda$ yields in the bins of interest.

After the event kinematics are reconstructed and the experimental yields are calculated, the extraction of the cross section is done using the Monte Carlo simulation package of Hall C, “SIMC”. The Monte Carlo simulation plays a central role in the data analysis for understanding the spectrometer acceptance in the experiment. In this simulation, $p(e,e'K^+)\Lambda$ events are generated in the phase-space of the experimental settings, and the propagation of the created particles through the spectrometers is simulated to construct the

SIMC yields. SIMC includes many aspects of the actual experiments, such as

- Particles' energy loss when traversing materials, such as the target itself and sections of detectors.
- Particles' energy loss by Bremsstrahlung, also known as radiative corrections, when interacting with the scattering nucleus.
- Multiple scattering of particles when traversing materials.
- Spectrometer acceptance.
- Kaon decay.

Physics quantities of the events were reconstructed in the same way as in the ENGINE, and several weights were assigned to the simulated events, among them the cross section of the reaction of interest. With that, the distributions of the simulated yields are created and can be directly compared to the measured yields. The initial cross section model of the $p(e,e'K^+)\Lambda$ reaction is then adjusted in SIMC and the simulation is run again iteratively, until the simulated yields are consistent with the measured yields of kaon electroproduction. This allowed the final extraction of the $p(e,e'K^+)\Lambda$ cross section.

Part of this cross section calibration for the E93-018 and the FPI-2 experiments was already done by Mohring [1999] and Horn [2006a] respectively. Their analyses were adapted for the different purposes of this work. For the E93-018 experiment, the cross section was fully separated into two bins in t and the kaon electromagnetic form factor was extracted.

For the FPI-2 experiment, the unseparated kaon cross section was extracted in the relatively high Q^2 kinematics of this experiment, of up to 2.07 GeV^2 . More details of the ENGINE algorithm to process each event and the usage of SIMC for the ultimate cross section extraction are discussed in the next sections.

3.2 Event reconstruction

Particles are detected in the spectrometers. However, to understand the physics and calculate cross sections, one needs to know the angles and momenta of the kaon and the scattered electron at the interaction vertex of the reaction, i.e., at the target. This was done by an offline analysis of the saved raw data for each event, using the standard Hall C analysis software (“ENGINE”). First, the particles’ track during their drift in the detector huts are reconstructed, and a set of matrices is used to project these tracks backwards through the spectrometers to the target, where the reaction occurred. Once the kaon and the scattered electron momenta and angles are reconstructed at the target, the kinematics quantities such as Q^2 , W , t , ϵ can be calculated.

Details of the tracking reconstruction and the spectrometers’ optics algorithm are discussed next.

3.2.1 Particle tracks in the detector hut of the spectrometers

The first part of the analysis to determine the kinematics of an event is to track the kaon and scattered electron in the detector hut of the spectrometers in order to determine their position and direction in these regions. As these are regions with negligible magnetic

fields, the tracking of the particles trajectories was done considering simple straight drifts of the particles in space. The information acquired by the drift chambers was used for retrieving this information.

As described in Sec. 2.5.1, the drift chambers of the SOS and the HMS are composed of two sets of wire detectors separated by a known drift distance. The algorithms for dealing with the signals from the SOS and the HMS drift chambers are similar in the ENGINE. For each event, the ENGINE first computes the position where particles hit each set of wire detectors of the drift chamber. This defines the coordinates of the particle's trajectory in the two sections of that drift chamber. By combining the coordinates of these two sections with the distance between them, the ENGINE algorithm finds the straight line that best fits the particle's direction, defining the particle's trajectory in the detector hut of the spectrometers.

The particle trajectory in the detector hut is defined by the position (x_{fp}, y_{fp}) and the direction (x'_{fp}, y'_{fp}) of the particle in the focal plane of the spectrometer, a plane situated at the center of the drift chambers. Here, x'_{fp} and y'_{fp} are the rate of change of the spectrometers' transverse coordinates x and y with the spectrometers' longitudinal coordinates z , i.e., $x'_{fp} = (dx/dz)|_{z=fp}$ and $y'_{fp} = (dy/dz)|_{z=fp}$.

It is important to note that the tracking algorithm requires at least five out of the six planes of the drift chamber to have registered a hit in order to determine the particle's trajectory. Events that do not meet this criterion are lost from the data analysis, and a correction factor to account for them is applied, as discussed in Sec. 3.4.2. Additionally,

there are events where multiple hits were registered in a drift chamber. For the FPI-2 experiments, multiple tracks could be reconstructed. In these cases, the track consistent with a hit in the calorimeter and closest to a hit in the second scintillator plane was selected. Other selection criteria were applied in some a special cases, and details are described by Tvaskis [2004].

3.2.2 Reconstruction of target coordinates

With the particles' trajectories fully defined in the detector hut of the spectrometers by the coordinates $(x_{fp}, y_{fp}, x'_{fp}, y'_{fp})$, a matrix transformation representing the optics of the spectrometer magnets is applied to propagate these trajectories backwards to the target. With that, the kaon and scattered electron angles and momenta for each event can be calculated at the interaction vertex, allowing for calculating the physics quantities of the reaction.

The matrix transformation to transport the particles' trajectories back to the target can be expressed in terms of the power series expansion

$$x_{tar}^{(i)} = \sum_{jklm}^N M_{jklm}^{(i)} (x_{fp})^j (y_{fp})^k (x'_{fp})^l (y'_{fp})^m, \quad (3.1)$$

where $x_{tar}^{(i)}$ represents the reconstructed elements $(x'_{tar}, y'_{tar}, y_{tar}, \delta)$ in the interaction vertex, and $M_{jklm}^{(i)}$ denotes the reconstruction matrix element. Here, x'_{tar} is defined as the out of plane scattering angle at the target, y'_{tar} is the in plane scattering angle relative to the spectrometer central angle, y_{tar} is the in plane position of the event in the spectrometer

coordinates, and δ is the fractional momentum deviation of the detected particle with respect to the central momentum of the hadron arm spectrometer p_c ($\delta = 100*(p_K - p_c)/p_c$). Finally, N is the order of the matrix ($N = 5$ for the HMS, and $N = 6$ for the SOS).

A special set of data was taken to build the elements of the reconstruction matrix in Eq. 3.1. For that, electrons were scattered off a thin foil of carbon, and a special “sieve slit” replaced the spectrometer collimator. The sieve slit is a thick plate with special pattern of holes drilled in it, as shown in Fig. 3.1.

Fig. 3.1: Schematic drawing of the sieve slit used for collimating the scattered beam in order to calibrate the optics of the focusing spectrometers. Figure extracted from Mohring [1999].

The sieve slit is made of HEVIMET, a special alloy composed of about 90% tungsten, 5% nickel, and 5% copper also known as “radiation stopper”. The scattered electrons passing through the hole pattern of the slit represents many beams on the spectrometer’s focal plane, allowing the fits of the reconstruction matrices according to the known position of the holes. More details about the optimization of the matrix elements are described by Arrington [1998].

3.3 Particle Identification

3.3.1 Electron Identification

Scattered electrons entering the electron arm spectrometer were identified using a combination of Cherenkov detectors and calorimeters. This procedure allowed for the separation of the electrons from the background of negatively charged particles, composed mostly of negative pions.

For the FPI-2 experiment, the SOS played the role of electron arm. The SOS gas Cherenkov detector and the SOS calorimeter were used for e/π^- separation. As discussed in Sec. 2.5.3, the SOS gas Cherenkov detector works as a threshold detector based on the velocity of the particles. In the momentum range of this experiment, electrons in the SOS are traveling above the threshold velocity of this detector and should radiate Cherenkov radiation. π^- , more massive particles, are traveling below the threshold velocity and no Cherenkov radiation is created, allowing for a e/π^- separation from a cut in the signal of this detector. The cut applied for identifying electrons in the SOS with the gas Cherenkov was $N > 0.5$ photoelectron for the analysis in this work.

Such a low cut in the SOS gas Cherenkov still allowed some π^- to pass this cut in the SOS, mainly due to δ -electrons created in this detector and triggering it. The SOS electromagnetic calorimeter described in Sec. 2.5.4 was thus also used for e/π^- separation to optimize the electron identification. An offline cut in the signal of the SOS calorimeter was placed, requiring that the fractional energy deposited $E_{\text{cal}}/E > 0.7$ for electron identification. Here, E_{cal} is the energy of the particle deposited in the calorimeter, while E is its total

energy as determined from tracking.

The efficiencies of these detectors to give a signal for electrons when the cuts defined above are applied were studied with elastic scattering, where a clean sample of electrons in the SOS can be obtained (in general with more than 99% of the particles being electrons). Here, the results from Horn [2006a] are used for FPI-2 as the efficiencies will not change in the present analysis. A summary of the results are presented in Tab. 3.1, and more details on this analysis for FPI-2 can be found in Horn [2006a]. This combination of both detectors allowed for a combined π^- rejection factor of 1:20 in the FPI-2 experiment.

In a similar fashion, the E93-018 experiment used the HMS gas Cherenkov and the HMS calorimeter for e/π^- in the electron arm (HMS). Unlike FPI-2, this experiment was primarily designed for measuring reactions involving K^+ production, and the trigger setup was different. One major difference was that the HMS gas Cherenkov and the HMS calorimeter were used in the trigger, and a hardware cut on these PID detectors was applied as discussed in Sec. 2.6. An additional offline cut on the number of photoelectrons in the HMS gas Cherenkov was placed, requiring that $N > 3.0$. The efficiencies of the electron identification detectors are shown in Tab. 3.1 [Niculescu, 1998b].

Tab. 3.1: Efficiencies for the PID detectors used for electron identification.

Experiment	e arm	Gas Cherenkov	Calorimeter
FPI-2	SOS	$99.92 \pm 0.02 \%$	$99.47 \pm 0.03 \%$
E93-018	HMS	$99.8 \pm 0.2 \%$	$99.7 \pm 0.2 \%$

3.3.2 Kaon Identification

For both FPI-2 and E93-018 experiments, one of the spectrometers was configured to detect positively charged particles, the HMS for FPI-2 and the SOS for E93-018. This spectrometer (the “hadron arm”) accepted all positively charged particles along with the kaons, the particle of interest for this work. A combination of Cherenkov detectors was used to identify events with kaon candidates in the experiments.

In the case of the FPI-2 experiment, the gas Cherenkov detector was used for selecting kaons in the HMS. In the momentum range of this experiment, kaons are traveling at velocities lower than the threshold velocity for Cherenkov radiation in this detector. A cut on this detector was placed requiring the number of photoelectrons in this detector $N < 1$ for accepting possible kaon events. As discussed by Horn [2006a], the efficiency of this cut was estimated from π^- production data where a clean sample of sub-threshold velocity hadrons could be selected. The efficiency of the HMS gas Cherenkov detector for the FPI-2 experiment is shown in Tab. 3.2.

The FPI-2 experiment had no dedicated K^+/π^+ separation beyond coincident time since the kaon contamination remaining after cuts was negligible in the pion phase space. In contrast, in the present analysis the aim is to isolate kaons. This requires a special treatment as the HMS aerogel detector cannot remove pions from the kaon sample. Both π^+ and K^+ are above the velocity threshold of this detector for all kinematic settings of the FPI-2. For this reason, the HMS aerogel detector was not used in this work. Instead, a more detailed study of π^+ background was carried out for the extraction of $p(e,e'K^+)\Lambda$

cross sections from the FPI-2 data, as discussed in Sec. 3.4.9.

For the E93-018 experiment, the π^+ background in the SOS (hadron arm) could be removed by means of the SOS aerogel detector. Good K^+/π^+ separation was achieved with a cut applied to the signal of the aerogel detector requiring the number of photoelectrons to be $N < 3.5$. Niculescu [1998b] studied the efficiency of this cut with π^+ production data, since this efficiency is related to the production of Cherenkov radiation (i.e., the charge and velocity of the particle only). The efficiencies for kaon identification are summarized in Tab. 3.2.

Tab. 3.2: Efficiencies for the PID detectors used for K^+ identification.

Experiment	Hadron arm	Detector	Efficiency
FPI-2	HMS	HMS gas Cherenkov	$99.6 \pm 0.05 \%$
E93-018	SOS	SOS aerogel	$97.4 \pm 0.3 \%$

3.3.3 Coincidence time between spectrometers

In the study of an exclusive reaction with two focusing spectrometers, the time difference between the detection of the electron in the electron arm and a hadron like the K^+ or π^+ in the hadron arm is an important tool to select events. Reactions with different final state hadrons are separated in time due to the different velocities with which those particles enter the hadron arm, triggering it at a later or earlier time compared to the electron arm.

Time-to-digital converters were used to register the relative time between the spectrometers, based on the signal of the hodoscopes paddles. Corrections to this time improved the separation of different reactions. These corrections take into account effects of the sig-

nal propagation time through different sections of the detector, effects of the pathlength of particles in the spectrometer, and also effects of pulse heights triggering the TDC (details of this analysis are explored by Horn [2006a]). With these corrections, the coincidence time resolution is 0.2 ns, which is smaller than the electron beam structure, allowing for a good separation of events with K^+ production from π^+ events.

The coincidence time between the spectrometers is used in the analysis of exclusive reactions to remove the background created by random coincidences between the electron and hadron arms. These random events are triggered by particles from different reactions that happened to reach the spectrometers in the coincidence time window of the $p(e,e'K^+)\Lambda$ events, and also passed all of the other cuts in the analysis. By shifting the time window of the coincidence time cut by a multiple of the beam time structure period, only random coincidences should be accepted by the cuts allowing for the estimation of the number of events that are expected to be from random coincidences in the real coincidence time window. In this analysis, several shifted time windows were considered, and the mean yield from these time windows is averaged for the subtraction of the random coincidences.

An example of the cuts applied for selecting real events and for studying random coincidences is shown in Fig. 3.2. The same procedure was used in the analysis of both FPI-2 and E93-018 experiments.

The experimental yields were obtained by subtracting the random coincidence yield from the real yield. The experimental yield Y is then calculated as

Fig. 3.2: Selection of the “real” coincidence time window where $p(e,e'K^+)\Lambda$ events were studied. In addition, yields of random (“rand”) coincidences were studied by selecting shifted time windows. The selection of shifted windows to the left or right of the main peaks was found to give the same results. Events where the coincidence time is consistent with protons and pions in the hadron arm were also detected, as illustrated by the “p” and “ π^+ ” peaks.

$$Y = Y_{real} - \frac{\sum_{i=1}^{N_w} Y_{i-random}}{N_w}, \quad (3.2)$$

where $Y_{i-random}$ is the yield in the i^{th} shifted time window containing random coincidence events only and N_w is the number of windows. For the final analysis of FPI-2 experiment, three shifted time windows were considered for the analysis of random coincidences (as shown in Fig. 3.2), while five shifted time windows were considered for the E93-018 experiment. An example of the yield distribution is shown in Fig. 3.5. Selecting shifted windows to the left or right of the main peaks or increasing the number of windows do not produce

any effect in the results.

As the coincidence time between the electron and the hadron spectrometers is part of the set of cuts to select $p(e,e'K^+)\Lambda$ events, inefficiencies due to this cut were also evaluated. A valid coincidence event is flagged when a Time-to-digital converter (TDC) is triggered by a hit in the HMS as a start and event(s) from the SOS stops this TDC in a time window within 100 ns. However, there are circumstances where an event of interest is not within the usual coincidence time window of real events due to another particle triggering the TDC instead. These events will be lost due to the coincidence time cut applied in the analysis, also known as “coincidence blocking” of the events. The correction for the coincidence blocking was previously studied by Horn [2006a] and it is used in present work.

3.3.4 Missing mass cuts

The exclusivity of the FPI-2 and the E93-018 experiments, i.e., the fact that the momenta of two out of three final state particles (scattered electron and K^+) are measured, allows for the reconstruction of the event’s missing mass (M_x). This calculation is based on the fundamental requirement of conservation of four-momentum (i.e., energy and momentum conservation) in the reaction. By applying a cut on the missing mass, kaons from the $p(e,e'K^+)\Lambda$ reaction could be selected by requiring the mass to be that of the produced hyperon, the Λ particle.

As will be further discussed in Chapter 5, the Λ peak in the missing mass distribution is asymmetric. The tail on the right of the peak is due to light charged particles, e.g., the incident and the scattered electron in the $p(e,e'K^+)\Lambda$ reaction, emitting photons when they

interact with the electromagnetic field of other particles in the medium (target, detector, etc). As the energy of these radiated photons is “missing”, i.e., not detected for the purpose of four-momentum conservation, the missing mass distribution of the $p(e,e'K^+)\Lambda$ is smeared mostly to the right of the Λ peak. For that reason, an asymmetric cut in M_x was applied in the analysis. Further details on radiative processes can be found in Sec. 4.2.5.

3.4 Systematic corrections to the experimental yields

Besides event reconstruction, the data analysis includes many other aspects, such as determination of detector calibration constants, trigger and tracking efficiencies, and computer and electronic lifetimes. The efficiencies are an important correction to the data, since the ultimate goal of this analysis is to obtain the kaon production cross section that is proportional to the number produced of $p(e,e'K^+)\Lambda$ events, and inefficiencies are considered as systematic corrections. These systematic corrections are discussed in the next sections, according to the particularities of each experiment.

3.4.1 Trigger efficiency

As discussed in Sec. 2.6, the pretrigger of the individual spectrometers during the FPI-2 and the E93-018 experiments is driven by the coincidence of the scintillator planes in each spectrometer. The coincidence logic of each spectrometer was configured to pretrigger when three out of the four scintillator planes gave a signal. With this logic, effects from possible inefficiencies of individual planes (for example in cases where particles hit the edge of a scintillator in one plane, therefore resulting in no signal in that scintillator) could be

reduced, resulting in an overall high efficiency (greater than 99%).

The trigger efficiency of each spectrometer is defined as the probability of at least three out of the four ($P_{3/4}$) scintillator planes registering a hit when a particle is incident in the spectrometer. This probability can be expressed as

$$\begin{aligned} P_{3/4} = & P_1 P_2 P_3 P_4 + P_1 P_2 P_3 (1 - P_4) + P_1 P_2 (1 - P_3) P_4 + \\ & + P_1 (1 - P_2) P_3 P_4 + (1 - P_1) P_2 P_3 P_4, \end{aligned} \quad (3.3)$$

where P_i is the probability of the i^{th} plane of scintillator to give a signal when a particle hits it. The probabilities were estimated from the data analysis on a run-by-run basis. First, a subset of events where three specific planes have hits registered is selected in that particular run. The probability P_i of the fourth plane is then calculated from the ratio of the number of events that have hits in the fourth plane over the total number of events in this subset. The typical trigger efficiency for both spectrometers during the E93-018 experiment was on the order of 100.0%, and about 98.6% during the FPI-2 experiment.

Additional detectors may be associated with the trigger, such as requiring a signal from the Gas Cherenkov for the electron arm pretrigger. The efficiencies of these additional components of the data acquisition were also considered in the analysis of the experiments, and they are discussed individually in the next sections of this chapter.

3.4.2 Tracking efficiency

The reconstruction of the position and direction of the particles inside the spectrometers is of fundamental importance for the analysis of the data, since these are used

to reconstruct the particles' four-momenta and, ultimately, the reaction kinematics. As presented in Sec. 2.5.1, the drift chambers of the SOS and the HMS spectrometers are the detectors used to track the particles' trajectories.

The spectrometer tracking efficiency can be divided in two classes: hardware and software efficiencies. The first is related to the capability of the drift chambers themselves to produce and collect a signal when a charged particle crosses their planes. After that, the tracking reconstruction algorithm in the REPLAY software calculates the particles' trajectories in the spectrometers.

For a track to be reconstructed, at least five planes of the drift chambers need to have registered a hit. If there are more than 25 hits in the chamber within the time window of a single event, the tracking is rejected by the analysis software. This, therefore, is the most important factor limiting the rates in the data taking.

To study the fraction of tracks in the drift chamber that the algorithm properly reconstructed, i.e. the tracking efficiencies, a clean sample of known particles was selected with the use of tight cuts using the scintillators. The tracking efficiency is dependent on the rate of particles in the spectrometer, as higher event rates increase the probabilities of multiple hits in the chamber. For that reason, the tracking efficiency was calculated and corrected for individual runs with different rates. The typical corrections for the E93-018 and the FPI-2 experiments are summarized in Tab. 3.3.

Other corrections related to the drift chambers were also considered, but they were analyzed apart from the tracking efficiency. For example, tracks that could not produce a

Tab. 3.3: Typical tracking efficiencies of the drift chambers and the tracking algorithm during the E93-018 and FPI-2 experiments.

Experiment	SOS	HMS
FPI-2	0.96-0.98	0.96-0.99
E93-018	0.83-0.93	0.91-0.98

valid trigger are considered by the electronic dead-time correction (Sec. 3.4.3). The computer dead-time takes into account tracks lost by the data acquisition, and this is explored in Sec. 3.4.4.

3.4.3 Electronic Dead Time

The electronic architectures of the modules used to discriminate signals from the detectors and create triggers for the HMS and SOS spectrometers have an inherent dead-time. Once a pretrigger is formed, the trigger gate bit is lifted and all of the information from the detectors of that spectrometer is recorded to register the event. For the duration of the trigger gate, all of the signals from the detectors are associated to that unique event and the electronic modules are blind (dead), i.e., no other hit can be recorded as a new event. The number of $p(e,e'K^+)\Lambda$ events detected need to be corrected for the events lost by the blindness of the electronic, the “electronic dead time”.

Since the duration of one event gate (τ) is fixed, the number of events lost due to electronic dead-time can be estimated from the random nature of the events in time. When independent events are produced with a mean rate R , a Poisson distribution of these events in time is expected. The probability $P(t)$ for events occurring within a time interval t is

$$P(t)dt = Re^{-Rt}dt. \quad (3.4)$$

The electronic dead time (EDT) can be estimated from Eq. 3.4 by integrating the probability of another event happening within this time interval. EDT can be evaluated as

$$EDT = \int_0^\tau Re^{-Rt}dt = 1 - e^{-R\tau}. \quad (3.5)$$

Typical rates in the individual spectrometers are on the order of hundreds of kHz. The gate width for the FPI-2 experiment was 60 ns (50 ns for E93-018 experiment), giving a small number for $R\tau$ ($\approx 10^{-6}$). For simplicity, Eq. 3.5 can be approximated as

$$EDT \approx R\tau. \quad (3.6)$$

As the true rate of events in the spectrometers is not known, Eq. 3.6 can be used as a first order approximation of the expected dead time during an experiment. A dedicated logic was prepared to measure the electronic dead-time of the experiments. This logic, built up of scalers, records the accumulated number of pretriggers that would have triggered each spectrometer using different gate widths. For FPI-2, the true number of events that hit the spectrometer from the different scalers was calculated here using

$$N_{true} = N_{measured} + \left(\frac{N_{PRE100} - N_{PRE150}}{50 \text{ ns}} \right) \cdot 60 \text{ ns}, \quad (3.7)$$

where N_{PRE100} and N_{PRE150} are the total number of events counted by the scalers configured with gate width of 100 ns and 150 ns respectively, and $N_{measured}$ is the number of events detected from the actual 60 ns gate logic used in this experiment. The electronic dead-time is then

$$EDT \approx \frac{6}{5} \cdot \left(\frac{N_{PRE100} - N_{PRE150}}{N_{PRE100}} \right), \quad (3.8)$$

where $N_{PRE100} \approx N_{true}$ was assumed.

The same principle was used for the E93-018 experiment, except that the gate width of this experiment was set to 50 ns and scalers with 30, 60, 90, and 120 ns were used to measure the electronic dead-time.

The electronic dead time correction factor was applied to the data on a run-by-run basis for both experiments. The ranges of electronic live times (defined as the complement of dead time, $1 - EDT$) are summarized in Tab. 3.4.

Tab. 3.4: Ranges of electronic live time during the E93-018 and FPI-2 experiments. The numbers denote the fraction of events expected to produce a valid pretrigger in the spectrometer.

Experiment	SOS	HMS
FPI-2	0.997-1.000	0.970-0.998
E93-018	0.973-0.990	0.996-1.000

3.4.4 Computer Dead Time

Once a coincidence of the hodoscope planes is formed in the electronics' logic, a pre-trigger is formed and the signals from the detectors are integrated over the time of the gate

width. The scalers of the spectrometers are incremented to register this pretrigger, and a signal is sent to the 8LM module indicating a new hit in the spectrometer. The recording of this hit as an event by the computers depends on whether the Trigger Supervisor was able to accept a new hit and save the event. If the Trigger Supervisor is busy processing an earlier event (or, in the case of the E93-018 experiment, if it has reached its limits of buffering events), a veto is sent to the 8LM unit and the new event is lost, since it cannot be processed.

The computer dead time (CDT) is a correction factor applied to the experimental yields to take into account the fraction of events lost by the 8LM, when it was vetoed by the Trigger Supervisor. The CDT can be directly estimated from

$$CDT = 1 - \frac{N_{trigger}}{N_{pretrigger}}, \quad (3.9)$$

the ratio of the number of triggers saved by the computer ($N_{trigger}$) over the number of pretriggers registered by the scalers ($N_{pretrigger}$). This is a good estimate of CDT given that the scalers are not affected by the veto from the Trigger Supervisor, being able to count all the hits created by the electronics.

The CDT is a correction factor independent of the EDT and both corrections were applied to the experimental yields. The electronic dead time is related to the electronic modules not being able to produce independent triggers for events happening too close to each other in time, with a time difference smaller than the gate width. This limits the data taking on the order of 50-60 ns, and both the pretrigger and the trigger lose this information.

On the other hand, the *CDT* is related to the capability of the data acquisition system to read and save the data from all of the TDCs and ADCs when a trigger is formed, registering the signal from all the detectors. Given the number of channels in the spectrometers, this process takes many microseconds to happen. Buffering the signals is a possibility to reduce *CDT*, as done during the E93-018 experiment, but *CDT* still played an important role in that case.

CDT is rate dependent and was calculated on a run-by-run basis. A summary of the typical *CDT* applied to the FPI-2 and E93-018 analysis are presented in Tab. 3.5.

Tab. 3.5: Ranges of computer dead time estimated for the E93-018 and FPI-2 experiments. This is the estimated fraction of events not recorded due to CDT, and changes in a run-by-run basis depending mostly on trigger rates.

Experiment	Computer dead time
FPI-2	0.01–0.40
E93-018	0.01–0.09

3.4.5 Role of aerogel detector in the analysis

The aerogel detector is a threshold detector based on the velocity of the particle, giving a signal for those particles traversing it with velocity above the threshold velocity of the aerogel. For the E93-018 experiment, the aerogel detector was part of the trigger in the hadron arm for π^+/K^+ separation. As discussed in Sec. 2.5.5, this threshold detector is expected to produce a signal for charged pions in the settings of this experiment. This allowed using the aerogel detector as a veto to the pretrigger of the SOS for π^+ , separating these events from charged kaons.

Although the kaons from the $p(e,e'K^+)\Lambda$ reaction were always below the threshold velocity of the aerogel for the settings of the E93-018, a fraction of all kaons produced a signal in the aerogel detector. The main reason for that is the production of δ -electrons in the spectrometer, which can be above the threshold velocity and produce a signal in the detector. In this case, a fraction of the $p(e,e'K^+)\Lambda$ events were also vetoed from the trigger.

To correct for events lost due to δ -electrons, a dedicated set of runs was configured for studying the aerogel detector response. Charged pions were used for this study since it was possible to select a clean sample of pions in the spectrometer without using the aerogel detector itself. To keep the conditions of this study similar to those of the actual $p(e,e'K^+)\Lambda$ production, the momentum of the spectrometer was set such that the velocity distribution of pions and kaons were similar (0.91c and 0.95c). With this condition set, the pions were not above the threshold velocity and should not have created a signal in the aerogel detector. The fraction of pion events that created a signal above 3.5 photoelectrons in the aerogel detector, mostly from the production of δ -electrons, was considered to be the efficiency correction for the aerogel detector. In the E93-018 experiment, this correction was on the order of 2.6 %.

As the FPI-2 experiment was not primarily designed for measuring K^+ in the hadron arm, the aerogel detector was not configured for π^+/K^+ separation and both π^+ and K^+ particles were above the threshold velocity for all kinematics settings. Therefore, the aerogel detector was not used in the analysis of $p(e,e'K^+)\Lambda$ events for FPI-2. A special analysis for subtracting the charged pion contribution was done in the present work based on the

missing mass distribution, as discussed in Sec. 3.4.9.

3.4.6 Target properties

The number of scattering centers in the path of the incident electrons is directly proportional to the target length and density. Uncertainties and systematic corrections of these properties were considered in the analysis of the cross sections of both experiments.

For the E93-018 experiment, the target of liquid hydrogen was measured to be 4.36 cm long, with an uncertainty of 0.2%. The density of liquid hydrogen for that experiment is 72.3 mg/cm^3 when no beam is present.

For the FPI-2 experiment, the cold LH2 target cell was 3.98 ± 0.01 cm in length. Effects changing the effective target length seen by the beam such as those from the rastering and from deviation due to beam alignment in different settings of the experiment were also considered on a run-by-run basis, as estimated by Horn [2006a]. All of these effects combined produced a correction with a maximum of $1.5\% \pm 0.05\%$ in the target length.

Finally, when the electron beam is incident on the target, the density of the liquid hydrogen decreases due to energy loss of electrons in the target. Electrons with few GeV of energy deposit on average 0.35 MeV/cm in liquid hydrogen, heating it up and reducing its density by $(3.8 \pm 0.4)\% / 100 \text{ } \mu\text{A} / \text{mm}$ of raster amplitude [Gustafsson, 1996]. Dedicated runs were taken for quantifying these effects on the specific conditions of the experiments by running the same experimental conditions with different beam currents. The corrections to account for target density reduction was applied in a run-by-run basis, depending on the beam currents, with typical values on the order of $0.8\% \pm 0.4\%$. Additionally, a correction

of $0.2\% \pm 0.2\%$ was also applied to correct for the purity of the target on E93-018 only.

3.4.7 Kaon absorption in the detector

The passage of kaons through different materials on their way towards the hadron spectrometer reduces the detection efficiency for $p(e,e'K^+)\Lambda$ events. This is because kaons can undergo nuclear interaction with the nuclei of these materials, and they can be “absorbed” by the components of the detectors themselves, the target, and some chamber windows. For this reason, a fraction of the kaon events will be lost before reaching the detectors needed to trigger the spectrometer.

An efficiency factor was estimated to correct the experimental yields for these lost events. The fundamental idea to calculate this factor is to compute the probability of a charged kaon to pass through a material of thickness t without being absorbed. This probability can be understood as a transmission factor T , defined as

$$T = e^{-t/\lambda}, \quad (3.10)$$

where λ is the absorption length of charged kaons in a particular material. The absorption length is defined as

$$\lambda = \left(\rho_V N_A \frac{\sigma_A}{A} \right)^{-1}, \quad (3.11)$$

where ρ_V is the density of the material, N_A is the Avogadro’s number, and σ_A is the kaon absorption cross section for a material with atomic mass A .

The concept of the kaon absorption cross section (σ_A) is related to the probability of a kaon to interact with a nucleus in a way that the event cannot be reconstructed anymore. σ_A was estimated from the kaon-nucleus interaction cross section from the Particle Data Group [Patrignani et al., 2016], where both the total interaction cross-section σ_{total} and the elastic cross section $\sigma_{elastic}$ are given. The total cross section is defined as the sum of all of the hadronic interaction between the kaon and the nucleus. However, describing the absorption cross section using σ_{total} alone would overestimate the number of events lost by kaon absorption. σ_{total} also includes the elastic cross section, and most of the kaon-nucleus elastic scattering is expected to be in the very forward direction, therefore likely to have the scattered K^+ still reaching the detector hut and be detected. In most of the cases of elastic interaction, the kaon practically does not change its momentum and still allows for the $p(e,e'K^+)\Lambda$ reconstruction. For that reason, the kaon-nucleus inelastic cross section ($\sigma_{inelastic} = \sigma_{total} - \sigma_{elastic}$) was considered as a better approximation for the evaluation of σ_A .

Finally, the A dependence of the cross section was parametrized as

$$\sigma_A = \sigma_0 \cdot A^\alpha, \quad (3.12)$$

where σ_0 and α were fit from experimental data. Experimental data available for the interaction of K^+ with heavier nuclei are scarce. For that reason, σ_0 was approximated as the inelastic cross section of the kaon-proton interaction, calculated from the subtraction of the total and elastic cross section shown in Fig. 3.3.

Fig. 3.3: Interaction cross section of K^+ with proton as function of kaon momentum (in the laboratory frame). Total and elastic data were extracted from Patrignani et al. [2016], and the lines denote polynomial fits of these cross sections.

σ_0 was fitted from the data shown in Fig. 3.3 with polynomial functions of the form $\sigma_0 = \sum a_j (p_K^{lab})^j$. The coefficients a_j are shown in Tab. 3.6.

Tab. 3.6: Coefficients of the polynomial fit to the kaon absorption cross section for K^+ -proton interaction ($\sigma_0 = \sum_i a_i (p_K)^i$). This fit was performed by approximating σ_0 to be the total K^+ -proton cross section subtracted of the elastic cross section data from Fig. 3.3.

	$p_K^{lab} \leq 0.75 \text{ GeV/c}$	$0.75 < p_K^{lab} \leq 1.5 \text{ GeV/c}$	$1.5 < p_K^{lab} \leq 8.0 \text{ GeV/c}$
a_0	0.0	-14.6	-2.00
a_1	—	129.1	11.14
a_2	—	-322.8	-3.10
a_3	—	243.1	0.40
a_4	—	93.3	-0.02
a_5	—	-177.1	—
a_6	—	53.4	—

The available experimental data for the A -scaling factor α shown in Fig. 3.4 span

several orders of magnitude in momentum. The calculation of the kaon absorption efficiency was performed with two values, for $\alpha = 0.78$ and 0.94 , and uncertainties were estimated to account for this range.

Fig. 3.4: Experimental data of the α coefficient used to parametrize the interaction of K^+ with heavier nuclei as described by Eq. 3.12. Data are from a recent JLab experiment [Nuruzzaman et al., 2011].

The total transmission of kaons through all of the materials in their path was then calculated as a combination of consecutive transmissions using Eq. 3.10 through the i^{th} material of the target, chambers, and spectrometer detectors. This results in a product of transmissions, that can be expressed as

$$T = \prod_i e^{-t_i/\lambda_i} = e^{-\sum_i t_i/\lambda_i}. \quad (3.13)$$

The different materials considered for the calculation of the kaon absorption correction

for the FPI-2 experiment are listed in Tab. 3.7, where the HMS was used as the hadron arm. For the E93-018, where the SOS was the hadron spectrometer, and the materials used in the kaon absorption calculation are listed in Tab. 3.8. The kaon absorption correction was applied on an event-by-event basis, since the momentum of K^+ in the acceptance of the spectrometers changes significantly. The kaon transmission in both experiments calculated for this analysis ranged between 96.0 and 99.9%, with a maximum uncertainty of 2.1%.

Tab. 3.7: List of materials in the path of charged kaons going towards the HMS spectrometer. These materials were included in the estimate of the fraction of events lost by absorption and for modeling multiple scattering in the HMS. Adapted from Horn [2006a].

Element	Component	Material	Thickness (cm)	Density (g/cm ³)	X0 (g/cm ²)	d/X0 (%)
Target cell	Target	LH2	3.968	0.0723	61.28	0.468
	Can window	Al	0.013	2.7	24.01	0.146
Target chamber	Window	Al	0.0406	2.7	24.01	0.457
	Air gap	Air	15	0.00121	36.66	0.050
HMS windows	Entrance window	Kevlar	0.0381	0.74	55.2	0.051
		Mylar	0.0127	1.39	39.95	0.044
	HMS exit window	Ti	0.0508	4.54	3.56	6.478
Dipole-DC gap	Air gap	Air	35	0.00121	36.66	0.116
HMS drift chamber	Window	Mylar	0.01	1.39	39.95	0.035
	Gas	Ar/C ₆ H ₆	1.8	0.00154	27.38	0.010
	Sense wires	Tungsten	0.00007068	19.3	6.76	0.020
	Field wires	Be/Cu	0.00648	5.4	38.88	0.090
HMS S1	X plane	Polystyrene	1.067	1.03	43.8	2.509
	Y plane	Polystyrene	1.067	1.03	43.8	2.509
HMS gas Cherenkov	Windows	Al	0.204	2.7	24.01	2.294
	Gas	C ₄ F ₁₀ 135	0.00246	23.7	1.401	
	Mirror support	Rohacell	1.8	0.05	40.88	0.220
DC-S2 gap	Air gap	Air	83.87	0.00121	36.66	0.277
HMS S2	X plane	Polystyrene	1.067	1.03	43.8	2.509
	Y plane	Polystyrene	1.067	1.03	43.8	2.509

Tab. 3.8: List of materials in the path of charged kaons going towards the SOS spectrometer. These materials were included in the estimate of the fraction of events lost by absorption and for modeling multiple scattering in the SOS. Adapted from Horn [2006a].

Element	Component	Material	Thickness (cm)	Density (g/cm ³)	X0 (g/cm ²)	d/X0 (%)
Target cell	Target	LH2	3.37	0.0723	61.28	0.389
	Can window	Al	0.013	2.7	24.01	0.146
Target chamber	Window	Al	0.0203	2.7	24.01	0.228
	Air gap	Air	15	0.00121	36.66	0.050
SOS windows	Entrance window	Kevlar	0.0127	0.74	55.2	0.017
		Mylar	0.0076	1.39	39.95	0.026
	Exit window	Kevlar	0.0381	0.74	3.56	0.792
		Mylar	0.0127	1.39	39.95	0.044
Dipole-DC gap	Air	149	0.00121	36.66	0.492	
SOS drift chamber	Cathode	Mylar	0.0088	1.39	39.95	0.031
	Gas	Ar/C ₆ H ₆	3.7068	0.00154	27.38	0.021
	Sense wires	Tungsten	0.0024	19.3	6.76	0.685
	Cathode	Mylar	0.0088	1.39	39.95	0.031
	Wires	Tungsten	0.0024	19.3	38.88	0.119
	Gas	Ar/C ₆ H ₆	3.7068	0.00154		
SOS S1	Y plane	Polystyrene	1.03	1.1	43.8	2.587
	X plane	Polystyrene	1.03	1.04	43.8	2.446
SOS gas Cherenkov	Windows	Al	0.06	1.39	24.01	0.347
	C ₄ F ₁₀	100	0.00246	23.7	1.038	
	Mirror support	Rohacell	9	0.05	40.88	1.101
SOS S2	X plane	Polystyrene	1.04	1.03	43.8	2.44
	Y plane	Polystyrene	1.1	1.03	43.8	2.587
Aerogel	Entrance window	Al	0.159	2.7	24.01	1.788
	Aerogel	SiO ₂	8	0.2	44.054	3.632

3.4.8 Dummy target subtraction

The incident electron beam interacts with all of the materials in its path, and any element in the region of the spectrometer pivot is expected to react with the beam and scatter particles into the spectrometer acceptance. The liquid hydrogen target used in the experiments is discussed in Sec. 2.3. The target has two aluminum windows in the path of the beam that scatter particles and produce events that can pass all of the cuts applied and impact the actual $p(e,e'K^+)\Lambda$ production yields.

A set of runs was taken to quantify the contribution of the target windows to the measured yields. The setting of the spectrometers was the same as in the actual $p(e,e'K^+)\Lambda$ production runs, but an empty target can - “dummy target” - was used to measure this effect. This dummy target was built with walls thicker than the actual target, which allows the data to be accumulated at higher rates than the actual $p(e,e'K^+)\Lambda$ production runs. The dummy target used during the FPI-2 experiment had windows 7.773 times thicker than the actual windows, while the dummy target windows used during the E93-018 experiment were 9.27 times thicker. Since the windows are thicker, the yields measured with the dummy targets were scaled down by these relative thicknesses to reproduce the yields of the actual liquid hydrogen (LH2) target can.

Analogous to the runs with LH2 target, particles produced from the dummy target are expected to create events in the real coincidence time window and in random coincidences. The same procedure described in Sec. 3.3.3 was applied to subtract the random coincidence contribution from the runs with dummy target, using Eq. 3.2. All of the other corrections

described before (dead-times, blocking efficiencies, etc) were calculated and applied in the calculation of the dummy target yields. The final $p(e,e'K^+)\Lambda$ production yield was then estimated as

$$Y_{exp} = Y_{LH2} - Y_{dummy}, \quad (3.14)$$

where Y_{LH2} is the random subtracted yield measured with the LH2 target, and Y_{dummy} is the thickness normalized and random subtracted yield measured with the dummy target. An example of the missing mass distribution for the E93-018 experiment, with events from the real coincidence time window and the random coincidences from both LH2 and dummy targets is shown in Fig. 3.5.

The contribution of the dummy target and the random coincidences from both LH2 and dummy targets to the yields was found to be in the range of 2-5% for both experiments.

3.4.9 Other background subtraction

For the FPI-2 experiment, a significant background exists in the region of the Λ missing mass that could not be completely removed with any cut applied to the data using an independent PID detector. Figure 3.6 shows an example of this background in the missing mass distribution for one experimental setting of the FPI-2.

The background under the Λ peak centered at the missing mass of 1.116 GeV/c² in Fig. 3.6 can be best separated with the use of the coincidence time between the spectrometers. Figure 3.7 shows the correlation between missing mass and coincidence time for the

Fig. 3.5: Typical missing mass distribution of events measured with the liquid hydrogen (LH2) target within the real and random coincidence time windows. In addition, the contribution to these distributions from the target window is shown by the dummy distributions, where events in the real and random coincidence time windows were analyzed.

data shown in Fig. 3.6. The region of missing mass and coincidence time considered for the selection of $p(e,e'K^+)\Lambda$ events is highlighted by the lines in this figure.

This background is composed of events where particles lighter than the K^+ entered into the acceptance of HMS, producing a peak in a coincidence time earlier than the $p(e,e'K^+)\Lambda$ events. An example could be pions from the decay of other particles created in the target.

As the FPI-2 experiment was originally designed for the measurement of pion electroproduction cross sections $p(e,e'\pi^+)n$, no dedicated detector for π^+/K^+ separation was used. Cuts applied to the missing mass distribution in the region of the neutron mass were sufficient to separate K^+ events from the $p(e,e'\pi^+)n$ reaction. Additionally, the pion

Fig. 3.6: Missing mass distribution for one setting of the FPI-2, showing the continuous background underneath the peak of the $p(e,e'K^+)\Lambda$ reaction.

Fig. 3.7: Two-dimensional distribution of events of one kinematic setting of the FPI-2 experiment, highlighting the region where events from the $p(e,e'K^+)\Lambda$ reaction are found.

electroproduction cross section is 2-3 orders of magnitude larger than the kaon electroproduction cross section in the phase-space of FPI-2. Any remaining K^+ contamination was thus negligible for the $p(e,e'\pi^+)n$ analysis. On the other hand, the absence of a detector for π^+/K^+ separation in the HMS is an important feature for this work, where the focus is on the $p(e,e'K^+)\Lambda$ reaction.

In contrast to the analysis of the $p(e,e'\pi^+)n$ reaction, a careful study of this background was necessary for the extraction of the yields of the $p(e,e'K^+)\Lambda$ reaction. This study was done investigating the dependence of the background with kinematic quantities important for the extraction of the cross sections, such as ϕ , Q^2 , t , and W .

To obtain the background, the missing mass distribution was fitted with functions representing the sum of the background and a peak centered at the Λ mass, as shown in Fig. 3.8 for one of the ϕ bins.

Three different functions were used to estimate the background: 1) a straight line added to a Gaussian peak; 2) a 3^{rd} degree polynomial added to a Gaussian peak; and 3) a 3^{rd} degree polynomial added to a Lorentzian peak. Once the missing mass distribution was fitted with each of these curves, the number of $p(e,e'K^+)\Lambda$ events was calculated by subtracting the total number of events in the (t,ϕ) bin and the fitted background (area under the background curve) within the limits of the missing mass cut.

Fig. 3.8: Fit of the background with different functional forms summed to the peak of the $p(e,e'K^+)\Lambda$ reaction. The experimental yield for $p(e,e'K^+)\Lambda$ is then the subtraction of the integrated yield highlighted in the red region from the fitted background underneath the same region of missing mass.

Chapter 4

Modeling the experimental yields with Monte Carlo simulation

This chapter details the simulation of the different aspects of the experiments using the JLab/Hall C Monte Carlo package, which is used in the extraction of the kaon electroproduction cross section from the experimental yields.

The extraction of the $p(e,e'K^+)\Lambda$ cross sections from the measured experimental yields involves many coupled degrees of freedom. The yield distributions built from the individual event kinematic reconstruction are directly related to the cross section of the reaction of interest, but effects such as radiative processes, particle decay, and spectrometer optics also play an important role in these distributions. Disentangling the reaction cross section from these other effects is not a linear problem, and Monte Carlo simulations have been shown

to be a powerful tool to do that. For the purpose of the analysis presented in this work, the SIMC simulation code was used for the analysis of both FPI-2 and E93-018 data.

SIMC is a Monte Carlo simulation package developed for the analysis of JLab-Hall C experiments. This code was adapted from the SIMULATE code, originally designed for the SLAC experiment NE18 [Makins, 1994]. After its first version had been adjusted to describe the Hall C spectrometers, SIMC has evolved, incorporating different aspects of the experiments in the Monte Carlo simulation. The latest version of SIMC allows for the simulation of rastering and energy spread of the beam, radiative processes, hadron decay, and finite resolution of the spectrometer tracking reconstruction. Moreover, SIMC can be used for simulating polarized and unpolarized target configurations, and several reactions such as $(e,e'p)$ and semi-inclusive and exclusive meson production. SIMC does not simulate the detectors' responses to the particles. SIMC is designed to simulate the production of events and propagation of particles through the spectrometers, and once the particles reach the detector hut of the spectrometers, the events are accepted and saved.

The basic principle of SIMC is first to create events for the reaction of interest by randomly generating sets of initial position and kinematic quantities of the electron-proton interaction in the target region. The created particles are then propagated through the target and sent to the spectrometer modules, where they drift through the different sections of the spectrometers and are bent and focused by the magnetic optics elements. Multiple scattering, energy loss, particle decay, and radiative effects are considered throughout the particles paths, allowing them to change direction and momentum as detailed in the fol-

lowing subsections. Finally, if the particles reach the detector hut of the spectrometer, the event's kinematics are reconstructed in a similar manner as the data and the event is saved. Each saved event is weighted by the model cross section corrected for radiative processes, luminosity factor, and a Jacobian to transform from cartesian to spherical angles.

The use of a Monte Carlo based simulation in this analysis was important to properly account for the effects of different aspects of the experiments in the simulated distributions of the kinematics variables. For example, multiple scattering spreads the angular distribution of the detected particles, smearing most of the kinematic distributions. Additionally, particle energy loss and radiative corrections are known to produce asymmetries in some of the kinematic distributions, since they produce an additional “missing energy” in the reaction. By accounting for these effects in SIMC, a better representation of the experiment could be achieved allowing for a precise extraction of the cross sections, and therefore also providing the possibility of studying these cross sections in different bins within the acceptance of the spectrometer.

4.1 Event generation

The SIMC event generator first creates the interaction point in the target using a beam-coordinate system (x_B, y_B, z_B) , taking into account the target length and the raster amplitudes. The electron beam energy E with its associated spread ΔE (considered as with an RMS value of 0.5% in this work) are taken as input, as well as the spectrometer angles and central momentum settings. Events with random electron and kaon angles and

momenta are created, completely defining all of the variables of the five-fold phase space of the $p(e,e'K^+)\Lambda$ production cross section.

The kinematics of the simulated events are generated in SIMC with a range broader than the spectrometer acceptance. Particles can undergo processes like multiple scattering and energy loss, and some events initially created outside of the acceptance of the spectrometers can still be admitted. This allows for a more realistic representation of the experiment yields.

4.2 Particle transport in the spectrometers

Once an event is created in SIMC, both the scattered electron and the kaon are transported out of the LH2 target, reaching the spectrometer entrance.

For each particle (electron or kaon) entering the respective spectrometer (SOS or HMS, depending on the experimental setting), the subroutines of SIMC are called to simulate the spectrometer optics. During the simulation of particle transport, several other processes are called to account for possible changes in their properties. Details of these processes are explored in the following sections.

4.2.1 Spectrometer Optics

The transport of particles through the spectrometers is done by two subroutines, depending on whether or not a magnetic field is present. In regions with no magnetic fields, the particles are drifted towards the next section of the spectrometer. For the sections representing the magnets of the spectrometers, the particles are transported using a COSY

INFINITY model [Berz, 1991]. Effects from multiple scattering and particle decay are also considered in both cases, as explained in the next sections.

The COSY model is built of matrix elements that transform the particle's position (x, y) , direction $(dx/dz, dy/dz)$, and fractional momentum (δ) at the entrance of a magnet region $(z=z_{in})$ to a new set of coordinates at the exit location of that region $(z=z_{out})$. This is done as

$$|x, y, dx/dz, dy/dz, \delta\rangle|_{z=z_{out}} = \left(\prod_i TM_i \right) \cdot |x, y, dx/dz, dy/dz, \delta\rangle|_{z=z_{in}}, \quad (4.1)$$

where up to five transformation matrices $(TM_i, i=1..5)$ are used to model particles transport. These matrices were originally constructed from magnet field maps and optimized with actual data.

4.2.2 Energy loss in materials

Particles propagating through the different sections of the spectrometers interact with the materials in their way and, among other effects, they lose energy. The energy loss is accounted for in SIMC in the process of transporting these particles.

The dominant energy loss mechanism of kaons in materials is the collision or ionization energy loss. In this process, the charged particle traveling through a material interacts with the electronic structure of the atoms and transfers part of its energy ionizing and exciting these atoms. The incoming and scattered electron also loses energy in the collision, but other

effects related to its small mass and the indistinguishability between the traveling electron and the electrons in the material were also accounted. For that, the modified Bethe-Bloch formalism was used to describe the collision energy loss for both the kaon and the electron. The most probable energy loss of the charged particle was modeled as

$$\langle E \rangle = \frac{\xi}{1000} \frac{Z_{eff} d}{A_{eff} \beta^2} \left(\ln(m_e/I^2) + 1.063 + 2 \ln(\gamma\beta) + \ln \left(0.1536 \frac{Z_{eff}}{A_{eff}} \frac{d}{\beta^2} \right) - \beta^2 - \delta \right), \quad (4.2)$$

where $\xi = \frac{2\pi N_A z^2 e^4}{m_e c^2}$, Z_{eff} and A_{eff} are the effective charge and effective mass of the material, d is the thickness traversed in g/cm², m_e is the electron mass, $\beta = v/c$ is the particle's velocity in units of speed of light, $\gamma = (1 - \beta^2)^{-1/2}$, and I is the mean excitation potential of the medium (21.8 eV for LH2), and ρ_V is the density of the material in g/cm³. Here, the energy density correction δ is most important for modeling the energy loss of electrons. It is defined as

$$\delta = \begin{cases} 0, & p/M \leq 1 \\ CO + \ln(\beta\gamma) + |CO/27| * (3 - \log_{10}(\beta\gamma))^3, & 1 < p/M < 10^3 \\ CO + \ln(\beta\gamma), & 10^3 < p/M \leq 10^{4.7} \\ CO + 10.82, & p/M > 10^{4.7}. \end{cases} \quad (4.3)$$

In this equation, $CO = \ln(h\nu_p) - \ln(I) + 0.5$, where $h\nu_p$ is the plasma frequency of the medium — $h\nu_p = 28.816 \cdot 10^{-06} * \sqrt{\rho_V * Z_{eff}/A_{eff}}$ - and I is its ionization potential

(21.8 eV for LH2, or $16 \cdot Z_{eff}^{0.9}$ eV for other materials).

The mean energy loss defined above was smeared in SIMC to account for the probabilistic nature of particle energy loss in matter and its effects on the overall yields distributions. This was done using a Landau distribution function of the form $\lambda_L = -2 \cdot \ln(x_L)$, where x_L is a random number created following a Gaussian distribution centered at zero and with unitary standard deviation. The energy loss E_{loss} is then created in SIMC as

$$E_{loss} = \langle E \rangle + \lambda_L \xi, \quad (4.4)$$

where $\langle E \rangle$ and ξ are defined with Eq. 4.2.

For the incoming and scattered electrons, the emission of radiation due to their deceleration in the medium (Bremsstrahlung) also plays a role in energy loss. The mean energy loss from this process is higher than the collision energy loss in liquid hydrogen when electrons are above 250 MeV. However, this effect has a much broader energy distribution than the collision process due to the nature of this particular interaction of the electrons with the medium. For this reason, Bremsstrahlung effects are accounted with a different approach in SIMC, similarly to other radiative channels of the $p(e,e'K^+)\Lambda$ diagram. This is discussed in Sec. 4.2.5.

The calculation of energy loss of the electrons and kaons was done for every element of the spectrometers and the target. Overall, the energy loss of the particles is on the order of several MeV, where most of it is lost in the target.

4.2.3 Multiple scattering

In addition to energy loss, the interaction of charged particles traversing a medium also produces a small deflection in their direction, an effect known as multiple scattering. This effect is a consequence of many instances of small angle Coulomb scattering of the particle from the nuclei in the medium. Statistically, the accumulation of many small deflections of the particle's direction can be described by a Gaussian distribution with the mean scattering angle of zero degree and a root mean square (RMS) width of θ_0 ,

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} \sqrt{d/X_0} [1 + 0.038 \cdot \ln(d/X_0)]. \quad (4.5)$$

The multiple scattering effect is calculated in SIMC for every segment of material that the particles traverse. The angles defining the direction of a particle traversing a material with thickness d/X_0 (units of radiation length) are changed by a factor $x_L \cdot \theta_0$, where x_L is a random number following a Gaussian distribution centered at zero and with unit width.

From Eq. 4.5, the effects of multiple scattering are seen to be more important for particles with lower momentum. Ultimately, the statistical nature of this process adds to the smearing of the yield distributions simulated by SIMC.

4.2.4 Kaon decay

The kaon has a mean lifetime τ_K of 12.4 ns in its rest frame. In this work, kaons of momenta of few GeV/c were measured in the laboratory frame. From special relativity, their lifetime is dilated by a factor of γ (for example, a kaon traveling at 1 GeV/c has

$\gamma = 2.25$). However, this dilated time is still short and a significant fraction of the produced kaons spontaneously decayed on their path toward the spectrometer's detectors and did not reach the detectors.

Particle decay follows a Poisson distribution, and hence the fraction of kaons with momentum p_K that travel a distance d without decaying is given by the exponential distribution

$$\frac{N(d)}{N_{created}} = e^{-\frac{m_K \cdot d}{p \cdot \tau_K}}, \quad (4.6)$$

where m_K is the kaon mass (493.667 GeV/c²). This equation can also be interpreted as the probability of a single kaon reaching a distance d in the lab frame. This allows for a probabilistic implementation of kaon decay in a Monte Carlo simulation, instead of weighting the events by a global weight factor. The particles created from the decay (daughter particles) can be treated through the simulation, and their contribution to the yields can also be studied.

The K^+ decay was implemented in SIMC to study the fraction of produced $p(e, e'K^+)\Lambda$ that effectively reached the detector hut and were detected. In the simulation of each event, the distance the kaon traveled (z_{decay}) is obtained from a random distance defining where this particular kaon would decay

$$z_{decay} = -\tau_K \beta \gamma \ln(1 - x_U), \quad (4.7)$$

where x_U is a random number generated between 0 and 1 with uniform distribution. In the simulation of each particular section of the spectrometer, the kaon is decayed at the position z_{decay} of that section if $z_{decay} < l$, i.e., if the decay is within the size of that section.

Once the kaon reaches the position z_{decay} , it is decayed according to its decay channels. There are many decay channels physically allowed for a kaon, but only the six most significant channels were implemented in this work. However, these six branches compose more than 99.98% of the phase-space of kaon decay, and the effects of other branches were estimated to be negligible for the purpose of this work. The channels for kaon decay are presented in Tab. 4.1 with the respective branching fraction.

Tab. 4.1: Most probable decay branches for K^+ and their respective branching ratio used in the simulation of kaon decay in SIMC. These channels compose more than 99.98% of the phase-space of kaon decay.

Decay channel	Branching ratio
$K^+ \rightarrow \mu^+ + \nu_\mu$	63.55%
$K^+ \rightarrow \pi^+ + \pi^0$	20.66%
$K^+ \rightarrow e + \pi^0 + \nu_e$	5.07%
$K^+ \rightarrow \mu^+ + \pi^0 + \nu_\mu$	3.353%
$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$	5.59%
$K^+ \rightarrow \pi^+ + \pi^0 + \pi^0$	1.761%

For the decay of a kaon in a particular simulated event, one of the branches from Tab. 4.1 is chosen according to the probability given by the branching ratios. At z_{decay} , the kaon is transformed into one of the charged daughter particles of the chosen branch. The energy and momentum of the original kaon are randomly divided among the daughter particles, and the chosen daughter particle continues to be transported through the sections of the spectrometer. If the daughter particle reaches the detector hut, the event is saved

with a decay flag.

The tracking of daughter particles is important to account for events mimicking kaons, the “mimicked events”. These are events that, in principle, would not be detected by the spectrometer since the original kaon has decayed. However, decayed particles can still create valid triggers, depending on whether the daughter particles passed the optics and acceptance of the spectrometer. Most of the events with daughter particles are eliminated by cuts applied on PID detectors and reconstructed kinematics. But still a fraction of the decayed events, especially those where the kaon decayed after the spectrometer magnets, can be within all of the cuts and contribute to the yields.

4.2.5 Radiative corrections

Charged particles such as e and K^+ can emit real or virtual photons when accelerated and, from conservation of energy, these particles lose energy in this process. This effect is known as “Bremsstrahlung”, a German word meaning “brake radiation”, related to the radiated photons from accelerated charged particles. This effect occurs in the interaction of traveling particles with electric fields, most importantly the fields of the atomic structure in the medium they traverse.

As the reconstructed kinematic quantities of the $p(e,e'K^+)\Lambda$ reaction depends on the energies of the particles in this system, losses due to Bremsstrahlung need to be accounted for an accurate extraction of the experimental cross sections.

The implementation of radiative corrections in SIMC is based on the formalism derived by Mo and Tsai [1969], where the inclusive elastic electron scattering off protons was studied.

Makins [1994] extended this formalism for coincidence elastic scattering reactions, but still scattering off the proton. For modeling the meson electroproduction reaction in SIMC, this formalism was modified such that the target particle is considered to be a stationary proton with the final meson being treated as an “off-shell” proton.

Radiative effects were modeled in two categories: external and internal Bremsstrahlung. The external Bremsstrahlung can be understood as a simple emission of real photons by the particle when interacting with fields along the medium they transverse. In this case, photons are emitted far from the interaction point of the $p(e, e'K^+)\Lambda$ reaction, and the total energy lost by the particle can be approximated by the sum of the energies of the radiated photons. On the other hand, internal Bremsstrahlung models photon radiated in the field of the nucleus where the reaction occurs. In this case, photons can be emitted close to each other (within few multiple of their wavelengths) due to the higher strength of the nuclear electric field, and electromagnetic interference between these photons need to be considered for properly accounting the total energy loss of the particle. Internal Bremsstrahlung is subdivided in two categories, known as soft and hard radiation. In the first, a real photon is emitted, lowering the energy of the radiating particle. Hard Bremsstrahlung radiation is represented by diagrams where the emitted photon is reabsorbed (“vertex corrections”) or in the case of vacuum polarization, where an electron positron pair is created (“one-loop” diagram). The diagrams considered for soft and hard internal corrections are represented in Figs. 4.1 and 4.2 respectively.

Aside from the changes in the kinematics of the particles by the radiative process, the

Fig. 4.1: Diagrams of the soft, first order Bremsstrahlung radiation contributing to radiative corrections used on the analysis of this work.

diagrams that contribute to the measured cross section are also modified. This effect was considered in SIMC by applying a weight factor R_{corr} to the first order kaon electroproduction cross section, where the soft photon and the extended peaking approximations were considered for evaluating its amplitude. The soft photon approximation considers that the energy of the emitted photon is small relative to the energy of the traveling particle, while the extended peaking approximation considers the photon to be emitted collinearly to the

Fig. 4.2: Diagrams of the hard, first order Bremsstrahlung radiation contributing to radiative corrections used on the analysis of this work.

particle direction. With these approximations, R_{corr} is expressed as

$$R_{corr} = (1 - \delta_{hard})\delta_{soft}\Phi_e^{ext}\Phi_{e'}^{ext}, \quad (4.8)$$

where δ represents the internal and external corrections and Φ are improved corrections to the external component. In the limit where Q^2 is greater than the mass squared of the radiating particle m^2 ,

$$\delta_{hard} = \frac{2\alpha}{\pi} \left(\left(1 - \frac{3}{4} \ln \left(\frac{Q^2}{m^2} \right) \right) + \left(\frac{5}{9} - \frac{1}{3} \ln \left(\frac{Q^2}{m^2} \right) \right) \right). \quad (4.9)$$

The soft correction factor, calculated considering the incident electron (e), the scattered electron (e'), and the kaon (K) was taken as

$$\begin{aligned}
\delta_{soft} = & \frac{1}{\Gamma(1+bt_e)} \frac{bt_e + \lambda_e}{k_e^{bt_e}} (k_e k'_e)^{-\lambda_e/2} \frac{1}{E_e^{1-\lambda_e-bt_e}} \cdot \\
& \cdot \frac{1}{\Gamma(1+bt_{e'})} \frac{bt_{e'} + \lambda_{e'}}{k_{e'}^{bt_{e'}}} (k_e k'_e)^{-\lambda_{e'}/2} \frac{1}{E_{e'}^{1-\lambda_{e'}-bt_{e'}}} \cdot \\
& \cdot \lambda_K (M_p k_K)^{-\lambda_K/2} \frac{1}{E_K^{1-\lambda_K}},
\end{aligned} \tag{4.10}$$

where bt_i is related to the screening properties of the material. In this equation, λ are calculated as

$$\begin{aligned}
\lambda_e &= \frac{\alpha}{\pi} \left(\ln \left(\frac{4k_e^2}{m_e^2} \right) - 1 + 2 \ln \left(\frac{k_e}{k_{e'}} \right) + \ln \left(\frac{1 - \cos(\theta_e)}{2} \right) \right) \\
\lambda_{e'} &= \frac{\alpha}{\pi} \left(\ln \left(\frac{4k_{e'}^2}{m_e^2} \right) - 1 + 2 \ln \left(\frac{k_e}{k_{e'}} \right) + \ln \left(\frac{1 - \cos(\theta_e)}{2} \right) \right) \\
\lambda_K &= \frac{\alpha}{\pi} \left(\ln \left(\frac{E_K}{E} + |p_K| E_K - |p_K| \right) - 2 \right).
\end{aligned} \tag{4.11}$$

In Eq. 4.8, the external radiation factors are written as

$$\Phi_i^{ext} = 1 - \frac{bt_i}{bt_i + \lambda_i} \frac{E_i}{|k_i|}, \tag{4.12}$$

where i stands for the incoming electron (e) or the scattered electron (e').

In SIMC, the radiative correction is applied to the incident electron, the scattered electron, and the kaon in a random way, following a distribution of the form

$$G = \frac{gE^{g-1}}{E_{max}^g - E_{min}^g}. \tag{4.13}$$

In this equation, E_{max} and E_{min} are the limits of the generated photon energy, and $g =$

$(\lambda_i + bt_i)$. Once the photon is radiated, the kinematics of the radiating particle is corrected. If the radiating particle is the incident electron, the reaction vertex kinematics are calculated according to the energy after the energy loss.

For the data analysis of both FPI-2 and E93-018 experiments, radiative corrections were important for the simulation of the tails in the yield distributions of missing mass. Energy losses of the particles broadened the Λ peak in this distribution asymmetrically. As the missing mass cut (discussed in Sec. 3.3.4) is defined at some position in the tail of the Λ peak, the simulation of radiative effects was very important for the accurate comparison of experimental and simulated yields. The stability of the extraction of the cross sections with the definition of the missing mass cut will be discussed in Sec. 5.4.3.

4.3 Monte Carlo Yields

The single events that are simulated in SIMC are transported along the various elements of the spectrometers, where forward matrices are used to represent the optics required to focus and bend the particles towards the region of the detector hut. Those events that do not decay along the transport and pass all of the apertures reaching the region of the detector hut are then saved by the simulation as good events. From there, the backwards trajectory reconstruction algorithm project the particles back to the target, calculating kinematics quantities of the event in a very similar way to what is done with the actual data.

After individual events are tracked in SIMC and saved, the distributions of simulated

yields (Y_{MC}) as a function of kinematics quantities can be created in a post-analysis for the extraction of the cross sections. In SIMC, the yields of kaon electroproduction (in units of number of events per millicoulomb of integrated beam current) can be expressed as

$$Y_{MC} = L_H \cdot \int_{cuts} \left(\frac{d^5\sigma}{dQ^2 dW d\phi_e d\Omega_K^*} \right)_{model} A(V) R_{corr}(V) dQ^2 dW d\phi_e d\Omega_K^*, \quad (4.14)$$

where $A(V)$ is a function describing the acceptance of coincidence events (including energy loss, kaon decay, etc), R_{corr} is the radiative correction weight factor, and $(d^5\sigma/dQ^2 dW d\phi_e d\Omega_K^*)$ is the modeled cross section. In this equation, L_H is part of the experimental luminosity, proportional to the number of scattering centers on the path of the beam described as

$$L_H = \frac{\rho_V t N_A}{M_P} N_e, \quad (4.15)$$

where ρ_V and t are the target density (g/cm^3) and thickness (cm), N_A is the Avogadro's number, M_P is the mass of the proton in amu, and N_e is the number of electrons in 1 mC ($10^{-3}/1.60218 \cdot 10^{-19}$).

Similarly to the experimental cross section, the five fold cross section for the model $(d^5\sigma/dQ^2 dW d\phi_e d\Omega_K^*)$ is decomposed in SIMC in terms of the virtual photon flux term and the virtual production cross section as

$$\frac{d^5\sigma}{dQ^2 dW d\phi_e d\Omega_K^*} = \Gamma_\nu(Q^2, W) \cdot \left(\frac{d^2\sigma}{d\Omega_K^*} \right), \quad (4.16)$$

where the photon flux Γ_ν is described by Eq. 1.8. The extraction of the production cross section is discussed in detail in Sec. 4.4.

As SIMC is based on the simulation of random events, the simulated SIMC yields are built from sums of discrete weighted events. For each simulation, a large number of events was created in order to properly sample the entire experimental phase-space considering all of the effects convoluted in Eq. 4.14 (energy loss, kaon decay, radiative corrections, etc). Events were created until a sufficiently large number of events could be reconstructed and saved from the simulation (in this work, $N_T=250,000$ events). To calculate the simulated yields from these events within a bin, Eq. 4.14 was discretized as

$$Y_{MC} = L_H \cdot \Delta^5 V_{gen} \cdot \sum_i \frac{W_i}{N_T}. \quad (4.17)$$

In this equation, $(L_H \cdot \Delta^5 V_{gen})$ is a normalization factor (“normfac”) to account for luminosity and the simulated phase-space, W_i is the weight of the i^{th} event, $W_i = (d^2\sigma/d\Omega_K^*)_i \cdot R_{corr}$, where $(d^2\sigma/d\Omega_K^*)_i$ is the cross section model evaluated for the kinematic of the i^{th} event.

4.4 Cross section modeling

The twofold production cross section $d^2\sigma/d\Omega_K^*$ in Eqs. 4.14 and 4.17 is the cross section for $p(e,e'K^+)\Lambda$ electroproduction described in Sec. 1.4. This cross section is the unknown parameter of the yields, and obtaining it is the ultimate goal of the simulations using SIMC. For that, the cross section model in SIMC is tuned for each particular experiment in order to describe the variations of the cross section across the acceptance of the spectrometers.

The SIMC cross section is tuned iteratively until the differences between the distributions of simulated and experimental yields for the different kinematic quantities are minimized. In particular, distributions of Q^2 , W , and t and also the spectrometer acceptance distributions are analyzed for this minimization process.

The cross section model for the analysis of the E93-018 experiment was tuned following the function form of Eq. 1.10, where the separated cross sections are dependent on kinematic parameters of the events. One function was used to fit each of the separated cross sections, with the form

$$\frac{d\sigma_i}{dtd\phi} = A_i \cdot e^{B_i \cdot |t-t_c|} \cdot \frac{1.0}{(1.0 + C_i * Q^2)/(1.0 + C_i * Q_c^2)} \quad (4.18)$$

where the parameters A_i , B_i , and C_i were fit for the separated cross sections, with $t_c = 0.4 \text{ GeV}^2$ and $Q_c^2 = 1.0 \text{ GeV}^2$. The index i refers to each of the separated cross sections: L, T, LT, and TT. Table 4.2 presents the resulting fitted parameters for the cross sections.

Tab. 4.2: Fitted coefficients for the function shown in Eq. 4.18 used to describe the separated cross section of the E93-018 experiment in SIMC.

Coefficient	σ_T	σ_L	σ_{LT}	σ_{TT}
A_i	2021.99	371.244	-199.112	0.0096
B_i	1.7549	-13.44	-5.11	15.91
C_i	0.973	1.084	1.375	0.726

For the FPI-2 experiment, the formulation for describing the cross sections in SIMC was slightly different. This was necessary because the hadron arm spectrometer was not aligned to the vector $\mathbf{q}$ of the measured $p(e,e'K^+)\Lambda$ reaction, since this experiment was aimed

at measuring the pion production reaction. For that reason, it was necessary to explicitly model the ϕ dependence of the cross section in order to disentangle it from the ϕ coverage of the spectrometers' acceptances. The L/T separated cross section could be obtained only after the unseparated cross sections were tuned. The equations used to describe the terms of the cross sections for this experiment were

$$\begin{aligned}
 \frac{d\sigma_{L,T}}{dtd\phi} &= D_i \cdot e^{-E_i \cdot |t|} \cdot \frac{1.0}{1.0 + F_i \cdot Q^2} \\
 \frac{d\sigma_{LT}}{dtd\phi} &= G_i \cdot e^{-H_i \cdot |t|} \cdot \frac{1.0}{1.0 + Q^4/I_i} \\
 \frac{d\sigma_{TT}}{dtd\phi} &= J_i \cdot e^{-K_i \cdot |t|} \cdot \frac{1.0}{1.0 + (Q^2/L_i)^2}
 \end{aligned} \tag{4.19}$$

The coefficients of Eq. 4.19 were fitted by iterating SIMC and finding the best set of coefficients that produce a simulated yield closer to the experimental yield. The coefficients found for this equation were

Tab. 4.3: Fitted coefficients for the functions of Eq. 4.19 used to describe the cross section of the FPI-2 experiment in SIMC.

Coefficient	Best fit
D_i	13.192
E_i	0.0191
F_i	14.455
G_i	0.13869
H_i	0.0017
I_i	10.237
J_i	4.3436
K_i	0.5250
L_i	0.6181

Finally, after the SIMC cross section was tuned to the equations above, minor differences between the experimental and simulated yields may still exist. To account for this,

the experimental cross section in a given bin was calculated as

$$\left(\frac{d^2\sigma_{EXP}}{dts\phi} \right) \Big|_{(Q^2=\langle Q^2 \rangle, t=\langle t \rangle)} = \frac{Y_{EXP}}{Y_{MC}} \left(\frac{d^2\sigma_{model}}{dtd\phi} \right) \Big|_{(Q^2=\langle Q^2 \rangle, t=\langle t \rangle)}, \quad (4.20)$$

where $\left(\frac{d^2\sigma_{model}}{dtd\phi} \right) \Big|_{(Q^2=\langle Q^2 \rangle, t=\langle t \rangle)}$ is the tuned SIMC cross section evaluated at the center of the bin of interest (from Eqs. 4.18 and 4.19). Then, the model cross section was normalized by the fraction of experimental and simulated yields in that bin. Further details of the resulting cross sections are presented in Chapter 5.

4.5 SIMC and data yields

The iterative tuning of the SIMC cross section was done to obtain the best agreement between SIMC and data yields. At the end of the optimization process, the $p(e, e'K^+)\Lambda$ cross section could be extracted in the different bins of the analysis, allowing for the extraction of the kaon form factor and its charge distribution as discussed in the next chapters. An example of the simulated SIMC yields for the E93-018 experiment is shown in Fig. 4.3 for some variables related to the angular and momentum acceptance of the spectrometers.

As can be seen in Fig. 4.3, the agreement between data and SIMC is within one standard deviation for most of the bins and, therefore, the tuning of the simulation was considered satisfactory. In addition, yields of important kinematic variables reconstructed from both the experimental and simulated events were compared during the cross section tuning. Figure 4.4 shows an example of these yields, comparing data and SIMC results.

Similarly to the acceptance variables, the distributions of the kinematic quantities were

in good agreement between data and simulation. Details of the cross section extraction from these yields are presented in the next chapter.

Fig. 4.3: Typical yield distributions showing the agreement between the reconstructed quantities of measured $p(e,e'K^+)\Lambda$ events (black points) and SIMC (red curve). This example is for the E93-018 experiment (with $Q^2 = 1.0$ (GeV/c) 2 , $\epsilon = 0.81$), where K^+ were detected in the SOS and e in the HMS.

Fig. 4.4: Typical distributions of $p(e,e'K^+)\Lambda$ kinematic yields showing the agreement between the measured events (black points) and SIMC (red curve). This example is for the E93-018 experiment (with $Q^2 = 1.0 \text{ (GeV/c)}^2$, $\epsilon = 0.81$), where K^+ were detected in the SOS and e in the HMS.

Chapter 5

Extraction of Kaon Electroproduction Cross Section

The extraction of the $p(e,e'K^+)\Lambda$ separated cross sections for the E93-018 and FPI-2 experiments are presented in this chapter. The definition of cuts in the analysis and studies for the estimation of systematic uncertainties are also discussed.

The deconvolution of the kaon electroproduction cross section out of the reaction yields measured with the Hall C spectrometers was done using SIMC. As described in detail in Chapter 4, this Monte Carlo-based program simulates the components of the experiment, including the production of $p(e,e'K^+)\Lambda$ events in the target, the propagation of particles through materials of the spectrometers considering effects of energy loss, multiple scattering, particle decay, radiative effects, and the spectrometer acceptance. Kinematic quantities are

reconstructed and physics quantities calculated in the same way as in the data analysis. The events generated in SIMC are also weighted by a cross section model of the reaction of interest, which was optimized for the experiment's kinematic regime. The SIMC yields were compared to those from the experimental data analysis. The events in both data and Monte Carlo yields were selected using a series of cuts. The cuts depend on the specifics of the experiment and also the kinematic region of interest. An iterative fit of the cross section model's parameters was performed until the yield ratio converged. The final cross section was then extracted using the optimized model cross section. A discussion of these procedures follows in the next sections.

5.1 Events selection: summary of cuts

A series of cuts was applied to both the experimental data and the SIMC simulation in order to select events for the analysis of the cross section. These cuts can be classified in two categories, those related to particle identification (PID) and those related to acceptance and physics quantities.

The particle identification system of the spectrometers is based on specialized detectors, as described in Sec. 2.5. The PID cuts were applied based on the signals read from these detectors, either as part of the trigger system that ran in real time with the experiment (also called *hardware* cuts, as for the aerogel detector in the E93-018 experiment - see Sec. 2.6.2) or during the offline data analysis (also known as *software* cut). In general, very low threshold cuts applied to PID detectors are observed to degrade the particle

Tab. 5.1: Summary of cuts used for selecting $p(e,e'K^+)\Lambda$ events in the E93-018 experiment.

Description	PID	cut
SOS momentum acceptance (%)	K^+	$ \delta_{\text{SOS}} < 17.0$
SOS out of plane angular acceptance (rad)	K^+	$ x'_{\text{tar}} < 0.04$
SOS in plane angular acceptance (rad)	K^+	$ y'_{\text{tar}} < 0.10$
SOS Aerogel (NPE)	K^+	$\text{saer_npe} < 3.5$
HMS momentum acceptance (%)	e^-	$ \delta_{\text{HMS}} < 8$
HMS out of plane angular acceptance (rad)	e^-	$ x'_{\text{tar}} < 0.06$
HMS in plane angular acceptance (rad)	e^-	$ y'_{\text{tar}} < 0.06$
HMS Gas Cherenkov (NPE)	e^-	$\text{hcer_npe} > 2.5$
Coincidence time and TOF (ns)	-	$ \beta_{\text{coin}} - \beta_K < 0.04$
Reconstructed missing mass (GeV)	-	$1.1 < M_x < 1.145$
Q^2/W		see Sec. 5.1.1

identification in the analysis, allowing particles other than those of interest to leak in. On the other hand, high threshold cuts can result in the loss of a significant number of good events, degrading the detection efficiency and reducing statistics. For this reason, the PID cuts were optimized to achieve good particle identification while keeping a reasonable detection efficiency. Since SIMC was not designed for simulating the individual detectors of the spectrometers, the PID cuts were applied to the experimental data only. The study of efficiencies which resulted from these cuts are discussed in Chapter 3.

In addition to the PID cuts, other cuts were applied to acceptance and physics quantities in the analysis. These cuts correspond to the angular and momentum acceptances of the spectrometers ($x'_{\text{tar}}, y'_{\text{tar}}, \delta$), and physics quantities such as Q^2 , W , t , ϕ , and the missing mass (M_x). These cuts were always applied to both simulation and experimental data. Lists of the cuts used for the analysis of the E93-018 and FPI-2 experiments are shown in Tab. 5.1 and Tab. 5.2 respectively.

Tab. 5.2: Summary of cuts used for selecting $p(e,e'K^+)\Lambda$ events in the FPI-2 experiment.

Description	PID	cut
SOS momentum acceptance (%)	e^-	$ \delta_{\text{SOS}} < 20$
SOS out of plane angular acceptance (rad)	e^-	$ x'_{\text{tar}} < 0.05$
SOS in plane angular acceptance (rad)	e^-	$ y'_{\text{tar}} < 0.07$
SOS calorimeter energy sum (GeV)	e^-	$\text{ssshsum} > 0.7$
SOS Gas Cherenkov (NPE)	e^-	$\text{scer_npe} > 0.5$
SOS calorimeter tracked (GeV)	e^-	$\text{sshtrk} > 0.7$
HMS momentum acceptance (%)	K^+	$ \delta_{\text{HMS}} < 10.0$
HMS angular acceptance (rad)	K^+	$ x'_{\text{tar}} < 0.09$
HMS angular acceptance (rad)	K^+	$ y'_{\text{tar}} < 0.055$
HMS Gas Cherenkov (NPE)	K^+	$\text{hcer_npe} < 1.0$
Coincidence time (ns)	-	$ \text{cointime} - 0.15 < 0.75$
Missing mass (GeV)	-	$ M_x - 1.12168 < 0.026$
Q^2/W		see Sec. 5.1.1

Some of the acceptance cuts varied depending on the kinematic setting of the experiment. In addition, kinematic binning, e.g. in t and ϕ needed for the L/T cross section separation in the present analysis, varies from experiment to experiment. This required different sets of binning cuts for the analysis of E93-018 and FPI-2 data. Details of these cuts are given along with the analysis of cross sections in the next sections. A discussion of effects of changing these cuts over the extraction of the cross sections follows as well.

5.1.1 Phase-space matching

For a complete pseudoscalar meson L/T cross section separation, data are taken for at least two different beam energies and at different angles with respect to the virtual photon propagation direction. Knowledge of the polarization of the virtual photon (ϵ) and the azimuthal angle ϕ allows for a complete L/T/LT/TT separation. However, the coverage of the phase-space is defined by the acceptance of the spectrometers, which can be slightly

different for each beam energy. For a consistent analysis of the data for the different ϵ settings and to minimize possible effects of systematic errors from the different coverage of the Q^2/W phase-space, a cut was designed to constrain the analysis to the overlapping phase-space region as illustrated in Fig. 5.1. The diamond-like colored regions show all of the events that passed the list of cuts from Tab. 5.1 and Tab. 5.2. Only events in the overlapping regions of the Q^2/W phase-space from two different settings were selected for the extraction of the separated cross sections.

Fig. 5.1: Q^2/W phase-space of all of the kinematic settings analyzed in this work, for both E93-018 and FPI-2 experiments. Only events in the overlapping regions of the Q^2/W phase-space from different ϵ settings were selected for the extraction of the separated cross sections. The diamond-like cut illustrated by the dashed/black lines was designed to select these events. An explanation on how these cuts were created can be found in the text.

A procedure was developed to define the edges of each diamond-like Q^2/W cut shown

in Fig. 5.1 by the dashed black lines. This procedure is based on the fit of the Q^2 and W distributions applied over a rotated and translated phase-space. An example of the Q^2/W cut definition procedure is shown in Fig. 5.2. Here, data from the high Q^2 setting of the FPI-2 experiment were used (red and blue distributions in Fig. 5.1, with all of the cuts from Tab. 5.2 already applied). The first step in defining the limits of the diamond cut was to rotate and translate the Q^2/W phase-space, finding new axes along the major and minor sides of this two dimensional distribution. The position of the central point of the covered Q^2/W region (referred as “O”) was determined from the low ϵ data by calculating it from the mean value of Q^2 and W . A straight line “ α ” was fit to these low ϵ data, and a major axis “M” was defined as having its origin at the point “O” and being parallel to “ α ”. Additionally, the minor axis “m” of the new rotated/translated phase-space was defined as being perpendicular to “M”, and with its origin at “O”.

Figure 5.2 (a) illustrates the definition of “O” and the M- and m-axis. The Q^2 and W values of each event were projected to the new “M” and “m” axes, where one-dimensional event distributions of these new quantities could be fitted with Gaussian curves, as shown in Figs. 5.2 (b) and (c). The cuts in the M/m frame were defined at two sigmas of the Gaussian fits (accepting therefore $\approx 95.5\%$ of the events in the low ϵ setting). The limits of the cut were determined in this M/m frame defining the points A, B, C, and D shown in Figs. 5.2 (b) and (c). A simple rotation/translation of these limits back to the Q^2/W phase-space established the vertices of the diamond-cut, as shown in Fig. 5.2 (d). This cut is then applied to both high and low ϵ kinematics to assure the same coverage of Q^2/W

Fig. 5.2: Procedure for defining the Q^2/W cut; example for the high Q^2 setting of the FPI-2 experiment. In (a), the Q^2/W phase space is rotated and translated by the definition of two new M - and m -axes. Gaussian curves are fit at the distribution of the low ϵ setting in (b) and (c), and the limits of the cut are defined at two sigmas away from the center of the Gaussian (points A, B, C, and D). Finally, the position of these points is reconstructed back into the Q^2/W phase-space, defining the vertices of the diamond-like cut as shown in (d).

phase-space during the extraction of the cross sections.

5.2 Cross section extraction in t and ϕ bins

For the L/T/LT/TT separation of the cross section, the data were binned in t and ϕ . The cross section model in SIMC was optimized to best represent the experimental yields in these bins. After the optimization, differences between model and experimental yields were generally lower than 10%. In the final cross section extraction, each bin was normalized by the ratio of experimental and SIMC yields. The final experimental cross section $\sigma_{exp} \equiv d\sigma_{exp}/dtd\phi$ was extracted as

$$\sigma_{exp} = \frac{Y_{exp}}{Y_{SIMC}} \sigma_{SIMC}, \quad (5.1)$$

where Y_{exp} is the experimental yield integrated over the phase-space and acceptance of the analyzed bin and corrected for the inefficiencies discussed in Sec. 3.4. Y_{SIMC} is the SIMC yield calculated as described in Sec. 4.3, integrated over the same phase-space and acceptance, and $\sigma_{SIMC} \equiv d\sigma_{SIMC}/dtd\phi$ is the cross section from SIMC evaluated at the center of the analyzed bin.

After the extraction of the cross section in each bin in ϕ , the dependence of the cross section on this angle was fitted following Eq. 5.2. The mean value of the cross sections in the ϕ bins is the unseparated cross section ($d\sigma_U/dtd\phi$ or, for simplicity, σ_U). The terms dependent on the first and second modes in ϕ (proportional to $\cos(\phi)$ and $\cos(2\phi)$) were simultaneously extracted from these fits. The following subsections present the results of

the cross sections obtained in each of the bins for both E93-018 and FPI-2 experiments.

5.2.1 Results from E93-018 experiment

The distributions of events in the variables t and ϕ for the kinematics of the E93-018 experiment are shown in Fig. 5.3 in a polar plot. The three different ϵ settings are shown in separated plots of this figure, where all of the cuts from Tab. 5.1 were applied.

Full coverage in ϕ can be observed for all of the ϵ settings of this experiment for values of t ranging from 0.38 to 0.50 (GeV/c)². In the analysis only the t/ϕ phase-space inside this region was considered.

The data were binned in two t bins with central values $t = 0.41$ (GeV/c)² and $t = 0.47$ (GeV/c)² and six ϕ bins between 0 and 360 degrees, with bin widths of 0.025 (GeV/c)² and 60 degrees respectively. Figure 5.4 and Fig. 5.5 present the unseparated cross section extracted for these two bins in t for each of the three different ϵ settings.

The dependence of the cross sections on ϕ was fitted to

$$\sigma = \sigma_U + \sigma_1 \cdot \cos(\phi) + \sigma_2 \cdot \cos(2\phi) \quad (5.2)$$

for each ϵ setting, considering the parameters σ_U , σ_1 , and σ_2 free and independent. The resulting fits are represented by the solid curves in Figs. 5.4 and 5.5.

With this binning, the statistical uncertainty in each bin is on the order of 6%. Fits of the ϕ -dependence were done for each ϵ setting independently, which allowed for a simultaneous extraction of the interference terms in Eq. 5.2. The results of σ_U from the fits in

Fig. 5.3: Distribution of events in the t/ϕ phase-space for the E93-018 experiment ($Q^2 = 1.0 \text{ GeV}^2$), for the three ϵ settings. The radial coordinate of these plots represent the four momentum transfer t of the events, while the angular coordinate is the angle ϕ between the scattering and reaction plane.

Figs. 5.4 and 5.5 are shown in Tab. 5.3.

The unseparated cross sections shown were fitted to the form

Fig. 5.4: Unseparated cross section for the lower bin in t of the E93-018 experiment, centered at $t=0.41 \text{ (GeV/c)}^2$. All three ϵ settings are shown, with a fit of the ϕ -dependence of the cross section following the form of Eq. 5.2 (solid red curves). The error bars represent statistical uncertainties only.

Fig. 5.5: Unseparated cross section for the higher bin in t of the E93-018 experiment, centered at $t=0.46 \text{ (GeV/c)}^2$. Fits of the ϕ -dependence of the cross sections follows the form of Eq. 5.2, for each of the three ϵ settings (red curves). The error bars represent statistical uncertainties only.

Tab. 5.3: Unseparated cross sections extracted from the E93-018 experiment by fitting the data shown in Figs. 5.4 and 5.5.

Center of t bin (GeV/c) ²	$\frac{d\sigma_U}{dt}$ nb/GeV ² ($\epsilon = 0.38$)	$\frac{d\sigma_U}{dt}$ nb/GeV ² ($\epsilon = 0.68$)	$\frac{d\sigma_U}{dt}$ nb/GeV ² ($\epsilon = 0.81$)
0.412	2146 ± 84	2433 ± 93	2252 ± 72
0.462	2317 ± 106	2264 ± 123	2437 ± 100

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}, \quad (5.3)$$

where the values from Tab. 5.3 were used as $\frac{d\sigma_U}{dt d\phi} = 2\pi \frac{d\sigma_U}{dt}$. The parameters of this linear fit, the slope and the intercept of the curve, are the separated longitudinal and transverse cross sections σ_L and σ_T respectively. The values of σ_U are shown in Fig. 5.6 for both t bins, with the linear fit for the extraction of the L/T separated cross section in these bins.

Finally, the separated longitudinal and transverse cross sections are plotted as a function of the central values of t in Fig. 5.7.

Although the number of t bins is limited, σ_L show an exponential fall off with t which could be interpreted as a hint of pole-like behavior. In addition, Mohring et al. [2003] extracted the cross section at this kinematic (for t_{min} only) for both $p(e,e'K^+)\Lambda$ and $p(e,e'K^+)\Sigma^0$ channels, and the ratio of the longitudinal cross sections for these two channels was found to be $\sigma_L^{\Sigma^0}/\sigma_L^{\Lambda} = 0.09 \pm 0.11$. This ratio is compatible with the ratio of the coupling constants $g_{K\Sigma N}^2/g_{K\Lambda N}^2 = 0.15$ [Guidal et al., 2000], that can also be interpreted as a hint of the dominance of the t channel process in σ_L that is required for the extraction of the kaon form factor using the method discussed in Sec. 1.4. With the assumption that the

Fig. 5.6: The ϵ dependence of the unseparated cross section from the E93-018 experiment. The red line denotes a fit to the form $\sigma_U = \sigma_T + \epsilon\sigma_L$ with the intercept and the slope denoting σ_T and σ_L respectively.

t channel is indeed dominant over σ_L , as suggested by these two evidences, the extraction of the kaon form factor was then performed and is presented in Chapter 6.

5.2.2 Results from FPI-2 experiment

The cross sections from the FPI-2 experiment were extracted in a similar way to those from the E93-018 experiment. However, the statistics collected during FPI-2 are lower and the coverage of the t/ϕ phase-space is limited in some settings, as can be seen in Fig. 5.8.

Fig. 5.7: Separated cross sections for the two bins in t of the E93-018 experiment. An exponential function of form $a.e^{-b \cdot |t|}$ was fitted to the data (blue dashed curve), which for σ_L could be interpreted as a pole-like behavior.

This was taken into account in the data analysis as explained below.

To achieve a broader coverage in ϕ , data in the FPI-2 experiment were collected at different angles with respect to the virtual photon propagation vector $\mathbf{q}$. This corresponded to running the experiment with additional hadron spectrometer angles, offset by ± 4 degrees with respect to the central setting. These different angles are represented by the different colors in Fig. 5.8.

The unseparated cross section in each ϕ bin was extracted using Eq. 5.1. After optimizing the cross section model, differences between data and simulated yields were less than 15%. The unseparated cross sections in the kinematics of the FPI-2 experiment are shown in Fig. 5.9 and Fig. 5.10, for a central value of $Q^2 = 1.37 \text{ GeV}^2$ and $Q^2 = 2.07 \text{ GeV}^2$

Fig. 5.8: t/ϕ phase-space coverage of $p(e,e'K^+)\Lambda$ for the kinematic settings of the FPI-2 experiment.

respectively.

The limited t/ϕ phase-space in the low ϵ setting does not allow for a full L/T/LT/TT cross section separation using this setting alone. The data from low and high ϵ settings were

Fig. 5.9: Cross section extracted in different bins in ϕ for the two ϵ settings of the low Q^2 setting ($Q^2 = 1.37 \text{ GeV}^2$) of the FPI-2 experiment.

thus fit simultaneously in a single fit using

$$\frac{d\sigma}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi, \quad (5.4)$$

where the fitted parameters were the values of $d\sigma_T/dt$, $d\sigma_L/dt$, $d\sigma_{TT}/dt$, and $d\sigma_{LT}/dt$. These

Fig. 5.10: Cross section extracted in different bins in ϕ for the two ϵ settings of the high Q^2 setting ($Q^2 = 2.07$ GeV²) of the FPI-2 experiment.

variables are constrained by factors dependent on ϵ only between the different ϵ settings of the same point in Q^2 and W , allowing for a simultaneous extraction of the L/T/LT/TT separated cross sections for each Q^2 .

5.3 Results on kaon electroproduction cross section

Figure 5.11 shows the separated longitudinal and transverse cross sections from both experiments as a function of Q^2 . Only the lowest t bin is shown for the E93-018 experiment.

Fig. 5.11: Separated cross sections for both E93-018 and FPI-2 experiments in function of Q^2 .

The world data for unseparated kaon electroproduction cross sections comes from many different experiments with disparate kinematic coverage. Kaon electroproduction data were taken at the Cambridge Electron Accelerator (CEA), as discussed by Brown et al. [1972] and Bebek et al. [1977b]. At the Wilson Synchrotron Laboratory (Harvard-Cornell), the kaon cross section was measured as well [Bebek et al., 1974, 1977b], and more recently, the E98-108 experiment at Thomas Jefferson National Accelerator Facility at Hall A [Coman et al., 2010]. Data for kaon photoproduction were also taken at the Deutsches Elektronen-Synchrotron (DESY). To compare data from these experiments to the results of

the present analysis, a global scaling in the invariant mass W is required since they were measured at different kinematics. The method of Bebek et al. [1977a] was used for this scaling,

$$\sigma_U \propto \frac{p_K^*}{W \cdot (W^2 - m_p^2)}, \quad (5.5)$$

where p_K^* is the center of mass momentum of the kaon and m_p is the mass of the proton. Only the highest available ϵ data were considered. The resulting comparison to the world data is shown in Fig. 5.12.

Fig. 5.12: Comparison of the unseparated cross section (σ_U) extracted in this work to high ϵ data available from other experiments, all scaled to the same invariant mass $W = 2.15$ GeV using Eq. 5.5. The grey-dashed curve shows a global fit of the form $a.(b + Q^2)^{-2}$, following the behavior of the unseparated cross section suggested by Bebek et al. [1977a] and Mohring [1999]. The red-solid line represents the prediction of the VR model [Vrancx et al., 2014] at $\epsilon = 0.88$ and is represented by the red-solid line.

Bebek et al. [1977a] and Mohring [1999] suggested a dipole-like scaling of the unseparated cross section with an asymptotic Q^{-4} behavior. The grey-dashed curve in Fig. 5.12 shows a fit of this form to the data. In addition, the predictions from the Regge model (“VR model”) by Vrancx et al. [2014] are shown by the red curve. This is a recent model developed for describing the kaon electroproduction reaction based on a Reggeized background model for photoproduction, giving a better description of previously available separated data, most importantly for σ_T .

The unseparated cross sections shown in Tab. 5.3 were fitted to the form of Eq. 5.3. The parameters of this linear fit, the slope and the intercept of the curve, are the separated longitudinal and transverse cross sections σ_L and σ_T respectively. The results of L/T separated cross sections are presented in Tab. 5.4.

Tab. 5.4: Separated cross sections extracted from the E93-018 and FPI-2 experiments.

Q^2 (GeV/c) ²	W GeV	t GeV ²	σ_L nb/GeV ²	σ_T nb/GeV ²
1.000	1.81	0.41	309±259	2067±172
1.000	1.81	0.46	231±340	2205±221
1.360	2.31	0.27	45.2±262	579±122
2.067	2.31	0.39	25.1±221	358±95

The uncertainties on the separated cross sections presented in Tab. 5.4 were estimated depending on the different sources of uncertainties of the unseparated cross section. Details on the sources of uncertainties and how they were treated for extracting σ_L and σ_T are presented in the next section.

5.4 Discussion of uncertainties

The uncertainties in the extraction of the separated cross sections arise from different sources, statistics and systematics. Uncertainties that are different between the high and low ϵ settings (i.e., uncorrelated) are amplified by a factor $1/\epsilon$ in the longitudinal cross section. This is a result of the ϵ dependence as can be seen in Eq. 1.12, as an example of two measured polarizations of the virtual photon as for the FPI-2 experiment (for the E93-018 experiment, the uncertainty arises from the fit of the three ϵ setting). The resulting uncertainty in σ_L and in σ_T in this case is given by

$$\begin{aligned}\Delta\sigma_L &= \frac{\sqrt{(\Delta\sigma_U(\epsilon_H))^2 + (\Delta\sigma_U(\epsilon_L))^2}}{\epsilon_H - \epsilon_L} \\ \Delta\sigma_T &= \frac{\sqrt{(\epsilon_L \cdot \Delta\sigma_U(\epsilon_H))^2 + (\epsilon_L \cdot \Delta\sigma_U(\epsilon_L))^2}}{\epsilon_H - \epsilon_L}.\end{aligned}\tag{5.6}$$

Statistical uncertainties are related to the random nature of any measurement. They are directly related to the number of events measured in a counting experiment. In general, a Poisson distribution is well suited for describing counting experiments and, therefore, statistical uncertainties are characterized by a $\sqrt{N}$ behavior, where N is the number of events measured. Detailed discussions on statistical uncertainties can be found in the literature, e.g., by Bevington and Robinson [2003].

Systematic uncertainties may come from a variety of experimental and data analysis factors. For example, systematic uncertainties assigned to the correction factors described in Sec. 3.4 quantify the understanding of the detector efficiencies and data acquisition li-

mitations. For L/T cross section separations, the systematic uncertainties are subdivided into random and scale uncertainties. Random systematic uncertainties affect each ϵ setting independently and are amplified by a factor $1/\epsilon$ in the extraction of σ_L . On the other hand, scale uncertainties affect all of the ϵ settings in the same way and produce a global shift in σ_U . They are not amplified in the extraction of σ_L .

Many corrections were applied to the experimental yields during the data analysis, as discussed in Sec. 3.4. The origin and detailed description of these corrections are described by Mohring [1999] and Horn [2006a]. Here, only details of the uncertainties that were treated differently in the present data analysis are presented, for example due to different binning. A summary of all of the systematic uncertainties applied in the present work are presented in Tabs. 5.5 and 5.6.

Among the systematic uncertainties presented in Tabs. 5.5 and 5.6, uncertainties from K^+ decay and from the variation in the cuts are the dominant sources of uncertainty. The first is amplified since a significant fraction of the kaons produced at the target ($> 30\%$) decay on their way to the spectrometer hut and do not reach all of the detectors. Quantifying how the daughter particles behave in their path, and accounting for the possibility of some kaons to trigger only part of the PID detectors are the main contributions to this uncertainty. Simulations with SIMC were done to study how the yields vary depending on K^+ decay; the resulting uncertainties were found to be of 1.2 and 1.0%, for the E93-018 and FPI-2 experiments respectively. Uncertainties due to cuts are discussed in more detail in the next section.

Tab. 5.5: Summary of uncertainties estimated for the E93-018 experiment from the different corrections used during the analysis. Random uncertainties are those that affect each ϵ setting independently, and they were summed in quadrature with the statistical uncertainties and amplified by a factor $1/\epsilon$ in the L-T separation of the cross sections. The scale uncertainties, on the other hand, affect all the ϵ settings in the same way. Therefore the scale uncertainties are not amplified in the L/T separation.

Effect	Random uncertainty (%)	Scale uncertainty (%)
HMS tracking efficiency	0.5	
HMS trigger efficiency	0.1	
HMS Gas Cherenkov efficiency	0.2	
HMS electronic live time	0.1	
HMS acceptance		2.0
SOS tracking efficiency	1.0	
SOS trigger efficiency	0.1	
SOS Aerogel efficiency		0.3
SOS electronic live time	0.1	
SOS acceptance		2.0
Coincidence efficiency	0.5	
Cointime cut efficiency	0.1	
Computer live time	0.3	
TOF β efficiency	0.7	
Cut variation	0.2	0.6–1.1
Radiative correction	1.0	1.0
Spectrometers angles	0.3	0.3–1.3
Kaon absorption	0.5	0.5
Kaon decay	1.2	3.0
Target length and density		0.4
Target density fluctuation	0.4	
Target purity		0.2
Charge measurement		1.5
Total	2.3	4.9

Tab. 5.6: Summary of uncertainties estimated for the FPI-2 experiment, considering the corrections used during the analysis. Random uncertainties and scale uncertainties are explained in the text and in the caption of Fig. 5.5.

Effect	Random uncertainty (%)	Scale uncertainty (%)
HMS tracking efficiency	0.4	1.0
HMS trigger efficiency	0.1	
HMS Cherenkov efficiency	0.1	
HMS electronic live time	0.07	
HMS acceptance	0.7	1.0
SOS tracking efficiency	0.1	0.5
SOS trigger efficiency	0.1	
SOS Cherenkov efficiency	0.02	
SOS Calorimeter efficiency	0.17	
SOS electronic live time	0.07	
SOS acceptance	0.7	1.0
Coincidence block	0.1	
Computer live time	0.2	
Cut variation	0.2	0.6–1.1
Radiative correction	0.4	2.0
Spectrometers angles	0.2	0.2–1.1
Kaon absorption	0.5	0.5
Kaon decay	1.0	3.0
Target length and density	0.2	0.8
Charge measurement	0.3	0.5
Total	1.70	4.49

5.4.1 Uncertainties from cuts

The list of cuts presented in Sec. 5.4 were defined for selecting good events balancing acceptance, PID, and statistics. The dependence of the cross section on the cuts applied was studied. Two examples of the stability of the extracted cross sections for different cuts are shown in the next sections, where the kaon electroproduction cross section was extracted from FPI-2 data for varying cuts on the fractional momentum acceptance of the HMS and the reconstructed missing mass respectively. A similar study was done for all of other cuts used in the analysis of this work and combined in quadrature. The overall uncertainty estimated from the cuts variation was found to be of up to 1.0%.

5.4.2 Stability of cross section with respect to HMS momentum acceptance

The first example illustrates how the extraction of the cross section depends on the HMS fractional momentum, δ_{HMS} . This study is of particular interest for FPI-2 kinematics as the detected kaons are not centered in the momentum acceptance of the HMS. Figure 5.13 illustrates the distribution of kaons during FPI-2.

The position of the lower limit of the δ_{HMS} cut is illustrated by the dashed curve in Fig. 5.13. Only events above this threshold are used for the cross section extraction. This lower limit was varied for the uncertainty study in steps of 0.5% and the kaon cross section was extracted for each lower limit. Results are shown in Fig. 5.14.

The cross sections extracted for different lower limits varying from -11% to 8% varied by less than 1.0%. A cut defined above -8 % would remove more than 40 % of the events in

Fig. 5.13: Distribution of the momentum fraction of particles δ_{HMS} in the HMS spectrometer for the four kinematics settings of the FPI-2 experiment. The dashed line represents the limit of the cut applied in HSDELTA for the cross section analysis. This limit was varied in a systematic study of the dependence of the cross section extraction to the cut definition, providing the uncertainty from cuts.

most of the kinematics. The lower limit of the δ_{HMS} cut was chosen as -10% for this work.

An uncertainty of $\pm 0.3\%$ was assigned from this study, reflecting the fact that the kaons are close to the edge of the momentum acceptance of the HMS.

5.4.3 Cross section dependence on missing mass

The cut position on the right side of the missing mass peak is of special importance since the missing mass distributions are asymmetric, having a long tail towards higher values. As discussed in Sec. 3.3.4, this asymmetry is a result of the energy loss of the scattered electron, produced by effects such as radiative processes (Bremstrahlung) and the

Fig. 5.14: Study of the dependence of the cross section extraction for FPI-2 kinematics in function of the definition of the cut in hsdelta. The final cut for the actual analysis was chosen to be in the flatness region of the curve.

interaction of the electron with materials of the target and the spectrometer. In principle, all of these effects are also simulated in SIMC, and the definition of the cut position should not play an important role in the cross section extraction. Nevertheless, a systematic study of the dependence of the cross section extraction with this cut was done to estimate the magnitude of this effect.

As an example, the missing mass distribution of the events from the data collected at $Q^2 = 2.06 \text{ GeV}^2$ (high epsilon setting) during the FPI-2 experiment is shown in Fig. 5.15. In this figure, the dashed lines represent the position of the missing mass cut applied to select $p(e,e'K^+)\Lambda$ events. Only events inside the range defined by these two limits were considered

in the analysis.

Fig. 5.15: Missing mass distribution of the high epsilon setting ($Q^2 = 2.06 \text{ GeV}^2$) of the FPI-2 experiment. Radiative processes make the peak asymmetric to higher values of missing mass, and a study of the effect on the definition of the missing mass cut was studied to the right of the peak.

The asymmetric distribution in the missing mass peak at the Λ mass is illustrated in Fig. 5.15. The tail to the right side of the peak is composed of events from the $p(e, e'K^+)\Lambda$ reaction and the cut in the missing mass removes some of these from the analysis. However, as the missing mass cut is applied both to the data and the SIMC simulation equally, the position of the cut is not expected to change the cross section extraction significantly. To study this effect, the kaon electroproduction cross section was re-extracted for different upper limits of the missing mass. The results of this study for the FPI-2 experiment is shown in Fig. 5.16.

Fig. 5.16: Cross section extracted for different positions of the right edge of the missing mass cut.

The dependence of the cross section extraction with the cut in missing mass is small when the right edge of the cut is defined above 1.15 GeV. As shown in Fig. 5.16, the cross section varies by 0.9% for cuts above this value. The right edge of the missing mass cut was defined at 1.15 GeV, and an uncertainty of 0.4% was assigned to represent the uncertainty from this cut.

Aside from the presented results on the sensitivity of the cross section extraction with cuts on δ_{HMS} and missing mass, the other cuts from Tab. 5.5 and Tab. 5.6 were investigated to estimate their impact on the extracted cross section. From all of these studies, an overall uncertainty of up to $\pm 1.0\%$ was estimated to represent the variation in the extraction of the cross section due to the applied cuts.

Chapter 6

Meson structure: Form Factor and Transverse Charge Density

In this chapter, the extraction of the kaon form factor from the E93-018 and FPI-2 experiments is presented at values of Q^2 of up to 2.07 GeV^2 . Further insights into charged meson structure are also assessed through the extraction of transverse charge densities of the pion and the kaon.

6.1 The Kaon form factor

The kaon form factor is extracted in this work by comparing the longitudinal cross section measured for $p(e,e'K^+)\Lambda$ in the E93-018 and FPI-2 experiments to the one calculated by the VGL Regge model, discussed in Sec. 1.4.2. This choice is motivated by the successful

application of this technique for studies of the pion form factor [Horn, 2006a; Horn et al., 2006b; Huber et al., 2008] and also by the model's ability to describe the previously available kaon data [Guidal et al., 2000].

The extraction of F_K was done by fitting the longitudinal cross section calculated by the VGL Regge model to the experimental data with the free parameters Λ_K^2 and $\Lambda_{K^*}^2$ (Sec. 1.4). Their best fit values were determined by calculating the values of the reduced χ^2 (χ_ν^2) for the calculated model's longitudinal cross section (σ_L^{VGL}) and the measured longitudinal cross section (σ_L^{data}) presented in Chapter 5. The calculation of χ_ν^2 was done using

$$\chi_\nu^2 = \frac{\chi^2}{\text{dof}} = \frac{1}{\text{dof}} \sum_{t \text{ bins}} \frac{(\sigma_L^{VGL} - \sigma_L^{\text{data}})^2}{(\Delta\sigma_L)^2}, \quad (6.1)$$

where $\Delta\sigma_L$ is the uncertainty of σ_L^{data} and dof is the number of degrees of freedom.

The resulting χ_ν^2 calculated for the kinematics of the E93-018 experiment is shown in Fig. 6.1. The minimum of χ_ν^2 , that represents the best agreement between model and data, was found by a fit of a second degree polynomial function (as shown by the red curve in Fig. 6.1). The corresponding Λ_K^2 is 0.591 GeV². From Eq. 1.14, this value of cutoff parameter corresponds to the kaon form factor $F_K = 0.37$ at $Q^2 = 1.0$ (GeV/c)².

The longitudinal cross sections from the E93-018 experiment are plotted in Fig. 6.2 along with the model using the best fit of $\Lambda_{K^+}^2$ from Fig. 6.1.

To estimate the effect of the K^* trajectory on the final result, the $\Lambda_{K^*}^2$ cutoff value was varied from 0.1 to 1.5 (GeV/c)². The resulting F_K was found to change by a maximum

Fig. 6.1: Reduced χ^2 calculated for different cutoff parameters with the VGL model and the longitudinal cross section extracted from the E93-018 experiment. The best value for fitting the experimental data is the minimum of χ^2_ν , at $\Lambda_{K^+}^2 = 0.591 \text{ GeV}^2$, that was obtained from the fit of a second degree polynomial shown by the red curve. The deviation of χ^2_ν by unity from the minimum is represented by the dashed-blue curve, giving an estimate of the uncertainty of $\Lambda_{K^+}^2$.

of 0.67% with the variation of $\Lambda_{K^*}^2$ in this range.

The kaon form factor was also extracted for the two kinematic settings of the FPI-2 experiment using the VGL Regge model. Similarly to the analysis of the E93-018 experiment, a χ^2 minimization was performed to determine the value of the cutoff parameters that results in the best agreement of the longitudinal cross section calculated from the model to the data.

The optimal cutoff parameters were found to be $\Lambda_K^2 = 0.105 \text{ (GeV/c)}^2$ and $\Lambda_K^2 =$

Fig. 6.2: Resulting longitudinal cross section from the fit of the VGL Regge model (red-dashed line) to the data of the E93-018 experiment (black points). The cutoff value that better describes the data with the VGL model was found to be $\Lambda_{K^+}^2 = 0.591 \text{ GeV}^2$, corresponding to $F_K = 0.37$.

0.135 (GeV/c)^2 for the settings with Q^2 of 1.36 GeV^2 and 2.07 GeV^2 respectively. From these results, the kaon form factor was extracted using Eq. 1.14, and found to be $F_K(Q^2 = 1.36) = 0.0716$ and $F_K(Q^2 = 2.067) = 0.0613$ respectively.

The uncertainty on F_K was estimated from the minimization shown in Fig. 6.1. A χ_ν^2 with one unity larger than the value of the minimum χ_ν^2 corresponds to one standard deviation of the fit. Therefore, the values of $\Lambda_{K^+}^2$ in which χ_ν^2 equals 1.02 (one unity above the minimum χ_ν^2 of 0.02) was considered as the uncertainty of the form factor for the setting of the E93-018 experiment. Table 6.1 summarizes the values of the K^+ form factor extracted in this work and Fig. 6.3 shows these values along with existing data of the kaon form factor

Tab. 6.1: F_K extracted by fitting the longitudinal cross section from the VGL Regge model to the experimental data presented in Chapter 5.

Q^2 (GeV ²)	W (GeV)	F_K	Experiment
1.000	1.81	0.370 ± 0.129	E93-018
1.360	2.31	0.072 ± 0.178	FPI-2
2.067	2.31	0.061 ± 0.190	FPI-2

from Fig. 1.4.

Fig. 6.3: Extracted kaon form factor from the E93-018 and FPI-2 experiments, together with the existing data of F_K at low Q^2 from elastic scattering experiments [Amendolia et al., 1986; Dally et al., 1980].

Niculescu [1998b] first attempted to extract F_K from the E93-018 data using the Chew-Low extrapolation method. This method has not been used anymore due to its large model uncertainties. The values of F_K extracted with this method was reported to be $F_K = 0.402 \pm 0.08$ and, within the uncertainties, is consistent with the results extracted in this

work.

Extending the range of Q^2 where F_K is measured by one order of magnitude allows for the evaluation of the kaon transverse charge density, as discussed in the next sections.

6.2 The transverse charge density of light mesons

The transverse charge densities of the pion and the kaon as a function of the impact parameter b ($\rho(b)$) were calculated using the formalism discussed in Sec. 1.7.2. Results for the pion were published by Carmignotto et al. [2014], and they are here reproduced for a broader comparison of the charge densities of both of these light mesons.

The extraction of the transverse charge density requires as an input the experimental value of $F(Q^2)$. The world data are shown in Fig. 6.4-(a) for the pion and Fig. 6.4-(b) for the kaon. The uncertainty on the calculation of $\rho(b)$ has two main sources: (1) experimental uncertainties on F_K from the individual measurements and combining data from different experiments in the Q^2 region where data exist ($Q^2 \leq Q_{max}^2$) and (2) uncertainties due to the lack of data in the region $Q^2 > Q_{max}^2$, where no measurements exist. Both sources of uncertainty are discussed in Sec. 6.2.2 and Sec. 6.2.3.

6.2.1 Parametrizing F_π and F_K

In order to extract the transverse charge density using Eq. 1.25, the world $F(Q^2)$ data were parametrized with a global fit of the form factor to enable the determination of uncertainties in the transverse charge density. A three parameter empirical fit form was used for both π and K^+ analysis as follows:

Tab. 6.2: Coefficients of Eq. 6.2 fitted to existing form factor data for both pion and kaon, used in the evaluation of the transverse charge density of these mesons.

Meson	A	B (GeV ⁻²)	C (GeV ⁻²)
π	0.384 ± 0.071	1.203 ± 0.101	1.054 ± 0.080
K^+	0.457 ± 0.011	1.541 ± 0.057	0.774 ± 0.074

$$F(Q^2) = A \cdot \frac{1}{(1 + B \cdot Q^2)} + (1 - A) \cdot \frac{1}{(1 + C \cdot Q^2)^2}. \quad (6.2)$$

The parameter A denotes the fractional contribution of the two terms to the overall fit. Equation 6.2 imposes the normalization condition $F_\pi(Q^2 = 0) = 1$. In order to obtain the present fit, all of the values of the parameters A , B , and C were varied simultaneously. Note that the slope of $F(Q^2)$ at $Q^2 \rightarrow 0$ GeV² is constrained by the world data set for low values of Q^2 and this fit incorporates this information. Curves with the form of Eq. 6.2 were fitted to pion and kaon data independently. For each fit, the experimental points were randomly recreated following a Gaussian distribution around their central values. The results of these fits are shown in the black/hatched band in Fig. 6.4 for both the pion and the kaon. The best coefficients for these fits are presented in Tab. 6.2.

To check the consistency of the extraction method, Eq. 6.2 was used to calculate the RMS radii of the pion and the kaon from its slope as $Q^2 \rightarrow 0$. Using the coefficients from Tab. 6.2, a value of r_π of 0.641 ± 0.023 fm and r_K of 0.601 ± 0.016 fm can be extracted, which are compatible with the values extracted from the world data $r_\pi = 0.672 \pm 0.008$ fm and $r_K = 0.560 \pm 0.031$ fm [Patrignani et al., 2016].

(a) Pion form factor

(b) Kaon form factor

Fig. 6.4: Empirical fit of the experimental form factor data (black/hatched band) used to evaluate the transverse charge densities of the pion and the kaon. The band represents the systematic uncertainties combining the different measurements. The curves are the same as shown in Fig. 1.3.

The present fit included F_π data up to $Q^2 = 9.8 \text{ GeV}^2$ and F_K data up to $Q^2 = 2.067 \text{ GeV}^2$. The impact of the higher Q^2 data points on the present fit is small due to their large experimental uncertainties. However, including these points here despite their large uncertainties is important for the truncation of the series expansion in Eq. 1.25 and the resulting incompleteness error. This error results from the region in Q^2 where no measurements exist at all. As Fig. 6.5 shows, the incompleteness error dominates over the experimental error.

6.2.2 Experimental Uncertainty

The kaon form factor at low values of Q^2 was measured with good precision from elastic scattering experiments, providing the kaon RMS radius. The measurement of the kaon form factor presented in this work allowed the expansion of the kinematic reach by one order of magnitude from these low- Q^2 data. Therefore, the form factor is considered to be measured up to $Q_{max}^2 = 2.07 \text{ GeV}^2$ for calculating the kaon charge density.

The pion form factor has been measured with high precision up to $Q^2 = 2.5 \text{ GeV}^2$. Data for values of $Q^2 = 3.3\text{--}9.8 \text{ GeV}^2$ with larger systematic uncertainties up to 50-70% also exist. Thus the pion form factor was considered to be a measured quantity for $Q^2 < Q_{max}^2 = 10 \text{ GeV}^2$.

The contribution of the experimental uncertainty to the transverse charge densities is illustrated by the black/hatched area shown in Fig. 6.5. The largest uncertainty on the calculated charge density occurs at $b = 0$, consistent with the uncertainty principle which relates distance to momentum.

The value of R used for this study was smaller than that used for the proton by

Fig. 6.5: Uncertainties in the transverse charge density $\rho(b)$ due to $F(Q^2)$ experimental data uncertainties σ_{EXP} (black/hatched band) discussed in Sec. 6.2.2 and the incompleteness error discussed in Sec. 6.2.3, when considering the upper band (blue band) or the lower band models (gray band).

Venkat et al. [2011], since both the pion and kaon radii are smaller than the corresponding value for the proton of either 0.84 or 0.87 fm [Pohl et al., 2013].

6.2.3 Incompleteness Error

Equation 1.25 uses the finite radius approximation requiring knowledge of the form factor in the full range of Q^2 for the calculation of the transverse charge density. Since the form factor measurements are limited to $Q^2 < Q_{max}^2$, the series expansion has to be truncated to Q_{max}^2 . The effects of this truncation in the calculation of $\rho(b)$ is estimated with the *incompleteness error*.

The basic transverse charge density of the pion is obtained using Eq. 1.25 for values of $Q^2 < 10 \text{ GeV}^2$ corresponding to values of $n_{max}^\pi=10$. A maximum error was estimated using two representative theoretical models with very different asymptotic behavior, which describe the existing F_π data well. Out of the available models, the monopole (with pion RMS radius $r_\pi = 0.672 \text{ fm}$) and the LF model from Hwang [2001] were chosen as an upper and lower bound, respectively. These models are both constrained by data. The region in between these two model predictions constitutes a band that includes the existing data and all other models of F_π mentioned in Sec. 1.3.1 that describe the data. The band thus also includes the true value of F_π in the region where no data exist.

For the evaluation of the kaon transverse charge density, a more limited scenario is found. Experimental form factor data is now available for Q^2 of up to 2.07 GeV^2 , allowing the calculation of the transverse charge density of the kaon with $n_{max}^K=4$ when using Eq. 1.25. From the models describing the kaon form factor presented in Sec. 1.3.2, the monopole was

chosen to represent the upper bound while the perturbative QCD asymptotic behavior represents the lower bound to represent F_K at higher Q^2 . The incompleteness error for the chosen models are estimated using Eq. 1.26.

The results are summarized in Fig. 6.5, which shows that the uncertainty due to the incompleteness error is much larger than that due to uncertainty in current data. The rapid increase of the incompleteness error estimated using the monopole model at $b=0$ fm is a consequence of the functional form's asymptotic behavior ($\sim 1/Q^2$), which results in a singularity at the center of the charge distribution [Miller, 2009; Miller et al., 2011]. As the monopole model was used to evaluate the pion and kaon transverse charge densities, the results shown in Fig. 6.5 are compatible with such a singularity within the uncertainty. The incompleteness error is likely overestimated as two extreme models were chosen, resulting in a conservative incompleteness error band. Future high Q^2 data like those discussed in Sec. 6.4 are anticipated to narrow the error band, by constraining the models and thus reducing the incompleteness error most importantly at intermediate and small distances.

6.3 Pion and Kaon transverse charge densities

Figure 6.6 shows the pion and the kaon charge densities evaluated using the series expansion of Eq. (1.25) with the experimental uncertainty based on the fits of $F(Q^2)$ (from Fig. 6.4) and with the incompleteness error estimated using the bound models as described by Eq. 1.26.

As these calculations were done in polar coordinates, the spatial transverse element of

Fig. 6.6: (1) The transverse charge density (red curve) calculated from the form factors. (2) The transverse charge density multiplied by the Jacobian b . The transverse charge density of the pion is shown in (a), while the kaon is shown in (b).

area is $d^2b = 2\pi b db$ for a given impact parameter b . Thus Fig. 6.6-(2) shows the transverse charge density multiplied by the Jacobian b , that can be interpreted as the charge density at a ring defined between the circumferences of radius b and $b + db$.

In the case of the pion, the uncertainties on $\rho(b)$ due to the incompleteness error for $b > 0.1$ fm are relatively small compared to the ones for the region $b < 0.1$ fm. This is because pion form factor data are readily available at low values of Q^2 (large values of impact parameter b) and, as a result, $\rho(b)$ is well determined in that region. On the other hand, in the region $b < 0.1$ fm the incompleteness error is very large, which is due to the lack of the pion form factor data at very large values of Q^2 .

The kaon transverse charge density has a larger uncertainty in the full range of b , especially from $b = 0$ to about 0.4 fm. This is due to the more limited availability of F_K , which directly increases the incompleteness error. An effect of the earlier truncation of the kaon series ($n_{max} = 4$) is that the most probable value (red curve) of the transverse charge density of the K^+ seems to be lower than that of the pion for lower values of b .

Finally, the oscillatory behavior of the transverse charge densities at $b > 0.25$ fm is a result of the truncation of the Bessel function series of Eq. 1.25. However, the choice of $R=2$ fm is not physically relevant for these results. Tests of the sensitivity of our results for R larger than 2.0 fm show no significant change in the transverse charge density within the uncertainties.

6.3.1 Meson cloud in the proton

Strikman and Weiss [2010] explored the proton transverse charge density finding that

the non-chiral core is dominant up to relatively large distances of ~ 2 fm. This suggests that there is a non-pionic core of the proton, as it is seen in the constituent quark or vector meson dominance models. One does not usually think of the pion having a meson cloud since a, e.g., $\rho\pi$ component would involve a high excitation energy. Therefore it is interesting to compare the proton and pion transverse charge densities as given by numerically stable series as in Eq. 1.25.

Figures 6.7 and 6.8 compare proton and pion transverse charge densities for different ranges of b . Figure 6.7 focuses on a region in b where the transverse charge density is expected to decrease from its value at the core while Fig. 6.8 illustrates a region in which the transverse charge density is expected to be significantly smaller than at the core and to approach zero.

For values of b less than about 0.2 fm the transverse charge density of the pion is larger than that of the proton. This higher density is expected because the pion's radius 0.672 fm is smaller than the proton's 0.84 fm. As noted by Miller [2009] and Miller et al. [2011], it is possible that the pion's transverse density is singular at $b = 0$. An interesting feature is that the curves seem to coalesce in the region $b > 0.3$ fm (at least within current uncertainties). According to Wilson et al. [2012] and Cloët and Miller [2012], similarity in binding forces could lead to a similarity in transverse densities. The result that the pion and proton transverse densities are similar in their core may be a first experimental glimpse at the transition between proton core and meson cloud, e.g., the “edge” of the proton.

The comparison between the pion and the proton transverse charge densities for

Fig. 6.7: **(1)** Comparison of the pion transverse charge density (red curve) to the proton transverse charge density (from Venkat et al. [2011]) shown in the green band. The uncertainties for the pion transverse charge density are as in Fig. 6.6. The green band for the proton includes both the experimental and incompleteness error. The proton error band is smaller as compared to the pion because the proton form factor is well known over a larger range in Q^2 . The two transverse charge density curves (red solid and green solid lines) coalesce in the region $b > 0.3$ fm within the uncertainty while the pion transverse charge density appears denser than that of the proton in the region $b < 0.2$ fm. **(2)** Ratio of pion to proton transverse charge densities (red solid curve). Here the error bands shown denote the uncertainty on the ratio of pion to proton charge density. The error band is dominated by the pion incompleteness error, so the same coloring and shading was used as of the panel (1) of this figure to indicate the individual uncertainty contributions.

Fig. 6.8: **(a)** Comparison of the pion transverse charge density to the proton transverse charge density as in Fig. 6.7, showing a range of higher impact parameter b , where the transverse charge density is expected to already be significantly smaller than at the core and to approach zero. **(b)** Ratio of pion to proton transverse charge densities. The curves and uncertainties are as in Fig. 6.7.

$b > 0.7$ fm is shown in Fig. 6.8. Both transverse charge densities in this higher region of b are significantly smaller than at the core. The uncertainties on the pion transverse charge density in this region are on the order of the charge density itself making it difficult to compare to the proton. However, the two densities are the same within current uncertainties. To explore any similarity of the pion and the proton transverse charge densities

in this region, precision pion form factor data at higher values of Q^2 like those discussed in Sect. 6.4 would be needed.

6.4 Impact of future experiments

Future data are anticipated to improve the extraction of the pion and kaon transverse charge densities, which in the region of $b > 0.3$ fm would be of great interest for further studies of the “edge” of hadrons.

Experiments at the 12 GeV JLab [JLab Experiment E12-06-101, 2006; JLab Experiment E12-07-105, 2007] will extend the Q^2 range of high precision pion form factor data to $Q^2 = 6$ GeV². Pion form factor extractions are also planned up to the highest achievable momentum transfer from the spectrometers at Hall C with 12 GeV beam, $Q^2 \sim 9$ GeV².

The kaon form factor is anticipated to be measured in an upcoming JLab experiment [JLab Experiment E12-09-011, 2009], which has the potential to extend F_K data to Q^2 of up to about 5.5 GeV².

Finally, the envisioned Electron-Ion Collider EIC could further extend this reach to Q^2 of about 25 GeV² [Huber, 2009]. This Q^2 region would add data into the region of interest for studying the hadron edge improving the precision of the extraction of the transverse charge densities.

Future measurements would add data into the far unmeasured regions of small b . The projected uncertainties of these future experiments are shown in Fig. 6.9 together with existing data and the models used in this work as bounds of the form factors.

(a) PION

(b) KAON

Fig. 6.9: F_π and F_K world data (red circles) and projected uncertainties for experiments at JLab at 12 GeV (blue diamonds) and those for measurements of the form factor with an EIC [Huber, 2009; JLab Experiment E12-06-101, 2006]. The projected uncertainties at the EIC are divided into three groups depending on the energy E_p of the ion beam (magenta triangles with $E_p=5$ GeV, green stars with $E_p=10$ GeV, brown squares with $E_p=15$ GeV).

With future data at values of higher Q^2 , the number of models describing the data will be better constrained over a wider kinematic range, and the uncertainty described by the band between the upper bound and lower bound will also become narrower. This is illustrated in Fig. 6.10 assuming that all data from both 12 GeV JLab and the EIC are measured.

Including the projected uncertainties of the future data, the precision of the kaon charge density are expected to be greatly improved with future F_K data, reaching values comparable to the current uncertainties of the pion. For the pion charge density, the estimated uncertainties would be better than 20% for $b > 0.1$ fm. This would greatly constrain the pion transverse charge density and determine if the proton and pion transverse charge densities are really the same for moderate values of b .

Fig. 6.10: Experimental and incompleteness error estimated on the transverse charge density considering (1) the existing data and (2) including the projected uncertainties of future data from 12 GeV experiments at JLab and the EIC. The pion is shown in (a), while the kaon is shown in (b).

Chapter 7

Conclusions

The L/T/LT/TT separated cross sections for kaon electroproduction were extracted from the E93-018 and FPI-2 experiments for the $p(e,e'K^+)\Lambda$ reaction. Data from the E93-018 experiment were analyzed in two bins in t , at Q^2 of 1.0 (GeV/c)^2 and invariant mass W of 1.81 GeV. The separated longitudinal cross section shows hints of an exponential fall-off with t , albeit with large uncertainties. This may suggest dominance of the t channel process. The compatibility between the σ_L cross section ratio for the $p(e,e'K^+)\Sigma^0$ and $p(e,e'K^+)\Lambda$ channels and the ratio $g_{K\Sigma N}^2/g_{K\Lambda N}^2$ also corroborates as a hint of the t channel dominance on σ_L , although also with large uncertainties involved.

With the observation of signs of t -channel dominance in σ_L , the kaon form factor was extracted from the data by carefully examining the behavior of the cross section at small values of t . In the present analysis this was done using the Regge model by Vanderhaeghen, Guidal, and Laget. The resulting kaon form factor values extend the kinematic reach in

Q^2 of the data up to 2.07 (GeV/c)^2 . Results show F_K to be systematically lower than the empirical monopole form, although still compatible with this form within the estimated uncertainties.

The kaon form factor results from the present work were combined with the world data of the space-like pion and kaon form factors and the transverse charge density of these mesons was extracted. The pion was found to have a larger transverse charge density than the proton in the region $b < 0.2 \text{ fm}$. The pion and the proton have the same transverse charge densities at values $0.3 \text{ fm} < b < 0.6 \text{ fm}$, which may be interpreted in terms of the spatial structure of the proton consisting of a core occupying most of the volume and a meson cloud dominating only at large impact parameters. The transverse charge density of the kaon has a larger uncertainty, since the available form factor data for the kaon is more limited. The kaon transverse charge density is compatible with both the pion and the proton, although there is an indication that it is higher than both the pion and the proton at values of $0.3 \text{ fm} < b < 0.6 \text{ fm}$. The uncertainties on these transverse charge densities are dominated by the incompleteness error. Future experiments to measure the form factor of pion and the kaon at higher values of Q^2 would play an important role in better defining the shape of the charge densities most importantly at lower values of b .

7.1 Opportunities with JLab at 12 GeV and beyond

With the 12 GeV upgrade of Jefferson Laboratory, many new opportunities for exploring hadronic and partonic physics are available. A dedicated 12 GeV experiment to study

the kaon production mechanism and, if possible, extract the kaon form factor is approved and scheduled to run during the earliest phases of Hall C new era [JLab Experiment E12-09-011, 2009]. Two other experiments are approved to probe the pion. These three experiments will extend the Q^2 range for L/T separated pion ($Q^2 \sim 9 \text{ GeV}^2$) and kaon ($Q^2 \sim 5.5 \text{ GeV}^2$) production cross section. In the context of the analysis discussed in the present work, E12-09-011 will allow for more precise tests of the relative contribution of longitudinal and transverse photons to the kaon cross section and determination of the contribution of the kaon exchange to σ_L . Assuming that the t channel process remains dominant in 12 GeV kinematics, the experiment will allow for kaon form factor extractions and verification of the results obtained in the present work, as well as the extension of the data to the highest possible Q^2 value. The experiment will also allow tests of the reaction mechanism at low values of Q^2 , which was not possible in the present work.

The 12 GeV upgrade of Jefferson Laboratory also provides the possibility to confirm the potential for nucleon imaging studies through exclusive processes including kaon electroproduction. The method of choice for nucleon imaging studies is typically deep exclusive photon production (DVCS). An experiment (E12-13-010) designed to fully disentangle all components of the DVCS cross section to the highest value of Q^2 through the energy dependence has been approved in Hall C. This experiment will also provide neutral pion cross sections. If the contribution of transverse photons can be shown to be dominant, this would allow for accessing a new way for nucleon imaging. Kaon electroproduction cross sections to the highest Q^2 play an important role in nucleon imaging studies as the kaon is the simplest

system experimentally accessible that contains strangeness.

Carrying out deep exclusive experiments with charged or neutral final states requires specialized particle detection. A new aerogel Cherenkov detector that will allow for kaon/proton separation in the new Super High Momentum Spectrometer (SHMS) in Hall C was constructed at CUA. This detector is a key instrument for several experiments where kaon identification is required using the SHMS. The details of the construction and initial commissioning of the SHMS aerogel Cherenkov detector are summarized by Horn et al. [2017]. Similar to the analysis of data from the E93-018 experiment described in the present work, the SHMS aerogel detector will serve as a particle identification detector. To detect neutral particles in combination with the magnetic spectrometers in Hall C a Neutral Particle Spectrometer (NPS) is currently being constructed at CUA. The NPS consists of a highly segmented PbWO_4 -based electromagnetic calorimeter preceded by a sweeping magnet.

Beyond the 12 GeV era, the Electron-Ion Collider (EIC) will allow the next significant improvement of world capabilities in exploring strong interaction providing the facilities to study hadron structure beyond the valence quark region accessible with 12 GeV at Jefferson Lab. The EIC could extend studies of the kaon electroproduction cross section discussed in the present work into the region where sea quarks dominate and may allow for studies of the transverse spatial structure of these sea quarks.

It is with great excitement that the author concludes this work, honored to have had the opportunity to contribute to unveiling features of hadron structure and to participate in the preparation for writing the next chapters of physics at JLab at 12 GeV and beyond.

Bibliography

Ackermann, H., Azemoon, T., Gabriel, W., Mertiens, H. D., Reich, H. D., Specht, G., Janata, F., and Schmidt, D.: Determination of the longitudinal and the transverse part in $\pi^\pm$ electroproduction, Nuclear Physics B137 (3), 294–300, doi: 10.1016/0550-3213(78)90523-0, 1978.

Adylov, G. T., Aliev, F. K., Bardin, D. Y., Gajewski, W., Ioan, I., Kulakov, B. A., Micelmacher, G. V., Niczyporuk, B., Nigmanov, T. S., Tsyganov, E. N., Turala, M., Vodopianov, A. S., Wala, K., Dally, E., Drickey, D. J., Liberman, A. D., Shepard, P. F., Tompkins, J. C., Buchanan, C. D., Poirier, J. A.: A measurement of the electromagnetic size of the pion from direct elastic pion scattering data at 50 GeV/c, Nuclear Physics B128 (3), 461–505, doi: 10.1016/0550-3213(77)90056-6, 1977.

Alcorn, J. et al.: Basic instrumentation for Hall A at Jefferson Lab, Nucl. Instrum. Meth. A522, 294–346, doi: 10.1016/j.nima.2003.11.415, 2004.

Amendolia, S. R., Badelek, B., Batignani, G., Beck, G. A., Bedeschi, F., Bellamy, E. H., Bertolucci, E., Bettoni, D., Bilokon, H., Bologna, G., Bosisio, L., Bradaschia, C., Budinich, M., Codino, A., Counihan, M. J., Dell’Orso, M., D’Ettorre Piazzoli, B., Fabbri, F. L., Fidecaro, F., Foà, L., Focardi, E., Frank, S. G. F., Giazotto, A., Giorgi, M. A., Green, M. G., Harvey, J., Heath, G. P., Landon, M. P. J., Laurelli, P., Liello, F., Mannonchi, G., March, P. V., Marrocchesi, P. S., Menasce, D., Menzione, A., Meroni, E., Milotti, E., Moroni, L., Picchi, P., Ragusa, F., Ristori, L., Rolandi, L., Saltmarsh, C. G., Saoucha, A., Satta, L., Scribano, A., Spillantini, P., Stefanini, A., Storey, D., Strong, J. A., Tenchini, R., Tonelli, G., Von Schlippe, W., Van Herwijnen, E., and Zallo, A.: A measurement of the pion charge radius, Physics Letters B146 (1), 116–120, doi: 10.1016/0370-2693(84)90655-5, 1984..

Amendolia, S. R., Batignani, G., Beck, G. A., Bellamy, E. H., Bertolucci, E., Bologna, G., Bosisio, L., Bradaschia, C., Budinich, M., Dell’orso, M., D’Ettorre Piazzoli, B., Fabbri, F. L., Fidecaro, F., Foa, L., Focardi, E., Frank, S. G. F., Gianetti, P., Giazotto, A.,

Giorgi, M. A., Green, M. G., Heath, G. P., Landon, M. P. J., Laurelli, P., Liello, F., Mannocchi, G., March, P. V., Marrocchesi, P. S., Menzione, A., Meroni, E., Picchi, P., Ragusa, F., Ristori, L., Rolandi, L., Scribano, A., Stefanini, A., Storey, D., Strong, J. A., Tenchini, R., Tonelli, G., Triggiani, G., von Schlippe, W., Zallo, A.: A measurement of the kaon charge radius, *Physics Letters B* 178 (4), 435–440, doi: 10.1016/0370-2693(86)91407-3, 1986.

Arrington, J. R.: Inclusive Electron Scattering From Nuclei at $x > 1$ and High Q^2 , Ph.D. thesis, California Institute of Technology, 1998.

Bebek, C. J., Brown, C. N., Herzlinger, M., Holmes, S., Lichtenstein, C. A., Pipkin, F. M., Sisterson, L. K., Andrews, D., Berkelman, K., Cassel, D. G., Hartill, D. L., and Hicks, N.: Electroproduction of K^+ Mesons in the Forward Direction, *Phys. Rev. Lett.* 32 (1), 21–24, doi: 10.1103/PhysRevLett.32.21, 1974.

Bebek, C. J., Brown, C. N., Herzlinger, M., Holmes, S. D., Lichtenstein, C. A., Pipkin, F. M., Raither, S., and Sisterson, L. K.: Determination of the pion form factor up to $Q^2 = 4 \text{ GeV}^2$ from single-charged-pion electroproduction, *Phys. Rev. D* 13 (1), 25–42, doi: 10.1103/PhysRevD.13.25, 1976.

Bebek, C. J., Brown, C. N., Kline, R. V., Pipkin, F. M., Raither, S. W., Sisterson, L. K., Browman, A., Hanson, K. M., Larson, D., and Silverman, A.: Scalar-transverse separation of electroproduced $K^+\Lambda$ and $K^+\Sigma^0$ final states, *Phys. Rev. D* 15 (11), 3082–3084, doi: 10.1103/PhysRevD.15.3082, 1977.

Bebek, C. J., Brown, C. N., Bucksbaum, P., Herzlinger, M., Holmes, S. D., Lichtenstein, C. A., Pipkin, F. M., Raither, S. W., and Sisterson, L. K.: Electroproduction of K^+ mesons along the virtual-photon direction, *Phys. Rev. D* 15 (3), 594–601, doi: 10.1103/PhysRevD.15.594, 1977.

Bebek, C. J., Brown, C. N., Holmes, S. D., Kline, R. V., Pipkin, F. M., Raither, S., Sisterson, L. K., Browman, A., Hanson, K. M., Larson, D., and Silverman, A.: Electroproduction of single pions at low ϵ and a measurement of the pion form factor up to $Q^2 = 10 \text{ GeV}^2$, *Phys. Rev. D* 17 (7), 1693–1705, doi: 10.1103/PhysRevD.17.1693, 1978.

Berryman, S.: Ancient Atomism, Stanford Encyclopedia of Philosophy, Editor: Edward N. Zalta, The Metaphysics Research Lab, ISSN 1095-5054, 2008.

Bevington, P. R. and Robinson, D. K.: Data reduction and error analysis for the physical sciences, 3rd edition, ISBN-10 0072472278, McGraw-Hill, 2003.

Berz, M.: The code COSY INFINITY, Conference Record of the 1991 IEEE Particle Accelerator Conference, San Francisco, CA, USA, 354-356, doi: 10.1109/PAC.1991.164299, 1991.

Blok, H. P., Horn, T., Huber, G. M., Beise, E. J., Gaskell, D., Mack, D. J., Tadevosyan, V., Volmer, J., Abbott, D., Aniol, K., Anklin, H., Armstrong, C., Arrington, J., Assamagan, K., Avery, S., Baker, O. K., Barrett, B., Bochna, C., Boeglin, W., Brash, E. J., Breuer, H., Chang, C. C., Chant, N., Christy, M. E., Dunne, J., Eden, T., Ent, R., Fenker, H., Gibson, E., Gilman, R., Gustafsson, K., Hinton, W., Holt, R. J., Jackson, H., Jin, S., Jones, M. K., Keppel, C. E., Kim, P. H., Kim, W., King, P. M., Klein, A., Koltenuk, D., Kovaltchouk, V., Liang, M., Liu, J., Lolos, G. J., Lung, A., Margaziotis, D. J., Markowitz, P., Matsumura, A., McKee, D., Meekins, D., Mitchell, J., Miyoshi, T., Mkrtchyan, H., Mueller, B., Niculescu, G., Niculescu, I., Okayasu, Y., Pentchev, L., Perdrisat, C., Pitz, D., Potterveld, D., Punjabi, V., Qin, L. M., Reimer, P., Reinhold, J., Roche, J., Roos, P. G., Sarty, A., Shin, I. K., Smith, G. R., Stepanyan, S., Tang, L. G., Tvaskis, V., van der Meer, R. L. J., Vansyoc, K., Westrum, D. Van and Vidakovic, S., Vulcan, W., Warren, G., Wood, S. A., Xu, C., Yan, C., Zhao, W.-X., Zheng, X., and Zihlmann, B.: Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2 I. Measurements of the cross section for the $^1\text{H}(e, e' \pi^+)n$ reaction, Phys. Rev. C78 (4), 045202, doi: 10.1103/PhysRevC.78.045202, 2008.

Bloom, E. D., Coward, D. H., DeStaebler, H., Drees, J., Miller, G., Mo, L. W., Taylor, R. E., Breidenbach, M., Friedman, J. I., Hartmann, G. C., and Kendall, H. W.: High-Energy Inelastic $e - p$ Scattering at 6° and 10° , Phys. Rev. Lett. 23 (16), 930–934, doi: 10.1103/PhysRevLett.23.930, 1969.

Brauel, P., Canzler, T., Cords, D., Felst, R., Grindhammer, G., Helm, M., Kollmann, W. D., Krehbiel, H., and Schädlich, M.: π^- Electroproduction off deuterium above the resonance region, Physics Letters B65 (2), 184–186, doi: 10.1016/0370-2693(76)90028-9, 1976.

- Brauel, P., Canzler, T., Cords, D., Felst, R., Grindhammer, G., Helm, M., Kollmann, W. D., Krehbiel, H., and Schädlich, M.: Separation of σ_L and σ_U in π^+ -electroproduction above the resonance region, *Physics Letters B* 69 (2), 253–256, doi: 10.1016/0370-2693(77)90656-6, 1977.
- Brauel, P., Canzler, T., Cords, D., Felst, R., Grindhammer, G., Helm, M., Kollmann, W. D., Krehbiel, H., Schädlich, M.: Electroproduction of $\pi+n$, $\pi-p$ and $K+\Lambda$, $K+2211;0$ final states above the resonance region, *Zeitschrift für Physik C Particles and Fields* 3 (2), 101–123, doi: 10.1007/BF01443698, 1979.
- Brown, C. N., Canizares, C. R., Cooper, W. E., Eisner, A. M., Feldman, G. J., Lichtenstein, C. A., Litt, L., Lockeretz, W., Montana, V. B., Pipkin, F. M., and Hicks, N.: Coincidence Measurements of Single K^+ Electroproduction, *Phys. Rev. Lett.* 28 (16), 1086–1089, doi: 10.1103/PhysRevLett.28.1086, 1972.
- Cabibbo, N.: Unitary Symmetry and Leptonic Decays, *Phys. Rev. Lett.* 10 (12), 531–533, doi: 10.1103/PhysRevLett.10.531, 1963.
- Cardarelli, F., Grach, I. L., Narodetskii, I. M., Pace, E., Salmè, G., and Simula, S.: Charge form factor of π and K mesons, *Phys. Rev. D* 53 (11), 6682–6685, doi: 10.1103/PhysRevD.53.6682, 1996.
- Carman, D. S., Park, K., Raue, B. A., Adhikari, K. P., Adikaram, D., Aghasyan, M., Amarian, M. J., Anderson, M. D., Anefalos Pereira, S., Anghinolfi, M., Avakian, H., Baghdasaryan, H., Ball, J., Baltzell, N. A., Battaglieri, M., Batourine, V., Bedlinskiy, I., Biselli, A. S., Bono, J., Boiarinov, S., Briscoe, W. J., Brooks, W. K., Burkert, V. D., Celentano, A., Chandavar, S., Charles, G., Cole, P. L., Contalbrigo, M., Cortes, O., Crede, V., D’Angelo, A., Dashyan, N., De Vita, R., De Sanctis, E., Deur, A., Djalali, C., Doughty, D., Dupre, R., El Alaoui, A., El Fassi, L., Eugenio, P., Fedotov, G., Fegan, S., Fersch, R., Fleming, J. A., Fradi, A., Gabrielyan, M. Y., Gevorgyan, N., Giovanetti, K. L., Girod, F. X., Goetz, J. T., Gohn, W., Gothe, R. W., Griffioen, K. A., Guegan, B., Guidal, M., Guo, L., Hafidi, K., Hakobyan, H., Hanretty, C., Harrison, N., Heddle, D., Hicks, K., Ho, D., Holtrop, M., Ilieva, Y., Ireland, D. G., Ishkhanov, B. S., Isupov, E. L., Jo, H. S., Joo, K., Keller, D., Khandaker, M., Khetarpal, P., Kim, A., Kim, W., Klein, A., Klein, F. J., Koirala, S., Kubarovsky, A., Kubarovsky, V., Kuleshov, S. V., Kvaltine, N. D., Lewis, S., Livingston, K., Lu, H. Y., MacGregor, I. J. D., Mao, Y., Martinez, D., Mayer, M., McKinnon, B., Mestayer, M. D., Meyer, C. A., Mineeva, T., Mirazita, M.,

Moiseev, V., Montgomery, R. A., Moutarde, H., Munevar, E., Munoz Camacho, C., Nadel-Turonski, P., Nasseripour, R., Nepali, C. S., Niccolai, S., Niculescu, G., Niculescu, I., Osipenko, M., Ostrovidov, A. I., Pappalardo, L. L., Paremuzyan, R., Park, S., Pasyuk, E., Phelps, E., Phillips, J. J., Pisano, S., Pogorelko, O., Pozdniakov, S., Price, J. W., Procureur, S., Prok, Y., Protopopescu, D., Puckett, A. J. R., Ricco, G., Rimal, D., Ripani, M., Rosner, G., Rossi, P., Sabatié, F., Saini, M. S., Salgado, C., Saylor, N. A., Schott, D., Schumacher, R. A., Seder, E., Seraydaryan, H., Sharabian, Y. G., Smith, G. D., Sober, D. I., Sokhan, D., Stepanyan, S. S., Stepanyan, S., Stoler, P., Strakovsky, I. I., Strauch, S., Taiuti, M., Tang, W., Taylor, C. E., Tian, Y., Tkachenko, S., Trivedi, A., Ungaro, M., Vernarsky, B., Voskanyan, H., Voutier, E., Walford, N. K., Watts, D., Weinstein, L. B., Wood, M. H., Zachariou, N., Zana, L., Zhang, J., Zhao, Z. W. and Zonta, I.: Separated structure functions for exclusive $K^+\Lambda$ and $K^+\Sigma^0$ electroproduction at 5.5 GeV measured with CLAS, Phys. Rev. C87 (2), 025204, doi: 10.1103/PhysRevC.87.025204, arXiv:1212.1336, 2013.

Carmignotto, M., Horn, T., and Albayrak, I.: Future instrumentation for Jefferson lab hall C at 12 GeV, AIP Conference Proceedings 1560 (1), 683–685, doi: 10.1063/1.4826871, 2013.

Carmignotto, M., Horn, T., and Miller, G.: Pion transverse charge density and the edge of hadrons, Phys. Rev. C 90, 025211, doi: 10.1103/PhysRevC.90.025211, arXiv:1404.1539, 2014.

Carroll, A. S., Chiang, I-H., Kycia, T. F., Li, K. K., Marx, M. D., Rahm, D. C., Baker, W. F., Earty, D. P., Giacomelli, G., Jonckheere, A. M., Koehler, P. F. M., Mazur, P. O., Rubinstein, R., Fackler, O.: Absorption cross section of $\pi^\pm$, $K^\pm$, p and p on nuclei between 60 and 280 GeV/c, Physics Letters B 80, 3, 319–322, doi: 10.1016/0370-2693(79)90226-0, 1979.

Chang, L., Cloët, I. C., Roberts, C. D., Schmidt, S. M., and Tandy, P. C.: Pion Electromagnetic Form Factor at Spacelike Momenta, Phys. Rev. Lett. 111 (14), 141802, doi: 10.1103/PhysRevLett.111.141802, 2013.

Chew, G. F. and Low, F. E.: Unstable Particles as Targets in Scattering Experiments, Phys. Rev. 113 (6), 1640–1648, doi: 10.1103/PhysRev.113.1640, 1959.

- Christenson, J. H., Cronin, J. W., Fitch, V. L., and Turlay, R.: Evidence for the 2π Decay of the K_2^0 Meson, Phys. Rev. Lett. 13 (4), 138–140, doi: 10.1103/PhysRevLett.13.138, 1964.
- Cloët, I. C., and Miller, G. A.: Nucleon form factors and spin content in a quark-diquark model with a pion cloud, Phys. Rev. C86 (1), 015208, doi: 10.1103/PhysRevC.86.015208, 2012.
- Coman, M., Markowitz, P., Aniol, K. A., Baker, K., Boeglin, W. U., Breuer, H., Bydžovský, P., Camsonne, A., Cha, J., Chang, C. C., Chant, N., Chen, J.-P., Chudakov, E. A., Cisbani, E., Cole, L., Cusanno, F., de Jager, C. W., De Leo, R., Deur, A. P., Dieterich, S., Dohrmann, F., Dutta, D., Ent, R., Filoti, O., Fissum, K., Frullani, S., Garibaldi, F., Gayou, O., Gilman, F., Gomez, J., Gueye, P., Hansen, J. O., Higginbotham, D. W., Hinton, W., Horn, T., Hu, B., Huber, G. M., Iodice, M., Jackson, C., Jiang, X., Jones, M., Kanda, K., Keppel, C., King, P., Klein, F., Kozlov, K., Kramer, K., Kramer, L., Lagamba, L., LeRose, J. J., Liyanage, N., Margaziotis, D. J., Marrone, S., McCormick, K., Michaels, R. W., Mitchell, J., Miyoshi, T., Nanda, S., Palomba, M., Pattichio, V., Perdrisat, C. F., Piasetzky, E., Punjabi, V. A., Raue, B., Reinhold, J., Reitz, B., Roche, R. E., Roos, P., Saha, A., Sarty, A. J., Sato, Y., Širca, S., Sotona, M., Tang, L., Ueno, H., Ulmer, P. E., Urciuoli, G. M., Uzzle, A., Vacheret, A., Wang, K., Wijesooriya, K., Wojtsekhowski, B., Wood, S., Yaron, I., Zheng, X., and Zhu, L.: Cross sections and Rosenbluth separations in $^1\text{H}(e, e' K^+)\Lambda$ up to $Q^2 = 2.35 \text{ GeV}^2$, Phys. Rev. C81 (5), 052201, doi: 10.1103/PhysRevC.81.052201, 2010.
- Dally, E. B., Hauptman, J. M., Kubic, J., Stork, D. H., Watson, A. B., Guzik, Z., Nigmanov, T. S., Riabtsov, V. D., Tsyganov, E. N., Vodopianov, A. S., Beretvas, A., Grigorian, A., Tompkins, J. C., Toohig, T. E., Wehmann, A. A., Poirier, J. A., Rey, C. A., Volk, J. T., Rapp, P. D., and Shepard, P. F.: Direct Measurement of the Negative-Kaon Form Factor, Phys. Rev. Lett. 45 (4), 232–235, doi: 10.1103/PhysRevLett.45.232, 1980.
- Dally, E. B., Drickey, D. J., Hauptman, J. M., May, C. F., Stork, D. H., Poirier, J. A., Rey, C. A., Wojslaw, R. J., Shepard, P. F., Lennox, A. J., Tompkins, J. C., Toohig, T. E., Wehmann, A. A., Ioan, I. X., Nigmanov, T. S., Tsyganov, E. N., and Vodopianov, A. S.: Measurement of the π^- form factor, Phys. Rev. D24 (7), 1718–1735, doi: 10.1103/PhysRevD.24.1718, 1981.
- da Silva, E. O., de Melo, J. P. B. C., El-Bennich, B., and Filho, Victo S.: Pion and

- kaon elastic form factors in a refined light-front model, *Phys. Rev. C* 86 (3), 038202, doi: 10.1103/PhysRevC.86.038202, (2012).
- Denisov, S. P., Donskov, S. V., Gorin, Y. P., Krasnokutsky, R. N., Petrukhin, A. I., Prokoshkin, Y. D., Stoyanova, D. A.: Absorption cross sections for pions, kaons, protons and antiprotons on complex nuclei in the 6 to 60 GeV/c momentum range, *Nuclear Physics B*, 61, 62–76, doi: 10.1016/0550-3213(73)90351-9 1973.
- Diehl, M.: Generalized parton distributions, *Physics Reports* 388 (2-4), 41–277, doi: 10.1016/j.physrep.2003.08.002, 2003.
- Dirac, P. A. M.: The Quantum Theory of the Emission and Absorption of Radiation, *Proc. R. Soc. Lond. A* 114, 243–265, doi: 10.1098/rspa.1927.0039, 1927.
- Eden, R. J.: Regge poles and elementary particles, *Reports on Progress in Physics* 34 (3), 995, doi: 10.1088/0034-4885/34/3/304, 1971.
- Feynman, R. P. and Zee, A.: *QED: The Strange Theory of Light and Matter*, Princeton University Press, ISBN-10: 0691125759, 2012.
- Gaskell, D.: Longitudinal Electroproduction of Charged Pions from Hydrogen, Deuterium, and Helium-3, Ph.D. Thesis, Oregon State University, 2001.
- Geiger, H. and Marsden, E.: The laws of deflexion of a particles through large angles, *Philosophical Magazine Series* 6, v. 25 (148), 604–623, doi: 10.1080/14786440408634197, 1913.
- Gell-Mann, M. and Pais, A.: Behavior of Neutral Particles under Charge Conjugation, *Phys. Rev.* 97 (5), 1387–1389, doi: 10.1103/PhysRev.97.1387, 1955.
- Gell-Mann, The interpretation of the new particles as displaced charge multiplets, *M. Nuovo Cim* 4 (Suppl 2), 848–866, doi: 10.1007/BF02748000, 1956.

- Guidal, M., Laget, J.-M., and Vanderhaeghen, M.: Pion and kaon photoproduction at high energies: forward and intermediate angles, *Nuclear Physics A* 627 (4), 645–678, doi: 10.1016/S0375-9474(97)00612-X, 1997.
- Guidal, M., Laget, J.-M., and Vanderhaeghen, M.: Electroproduction of strangeness above the resonance region, *Phys. Rev. C* 61 (2), 025204, doi: 10.1103/PhysRevC.61.025204, arXiv:hep-ph/9904511, 2000.
- Guidal, M., Laget, J.-M., and Vanderhaeghen, M.: Exclusive electromagnetic production of strangeness on the nucleon: Regge analysis of recent data, *Phys. Rev. C* 68 (5), 058201, doi: 10.1103/PhysRevC.68.058201, 2003.
- Gustafsson, K.: Cryotarget density dependence of beam current, JLab Hall C internal report (unpublished), 1996.
- He, J., Juliá-Daz, B., and Dong, Yb.: Electroweak properties of the π , K and $K^*(892)$ in the three forms of relativistic kinematics, *Eur. Phys. J. A* 24, 411, doi: 10.1140/epja/i2005-10013-7, 2005.
- Hofstadter, R. and McAllister, R. W.: Electron Scattering from the Proton, *Phys. Rev.* 98 (1), 217–218, doi: 10.1103/PhysRev.98.217, 1955.
- Hofstadter, R.: Electron Scattering and Nuclear Structure, *Rev. Mod. Phys.* 28 (3), 214–254, doi: 10.1103/RevModPhys.28.214, 1956.
- Hofstadter, R.: Nuclear and Nucleon Scattering of High-Energy Electrons, *Annu. Rev. Nucl. Sci.* 7, 231, doi: 10.1146/annurev.ns.07.120157.001311, 1957.
- Horn, T.: The Pion Charge Form Factor Through Pion Electroproduction, Ph.D. thesis, University of Maryland, 2006.
- Horn, T., Aniol, K., Arrington, J., Barrett, B., Beise, E. J., Blok, H. P., Boeglin, W., Brash, E. J., Breuer, H., Chang, C. C., Christy, M. E., Ent, R., Gaskell, D., Gibson, E., Holt, R. J., Huber, G. M., Jin, S., Jones, M. K., Keppel, C. E., Kim, W., King, P. M., Kovaltchouk, V., Liu, J., Lolos, G. J., Mack, D. J., Margaziotis, D. J.,

Markowitz, P., Matsumura, A., Meekins, D., Miyoshi, T., Mkrtchyan, H., Niculescu, I., Okayasu, Y., Pentchev, L., Perdrisat, C., Potterveld, D., Punjabi, V., Reimer, P., Reinhold, J., Roche, J., Roos, P. G., Sarty, A., Smith, G. R., Tadevosyan, V., Tang, L. G., Tvaskis, V., Vidakovic, S., Volmer, J., Vulcan, W., Warren, G., Wood, S. A., Xu, C., and Zheng, X.: Determination of the Pion Charge Form Factor at $Q^2 = 1.60$ and 2.45 (GeV/c)^2 , Phys. Rev. Lett. 97, 19, 192001, doi: 10.1103/PhysRevLett.97.192001, 2006.

Horn, T., Qian, X., Arrington, J., Asaturyan, R., Benmokthar, F., Boeglin, W., Bosted, P., Bruell, A., Christy, M. E., Chudakov, E., Clasie, B., Dalton, M. M., Daniel, A., Day, D., Dutta, D., El Fassi, L., Ent, R., Fenker, H., Ferrer, J., Fomin, N., Gao, H., Garrow, K., Gaskell, D., Gray, C., Huber, G. M., Jones, M. K., Kalantarians, N., Keppel, C. E., Kramer, K., Li, Y., Liang, Y., Lung, A. F., Malace, S., Markowitz, P., Matsumura, A., Meekins, D., Mertens, T., Miyoshi, T., Mkrtchyan, H., Monson, R., Navasardyan, T., Niculescu, G., Niculescu, I., Okayasu, Y., Opper, A. K., Perdrisat, C., Punjabi, V., Rauf, A. W., Rodriguez, V., Rohe, D., Seely, J., Segbefia, E., Smith, G. R., Sumihama, M., Tadevosyan, V., Tang, L. G., Tvaskis, V., Villano, A., Vulcan, W., Wesselmann, F. R., Wood, S. A., Yuan, L., and Zheng, X. C.: Scaling study of the pion electroproduction cross sections, Phys. Rev. C78 (5), 058201, doi: 10.1103/PhysRevC.78.058201, 2008.

Horn, T.: Global analysis of exclusive kaon and pion electroproduction, Phys. Rev. C85 (1), 018202, doi: 10.1103/PhysRevC.85.018202, 2012.

Horn, T. and Roberts, C. D.: The pion: an enigma within the Standard Model, J. Phys. G: Nucl. Part. Phys. 43, 073001, doi: 10.1088/0954-3899/43/7/073001, 2016.

Horn, T., Mkrtchyan, H., Ali, S., Asaturyan, A., Carmignotto, M., Dittmann, A., Dutta, D., Ent, R., Hlavin, N., Illieva, Y., Mkrtchyan, A., Nadel-Turonski, P., Pegg, I., Ramos, A., Reinhold, J., Sapkota, I., Tadevosyan, V., Zhamkochyan, S., and Wood, S. A.: The Aerogel Čerenkov detector for the SHMS magnetic spectrometer in Hall C at Jefferson Lab, NIM-A 842, 28–47, doi: 10.1016/j.nima.2016.10.039, arXiv:1607.05264, 2017.

Hu, Z., Li, G., Xie, H., Hua, T., Chen, P., and Huang, F.: Measurement of Young’s modulus and Poisson’s ratio of human hair using optical techniques, Proc. SPIE 7522, Fourth International Conference on Experimental Mechanics, 75222Q, doi: 10.1117/12.851415, 2010.

Huber, G. M., Blok, H. P., Horn, T., Beise, E. J., Gaskell, D., Mack, D. J., Tadevosyan, V., Volmer, J., Abbott, D., Aniol, K., Anklin, H., Armstrong, C., Arrington, J., Assamagan, K., Avery, S., Baker, O. K., Barrett, B., Bochna, C., Boeglin, W., Brash, E. J., Breuer, H., Chang, C. C., Chant, N., Christy, M. E., Dunne, J., Eden, T., Ent, R., Fenker, H., Gibson, E. F., Gilman, R., Gustafsson, K., Hinton, W., Holt, R. J., Jackson, H., Jin, S., Jones, M. K., Keppel, C. E., Kim, P. H., Kim, W., King, P. M., Klein, A., Koltenuk, D., Kovaltchouk, V., Liang, M., Liu, J., Lolo, G. J., Lung, A., Margaziotis, D. J., Markowitz, P., Matsumura, A., McKee, D., Meekins, D., Mitchell, J., Miyoshi, T., Mkrtchyan, H., Mueller, B., Niculescu, G., Niculescu, I., Okayasu, Y., Pentchev, L., Perdrisat, C., Pitz, D., Potterveld, D., Punjabi, V., Qin, L. M., Reimer, P. E., Reinhold, J., Roche, J., Roos, P. G., Sarty, A., Shin, I. K., Smith, G. R., Stepanyan, S., Tang, L. G., Tvaskis, V., van der Meer, R. L. J., Vansyoc, K., Westrum, D. Van and Vidakovic, S., Vulcan, W., Warren, G., Wood, S. A., Xu, C., Yan, C., Zhao, W.-X., Zheng, X., and Zihlmann, B.: Charged pion form factor between $Q^2 = 0.60$ and 2.45GeV^2 . II. Determination of, and results for, the pion form factor, Phys. Rev. C78, 4, 045203, 16, doi: 10.1103/PhysRevC.78.045203, 2008.

Huber, G.: Institute of Nuclear Theory Workshop, Seattle, WA, (unpublished), 2009.

Hutauruk, P. T. P., Cloët, I. C., Thomas, A. W.: Flavor dependence of the pion and kaon form factors and parton distribution functions, Phys. Rev. C 94 (3), 035201, doi: 10.1103/PhysRevC.94.035201, 2016.

Hwang, C. W.: Consistent treatment for pion form factors in spacelike and timelike regions, Phys. Rev. D64 (3), 034011, doi: 10.1103/PhysRevD.64.034011, 2001.

JLab Experiment E12-06-101 – Huber, G. et al.: Measurement of the Charged Pion Form Factor to High Q^2 , Approved Jefferson Lab experiment E12-06-101, http://www.jlab.org/exp_prog/proposals/06/PR06-101.pdf, 2006.

JLab Experiment E12-07-105 – Horn, T. et al.: Scaling Study of the L-T Separated Pion Electroproduction Cross Section at 11 GeV, Approved Jefferson Lab experiment E12-07-105, http://www.jlab.org/exp_prog/proposals/07/PR12-07-105.pdf, 2007.

JLab Experiment E12-09-011 – Horn, T. et al.: Studies of the L-T Separated Kaon Electroproduction Cross Section from 5-11 GeV, Approved Jefferson

Lab experiment, http://www.jlab.org/exp_prog/PACpage/PAC38/proposals/Previously_Aproved/E12-09-011_Update.pdf, 2009.

<https://www.jlab.org/sites/default/files/documents/hr/Jefferson%20Lab%20Mission%20Statement.pdf>, consulted in October 2015.

King's College London Archives, Library Services, S3.02, Strand Building, London WC2R 2LS.

Kobayashi, M. and Maskawa, T.: CP-Violation in the Renormalizable Theory of Weak Interaction, *Prog. Theor. Phys.* 49 (2), 652–657, doi: 10.1143/PTP.49.652, 1973.

Lattes, C. M. G., Muirhead, H., G. P. S. Occhialini, G. P. S., and Powell, C. F.: Processes involving charged mesons, *Nature* 159, 694–697, doi: 10.1038/159694a0, 1947.

Leemann, C. W., Douglas, D. R., and Krafft, G. A.: The Continuous Electron Beam Accelerator Facility: CEBAF at Jefferson Laboratory, *Annu. Rev. Nucl. Part. Sci.*, 51:413–450, doi: 10.1146/annurev.nucl.51.101701.132327, 2001.

Leicester, H. M.: *Source Book in Chemistry, 1900-1950*, Cambridge/MA, Harvard UP, 1968.

Mack, D. J., Blok, H. P., Huber, G. M., et al.: Proposal of experiment to Jefferson Lab Hall C E01-004 - Extension to E93-021: The Charged Pion Form Factor, 2000.

Makins, N. C. R.: Measurement of the nuclear dependence and momentum transfer dependence of quasielastic (e,e'p) scattering at large momentum transfer, Ph.D. Thesis, Massachusetts Institute of Technology, Dept. of Physics, <http://hdl.handle.net/1721.1/28133>, 1994.

Maris, P. and Tandy, P. C.: π , K^+ , and K^0 electromagnetic form factors, *Phys. Rev. C* 62 (5), 055204, doi: 10.1103/PhysRevC.62.055204, 2000.

Masjuan, P., Peris, S., and Sanz-Cillero, J. J.: Vector meson dominance as a first step in a systematic approximation: The pion vector form factor, *Phys. Rev. D* 78 (7), 074028, doi: 10.1103/PhysRevD.78.074028, 2008.

- McAllister, R. W. and Hofstadter, R.: Elastic Scattering of 188-Mev Electrons from the Proton and the Alpha Particle, *Phys. Rev.* 102 (3), 851–856, doi: 10.1103/PhysRev.102.851, 1956.
- Miller, G. A.: Singular charge density at the center of the pion?, *Phys. Rev. C* 79 (5), 055204, doi: 10.1103/PhysRevC.79.055204, 2009.
- Miller, G. A.: Transverse charge densities, *Annu. Rev. Nucl. Part. Sci.* 60, 1–25, doi: 10.1146/annurev.nucl.012809.104508, 2010.
- Miller, G. A., Strikman, M., and Weiss, C.: Pion transverse charge density from timelike form factor data, *Phys. Rev. D* 83 (1), 013006, doi: 10.1103/PhysRevD.83.013006, 2011.
- Mo, L. W. and Tsai, Y. S.: Radiative Corrections to Elastic and Inelastic ep and up Scattering, *Rev. Mod. Phys.* 41 (1), 205–235, doi: 10.1103/RevModPhys.41.205, 1969.
- Mohring, R. M.: A Comparison of Longitudinal and Transverse Cross Sections in the $p(e,e'K^+)\Lambda$ and $p(e,e'K^+)\Sigma^0$ Reactions, Ph.D. thesis, University of Maryland, 1999.
- Mohring, R. M., Abbott, D., Ahmidouch, A., Amatuni, Ts. A., Ambrozewicz, P., Angelescu, T., Armstrong, C. S., Arrington, J., Assamagan, K., Avery, S., Bailey, K., Beard, K., Beedoe, S., Beise, E. J., Breuer, H., Carlini, R., Cha, J., Chang, C. C., Chant, N., Cisbani, E., Collins, G., Cummings, W., Danagouliau, S., De Leo, R., Duncan, F., Dunne, J., Dutta, D., Eden, T., Ent, R., Eyraud, L., Ewell, L., Finn, M., Fortune, T., Frolov, V., Frullani, S., Furget, C., Garibaldi, F., Gaskell, D., Geesaman, D. F., Gueye, P., Gustafsson, K. K., Hansen, J.-O., Harvey, M., Hinton, W., Hungerford, E., Iodice, M., Jackson, C., Keppel, C., Kim, W., Kino, K., Koltenuk, D., Kox, S., Kramer, L., Leone, T., Lung, A., Mack, D., Madey, R., Maeda, M., Majewski, S., Markowitz, P., Mart, T., Martoff, C. J., Meekins, D., Mihul, A., Mitchell, J., Mkrtchyan, H., Mtingwa, S., Niculescu, I., Perrino, R., Potterveld, D., Price, J. W., Raue, B. A., Real, J.-S., Reinhold, J., Roos, P., Saito, T., Savage, G., Sawafuta, R., Segel, R., Stepanyan, S., Stoler, P., Tadevosian, V., Tang, L., Teodorescu, L., Terasawa, T., Tsubota, H., Urciuoli, G. M., Volmer, J., Vulcan, W., Welch, T. P., Williams, R., Wood, S., Yan, C., and Zeidman, B.: Separation of the longitudinal and transverse cross sections in the $^1\text{H}(e, e' K^+)\Lambda$ and $^1\text{H}(e, e' K^+)\Sigma^0$ reactions, *Phys. Rev. C* 67 (5), 055205, doi: 10.1103/PhysRevC.67.055205, 2003.

Mkrtchyan, H., Carlini, R., Tadevosyan, V., Arrington, J., Asaturyan, A., Christy, M. E., Dutta, D., Ent, R., Fenker, H. C., Gaskell, D., Horn, T., Jones, M. K., Keppel, C. E., Mack, D. J., Malace, S. P., Mkrtchyan, A., Niculescu, M. I., Seely, J., Tvaskis, V., Wood, S. A., and Zhamkochyan, S.: The lead-glass electromagnetic calorimeters for the magnetic spectrometers in Hall C at Jefferson Lab, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 719, 85–100, doi: 10.1016/j.nima.2013.03.070, 2013.

Nash, L. K.: The Origin of Dalton’s Chemical Atomic Theory, *Isis* 47 (2), 101–116, 1956.

Ne’eman, Y.: Derivation of strong interactions from a gauge invariance, *Nuclear Physics* 26 (2), 222–229, doi: 10.1016/0029-5582(61)90134-1, 1961.

Neil, G. R., Behre, C., Benson, S. V., Bevins, M., Biallas, G., Boyce, J., Coleman, J., Dillon-Townes, L. A., Douglas, D., Dylla, H. F., Evans, R., Grippo, A., Gruber, D., Gubeli, J., Hardy, D., Hernandez-Garcia, C., Jordan, K., Kelley, M. J., Merminga, L., Mammossner, J., Moore, W., Nishimori, N., Pozdeyev, E., Preble, J., Rimmer, R., Shinn, M., Siggins, T., Tennant, C., Walker, R., Williams, G. P., and Zhang, S.: The JLab high power ERL light source, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 557(1), 9–15, doi: 10.1016/j.nima.2005.10.047, 2006.

Niculescu, G., Mohring, R. M., Gueye, P., Abbott, D., Ahmidouch, A., Amatuni, Ts. A., Ambrozewicz, P., Angelescu, T., Armstrong, C. S., Assamagan, K., Avery, S., Bailey, K., Baker, O. K., Beard, K., Beedoe, S., Beise, E., Breuer, H., Carlini, R., Cha, J., Chang, C. C., Chant, N., Cisbani, E., Collins, G., Cummings, W., Danagouliau, S., De Leo, R., Duncan, F., Dunne, J., Dutta, D., Eden, T., Ent, R., Eyraud, L., Ewell, L., Finn, M., Fortune, T., Frolov, V., Frullani, S., Furget, C., Garibaldi, F., Gaskell, D., Geesaman, D. F., Gustafsson, K. K., Hansen, J.-O., Harvey, M., Hinton, W., Hungerford, E., Iodice, M., Jackson, C., Keppel, C., Kim, W., Kino, K., Koltenuk, D., Kox, S., Kramer, L., Leone, T., Lolos, G., Lung, A., Mack, D., Madey, R., Maeda, M., Majewski, S., Markowitz, P., Martoff, C. J., Meekins, D., Mihul, A., Mitchell, J., Mkrtchyan, H., Mtingwa, S., Niculescu, I., Perrino, R., Potterveld, D., Price, J. W., Raue, B. A., Real, J.-S., Reinhold, J., Roos, P., Saito, T., Savage, G., Sawafta, R., Segel, R., Stepanyan, S., Stoler, P., Tadevosian, V., Tang, L., Teodorescu, L., Terasawa, T., Tsubota, H., Urciuoli, G. M., Volmer, J., Vulcan, W., Welch, P., Williams, R., Wood, S., Yan, C., and

- Zeidman, B.: Longitudinal and Transverse Cross Sections in the $^1\text{H}(e, e' K^+)\Lambda$ Reaction, Phys. Rev. Lett. 81 (9), 1805–1808, doi: 10.1103/PhysRevLett.81.1805, 1998a.
- Niculescu, G.: First measurement of the longitudinal and transverse cross section in $^1\text{H}(e, e' K^+)\Lambda$, Ph.D. thesis, Hampton University, 1998b.
- Ninomiya, Y., Bentz, W., and Cloët, I. C.: Dressed quark mass dependence of pion and kaon form factors, Phys. Rev. C 91 (2), 025202, doi: 10.1103/PhysRevC.91.025202, 2015.
- Nuruzzaman, Dutta, D., Arrington, J., Asaturyan, R., Benmokhtar, F., Boeglin, W., Bosted, P., Bruell, A., Clasie, B., Christy, M. E., Chudakov, E., Dalton, M. M., Daniel, A., Day, D., Fassi, L. El, Ent, R., Fenker, H. C., Ferrer, J., Fomin, N., Gao, H., Garrow, K., Gaskell, D., Gray, C., Horn, T., Huber, G. M., Jones, M. K., Kalantarians, N., Koppel, C. E., Kramer, K., Li, Y., Liang, Y., Lung, A. F., Malace, S., Markowitz, P., Matsumura, A., Meekins, D. G., Mertens, T., Miyoshi, T., Mkrtchyan, H., Monson, R., Navasardyan, T., Niculescu, G., Niculescu, I., Okayasu, Y., Opper, A. K., Perdrisat, C., Punjabi, V., Qian, X., Rauf, A. W., Rodriguez, V. M., Rohe, D., Seely, J., Segbefia, E., Smith, G. R., Sumihama, M., Tadevosyan, V., Tang, L., Tvaskis, V., Vulcan, W. F., Wesselsmann, F. R., Wood, S. A., Yuan, L., Zheng, X. C.: Nuclear transparency and effective kaon-nucleon cross section from the $\text{A}(e, e' K^+)$ reaction, Phys. Rev. C, 84 (1), 015210, doi: 10.1103/PhysRevC.84.015210, 2011.
- O'Connell, H. B., Pearce, B. C., Thomas, A. W., and Williams, A. G.: Rho-omega mixing and the pion electromagnetic form-factor, Physics Letters B354 (1), 14-19, doi: 10.1016/0370-2693(95)00642-X, 1995.
- Patrignani, C. et al. (Particle Data Group), Chin. Phys. C, 40, 100001, doi: 10.1088/1674-1137/40/10/100001, 2016.
- Pohl, R., Gilman, R., Miller, G. A., and Pachucki, K.: Muonic Hydrogen and the Proton Radius Puzzle, Ann. Rev. Nucl. Part. Sci. 63, 175, doi: 10.1146/annurev-nucl-102212-170627, arXiv:1301.0905, 2013.
- Povh, B., Rith, K., Scholz, C., and Zetsche, F.: Particles and Nuclei - An Introduction to the Physical Concepts, Springer Nature, doi: 10.1007/978-3-540-79368-7, 2008.

- Rocha, G. R.: A história do atomismo - A construção e a desconstrução de uma imagem sintático-semântica do conhecimento científico, MSc. dissertation from the Universidade Federal de Minas Gerais, 2006.
- Rocha-Lima, A., Carmignotto, M. A. P., and Andrade, M. G.: Relatório de difração do curso de Física Experimental IV, Physics Institute of the University of São Paulo, unpublished, 2004.
- Rochester, G. D. and Butler, C. C.: Evidence for the Existence of New Unstable Elementary Particles, *Nature* 160, 855–857, doi: 10.1038/160855a0, 1947.
- Rosenbluth, M. N.: High Energy Elastic Scattering of Electrons on Protons, *Phys. Rev.* 79 (4), 615–619, doi: 10.1103/PhysRev.79.615, 1950.
- Strikman, M. and Weiss, C.: Quantifying the nucleon’s pion cloud with transverse charge densities, *Phys. Rev. C* 82 (4), 042201, doi: 10.1103/PhysRevC.82.042201, 2010.
- Tadevosyan, V., Blok, H. P., Huber, G. M., Abbott, D., Anklin, H., Armstrong, C., Arrington, J., Assamagan, K., Avery, S., Baker, O. K., Bochna, C., Brash, E. J., Breuer, H., Chant, N., Dunne, J., Eden, T., Ent, R., Gaskell, D., Gilman, R., Gustafsson, K., Hinton, W., Jackson, H., Jones, M. K., Keppel, C., Kim, P. H., Kim, W., Klein, A., Koltenuk, D., Liang, M., Lolos, G. J., Lung, A., Mack, D. J., McKee, D., Meekins, D., Mitchell, J., Mkrtchyan, H., Mueller, B., Niculescu, G., Niculescu, I., Pitz, D., Potterveld, D., Qin, L. M., Reinhold, J., Shin, I. K., Stepanyan, S., Tang, L. G., Meer, R. L. J. van der and Vansyoc, K., Westrum, D. Van and Volmer, J., Vulcan, W., Wood, S., Yan, C., Zhao, W.-X., and Zihlmann, B.: Determination of the pion charge form factor for $Q^2 = 0.60\text{--}1.60\text{ GeV}^2$, *Phys. Rev. C* 75 (5), 055205, doi: 10.1103/PhysRevC.75.055205, 2007.
- Tvaskis, V., Ph.D. thesis from the Vrije Universiteit Amsterdam & NIKHEF, 2004.
- Terburg, B.: High energy two-body deuteron photodisintegration, Ph.D. thesis (unpublished), 1999.
- Unser, K. B.: The parametric current transformer, a beam current monitor developed for LEP, *AIP Conference Proceedings* 252, 266–275, doi: 10.1063/1.42124, 1992.

- Vanderhaeghen, M., Guidal, M., and Laget, J.-M.: Regge description of charged pseudoscalar meson electroproduction above the resonance region, *Phys. Rev. C* 57 (3), 1454–1457, doi: 10.1103/PhysRevC.57.1454, 1998.
- Venkat, S., Arrington, J., Miller, G. A., and Zhan, X.: Realistic transverse images of the proton charge and magnetization densities, *Phys. Rev. C* 83 (1), 015203, doi: 10.1103/PhysRevC.83.015203, 2011.
- Volmer, J.: The pion charge form factor via pion electroproduction on the proton, Ph.D. Thesis, Vrije Universiteit Amsterdam, 2000.
- Volmer, J., Abbott, D., Anklin, H., Armstrong, C., Arrington, J., Assamagan, K., Avery, S., Baker, O. K., Blok, H. P., Bochna, C., Brash, E. J., Breuer, H., Chant, N., Dunne, J., Eden, T., Ent, R., Gaskell, D., Gilman, R., Gustafsson, K., Hinton, W., Huber, G. M., Jackson, H., Jones, M. K., Keppel, C., Kim, P. H., Kim, W., Klein, A., Koltenuk, D., Liang, M., Lolos, G. J., Lung, A., Mack, D. J., McKee, D., Meekins, D., Mitchell, J., Mkrtchyan, H., Mueller, B., Niculescu, G., Niculescu, I., Pitz, D., Potterveld, D., Qin, L. M., Reinhold, J., Shin, I. K., Stepanyan, S., Tadevosyan, V., Tang, L. G., van der Meer, R. L. J., Vansyoc, K., Van Westrum, D., Vulcan, W., Wood, S., Yan, C., Zhao, W.-X., and Zihlmann, B., Measurement of the Charged Pion Electromagnetic Form Factor, *Phys. Rev. Lett.* 86 (9), 1713–1716, doi: 10.1103/PhysRevLett.86.1713, 2001.
- Vrancx, T., Ryckebusch, J., and Nys, J.: $K^+\Lambda$ electroproduction above the resonance region, *Phys. Rev. C* 89 (6), 065202, doi: 10.1103/PhysRevC.89.065202, 2014. - Online calculator available in December/2016 at <http://rprmodel.ugent.be/calc/>.
- Watson, J. D. and Crick, F. H. C.: Molecular structure of nucleic acids, *Nature* 171, 737–738, doi: 10.1038/171737a0, 1953.
- Wilson, D. J., Cloët, I. C., Chang, L., and Roberts, C. D.: Nucleon and Roper electromagnetic elastic and transition form factors, *Phys. Rev. C* 85 (2), 025205, doi: 10.1103/PhysRevC.85.025205, 2012.
- Wróblewski, A. K.: Hypernuclei (and Strange Particles) - How It All Began?, *ACTA PHYSICA POLONICA SERIES B* 35 (3), 901–928, 2004.

Xiao, B.-W., Qian, X., and Ma, B.-Q.: The kaon form factor in the light-cone quark model, Eur. Phys. J. A 15 (4), 523-527, doi: 10.1140/epja/i2002-10059-y, 2002.

Zweig, G.: An $SU(3)$ model for strong interaction symmetry and its breaking, Version 2, Developments in the Quark Theory of Hadrons, Volume 1, edited by D. Lichtenberg and S. Rosen., 22–101, CERN-TH-412, NP-14146, PRINT-64-170, 1964..