Analyticity Constraints for Hadron Amplitudes: Going High to Heal Low Energy Issues

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Analyticity constitutes a rigid constraint on hadron scattering amplitudes. This property is used to relate models in different energy regimes. Using meson photoproduction as a benchmark, we show how to test contemporary low energy models directly against high energy data. This method pinpoints deficiencies of the models and treads a path to further improvement. The implementation of this technique enables one to produce more stable and reliable partial waves for future use in hadron spectroscopy and new physics searches.

Introduction.— Determination of various hadronic effects represents a major challenge in searches for New Physics through precision measurements [1–5]. For example, the possible identification of Beyond Standard Model signals in B meson decays is hindered by uncertainties in hadronic final state interactions. The strongly coupled nature of QCD prevents us from computing these effects directly from the underlying microscopic formulation. Nevertheless, one can use the first principles of S-matrix theory to impose stringent constraints on hadron scattering amplitudes [6–8]. These approaches are encountering a renewed interest even in the more formal context of strongly coupled theories [9–11].

In this Letter, we show how to use analyticity to relate the amplitudes at high energies to the physics at low energies, where resonance effects dominate. This is not only important for reducing hadronic uncertainties in the aforementioned processes, but is of interest on its own merits for unraveling the spectrum of QCD. The current spectrum summarized in the Particle Data Group (PDG) is far from complete [12], according to phenomenological predictions and lattice QCD simulations. For example, the recent discoveries of unexpected peaks in data indicate that the true hadron spectrum is far more complex than predicted [13–18]. As a working case, we focus here on the baryon sector in the intermediate energy range. In the PDG these N^* and Δ resonances are referred to as "poorly known" [12], despite the large amount of data available. The ambiguities encountered when identifying resonances are related to the fact that, as the center of mass energy increases, so does the number of contributing partial waves, vastly complicating the reaction models used in data analysis. The $2-3\,\mathrm{GeV}$ mass region is of particular interest for baryon spectroscopy since, besides the ordinary quark model multiplets, it is expected to contain a new form of exotic light quark matter that is dominated by excitations of the gluon field [16, 19]. The recent upgrade at Jefferson Lab [20–24] is providing high statistics data on hadron photoproduction. New amplitude analysis methods are a prerequisite to achieve a robust extraction of hadron resonance parameters.

Many research groups carry out low energy, coupled channel, partial wave analyses (PWA) for baryon spectroscopy. Currently, the most active are ANL-Osaka [25], Bonn-Gatchina [26, 27], JPAC [28, 29], Jülich-Bonn [30, 31], MAID [32], and SAID [33, 34]. These groups perform global fits to hadro- and/or photoproduction data using a finite set of partial waves to extract baryon resonance properties [35, 36]. In these approaches the high energy data are largely ignored. As we show in this Letter, these data can greatly impact the baryon spectrum analyses through analyticity. Specifically, we implement Finite Energy Sum Rules derived from dispersion relations [37], and use simple approximations to describe the high energy data. The sum rules relate the amplitudes in the baryon resonance region to the high energy dynamics, which are governed by meson exchanges [38] described by Regge theory. We apply our method to the existing data on π^0 and η photoproduction [39–41]. These cases constitute a first step towards a straightforward and systematic implementation of high energy constraints into low energy amplitudes, and provide a template for further application in data analysis.

Analyticity constraints for photoproduction.— The reaction $\gamma p \to x p$, where $x = \pi^0, \eta$ is completely described in terms of four independent scalar amplitudes $A_i(s,t)$. These are analytic functions of the Mandelstam variables s (the square of the center of mass energy) and t (the square of the momentum transfer) [42]. At fixed t, each $A_i(s,t)$ satisfies a dispersion relation involving the dis-

continuity with respect to s along the unitarity cut and the crossed-channel unitarity cut in $u = 2m_p^2 + m_x^2 - s - t$. Charge conjugation symmetry relates the discontinuity along the crossed channel cut to that of the direct channel. This symmetry is made explicit by writing the amplitude as a function of the variable[1] $\nu \equiv (s-u)/2$. For large $|\nu|$ and small t kinematics, the amplitudes are well approximated by Regge poles, *i.e.* via crossed channel exchanges. In this region, the amplitudes take the form

$$\mathrm{Im}\, A_i(\nu, t) = \sum_n \beta_i^{(n)}(t)\, \nu^{\alpha^{(n)}(t)-1}. \qquad (1)$$

The Regge poles are determined by the trajectories $\alpha^{(n)}(t)$ and the residues $\beta_i^{(n)}(t)$. The index n runs over all possible exchanges. This approximation holds only if $|\nu|$ is greater than some cutoff Λ above the resonance region. For $|\nu| < \Lambda$, the amplitude is dominated by direct channel resonances, and thus it can be well approximated by a finite number of partial waves. One can write a dispersion relation using the contour in Fig. 1, and calculate explicitly the integral in the region $|\nu| = \Lambda$ [38],

$$\int_0^\Lambda \mathrm{Im}\, A_i(\nu, t)\; \nu^k\, \mathrm{d}\nu = \sum_n \beta_i^{(n)}(t)\, \frac{\Lambda^{\alpha^{(n)}(t)+k}}{\alpha^{(n)}(t)+k}. \qquad (2)$$

The amplitudes $A_{1,2,4}$ and A_3 are even and odd functions of ν, respectively. Here k is an arbitrary positive integer, odd for $A_{1,2,4}$ and even for A_3. We give the value of Λ in terms of an energy cutoff $s_{\max}$, which introduces additional t dependence $\Lambda = s_{\max} + (t - 2m_p^2 - m_x^2)/2$. We restrict the sum on the right hand side (*rhs*) of Eq. (2) to the dominant t-channel Regge poles. Each A_i receives a contribution from both isoscalar and isovector exchanges. Natural parity exchanges (with $P = (-)^J$) dominate A_1 and A_4, while the unnatural ones (with $P = (-)^{J+1}$) dominate A_2 and A_3. More specifically, the $n = \rho, \omega$, Regge poles contribute to A_1 and A_4, while A_2 and A_3 are determined by exchanges of the $n = b, h, \rho_2, \omega_2$.[2]

The trajectories are nearly degenerate for all the natural exchanges [45], and in the kinematical region of interest, they can be well approximated by

$$\alpha(t) \equiv \alpha^{(\rho)}(t) = \alpha^{(\omega)}(t) = 1 + 0.9\,(t - m_\rho^2), \qquad (3)$$

for $i = 1, 4$. For the unnatural exchanges, $\alpha(t) = 0.7\,(t - m_\pi^2)$ for $i = 2, 3$. The contribution of unnatural versus natural exchanges to observables in the forward direction is suppressed at high energy. For example, with a beam energy of 9 GeV, the suppression is expected to be $\nu^{2(0.9m_\rho^2 - 0.7m_\pi^2 - 1)} \sim 7\%$. This can be compared with polarization observables, such as the beam asymmetry Σ,[3] which are sensitive to the interference between the natural and the unnatural Regge poles. If one neglects the unnatural contributions, $\Sigma = 1$. The recent measurement of π^0 and η beam asymmetries at GlueX [23] confirms that $\Sigma > 0.9$, so that the unnatural exchanges contribute $\lesssim 5\%$ to the observables. In the following, we will consider the amplitudes dominated by natural exchanges, A_1 and A_4, only. We use low energy models as input to determine the left hand side of Eq. (2), and use it to predict the residues. To this aim we define the effective residues,

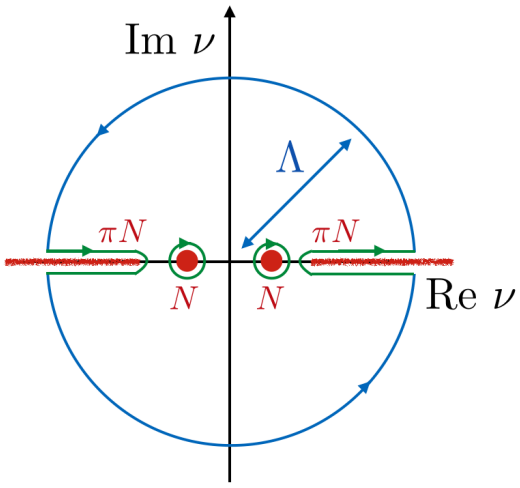

FIG. 1. Contour in the complex ν-plane used in the derivation of the sum rules in Eq. (2). The radius Λ must be taken sufficiently large, for the single Regge pole approximation to hold at $|\nu| = \Lambda$. The nucleon pole and the πN cuts are shown on the real axis.

$$\widehat{\beta}_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}} \int_0^\Lambda \mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu, t)\, \nu^k\, \mathrm{d}\nu, \qquad (4)$$

where A_i^{PWA} is the amplitude calculated from low-energy models.

Because of Regge trajectory degeneracy, the $\widehat{\beta}_i(t)$'s describe the sum of the contribution of both isovector and isoscalar exchanges. Consistency of the single pole hypothesis requires the *rhs* of Eq. (4) to be independent of k.[4] For $|\nu| > \Lambda$, the amplitudes can be expressed in terms of the effective residues as [38]

$$\widehat{A}_i(\nu, t) = \left[i + \tan\frac{\pi}{2}\alpha(t) \right] \widehat{\beta}_i(t)\, \nu^{\alpha(t)-1}. \qquad (5)$$

[1] As customary, all dimensional variables are given in units of 1 GeV.

[2] Even though there are some experimental indications of the existence of ρ_2 and ω_2 [43, 44], they have been observed by one single group, and thus need further confirmation [12].

[3] The beam asymmetry is $\Sigma \equiv (\mathrm{d}\sigma_\perp - \mathrm{d}\sigma_\parallel)/(\mathrm{d}\sigma_\perp + \mathrm{d}\sigma_\parallel)$, with $\mathrm{d}\sigma_{\perp(\parallel)}$ the differential cross section of the photon polarized perpendicular (parallel) to the reaction plane.

[4] For example, if one added another nondegenerate trajectory $\alpha_2 < \alpha$, the effective residue would depend on k as $\hat{\beta}_i = \beta_i + \frac{\beta_{i,2}}{\Lambda^{\alpha-\alpha_2}} \frac{\alpha+k}{\alpha_2+k}$. The latter becomes negligible for Λ sufficiently large.

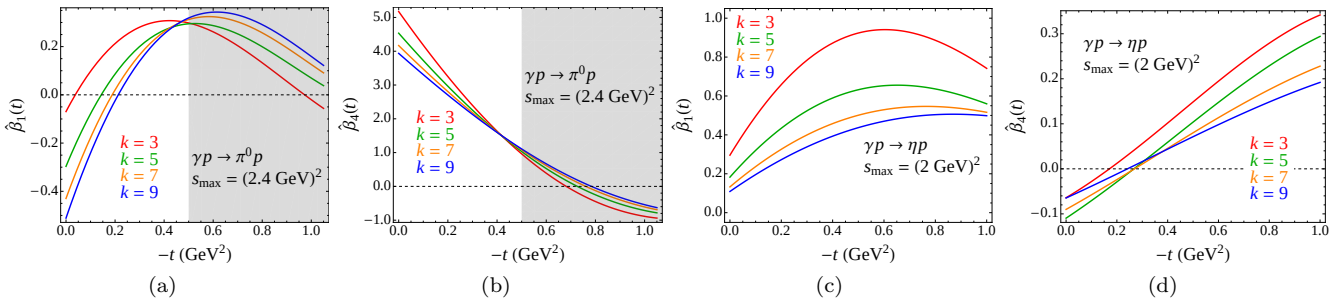

FIG. 2. Effective residues computed from the low energy models using Eq. (4). (a) and (b): π^0 photoproduction using SAID [33]. (c) and (d): η photoproduction using η-MAID [32]. For π^0, the single pole approximation is valid for $-t \lesssim 0.5$ GeV^2, as explained in the text. The dispersion in k is small for π^0, while the large variation with k for η indicates issues with the low energy model.

The $\widehat{A}_i(\nu, t)$ are the high energy amplitudes calculated from the low energy models entering in the $\widehat{\beta}_i(t)$. Comparing the observables calculated with these to data allows us to check the quality of the low energy models. In the high energy limit, the differential cross section becomes

$$\frac{\mathrm{d}\widehat{\sigma}}{\mathrm{d}t} \simeq \frac{1}{32\pi}\left[\left|\widehat{A}_1\right|^2 - t\left|\widehat{A}_4\right|^2\right] = \frac{\nu^{2\alpha(t)-2}}{32\pi}\left[1+\tan^2\frac{\pi}{2}\alpha(t)\right]\left[\widehat{\beta}_1^2(t) - t\,\widehat{\beta}_4^2(t)\right]. \quad (6)$$

Results.— We next discuss what these constraints can tell us about the existing low energy analyses. We consider $\widehat{\beta}_i(t)$ for $k = 3, 5, 7, 9$. For π^0, we use the SAID partial wave model which is valid up to $s_{\rm max} = (2.4\,\mathrm{GeV})^2$ [33]. For η, the amplitudes need to be extrapolated below the physical ηN threshold, down to the πN threshold (see Fig. 1). Among the various models, only η-MAID [32], valid only up to $s_{\rm max} = (2\,\mathrm{GeV})^2$, is given in terms of analytical functions that allow for this continuation [46].

The two effective residues $\widehat{\beta}_{1,4}(t)$ are shown in Figs. 2(a) and 2(b) for π^0 and in Figs. 2(c) and 2(d) for η, respectively. In the case of π^0, we restrict the analysis to the $0 \le -t \le 0.5\,\mathrm{GeV}^2$ region, because of subleading Regge cut contributions which are known to dominate the cross section at higher $-t$ [47]. We note that the residues are fairly independent of k. Conversely, the dependence on k for η is large. This points to a problem in the low energy model. Possible reasons can be that the resonant content for energies less than 2 GeV is underestimated, or the $2-3$ GeV resonances are relevant. In either case the low energy model can be improved using these constraints.

In Fig. 3(a) we predict the high energy π^0 differential cross section computed in Eq. (6) using the effective residues $\widehat{\beta}_i(t)$, and compare with data. The overall agreement is impressive, both in the magnitude and shape of the t dependence. The energy dependence is given by the trajectories in Eq. (3). In the region of interest, the t dependence is fully determined by the low energy amplitudes through the integral over the imaginary part, see Eq. (2). There is a dip in the cross section data near $-t = 0.5\,\mathrm{GeV}^2$, which can be traced to the zero in the dominant $\widehat{\beta}_4(t)$ at $-t \simeq 0.7\,\mathrm{GeV}^2$ in Fig. 2(b). The predictions are almost independent of the moment k. The t dependence is identical for moments up to $k = 9$, and the overall normalization changes by a maximum of 20%.

The predictions for η are shown in Fig. 3(b). Since the $\widehat{\beta}_i(t)$ computed from the low energy model have significant k dependence, we show the cross section for fixed, $k = 3$, which happens to have the correct overall normalization. The prediction agrees very well with data up to somewhat higher $-t$, but it underestimates the cross section in the forward, $-t < 0.25\,\mathrm{GeV}^2$ region. This effect originates from the small value of $\widehat{\beta}_4$ in this region, as can be seen on Figs. 2(c) and 2(d). It is worth noting that the available PWA models [25, 27, 31, 32] strongly disagree in this specific t region. In particular, in η-MAID there is a peculiar cancellation between isoscalar and isovector exchanges, which results in a smaller effective residue [46]. This illustrates how the implementation of our approach can impact on the low energy analyses.

Conclusions.— We discussed a technique which uses analyticity to constrain low energy hadron effects with the high energy data. We have benchmarked it against meson photoproduction, one of the main reactions to study hadron spectroscopy. In this specific case, we showed the effectiveness of the approach in identifying potential deficiencies in the low energy models. We showed explicitly how the baryon spectrum determines the seemingly unrelated meson exchanges dominating forward scattering at high energies, and *vice versa*. Experiments at Jefferson Lab are currently exploring meson photoproduction above the baryon resonance region. The technique presented here can be applied to these forthcoming data, and make a significant impact on baryon spectroscopy research. The approach can be extended to

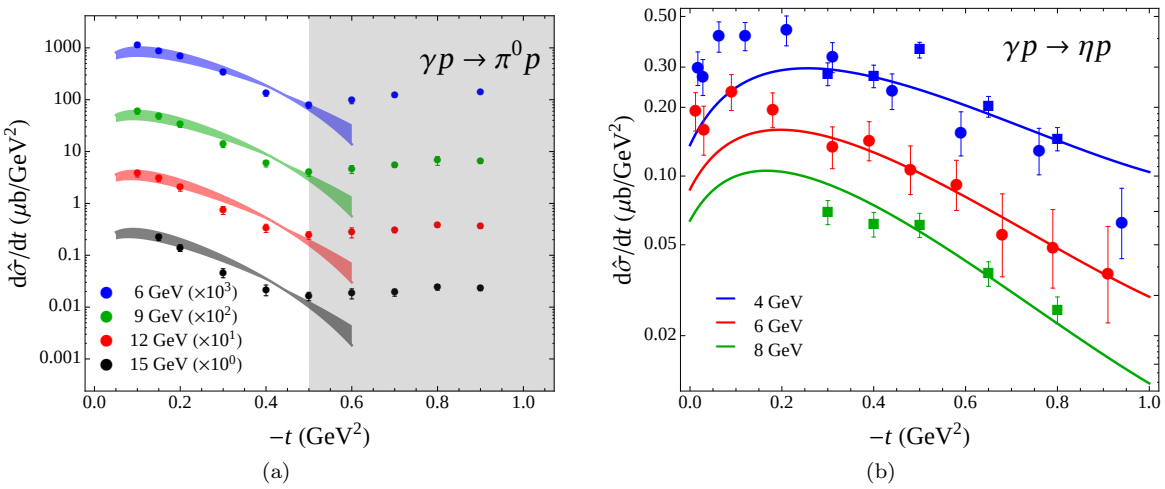

FIG. 3. Differential cross sections computed from the low energy models using Eq. (6). (a) π^0 photoproduction using SAID [33]. The prediction is restricted to $-t < 0.5\,\mathrm{GeV}^2$, as explained in the text. The error band takes into account the (small) dispersion with k. The legend indicates the beam energy in the laboratory frame and the scaling factors. Data are from [40]. (b): η photoproduction using η-MAID [32]. Prediction is shown for $k = 3$, as explained in the text. The legend indicates the beam energy in the laboratory frame. Data are from [39] (circles) and [41] (squares). For π^0 the prediction agrees with data, while for η the depletion in the forward $-t < 0.25\,\mathrm{GeV}^2$ is a marker for an inconsistency of the low energy model.

other hadron reactions, and help control the hadronic effects that drive the uncertainties in New Physics searches, especially in the heavy flavor sector.

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