

# Amplitude analyses for extraction of $N^*$ parameters and JPAC effort on JLab Physics

Vincent MATHIEU

Indiana University

-

Joint Physics Analysis Center

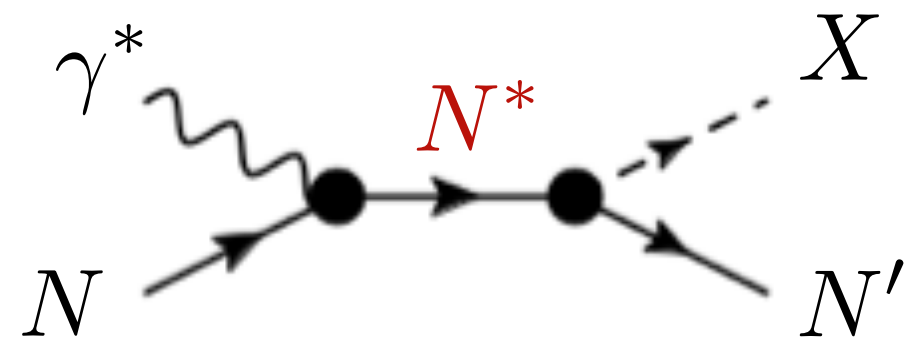
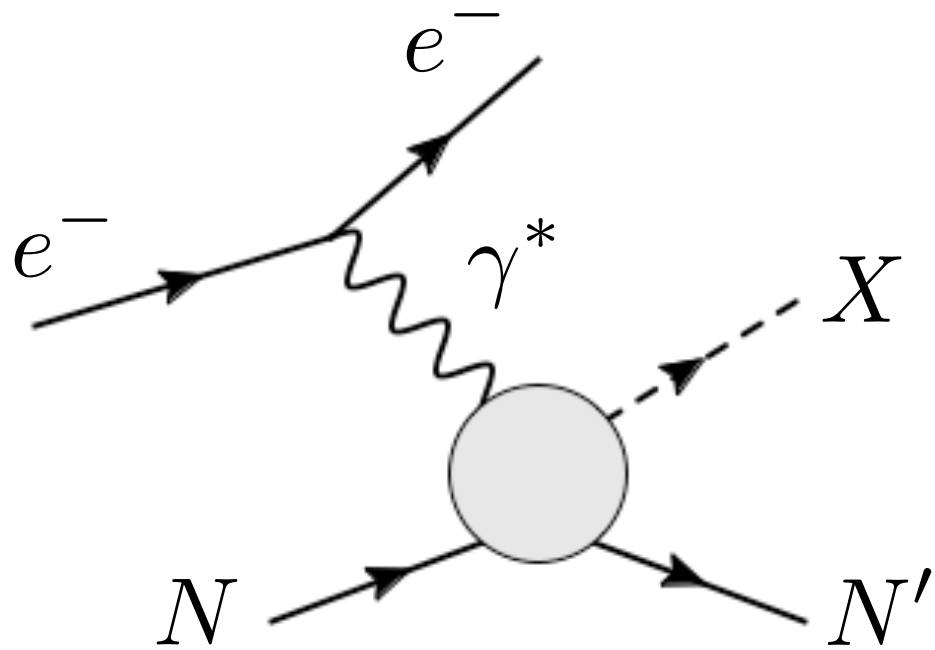
$N^*$  meeting  
JLab - June 2017



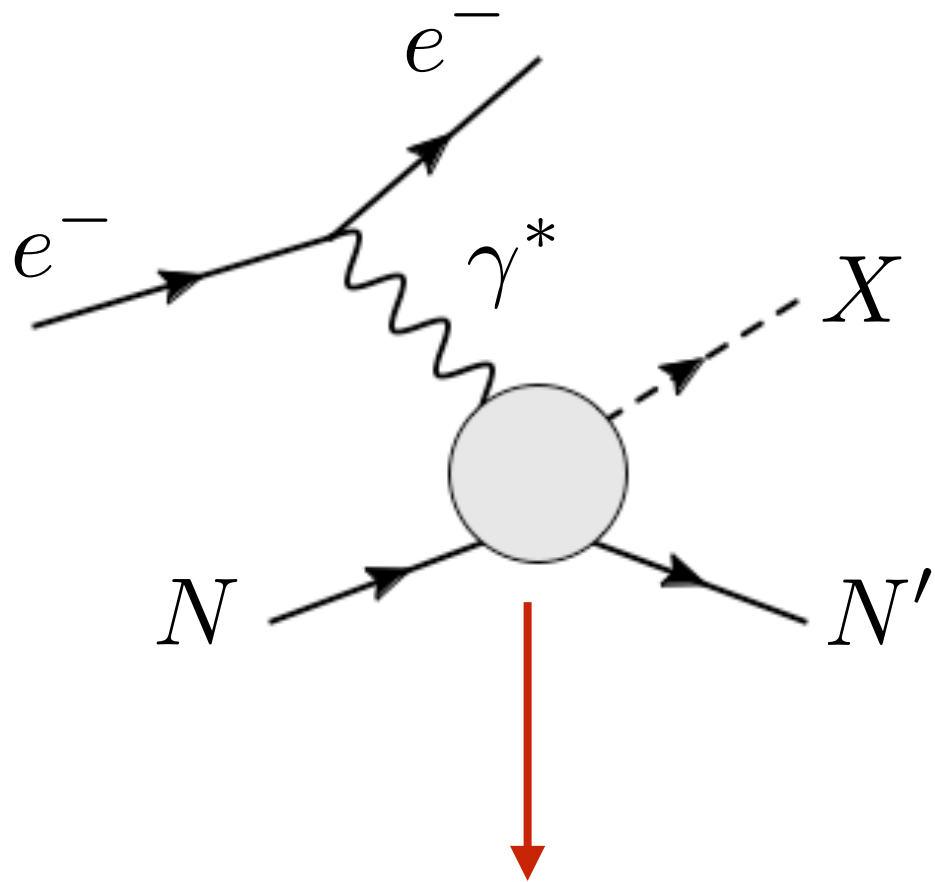
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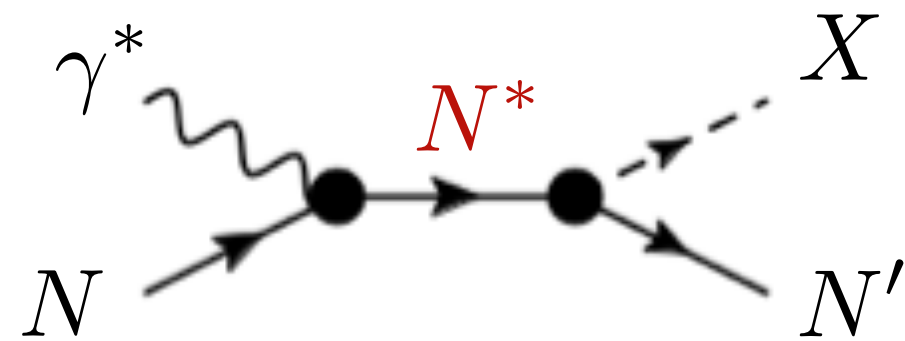
# Motivations



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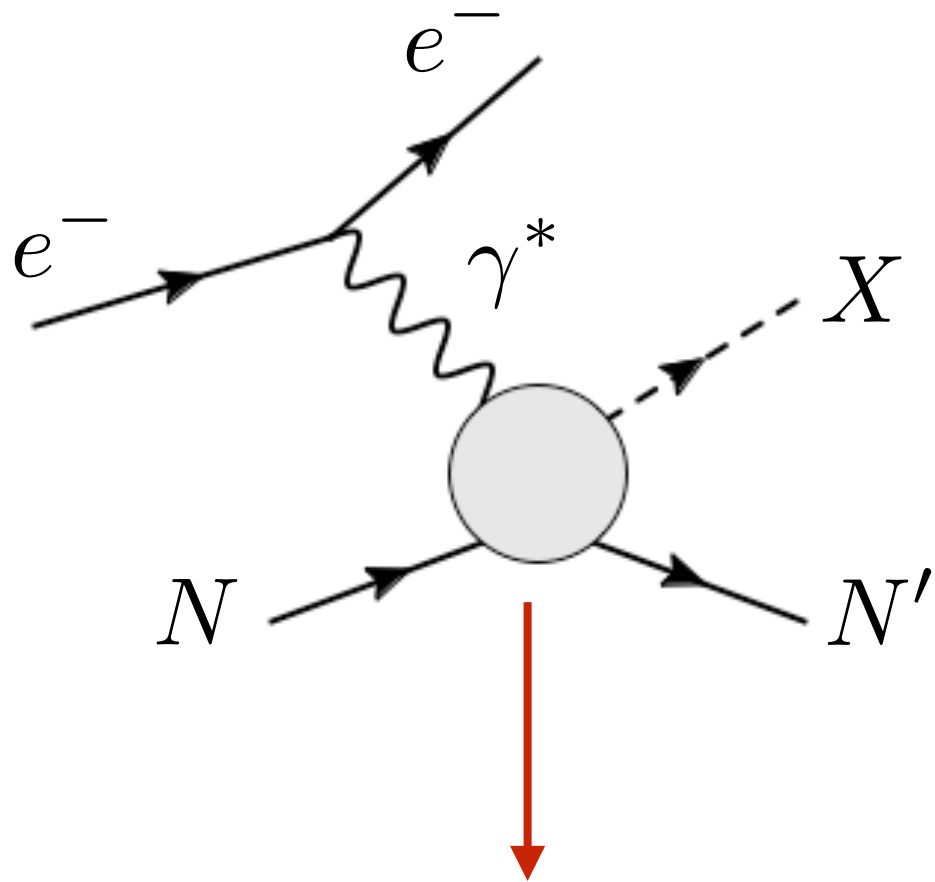
$$A_{\lambda_\gamma, \lambda_N; \lambda'_N}(q^2, E, \theta)$$



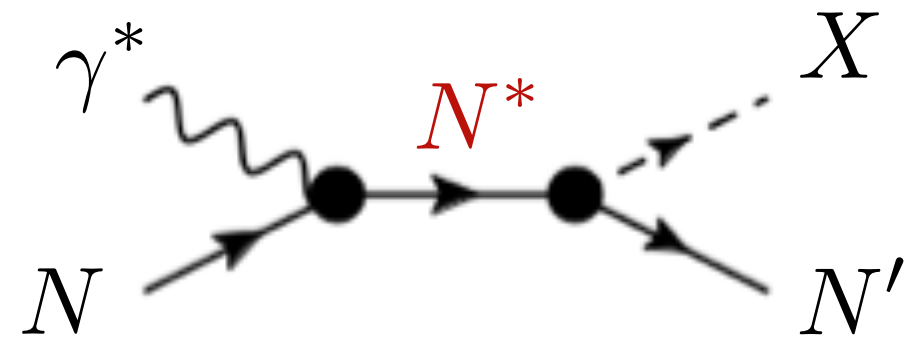
**6 functions of 3 variables**

→ **Need data and constraints**

# Motivations



$$A_{\lambda_\gamma, \lambda_N; \lambda'_{N'}}(q^2, E, \theta)$$

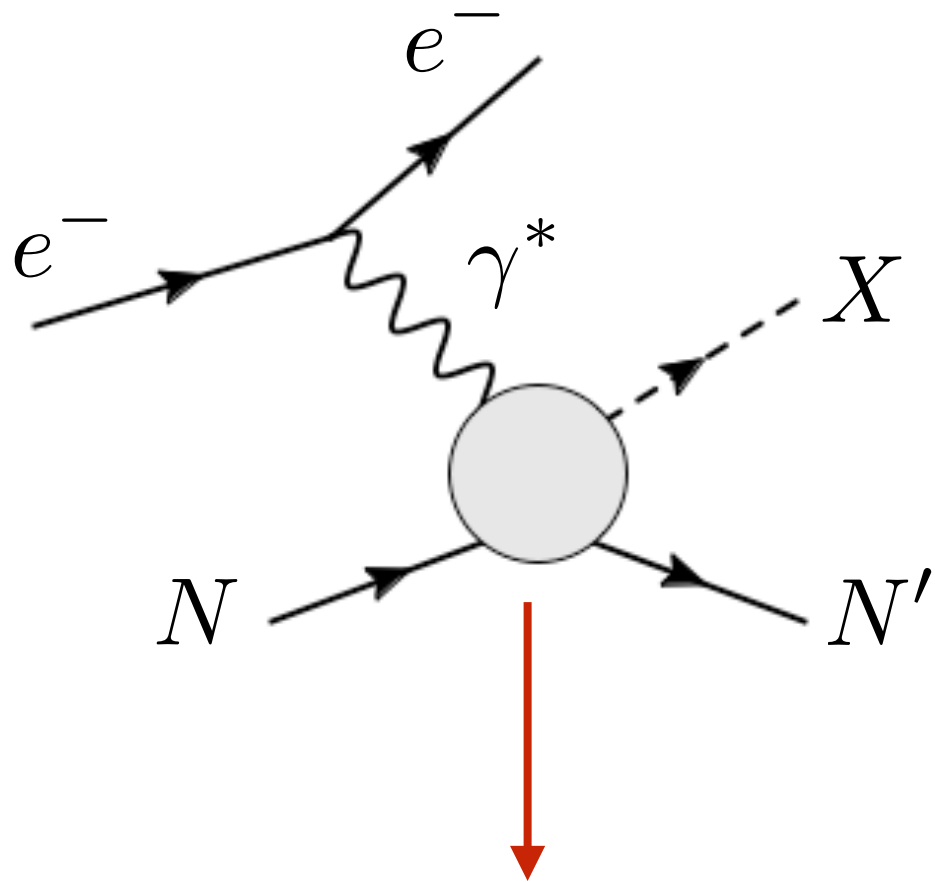


**6 functions of 3 variables**

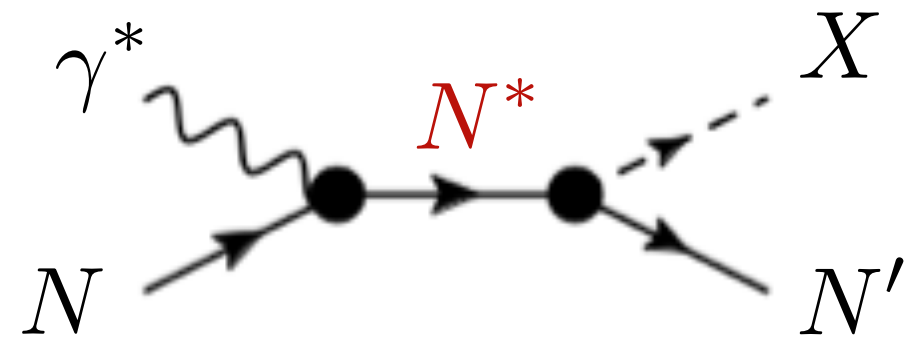
→ **Need data and constraints**

**Constraints from S-Matrix principles and from QCD**

# Motivations



$$A_{\lambda_\gamma, \lambda_N; \lambda'_{N'}}(q^2, E, \theta)$$



**6 functions of 3 variables**

→ **Need data and constraints**

**Constraints from S-Matrix principles ~~and from QCD~~**  
 cf other talks

# Outline

## Constraints from S-Matrix principles:

- **Unitarity**

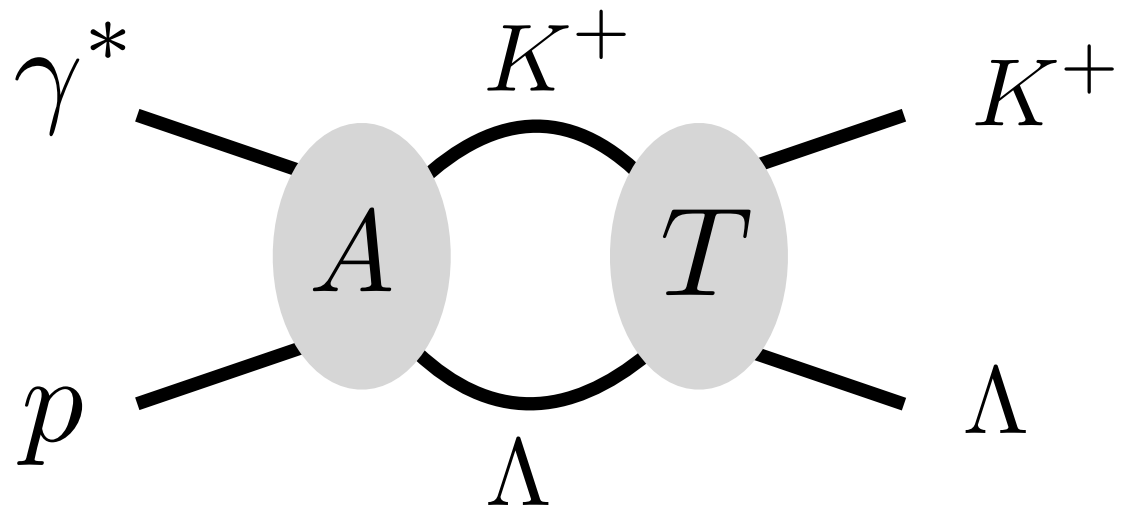
implemented in analysis

- **Analyticity**

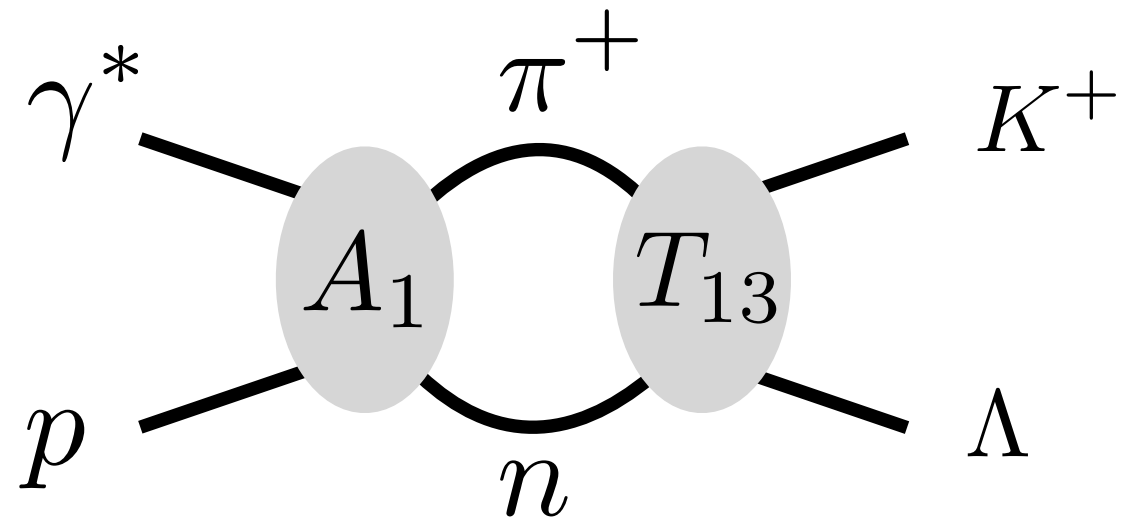
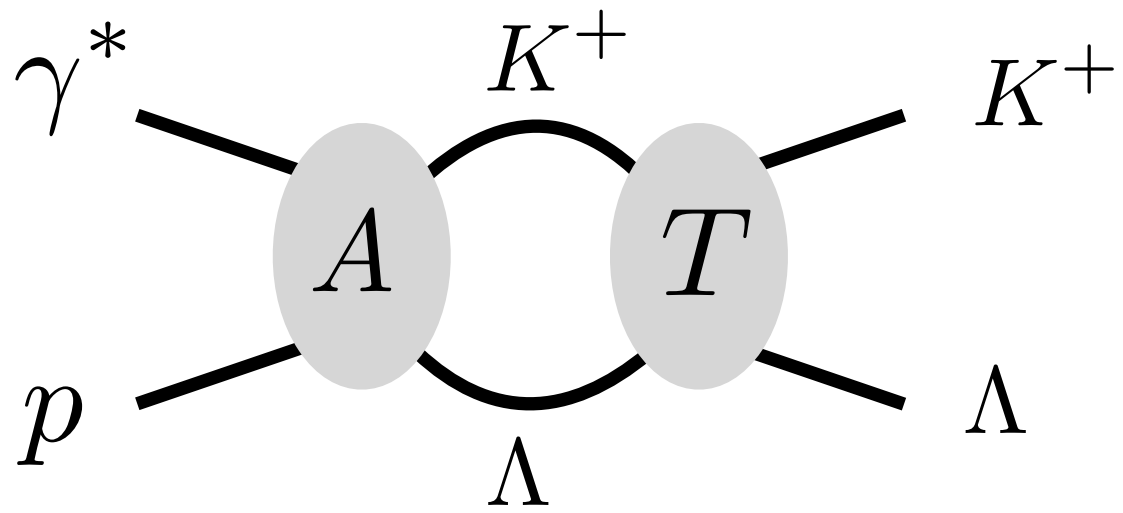
not implemented in analysis  
examples:  $\gamma p \rightarrow \pi^0 p, \eta p$

- **N\* trajectories**

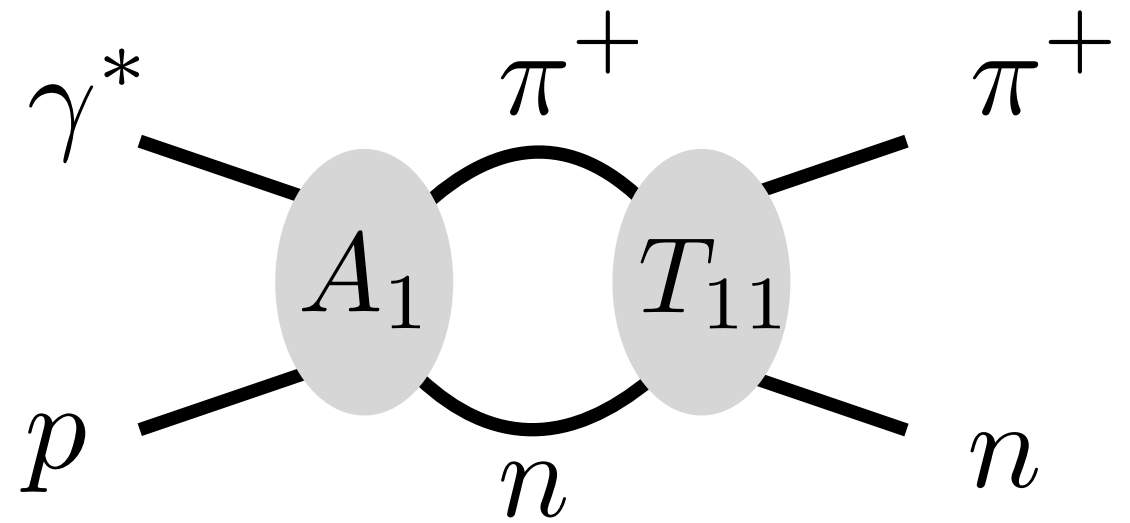
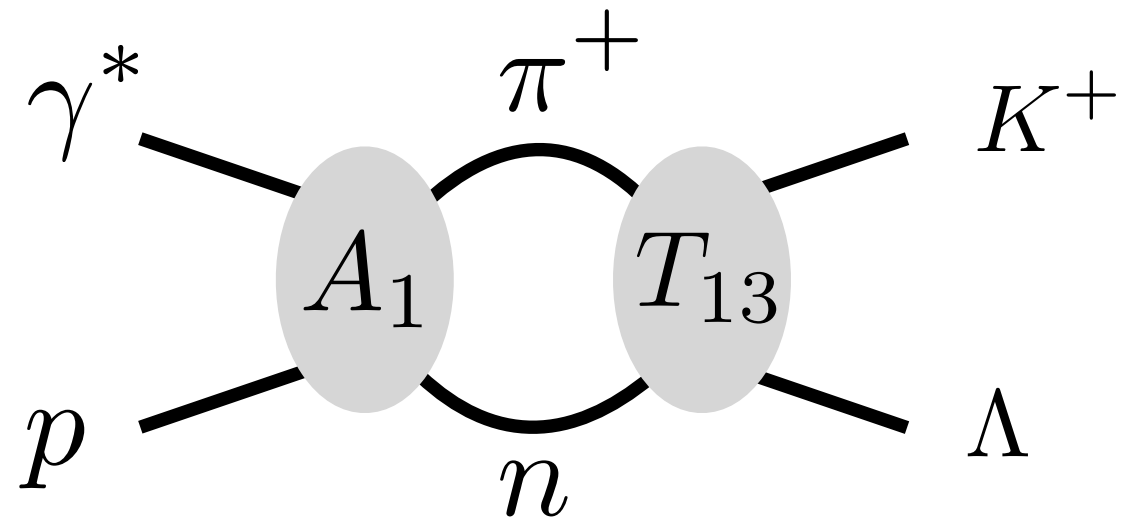
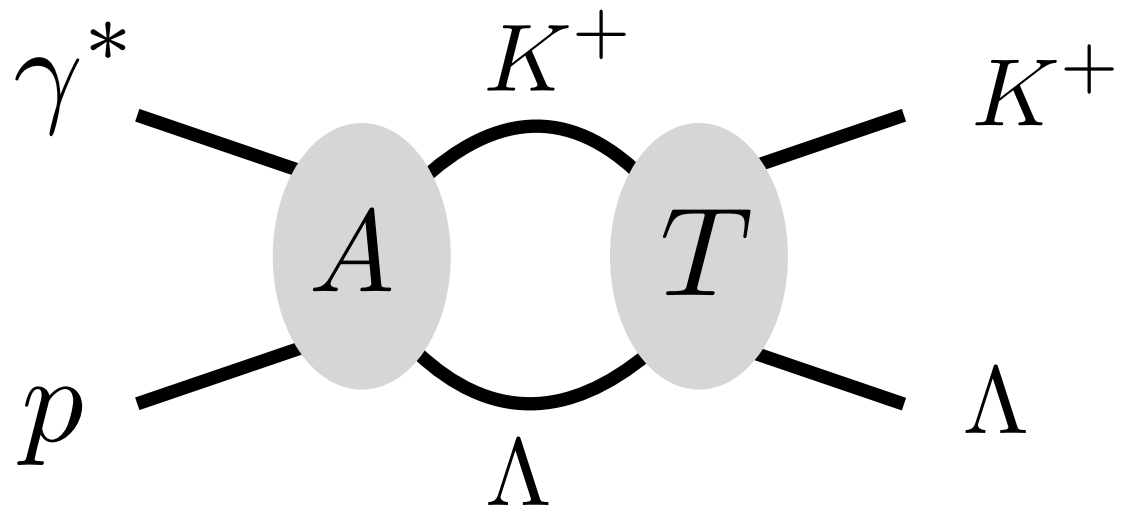
# Unitarity



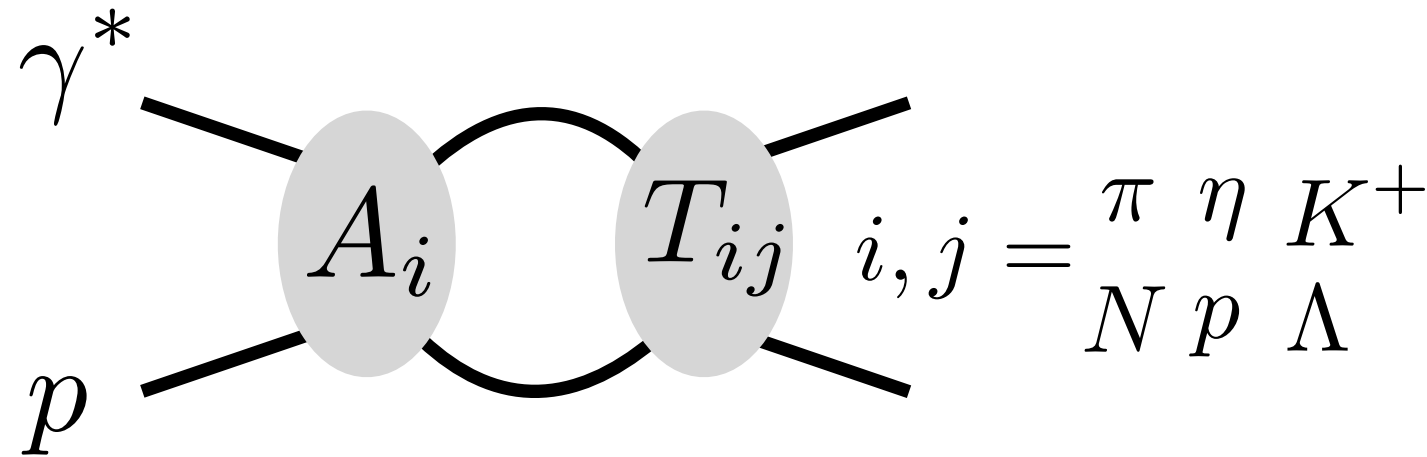
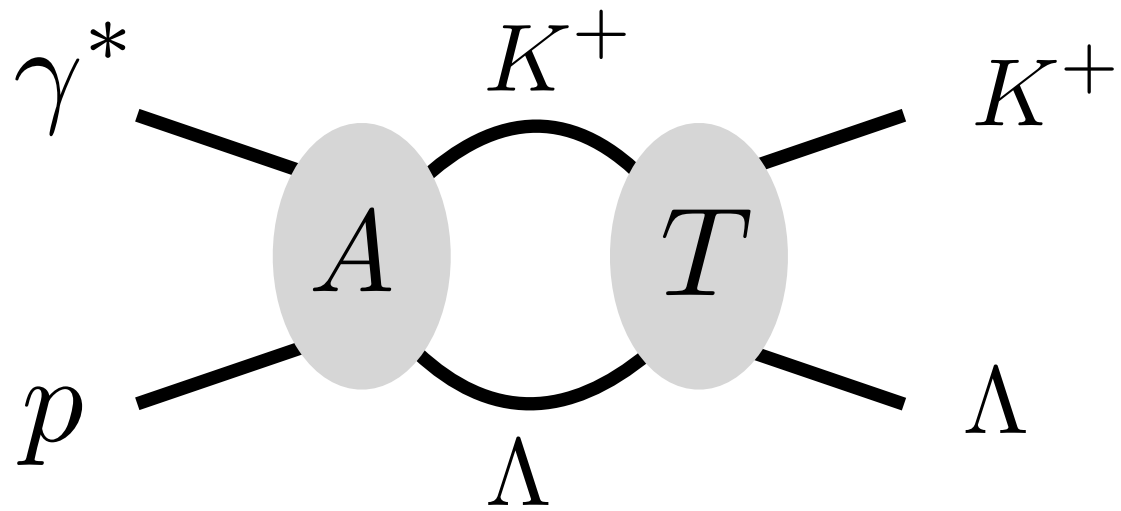
# Unitarity



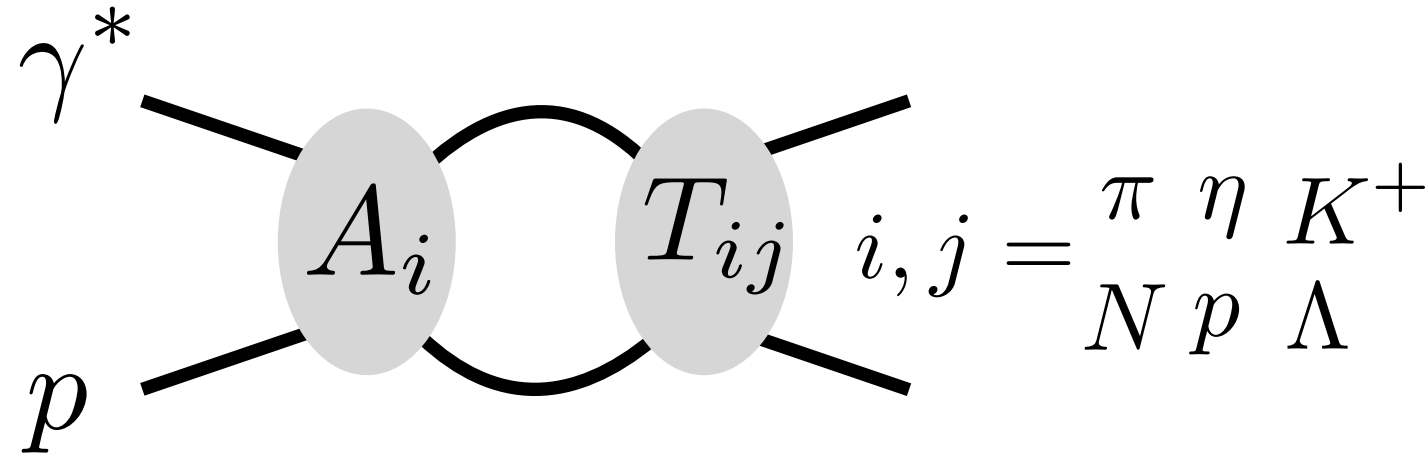
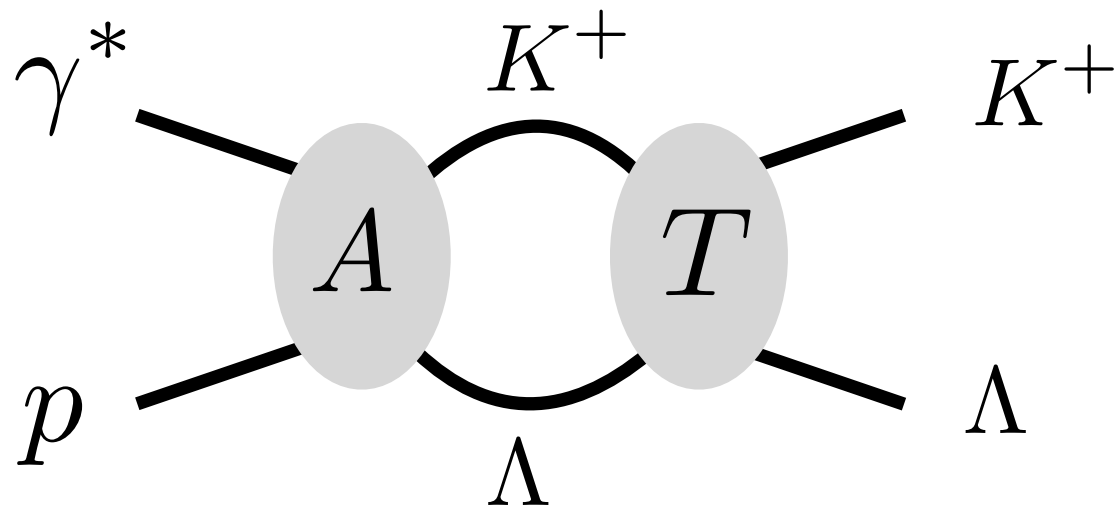
# Unitarity



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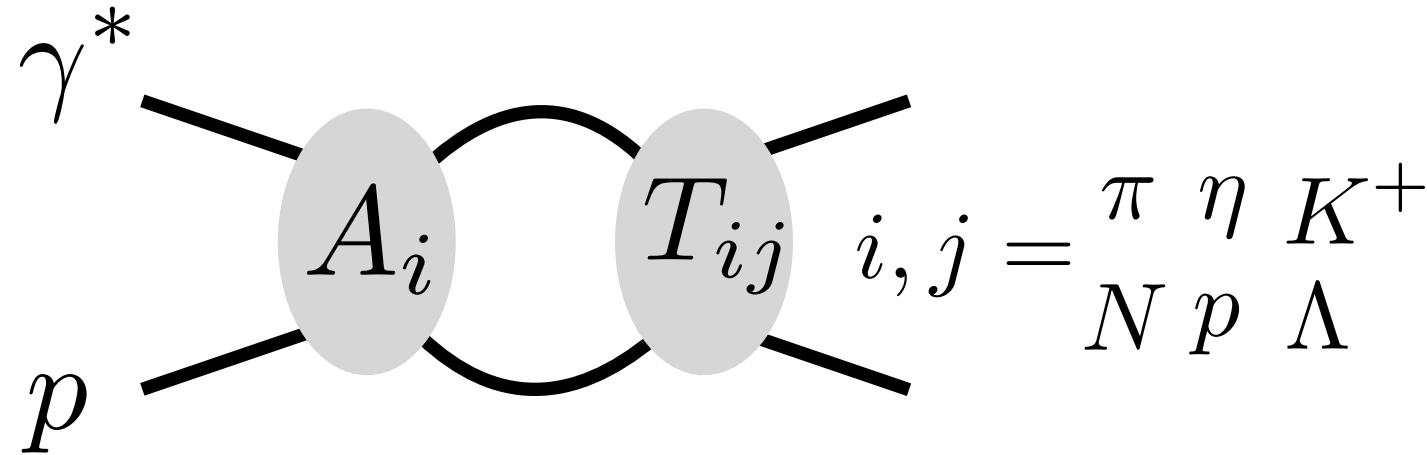
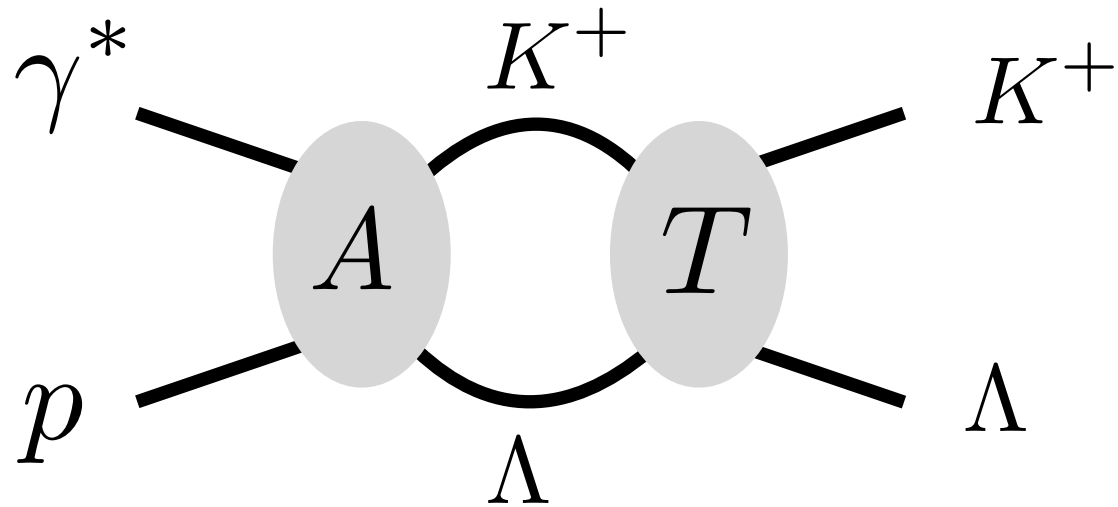


## Partial wave expansion

$$T_{\lambda; \lambda'}^{ij}(s, q^2, \theta) : f_{\ell \pm}^{ij}(s, q^2) \quad (s = E_{\text{cm}}^2)$$

$$A_{\lambda_\gamma \lambda; \lambda'}^i(s, q^2, \theta) : E_{\ell \pm}^i(s, q^2), M_{\ell \pm}^i(s, q^2), S_{\ell \pm}^i(s, q^2)$$

# Unitarity



## Partial wave expansion

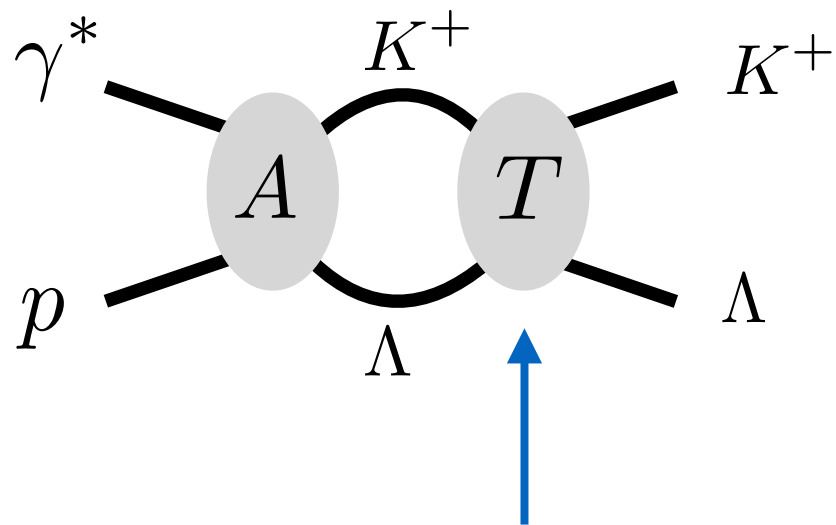
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$$\text{Im } E_{\ell \pm}^j(s, q^2) = \sum_i E_{\ell \pm}^i(s, q^2) f_{\ell \pm}^{ji*}(s) \theta(s - s_i)$$

$s_n =$  threshold channel n

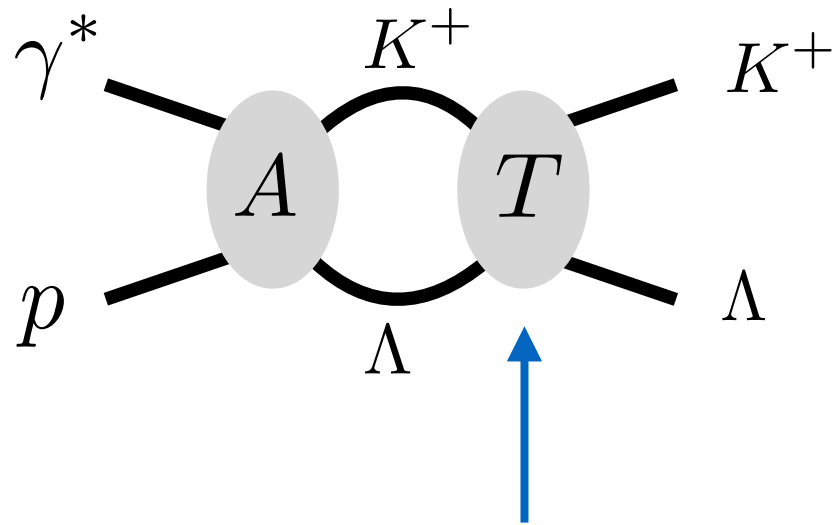
# Unitarity



$$\text{Im } E_{\ell\pm}^j(s, q^2) = \sum_i E_{\ell\pm}^i(s, q^2) f_{\ell\pm}^{ji*}(s) \theta(s - s_i)$$

hadronic amplitudes as input

# Unitarity



$$\text{Im } E_{\ell\pm}^j(s, q^2) = \sum_i E_{\ell\pm}^i(s, q^2) f_{\ell\pm}^{ji*}(s) \theta(s - s_i)$$

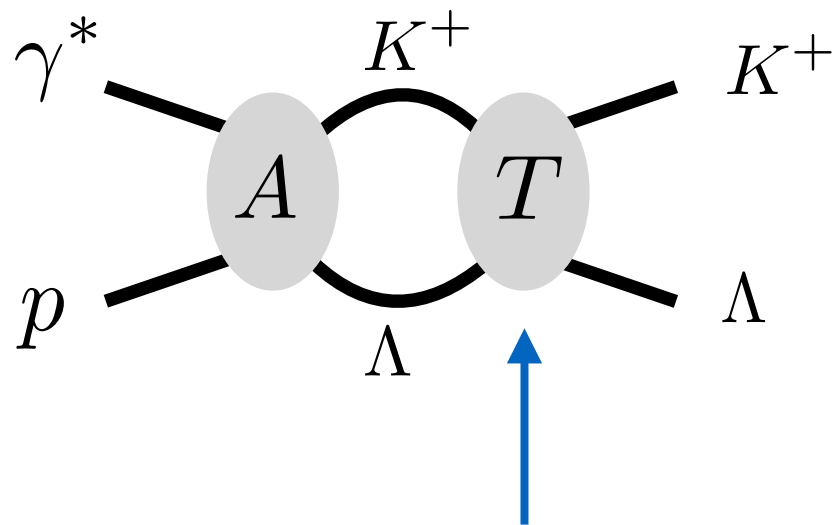
**Possible solution of unitarity**

$$E_{\ell\pm}^j(s, q^2) = e_{\ell\pm}(s, q^2) f_{\ell\pm}^{1j}(s)$$

**pole positions (mass & width)  
do not change**

$$= e_{\ell\pm}(s, q^2) \times$$

# Unitarity



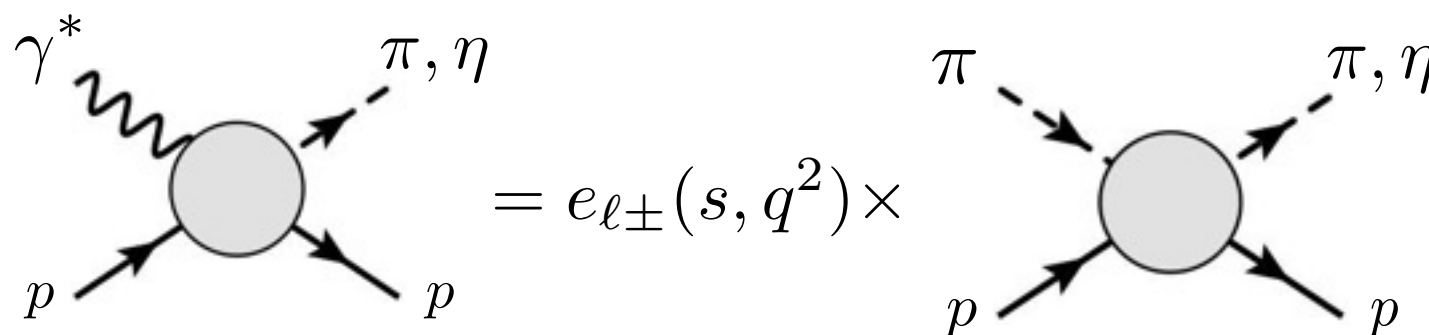
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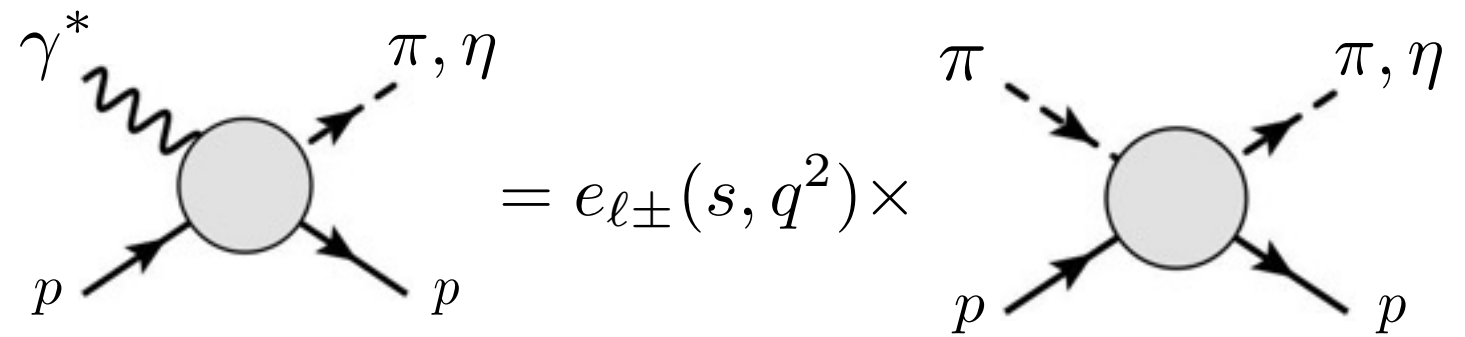
$$e_{\ell\pm}(s, q^2) = e_{\ell\pm}(0, q^2) \text{ polyn.}(s)$$



# Unitarity

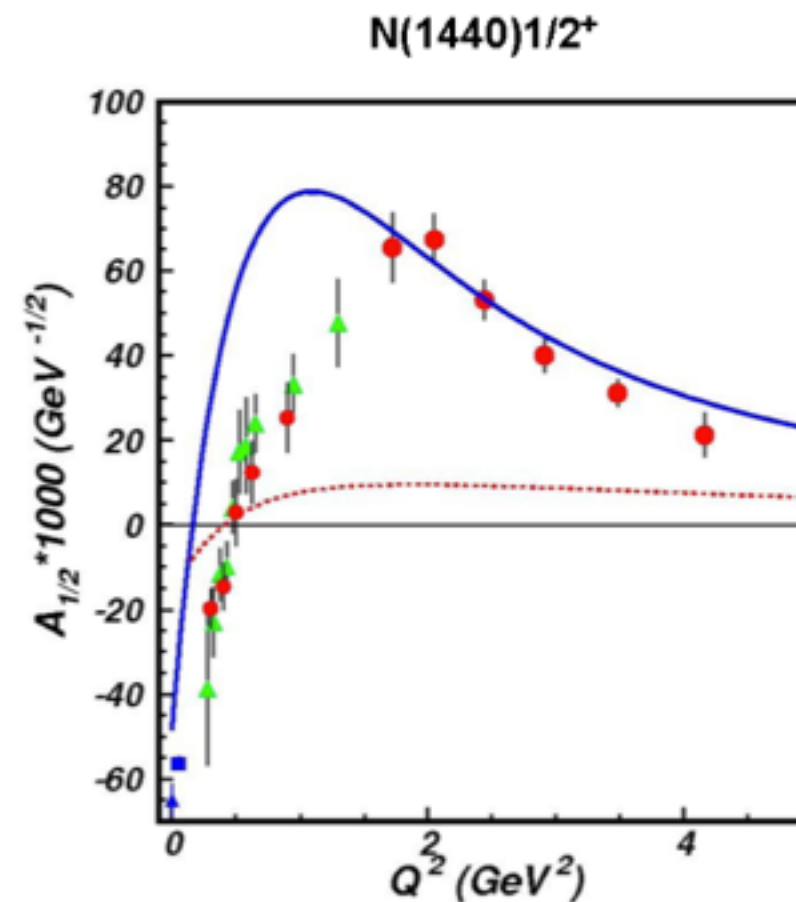
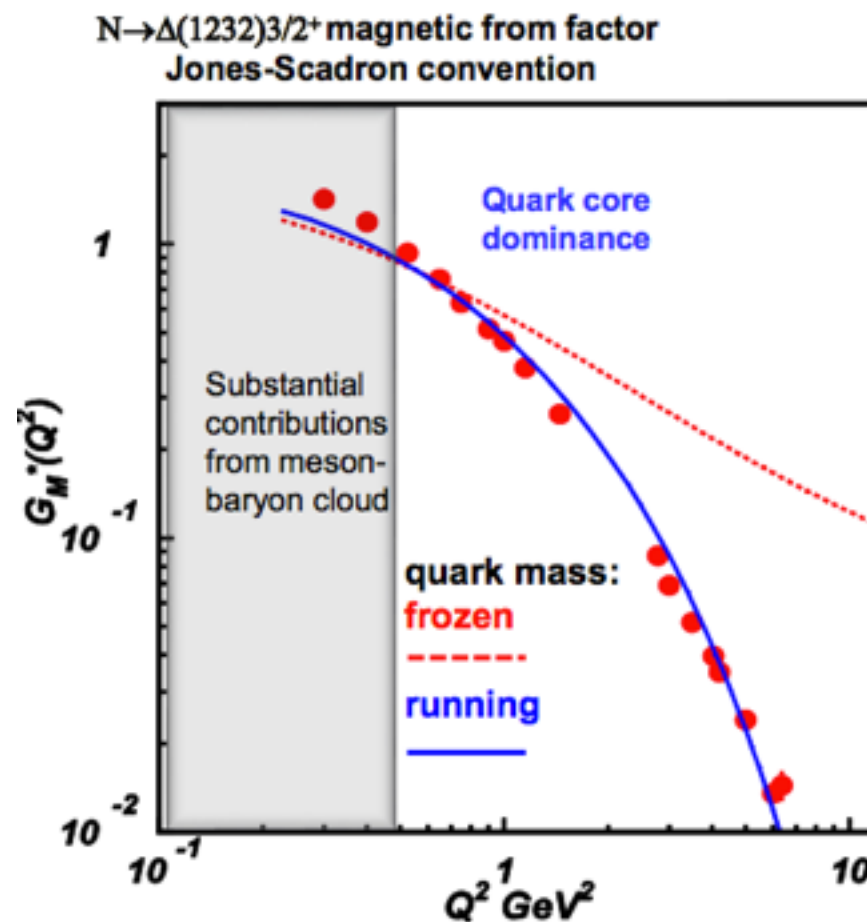
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# CLAS: Present & Future

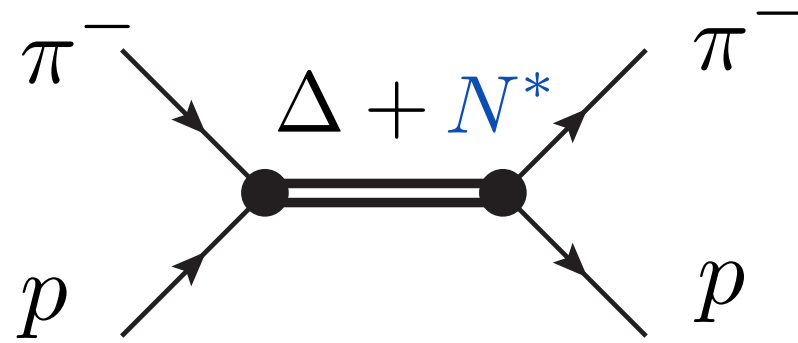
| Hadronic final state | Covered W-range, GeV                       | Covered $Q^2$ -range, $\text{GeV}^2$       | Measured observables                                                                  |
|----------------------|--------------------------------------------|--------------------------------------------|---------------------------------------------------------------------------------------|
| $\pi^+n$             | 1.1-1.38<br>1.1-1.55<br>1.1-1.7<br>1.6-2.0 | 0.16-0.36<br>0.3-0.6<br>1.7-4.5<br>1.8-4.5 | $d\sigma/d\Omega$<br>$d\sigma/d\Omega$<br>$d\sigma/d\Omega, A_b$<br>$d\sigma/d\Omega$ |
| $\pi^0p$             | 1.1-1.38<br>1.1-1.68<br>1.1-1.39           | 0.16-0.36<br>0.4-1.8<br>3.0-6.0            | $d\sigma/d\Omega$<br>$d\sigma/d\Omega, A_b, A_t, A_{bt}$<br>$d\sigma/d\Omega$         |
| $\eta p$             | 1.5-2.3                                    | 0.2-3.1                                    | $d\sigma/d\Omega$                                                                     |
| $K^+\Lambda$         | thresh-2.6                                 | 1.40-3.90<br>0.70-5.40                     | $d\sigma/d\Omega$<br>$P^0, P'$                                                        |

|                                                      |                                                                                                                                                                                                                                       |
|------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Hybrid Baryons</b><br>E12-16-010                  | Search for hybrid baryons (qqqg) focusing on $0.05 \text{ GeV}^2 < Q^2 < 2.0 \text{ GeV}^2$ in mass range from 1.8 to 3 GeV in $K\Lambda, N\pi\pi, N\pi$ (A. D'Angelo, et al.)                                                        |
| <b>KY</b><br><b>Electroproduction</b><br>E12-16-010A | Study $N^*$ structure for states that couple to KY through measurements of cross sections and polarization observables that will yield $Q^2$ evolution of electrocoupling amplitudes at $Q^2 < 7.0 \text{ GeV}^2$ (D. Carman, et al.) |

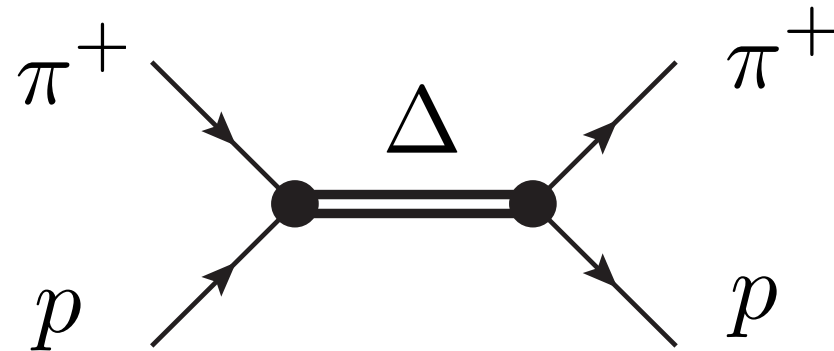
|                                          |
|------------------------------------------|
| <b>Run Group conditions:</b>             |
| $E_b = 6.6 \text{ GeV}, 50 \text{ days}$ |
| $E_b = 8.8 \text{ GeV}, 50 \text{ days}$ |

# Analyticity

$\pi^- p :$

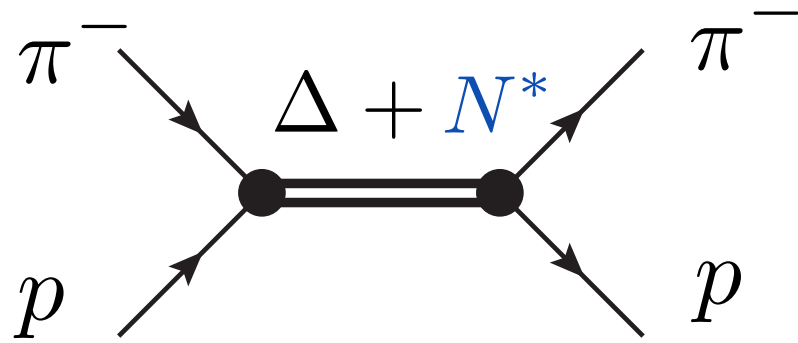


$\pi^+ p :$

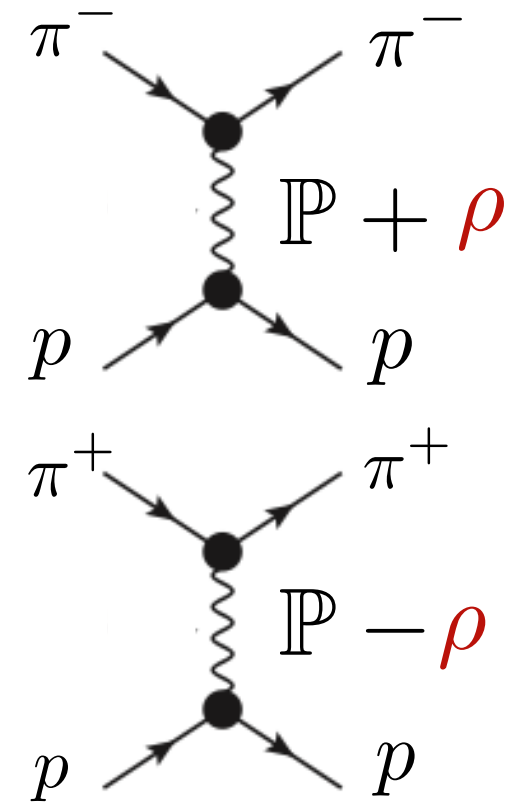
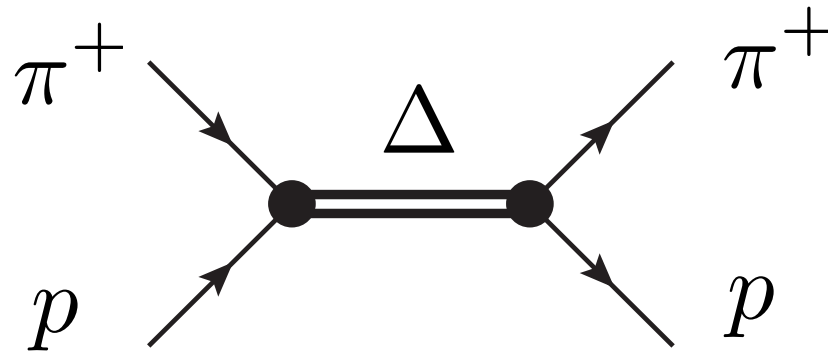


# Analyticity

$\pi^- p :$

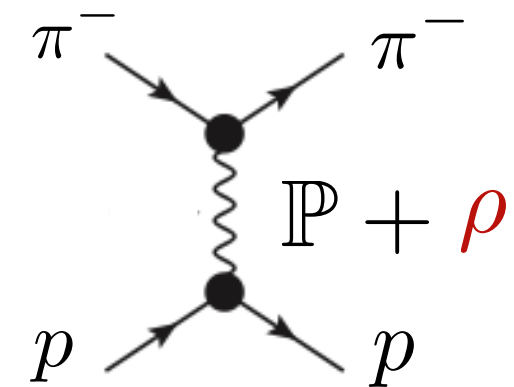
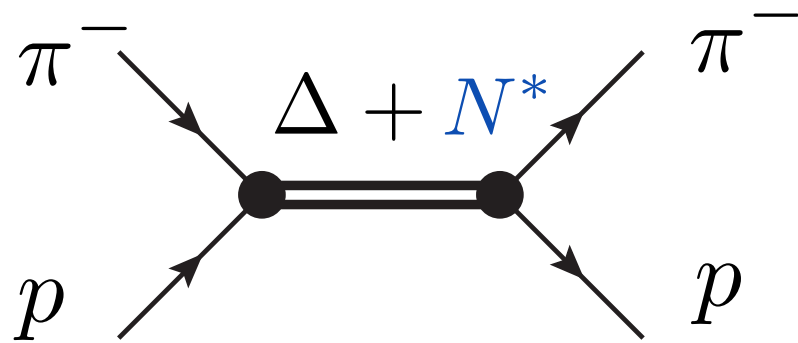


$\pi^+ p :$

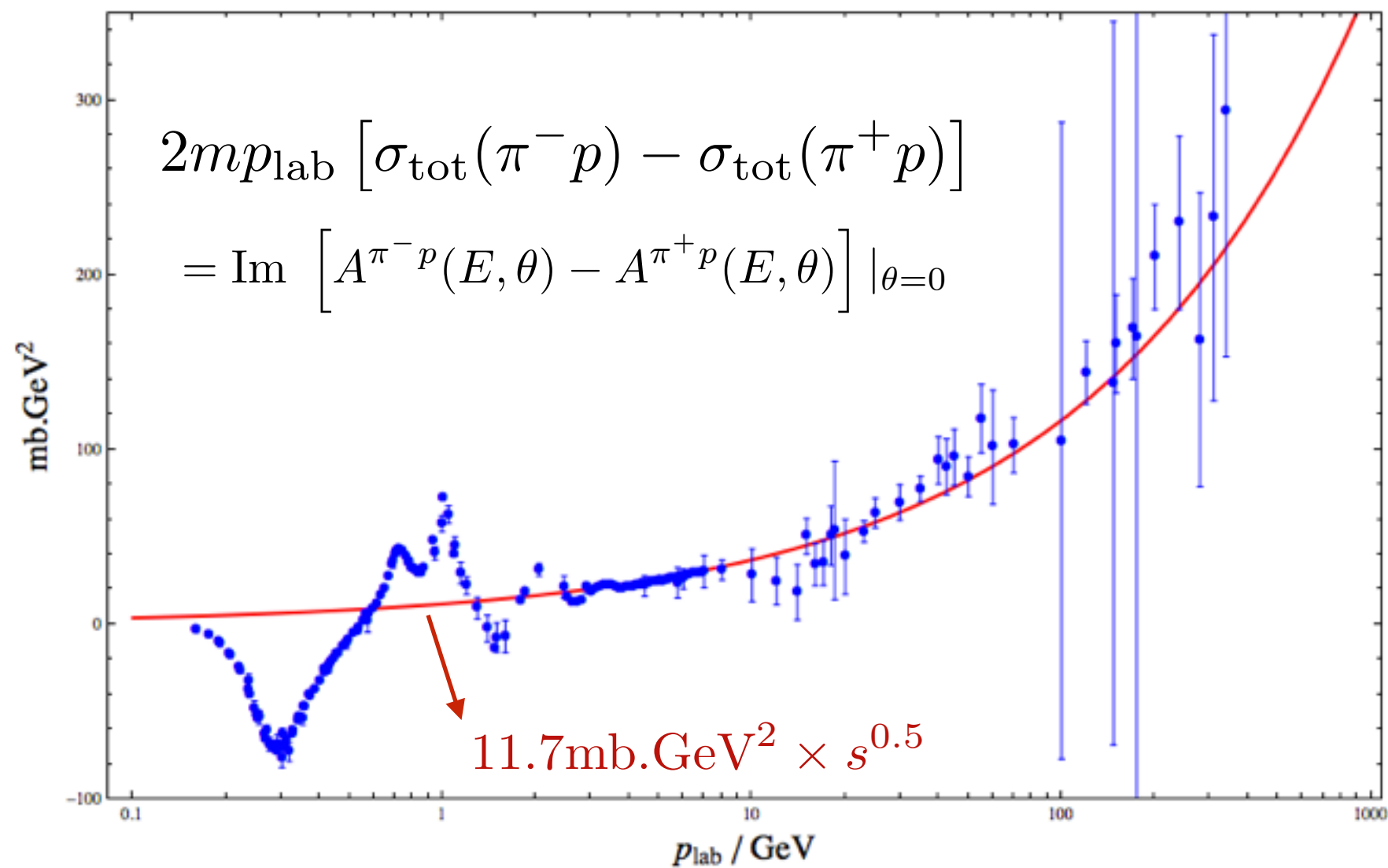
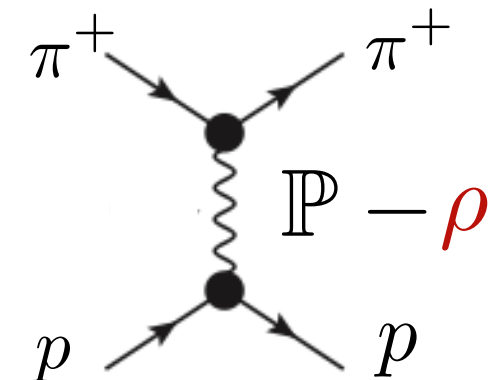
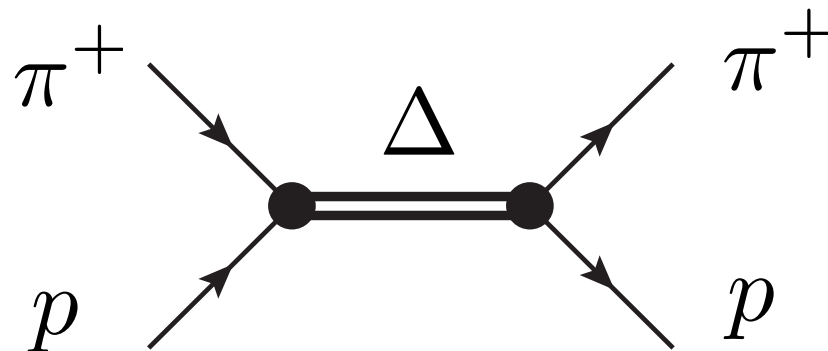


# Analyticity

$\pi^- p :$



$\pi^+ p :$

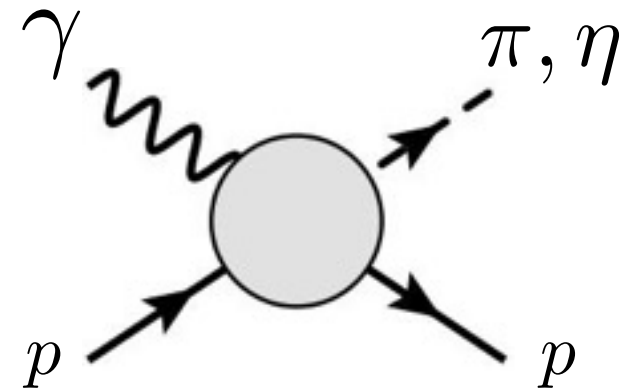


$$\langle N^* \rangle \stackrel{?}{=} \langle \rho \rangle$$

# Analyticity

$$\int_0^\Lambda \text{Im } A_i(\nu, t) \nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha_i(t)+k}}{\alpha_i(t) + k + 1} + \dots$$

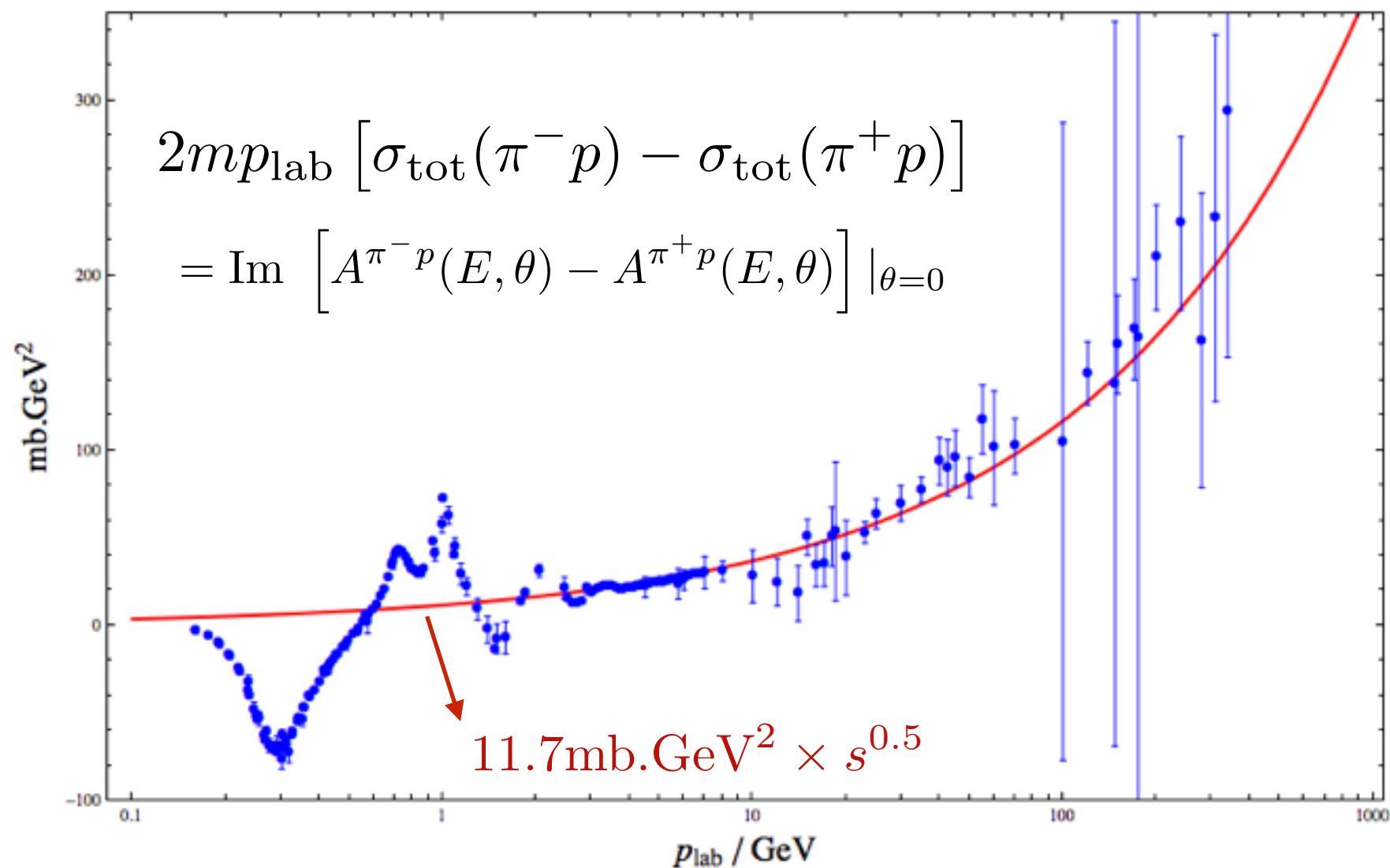
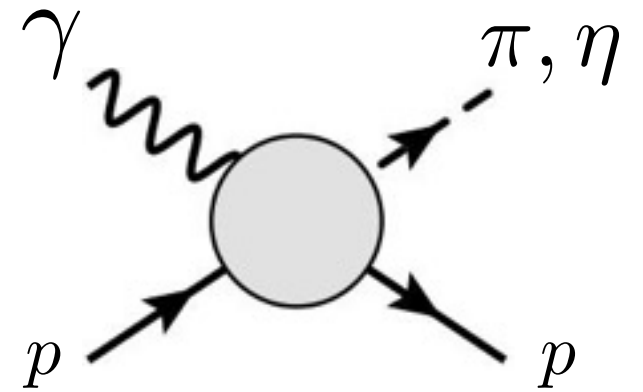
$$\nu = (s - u)/2$$



# Analyticity

$$\int_0^\Lambda \text{Im } A_i(\nu, t) \nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha_i(t)+k}}{\alpha_i(t) + k + 1} + \dots$$

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$$\langle N^* \rangle \stackrel{?}{=} \langle \rho \rangle$$

# Finite Energy Sum Rules

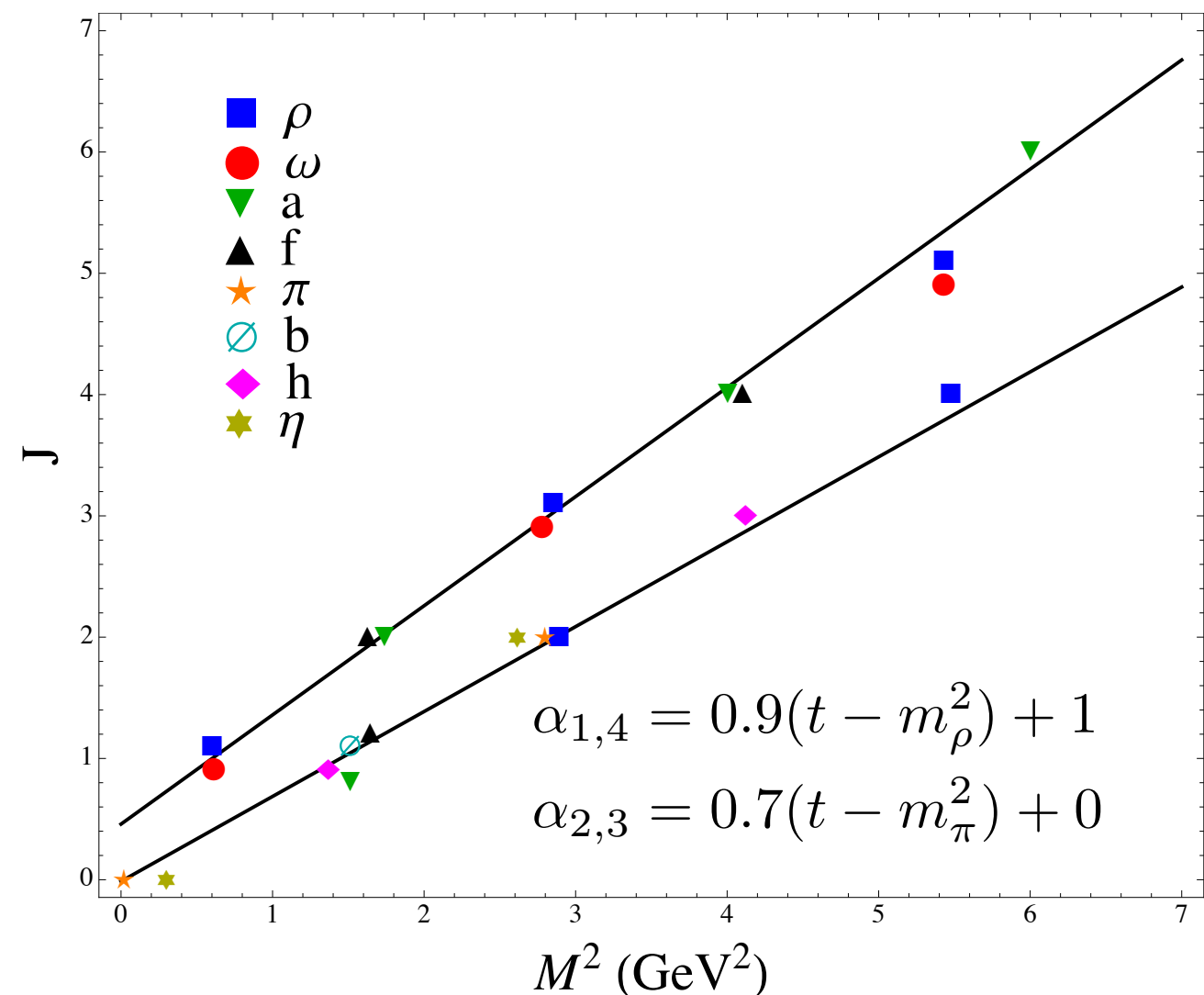
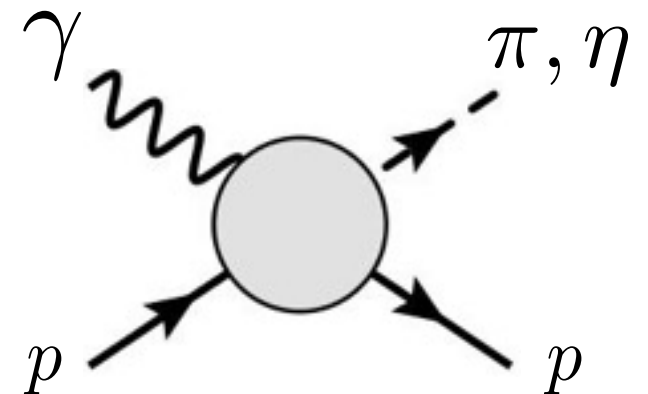
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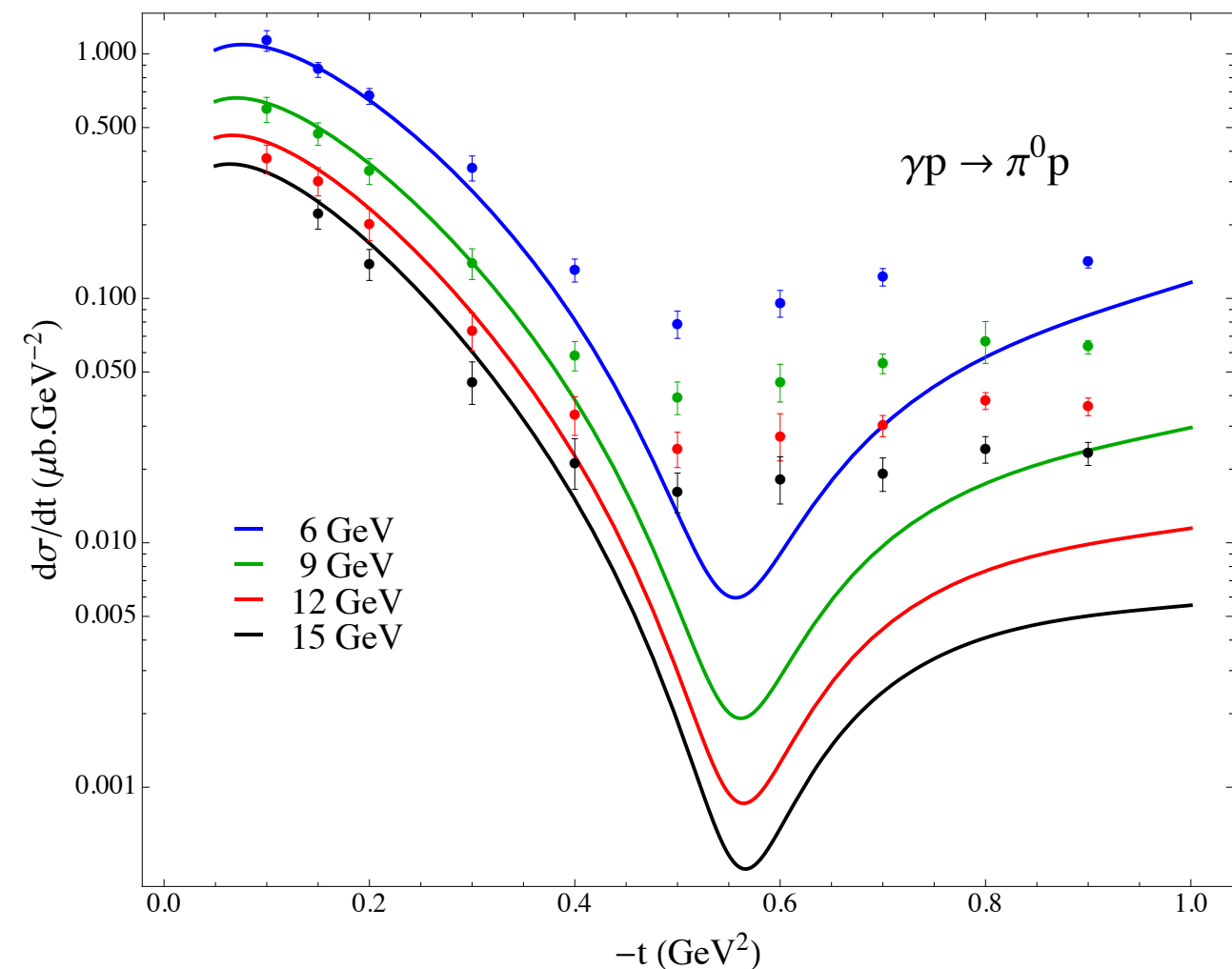
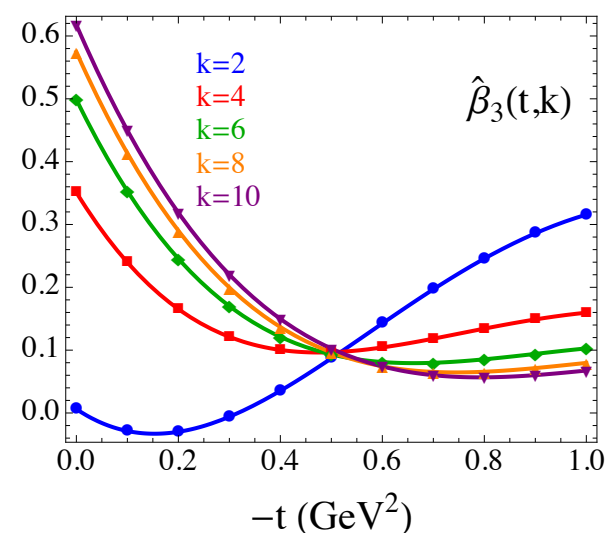
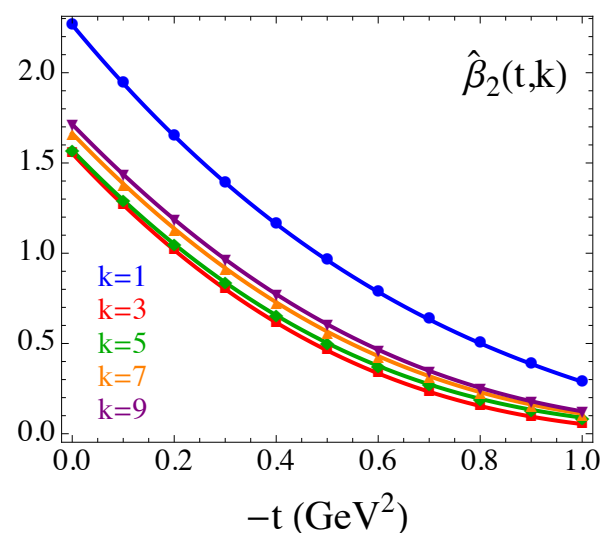
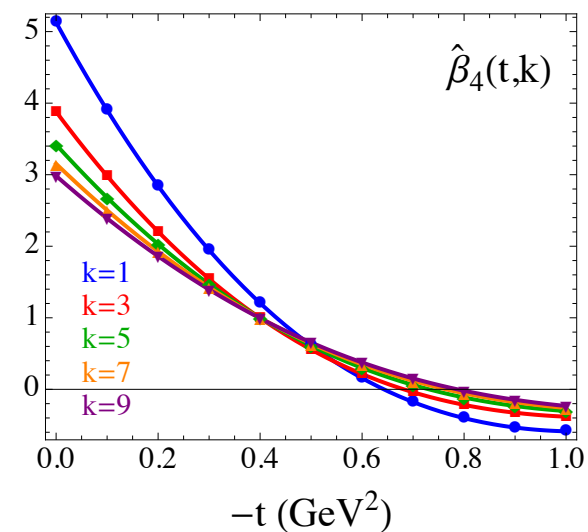
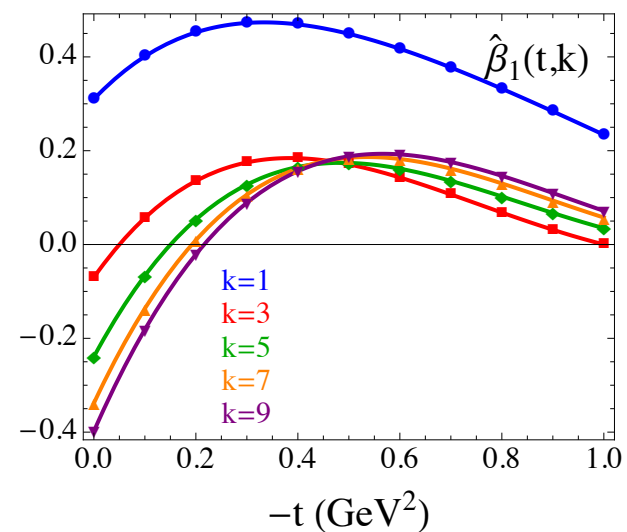
$$S_i(t, k)$$

Can we predict the residues ?

$$\begin{aligned} \hat{\beta}_i(t) &= S_i(t, k) \frac{\alpha_i(t) + k + 1}{\Lambda^{\alpha_i(t)+k}} \\ &= \beta_i(t) + \mathcal{O}(1/\Lambda) \end{aligned}$$



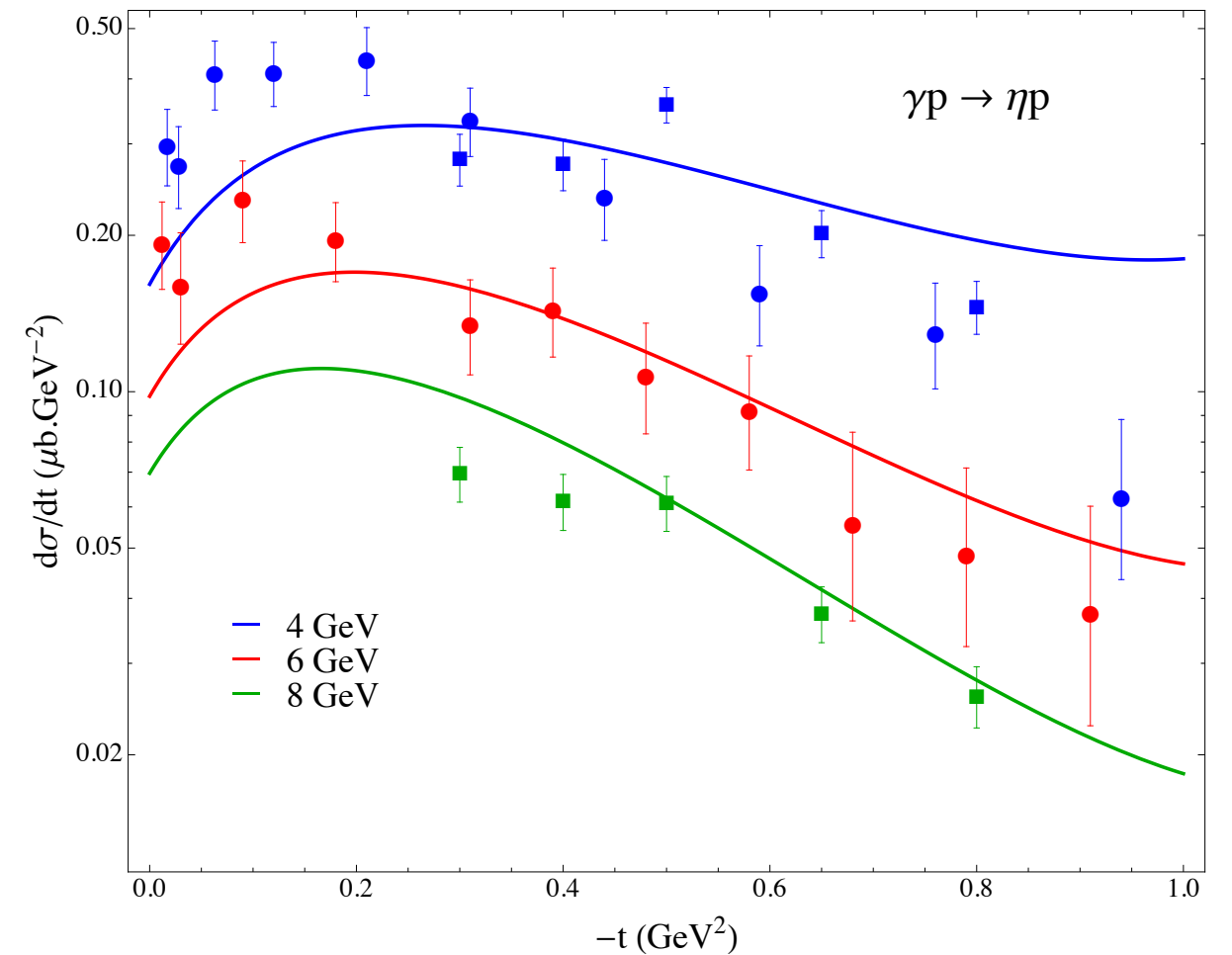
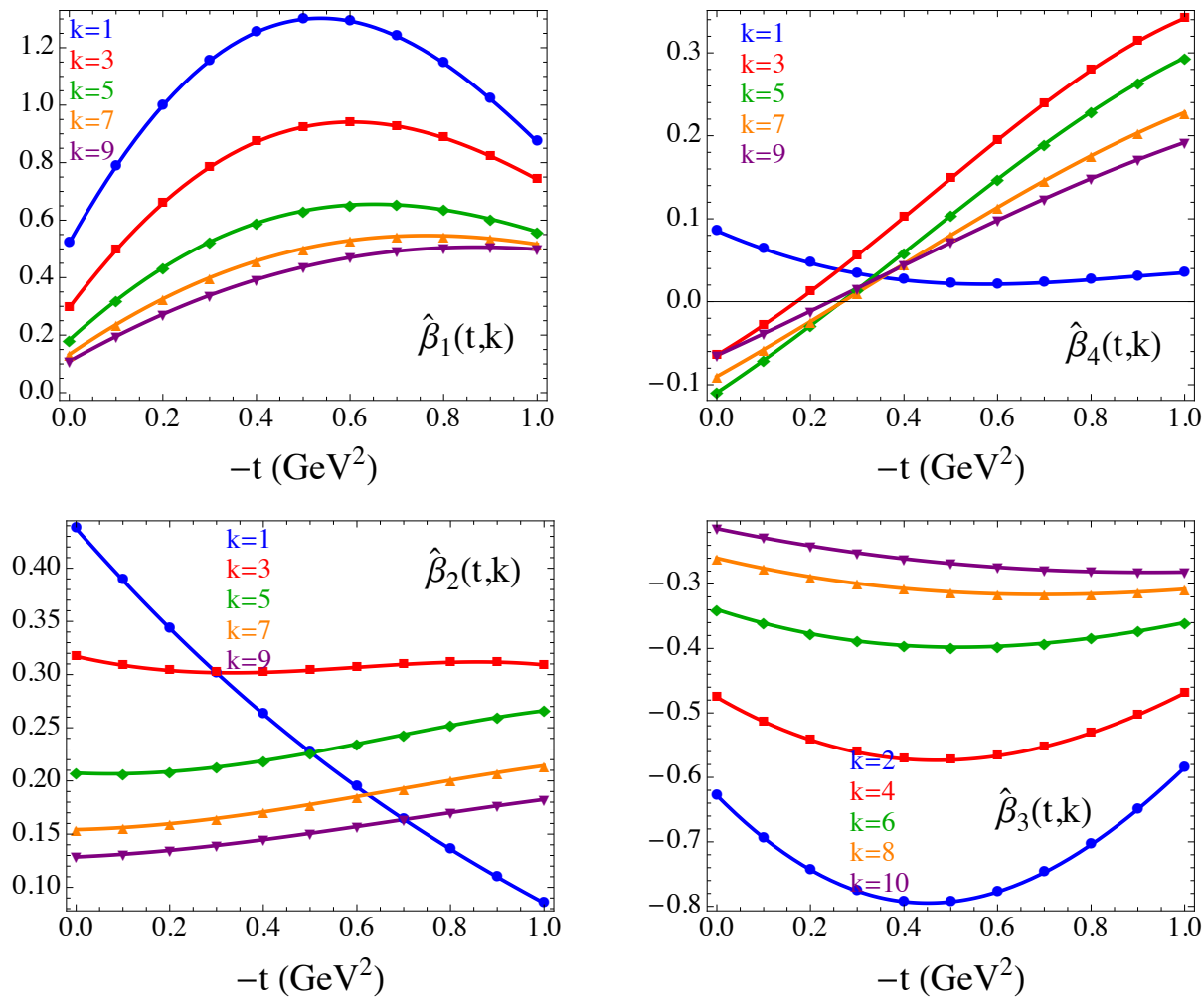
$$\gamma p \rightarrow \pi^0 p$$



**SAID (R. Workman et al) used for the low energy models**

**effective residues almost k-independent**

**Good prediction of t-dependence at high energy**



**MAID (L. Tiator et al) used for the low energy models**

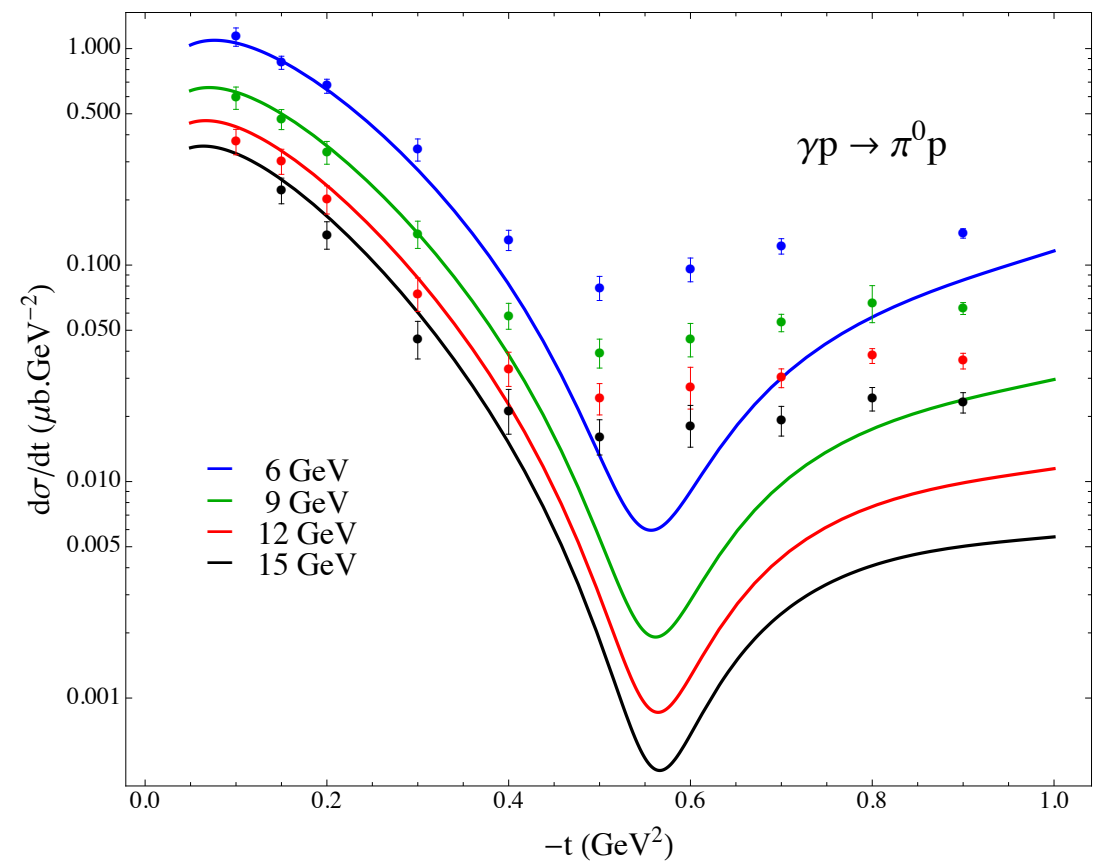
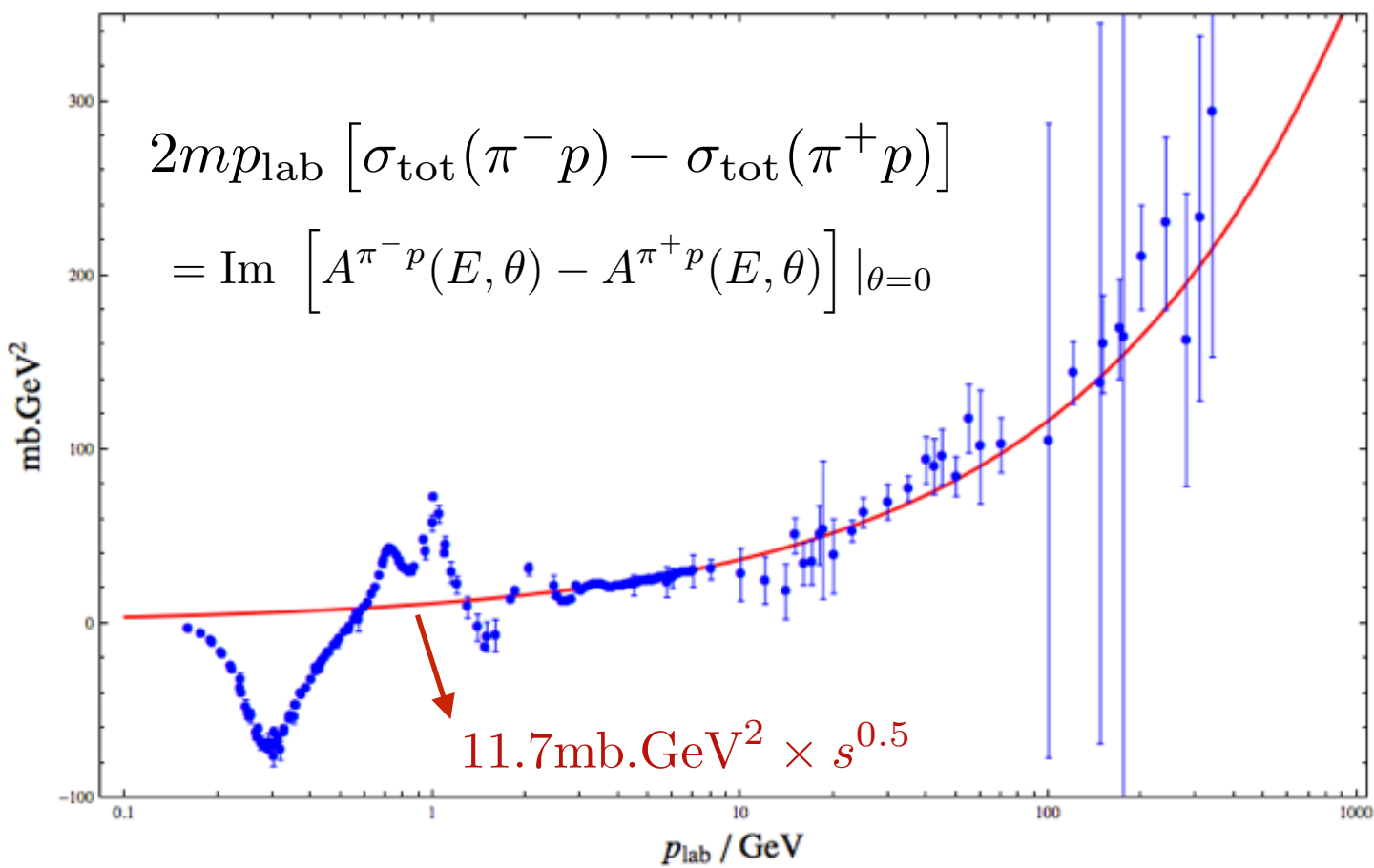
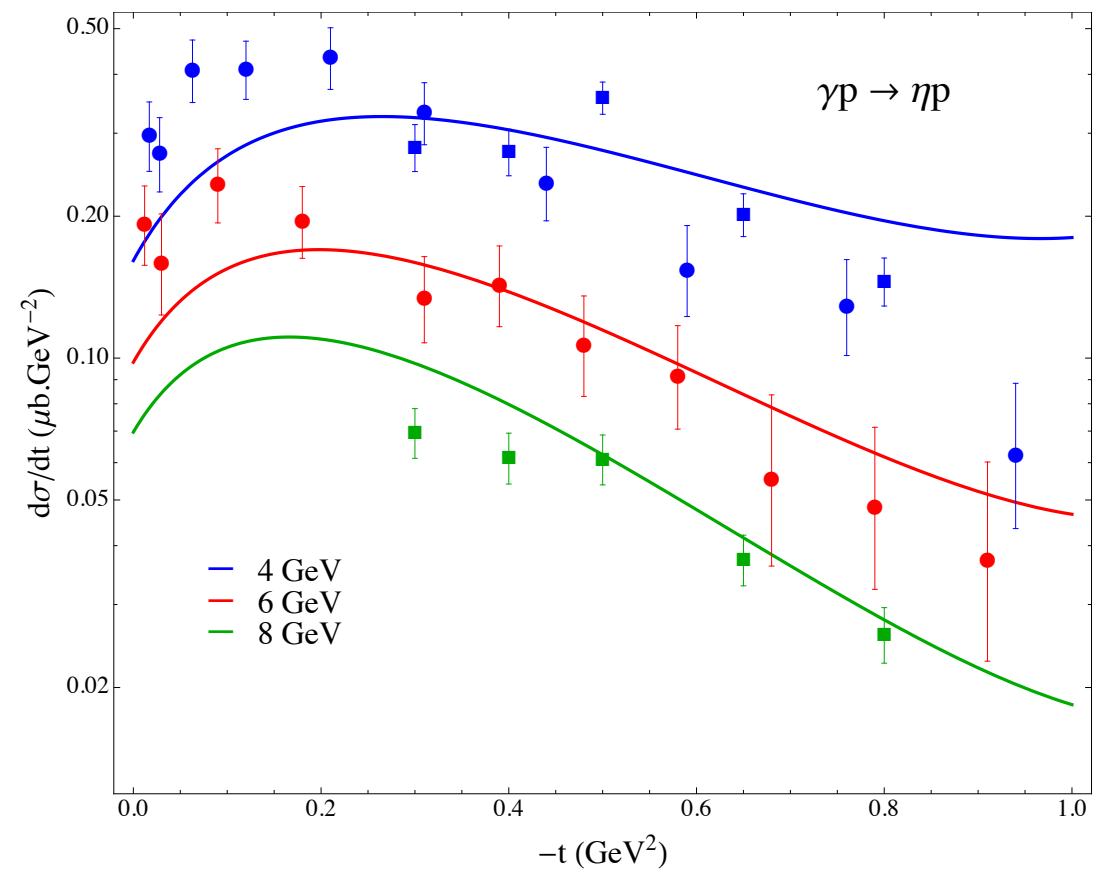
**effective residues not k-independent**

**Good prediction of t-dependence at high energy**

input

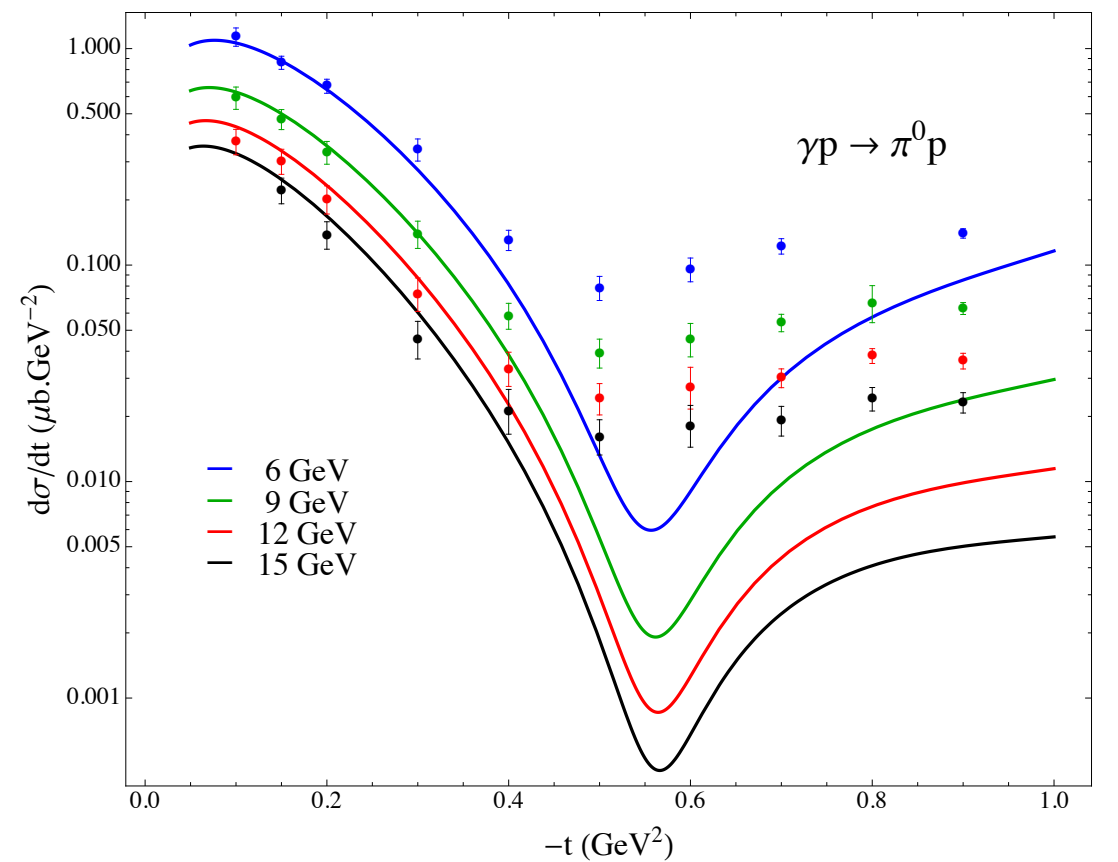
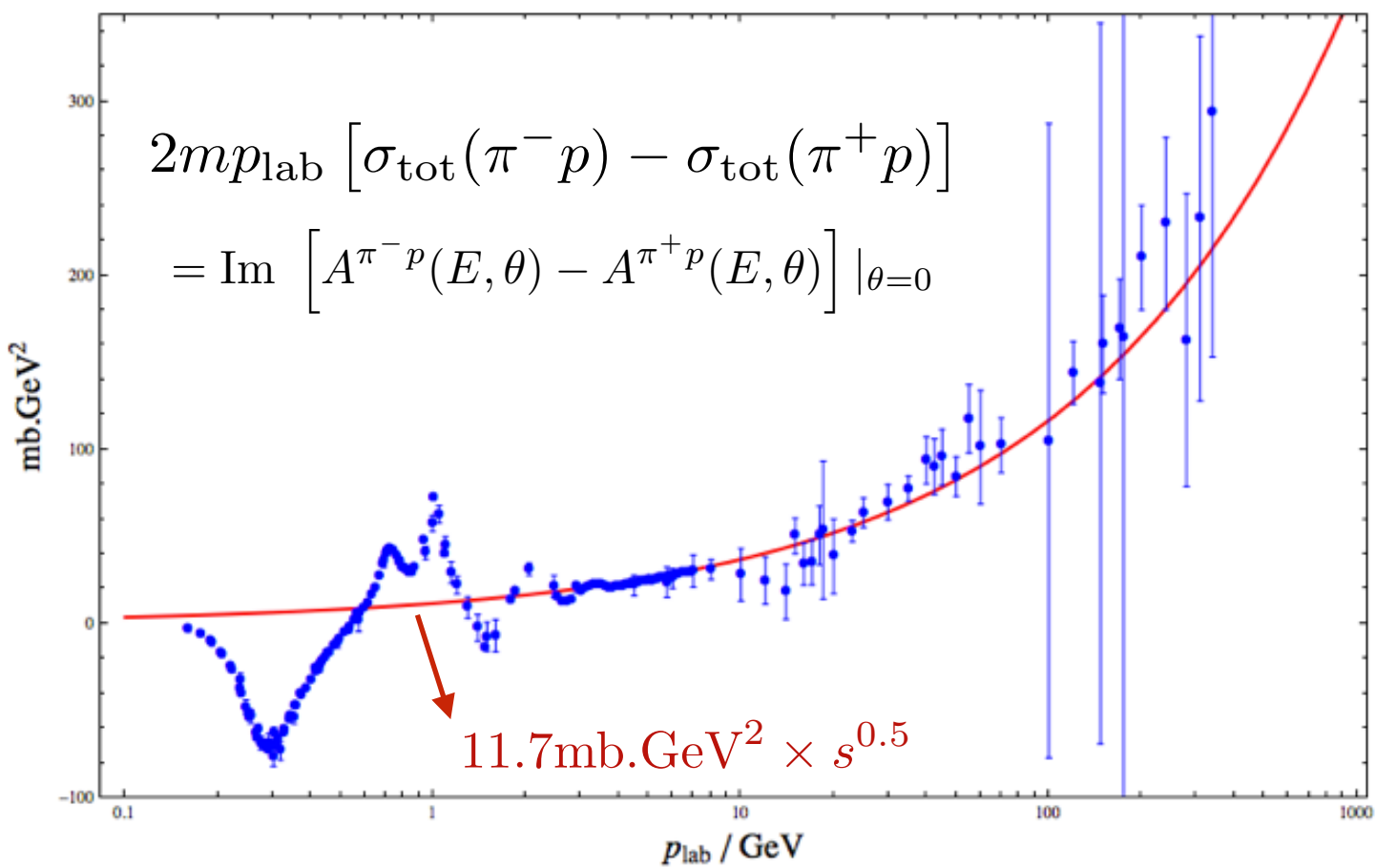
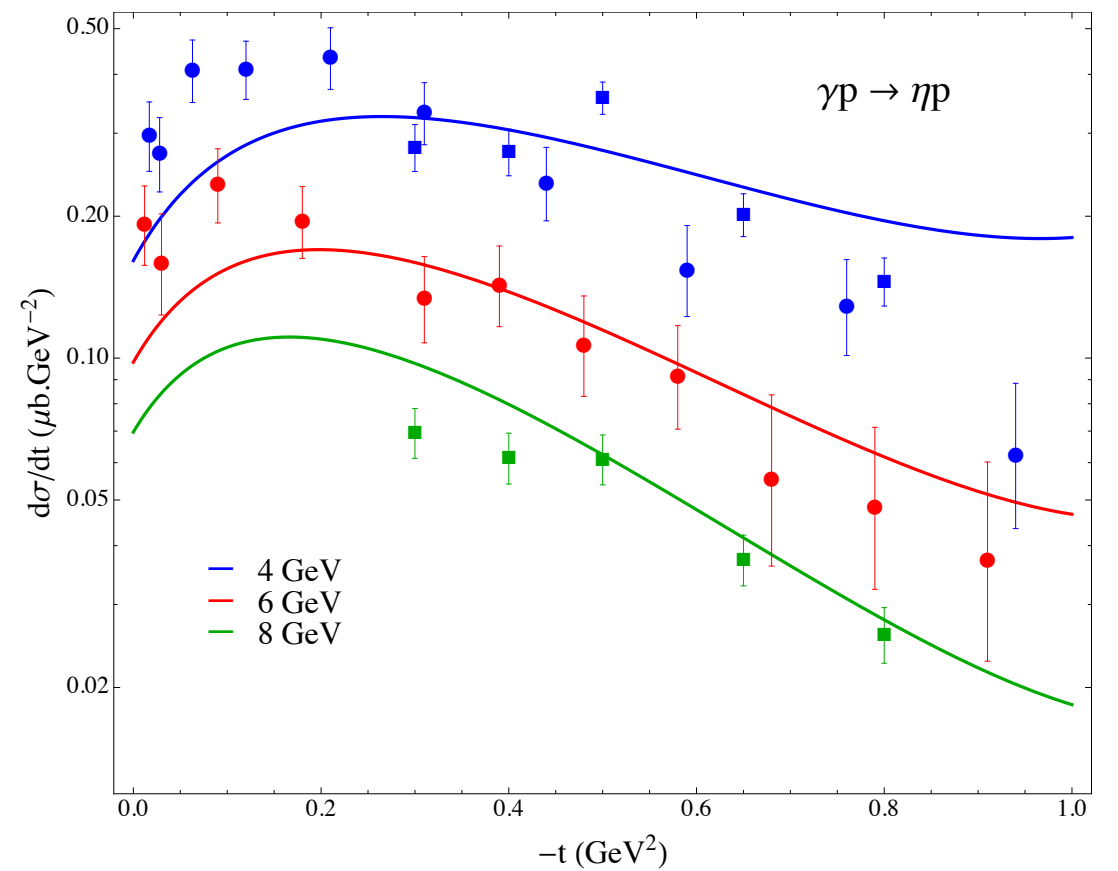
prediction

$$\int_0^\Lambda \text{Im } A_i(\nu, t) \nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha_i(t)+k}}{\alpha_i(t) + k + 1}$$



$$\int_0^\Lambda \text{Im } A_i(\nu, t) \nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha_i(t)+k}}{\alpha_i(t) + k + 1}$$

input input



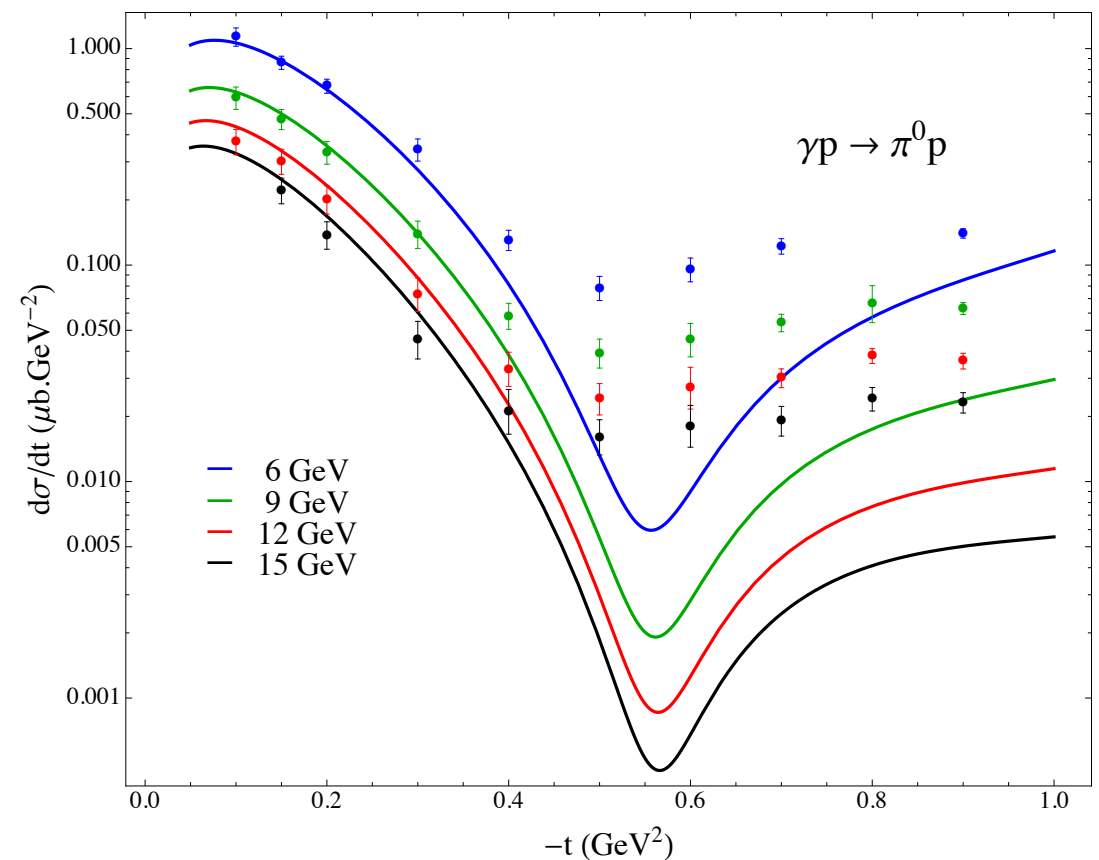
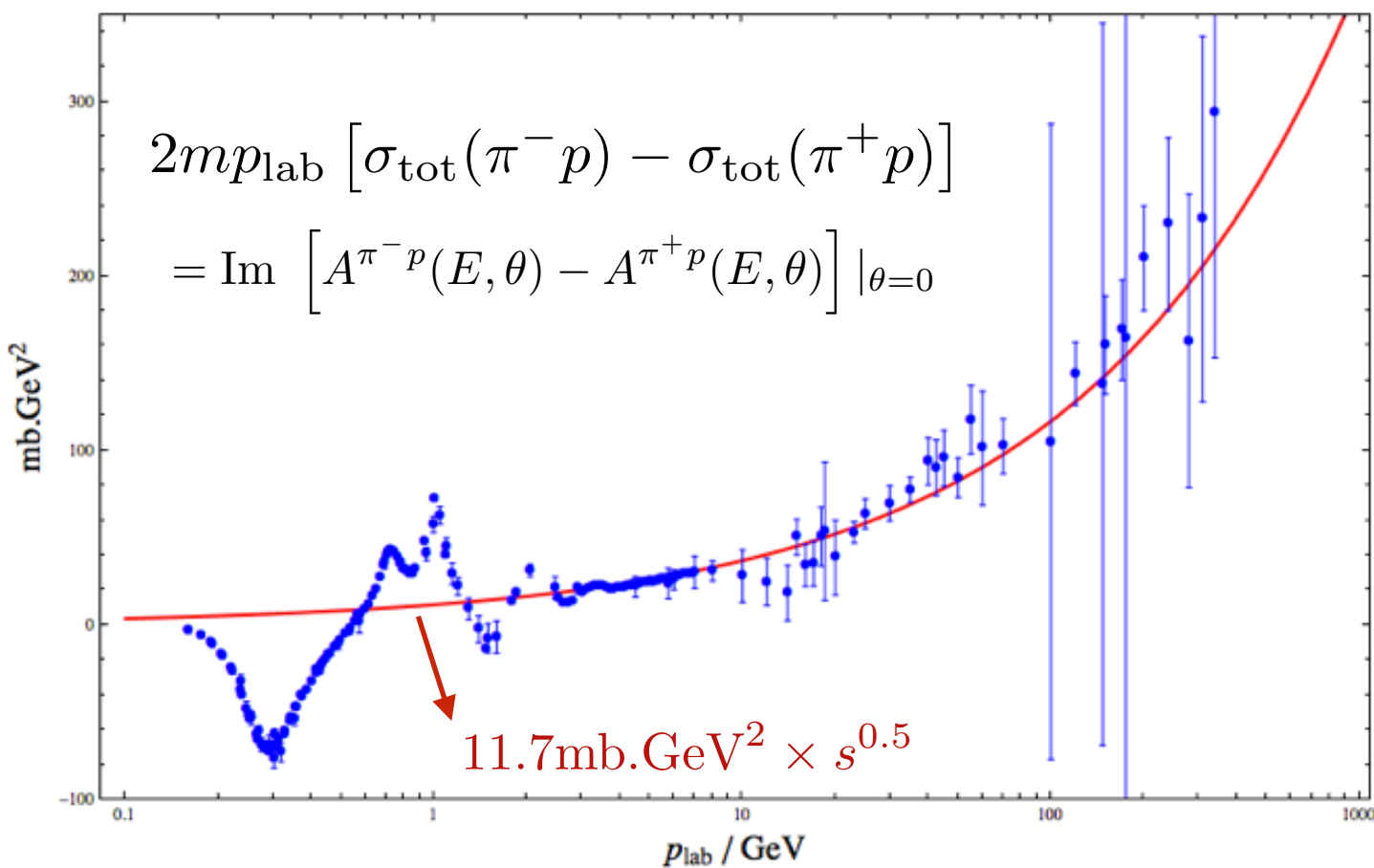
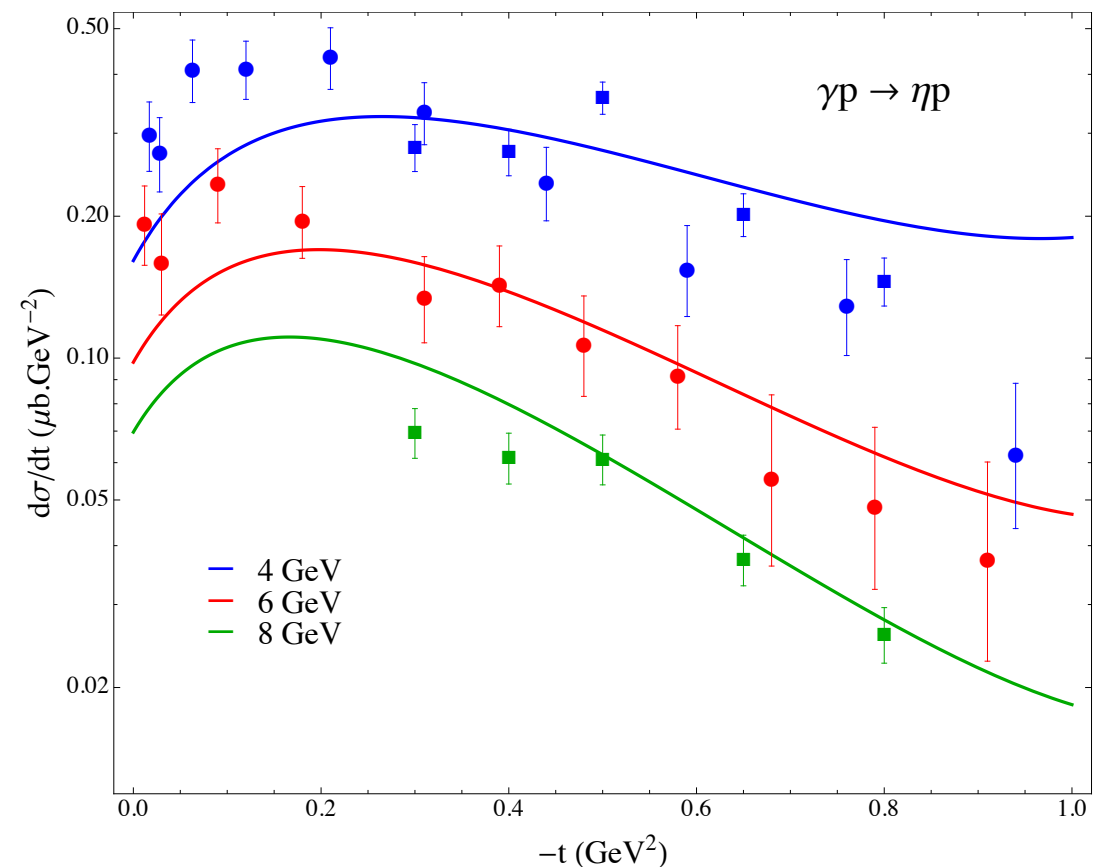
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input

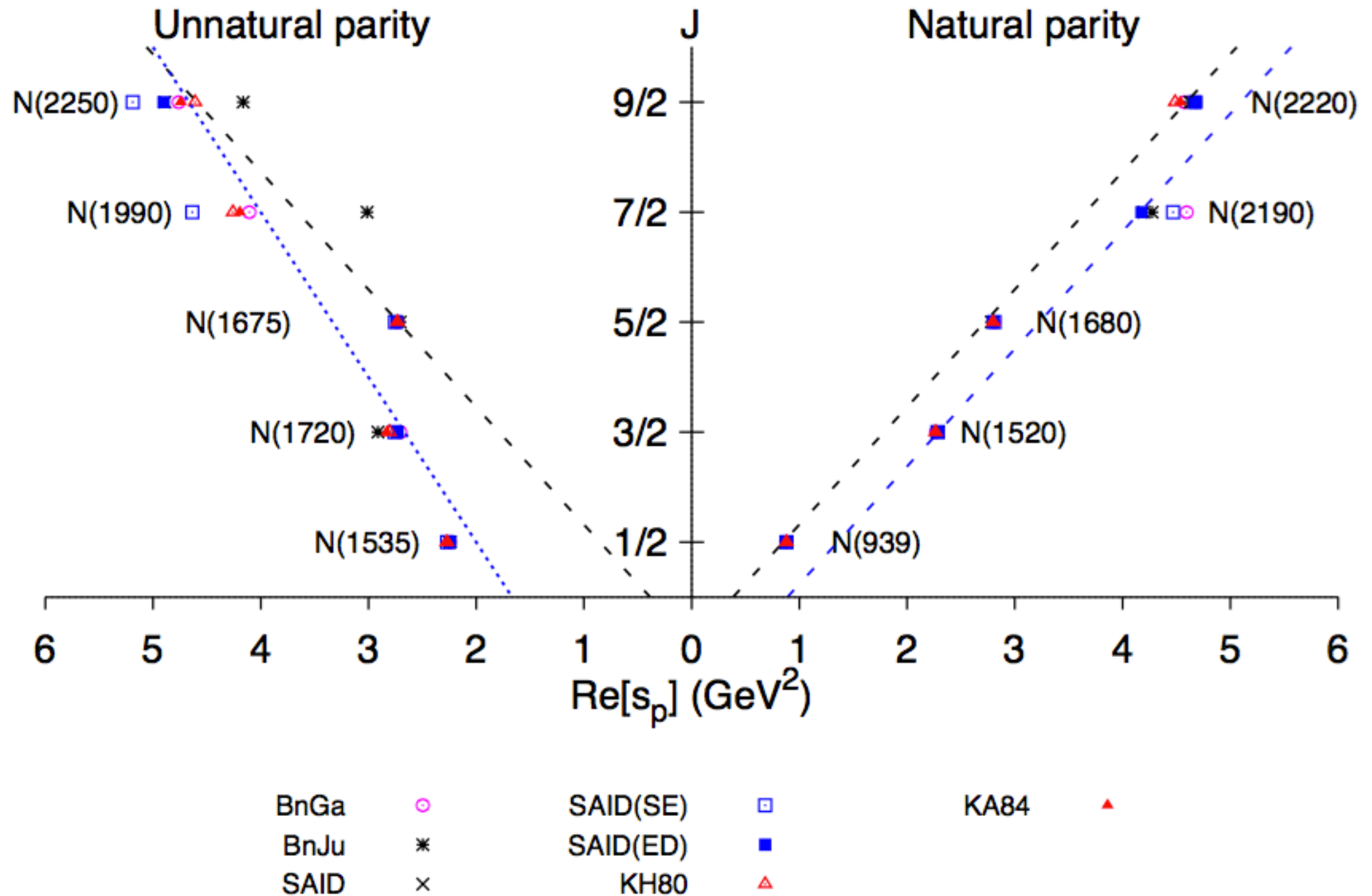
$$\int_0^\Lambda \text{Im } A_i(\nu, t) \nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha_i(t)+k}}{\alpha_i(t) + k + 1}$$

Use high energy data **to constraint**  
low energy models

FESR as a penalty in the fit



# Regge Trajectories



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### Photoproduction:

1. High energy model for  $\eta'$  beam asymmetry photoproduction:  $\gamma p \rightarrow \eta^{(\prime)} p$  page
2. High energy model for  $\eta$  photoproduction:  $\gamma p \rightarrow \eta p$  page
3. High energy model for  $\pi^0$  photoproduction:  $\gamma p \rightarrow \pi^0 p$  page
4. High energy model for  $J/\psi$  photoproduction:  $\gamma p \rightarrow J/\psi p$  page

### Hadroproduction:

1. Pion-nucleon scattering:
  - Amplitudes  $\pi N \rightarrow \pi N$  amplitude page
  - Finite energy sum rules  $\pi N \rightarrow \pi N$  FESR page
2. Kaon-nucleon scattering:  $\bar{K} N \rightarrow \bar{K} N$  page

### Light meson Decay:

1.  $\eta$  meson into three pions:  $\eta \rightarrow 3\pi$  page
2. vector meson into three pions:  $\omega, \phi \rightarrow 3\pi$  page



Interactive webpage:

<http://www.indiana.edu/~jpac/index.html>



## Resources

- **Publication:** [Nys16]
- **C/C++ observables:** C-code main, Input file, C-code source, C-code header, Eta-MAID 2001 multipoles
- **C/C++ minimal script to calculate the amplitudes:** C-code zip
- **Data:** Dewire , Braunschweig
- **Contact person:** Jannes Nys
- **Last update:** November 2016

## Run the code

$E_\gamma$  in GeV

☒ t ☐ cos

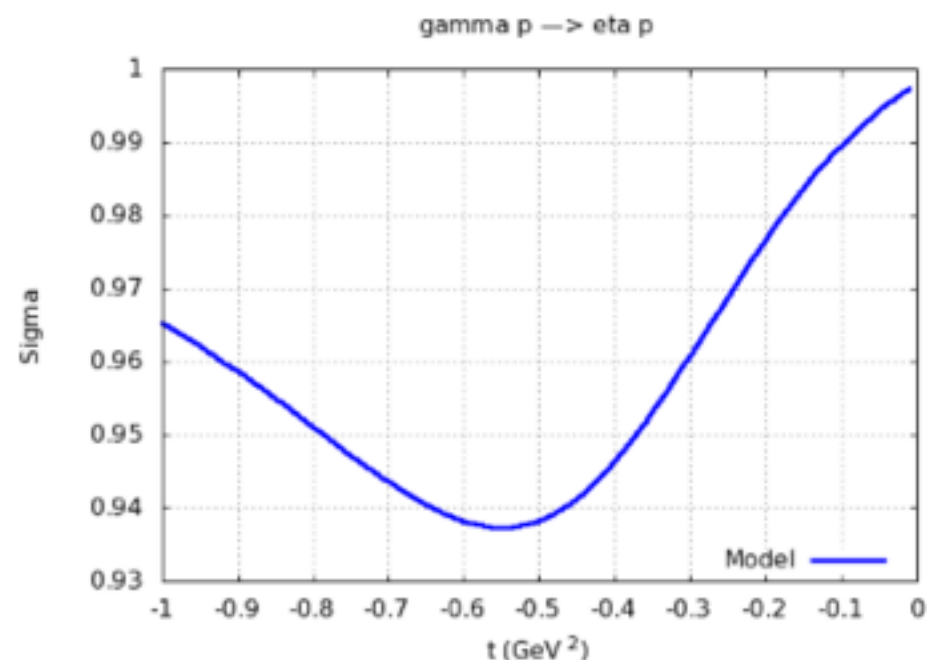
$t$  in  $\text{GeV}^2$  (min max step)

$\cos \theta$  (min max step)

$$\gamma p \rightarrow \eta p$$

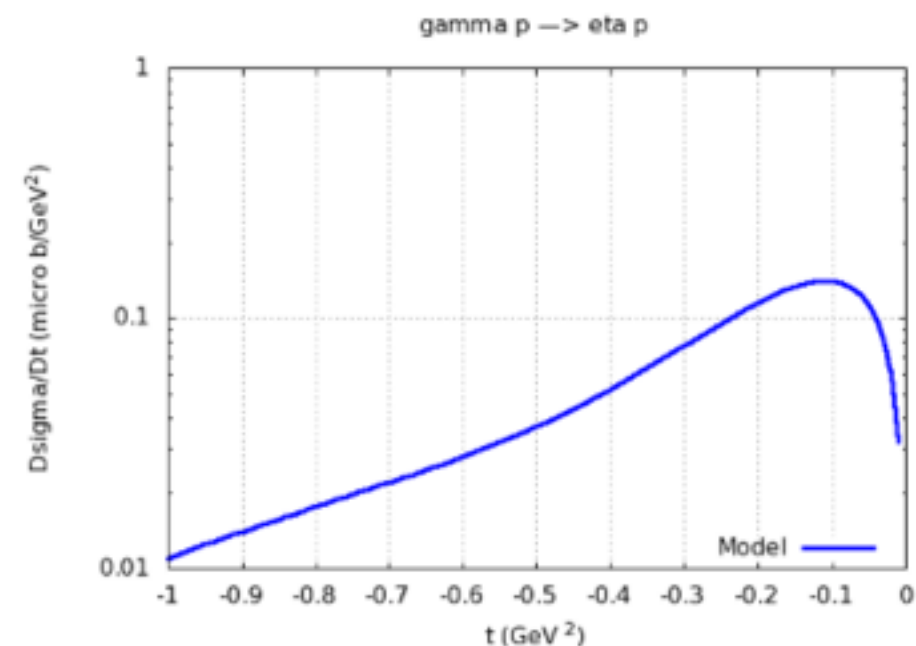
**Observable: photon beam asymmetry**

Download the the plot with  $O_x=t$  , the plot with  $O_x=\cos$  .



**Observable: differential cross section**

Download the the plot with  $O_x=t$  , the plot with  $O_x=\cos$  .





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